# HETEROGENEOUS FIRMS, HETEROGENEOUS MODELS

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# Introduction

Models of heterogeneous firms: What have we learned?

[Melitz (2003)]

1. Selection effects
2. Competition effects
3. Matching the size distribution of firms
4. Superstar firms

# Background


MRÁZOVÁ, M., AND J. P. NEARY (2011): "Selection Effects with Heterogeneous Firms," Discussion Paper No. 588, Department of Economics, University of Oxford.

——— (2013): "Not So Demanding: Preference Structure, Firm Behavior, and Welfare," Discussion Paper No. 691, Department of Economics, University of Oxford.

MRÁZOVÁ, M., J. P. NEARY, AND M. PARENTI (2014): "Demand, Technology, and the Size Distribution of Firms," in preparation.

NEARY, J. P. (2010a): "International Trade in General Oligopolistic Competition," Working Paper, University of Oxford.

——— (2010b): "Two and a Half Theories of Trade," *The World Economy*, 33(1), 1–19.

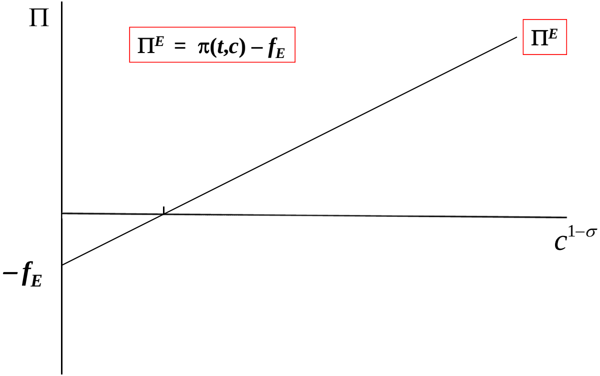
# Related Literature

1. Selection effects:
   - Bertoletti-Epifani (2014), Mrázová-Neary (2011), Bache-Laugesen (2013)
2. Competition effects:
   - ZKPT (2013), Bertoletti-Epifani (2014)
   - Alternatives to CES:
     - Quadratic preferences: Melitz-Ottaviano (2008)
     - Stone-Geary LES: Simonovska (2010)
     - Translog: Feenstra-Weinstein (2010)
     - Negative exponential/CARA: Behrens-Murata (2007)
     - Bulow-Pfleiderer: Atkin-Donaldson (2012)
     - QMOR: Feenstra (2014)
3. Matching the size distribution of firm sales:
   - Pareto: Helpman-Melitz-Yeaple (2004), Chaney (2006)
   - Mixture of thin- and fat-tailed Pareto: Edmonds et al. (2012)
   - Log-normal: Head-Mayer-Thoenig (2014), Bee-Schiavo (2014)
   - Piecewise log-normal-Pareto: Luttmer (2007), Eaton et al. (2011)
4. Superstar Firms in Oligopoly
   - Neary (2010), Parenti (2013)

# Outline

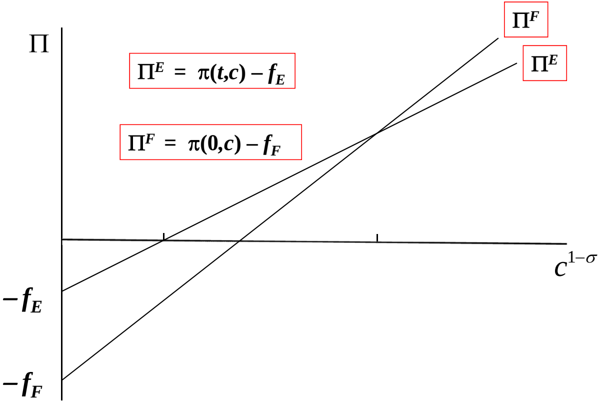
# Outline

# Which Firms Export?



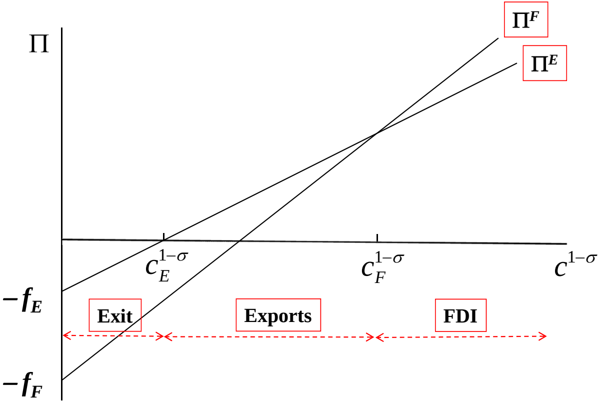

- More productive firms select into exporting
- Very robust result: Not sensitive to CES
  - Requires only that ex post profits $\pi$ are decreasing in $c$
- Counter-examples can be explained in other ways: e.g., Lu (2011)

