Short-cut demand elasticities

by
Ken Clements*
Economics, UWA
August 2025

Abstract

Suppose you don't know the price elasticity of demand but need some value. The rule of ½ states that the elasticity is equal to -½. This presentation

(i) highlights the precise conditions under which the rule holds,

(ii) indicates its practical importance, and

(iii) points to related results on simplifications of demand elasticities.

When nothing much is known about the elasticity, -½ is likely to be a good starting point for certain types of goods.

*This is mostly based on the book by K. W. Clements, H. Liu, J. M. Mariano, E. A. Selvanathan, S. Selvanathan and G. Verikios (2025), Short-cut demand elasticities and other convenient approaches to consumer demand (Springer). In addition to my co-authors of the book, for helpful comments and support, I also thank Simon Chang, Shawn Xiaoguang Chen, Tom Hertel, Haiyan Liu, Anda Nugroho and seminar participants at UWA.

Booze consumption

Price elasticities of demand, imported booze, USA:

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	**−0.54** (0.06)***	−0.20 (0.06)***	−0.32 (0.06)***	−0.14 (0.04)***	0.18 (0.06)***	0.05 (0.03)**
Whiskey	−0.12 (0.04)***	**−0.41** (0.08)***	−0.11 (0.05)**	−0.13 (0.04)***	−0.09 (0.05)**	0.00 (0.02)
Vodka	−0.36 (0.07)***	−0.24 (0.09)***	**−0.63** (0.12)***	−0.05 (0.06)	−0.29 (0.08)***	0.02 (0.04)
Liqueurs	−0.27 (0.06)***	−0.38 (0.09)***	−0.10 (0.08)	**−0.73** (0.09)***	−0.04 (0.07)	−0.05 (0.03)
Brandy	0.05 (0.05)	−0.30 (0.07)***	−0.39 (0.08)***	−0.09 (0.05)*	**−0.71** (0.09)***	−0.03 (0.03)
Other	0.07 (0.05)	−0.04 (0.07)	0.05 (0.07)	−0.04 (0.05)	0.03 (0.06)	**−1.02** (0.05)***

Source: A. Muhammad (2025). “Trump Tariffs 2.0: Assessing the Impacts on US Distilled Spirits Imports.” Agribusiness, 1-7

Own-price elasticities around - ½

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	**−0.54** (0.06)***	−0.20 (0.06)***	−0.32 (0.06)***	−0.14 (0.04)***	0.18 (0.06)***	0.05 (0.03)**
Whiskey	−0.12 (0.04)***	**−0.41** (0.08)***	−0.11 (0.05)**	−0.13 (0.04)***	−0.09 (0.05)**	0.00 (0.02)
Vodka	−0.36 (0.07)***	−0.24 (0.09)***	**−0.63** (0.12)***	−0.05 (0.06)	−0.29 (0.08)***	0.02 (0.04)
Liqueurs	−0.27 (0.06)***	−0.38 (0.09)***	−0.10 (0.08)	**−0.73** (0.09)***	−0.04 (0.07)	−0.05 (0.03)
Brandy	0.05 (0.05)	−0.30 (0.07)***	−0.39 (0.08)***	−0.09 (0.05)*	**−0.71** (0.09)***	−0.03 (0.03)
Other	0.07 (0.05)	−0.04 (0.07)	0.05 (0.07)	−0.04 (0.05)	0.03 (0.06)	**−1.02** (0.05)***

Small cross-price elasticities

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	**−0.54** (0.06)***	−0.20 (0.06)***	−0.32 (0.06)***	−0.14 (0.04)***	0.18 (0.06)***	0.05 (0.03)**
Whiskey	−0.12 (0.04)***	**−0.41** (0.08)***	−0.11 (0.05)**	−0.13 (0.04)***	−0.09 (0.05)**	0.00 (0.02)
Vodka	−0.36 (0.07)***	−0.24 (0.09)***	**−0.63** (0.12)***	−0.05 (0.06)	−0.29 (0.08)***	0.02 (0.04)
Liqueurs	−0.27 (0.06)***	−0.38 (0.09)***	−0.10 (0.08)	**−0.73** (0.09)***	−0.04 (0.07)	−0.05 (0.03)
Brandy	0.05 (0.05)	−0.30 (0.07)***	−0.39 (0.08)***	−0.09 (0.05)*	**−0.71** (0.09)***	−0.03 (0.03)
Other	0.07 (0.05)	−0.04 (0.07)	0.05 (0.07)	−0.04 (0.05)	0.03 (0.06)	**−1.02** (0.05)***

Trouble-makers

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	**−0.54** (0.06)***	−0.20 (0.06)***	−0.32 (0.06)***	−0.14 (0.04)***	0.18 (0.06)***	0.05 (0.03)**
Whiskey	−0.12 (0.04)***	**−0.41** (0.08)***	−0.11 (0.05)**	−0.13 (0.04)***	−0.09 (0.05)**	0.00 (0.02)
Vodka	−0.36 (0.07)***	−0.24 (0.09)***	**−0.63** (0.12)***	−0.05 (0.06)	−0.29 (0.08)***	0.02 (0.04)
Liqueurs	−0.27 (0.06)***	−0.38 (0.09)***	−0.10 (0.08)	**−0.73** (0.09)***	−0.04 (0.07)	−0.05 (0.03)
Brandy	0.05 (0.05)	−0.30 (0.07)***	−0.39 (0.08)***	−0.09 (0.05)*	**−0.71** (0.09)***	−0.03 (0.03)
Other	0.07 (0.05)	−0.04 (0.07)	0.05 (0.07)	−0.04 (0.05)	0.03 (0.06)	**−1.02** (0.05)***

Simplifications in 3 steps

Simplification I: Two groups

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila						
Whiskey						
Vodka						
Liqueurs						
Brandy						
Other						

Step II

Simplification I: Two groups

Product	**Tequila**	**Whiskey**	**Vodka**	**Liqueurs**	**Brandy**	**Other**
Tequila						
Whiskey						
Vodka						
Liqueurs						
Brandy						
Other						

Simplification II: First group = -½ × δ_{ij}

Product	**Tequila**	**Whiskey**	**Vodka**	**Liqueurs**	**Brandy**	**Other**
Tequila	−½	0	0			
Whiskey	0	−½	0			
Vodka	0	0	−½			
Liqueurs						
Brandy						
Other						

Step III

Simplification I: Two groups

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila						
Whiskey						
Vodka						
Liqueurs						
Brandy						
Other						

Simplification II: First group = -½ × δ_{ij}

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	−½	0	0			
Whiskey	0	−½	0			
Vodka	0	0	−½			
Liqueurs						
Brandy						
Other						

Simplification III: Zero off-diagonals

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	−½	0	0	0	0	0
Whiskey	0	−½	0	0	0	0
Vodka	0	0	−½	0	0	0
Liqueurs	0	0	0			
Brandy	0	0	0			
Other	0	0	0			

Trouble-makers undisturbed

Simplification I: Two groups

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila						
Whiskey						
Vodka						
Liqueurs						
Brandy						
Other						

Simplification II: First group = -½ × δ_{ij}

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	−½	0	0			
Whiskey	0	−½	0			
Vodka	0	0	−½			
Liqueurs						
Brandy						
Other						

Simplification III: Zero off-diagonals

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	−½	0	0	0	0	0
Whiskey	0	−½	0	0	0	0
Vodka	0	0	−½	0	0	0
Liqueurs	0	0	0			
Brandy	0	0	0			
Other	0	0	0			

Second group, trouble-makers remain

Product	Tequila	Whiskey	Vodka	Liqueurs	Brandy	Other
Tequila	−½	0	0	0	0	0
Whiskey	0	−½	0	0	0	0
Vodka	0	0	−½	0	0	0
Liqueurs	0	0	0	η_{44}	η_{45}	η_{46}
Brandy	0	0	0	η_{54}	η_{55}	η_{56}
Other	0	0	0	η_{64}	η_{65}	η_{66}

Payoff from simplification

- Own-price elasticities = -½ for first group
- Cross-price elasticities = 0 within first group and between 2 groups
- Not ok for second group
- $6 \times 6 = 36$ elasticities reduced to $1 + (3 \times 3) = 10$
- Substantial simplification

Rule of ½

Price elasticities = -½

- Demands for n commodities:

$$\log q_i = \eta_i \log Q + \sum_{j=1}^{n} \eta_{ij} \log p_j\,, \qquad i = 1, \cdots, n$$

- Under rule of ½ this simplifies to

$$\log q_i \approx \eta_i \log Q - \frac{1}{2} \log p_i\,, \qquad i = 1, \cdots, n$$

- n^2 price elasticities reduced to 1
- Big claim!

Useful?

- When not much is known
- Reduces dimensionality of large models

Utility and demands

$$u = u(q_1, \cdots, q_n), \qquad \sum_{i=1}^{n} p_i q_i = M$$

$$q_i = q_i(M, p_1, \cdots, p_n), \qquad i = 1, \cdots, n$$

$$\underbrace{\frac{\partial q_i}{\partial p_j}}_{\text{Total effect}} = \underbrace{\frac{\partial q_i^*}{\partial p_j}}_{\text{Substition effect}} - \underbrace{q_j \frac{\partial q_i}{\partial M}}_{\text{Income effect}}$$

$$\underbrace{\frac{\partial q_i^*}{\partial p_j}}_{\text{Substition effect}} = \underbrace{\lambda u^{ij}}_{\text{Specific SE}} - \underbrace{\frac{\lambda}{\partial \lambda / \partial M} \cdot \frac{\partial q_i}{\partial M} \cdot \frac{\partial q_j}{\partial M}}_{\text{General SE}}$$

λ = MU of income, $\qquad \left[u^{ij}\right] = \left[\frac{\partial^2 u}{\partial q_i \partial q_j}\right]^{-1}$ $(n \times n \text{ matrix})$

More familiar elasticity form

Compensated elasticity of good i w.r.t. price of j is $\eta^*_{ij} = \frac{\partial q^*_i}{\partial p_j}\frac{p_j}{q_i}$

From

$$\frac{\partial q^*_i}{\partial p_j} = \lambda u^{ij} - \frac{\lambda}{\partial\lambda/\partial M}\cdot\frac{\partial q_i}{\partial M}\cdot\frac{\partial q_j}{\partial M},$$

to

$$\boxed{\eta^*_{ij} = F_{ij} - \phi\eta_i w_j \eta_j}$$

$F_{ij} =$ Frisch price elasticity $\left(\left.\frac{\partial \log q_i}{\partial \log p_j}\right|_{d\lambda=0}\right)$

$\phi = \left(\frac{\partial \log \lambda}{\partial \log M}\right)^{-1}$ inverse of income elasticity of λ ("income flexibility")

$\eta_i = \frac{\partial \log q_i}{\partial \log M}$ income elasticity of good *i*

$w_j = \frac{p_j q_j}{M}$ budget share of good *j*

Preference independence

$$u(q_1, \cdots, q_n) = \sum_{i=1}^{n} u_i(q_i), \quad u_i(\cdot) = i^{th} \text{ subutility function}$$

Broad aggregates – food, clothing, housing, etc.

