A Ricardian Trade Structure in CGE: Modeling Eaton-Kortum Base Trade with GTAP

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Background and outline

- Most CGE-models use the Armington assumption (product differentiation by country of origin) to model international trade
- The Armington structure was one of the criticisms of CGE models in trade academia
  - Caliendo and Parro (2015, RESTAT) for example state: "These models have been criticized for their complexity, lack of transparency and analytical foundations, and the arbitrary choice of the value of key parameters."
- Eaton and Kortum (2002, Ectrica), EK, introduce a model of comparative advantage model with multiple products and multiple countries employing a probabilistic formulation for productivity
- Arkolakis, Costinot, Rodriguez-Clare (2012, AER) show that subject to a set of macro-restrictions Eaton-Kortum and Armington generate the same reduced form equations for international trade (expressions for demand and price index)
  - Possible differences in more elaborate model like GTAP

Background and outline

- This paper incorporates the EK trade structure in the GTAP model and analyses the differences. In this presentation we go into the following topics:
  - Overview of the differences between new quantitative trade (NQT) and CGE models
  - Exposition of the EK model
  - Implementation of the EK trade structure in the GTAP model:
    - Technical issues and coding
    - Comparison of EK and Armington structure with a set of stylized counterfactual experiments
  - The comparison generates three main insights
    - The impact of trade policy experiments on real income is very similar. Differences are driven by a role for the transportation sectors
    - Impact of trade cost changes on the volume of trade is smaller in EK than Armington if the models are calibrated to the same trade elasticity, the elasticity of the value of trade wrt trade costs
    - The terms of trade gains from raising tariffs are not uniformly larger in the EK model

CGE versus NQT models

- About 20 years after the large scale implementation of CGE models to analyse counterfactual trade policy experiments, academic trade economists developed so-called new quantitative trade models (NQT) consisting of two types:
  - Structural gravity (SG) models: inspired by Anderson and Van Wincoop (2003, AER) employing an Armington structure, counterfactuals are conducted with the structural gravity equation on predicted trade values
  - Models applying exact hat algebra (EHA): inspired by work of Eaton and Kortum (2002, Econometrica) employing a Ricardian structure of trade. Dekle, Eaton and Kortum (2008, IMF Staff Papers) proposed EHA to calculate counterfactuals

CGE versus NQT: five systematic differences

1. Scope of the model
   - CGE models: more extensive with economic and institutional details
   - NQT models: more compact and parsimonious models
2. Baseline calibration
   - SG models calibrate baseline to predicted values
   - CGE models calibrate baseline to actual, observed values
   - Also NQT models applying EHA calibrate to actual, observed values
3. Structural estimation
   - NQT models are more rigorous on structural estimation: estimating all parameters of the model used to run counterfactual experiments based on the same dataset as used for the counterfactual experiments
   - CGE models: more flexible also taking parameters from the literature
4. Solution method
   - CGE-in-levels (GAMS) and SG: solve baseline equilibrium in levels, solve counterfactual equilibrium in levels, and compare
   - CGE-in-relative-changes (GEMPACK) and EHA: calculate percentage changes in multiple steps or ratios of baseline and counterfactual
5. Build-on approach (CGE) versus starting from scratch (NQT)

CGE versus NQT: scope of the model

- CGE models and models applying EHA converged on use of:
  - Multiple sectors, multiple factors of production, intermediate linkages
- SG models:
  - Single factor, mostly single sector, and oftentimes no intermediate linkages
- Features present in CGE models and absent in NQT models with EHA:
  - Different import demand shares by end user (private households, government, and firms)
  - Savings, investment, and capital
  - Non-homothetic preferences in private household demand
  - Substitution elasticities deviating from 1 in the choice between intermediates and between factors of production
  - Export subsidies, other tax instruments, and a separate transport sector.

CGE versus NQT: structural estimation

- CGE models:
  - Because of dimensionality not all parameters are estimated based on dataset used to run simulations
- NQT models:
  - Only trade elasticities needed
  - Other nests are typically set at Cobb-Douglas, i.e. substitution elasticities equal to 1
- Convergence: CGE-studies like Egger et al. (2015, EP, study on TTIP) also estimate trade elasticities nowadays
- Divergence:
  - CGE-community convinced about using parameters from other studies
  - NQT-community in general refuses the use of parameters from other studies and prefers to use Cobb-Douglas nests, unless all parameters can be estimated in more complicated settings
  - Compare approach in Costinot Donaldson and Smith (2016, JPE) and Gouel and Laborde (2022, JEEM)

Introduction Eaton-Kortum model

- Original Ricardian model features two countries, two goods and one factor of production
- Extension to multiple countries is easy, see any undergrads book
- Extension to multiple goods: Dornbusch, Fischer and Samuelson (1977)
- Extension to multiple goods and multiple countries: Eaton and Kortum (2002), *Econometrica*
- EK develop model with continuum of goods and probabilistic formulation of productivity: each country draws productivity of each good on the continuum and cheapest country (most productive corrected for trade costs) supplies certain good
- EK model can be characterized by three parameters:
  - Each country's state of technology, governing absolute advantage
  - The heterogeneity of productivity, governing comparative advantage
  - Geographic barriers

Model Setup

- N countries and continuum of goods ranging from 0 to 1
- Productivity varies across countries and across goods
- Country i's efficiency in producing good $j \in [0, 1]$ is denoted by $z_i(j)$
- Input cost in country i is denoted as c_i (equal across sectors)
- With constant returns to scale, cost of producing good j in country i is thus $c_i / z_i(j)$
- Geograhic barriers introduced with iceberg trade costs: delivering a unit from country i to country n requires producing d_{ni} units in i
- By assumption $d_{ii} = 1$ and trade costs obey triangular inequality: $d_{ni} \leq d_{nk} d_{ki}$

