HETEROGENEOUS FIRMS, HETEROGENEOUS MODELS

J. Peter Neary
University of Oxford, CEPR, and CESifo

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Models of heterogeneous firms: What have we learned? [Melitz (2003)]

1. Selection effects
2. Competition effects
3. Matching the size distribution of firms
4. Superstar firms
Background


Related Literature

1. **Selection effects:**

2. **Competition effects:**
   - ZKPT (2013), Bertoletti-Epifani (2014)
   - Alternatives to CES:
     - Bulow-Pfleiderer: Atkin-Donaldson (2012)
     - QMOR: Feenstra (2014)

3. **Matching the size distribution of firm sales:**
   - Mixture of thin- and fat-tailed Pareto: Edmonds et al. (2012)

4. **Superstar Firms in Oligopoly**
   - Neary (2010), Parenti (2013)
Outline

1. Selection Effects
2. Competition Effects
3. A Firm’s-Eye View of Demand
4. Matching the Size Distribution of Firms
5. Superstar Firms and Market Structure
6. Conclusion
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Which Firms Export?

- More productive firms select into exporting
- Very robust result: Not sensitive to CES
  - Requires only that ex post profits $\pi$ are decreasing in $c$
- Counter-examples can be explained in other ways: e.g., Lu (2011)

\[ \Pi^E = \pi(t,c) - f_E \]
Exports versus FDI

\[ \Pi^E = \pi(t,c) - f_E \]

\[ \Pi^F = \pi(0,c) - f_F \]

Which Firms Export and Which Engage in FDI?

\[
\begin{align*}
\Pi &= \Pi^F - f_F C^{1-\sigma} - f_E C^{1-\sigma} \\
\Pi^E &= \Pi^E - f_F C^{1-\sigma} - f_E C^{1-\sigma}
\end{align*}
\]
First- and Second-Order Selection Effects

- Second-order selection effects less robust
  - Only guaranteed if $\Pi^F$ is steeper than $\Pi^E \iff \pi_c(c, 0) < \pi_c(c, \tau)$
  - i.e. $\pi_c(c, \tau)$ supermodular in $\{c, \tau\} \iff \pi_{c\tau} > 0$
  - $\iff$ Elasticity of output with respect to marginal cost (MCEO) $> 1$
Selection into FDI by large firms requires $\text{MCEO} > 1$

- **CES:** $\text{MCEO} > 1$: 10% fall in $c \Rightarrow > 10\%$ rise in output
  - So more efficient firms have higher profits when they engage in FDI
- **Linear demands:** $\text{MCEO} < 1$ for larger firms: Reverse selection effects
- **“CEMR” demands:** $\text{MCEO} = 1 \Rightarrow$ No selection effects

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(a) CES

(b) Linear

(c) CEMR = 1
Outline

1. Selection Effects

2. Competition Effects
   - Two Kinds of Competition Effects
   - Globalization as a Two-Edged Sword

3. A Firm’s-Eye View of Demand

4. Matching the Size Distribution of Firms

5. Superstar Firms and Market Structure

6. Conclusion
Two Kinds of Competition Effects

- Competition Effects of Globalization:
  1. Squeeze on markups
  2. “Matthew Effect” on Profit Profile
     - “To those who have, more shall be given”

- Both occur IFF demand is subconvex:
  1. Squeeze on markups:
     - \( m \equiv \frac{p}{c} = \frac{\varepsilon(x)}{\varepsilon(x) - 1} \)
     - \( m \) increasing in \( x \) IFF \( \varepsilon \) is decreasing in \( x \)
     - \( \Rightarrow \) \{ Cross-Section: Larger firms have higher markups \}
       \{ Time Series: Globalization squeezes incumbents’ markups \}
  2. “Matthew Effect” on Profit Profile
Globalization as a Two-Edged Sword

Effects of globalization on every firm’s profits (including the threshold firm’s):

