Chapter 8

Public consumption

8.1. Introduction

In Chapter 6 we ignored public consumption by assuming that the public household buys a single good, the market for which could be disregarded in view of Walras' Law. In Chapter 7 public consumption was neglected completely by assuming that all tax revenues are channelled to private households. In this chapter these shortcomings will be removed. Here, we focus upon the provision of public goods and in particular upon the composition of the basket of public goods. We argue that the determination of the optimal basket should be based upon individual preferences for public goods, while, once determined collectively, the benefits of their provision depend, in turn, on the same individual preferences. The main part of this chapter is therefore devoted to the formalization of two concepts: the public utility function and the benefits of public goods.

In section 2 we repeat the main definitions of public goods, the public household, and the fisc and elaborate on the relations between them. In section 3 we focus on the provision of public goods and the determination of the public utility function. Section 4 describes how, under the assumptions made, the concept of compensating variation can be used to measure the benefit of public goods. Using this measure we show in section 5 how the so-called "budget incidence" and the "differential incidence" could be determined. We end with a summary.

8.2. The private households, the public household, and the fisc

Let us first repeat some of the basic definitions as presented in Chapter 2. There are two types of goods in our economy, namely private and
public goods. Private goods are characterized by rival consumption while public goods are characterized by non-rival consumption. This means that the benefits of public goods apply to all individuals while the consumption by one individual does not reduce the benefits derived by others.\(^1\) Private goods are provided through the market mechanism and every private good has a price. Public goods are provided by the public household by transforming private goods. We assumed that the benefits of public goods apply to private households, but not to firms. The provision of public goods is subject to a budget, which is determined by the fisc. The fisc collects the tax revenue (i.e., the proceeds of transaction taxes, net of transaction subsidies) and returns this as lump-sum payments (i.e., transfers, net of lump-sum taxes) to households. The transfer to the public household constitutes the public budget.

Since the introduction of the fisc implies that the public household does not collect and distribute tax revenues, we wonder what it stands for. Well, in our framework the public household is in the first place a household like a private household: it demands private goods in order to satisfy some needs. This is called public consumption. The difference between the public and the private household lies in the difference of needs: private households try to satisfy their own, private needs while the public household tries to satisfy the collective, or public needs of private households. Notice that the public household is assumed to provide only public goods, no private goods. Public production of private goods will be considered as an activity of the private sector in so far as it can be represented by the actions of an ordinary firm, possibly with special tax (or subsidy) treatments.

There is another difference between public and private households which is more gradual and less fundamental: while the income of the public household consists of tax revenues received (as a lump-sum transfer) from the fisc, the private household’s income will, in general, consist for the greater part of the proceeds of goods sold (e.g., income from labour services supplied). The reason behind this distinction is the lack of earning power of the public household, in particular the lack of labour potential: each dollar spent by the public household should, somehow, be earned in the private sector. However, this does not have to be a problem, since it can be adequately described by differences in the income shares of both households.

\(^1\)On public goods see Samuelson (1954, 1969), Musgrave (1969), and Musgrave and Musgrave (1976, ch. 3).

While the difference between the public household and the fisc follows from the descriptions given above, it seems right to emphasize the difference between both institutions with respect to the problem of tax incidence. The actions of the fisc are, in our model, assumed to be exogenous, i.e., given outside the model, while the actions of the other agents (households, including the public one, and firms) are assumed to be endogenous, i.e., determined by the model. Hence we are interested in the consequences of a tax change (an action of the fisc), given the behavioural rules of all other agents as specified in the model. This assumption implies that public consumption is not an instrument, the effects of which might be examined by means of our model, while taxes (including transfers, subsidies, etc.) are. This is a consequence of the definition of our problem: to examine the impact of tax changes. In line with the distinction in operation of the fisc and the public household, we assume that the public household, like a public household, considers all (net) prices and the transfer received from the fisc (the public budget) to be exogenous. As a consequence the public household decides only upon the composition of the basket of public goods, not upon the total budget involved. The latter decision is relegated to the fisc. Finally, as mentioned in Chapter 2, we assume a balanced budget for the public sector, implying a balance between total tax revenue and total transfer expenditure, for part of the fisc, and between the public budget and total expenditures on public consumption, for part of the public household.

8.3. The provision of public goods

Now we are ready to consider the objective of the public household: the provision of public goods. Since the basic idea behind the provision of public goods is the satisfaction of the needs of private households, we start with a description of the private preferences for public goods. However, before we can do that we should identify public goods. Public goods are provided by transforming private goods. For example, the public good “national defence” is provided by transforming private goods like labour and capital services, materials, etc. into something like an army. This transformation is assumed to be performed inside the public household. Unless we specify an explicit transformation function it is difficult to determine the quantities of public goods obtained in this way. Therefore, we will simplify the analysis by assuming that each private good demanded by the public household immediately turns into a
public good as soon it has become available to the public household. For example, labour services, although a private good in principle, become a public good when "consumed" by the public household: each employee of the public household becomes, literally, a "public servant" whose services enter in the utility functions of all private households.