Model Setup

- Delivering a good j from country i to country n costs:

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)}\right) d_{ni} \tag{1}$$

- Under perfect competition $p_{ni}(j)$ is also the price for consumers in country n.
- Consumers in country n buy the cheapest good (no search costs). So the price of good j in country n, $p_n(j)$, is the minimum across all sources:

$$p_n(j) = \min\{p_{ni}(j); i = 1, ..., N\} \tag{2}$$

- Utility across continuum of goods is CES with substitution elasticity σ:

$$U = \left[\int_0^1 Q(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

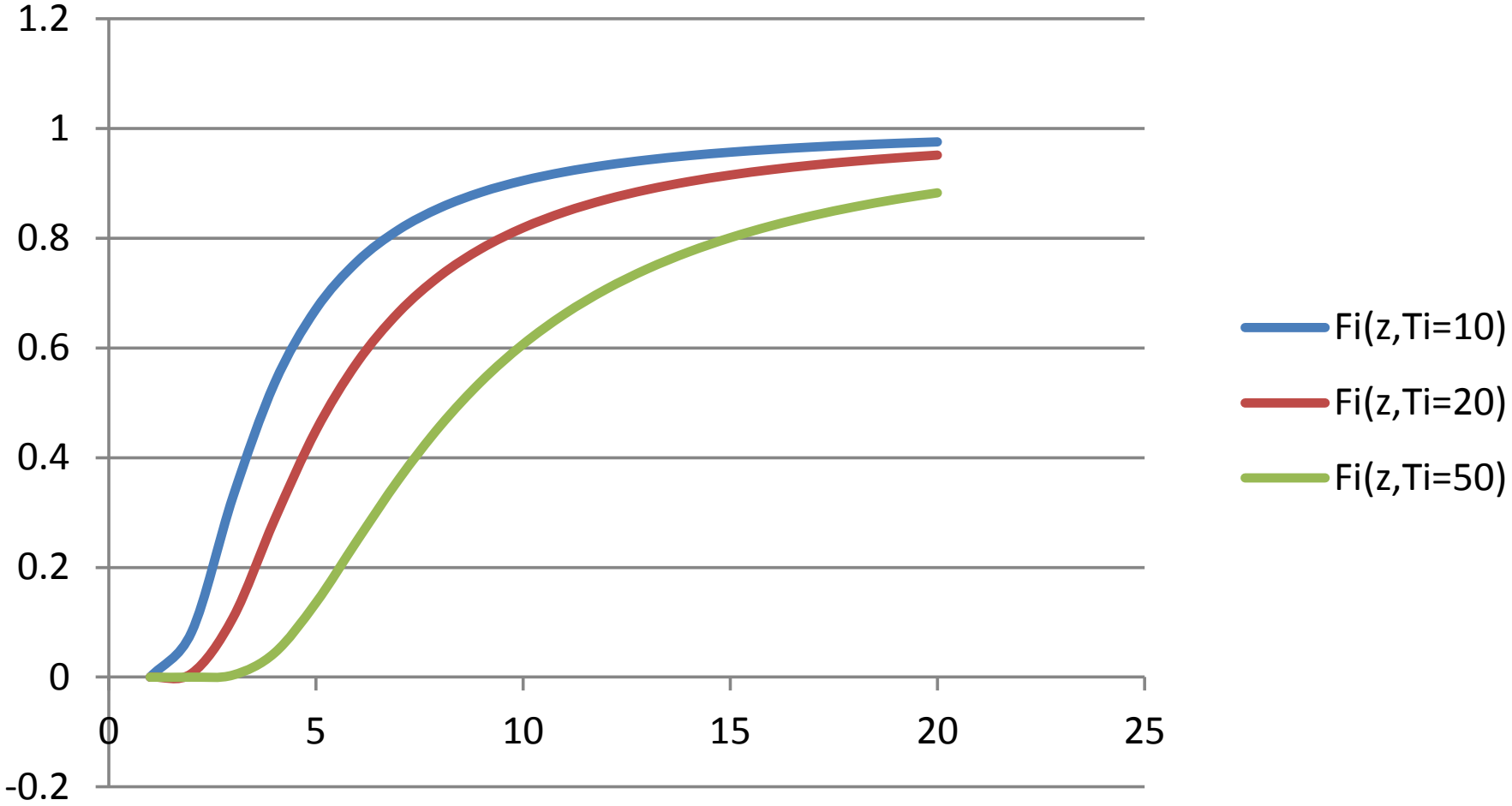
Technology

- Technology is stochastic: country i's efficiency in producing good j is the realization of a random variable Z_i (independent across j) with distribution function $F_i(z) = \Pr(Z_i \leq z)$
- Using a continuum of goods, by the law of large numbers $F_i(z)$ is also the fraction of goods for which country i's efficiency is smaller than z
- Working with a Frechet distribution for efficiency generates a simple expression for the cumulative distribution function of z:

