Implicit Utility and the Canonical Gravity Model

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23rd Annual Conference on Global Economic Analysis

June 2020

Gravity Trade Puzzle

- Structural estimation of gravity models of trade transparently maps regression coefficients to structural parameters.
- Puzzle of separating trade costs from trade responses.
- Visually, e.g., following coefficients on log distances:

$$\boldsymbol{\rho}(\mathbf{1}-\boldsymbol{\sigma})\log d_{ij}.$$

- Yang (2019) shows identification using the CDE preferences with easily accessible population data.

Question and lessons studied

- HY discussions: Can we do this in CES-gravity models?
- Theoretical demand structure is alone sufficient:

$$(\mathbf{1}-\boldsymbol{\sigma})\log[\quad ? \quad] \longrightarrow \boldsymbol{\rho}(\mathbf{1}-\boldsymbol{\sigma})\log d_{ij}.$$

Related Papers

- Hanoch (1975): implicitly additive demand system;
- Anderson (1979): gravity model theory;
- McCallum (1995): high border costs;
- Hummels (1999): *ad valorem* freight charges + tariff rates;
- Eaton and Kortum (2002): Ricardian Framework;
- Anderson and van Wincoop (2003)/AvW: $\sigma = 5, 10, 20$;
- Balistreri and Hillberry (2007)/BH: $\sigma = 5$;
- Simonovska and Waugh (2014): disaggregate prices + trade-flow data $\longrightarrow \sigma \approx 4$;
- Caliendo and Parro (2015): tariff data + asymmetric border;
- Heid, Larch and Yotov (2017): non-discriminatory trade-policy variables $\longrightarrow \sigma \in [4.3, 6.9]$;
- Feenstra (1994), Broda and Weinstein (2006), Soderbery (2015): imported quantities + unit values $\longrightarrow \sigma$ at the product-level;
- Preckel, Cranfield and Hertel (2010), Yang, Gouel and Hertel (2018), Gouel and Guimbard (2019): implicit additive direct models (MAIDADS) + MLE $\longrightarrow \kappa$, $\boldsymbol{U_j}$'s;
- Yang (2019): CDE + population + MPEC $\longrightarrow \sigma_i$'s, $\boldsymbol{U_j}$'s;
- Yang and Preckel (2020): CDE + MLE $\longrightarrow \boldsymbol{U_j}$'s;
- Hillberry and Yang (2020): CES + MPEC $\longrightarrow \boldsymbol{\sigma}$.

A Solution from the Demand Theory

We bridge Hanoch (1975)'s demand theories with trade.

The representative consumer preferences in a region j are modeled by the following direct CES function: Demand Parameterization (click here)

$$U_j \equiv \left[\sum_i \alpha_i^{(1-\sigma)/\sigma} \left(\frac{T_{ij}}{t_{ij}} \right)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} . \quad (1)$$

- T_{ij} is the quantity of shipment from i to j, which is
- melted by the iceberg trade cost variable $t_{ij} > 0$;
- U_j's are the representative agent's utility in region j;
- α_i's > 0 are taste parameters;
- $\sigma > 0$ is the constant elasticity of substitution.

Trade Flows with CES price index

Then under region j's national budget constraint, the nominal trade flow equation is given by:

$$FOB_iT_{ij} = Y_j\left(\frac{\alpha_i FOB_i t_{ij}}{P_j}\right)^{1-\sigma}, \tag{2}$$

- FOB_i is the domestic price of output units in region i;
- Y_j is the national income in region j;
- P_j is the (aggregate) consumer price index in region j.

P_j equals the inverse of shadow price resulted from the utility maximization, and for any CES demand function:

$$U_j = \frac{Y_j}{P_j}. \tag{3}$$

Implicit Trade Flows

Writing Eq. (3) into (2) allows us to rewrite Eq. (2) that is embedded with utility U_j:

$$FOB_iT_{ij} = U_j^{1-\sigma}(\alpha_i t_{ij})^{1-\sigma} FOB_i^{1-\sigma} Y_j^{\sigma}. \tag{4}$$

Motivations of Eq. (4):

- σ controls counterfactuals of T_{ij}, FOB_i(or t_{ij}) and Y_j;
- Furthermore, $T_{ij} = (\alpha_i U_j)^{1-\sigma}(FOB_i t_{ij})^{-\sigma} Y_j^{\sigma}$;
- With $\overline{U}_j$: $1\% \uparrow$ of FOB_i or $t_{ij} \implies \sigma\% \downarrow$ of T_{ij};
- Due to the generic σ, in order to hold U_j and T_{ij} fixed while prices $\uparrow$ by 1%, the region-j consumer must be compensated by exactly a $1\% \uparrow$ in Y_j;
- Holding FOB_i's and U_j's fixed, the econometric exercise exploits variations in T_{ij} and Y_j.