Let us assume that there are $H$ households in the economy. The index $h = 1$ refers to the public household, while $h = 2, \ldots, H$ refers to private households. Following the notation of Chapter 4, Section 6, we define $q_{hn}$ as the quantity of net demand of private good $n$ by household $h$. As usual, there are $N + 1$ private goods. Since the public household may demand all private goods, there are also $N + 1$ public goods (each corresponding to one of the $N + 1$ private goods). Denoting the quantity of the $n$th public good by $q_{Gn}$ (with the subscript $G$ referring to government) we have

$$q_{Gn} = q_{hn}^n, \quad n = 0, 1, \ldots, N,$$  \hspace{1cm} (8.1)

since the quantity $q_{hn}^n$ represents the net demand of the public household.

Private households derive utility from private as well as public goods. Given the notation as outlined above, we can formulate utility $u^{nh}$ of private household $h$ as a function of the net quantities of private goods demanded and the quantities of public goods provided by the public household:

$$u^{nh} = u^{nh}(q_{h0}^h, q_{h1}^h, \ldots, q_{hN}^h, q_{G0}, \ldots, q_{GN}), \quad h = 2, \ldots, H.$$ \hspace{1cm} (8.2)

We assume that all the usual assumptions concerning first- and second-order derivatives (see Chapter 4, section 2) hold also for $u^{nh}$. Equation (8.2) forms the starting point of our analysis of the provision of public goods. Using this specification and some additional assumptions, we will try to formulate a preference function for the public household. Additionally, by specifying the preferences for public goods at the individual level, we are able to assess the benefits of public consumption for the private household. In our opinion this duality in causality is important: individual preferences determine the preferences of the public household while the resulting provision of public goods determines, in turn, individual welfare. In order to formalize both parts (the formulation of public preferences and the evaluation of public consumption) we have to make two restrictive (but rather unavoidable) assumptions to which we now turn.

First we will assume that private preferences are separable with respect to private and public goods. In other words, we assume that the marginal rate of substitution between private (public) goods is unaffected by the amount of public (private) goods. Our second assumption concerns the preferences for different public goods: we assume that the marginal rates of substitution between public goods are the same for all private households. In formulae:

$$u^{*h} = u^{*h}(u^h(q_{h0}^h, \ldots, q_{hN}^h), u^0_G(q_{G0}, \ldots, q_{GN}), \quad h = 2, \ldots, H),$$  \hspace{1cm} (8.3)

$$u^0_G(q_{G0}, \ldots, q_{GN}) = u^0_G(q_{G0}, \ldots, q_{GN}), \quad h = 2, \ldots, H.$$  \hspace{1cm} (8.4)

Equation (8.3) shows that $u^{*h}()$ is "weakly" separable with respect to private and public goods. $u^h()$ represent the utility derived from private goods, while $u^G()$ represent the utility derived from public goods. The second expression, eq (8.4), shows the equality of preferences for public goods among different private households. The marginal rates of substitution between two private or two public goods are now, respectively, for $n, m = 0, 1, \ldots, N$:

$$\frac{\partial u^{nh}}{\partial q_{hn}^n} \cdot \frac{\partial q_{hn}^n}{\partial q_{hm}^m} = \frac{\partial u^h}{\partial q_{hm}^m}, \quad h = 2, \ldots, H,$$  \hspace{1cm} (8.5)

$$\frac{\partial u^h}{\partial q_{hm}^m} \cdot \frac{\partial q_{hm}^m}{\partial q_{Gn}} = \frac{\partial u^0_G}{\partial q_{Gn}}, \quad h = 2, \ldots, H.$$  \hspace{1cm} (8.6)

which confirms that the marginal rate of substitution between private (public) goods is unaffected by the amount of public (private) goods, since $u^h()$ and $u_G()$ are only functions of $q_{hn}^n$ and $q_{Gn}$, respectively, while the marginal rate of substitution between any two public goods is the same for all private households.

What are the implications of these assumptions? Well, the first assumption implies that, for example, the demand for a car is unaffected by the public provision of highways (assumed to be a public good for the moment). The second assumption implies that, although preferences for the aggregate of public goods may differ among private households (as different households might assign different "weights" to $u^h$ and $u^G$),

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2Although this assumption might seem rather restrictive, we wonder if it is a restriction at all, since we can replace each combination of a transformation function (transforming private goods into public goods) and a utility function (on these public goods) by a more comprehensive utility function defined on the "primary inputs", the private goods.