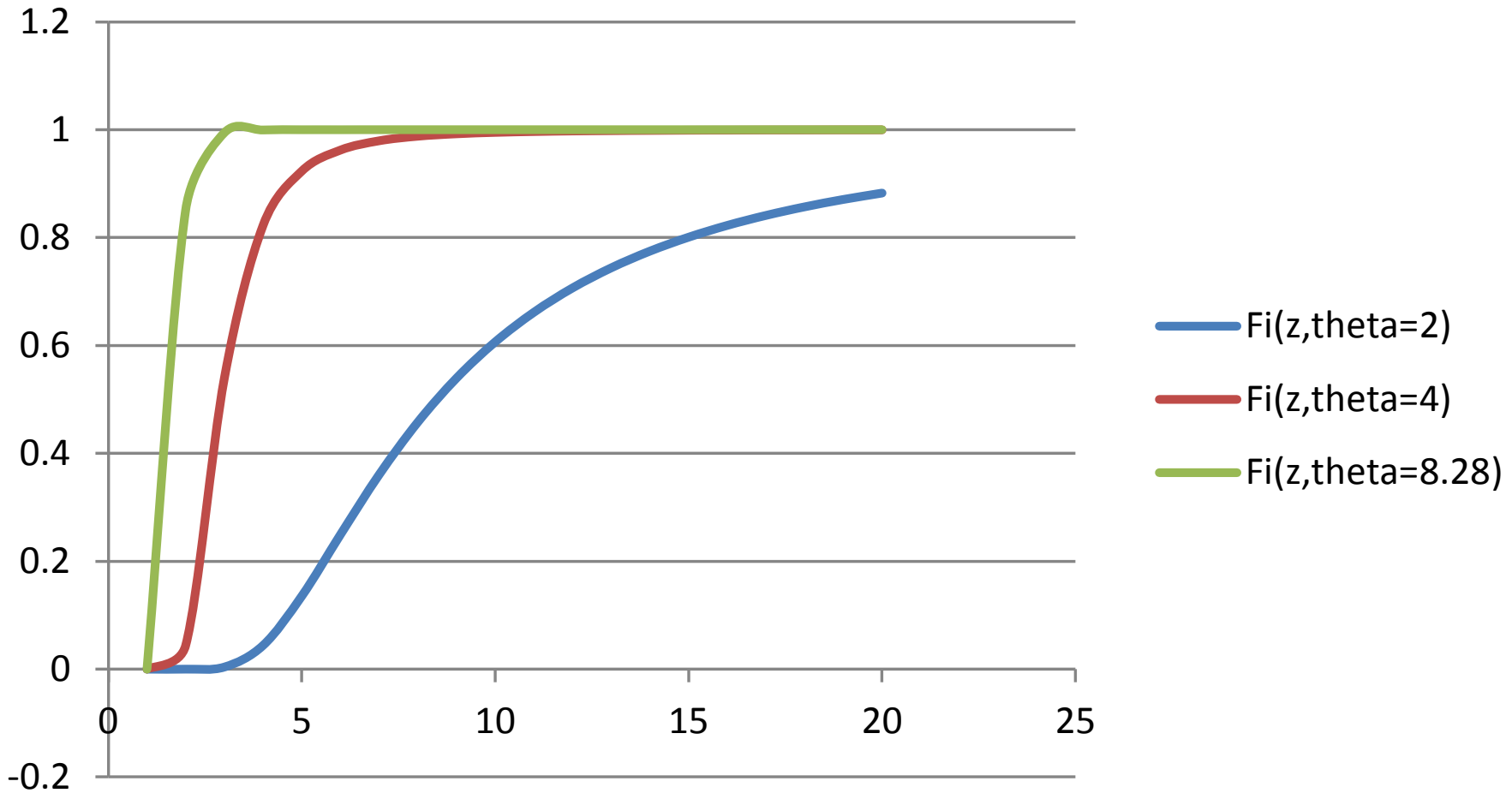
$$F_i(z) = e^{-T_i z^{-\theta}} = \frac{1}{e^{\frac{T_i}{z_i^{\theta}}}} \tag{4}$$

- T_i governs the location of the distribution: a bigger T_i implies that a high efficiency draw for any good j is more likely.
- θ measures the variation of the efficiency distribution. A bigger θ implies less variability. With a very large θ, probabilities for all z's become equal.

Distribution function for varying Ti's



Distribution function for varying theta's



Prices

- T_i is a measure of absolute advantage of country i
- θ is a measure of comparative advantage. With a lower value of θ there is more heterogeneity in efficiency across goods
- Substituting the expression for p_{ni} into the distribution of efficiency, implies that country i presents country n with a distribution of prices $G_{ni}(p) = \Pr(p_{ni} \leq p) = 1 - F_i\left(\frac{c_i d_{ni}}{p}\right)$:

$$G_{ni}(p) = 1 - e^{-\left[T_i (c_i d_{ni})^{-\theta}\right] p^{\theta}} \tag{5}$$

- The lowest price distribution in country n among all importers i can be calculated from the fact that the probability that the price is smaller than p in country n is equal to one minus the probability that all countries supply the good at a price larger than p:

$$G_n(p) = 1 - \prod_{i=1}^{N} \left[1 - G_{ni}(p)\right] \tag{6}$$

Prices

- Substituting equation (5) into equation (6) leads to the following price distribution in country n:

$$G_n(p) = 1 - \prod_{i=1}^{N} e^{-\left[T_i (c_i d_{ni})^{-\theta}\right] p^{\theta}} = 1 - e^{-\Phi_n p^{\theta}} \tag{7}$$

- Φ_n is defined as:

$$\Phi_n = \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta} \tag{8}$$

- Φ_n summarizes how prices in each country n are governed by:
  - States of technology T_i around the world
  - Inputs costs c_i around the world
  - Geographic barriers d_{ni}
- In a zero gravity world without geographic barriers ($d_{ni} = 1$ for all n and i), Φ is the same everywhere and the law of one price holds
- Under autarky ($d_{ni} \to \infty$ for $n \neq i$), Φ_n reduces to $T_n c_n^{-\theta}$, so is determined by technology and input costs at home