Basic wants – sustenance, warmth, shelter

Under preference independence, no utility interactions $u^{ij} = F_{ij} = 0, i \neq j$, and

$$\eta_{ij}^* = F_{ij} - \phi \eta_i w_j \eta_j$$

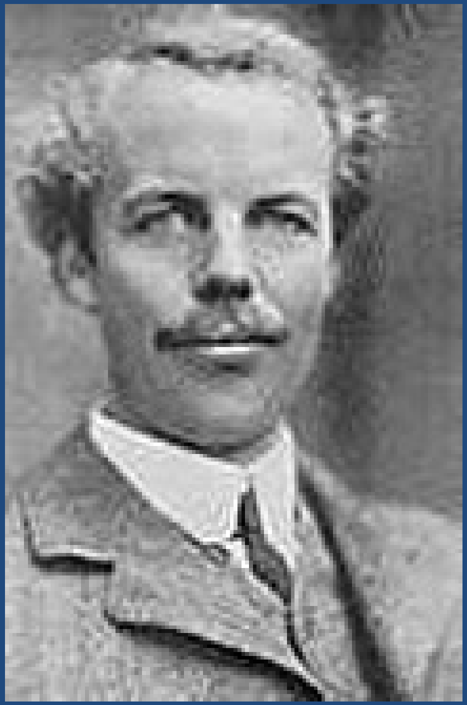
becomes

$$\eta_{ij}^* = \begin{cases} \phi \eta_i - \phi \eta_i w_i \eta_i, & i = j \\ -\phi \eta_i w_j \eta_j, & i \neq j \end{cases}$$

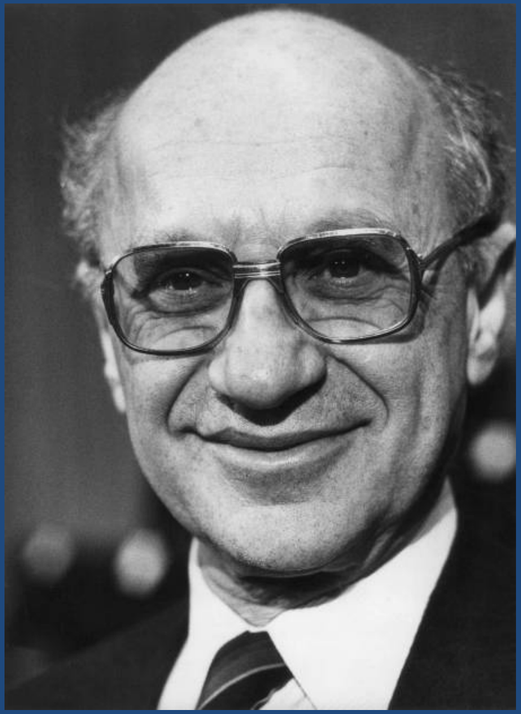
As $\phi \eta_i w_i \eta_i \approx 0$,

$$\boxed{\eta_{ii}^* \approx \phi \eta_i, \qquad i = 1, \cdots, n, \qquad \text{"Pigou's law"}}$$

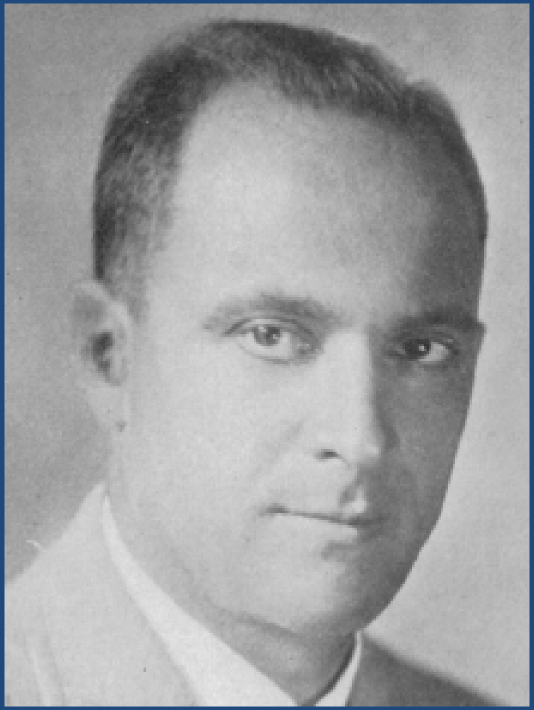
Three (very) different economists
What's the link?



Arthur Pigou
1877-1959



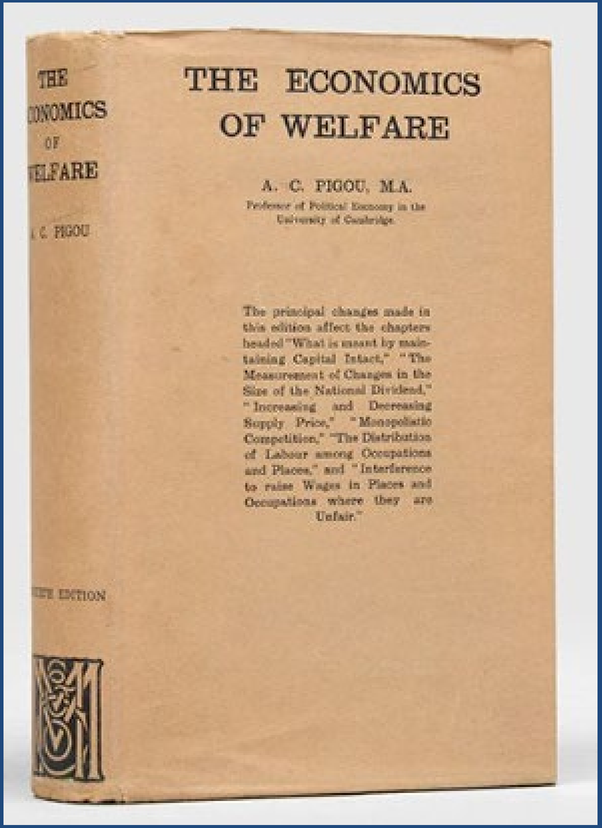
Milton Friedman
1912-2006



Henry Schultz
1893-1938

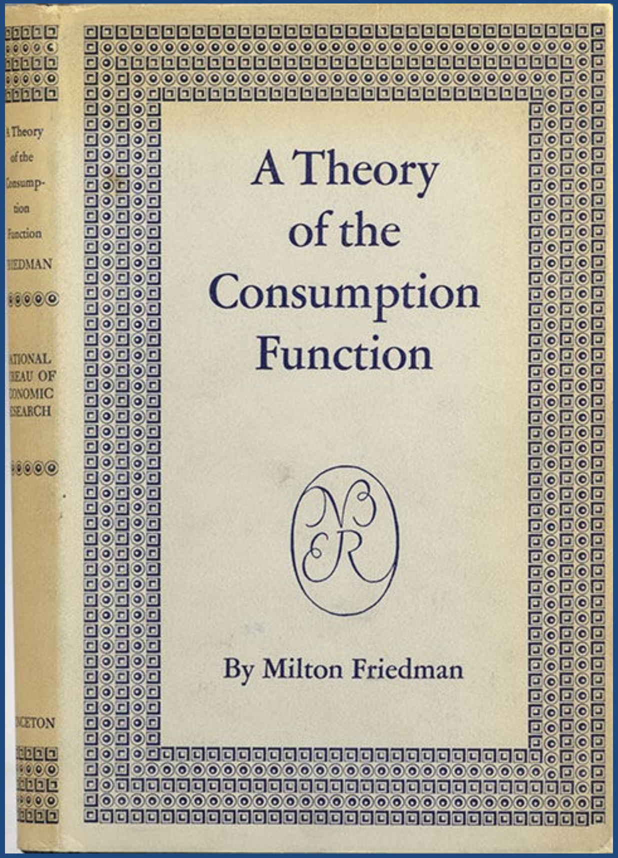
Early contributors to preference independence

- Pigou (1910). “A method of determining the numerical value of the elasticities of demand.” <u>Economic Journal</u>



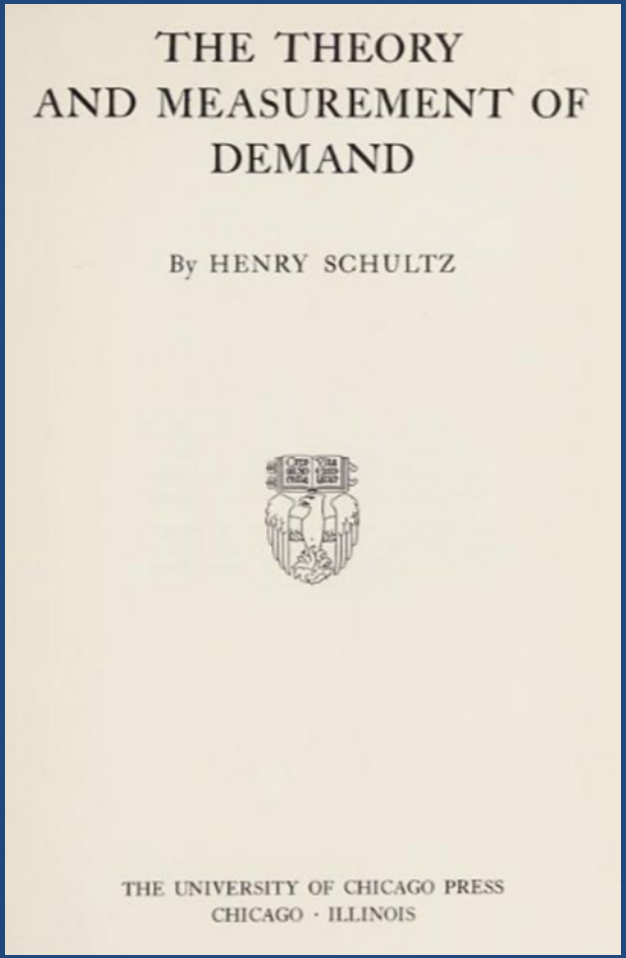
Early contributors to preference independence

- Pigou (1910). “A method of determining the numerical value of the elasticities of demand.” Economic Journal
- Friedman (1935). “Professor Pigou’s method for measuring elasticities of demand from budgetary data.” Quarterly Journal of Economics



Early contributors to preference independence

- Pigou (1910). “A method of determining the numerical value of the elasticities of demand.” Economic Journal
- Friedman (1935). “Professor Pigou’s method for measuring elasticities of demand from budgetary data.” Quarterly Journal of Economics
- Schultz (1938). The theory and measurement of demand. University of Chicago Press.