3On separability, see Green (1964), Berndt and Christensen (1973), and Matal (1973).
depending on the specification of \( u^{*4} \), the preference structure for mutual public goods is the same for each private household.

Why should we make these two restrictive assumptions? Since consumption of public goods is non-rival the benefits of it apply to all households, independent of their willingness to pay a price for the provision of these goods. This is the well-known free-rider problem. Therefore there is little chance that individual households will voluntarily reveal their preferences for public goods, as in the case of private goods where households have to pay a price for their provision. But if we do not know the individual preferences for public goods, as we cannot reveal the preferences for private goods either, unless we assume public and private preferences to be separable. This motivates our assumptions of separability. But even under this assumption it is impossible to measure the total burden of taxes, i.e. inclusive of the benefits of public goods, if preferences for public goods are different among individuals and unknown. And, what is worse, we cannot apply differential incidence (see section 5 below) since it is not possible then to keep the aggregate benefit of all public goods unchanged simply because there exists then no such aggregate which is the same for all individuals. Finally, if “public” preferences are different among households, there is also no obvious way of aggregating across individuals to arrive at a consistent preference function for the public household, as Arrow (1951) has demonstrated so famously. Therefore, we will subsequently assume that preferences for public goods are separable from preferences for private goods, while marginal rates of substitution for public goods are equal among private households. And although we realize that both assumptions seem far from realistic, we hope to show that there is still some room for interesting conclusions.

Given the individual preference structure as expressed by [see eqs. (8.3) and (8.4)]

\[
u^{*4} = u^{*4}(u^{h}(q_{h}^{110}, \ldots, q_{h}^{1N}), u_{c}(q_{010}, \ldots, q_{01N})), \quad h = 2, \ldots, H.
\]

we can easily determine the preference function for the public household by making reference to a social choice procedure.\(^4\) Since “public preferences” \( u_{c}(\cdot) \) are the same across individuals, a simple voting mechanism will do. Therefore, we assume that each individual tries to maximize its utility by (1) demanding or supplying private goods and (2) voting for a distribution of public goods, thereby acting as if his preferred basket of public goods were actually provided. Since the actions of the fisc are exogenous in our model, transaction taxes and the transfer distribution are given; in particular, the distribution of transfers \( \lambda^{h} \) between the private sectors \( (h = 2, \ldots, H) \) and the public sector \( (h = 1) \) is assumed to be fixed. Furthermore, we assume that all households (including the public) pay the same price \( p_{h}^{1} \) for private good \( n \). In maximizing utility, each private household takes into account the budget restrictions on both private and public goods. Given the preference structure as specified in (8.7) the optimization problem of private household \( h \) becomes, for \( h = 2, \ldots, H \)

\[
\max u^{*h}(u^{h}(q_{h}^{101}, \ldots, q_{h}^{1N}), u_{c}(q_{101}, \ldots, q_{1N})))
\]

subject to

\[
\sum_{n=1}^{N} p_{n}^{1} q_{n}^{1} = \lambda^{h},
\]

and

\[
\sum_{n=1}^{N} p_{n}^{1} q_{n}^{1} = \lambda^{1}.
\]

By virtue of the assumptions of separability and equality of marginal rates of substitution for public goods, we may separate this objective into two parts: first, the private objective function, for \( h = 2, \ldots, H \)

\[
\max u^{h}(q_{h}^{101}, \ldots, q_{h}^{1N})
\]

subject to

\[
\sum_{n=1}^{N} p_{n}^{1} q_{n}^{1} = \lambda^{h},
\]

and secondly, as a result of unanimous voting (since \( u_{c}(\cdot) \) is the same across individuals), the public objective function

\[
\max u_{c}(q_{101}, \ldots, q_{1N})
\]

subject to

\[
\sum_{n=1}^{N} p_{n}^{1} q_{n}^{1} = \lambda^{1}.
\]

\(^4\) A “classic” on social choice is Arrow (1951); see also Quirk and Saposnik (1968, ch. 4), and Musgrave and Musgrave (1976, ch. 6). Besides the simple majority voting mechanism, some new processes for making social choices have recently attracted attention, see Tullock (1976), Green and Laffont (1977), and Groves and Ledyard (1977). An ingenious procedure to make preferences for public goods observable is suggested by Bradford and Hidebrandt (1977).
Under the simplifying assumption as stated in eq. (8.1) we can reformulate the public objective completely analogously to the private objective; substitution of (8.1) into (8.10) yields

$$\text{max } u'(q_{1H}, \ldots, q_{1N})$$

subject to

$$\sum_{h=0}^{N} p_{h} q_{1h} = A'$$, \hspace{1cm} (8.11)

where we write $u'(\cdot)$ instead of $u_0(\cdot)$ in order to emphasize the correspondence with the private households.