Implicit Utility into the GE Framework

In equilibrium, each region j's aggregate income must equal the total value of **CIF** goods purchased by region j's consumer:

$$Y_j = \sum_i FOB_i T_{ij}. \tag{5}$$

Combining Eq. (5) with (4), it can be shown that

$$Y_j = \sum_i U_j^{1-\sigma} (\alpha_i t_{ij})^{1-\sigma} FOB_i^{1-\sigma} Y_j^{\sigma}. \tag{6}$$

It implies a representation of the implicit additive indirect function associated with the **AvW's gravity** + **Hanoch's *implicit utility***:

$$\sum_i \alpha_i^{1-\sigma} U_j^{1-\sigma} \left(\frac{FOB_i t_{ij}}{Y_j} \right)^{1-\sigma} \equiv 1. \tag{7}$$

Trade costs into the GE Framework

Following BH, we account for both distance and asymmetric border effects on trade costs:

$$t_{ij} = d_{ij}^{\rho} \left[\exp \left(\frac{\mathbf{A}}{1-\sigma} \right) \right]^{1-\delta_{ij}}. \quad (8)$$

- d_{ij} is the distance between i and j observed from the data;
- ρ is the elasticity of trade costs with respect to distance;
- $\mathbf{A} = (1-\sigma) \ln b_{ij}$ are the border coefficients;
- b_{ij}'s equal 1 plus tariff equivalent of border costs: $TR^{equiv} = b_{ij} - 1$.
- δ_{ij}'s are the dummy variables equaling 0 if shipments cross border, equaling 1 otherwise.

Identification

Let $X_{ij} = FOB_i T_{ij}$ denote the value of flows between i and j, it can be shown that the empirical form of gravity equation is given by

$$\log(X_{ij}) = (1-\boldsymbol{\sigma})\log\alpha_i + (1-\boldsymbol{\sigma})\log \boldsymbol{U_j} + (1-\boldsymbol{\sigma})\log \mathrm{FOB}_i + \boldsymbol{\rho}(1-\boldsymbol{\sigma})\log d_{ij} + (1-\delta_{ij})(1-\boldsymbol{\sigma})b_{ij} + \boldsymbol{\sigma}\log Y_j. \quad (9)$$

Identification:

- If we know the cardinal value of U_j, then we can identify σ;
- With σ being pinned down, we can obtain the estimates of α_i's, b_{ij} and ρ given the information on bilateral distances.
- Impossible in reduced-form approaches with U_j unobserved.
- This is because (standard CES-gravity) fixed effects would sweep out Y_j and U_j, and leave the product of ρ and $1-\sigma$ unidentified.
- For this reason, we follow Yang (2019)'s structural approach to evaluate U_j simultaneously, using an MPEC algorithm.

BH (JIE, 2007)'s Computable General Equilibrium

BH constructs a 4n system of equations that is an operational GE.

(1) Income definition:

$$Y_i = FOB_i E_i^0.$$

(2) Goods market-clearing condition:

$$E_i^0 = \sum_j \left[\frac{Y_j}{FOB_i} \left(\frac{\alpha_i FOB_i t_{ij}}{P_j} \right)^{1-\sigma} \right].$$

(3) Unit expenditure function:

$$P_j = \left[\sum_i (\alpha_i FOB_i t_{ij})^{1-\sigma} \right]^{1/(1-\sigma)}.$$

(4) Income balance:

$$U_i P_i = Y_i.$$

Linkage with HY's 3n CGE in the Estimation System:

- BH model the 4n-system equations s.t. LS in their estimation system.
- We apply Hanoch's implicit utility theorem and reduce the system to 3n.
- We explicitly evaluate U given the system of constraints (next slide).

3n GE System of Equations as Constraints

(1) National income definition (**income**):

$$Y_i = FOB_i E_i^0 \quad \longrightarrow \quad \text{MCP.Y}$$

(2) National endowment identity (**supply**):

$$E_i^0 = \sum_j U_j^{1-\sigma}(\alpha_i t_{ij})^{1-\sigma} FOB_i^{-\sigma} Y_j^{\sigma} \quad \longrightarrow \quad \text{MCP.FOB}$$

(3) Preferences definition (**demand**):

$$\sum_i \alpha_i^{1-\sigma} U_j^{1-\sigma} \left(\frac{FOB_i t_{ij}}{Y_j}\right)^{1-\sigma} \equiv \kappa \quad \longrightarrow \quad \text{MCP.P} \qquad \left[\text{MCP.U} = \frac{Y}{P}\right]$$

- E_i^0 is region i's fixed endowment; red: complementary variables;
- Equation (3) is both a GE environment and normalization;
- We let the computation algorithm determine the data-generating scaling factor of U_j, expressed by some $\kappa > 0$;
- This system formulates a mixed complementarity problem (**MCP**).