THE THEORY
AND MEASUREMENT OF
DEMAND

By HENRY SCHULTZ

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO · ILLINOIS

Pigou's law,
price and income elasticities linked

$$\eta_{ii}^{*} \approx \phi \eta_i, \qquad i = 1, \cdots, n$$

- Price elasticities are proportional to income elasticities
- Luxuries are more price elastic than necessities
- Puzzling? Income and price elasticities deal with different dimensions of the good
- Common usage of language:

Salt – essential (few substitutes) and viewed as a necessity (low income elasticity)

Foreign vacation -- not essential and not a necessity

Elasticity = ½ in two more steps

To go from

$$\boxed{\eta_{ii}^{*} \approx \phi\eta_i \;\text{ to }\; \eta_{ii}^{*} \approx -½}$$

need two steps:

<u>Step 1: Income flexibility $\phi = -½$</u>

Evidence that $\phi \approx -½$:

(i) Time-series estimates

(ii) Cross-commodity estimates

(iii) The previous literature

Still some controversy, however

More later

<u>Step 2: $\eta_i = 1$</u>

Based on $\sum_{i=1}^{n} w_i\eta_i = 1$

Summary of rule of ½

1. Preference independence
2. Neglect general substitution effect
3. Income flexibility $\phi = -½$
4. Income elasticity $\eta_i = 1$

Strong conditions

- Prior rejections of preference independence
- Small-sample biases with usual asymptotic tests
- Pigou's law criticised by Deaton (1974)
- Saroja Selvanathan (1993) unable to reject Pigou for 15 out of 18 OECD countries

Three sources of ϕ-values

1. Time-series, multiple countries
2. Cross-commodity regressions
3. Prior studies/surveys

Source 1 of ϕ-values
Time-series evidence

Rotterdam model for $t = 2, \cdots, T$ periods:

Coefficients

$$\bar{w}_{it}\,\Delta\log q_{it} = \theta_i\,\Delta\log Q_t + \sum_{j=1}^{n} \pi_{ij}\,\Delta\log p_{jt}\,, \qquad i = 1, \cdots, n$$

Average w_i Log–change of q_{it} Log–change of p_{jt}

Divisia index of income, $\Delta\log Q_t = \sum_{j=1}^{n} \bar{w}_{jt}\Delta\log q_{jt}$

Preference independence version:

Frisch price index, $\Delta\log P_t' = \sum_{j=1}^{n} \theta_j\,\Delta\log p_{jt}$

$$\bar{w}_{it}\,\Delta\log q_{it} = \theta_i\,\Delta\log Q_t + \phi\theta_i\,\underbrace{(\Delta\log p_{it} - \Delta\log P_t')}_{\text{Own relative price}}$$

Income flexibility Marginal share

Preference independent Rotterdam

$$\bar{w}_{it}\,\Delta\log q_{it} = \theta_i\,\Delta\log Q_t + \phi\theta_i(\Delta\log p_{it} - \Delta\log P'_t), \qquad \Delta\log P'_t = \sum_{j=1}^{n} \theta_j\,\Delta\log p_{jt}$$

Income and own-price elasticities:

$$\eta_i = \frac{\theta_i}{\bar{w}_{it}}, \qquad \eta^*_{ii} = \frac{\phi\theta_i}{\bar{w}_{it}} - \frac{\phi\theta_i^2}{\bar{w}_{it}}$$

Total SE = Specific SE + General SE

Preference independent Rotterdam

$$\bar{w}_{it}\,\Delta\log q_{it} = \theta_i\,\Delta\log Q_t + \phi\theta_i(\Delta\log p_{it} - \Delta\log P'_t), \qquad \Delta\log P'_t = \sum_{j=1}^{n}\theta_j\,\Delta\log p_{jt}$$

Income and own-price elasticities:

$$\eta_i = \frac{\theta_i}{\bar{w}_{it}}, \qquad \eta^*_{ii} = \frac{\phi\theta_i}{\bar{w}_{it}} - \frac{\phi\theta_i^2}{\bar{w}_{it}}$$

Marginal share: $\theta_i = \dfrac{\partial(p_i q_i)}{\partial M}$, $\quad 0 < \theta_i < 1$

Income $M = \sum_{i=1}^{n} p_i q_i$

$$\frac{\phi\theta_i^2}{\bar{w}_{it}} \approx 0, \qquad \eta^*_{ii} \approx \frac{\phi\theta_i}{\bar{w}_{it}} = \phi\eta_i$$

$$\boxed{\eta^*_{ii} \approx \phi\eta_i\,,\ i = 1, \cdots, n}$$

Price elasticities proportional to income elasticities, Pigou’s law

Estimating ϕ

Preference-independent Rotterdam:

$$\bar{w}_{it}\,\Delta\log q_{it} = \theta_i\,\Delta\log Q_t + \phi\theta_i(\Delta\log p_{it} - \Delta\log P_t'), \qquad i = 1, \cdots, n$$

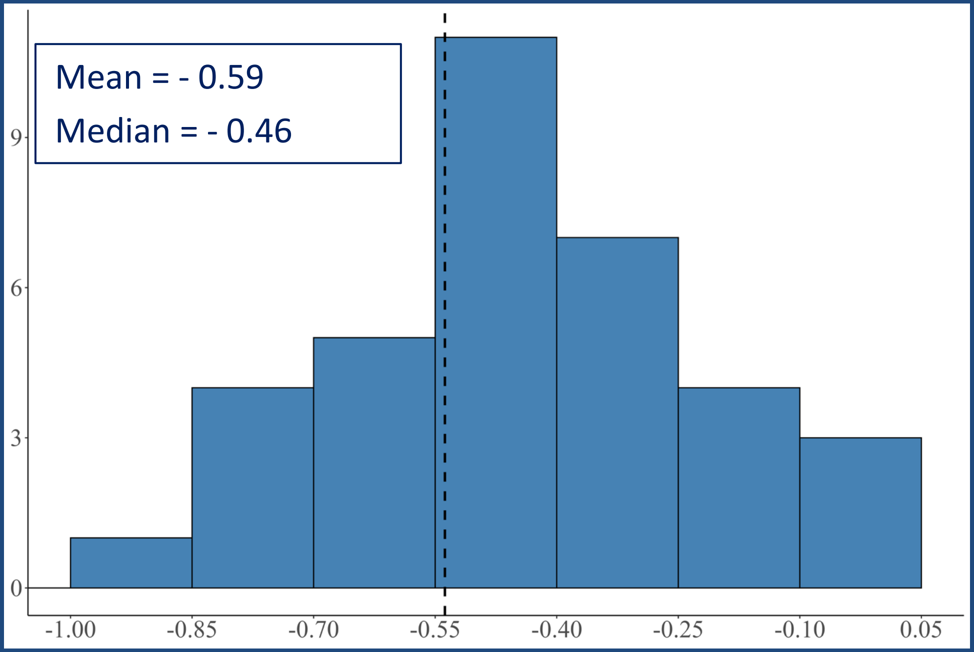
ϕ = constant coefficient (income flexibility)

Estimate with time-series data for 37 OECD countries separately with $n = 9$ goods

Obtain $\hat{\phi}_1, \cdots, \hat{\phi}_{37}$

Time-series estimates of income flexibility ϕ

- 37 ϕ-estimates, one for each of 37 OECD countries
- Some dispersion but not too far from -½



Source 2 of ϕ-values
Cross-commodity regressions

- Preference independence version of Rotterdam:

$$\bar{w}_{it}\,\Delta\log q_{it} = \theta_i\,\Delta\log Q_t + \phi\theta_i(\Delta\log p_{it} - \Delta\log P_t'), \qquad i = 1, \cdots, n$$

ϕ common to each equation

- Define

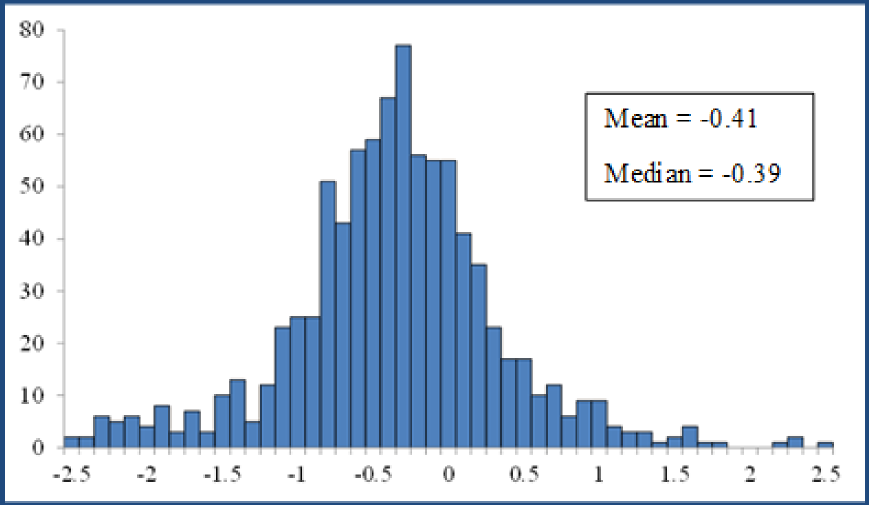
$$y_{it} = \bar{w}_{it}\,\Delta\log q_{it} - \theta_i\,\Delta\log Q_t\,, \qquad x_{it} = \theta_i(\Delta\log p_{it} - \Delta\log P_t')$$

- When θ_i known, estimate cross-commodity regression for each t:

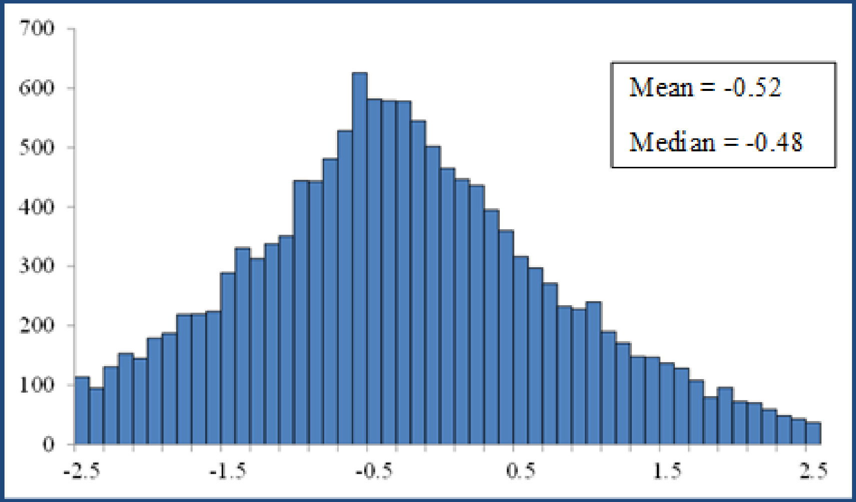
$$\boxed{y_{it} = \alpha_t + \phi_t x_{it}\,,\ \ i = 1, \cdots, 9}$$

to give $\hat{\phi}_1, \cdots, \hat{\phi}_T$

Cross-commodity estimates of ϕ, 37 countries, each year



Cross-commodity estimates of ϕ, 176 ICP countries, each pair



Centre of gravity not too far from -½, but note dispersion

Source 3 of ϕ-values
Previous studies

- Prior studies/surveys
- Mostly centred around -½
- But still not completely settled
- Details in Appendix

Uncertainty of ϕ

- All three sources suggest ϕ-estimates centred around $-½$
- Some genuine uncertainty, however
- Why?