One final word about the specification of the public preferences, as represented by $u'(\cdot)$, seems to be in order. The advantage of the procedure outlined above is that the preferences for public goods are revealed by the behaviour of the public household, just as the preferences for private goods are revealed by the behaviour of private households. Therefore, we might use the same (econometric) techniques for the determination of $u'(\cdot)$ as we do for the determination of $u_0(\cdot)$, $h = 2, \ldots, H$ (e.g. parameter estimation using time series data). Although perfectly legitimate, we do not think that this procedure is found to be attractive in the eyes of the policy-maker: according to him public consumption is just as exogenous as taxes are. However, we cannot change both public consumption and taxes simultaneously, without taking into account the public budget constraint. We might change taxes, given a certain policy to change public consumption as prices and revenue change, or we might change public consumption, given a certain policy to change taxes in order to keep the budget balanced. Since our interest lies in the first problem, the consumption behaviour of the public household should be endogenous, although public consumption and therefore the public preference structure might be determined by the same policy-maker who changes the taxes. An example might illustrate this point. The policy-maker is interested in the effects of a change in, say income tax. In addition to this information, he also specifies that the resulting change in tax revenue should be handed over to the public household which allocates its budget according to fixed budget-shares. It can easily be shown that this kind of public behaviour corresponds to maximizing a simple Cobb-Douglas utility function to one-level CES utility function with elasticity of substitution equal to one. Another example is if the public household is assumed to allocate its budget according to fixed volume shares: the corresponding utility function is the so-called Leontief function (a one-level CES utility function with elasticity of substitution equal to zero). Notice that, even if the policy-maker determines that all additional revenue should be channelled to private households, he has to say something about public consumption, since price changes might call for a reallocation of the existing public budget in order to keep the budget balanced. In our approach he does so by specifying the public utility function.5

8.4. The benefits of public goods

In this section we examine the possibility of expressing the benefits of public goods in the same way as we expressed the burden of taxes: by using the compensating variation (see Chapter 7, section 4). The idea amounts to the following: after a change in taxes (including transfers) and public goods we ask the private household how much additional transfer it needs in order to be restored to the original (pre-change) utility level. This amount, the so-called compensating variation, will be used as an integral measure of the change in utility, including the benefits of public goods and the burden of taxes. Again, as was the case in Chapter 7, we concentrate on the marginal (instead of average) burden by comparing post- and pre-change situations, where the changes in taxes are assumed to be small. As usual, negative burdens correspond to benefits.

The utility function of private household $h = 2, \ldots, H$ might be written as [see eqs. (8.7) and (8.1)]

$$u^*(u^*(q^h_{1H}, \ldots, q^h_{1N}), u^i(q^h_{1H}, \ldots, q^i_{1N}))$$, \hspace{1cm} (8.12)

where $u^i$ stands for $u_0$. The compensating transfer should be such that, after a change in taxes (resulting in a change in prices, transfers and public goods) utility is unchanged. For $h = 2, \ldots, H$:

$$\Delta u^h = \sum_{n=0}^{N} \left( \frac{\partial u^h}{\partial q^h_{1n}} \Delta q^h_{1n} + \frac{\partial u^h}{\partial q^i_{1n}} \Delta q^i_{1n} \right) = 0.$$ \hspace{1cm} (8.13)

Since private households maximize $u^h(h \neq 1)$, while the public household

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5For a general discussion of the other problem, i.e. that of expenditure incidence, we refer to McLure (1974a) and van Mierlo (1978).
maximizes $u^h$, both subject to the relevant budget constraints, we know that for $h = 1, 2, \ldots, H$

$$
\frac{\partial u^h}{\partial q^h_{i0}} = \psi^h p^h_{i0}, \quad n = 0, 1, \ldots, N.
$$  

(8.14)

where $\psi^h (h = 1, \ldots, H)$ are Lagrange multipliers corresponding to the budget constraints

$$
\sum_{n=0}^{N} p^h_{i0} q^n_{i0} = \lambda^h, \quad h = 1, \ldots, H.
$$  

(8.15)

Substituting (8.14) into (8.13) we have the equivalent condition. For $h = 2, \ldots, H$

$$
\sum_{n=0}^{N} \left( p^h_{i0} d q^n_{i0} + \xi^h p^h_{i0} d q^1_{i0} \right) = 0,
$$  

(8.16)

where

$$
\xi^h = \frac{\partial u^{nh}}{\partial u^{nh}} \psi^h, \quad h = 2, \ldots, H.
$$  