Prices

- There are three useful properties of the price distribution in equation (7):
  1. The probability that country i provides a good at the lowest price in country n (and with a continuum of goods also the fraction of goods provided by i in n) is equal to i's contribution to n's price parameter: Proof Property 1

$$\pi_{ni} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\Phi_n} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\sum_{j=1}^{N} T_j \left(c_j d_{nj}\right)^{-\theta}} \tag{9}$$

  2. The price of a good that country n actually buys from any country i also has the distribution $G_n(p)$. Proof Property 2
  3. The exact price index for the CES objective function, assuming $\sigma < \theta + 1$: Proof Property 3

$$p_n = \gamma \Phi_n^{-\frac{1}{\theta}} \tag{10}$$

Interpretation Gravity Equation

$$X_{ni} = d_{ni}^{-\theta} \frac{p_n^{\theta}}{\sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m}} Q_i X_n \tag{11}$$

- This gravity equation is very similar to the Anderson&Van Wincoop (AvW) gravity equation with Armington preferences:
  - Trade from country i to country n rises proportionally in total sales in the exporter, Q_i and total expenditure in the importer, X_n
  - Trade falls in trade costs d_{ni} with an elasticity θ. In AvW the elasticity of trade wrt trade costs is $\sigma - 1$: a larger substitution elasticity leads to more substitution away from countries with high trade costs (intensive margin adjustment)
  - In EK larger trade costs d_{ni} lead to substitution away from goods produced in country i (extensive margin adjustment). A larger θ corresponds with less variability in productivity distribution. Higher trade costs exert a stronger effect on trade flows, as productivities in countries are closer to each other.

Calibration of EK structure in the GTAP model: preliminary observations

- Although EHA-type models typically employ the Eaton-Kortum model, the original Eaton Kortum paper does not calibrate the baseline to actual values as in EHA-type models, but to estimated values
- Consistent with this, iceberg trade costs and technology parameters are estimated separately in the EK paper, whereas in EHA-type models and CGE models the baseline is calibrated directly from observed trade values (for example the GTAP Data Base version 11 with base year 2017) and technology parameters and iceberg trade costs are not needed.

Implementation of EK structure in the GTAP model

- Most changes to the code are related to the fact that bilateral sectoral prices do not vary by origin in the Eaton-Kortum model.
  - Hence, the sectoral price in destination country d is the same for goods from any country of origin s.
  - The reason is that a country of origin displaying higher trade costs will export less varieties to a specific country of destination.
  - Hence, if one country of origin has higher costs than another country, the higher cost country exports less varieties with higher costs
  - Under a Frechet distribution the intensive and extensive margin impacts of higher costs exactly cancel out
- We have to make five sets of changes to the code, discussed in turn:
  1. Changes in the update statements of values related to trade
  2. Changes in the trade elasticities, i.e. ESUBD and ESUBM
  3. Changes in the expressions for import demand and price indices
  4. Changes in the goods market equilibrium condition
  5. Changes in the expressions for tax revenues

Changes in update statements

- In the conventional GTAP model all values are updated multiplying price by quantity
- In the EK model we make a distinction between values used to calculate shares in defining price indices and values of trade and domestic sales used to calculate tax revenues and income
  - Values used to calculate shares in price indices are updated only with quantity shares, since quantity shares have to be used when hat differentiating expressions for price indices
  - Values employed in calculating tax revenues are updated with price times quantity, using bilateral prices that do not vary by country of origin
  - For the case of private import demand this leads to the set of code on the following slide

Changes in update statements

Listing 15. Value of household expenditure and update statements

```
!< gtapv7-ek: define domestic and import price variables >!
Variable (orig_level=1.0)(all,c,COMM)(all,r,REG)
   ppmek(c,r) # price of imported c purchased by household in r, net of tax #;

Coefficient (ge 0)(all,c,COMM)(all,r,REG)
   VMPP(c,r) # private hhld expenditure on imp. c in r at producer prices #;
Read
   VMPP from file GTAPDATA header "VMPP";
!< gtapv7-ek: Modify update statement by changing price from ppm to ppa >!
Update (all,c,COMM)(all,r,REG)
   VMPP(c,r) = ppa(c,r) * qpm(c,r);
Coefficient (ge 0)(all,c,COMM)(all,r,REG)
   VMPB(c,r) # private household expenditure on imp. c in r at basic prices #;
Read
   VMPB from file GTAPDATA header "VMPB";
!< gtapv7-ek: Modify update statement from pms to ppmek >!
Update (all,c,COMM)(all,r,REG)
   VMPB(c,r) = ppmek(c,r) * qpm(c,r);

!< Expenditures at producer prices have a uniform price in the EK-model >!
Coefficient (ge 0)(all,c,COMM)(all,r,REG)
   VMPPEK(c,r) # private hhld expenditure on domestic c in r at purchaer's
   prices, EK #;
!< Update based on quantity shares >!
Formula (initial)(all,c,COMM)(all,r,REG)
   VMPPEK(c,r) = VMPP(c,r);
Update (all,c,COMM)(all,r,REG)
   VMPPEK(c,r) = qpm(c,r);

Equation E_ppmek
# EK household consumption prices for imported com. c, net of tax #
(all,c,COMM)(all,r,REG)
   ppmek(c,r) = ppa(c,r) - tpm(c,r);
```