- $\phi = \left(\frac{\partial \log \lambda}{\partial \log M}\right)^{-1}$, involves third-order derivative utility function:

  First-order -- MU of income, $\lambda = \frac{\partial u_I}{\partial M}$ $\quad (u_I = \text{indirect utility function})$

  Second-order -- Income elasticity of λ, $\frac{1}{\phi} = \frac{\partial \log \left(\frac{\partial u_I}{\partial M}\right)}{\partial \log M}$

  Third-order -- Change in elasticity, $\frac{\partial}{\partial \log M}\left(\frac{1}{\phi}\right) = \frac{\partial}{\partial \log M}\left\{\frac{\partial \log \left(\frac{\partial u_I}{\partial M}\right)}{\partial \log M}\right\}$

- Higher-order effects difficult to pin down

Possible objections

Rule of ½

1. $\eta_{ii}^{*} \approx -½$ based on unity income elasticities

   No necessities, no luxuries

   No Engel's law

2. Preference independence

   Unlikely for micro goods

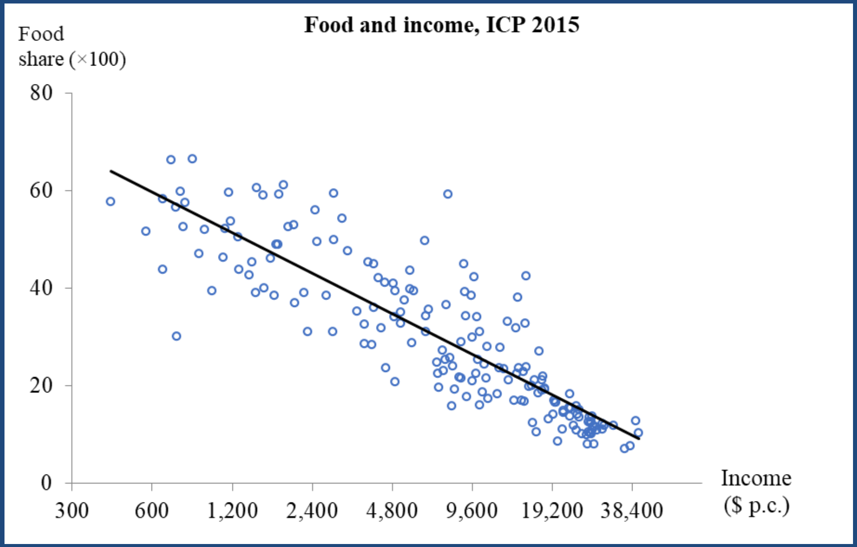
Objection 1:
No necessities, no luxuries

Engel's law

- Food share falls with income
- Strong empirical regularity
- Food is a necessity, income inelastic, $\eta_i < 1$

Rule of ½

- $\eta_{ii}^* \approx -½$ based on $\eta_i = 1$
- Violates Engel's law
- Cobb-Douglas utility, $u(q_1, \cdots, q_n) = \sum_{i=1}^{n} \beta_i \log q_i$
- Goods all neutral
- Constant budget shares
- That's bad



Response 1: Allowing for necessities & luxuries

Income elasticities

- From the budget constraint

$$\sum_{i=1}^{n} w_i \eta_i = 1$$

- If $\eta_i < 1,\ i = \text{food}$, then some other $\eta_i > 1$

Non-homotheticity

- Replace

$$\eta_{ii}^{*} \approx -\frac{1}{2} \quad \text{with} \quad \eta_{ii}^{*} \approx -\frac{1}{2}\,\eta_i$$

$$|\eta_{ii}^{*}| = \begin{cases} > \dfrac{1}{2}, & \text{if } \ \eta_i > 1, \text{luxury} \\ < \dfrac{1}{2}, & \text{if } \ \eta_i < 1, \text{necessity} \end{cases}$$

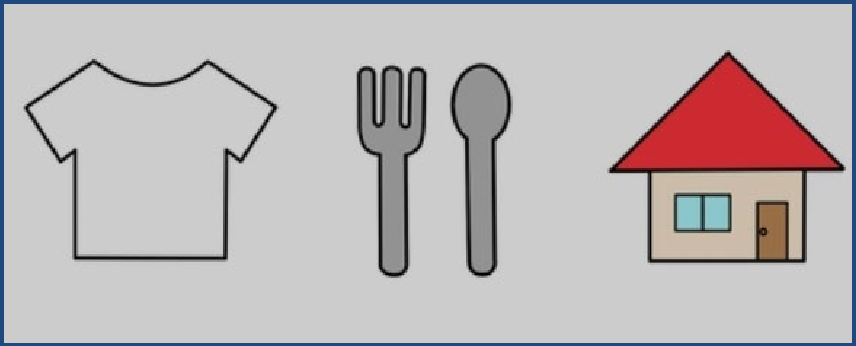
- Luxuries more price elastic than necessities
- Pigou's law

Objection 2:
Micro goods not preference independent

- Food, clothing house – broad aggregates
- Little substitutability
- Preference independent probably ok

$$u(q_1, \cdots, q_n) = \sum_{i=1}^{n} u_i(q_i)$$

- But not for more micro products such as
  - Individual food items
  - Individual brands of same good



Response 2:
Aggregate micro goods into groups

- Divide the n micro goods into $G < n$ groups of goods, $\boldsymbol{q}_1, \cdots, \boldsymbol{q}_G$

$$u(q_1, \cdots, q_n) = \sum_{g=1}^{G} u_g(\boldsymbol{q}_g), \qquad u_g(\cdot) = \text{subutility function for group } g$$

- Preference independence applied to groups. A.k.a. block-independent preferences
- Under block independence,

$$\eta_{ii}^* \approx \phi \eta_i, \qquad i = 1, \cdots, n, \qquad \text{becomes} \qquad \mathrm{N}_{gg}^* \approx \phi \mathrm{N}_g, \qquad g = 1, \cdots, G$$

Here,

$\mathrm{N}_{gg}^* = \sum_{i \in \boldsymbol{s}_g} \sum_{j \in \boldsymbol{s}_g} \frac{w_i}{W_g} \eta_{ij}^* =$ price elasticity of g $\left(W_g = \sum_{i \in \boldsymbol{s}_g} w_i \,; \boldsymbol{S}_g = \text{goods in set } g\right)$

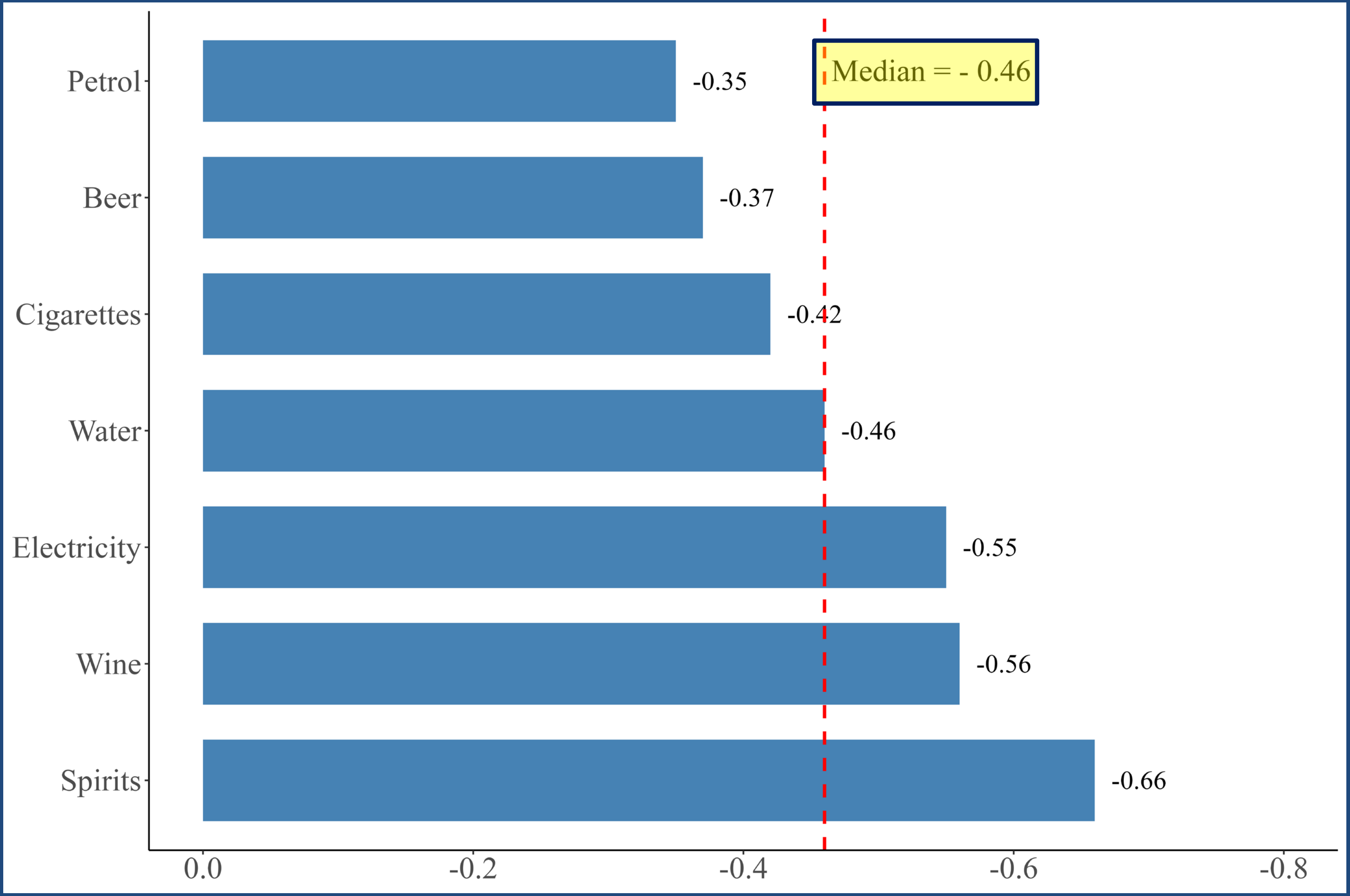
$\mathrm{N}_g = \sum_{i \in \boldsymbol{s}_g} \frac{w_i}{W_g} \eta_i =$ income elasticity of g

- Groupwise version of previous results

Direct evidence

- So much for theory and indirect evidence
- What about actual price elasticities?
- Are they $\approx -\frac{1}{2}$?

Price elasticities, broad aggregates



Sources of elasticities

Beer, wine and spirits

Clements, Mariano, Verikios and Wong (2022). "How Elastic is Alcohol Consumption?" Economic Analysis and Policy

Elder, Lawrence, Ferguson, Naimi, Brewer, Chattopadhyay, Toomey and Fielding (2010). "The Effectiveness of Tax Policy Interventions for Reducing Excessive Alcohol Consumption and Related Harms." American Journal of Preventive Medicine

Fogarty (2010). "The Demand for Beer, Wine and Spirits: A Survey of the Literature." Journal of Economic Surveys

Gallet (2007). "The Demand for Alcohol: A Meta-Analysis of Elasticities." Australian Journal of Agricultural and Resource Economics

Nelson (2013a). "Robust Demand Elasticities for Wine and Distilled Spirits: Meta-Analysis with Corrections for Outliers and Publication Bias." Journal of Wine Economics

Nelson (2013b). "Meta-Analysis of Alcohol Price and Income Elasticities – with Corrections for Publication Bias." Health Economics Review

Nelson (2014). "Estimating the Price Elasticity of Beer: Meta-Analysis of Data with Heterogeneity, Dependence, and Publication Bias." Journal of Health Economics

Selvanathan and Selvanathan (2005). The Demand for Alcohol, Tobacco and Marijuana: International Evidence

Sornpaisarn, Shield, Cohen, Schwartz and Rehm (2013). "Elasticity of Alcohol Consumption, Alcohol-Related Harms, and Drinking Initiation in Low- and Middle-Income Countries: A Systematic Review and Meta-Analysis." International Journal of Alcohol and Drug Research

Wagenaar, Salois and Komro (2009). "Effects of Beverage Alcohol Price and Tax Levels on Drinking: A Meta-Analysis of 1003 Estimates from 112 Studies." Addiction

Water

Dalhuisen, Florax, de Groot, and Nijkamp (2003). "Price and Income Elasticities of Residential Water Demand: A Meta-Analysis." Land Economics

Espey, Espey and Shaw (1997). "Price Elasticity of Residential Demand for Water: A Meta-Analysis." Water Resources Research

Cigarettes

Gallet and List (2003). "Cigarette Demand: A Meta-Analysis of Elasticities." Health Economics

Petrol

Espey (1996). "Explaining the Variation in Elasticity Estimates of Gasoline Demand in the United States: A Meta-Analysis." Energy Journal

Espey (1998). "Gasoline Demand Revisited: An International Meta-Analysis of Elasticities." Energy Economics