(8.17)

For the moment we postpone the interpretation of $\xi^h$. In the hypothetical situation of compensation we have, from the budget constraint of the private household $h = 2, \ldots, H$,

$$
\sum_{n=0}^{N} \left( p^h_{i0} d q^n_{i0} + dp^h_{i0} dq^1_{i0} \right) = d \lambda^h + d \beta^{*h},
$$  

(8.18)

where $d \lambda^h$ equals the transfer change resulting from the tax change, while $d \beta^{*h}$ equals the compensating variation, chosen to accomplish condition (8.16). Elimination of $\sum p^h_{i0} dq^n_{i0}$, from (8.16) and (8.18) leads to the following expression for the compensating variation $d \beta^{*h}$:

$$
d \beta^{*h} = d \lambda^h + \sum_{n=0}^{N} (dp^h_{i0} q^n_{i0} - \xi^h p^h_{i0} dq^1_{i0}), \quad h = 2, \ldots, H.
$$  

(8.19)

which might be written as

$$
d \beta^{*h} = \left( \sum_{n=0}^{N} dp^h_{i0} q^n_{i0} - d \lambda^h \right) + \xi^h \left( \sum_{n=0}^{N} dp^h_{i0} q^n_{i0} - d \lambda^h \right),
$$  

(8.20)

using the total differential of the budget constraint for the public sector. Using the notation $d \beta^{*h}$ for the compensation of $u^h$ (as opposed to $d \beta^{**h}$ for the compensation of $u^{**h}$) we express the compensation $d \beta^{*h}$ for private household $h$ as

$$
d \beta^{*h} = d \beta^* + \xi^h d \beta^1, \quad h = 2, \ldots, H.
$$  

(8.21)

where

$$
d \beta^{*h} = \sum_{n=0}^{N} dp^h_{i0} q^n_{i0} - d \lambda^h, \quad h = 2, \ldots, H.
$$  

(8.22)

By virtue of the assumptions of separability and equality of marginal rates of substitution for public goods, the compensating variation $d \beta^{**h}$ can be expressed as a sum of the compensation needed to restore private utility $u^*$ (the utility of private goods) plus a fraction $\xi^h$ (depending on the particular household $h$) of the compensation needed to restore public utility $u^1$ (the utility of public goods). The fraction $\xi^h$ reflects two things [see eq. (8.17)]. First, the differences between private households in the valuation of the aggregate bundle of public goods as compared to the aggregate bundle of private goods. This determines the marginal rate of substitution

$$
\frac{\partial u^{*h}}{\partial u^{*h}} / \frac{\partial u^1}{\partial u^1}, \quad h = 2, \ldots, H.
$$  

(8.23)

Secondly, it reflects differences in the marginal utility of money, as expressed by the Lagrange multipliers $\psi^h$. To be more specific, the ratio $\psi^h / u^1$ in the definition of $\xi^h$ equals the ratio of the marginal "public" utility of money to the marginal "private" utility of money:

$$
\frac{\psi^1}{\psi^h} = \frac{\partial u^1}{\partial u^h} / \frac{\partial u^1}{\partial \lambda^h}, \quad h = 2, \ldots, H.
$$  

(8.24)

Thus, if we assume that, for a fixed marginal "public" utility of money, the marginal "private" utility of money is a decreasing function of income, the fraction $\xi^h$ will be larger for the rich man than for the poor man, other things [such as the marginal rate of substitution in (8.23)] being the same. The reason for this outcome is that in order to compensate for a loss of public goods (represented by a decrease in $u^1$), the rich man needs more money than the poor one, since the relative increase in private utility resulting from a given compensation will be smaller. Notice that both marginal utilities of money, $\psi^1$ and $\psi^h$, are not invariant to separate transformations of $u^1$ and $u^h$, respectively. Their ratio, however, is invariant with respect to the same transformation applied simultaneously to both $u^1$ and $u^h$, as it is with respect to (monotonically increasing) transformations of the total utility function $u^{**h}$. Therefore, "total" utility $u^{**h}$ is still an ordinal concept.
As in Chapter 7 we consider the compensating variation as a measure of the burden of a tax change. Since \( d\beta^h \) measures the burden including the benefit of the change in public consumption, we call it the total burden. It will be convenient to give names to the two components of the total burden: the first component, \( d\beta^h \) (the compensation of private household \( h \) for the loss of "private" utility \( u^h \)) will be called the direct burden while the second component, \( \xi^h d\beta^h \) (the compensation of private household \( h \) for the loss of "public" utility \( u^i \)) will be called the indirect burden. The indirect burden for private household \( h \) is a fraction \( \xi^h \) of the "burden" of the public household, \( d\beta^h \). Subsequently, we shall call \( d\beta^h \) the public burden. Notice that the burden is directly observable by inspection of the price and transfer changes relevant for the public household.