Changes in trade elasticities

- We calibrate *THETA* to the estimated elasticities in the GTAP Data Base assuming that the same trade elasticity holds in the Armington and Eaton Kortum model.
- In the gravity equation based on the Armington model the substitution elasticity, σ_c, is equal to one plus the tariff elasticity η_c^{tar}, $\sigma_c = 1 + \eta_c^{tar}$ (gravity equation estimated using tariff-inclusive values).
- In the Eaton-Kortum model the tariff elasticity is equal to the dispersion parameter, so we have $\theta_c = \eta_c^{tar}$:

$$x_{csd} = \exp\left\{d_{cs} + d_{cd} - \theta_c \ln t_{csd}^{imp} itm_{csd} t_{csd}^{\exp} + \xi_c \ln \mathbf{grav}_{csd}\right\} \varepsilon_{csd} \tag{12}$$

- We thus calibrate *THETA* based on the estimated tariff elasticities from the GTAP Data Base using $\theta_c = \eta_c^{tar} = \sigma_c - 1$ implying $THETA(c,d) = ESUBM(c,d) - 1$.
- Effectively, this means that the trade elasticity parameter changes from *ESUBM* into $THETA = ESUBM - 1$.

Changes in import demand and the price index

- The GTAP model has a two-level nested structure of import demand with different elasticities
- In the EK model the trade elasticities between domestic and imported and between imported goods from different sources are identical
- We stick with the nested structure of import demand, because import demand shares vary across the groups of end users in the data.
- The expressions for import demand remain the same as in GTAP model, except for the use of *THETA* instead of ESUBM and ESUBD
- The expression for the importer price index also remains the same, observing that pmds and pms do not denote prices but costs.
- Different from the GTAP model is that the share employed in the aggregate cost equation, MSHRS, is based on quantity shares instead of value shares.
- In the code MSHRS is updated directly instead of the underlying values to consider possible changes in iceberg trade costs
- The definition of the aggregate cost index for the four groups of agents, for example ppa, follows the same logic, thus employing quantity shares

Changes in import demand and the price index

Listing 16. Import cost equations

```
Coefficient (parameter)(all,c,COMM)(all,s,REG)(all,d,REG)
   VMSBEK(c,s,d) # initial value of imports of c from s to d at domestic (basic
   ) prices #;
Formula (initial) (all,c,COMM)(all,s,REG)(all,d,REG)
   VMSBEK(c,s,d) = VMSB(c,s,d);
Coefficient (all,c,COMM)(all,s,REG)(all,d,REG)
   MSHRS(c,s,d) # share of imports from s in imp. bill of r at basic prices #;
Formula (initial)(all,c,COMM)(all,s,REG)(all,d,REG)
   MSHRS(c,s,d) = VMSBEK(c,s,d) / sum{ss,REG, VMSBEK(c,ss,d)};
Update (all,c,COMM)(all,s,REG)(all,d,REG)
   MSHRS(c,s,d) = qxs(c,s,d) * ams(c,s,d) * qmsn(c,d);
Update (explicit)(all,c,COMM)(all,s,REG)(all,d,REG)
   VMSB(c,s,d) = MSHRS(c,s,d) * VMB(c,d);
Equation E_pms
# price for aggregate imports #
(all,c,COMM)(all,d,REG)
   pms(c,d) = sum{s,REG, MSHRS(c,s,d) * [pmds(c,s,d) - ams(c,s,d)]};
Equation E_qmsn
# negative of aggregate imports of c in region r, basic price weights #
(all,c,COMM)(all,r,REG)
   qmsn(c,r) = -1 * [qms(c,r)];
```

Changes in tax revenues and goods market equilibrium

- The expressions for the ratio of import and export tax revenues to total income changes because the import price is independent of the origin country which complicates the calculation of the tax base
- To calculate the cif price the landed price per agent ag is divided by both ag-specific import taxes and normal import taxes.
- The tax base for imports calculated by multiplying this cif price by the quantity imported.
- For the export tax the cif price has to be divided by the transportation margin and export taxes
- Whereas goods market equilibrium in the GTAP model is defined using quantities, in the Eaton-Kortum specification it is reformulated in terms of values—i.e., prices times quantities.
- Prices in destination markets are only destination specific and thus source-independent, implying that the price of goods sold to different destinations is different.