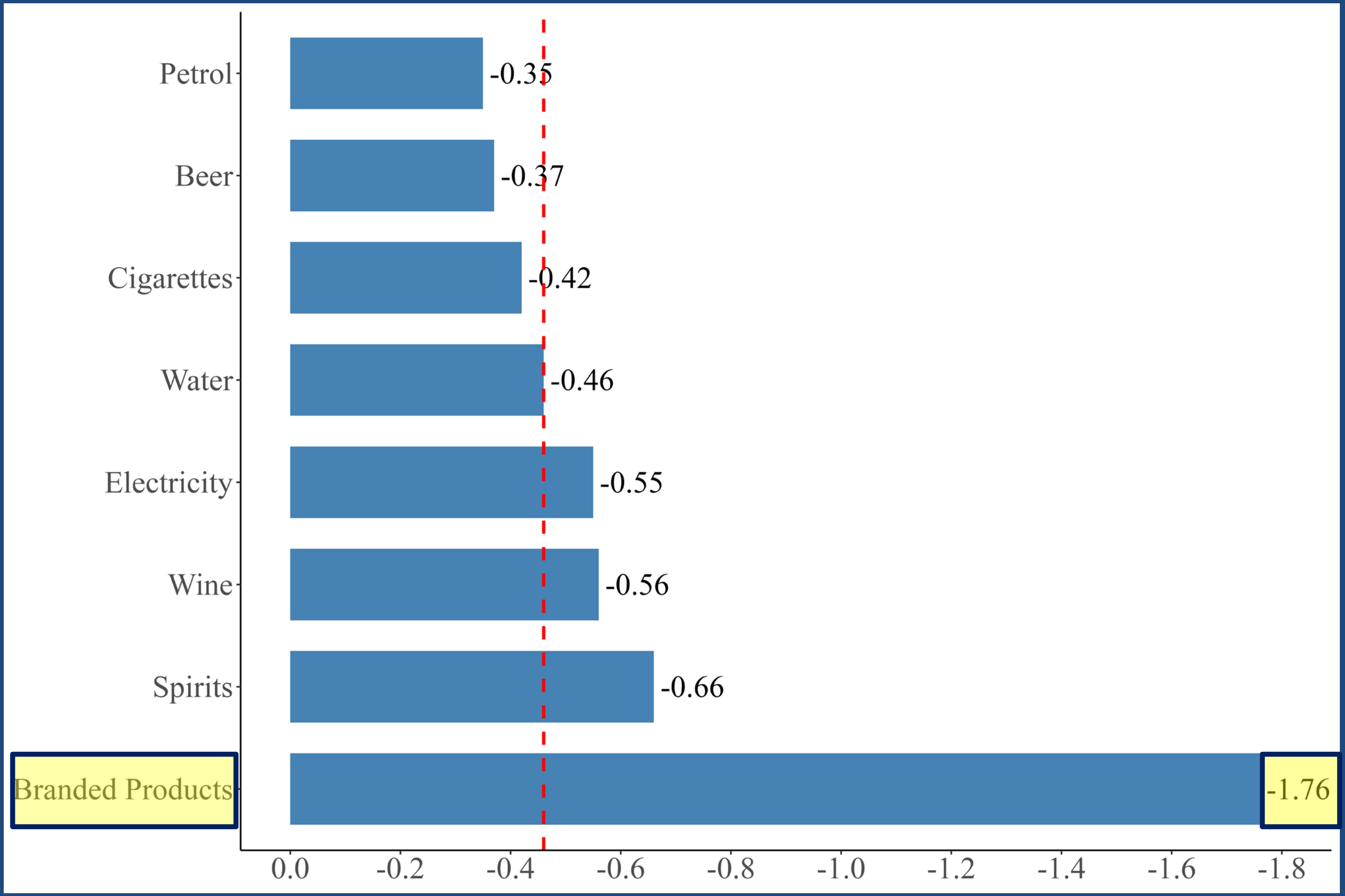
Goodwin, Dargay and Hanly (2004). "Elasticities of Road Traffic and Fuel Consumption with Respect to Price and Income: A Review." Transport Reviews

Graham and Glaister (2002). "Review of Income and Price Elasticities of Demand for Road Traffic." Working Paper, Imperial College

Electricity

Espey and Espey (2004). "Turning on the Lights: A Meta-Analysis of Residential Electricity Demand Elasticities." Journal of Agricultural and Applied Economics

Micro products more elastic



Source: Tellis (1988). "The Price Elasticity of Selected Demand: A Meta-analysis of Econometric Models of Sales." Journal of Marketing Research

Wrap up, rule of ½

- **Idiot's law of elasticities** (Dawkins, Srinivasan and Whalley, 2001)
- "All elasticities are 1 until someone shows them to be otherwise, or 'coffee-table elasticities' where informal discussions and opinions around the coffee table determine whether a value of, say, ½ or 2 is chosen."

- ½ has more than a coffee-table, let alone an idiot's, pedigree

- Qualifications to rule ½:

  (i) Applicable to broad aggregates only

  (ii) ½ a guideline, not a law

  (iii) Based on approximations

"Approximations are the soul of science" (von Neumann)

Linear expenditure system

$$p_i q_i = p_i \gamma_i + \beta_i \left(M - \sum_{j=1}^{n} p_j \gamma_j \right), \qquad i = 1, \cdots, n, \qquad M = \sum_{i=1}^{n} p_i \, q_i$$

Beautifully simple model

1. Linear in variables. Mildly nonlinear in parameters
2. Clear interpretation
3. Parsimony of parameters, *n* marginal shares, $\beta_1, \cdots, \beta_n$, and *n* subsistence parameters, $\gamma_1, \cdots, \gamma_n$
4. Consistent with Stone-Geary utility maximisation

$$u(q_1, \cdots, q_n) = \sum_{i=1}^{n} \beta_i \log(q_i - \gamma_i)$$

Under-appreciated problem: LES and food

LES Engel curve

$$p_i q_i = \alpha_i + \beta_i M,$$

$$\alpha_i = \sum_{j=1}^{n} (\delta_{ij} - \beta_i)\, p_j \gamma_j, \quad \delta_{ij} = \text{kronecker}, \;\; \beta_i = \text{marginal share}$$ (a constant!)

Income elasticity: $\eta_i = \frac{\beta_i}{w_i}, \qquad d \log \eta_i = -d \log w_i$

$$\log \eta_{F,Rich} - \log \eta_{F,Poor} = -(\log w_{F,Rich} - \log w_{F,Poor})$$

$$\eta_{F,Rich} = \frac{\eta_{F,Poor}}{(w_{F,Rich}/w_{F,Poor})}$$

Numerical example:

Country	Budget share
Poor	$w_{F,Poor} = 50\%$
Rich	$w_{F,Rich} = 10\%$

Under-appreciated problem: LES and food

LES Engel curve

$$p_i q_i = \alpha_i + \beta_i M,$$

$$\alpha_i = \sum_{j=1}^{n} (\delta_{ij} - \beta_i)\, p_j \gamma_j, \quad \delta_{ij} = \text{kronecker}, \;\; \beta_i = \text{marginal share}$$ (a constant!)

Income elasticity: $\eta_i = \frac{\beta_i}{w_i}$, $\quad d \log \eta_i = -d \log w_i$

$$\log \eta_{F,Rich} - \log \eta_{F,Poor} = -(\log w_{F,Rich} - \log w_{F,Poor})$$

$$\eta_{F,Rich} = \frac{\eta_{F,Poor}}{(w_{F,Rich}/w_{F,Poor})}$$

Numerical example:

Country	Budget share	Income elasticity
Poor	$w_{F,Poor} = 50\%$	$\eta_{F,Poor} = 0.3$
Rich	$w_{F,Rich} = 10\%$	$\eta_{F,Rich} = \frac{\eta_{F,Poor}}{(w_{F,Rich}/w_{F,Poor})} = \frac{0.3}{10/50} = 1.5$

Food luxury for rich!

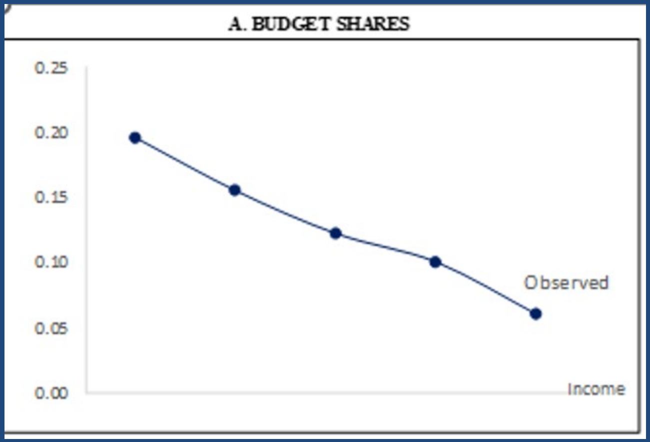
Warning
LES income elasticities

- LES income elasticity: $\eta_i = \frac{\beta_i}{w_i}$
- $d \log \eta_i = -d \log w_i = -(\eta_i - 1)\, d \log M$
- $d \log \eta_i = \begin{cases} > 0 & \text{if} \quad \eta_i < 1, \text{necessity} \\ < 0 & \text{if} \quad \eta_i > 1, \quad \text{luxury} \end{cases}$
- Income elasticities inversely proportional to budget shares
- Problematic for food when there are large changes in income
- Villain: Constant marginal shares, β_i's
- Warning applies to Rotterdam as well as LES
- More on LES in Appendix

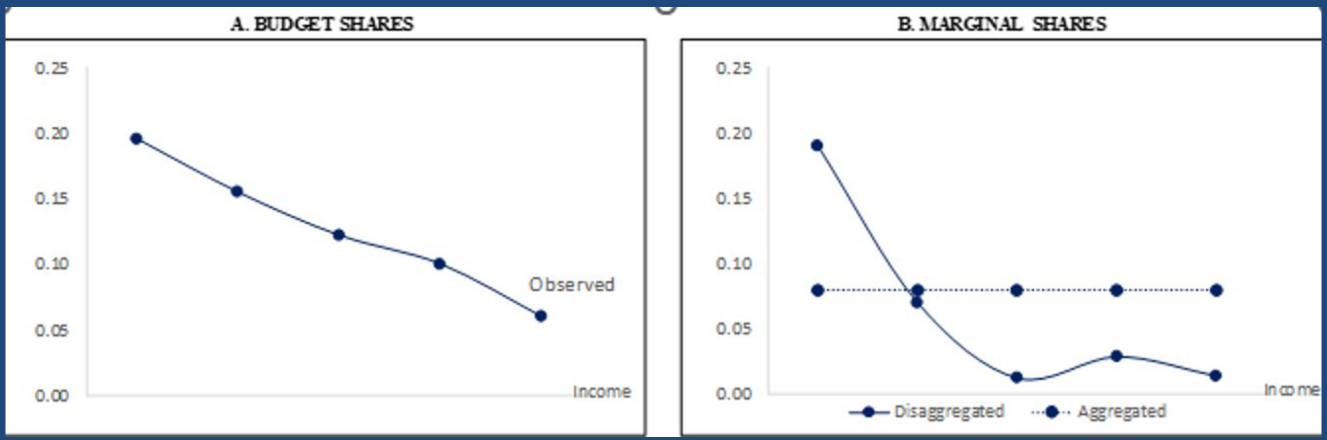
One solution:
Piecewise LES

- Piecewise linear approximation to underlying nonlinear Engel curves
- Estimate LES for each income quintile separately
- Data: Australian Household Expenditure Survey 2015-16. Representative sample of 10,000 households