# Exports versus FDI

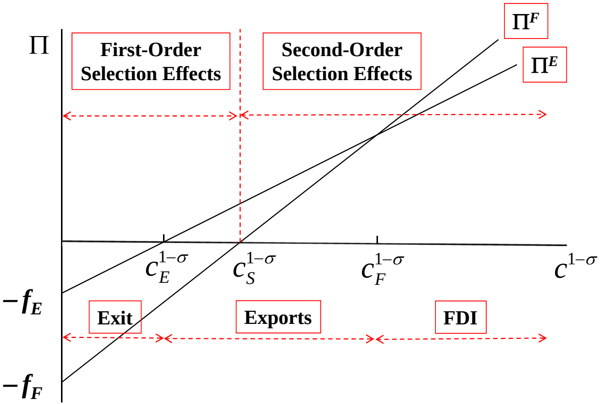



Helpman-Melitz-Yeaple (2004)

# Which Firms Export and Which Engage in FDI?

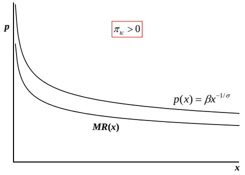


# First- and Second-Order Selection Effects

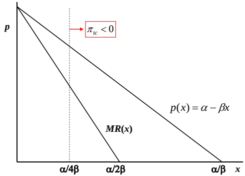



- Second-order selection effects less robust
  - Only guaranteed if $\Pi^F$ is steeper than $\Pi^E$ $\Leftrightarrow$ $\pi_c(c, 0) < \pi_c(c, \tau)$
  - i.e. $\pi_c(c, \tau)$ *supermodular* in $\{c, \tau\}$ $\Leftarrow$ $\pi_{c\tau} > 0$
  - $\Leftrightarrow$ Elasticity of output with respect to marginal cost (MCEO) $> 1$

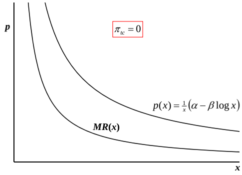
# Second-Order Selection Effects and Demand




(a) CES




(b) Linear




(c) CEMR $= 1$

- Selection into FDI by large firms requires MCEO $> 1$
  - CES: MCEO $> 1$: 10% fall in $c \Rightarrow > 10\%$ rise in output
    - So more efficient firms have higher profits when they engage in FDI
  - Linear demands: MCEO $< 1$ for larger firms: Reverse selection effects
  - "CEMR" demands: MCEO $= 1 \Rightarrow$ No selection effects

General Condition | Illustration

# Outline

# Two Kinds of Competition Effects

- Competition Effects of Globalization:
  1. Squeeze on markups
  2. "Matthew Effect" on Profit Profile
     - "To those who have, more shall be given"
- Both occur IFF demand is *subconvex*:
  1. Squeeze on markups:
     - $m \equiv \frac{p}{c} = \frac{\varepsilon(x)}{\varepsilon(x)-1}$
     - $m$ increasing in $x$ IFF $\varepsilon$ is decreasing in $x$
     - $\Rightarrow$ $\begin{cases} \text{Cross-Section: Larger firms have higher markups} \\ \text{Time Series: Globalization squeezes incumbents' markups} \end{cases}$
  2. "Matthew Effect" on Profit Profile

# Globalization as a Two-Edged Sword

Effects of globalization on every firm's profits (including the threshold firm's):

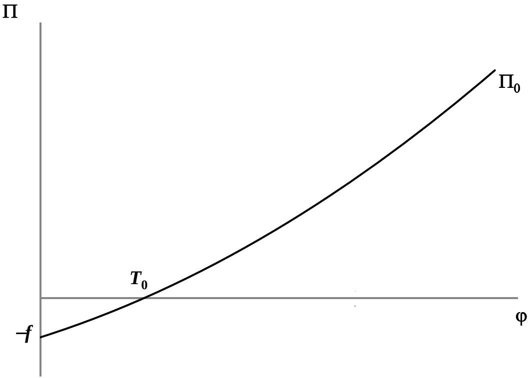
1. Direct impact: Market Expansion
   - Raises its profits $\Rightarrow$ Threshold productivity tends to $\uparrow$
2. Indirect impact: Competition
   - Raises *all* firms' profits $\Rightarrow$ Increases competition
     $\Rightarrow$ Reduces profits of marginal firm
     $\Rightarrow$ Threshold productivity tends to $\downarrow$
3. The Matthew Effect with Subconvexity:

$$\hat{\pi}_i = \left(1 - \frac{\varepsilon_i}{\bar{\varepsilon}}\right)\hat{k}$$