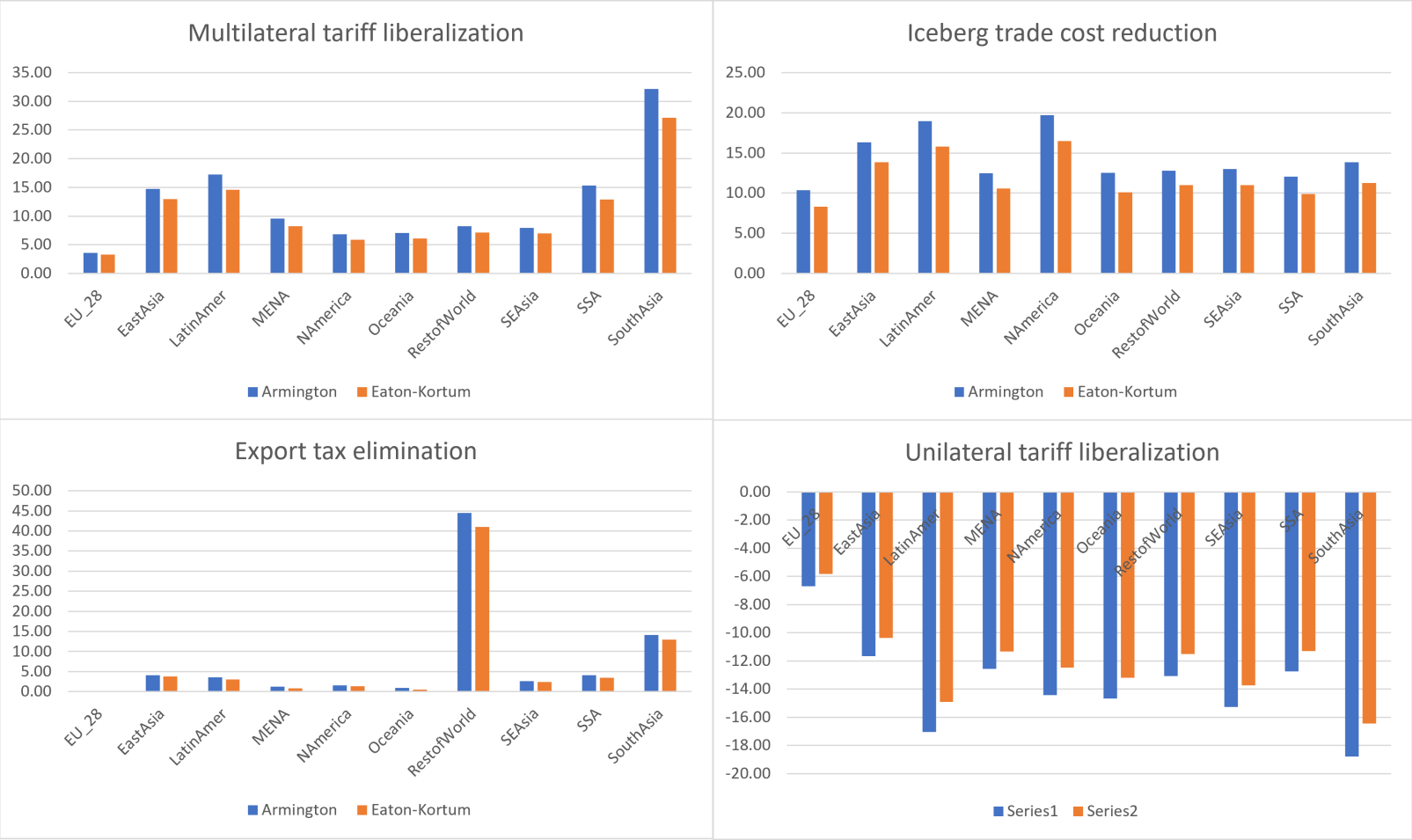
Simulation design

- To compare simulations outcomes of the EK model and the standard GTAP model the GTAP Data Base, Version 10, is aggregated to 10 regions, 10 sectors, and 5 factors of production.
- To make the two models comparable ESUBD=ESUBM in the GTAP model and the same trade elasticity is employed in the estimated gravity model, implying $THETA = ESUBM - 1$.
- We conduct four sets of experiment:
  1. Global tariff liberalization: Eliminate tariffs in all regions.
  2. Global iceberg trade cost cut: 5% decrease in iceberg trade costs in all regions.
  3. Global export tax liberalization: Eliminate export taxes and subsidies.
  4. Unilateral tariff increases: Ten experiments with each region increasing (the power of) tariffs by 10% vis-a-vis other regions.
- The fourth experiment is included to explore how terms of trade effects of tariffs differ between the two models
- Since the EK code is highly non-linear a large number of steps is required to make Walraslack marginal. We solve the model with Euler 100-300-500 steps, although the solutions are virtually identical with a smaller number of steps

Simulation results: real income effects

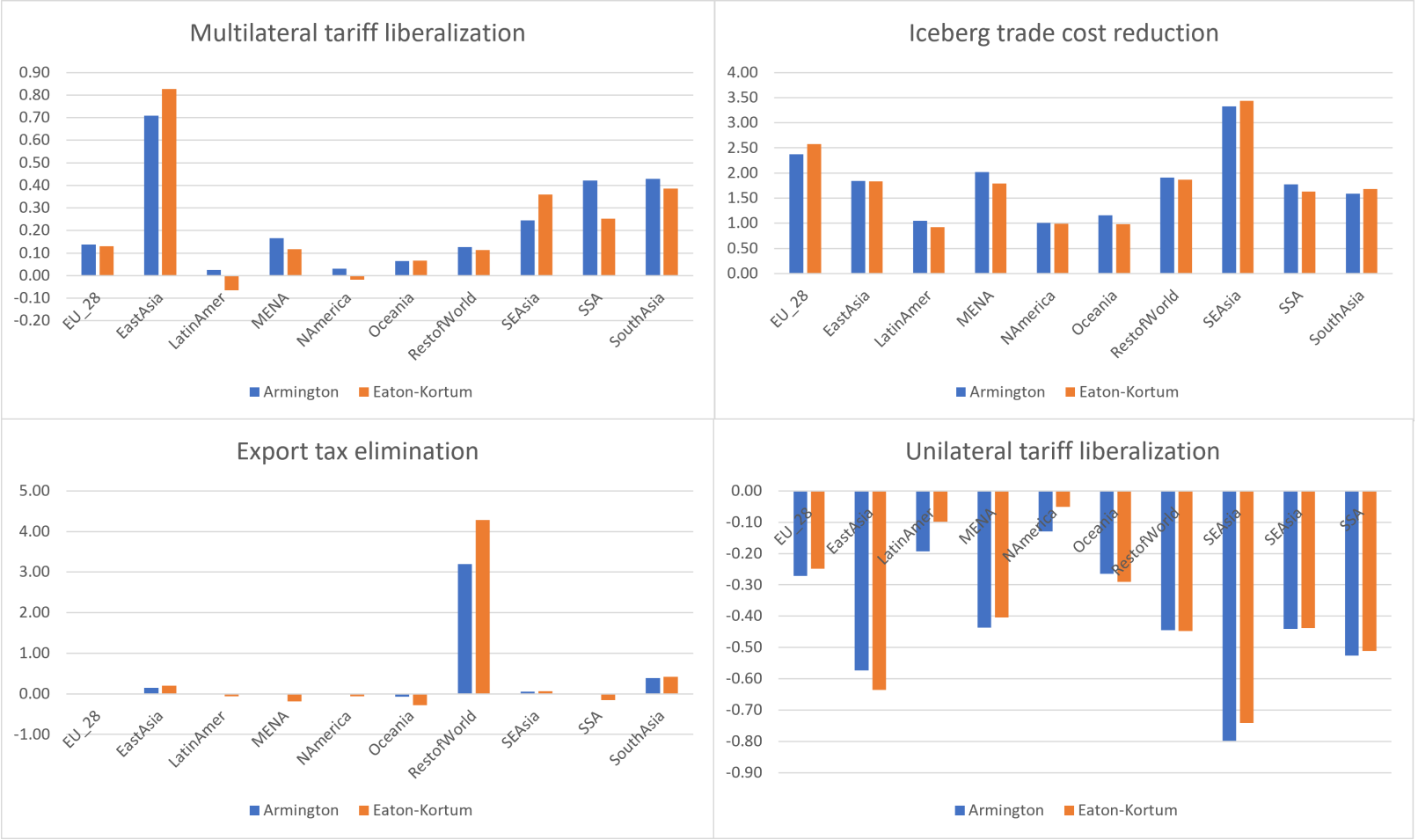
- The difference in real income effects is very small, as expected from the theory.