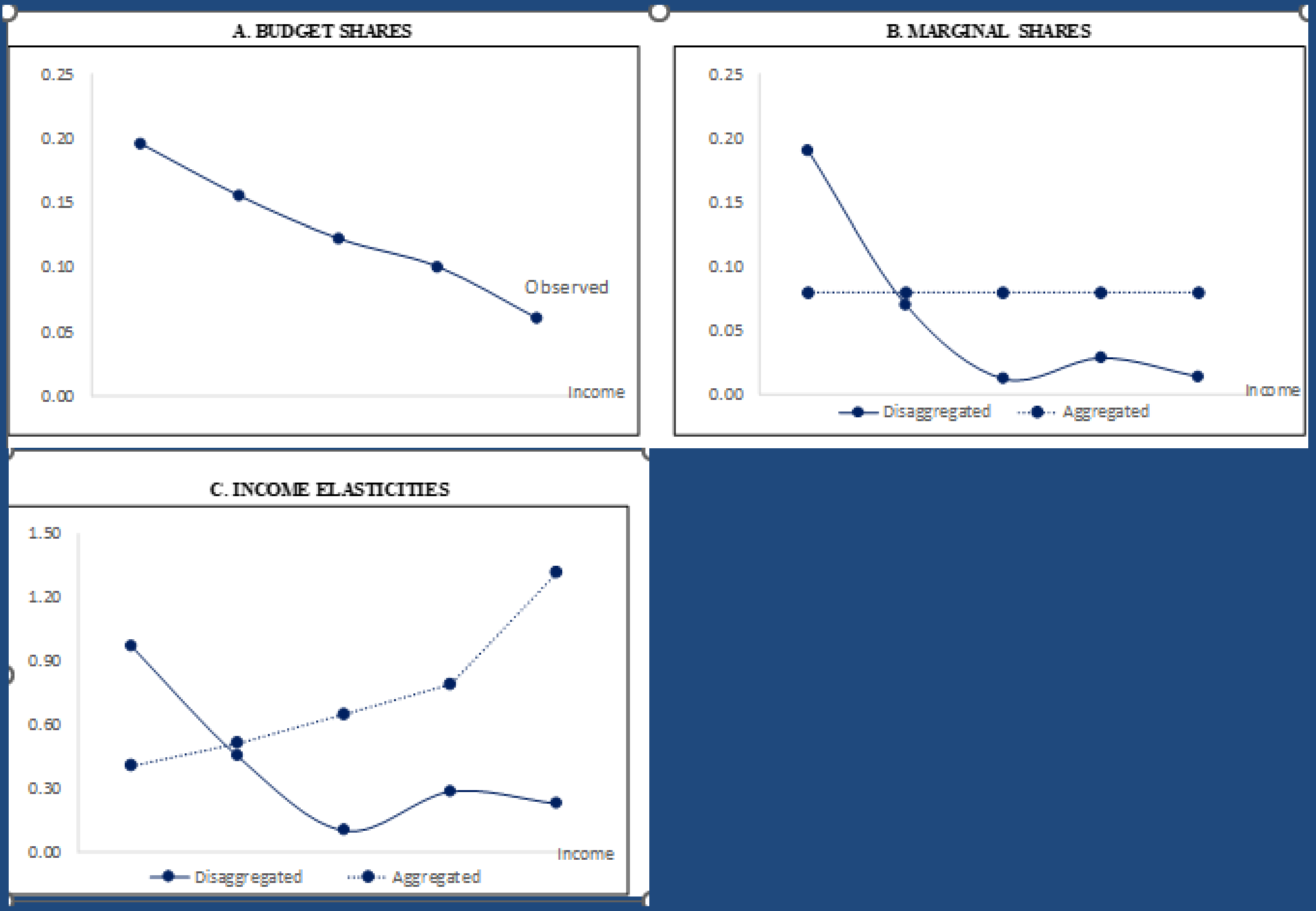
Food and income quintile LES, Australia HES



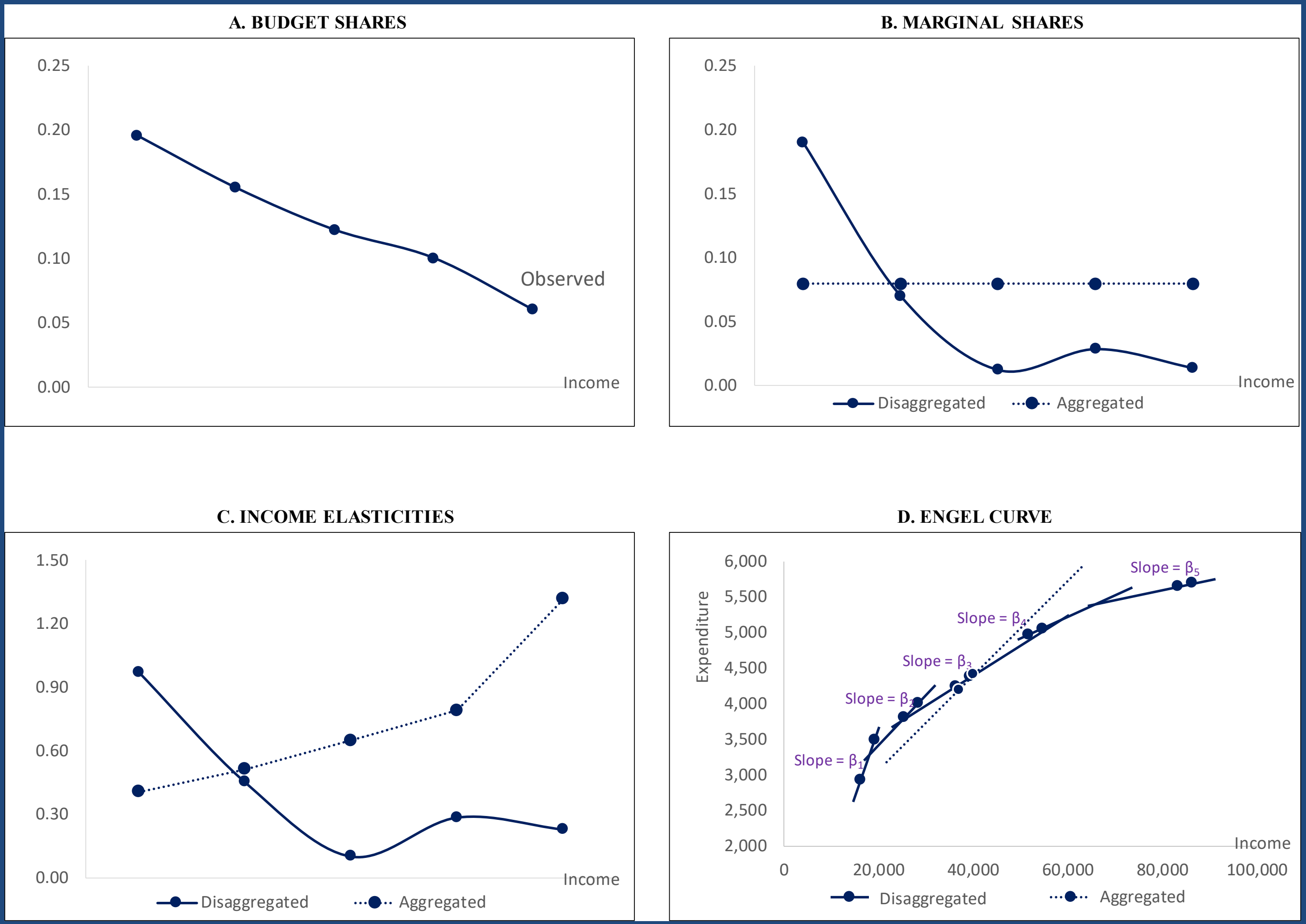
Food and income quintile LES, Australia HES



Food and income quintile LES, Australia HES



Food and income quintile LES, Australia HES



CGE applications

1. Multiple consumers with piecewise LES
2. Estimate σ for intermediate inputs
3. Estimate CES demand for foreign and domestic varieties

Substantial impact on results compared to standard default values

Details in our book

Thanks for your attention

References

Below is a list of references for the presentation, as well useful sources on applied demand analysis.

Barten, A. P. (1964). "Consumer Demand Functions under Conditions of Almost Additive Preferences." Econometrica 32: 1-38.

Bewley, R. A. (1986). Allocation Models: Specification, Estimation and Applications. Ballinger.

Baumol, W. J. (1952). "The Transactions Demand for Cash: An Inventory Theoretic Approach." Quarterly Journal of Economics 66: 545-56.

Brown, A., and A. Deaton (1972). "Surveys in Applied Economics: Models of Consumer Behaviour." Economic Journal 82: 1145-236.

Clements, K. W. (2008). "Price Elasticities of Demand are Minus One-Half." Economics Letters 99: 490-93.

Clements, K. W., H. Liu, J. M. Mariano, E. A. Selvanathan, S. Selvanathan and G. Verikios (2025). Short-cut Demand Elasticities and other Convenient Approaches to Consumer Demand. Springer.

Clements, K. W., and J. Si (2017). "Notes on the Pattern of OECD Consumption." Unpublished working paper, UWA Business School.

Clements, K. W., and J. Si (2018). "Divisia and Frisch are Friends." UWA Economics Discussion Paper 18-05.

Clements, K. W., J. Si, E. A. Selvanathan and S. Selvanathan (2020). "Demand Elasticities for 9 Goods in 37 Countries." Applied Economics 52: 2636-55.

Dawkins, C., T. Srinivasan and J. Whalley (2001). "Calibration." In J. J. Heckman and E. Leamer (Eds.), Handbook of Econometrics, vol. 5. Elsevier.

Deaton, A. S. (1974). "A Reconsideration of the Empirical Implications of Additive Preferences." Economic Journal 84: 338-48.

References (cont'd)

Deaton, A. S., and J. Muellbauer (1980). Economics and Consumer Behaviour. Cambridge University Press.

Friedman, M. (1935). "Professor Pigou's Method for Measuring Elasticities of Demand from Budgetary Data." Quarterly Journal of Economics 50: 151-63.

Frisch, R. (1959). "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors." Econometrica 27: 177-96.

Frisch, R. (1932). New Methods of Measuring Marginal Utility. J. C. B. Mohr.

Goldberger, A. S. (1987). Functional Form and Utility: A Review of Consumer Demand Theory. Westview Press.

Gorman, W. M. (1959). "Separability Utility and Aggregation." Econometrica 27: 469-81.

Houthakker, H. S. (1957). "An International Comparison of Household Expenditure Patterns, Commemorating the Centenary of Engel's Law." Econometrica 25: 532-51.

Houthakker, H. S. (1960). "Additive Preferences." Econometrica 28: 244-57.

Lluch, C., A. A. Powell and R. A. Williams (1977). Patterns in Household Demand and Savings. Oxford University Press.

Muhammad, A. (2025). "Trump Tariffs 2.0: Assessing the Impacts on US Distilled Spirits Imports." Agribusiness: 1-7.

Pigou, A. C. (1910). "A Method of Determining the Numerical Value of Demand Elasticities." Economic Journal 20: 636-40.

References (cont'd)

Pollak, R. A., and T. J. Wales (1992). Demand System Specification and Estimation. Oxford University Press.

Powell, A. A. (1974). Empirical Analytics of Demand Systems. DC Heath.

Selvanathan, S. (1993). A System-Wide Analysis of International Consumption Patterns. Kluwer.

Stone, R. (1954). "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand." Economic Journal 64: 511-27.