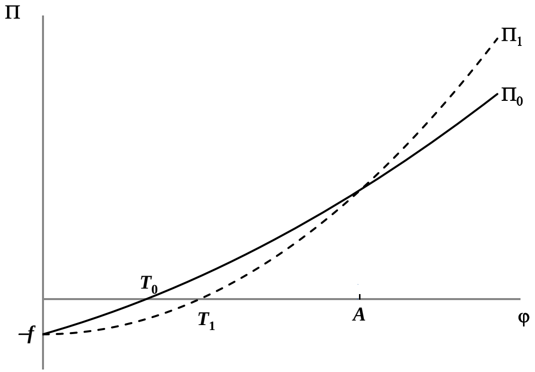
   - The direct, market expansion, impact dominates for larger firms
   - The indirect, competition, impact dominates for smaller firms
   - The threshold firm ceases to be profitable and drops out
   - The average productivity of exporters rises

Details

# The Matthew Effect of Globalization



# The Matthew Effect of Globalization




- Large firms expand
- Smaller firms contract, some exit
- On average, exporters become more productive

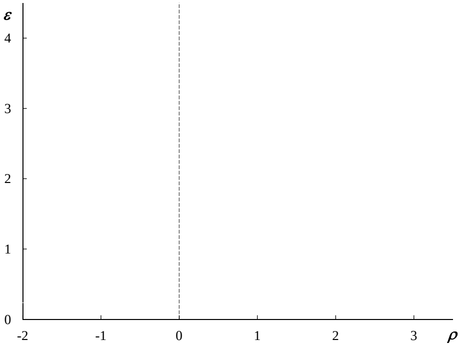
# Outline

# A Firm's-Eye View of Demand

- Perceived inverse demand function:

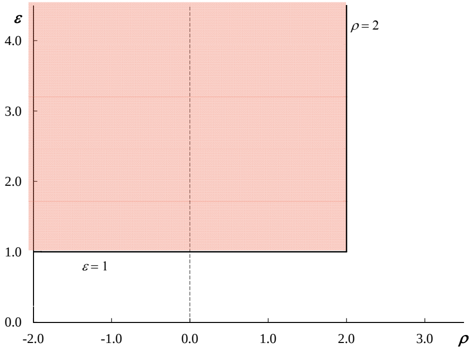
$$p = p(x) \qquad p' < 0$$

- Two key demand parameters:
  1. Slope/Elasticity: $\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0$
  2. Curvature/Convexity: $\rho(x) \equiv -\frac{xp''(x)}{p'(x)}$



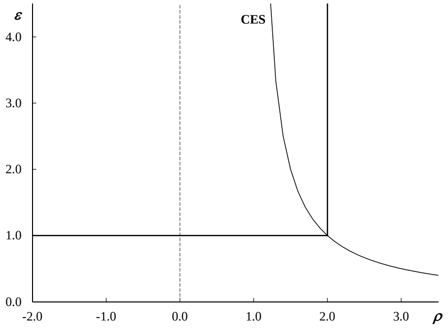
# The Admissible Region

- For a monopoly firm:
  - First-order condition:
    $p + xp' = c \geq 0 \quad \Rightarrow \quad \varepsilon \geq 1$
  - Second-order condition:
    $2p' + xp'' < 0 \quad \Rightarrow \quad \rho < 2$



# CES Demands

- In general, both $\varepsilon$ and $\rho$ vary with sales
- Exception: CES/iso-elastic case:
  - $p = \beta x^{-1/\sigma}$
  - $\Rightarrow \quad \varepsilon = \sigma, \ \ \rho = \frac{\sigma+1}{\sigma} > 1$
  - $\Rightarrow \quad \varepsilon = \frac{1}{\rho-1}$

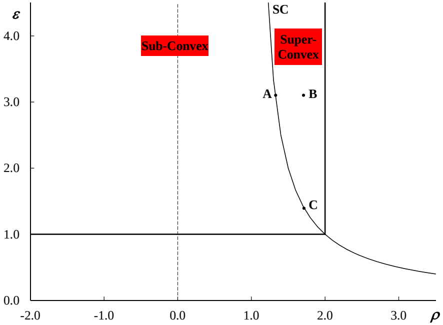



- Cobb-Douglas: $\varepsilon = 1, \rho = 2$; just on boundary of both FOC and SOC