1. Direct impact: Market Expansion
   - Raises its profits $\Rightarrow$ Threshold productivity tends to $\uparrow$

2. Indirect impact: Competition
   - Raises all firms’ profits $\Rightarrow$ Increases competition
     $\Rightarrow$ Reduces profits of marginal firm
     $\Rightarrow$ Threshold productivity tends to $\downarrow$

3. The Matthew Effect with Subconvexity:
   \[ \hat{\pi}_i = \left(1 - \frac{\varepsilon_i}{\bar{\varepsilon}}\right) \hat{k} \]
   - The direct, market expansion, impact dominates for larger firms
   - The indirect, competition, impact dominates for smaller firms
   - The threshold firm ceases to be profitable and drops out
   - The average productivity of exporters rises
The Matthew Effect of Globalization

Rank firms by their productivity

Export profits are increasing in productivity

So: More productive firms select into exporting
The Matthew Effect of Globalization

- Large firms expand
- Smaller firms contract, some exit
- On average, exporters become more productive
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Perceived inverse demand function:

\[ p = p(x) \quad p' < 0 \]

Two key demand parameters:

1. **Slope/Elasticity:**
   \[ \varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0 \]

2. **Curvature/Convexity:**
   \[ \rho(x) \equiv -\frac{xp''(x)}{p'(x)} \]
For a monopoly firm:

- First-order condition:
  \[ p + xp' = c \geq 0 \quad \Rightarrow \quad \varepsilon \geq 1 \]

- Second-order condition:
  \[ 2p' + xp'' < 0 \quad \Rightarrow \quad \rho < 2 \]
In general, both $\epsilon$ and $\rho$ vary with sales.

Exception: CES/iso-elastic case:

- $p = \beta x^{-1/\sigma}$
- $\Rightarrow \epsilon = \sigma, \quad \rho = \frac{\sigma+1}{\sigma} > 1$
- $\Rightarrow \epsilon = \frac{1}{\rho-1}$

Cobb-Douglas: $\epsilon = 1, \rho = 2$; just on boundary of both FOC and SOC.
$p(x)$ is subconvex at $x^0$ IFF:
- $p(x)$ is less convex than a CES demand function with the same elasticity: $\rho > \frac{\epsilon + 1}{\epsilon}$
- $\varepsilon$ is decreasing in sales:
  - $\varepsilon x = \frac{\varepsilon}{x} [\rho - \frac{\epsilon + 1}{\epsilon}]$

“Globalization” $[x \downarrow]$ leads to competition effects $[\frac{p}{c} \downarrow]$
- Because the mark-up is decreasing in elasticity: $\frac{p}{c} = \frac{\varepsilon}{\varepsilon - 1} = 1 + \frac{1}{\varepsilon - 1}$
Lower-c firms choose FDI IF:
- $\pi(t, c)$ supermodular in \{t, c\}
- $\Leftrightarrow$ MCEO $> 1$
- $\Leftrightarrow \varepsilon + \rho > 3$

Clearly: SupC $\Rightarrow$ SupM
Can We Have Subconvexity and Supermodularity?

- Demand functions represented in \( \{\varepsilon, \rho\} \) space by their Demand Manifold
- Most common demand functions are:
  - Subconvex \( \Rightarrow \) Competition effects
  - Submodular for high output \( \Rightarrow \) Reverse selection effects
Can We Have Subconvexity and Supermodularity?

- Demand functions represented in \{\varepsilon, \rho\} space by their Demand Manifold
- Most common demand functions are:
  - Subconvex \Rightarrow Competition effects
  - Submodular for high output \Rightarrow Reverse selection effects
- Exception: AIDS/Translog
Matching the Size Distribution of Firms

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Recall: The Canonical Model

Pareto Productivities + CES Demands → Pareto Sales
Matching the Size Distribution of Firms

Back ing Out Demands

Pareto Productivities + CES Demands → Pareto Sales

Pareto Productivities → ? ← Pareto Sales
Which Demands are Consistent with Pareto?