Tellis, G. J. (1988). "The Price Elasticity of Selected Demand: A Meta-analysis of Econometric Models of Sales." Journal of Marketing Research 25: 331–41.

Theil, H. (1965). "The Information Approach to Demand Analysis." Econometrica 33: 67-87.

Theil, H. (1975/76). Theory and Measurement of Consumer Demand. Two Volumes. North-Holland.

Theil, H. (1980). The System-Wide Approach to Microeconomics. The University of Chicago Press.

Tobin, J. (1956). "The Interest Elasticity of the Transactions Demand for Cash." Review of Economics and Statistics 38: 241-47.

Appendix

This contains elaborations of the material in the main slides and some further results. Optional reading!

Glossary and key results, demand analysis and ½

Symbol/Result	Meaning
q_i	Quantity consumed of good i, $i = 1, \dots, n$
p_i	Price of good i
$u(q_1, \cdots, q_n)$	Utility function
$\frac{\partial u}{\partial q_i}$	Marginal utility of good i
$\sum_{i=1}^{n} p_i q_i = M$	Budget constraint where M is income
$w_i = \frac{p_i q_i}{M}$	Budget share of i
$\eta_i = \frac{\partial \log q_i}{\partial \log M}$	Income elasticity of i
$\eta_{ij} = \frac{\partial \log q_i}{\partial \log p_j}$	Uncompensated elasticity of demand for i with respect to price of j, $i, j = 1, \dots, n$. Money income held constant.
$\eta_{ij}^* = \eta_{ij} + w_j \eta_i$	Compensated elasticity of demand for i with respect to price of j, $i, j = 1, \dots, n$. Real income held constant.
$\lambda = \frac{\partial u_I}{\partial M}$	Marginal utility of income with $u_I(M, \boldsymbol{p})$ indirect utility function
$q_i = q_i(M, p_1, \cdots, p_n)$	Demand function for i
$\phi = \left(\frac{\partial \log \lambda}{\partial \log M}\right)^{-1} < 0$	Income flexibility, the reciprocal of the income elasticity of the marginal utility of income
$\eta_{ij} = \phi \eta_i (\delta_{ij} - w_j \eta_j) - \eta_i w_j$	Uncompensated elasticity of demand for i with respect to price of j. Holds under independent marginal utilities, $\frac{\partial(\partial u / \partial q_i)}{\partial q_j} = 0, i \neq j$.
$\eta_{ii}^* \approx \phi \eta_i$	As an approximation, compensated own-price elasticity (η_{ii}^*) a multiple ϕ of corresponding income elasticity. Holds under independent marginal utilities.
$u(q_1, \cdots, q_n) = \sum_{i=1}^{n} u_i(q_i)$	Preference independent utility function with $u_i(\cdot)$ the sub-utility function i depending on consumption of good i only

Glossary and key results, LES

Symbol/Equation	Meaning/Name	Constraints
A. Notation		
q_i	Quantity consumed of good *i*, $i = 1, \ldots, n$	
p_i	Price of good i	
$M = \sum_{i=1}^{n} p_i q_i.$	Total expenditure, "income" for short	
$w_i = \frac{p_i q_i}{M}$	Budget share of *i*	$w_i > 0, \sum_{i=1}^{n} w_i = 1$
B. Expenditure System		
$p_i q_i = p_i \gamma_i + \beta_i \left(M - \sum_{j=1}^{n} p_j \gamma_j \right)$	LES, $i = 1, \ldots, n$	
$p_i \gamma_i$	Cost of subsistence of *i*	$p_i \gamma_i < p_i q_i$
$\beta_i = \frac{\partial (p_i q_i)}{\partial M}$	Marginal share of *i*	$\beta_i > 0 \sum_{i=1}^{n} \beta_i = 1$
$M - \sum_{j=1}^{n} p_j \gamma_j$	Supernumerary income	$M - \sum_{j=1}^{n} p_j \gamma_j > 0$

(Continued next page)

Glossary and key results, LES (cont'd)

Symbol/Equation	Meaning/Name	Constraints
C. Demand elasticities		
$\eta_i = \frac{\partial \log q_i}{\partial \log M} = \frac{\beta_i}{w_i}$	Income elasticity of *i*	$\eta_i > 0, \sum_{i=1}^{n} w_i \eta_i = 1$
$\eta_{ij} = \frac{\partial \log q_i}{\partial \log p_j} = \delta_{ij}\left(\frac{s_i}{w_i} - 1\right) - \frac{\beta_i}{w_i} s_j$	Uncompensated elasticity of demand for i with respect to price of j, $i, j = 1, \ldots, n$, with $s_i = \frac{p_i \gamma_i}{M}$ the subsistence share of *i*. Money income constant	$\eta_i + \sum_{j=1}^{n} \eta_{ij} = 0$ Homogeneity
$\eta_{ij}^* = \frac{\partial \log q_i}{\partial \log p_j} = \delta_{ij}\left(\frac{s_i}{w_i} - 1\right) + \frac{\beta_i}{w_i}(w_j - s_j)$	$(i,j)^{th}$ compensated price elasticity. Real income constant	$\sum_{j=1}^{n} \eta_{ij}^* = 0$ Homogeneity; $w_i \eta_{ij}^* = w_j \eta_{ij}^*$ Slutsky symmetry
D. Uncompensated price elasticities with positive gammas		
η_{ii}	Uncompensated own-price elasticity	$\eta_{ii} = \frac{p_i \gamma_i}{p_i q_i}(1 - \beta_i) - 1$; $-1 < \eta_{ii} < 0$, price inelastic
$\eta_{ij}, i \neq j$	Uncompensated cross-price elasticity	$\eta_{ij} = -\beta_i \frac{p_j \gamma_j}{p_i q_i} < 0$; *i, j* gross complements

(Continued next page)

Glossary and key results, LES (cont'd)

Symbol/Equation	Meaning/Name	Constraints
E. Compensated price elasticities with positive gammas		
η_{ii}^*	Compensated own-price elasticity	$\eta_{ii}^* = \eta_{ii} + \beta_i$ $\eta_{ii}^* > \eta_{ii} < -1$ η_{ii}^* not unambiguously < -1
$\eta_{ij}^*, i \neq j$	Compensated cross-price elasticity	$\eta_{ij}^* = \eta_{ij} + \frac{\beta_i}{w_i} w_j$ $\eta_{ij}^* > \eta_{ij} > 0$ *i, j* net substitutes
F. Utility		
$u(q_1, \cdots, q_n) = \sum_{i=1}^{n} \beta_i \log(q_i - \gamma_i)$	Stone-Geary utility	$0 < \beta_i < 1, \sum_{i=1}^{n} \beta_i = 1, \gamma_i < q_i$
$\lambda = \frac{1}{M - \sum_{i=1}^{n} p_i \gamma_i}$	Marginal utility of income	$\lambda > 0$
$\omega = \frac{\partial \log \lambda}{\partial \log M} = -\frac{M}{M - \sum_{i=1}^{n} p_i \gamma_i}$	Frisch parameter -- income elasticity of marginal utility of income	$\omega < -1$

Measurement of income flexibility ϕ

The starting point on the measurement of the income flexibility is Frisch's (1959) famous conjecture regarding its inverse, the income elasticity of the marginal utility, $\omega = \frac{1}{\phi}$, known as the Frisch parameter. Quoting Frisch, the conjecture is:

> We may, perhaps, assume that in most cases the [Frisch parameter] has values of the order of magnitude given below.
>
> $\omega = -10$ for the extremely poor and apathetic part of the population.
>
> $\omega = -4$ for the slightly better off but still poor part of the population with a fairly pronounced desire to become better off.
>
> $\omega = -2$ for the middle-income bracket, "the median part" of the population.
>
> $\omega = -0.7$ for the better off part of the population.
>
> $\omega = -0.1$ for the rich part of the population with ambitions towards "conspicuous consumption".

As ω falls in absolute value as income rises, its reciprocal $|\phi|$ increases

Evidence that $\phi \approx -½$

Obviously, Frisch views the income flexibility $\phi = \frac{1}{\omega}$ as varying across the income distribution. By contrast, ϕ is a constant parameter in the Rotterdam model. This model and its variants have been employed in a number of studies. Selvanathan (1993) uses time-series data for 15 OECD countries and when the data are pooled $\hat{\phi} = -0.45$. Then, using cross-commodity regressions, as a weighted mean she obtains $\hat{\phi} = -0.46$. Using data for 30 countries, Theil (1987) obtains an estimate of ϕ of –0.53.

Chen (1999) estimates a demand system for 42 countries to give $\hat{\phi} = -0.42$ when there are intercepts in his differential demand equations, which play the role of residual trends in consumption. However, when there are no intercepts, $\hat{\phi} = -0.29$, pointing to some uncertainty in the true value of the income flexibility

In a review paper, Brown and Deaton (1972) state "there would seem to be fair agreement on the use of a value for ϕ around minus one half".

Linear expenditure system

$$p_i q_i = p_i \gamma_i + \beta_i \left(M - \sum_{j=1}^{n} p_j \gamma_j \right), \qquad i = 1, \cdots, n, \qquad M = \sum_{i=1}^{n} p_i\, q_i$$

- $2n$ parameters: $\beta_1, \cdots, \beta_n$ and $\gamma_1, \cdots, \gamma_n$
- Get $\beta_i{}'s$ from Engel-curve regressions:

$$p_i q_i = \alpha_i + \beta_i M, \qquad \alpha_i = \sum_{j=1}^{n} (\delta_{ij} - \beta_i)\, p_j \gamma_j, \qquad \delta_{ij} = \text{kronecker}$$

- Identifying $\gamma_j{}'\text{s}$:

$$p_i \gamma_i = \alpha_i + \beta_i \left(1 + \frac{1}{\omega} \right) M$$

$$\omega = -\left(\frac{M}{M - \sum_{i=1}^{n} p_i q_i} \right) < 0, \qquad \text{"Frisch parameter"}$$

ω = income elasticity of MU of income $\left(\frac{1}{\omega} = \phi \right)$

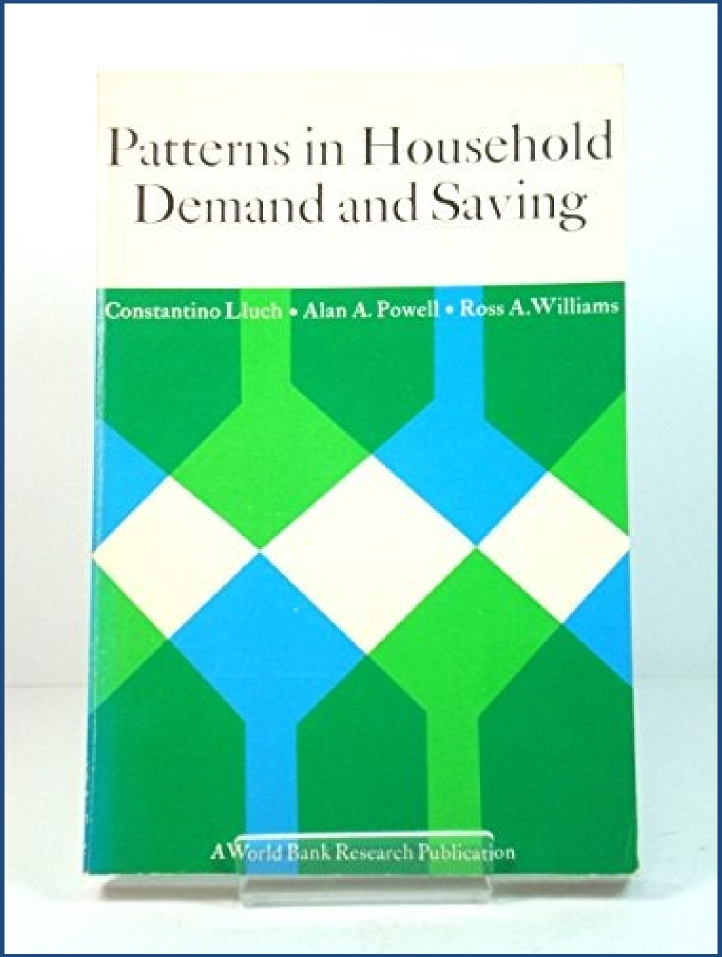
Lluch, Powell and Williams (1977), influential book

- Rich source of elasticities for many countries based on ELES/LES
- Elasticities used in CGE models
- Cross-country regression:

$$\log|\omega_c| = a + b\log Y_c + \varepsilon_c$$

$\hat{b} = -0.36$ and significant

- Supports “Frisch conjecture”
- But contradicts the previous conclusion $\phi = \frac{1}{\omega} = \text{constant} = -\frac{1}{2}$



LES and Frisch

In the linear expenditure system, the Frisch parameter takes the form

$$\omega = -\left(\frac{M}{M-\sum_{i=1}^{n} p_i\gamma_i}\right), \quad \sum_{i=1}^{n} p_i\gamma_i = \text{cost of substistence.}$$

If, for example, subsistence costs one-half of income, $\omega = -2$. The marginal utility of income declines twice as fast as income increases, which seems reasonable. Note that in LES $\omega < -1$.