# Competition Effects: Subconvexity

$p(x)$ is subconvex at $x^0$ IFF:

- $p(x)$ is less convex than a CES demand function with the same elasticity: $\rho > \frac{\varepsilon+1}{\varepsilon}$
- $\varepsilon$ is decreasing in sales:
  - $\varepsilon_x = \frac{\varepsilon}{x}\left[\rho - \frac{\varepsilon+1}{\varepsilon}\right]$



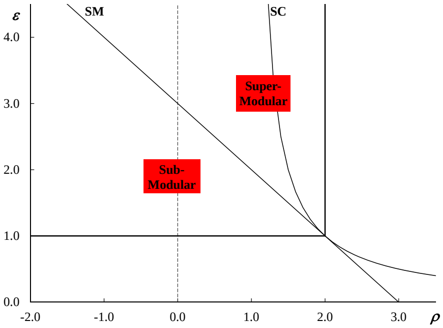

- "Globalization" $[x \downarrow]$ leads to competition effects $\left[\frac{p}{c} \downarrow\right]$
  - Because the mark-up is decreasing in elasticity: $\frac{p}{c} = \frac{\varepsilon}{\varepsilon-1} = 1 + \frac{1}{\varepsilon-1}$

# Second-Order Selection Effects: Supermodularity

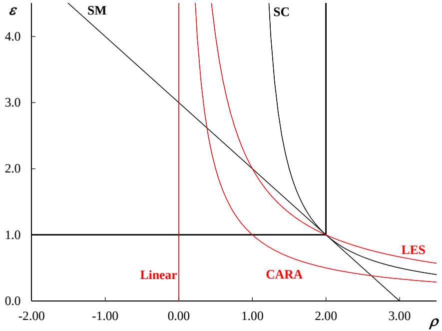
- Lower-$c$ firms choose FDI IF:
  - $\pi(t, c)$ supermodular in $\{t, c\}$
  - $\Leftrightarrow$ MCEO $> 1$
  - $\Leftrightarrow \ \varepsilon + \rho > 3$

Recall Demand Functions

- Clearly: SupC $\Rightarrow$ SupM

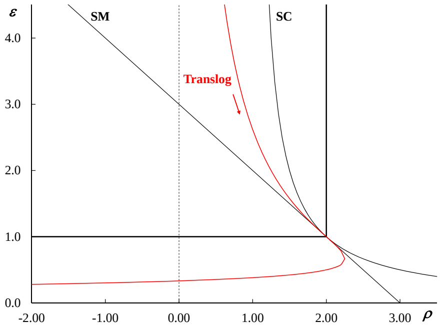


# Can We Have Subconvexity and Supermodularity?




- Demand functions represented in $\{\varepsilon, \rho\}$ space by their *Demand Manifold*
- Most common demand functions are:
  - Subconvex $\Rightarrow$ Competition effects
  - Submodular for high output $\Rightarrow$ Reverse selection effects

# Can We Have Subconvexity and Supermodularity?




- Demand functions represented in $\{\varepsilon, \rho\}$ space by their *Demand Manifold*
- Most common demand functions are:
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- Exception: AIDS/Translog

# Outline

# Recall: The Canonical Model

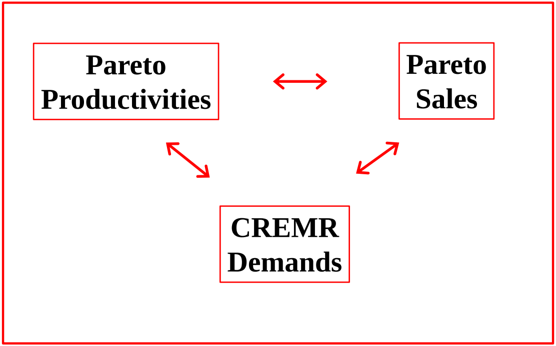
**Pareto Productivities** + **CES Demands** → **Pareto Sales**

# Backing Out Demands

**Pareto Productivities** + **CES Demands** → **Pareto Sales**

**Pareto Productivities** → **?** ← **Pareto Sales**

# Which Demands are Consistent with Pareto?




Proposition: Any two imply the third

# CREMR Demands

- "CREMR": "Constant Revenue Elasticity of Marginal Revenue"
  - MR=MC $\Rightarrow \quad \varphi = c^{-1} = (r')^{-1}$
  - So: Constant elasticity of sales with respect to productivity

$$p(x) = \frac{\beta}{x}(x-\gamma)^{\frac{\sigma-1}{\sigma}}, \quad 1 < \sigma < \infty,\ x > \gamma\sigma,\ \beta > 0$$