Simulation results: trade effects



- Changes in trade volumes are smaller in EK than standard GTAP (Armington), because the elasticity of trade volumes with respect to trade costs is smaller in the EK model, $THETA = ESUBM - 1$.

Simulation results: real GDP effects

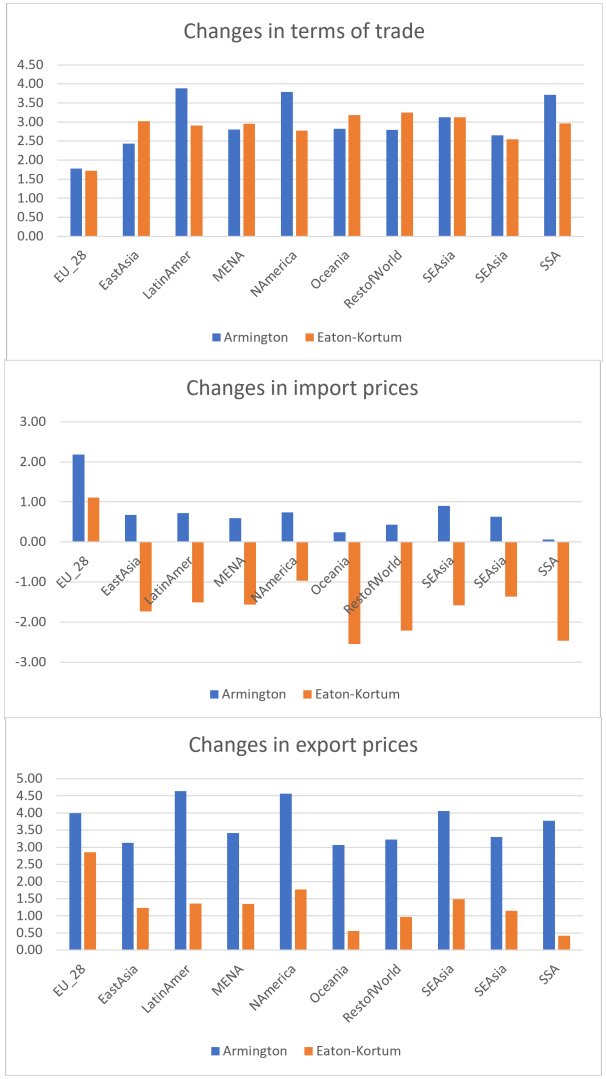


- The change in GDP differs more between the two models than the change in real income, because the changes in real exports and real imports differ between the two models.

Simulation results: terms of trade effects

- Hypothesis: the terms of trade gains of raising tariffs are larger in the Eaton-Kortum specification, because pre-tariff prices can be driven down more since the tariff-inclusive price changes less and exporters thus pay a larger part of tariff increases.
- Intuitively, higher tariffs imply that a share of firms will stop importing and this extensive margin adjustment will imply that the highest cost varieties will drop out thus reducing the pre-tariff import price.
- But in a general equilibrium setting, the imposition of tariffs also implies that the price level in the importing country increases leading to higher export prices thus raising terms of trade and the increase in export prices is also smaller in the Eaton-Kortum than in the Armington specification.
- We test the hypothesis with 10 experiments increasing the power of tariffs in each of the regions by 10 per cent vis-a-vis all other regions.
- The results shows that in some regions the terms of trade improvement is larger under Armington and in other regions under EK.
- In line with the hypothesis the reduction in (pre-tariff) import prices is larger under EK, while the increase in export prices is smaller under EK.

Simulation results: terms of trade effects



Concluding remarks

- The simulations with the GTAP-EK model generated three main insights.
  1. The real income effects are virtually identical in the Eaton-Kortum and Armington versions of the GTAP model.
  2. Changes in the volume of trade are smaller in response to trade cost changes in the Eaton-Kortum than in the Armington specification, whereas changes in the value of trade are (virtually) identical.
  3. The terms of trade gains of imposing tariffs differ between EK and Armington with pre-tariff import prices driven down more in the Eaton-Kortum specification with export prices also rising less.