Up till now we have dealt with the fraction \( \xi^h \); however, one might wonder whether it really is a proper fraction, i.e. something between zero and one. Assuming that every private household values public goods positively, it is not difficult to show [see (8.17)] that

\[
\xi^h > 0, \quad h = 2, \ldots, H. \tag{8.25}
\]

With respect to a possible upper bound of \( \xi^h \), a somewhat more complicated reasoning is required. Imagine that we increase the transfer received by the public household, \( \lambda^1 \), by a very small amount, while simultaneously decreasing the transfers received by the private households, \( \lambda^h \), in order to finance the increase in the public budget:

\[
d\lambda^1 = - \sum_{h=2}^{H} d\lambda^h. \tag{8.26}
\]

As a result of the increase in \( \lambda^1 \), public consumption will increase, thereby increasing the benefits of public goods for all private households. At the same time private households suffer a burden due to the additional lump-sum tax (equivalent to the decrease in the lump-sum transfer). Since burdens and benefits are expressed in money terms by use of the compensating variation, we might add up all burdens and benefits over private households. Acting in this way we forget all distributional aspects and concentrate on efficiency only: we wonder if the aggregate net compensation is negative, zero, or positive. Assume for the moment that prices \( p_{hi} \) remain unchanged. Then the aggregate total burden (the excess burden) equals

\[
\sum_{h=2}^{H} d\beta^h = \sum_{h=2}^{H} \left( - d\lambda^h - \xi^h d\lambda^1 \right) = \left( 1 - \sum_{i=2}^{H} \xi^i \right) d\lambda^1. \tag{8.27}
\]

Depending on the sum \( \sum \xi^i \) being equal to or smaller or greater than one, the aggregate total burden is zero, positive, or negative. If the sum is greater than one it is efficient to increase public consumption by increasing the public budget \( \lambda^1 \) since the lump-sum tax necessary to finance this increased consumption is outweighed by the aggregate benefit of the increase of public goods. The opposite case holds if the sum is smaller than one. The optimal public budget \( \lambda^1 \) might be defined as the budget where the aggregate total burden is equal to zero in the margin, i.e. where

\[
\sum_{i=2}^{H} \xi^i = 1. \tag{8.28}
\]

In this case the aggregate marginal costs of public consumption equals the aggregate marginal benefit of public consumption. It will be clear that if the public budget is optimal, conditions (8.25) and (8.28) imply that each \( \xi^i \) is between zero and one, while their sum equals one, i.e. they are true fractions. Then the public burden \( d\beta^h \) is distributed over private households according to the fractions \( \xi^h \), thereby providing an observable measure of the aggregate burden of the change in public consumption. However, if the public budget is not optimal, the public burden \( d\beta^h \) distributed over private households, \( \lambda^h \), in order to finance the increase in the public budget:

As a conclusion we find that owing to our assumptions of separability and equality of public preferences we are able to express the total burden \( d\beta^h \) of a private household as the sum of the direct burden \( d\beta^h \) corresponding to the change in private consumption and a fraction \( \xi^h \) of the burden \( d\beta^h \) of the public household; the last component represents the indirect burden of the change in public goods. The fraction \( \xi^h \) depends on the individual preferences for the aggregate bundle of public goods and on the marginal "private" utility of money. Only if the public budget is optimal, i.e. if the marginal costs of public consumption equal the aggregate marginal benefits, do the fractions \( \xi^h \) sum to one across private households.

8.5. Balanced-budget and differential incidence

If we change taxes, public consumption will, in general, change also. This will be the case even if the public budget remains unchanged, since
changes in prices will, in general, ask for a reallocation of public expenditures. The distribution of the total burden then reflects the so-called balanced-budget incidence, i.e., the overall incidence of a change in taxes, including the incidence of the induced change in public consumption.

In order to arrive at operational measures of balanced-budget incidence, some additional assumptions are required. First, we assume that in the initial situation the public budget is optimal. Secondly, we assume that the marginal rate of substitution between public and private goods, as expressed in eq. (8.23), is the same for all households. Then, we have for two private households $h$ and $i$

$$\xi^h / \xi^i = \phi^h / \phi^i, \quad \text{while} \sum_{i=2}^{n} \xi^i = 1$$  \hspace{1cm} (8.29)

stating that the fractions $\xi^h$ are in inverse proportion to the marginal "private" utilities of money. Together with some rule to relate the marginal "private" utility of money to observable characteristics of private households (e.g., to income) we are able to allocate the benefits of public goods and to determine the distribution of the total burden.