- CES a special case: $\gamma = 0 \quad \Rightarrow \quad p(x) = \beta x^{-1/\sigma}$
  - CREMR elasticity of demand: $\varepsilon(x) = \frac{x-\gamma}{x-\gamma\sigma}\sigma$
- "CREMR":

$$\hat{r'} = -\frac{1}{\sigma-1}\hat{r}$$

  - $r(x) \equiv xp(x), \ \ r'(x) = p(x) + xp'(x), \ \ \hat{r} \equiv d\log r = \frac{dr}{r} \ (r \neq 0)$

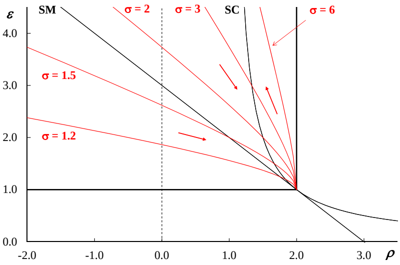
# CREMR Demands: Pros and Cons

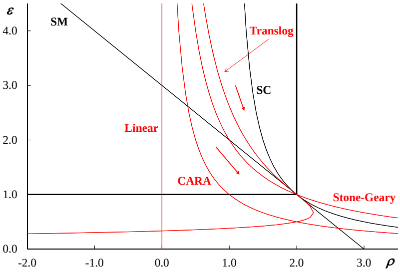
$$p(x) = \frac{\beta}{x}(x-\gamma)^{\frac{\sigma-1}{\sigma}}, \quad 1 < \sigma < \infty,\ x > \gamma\sigma,\ \beta > 0$$

- Features of CREMR demands:
  - A rich range of properties:
    - Nests CES: converges to CES as $x \to \infty$
    - Variable mark-ups: For any $\gamma \neq 0$
    - Competition effects: "Subconvex" IFF $\gamma > 0$
    - "Normal" selection effects: Profit function "supermodular" IFF $\sigma \geq 2$
  - Very different from standard demand functions
  - Inconsistent with a choke price
  - Utility function is analytic and can be simulated, but hard to work with

Next section

# CREMR Very Different from Other Demands




$$p(x) = \frac{\beta}{x}(x-\gamma)^{\frac{\sigma-1}{\sigma}}$$

$$\Rightarrow \quad \bar{\rho}(\varepsilon) = 2 - \frac{1}{\sigma-1}\frac{(\varepsilon-1)^2}{\varepsilon}$$

Compare CEMR Demand Manifolds

# Outline

# But, in practice, it is large firms that matter

- So far: Monopolistic competition
  - Firms heterogeneous in size ...
  - ... but qualitatively identical:
    - Infinitesimal: No market power
    - Probability of exit/death independent of productivity
- By contrast: Firms that dominate world trade are:
  - Super-large
  - Old
  - Multi-product
  - Multi-division

# Outline

# Conclusion

- Quantification is in vogue in international trade!
- We have learnt so much from recent work on heterogeneous firms ...
  - CES + Pareto a valuable and highly tractable workhorse
  - But for many purposes we need alternatives
  - Heterogeneous models: No single parametric functional form can capture all the features we would like
- ... And there is lots more to learn!
  - Alternatives to CES?
  - Alternatives to Pareto?
  - Alternatives to monopolistic competition?

# Thanks and Acknowledgements*

Thank you for listening. Comments welcome!

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# Supermodularity and the MCEO

Back to Text

- Sufficient condition for second-order selection effects:
  - $\Pi^F$ is steeper than $\Pi^E$
  - $\Leftrightarrow \;\; \pi_c(c, 0) < \pi_c(c, \tau)$
  - $\Leftarrow \;\; \pi_{c\tau} > 0$
- Necessary and sufficient condition for $\pi_{c\tau} > 0$:
  - $\pi(c, \tau) = [p(x) - \tau c]x, \quad x$ optimal
  - $\Rightarrow \;\; \pi_c = -\tau x$
  - $\Rightarrow \;\; \pi_{c\tau} = -x - \tau \frac{\partial x}{\partial \tau} = -x \left(1 + \frac{c}{x} \frac{\partial x}{\partial c}\right)$
  - $\Rightarrow \;\; \pi_{c\tau} > 1$ IFF $-\frac{c}{x} \frac{\partial x}{\partial c} > 1$