Proposition: Any two imply the third
CREMR Demands

- “CREMR”: “Constant Revenue Elasticity of Marginal Revenue”
  - MR=MC  ⇒  ϕ = c^{-1} = (r')^{-1}
  - So: Constant elasticity of sales with respect to productivity
    \[ p(x) = \frac{\beta}{x} (x - \gamma) \frac{\sigma - 1}{\sigma}, \quad 1 < \sigma < \infty, \ x > \gamma \sigma, \ \beta > 0 \]

- CES a special case: \( \gamma = 0 \)  ⇒  \( p(x) = \beta x^{-1/\sigma} \)

- CREMR elasticity of demand: \( \varepsilon(x) = \frac{x - \gamma}{x - \gamma \sigma} \sigma \)

- “CREMR”:
  \[ \hat{r}' = -\frac{1}{\sigma - 1} \hat{r} \]

- \( r(x) \equiv xp(x), \ r'(x) = p(x) + xp'(x), \ \hat{r} \equiv d \log r = \frac{dx}{r} \ (r \neq 0) \)
CREMR Demands: Pros and Cons

\[ p(x) = \frac{\beta}{x} (x - \gamma)^{\frac{\sigma - 1}{\sigma}}, \quad 1 < \sigma < \infty, \quad x > \gamma \sigma, \quad \beta > 0 \]

Features of CREMR demands:

- A rich range of properties:
  - Nests CES: converges to CES as \( x \to \infty \)
  - Variable mark-ups: For any \( \gamma \neq 0 \)
  - Competition effects: “Subconvex” IFF \( \gamma > 0 \)
  - “Normal” selection effects: Profit function “supermodular” IFF \( \sigma \geq 2 \)

- Very different from standard demand functions
- Inconsistent with a choke price
- Utility function is analytic and can be simulated, but hard to work with
Matching the Size Distribution of Firms

CREMR Very Different from Other Demands

\[ p(x) = \frac{\beta}{x} (x - \gamma)^{\frac{\sigma - 1}{\sigma}} \]

\[ \Rightarrow \bar{\rho}(\varepsilon) = 2 - \frac{1}{\sigma - 1} \frac{(\varepsilon - 1)^2}{\varepsilon} \]

Compare CEMR Demand Manifolds
But, in practice, it is large firms that matter

- So far: Monopolistic competition
  - Firms heterogeneous in size ...
  - ... but qualitatively identical:
    - Infinitesimal: No market power
    - Probability of exit/death independent of productivity
- By contrast: Firms that dominate world trade are:
  - Super-large
  - Old
  - Multi-product
  - Multi-division
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Quantification is in vogue in international trade!

We have learnt so much from recent work on heterogeneous firms ...

- CES + Pareto a valuable and highly tractable workhorse
- But for many purposes we need alternatives
- Heterogeneous models: No single parametric functional form can capture all the features we would like

... And there is lots more to learn!

- Alternatives to CES?
- Alternatives to Pareto?
- Alternatives to monopolistic competition?
Thank you for listening. Comments welcome!

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Supermodularity and the MCEO

Sufficient condition for second-order selection effects:
- $\Pi^F$ is steeper than $\Pi^E$
- $\Leftrightarrow \pi_c(c, 0) < \pi_c(c, \tau)$
- $\Leftrightarrow \pi_{c\tau} > 0$

Necessary and sufficient condition for $\pi_{c\tau} > 0$:
- $\pi(c, \tau) = [p(x) - \tau c]x$, \quad x optimal
- $\Rightarrow \pi_c = -\tau x$
- $\Rightarrow \pi_{c\tau} = -x - \tau \frac{\partial x}{\partial \tau} = -x \left(1 + \frac{c}{x} \frac{\partial x}{\partial c}\right)$
- $\Rightarrow \pi_{c\tau} > 1 \iff -\frac{c}{x} \frac{\partial x}{\partial c} > 1$