Two objections might be raised to the procedure outlined above. First, the assumption of equal marginal rates of substitution, eq. (8.23), removes the last possibility of different preferences for public goods among individuals. Secondly, one might have doubts about the possibilities of expressing a relationship between the marginal "private" utility of money and observable characteristics such as income.

In view of these objections we suggest an alternative procedure. Instead of making absolute statements about the impact of a change in tax instruments (and of the corresponding changes in public consumption) we might consider the relative impact of a simultaneous change in two tax instruments such that "public" utility remains the same. In this case one tax is substituted for another while the volume of public consumption (as measured by $u'$) is held constant. The resulting distribution of the total burden equals the distribution of the direct burden, since the indirect burden is zero in this case. This is called differential incidence.

In the case of more than two tax instruments, the examination of all possible differential incidence patterns may become somewhat complicated; if there are $K$ different tax instruments, we have to examine $K^2$ combinations. However, our marginal analysis now turns out to be very convenient; owing to the linearity of the model we know that the effect of a linear combination of tax instruments equals the linear combination of the effects of each instrument separately. Therefore, if we normalize each change in tax so that the resulting change in public burden is the same, we can make pairwise comparisons of the effects of different tax instruments simply by comparing the distributions of the direct burden, since the distributions of the indirect burden will be the same for all instruments normalized in this way.

Another consequence of this normalization is that we can compare aggregate total burdens for different tax instruments by comparing the aggregate direct burdens. Hence, the normalization enables us to make relative statements about total burdens without the need to know the exact distribution of the benefits of public goods.

We can easily formalize the idea of differential incidence as follows: indicating a burden of a unit change in tax $k$ by the subscript $k$, we have

$$d\beta_k^{+} = d\beta_k^{+} + \xi^k d\beta^k, \quad h = 2, \ldots, H.$$  \hspace{1cm} (8.30)

The normalized burden for household $h$ (normalized so that the public burden is unity) is obtained by dividing eq. (8.30) through $d\beta^k$. The total burden of a normalized change in tax $k$ is therefore

$$d\beta_k^{+} / d\beta^k = \xi^h + \xi^k, \quad h = 2, \ldots, H; \hspace{1cm} \text{(8.31)}$$

so that the difference in the normalized total burdens of two tax instruments ($k = 1, 2$, say), for household $h$ equals

$$d\beta_k^{+} - d\beta_l^{+} = d\beta_k^{+} / d\beta^k - d\beta_l^{+} / d\beta^l, \quad h = 2, \ldots, H; \hspace{1cm} \text{(8.32)}$$

stating that the difference in total burden equals the difference in direct burden, after normalization. Expression (8.32) reflects the (total) burden of household $h$ when tax 1 is increased and tax 2 is decreased so that the

\begin{itemize}
  \item[\text{This is the procedure suggested by Aaron and McGuire (1970); see also Maital (1973). It amounts to equal proportional benefit of public goods in terms of utility; in terms of money, however, the distribution is not proportional. The analogy on the other side, namely equal proportional sacrifice, is an important issue in the discussion on the progressivity of income tax rates. For this topic, see Musgrave and Musgrave (1976, ch. 9), and Keller and Hartog (1977).}]
  \item[\text{See Musgrave (1959, p. 212). Nice examples of empirical work on differential incidence can be found in Whalley (1977b) and Shoven and Whalley (1977).}]
\end{itemize}
volume of public consumption (as measured by $d\beta^*$) remains unchanged. By subtracting one normalized total burden from the other, we "lose" the unknown fraction $\xi^p$ and, therefore, the unknown indirect burden. It is this feature that makes differential incidence so attractive.

The aggregate total burdens (i.e. the excess burdens) for normalized tax changes are

$$\frac{d\beta^*}{d\beta^1} = \sum_{h=1}^{n} \frac{d\beta^h}{d\beta^1} + \sum_{h=1}^{n} \xi^p,$$

so that the difference in aggregate total burden of both taxes equals, after normalization

$$\frac{d\beta^*}{d\beta^1} - \frac{d\beta^*}{d\beta^1} = \frac{d\beta_1}{d\beta^1} - \frac{d\beta_1}{d\beta^1},$$

(8.33)

where

$$d\beta_1 = \sum_{h=1}^{n} d\beta^h_i,$$

(8.34)

representing the aggregate direct burden including the "direct" burden $d\beta^1_i$ of the public household. Hence, the difference in aggregate total burden equals the difference in aggregate direct burden, after normalization. And since the aggregate direct burden is observable, this enables us to make statements about relative efficiency even if the sum $\Sigma \xi^p$ is unknown. Therefore it is not necessary to assume an optimal public budget, i.e. the sum $\Sigma \xi^p$ might be different from one in the case of differential incidence. Notice that, if the public budget is optimal, the aggregate total burden equals the aggregate direct burden, which is observable:

$$d\beta^* = d\beta_1, \quad \text{if} \quad \sum_{h=1}^{n} \xi^p = 1.$$

(8.35)