The work in this paper can be extended in at least three directions.

1. Projections of the impact of counterfactual experiments on volumes of trade and trade prices can be compared with empirical estimates of this response in the data to test which model performs best
2. The validity of the EK model can be tested based on its prediction that tariff-inclusive bilateral import prices are identical across all source countries
3. The EK model can be employed in recursive-dynamic applications to evaluate whether long-run projections are different in the Armington and EK frameworks.

Trade Flows and Gravity

- The probability that country i provides a good to country n is equal to:

$$
\begin{aligned}
\pi_{ni} &= P\left(p_{ni}(j) \leq \min\{p_{ns}(j)\,;\, s \neq i\}\right) \\
&= \int_0^\infty \prod_{s \neq i} \left(1 - G_{ns}(p)\right) dG_{ni}(p) \\
&= \int_0^\infty \prod_{s \neq i} \left(e^{-T_s (d_{ns} c_s)^{-\theta} p^\theta}\right) T_i (d_{ni} c_i)^{-\theta} e^{-T_i (d_{ni} c_i)^{-\theta} p^\theta} dp \\
&= T_i (d_{ni} c_i)^{-\theta} \int_0^\infty \prod_{i=1}^{N} e^{-T_i (d_{ni} c_i)^{-\theta} p^\theta} dp \\
&\overset{t=p^\theta}{=} T_i (d_{ni} c_i)^{-\theta} \int_0^\infty e^{-\sum_{i=1}^{N} T_i (d_{ni} c_i)^{-\theta} t} dt \\
&= -\frac{T_i (d_{ni} c_i)^{-\theta}}{\Phi_n} e^{-\Phi_n t} \Big|_0^\infty = \frac{T_i (d_{ni} c_i)^{-\theta}}{\Phi_n}
\end{aligned}
$$

Trade Flows and Gravity

- To show property 2 we show that the distribution of prices of goods sourced from i in country n given that goods are actually sourced from country i is:

$$
\begin{aligned}
G_{ni}(p) &= \frac{1}{\pi_{ni}} \int_0^p \prod_{s \neq i} \left(1 - G_{ns}(q)\right) dG_{ni}(q) \\
&= \frac{1}{\frac{T_i (d_{ni} c_i)^{-\theta}}{\Phi_n}} \int_0^p \prod_{s \neq i} \left(e^{-T_s (d_{ns} c_s)^{-\theta} q^\theta}\right) T_i (d_{ni} c_i)^{-\theta} e^{-T_i (d_{ni} c_i)^{-\theta} q^\theta} dq \\
&= \frac{1}{\frac{T_i (d_{ni} c_i)^{-\theta}}{\Phi_n}} \int_0^p \prod_{s \neq i} \left(e^{-T_s (d_{ns} c_s)^{-\theta} q^\theta}\right) T_i (d_{ni} c_i)^{-\theta} e^{-T_i (d_{ni} c_i)^{-\theta} q^\theta} dq \\
&\overset{t=q^\theta}{=} \Phi_n \int_0^{p^\theta} e^{-t \sum_{s=1}^{N} T_i (d_{ni} c_i)^{-\theta}} dt \\
&= \frac{\Phi_n}{\Phi_n} e^{-\Phi_n t} \Big|_0^{p^\theta} = 1 - e^{-\Phi_n p^\theta}
\end{aligned}
$$

Trade Flows and Gravity

- We derive the price index as follows:

$$
\begin{aligned}
(p_n)^{1-\sigma} &= \int_0^\infty p^{1-\sigma} dG_n(p) = \int_0^\infty p^{1-\sigma} d\left(1 - e^{-\Phi_n p^\theta}\right) \\
&\overset{t=\Phi_n p^\theta}{=} \int_0^\infty \left(\frac{t}{\Phi_n}\right)^{\frac{1-\sigma}{\theta}} d\left(1 - e^{-t}\right) \\
&= (\Phi_n)^{\frac{\sigma-1}{\theta}} \int_0^\infty t^{\frac{1-\sigma}{\theta}} e^{-t} dt = (\Phi_n)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{\theta - \sigma + 1}{\theta}\right) \qquad (13)
\end{aligned}
$$

With $\Gamma(r)$ the gamma function, $\Gamma(r) = \int_0^\infty t^{r-1} e^{-t} dt$. Hence the price index is defined as:

$$
p_n = (\Phi_n)^{-\frac{1}{\theta}} \left(\Gamma\left(\frac{\theta - \sigma + 1}{\theta}\right)\right)^{\frac{1}{1-\sigma}} \qquad (14)
$$

Trade Flows and Gravity

- The fraction of goods bought from country i, π_{ni}, is also the fraction of its expenditures on goods from i:

$$\frac{X_{ni}}{X_n} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\sum_{j=1}^{N} T_j \left(c_j d_{nj}\right)^{-\theta}} \tag{15}$$

- Explanation: Since the prices paid on goods from any source country are equal, average expenditures on goods from each sourcing country are identical
- Equation (15) can be rewritten using the expression for total sales of country i, Q_i:

$$Q_i = \sum_{m=1}^{N} X_{mi} = T_i c_i^{-\theta} \sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m} \tag{16}$$

- Solving for $T_i c_i^{-\theta}$, substituting the result into equation (15), and using equation (10) gives the following gravity equation:

$$X_{ni} = d_{ni}^{-\theta} \frac{p_n^{\theta}}{\sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m}} Q_i X_n \tag{17}$$