As

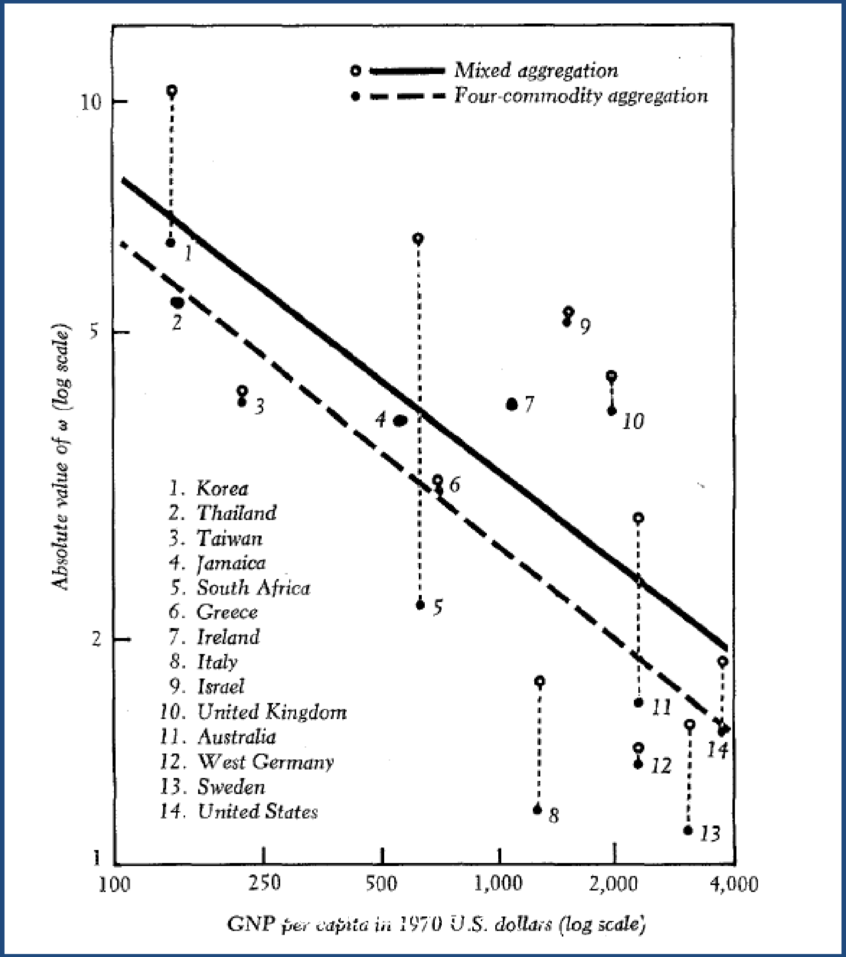
$$\frac{\partial(M/(M-\sum_{i=1}^{n} p_i\gamma_i))}{\partial M} < 0,$$

$|\omega|$ falls as income rises, in agreement with Frisch's conjecture. Thus, for a given LES – that is, for a given value of subsistence expenditure $\sum_{i=1}^{n} p_i\gamma_i$ -- the conjecture is built-in to LES.

Lluch et al. (1977) estimate ELES, a variant of LES involving savings, for 14 countries. Since ELES is estimated separately for each country, 14 independent estimates of ω are obtained, which can be used to test Frisch. The graph to the right reproduces their plot of $|\omega|$ against GDP per capita. As noted on previous slide, Lluch and co estimate

$$|\omega| = 36 \times \text{GDP}^{-0.36}\,.$$

Other studies extrapolate this equation to give an implied ω-value, from which price elasticities are derived.



Source: Lluch et al. (1977, p. 77).

Some weaknesses with LES

1. Utility function might be wrong,

$$u(q_1, \cdots, q_n) = \sum_{i=1}^{n} \beta_i \log(q_i - \gamma_i)$$

2. Additive utility -- broad groups only

3. No inferior goods, all normal $(\beta_i > 0)$

4. Pairwise substitutes only

Remember

“All models are imperfect, but some are useful” (George Box)

“Models are to be used, not believed” (Henri Theil)

Other estimates of ϕ

DeJanvry et al. (1972) and Gao (2012, p. 106) also find support for Frisch, conflicting with treating the income flexibility (the inverse of ω) as a constant.

Other sources of estimates ϕ, or its inverse, include Acland and Greenberg (2023), Evans (2005), Layard et al. (2008), Groom and Maddison (2019) and Hjertstrand (2025). See also Clements et al. (2022, online appendix) and Stern (1977).

In summary, there is still some uncertainty surrounding the validity of the Frisch conjecture and the actual value of ϕ. Related is the higher-order effect of the Frisch conjecture, mentioned previously, that makes precise estimation difficult.

CES and −½

- CES utility function:

$$u(q_1, \cdots, q_n) = \left(\sum_{i=1}^{n} \alpha_i q_i^{-\rho}\right)^{-\frac{1}{\rho}}, \sigma = \frac{1}{1+\rho} > 0$$

$$q_i = \alpha_i^{\sigma} \frac{M}{P^*} \left(\frac{p_i}{P^*}\right)^{-\sigma}, P^* = \left(\sum_{j=1}^{n} \alpha_j^{\sigma} p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \eta_{ii} = -\sigma(1 - w_i)$$

- Budget-share weighted average of $\eta_{ii} \in \left[-\sigma\left(1 - \frac{1}{n}\right), 0\right]$
- Average of the upper and lower limits is $-½\sigma\left(1 - \frac{1}{n}\right) \approx -½\sigma$
- "Average" price elasticity implied by CES $\approx -½\sigma$

½ and monetary economics

Money demand function:

$$m = f(y, i), \quad m = \text{real money balances};\ y = \text{income};\ i = \text{interest rate}$$

Baumol–Tobin model:

$$m = \sqrt{\frac{cy}{2i}},\ c = \text{constant} > 0$$

Interest elasticity:

$$\boxed{\frac{\partial(\log m)}{\partial(\log i)} = -½}$$

Price stabilisation and ½

Demand for a major exported commodity

$$\log q = k' + \eta \log p\,,\ \eta < 0,\ \log p = k + \tfrac{1}{\eta}\log q$$

Income

$$M = p \cdot q$$

$$\log M = \log p + \log q = \left(k + \tfrac{1}{\eta}\log q\right) + \log q = k + \left(1 + \tfrac{1}{\eta}\right)\log q$$

Dispersion

(1) $$\text{var}\,(\log M) = \left(1 + \tfrac{1}{\eta}\right)^2 \text{var}\,(\log q)$$

Price stabilisation, $\Delta(\log p) = 0$

From $\log M = \log p + \log q,$ when $\text{var}\,(\log p) = 0,$

(2) $$\text{var}\,(\log M) = \text{var}\,(\log q)$$

Change in dispersion of income, eq. (2) – (1)

$$\Delta\text{var}\,(\log M) = \text{var}\,(\log q) - \left(1 + \frac{1}{\eta}\right)^2 \text{var}\,(\log q)$$

$$= \left\{1 - \left(1 + \frac{1}{\eta}\right)^2\right\}\text{var}\,(\log q) = \begin{cases} > 0 \text{ if } |\eta| > \dfrac{1}{2} \\ < 0 \text{ if } |\eta| < \dfrac{1}{2} \end{cases}$$

Source: Newbery and Stiglitz (1981)

References for appendix

Acland, D., and D. H. Greenberg (2023). "The Elasticity of Marginal Utility of Income for Distributional Weighting and Social Discounting: A Meta-analysis." Journal of Benefit-Cost Analysis 14: 386-405.

Arrow, K. J., H. B. Chenery, B. S. Minhas and R. M. Solow (1961). "Capital-Labor Substitution and Economic Efficiency." Review of Economics and Statistics 43: 225-50.

Brown, A., and A. Deaton (1972). "Surveys in Applied Economics: Models of Consumer Behaviour." Economic Journal 82: 1145-236.

Chen, D. L. (1999). World Consumption Economics. World Scientific.

Clements, K. W., H. Liu, J. M. Mariano, E. A. Selvanathan, S. Selvanathan and G. Verikios (2025). Short-cut Demand Elasticities and other Convenient Approaches to Consumer Demand. Springer.

Clements K. W., M. J. Mariano and G. Verikios (2022). "Expenditure Patterns, Heterogeneity, and Long-term Structural Change." Economic Modelling. 113: 105888.

DeJanvry, A., J. Bieri and A. Nunez (1972). "Estimation of Demand Parameters Under Consumer Budgeting: An Application to Argentina." American Journal of Agricultural Economics 54: 422-30.

Evans, D. J. (2005). "The Elasticity of Marginal Utility of Consumption: Estimates for 20 OECD Countries." Fiscal Studies 26: 197-224.

Frisch, R. (1959). "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors." Econometrica 27: 177-96.

Gao, G. (2012). The Economic Behaviour of Seven Billion Consumers: Food, Functional Form and Quality. PhD thesis, The University of Western Australia.

References for appendix (cont'd)

Groom, B., and D. Maddison (2019). "New Estimates of the Elasticity of Marginal Utility for the UK." Environmental and Resource Economics 72: 1155-82.

Hjertstrand, P. (2025). "The Marginal Utility of Income and Homogeneous Demand Systems." Journal of Economic Behavior and Organization 229: 106853.

Layard, R., G., Mayraz and S. Nickell (2008). "The Marginal Utility of Income." Journal of Public Economics 92: 1846-57.

Newbery, D. M. G., and J. E. Stiglitz (1981). The Theory of Price Stabilisation: A Study in the Economics and Risk. Oxford University Press.

Selvanathan, S. (1993). A System-Wide Analysis of International Consumption Patterns. Kluwer.

Stern, N. (1977). "The Marginal Valuation of Income." In M. Artis and R. Norbay (eds), Studies in Modern Economic Analysis. Blackwell.

Theil, H. (1987). "Evidence from International Consumption Comparisons" Chapter 2 in H. Theil and K. W. Clements Applied Demand Analysis: Results from System-Wide Approaches. Ballinger.