Instead of burdens in nominal terms (e.g. in dollars) we often prefer to present the burden of household $h$ relative to income $\nu^h$ of household $h$: the relative direct burden, for example, equals

$$\hat{\beta}^h = \frac{d\beta^h_i}{d\nu^h} = \sum_{\pi=0}^{H} \rho_{\pi} c^h_{\pi} - \bar{\lambda}^h, \quad h = 1, \ldots, H,$$

(8.36)

where $c^h_{\pi}$ represents the income share of good $\pi$ for household $h$, while $\bar{\lambda}^h$ represents the change in transfer of household $h$, relative to income $\nu^h$. The aggregate direct burden relative to total income, $\beta_1$, is defined by

$$\hat{\beta} = \sum_{h=1}^{n} \frac{d\beta^h_i}{d\nu^h} \xi^p = \sum_{h=1}^{n} \gamma^h \delta^h.$$

(8.37)

where $\gamma^h$ represents the income share of household $h$ in aggregate income $\nu$:

$$\gamma^h = \nu^h / \nu, \quad \text{where} \quad \nu = \sum_{h=1}^{n} \nu^h.$$  

(8.38)

The corresponding normalized burdens are found simply by dividing the burdens by the relative public burden, $\beta_1$.

### 8.6. Summary

In this chapter we consider public goods. Public goods are distinct from private goods in that their benefits apply to all individuals while the consumption by one individual does not reduce the benefits derived by others. Public goods are supposed to be provided by the public household by transforming private goods. We assume that each private good demanded by the public household turns into a public good. The public collects the proceeds of transaction taxes and returns this as transfers to households. The transfer to the public household constitutes the public budget, which is assumed to be exogenous to the public household.

In general each change of taxes will result in a change in public consumption of private goods: the balanced-budget assumption implies that there has to be some change in public consumption if prices and/or the public budget changes. Since a change in public consumption affects aggregate demand, we need to have some knowledge of how public consumption changes if prices and budget change. We assume here that public consumption is determined by maximization of a utility function for public goods, just as private consumption is determined by the maximization of a utility function for private goods. This "public utility function" has to be derived from the preferences of individuals for public goods. The problem of specifying a collective utility function for public goods, given individual preferences, is strongly related to the problem of deriving the individual benefit of the collectively determined public consumption. Like many others, we have not succeeded in solving both problems in a way which comes close to reality. We have to make rather restrictive assumptions concerning the individual preferences for public goods.

First, we assume that preferences for public goods are separable from
preferences for private goods. This means that individual utility can be written as a function of two components: the utility of private goods and the utility of public goods. This implies that the marginal rates of substitution between private goods are unaffected by the amount of public goods available, and vice versa. Secondly, we assume that the utility function for public goods is identical across individuals. This implies that the marginal rates of substitution between public goods are the same for all private households. Preferences for public goods in general, however, may differ among households, since different "weights" can be assigned to the private and public part of the individual utility function.

Given these assumptions, we can easily arrive at a public utility function by making reference to a simple social choice mechanism. The same assumptions enable us to quantify the individual burden of changes in the provision of public goods, by using the concept of compensating variation as introduced in Chapter 7. The individual burden involved with the change in private goods is called the "direct" burden of a tax change, while the one corresponding to the induced change in public goods is called the "indirect" burden. "Balanced-budget incidence" is concerned with the distribution of the combined burden, called the "total" burden. Under the assumptions made, the indirect burden of a private household is proportional to the burden of the public household ("the public burden"), which is measured by the compensation needed to restore public utility. Only if the public budget is optimal (i.e. if the aggregate marginal cost and benefit of public consumption are equal) is the sum over private households of the indirect burdens equal to the public burden.

Since the determination of the distribution of the indirect burden over private households is rather difficult, the alternative of "differential incidence" is suggested. In this case, each change in tax is normalized so that the resulting change in public burden is the same. Then, we can make pairwise comparisons of the effects of different tax instruments simply by comparing the distribution of the direct burden, since the distribution of the indirect burden is the same for all instruments normalized this way. Additionally, we can find the differences in efficiency (i.e. the excess burden) by comparing the aggregate direct burdens of normalized tax changes. Therefore, the normalization enables us to confine our interest to the comparison of the direct burdens without having to bother about the indirect burdens.