MCP1

MPEC and MCPs

- We formulate the constrained optimization problem using the algorithm of mathematical programming with equilibrium constraints (MPEC). A PPML-MPEC constrained optimization (click here)
- Popular in solving optimization of engineering problems.
- An appropriate candidate of solving constraints that are highly non-linear and are MCPs.
- The MCPs, in turn, verify that our 3n GE system is operational, via homogeneity test and check of Walras' Law.
- Recent literature using MPEC to solve GE gravity models of trade include: BH, Balistreri, Hillberry and Rutherford (2011), Tan (2012), Yang (2019).
- The MPEC program is implemented in GAMS version 31.1.1 with the help of the preprocessor using GAMS-F tool.

Empirical Verification of Theoretical Approach

- To verify that our approach is consistent with empirical findings, we replicate BH's results who structurally estimate AvW's coefficient $a1 = -1.44$.
- The authors use least squares (LS) as objective, and fixed $\sigma = 5$. Since $a1 = \rho(1 - \sigma)$, this implies $\rho = 0.36$.
- Thus, our theoretical structure must hypothetically yield exactly the **same result**, provided that we use the same data, econometric models, and identically exogenize the level of σ.

Relevant procedures:

- Step 1: we first replicate BH's model using explicit direct CES.
- Step 2: we replicate the model again using our implicit model.
- Step 3: confirmed that our model consistently yields $\rho = 0.36$.
- Step 4: change objectives to PPML, and repeat step 1 and 2.
- Step 5: verified empirical consistency again in both models.

Freeing σ with Symmetric Border Costs

- We directly solve the canonical AvW's model of aggregate trade.
- In this step, we release σ and directly estimate σ, ρ, and b under the same equilibrium constraints and normalization scheme that would have replicated BH/AvW if σ were fixed;
- In this exercise, we sill use LS estimator as in BH;
- Defining the fitted value

$$\begin{aligned}\hat{z}_{ij} &= \log\left(\frac{X_{ij}}{Y_j}\right) \\ &= (1-\sigma)\left[\log\alpha_i + \log(U_j) + \log(FOB_i) + \log t_{ij} - \log(Y_j)\right];\end{aligned}$$

- Econometric specification given by min $\sum_i \sum_j [z_{ij} - \hat{z}_{ij}]^2$;
- Constrained by (i): Eq (1) - (3);
- and (ii) AvW/BH's normalization for the scale of utility;
- **AcW's data**: 30 US states, 10 Canadian provinces, 1 rest of the US, 1,551 non-zero trade-flows observations.

Check the Excess Degrees of Freedom

- Result 1: replicated that $\rho = 0.36$, when $\sigma = 5$ is held fixed;
- Result 2: with LS, $\sigma = 1.62$, $\rho = 2.31$, $b = 2.96$.
- We stop here, and following Yang and Preckel (2020) to ask whether there are additional degrees of freedom in the parameter spaces that can be removed.
- That is, whether additional normalization is needed as we are moving to freeing σ? Latitude for changing parameters?
- Thus, we construct a new problem by maximizing and minimizing σ while including the original estimation problem and requiring that the sum of squared residual is at least as small as computed from the estimation problem.
- We then repeat this procedure for ρ and b.
- Conclusion: No, the parameter values cannot be changed significantly without increasing the residuals.

Structural Estimation and Bootstrapping

Table 1: Structural estimation with implicit and explicit representation

	BH replication with explicit representation	BH replication with implicit representation	Structural estimation with implicit representation	Implicit representation with regional specific $\sigma's$
	(1)	(2)	(3)	(4)
$a1 = (1-\sigma)\rho$	-1.44	-1.44	-1.44	
$a2 = (1-\sigma)\ln b_{US-CA}$	-1.85	-1.85	-1.85	
$a3 = (1-\sigma)\ln b_{CA-US}$	-1.85	-1.85	-1.85	
$\bar{a1} = (1-\bar{\sigma})\rho$				-1.47
$\bar{a2} = (1-\bar{\sigma})\ln b_{US-CA}$				-1.39
$\bar{a3} = (1-\bar{\sigma})\ln b_{CA-US}$				-1.39
σ	5 (assigned)	5 (assigned)	1.62	
			(0.005)	
$\bar{\sigma}$				1.81
				(0.02)
ρ	0.36	0.36	2.31	1.82
	(0.005)	(0.005)	(0.03)	(0.06)
$\ln b_{US-CA}$	0.46	0.46	2.96	1.72
	(0.02)	(0.02)	(0.14)	(0.12)
$\ln b_{CA-US}$	0.46	0.46	2.96	1.72
	(0.02)	(0.02)	(0.14)	(0.12)
N	1511	1511	1511	1511
Sum of squared residuals	2262.84	2262.84	2262.84	1286.03

Standard errors across columns in "()" obtained from 2,000 bootstrap resamples.

Fitted into PPML with Trade Flows Objectives

- Following Gourieroux, Monfort and Trognon (1984) and Santos Silva and Tenreyro (2006), with asymmetric border costs b_{ij}, the PPML estimator is given by

$$
\begin{aligned}
X_{ij} = \exp\Big\{ \left|(1-\sigma)\log \alpha_i\right| + (1-\sigma)\log U_j + (1-\sigma)\log FOB_i \\
+ \rho(1-\sigma)\log d_{ij} + (1-\delta_{ij})(1-\sigma)b_{ij} + \sigma \log Y_j \Big\} + \varepsilon_{ij} \\
= \exp(x_{ij} g_i) + \varepsilon_{ij}.
\end{aligned}
$$

- $\mathrm{x}_{ij} g_i$ is a proxy representing everything inside the curved bracket;
- ε_{ij} is the disturbance term;
- we repeat the check of robustness and bootstrapping as for LS;
- we release the symmetric border assumption and repeat all steps.

A PPML-MPEC constrained optimization (click here)

Concluding Remarks

- We bridge the demand theory with trade literature and show that an alternative but identical CES-gravity can achieve identification via an application of a canonical gravity model.
- We demonstrate that theoretical structure is alone sufficient for identifying σ, ρ and b, without adding any more data.
- We generalize the standard trade flows to implicit trade flows.
- The procedure allows evaluation of the utility index, which is critical to identifying structural parameters:

$$(\mathbf{1} - \boldsymbol{\sigma}) \log[\boldsymbol{U_j}] \longrightarrow \boldsymbol{\rho}(\mathbf{1} - \boldsymbol{\sigma}) \log d_{ij}.$$

- We show that the MPEC algorithm is useful to calculate cardinal value of utility (even with the CES function) with MCPs.

Thank You!

Questions or comments?

Demand Parameterization

- Hanoch (1975): $G(\frac{p}{w}, u) = \sum_i \beta_i u^{e_i(1-\alpha_i)} (\frac{p_i}{E})^{1-\alpha_i} \equiv 1$
- The following indirect CES function is its special case:

$$U \equiv \left[\sum_i \beta_i \left(\frac{p_i}{E} \right)^{1-\sigma} \right]^{1/\sigma - 1},$$

- which is also a parametric transformation from the preferences in Balistreri and Hillberry (2007):

$$U \equiv \left[\sum_i \alpha_i^{(1-\sigma)/\sigma} Q_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)},$$

$$\alpha_i^{1-\sigma} = \beta_i > 0 \ \forall \ i.$$

Head back (click here)

A PPML Constrained Optimization

The PPML-MPEC problem is formulated as follows:

$$\max_{\mathbf{g}_i=\{\alpha_i,\sigma,\rho\},U_j,b_{ij},0<\kappa<c} L(b_i) = \text{constant} - \sum_i \sum_j exp(x_{ij}g_i) + \sum_i \sum_j y_{ij}x_{ij}g_i$$

$$\begin{aligned} s.t. \quad & \text{(i)} \quad \mathrm{GE}(\mathbf{g}_i, U_j, b_{ij}) \ [\text{set of GE constraints}] \\ & \text{(ii)} \quad \mathrm{CES}(\mathbf{g}_i, U_j, b_{ij}) \equiv \kappa \ [\text{CES additivity constraint}] \\ & \text{(iii)} \quad \alpha_i, \sigma > 0 \\ & \text{(iv)} \quad U_j, \rho > 0 \\ & \text{(v)} \quad b_{ij} \geq 0 \\ & \text{(vi)} \quad \left(\frac{Y_{\text{Alabama}}}{U_{\text{Alabama}}}\right)^{1-\sigma} = \sum_i \left[\frac{U_i t_{\text{Alabama},j}}{\sum_j Y_j}\right]^{1-\sigma}. \end{aligned}$$

- The last equilibrium constraint is a specific normalization that is equivalent to AvW and BH.

Head back to MPEC (click here) Head back to PPML (click here)

Inequality Constraints as an MCP1

The market clearing conditions imply that the strict equalities would hold if and only if the associated goods are free of charge

$$E_i^0 \geq \sum_j U_j^{1-\sigma} (\alpha_i t_{ij})^{1-\sigma} FOB_i^{-\sigma} Y_j^{\sigma} \quad \perp \quad FOB_i \geq 0.$$

Head back (click here)