Incorporating Labor-Leisure Choice into a Static General Equilibrium Model

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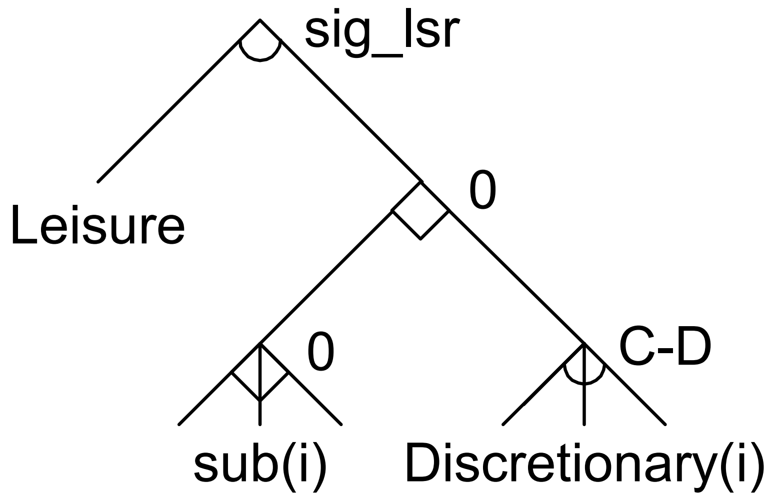
Introduction

- USITC’s U.S. Model
- Introduce labor-leisure choice for household
- Parameterization is critical to sensible results
- Follows Ballard (1999)

Overview of the Consumer Model



- Original formulation used LES
  - Subsistence consumption: sub(i)
  - Discretionary consumption: Discretionary(i)
- Leisure added as top nest of CES with composite

Derivation of the Behavioral Parameters

- We need to establish the relationship between the parameters to pin down leisure
- Start with utility maximization
- Find demands
- Establish indirect utility and unit expenditure
- Derive compensated and uncompensated elasticities of leisure demand and labor supply

Equations of Model (I)

$$U = \left[\theta \left(\frac{L}{L_0} \right)^{\rho} + (1-\theta) \left(\frac{X}{X_0} \right)^{\rho} \right]^{1/\rho}, \text{ such that } M + W\overline{L} = WL + P_X X \,. \tag{1}$$

$$\theta = \frac{W_0 L_0}{M_0 + W_0 \overline{L}} \tag{2}$$

$$L^* = L_0 \left(\frac{M + W\overline{L}}{M_0 + W_0 \overline{L}} \right) \left(\frac{W}{W_0} \right)^{-\sigma} \left[\theta \left(\frac{W}{W_0} \right)^{1-\sigma} + (1-\theta) \left(\frac{P_X}{P_{X0}} \right)^{1-\sigma} \right]^{-1} \tag{3a}$$

$$X^* = X_0 \left(\frac{M + W\overline{L}}{M_0 + W_0 \overline{L}} \right) \left(\frac{P_X}{P_{X0}} \right)^{-\sigma} \left[\theta \left(\frac{W}{W_0} \right)^{1-\sigma} + (1-\theta) \left(\frac{P_X}{P_{X0}} \right)^{1-\sigma} \right]^{-1} \tag{3b}$$

Equations of Model (II)

$$V(P_X, W, M) = \frac{M + W\bar{L}}{M_0 + W_0\bar{L}} \cdot \frac{1}{e(W, P_X)} \qquad (4)$$

$$e(W, P_X) = \left[\theta\left(\frac{W}{W_0}\right)^{1-\sigma} + (1-\theta)\left(\frac{P_X}{P_{X0}}\right)^{1-\theta}\right]^{\frac{1}{1-\sigma}}. \qquad (5)$$

$$\eta_L |_{\bar{u}} = \sigma\left[\theta\left(\frac{W_0 e(P_X, W)}{W}\right)^{\sigma-1} - 1\right] \quad \text{At benchmark, } \eta_L |_{\bar{u}} = \sigma[\theta - 1]. \qquad (6)$$

$$\varepsilon_L |_{\bar{u}} (\bar{L} - L) + \eta_L |_{\bar{u}} L = 0 \qquad (7a)$$

$$\varepsilon_L |_{\bar{u}} = -\eta_L |_{\bar{u}} \frac{L}{\bar{L} - L} \qquad (7b)$$

Equations of Model (III)

$$\eta_L = \underbrace{\frac{W\bar{L}}{M + W\bar{L}} - \theta\left[\frac{W_0}{W}\cdot e(P_X, W)\right]^{\sigma-1}}_{\eta_I} + \underbrace{\sigma\left[\theta\left(\frac{W_0}{W}\cdot e(P_X, W)\right)^{\sigma-1} - 1\right]}_{\eta_L|_{\bar{u}}} \qquad (8)$$

$$\eta_L L + \varepsilon_L(\bar{L} - L) = 0 \qquad (9a)$$

$$\varepsilon_L = -\eta_L \frac{L}{\bar{L} - L} \qquad (9b)$$

$$\eta_I = \frac{\bar{L} W_0}{M_0 + \bar{L} W_0} - \theta \qquad (10a)$$

$$\eta_I = \theta\left(\frac{1-\lambda}{\lambda}\right), \qquad \text{where } \lambda = L/\bar{L} \qquad (10b)$$

Equations of the Model (IV)

$$\varepsilon_L = \frac{-\lambda}{1-\lambda}\left[\eta_I + \eta_L \,|_{\bar{u}}\right] \qquad (11)$$

$$\varepsilon_L = \varepsilon_L\big|_{\bar{u}} - \theta \qquad \mathbf{(12)}$$

$$L = \frac{\theta(M+H)}{1-\theta} \qquad \mathbf{(13)}$$

$$\eta_L\big|_{\bar{u}} = \sigma(\theta - 1) \qquad \mathbf{(14)}$$

$$\sigma = \frac{\varepsilon_L\big|_{\bar{u}}}{1-\theta} \cdot \frac{\bar{L} - L}{L} \qquad \mathbf{(15)}$$

Implementation in MPSGE/GAMS

```
theta_l = eps_l_u - eps_l;

lsr     = sum(i,fd(i,"hhld")) * theta_l / (1-theta_l);

l_bar = (ylab0+labsav0-ltaxt0-lytaxt0) + lsr;

sig_lsr = (eps_l_u/(1- theta_l)) * ((l_bar - lsr)/lsr);
```

- Incorporate equations (12)-(15) in code
- Add top CES nest of leisure and consumption goods

```
$PROD:U s:sig_lsr les:0 cd(les):1
            O:cpi    Q:(sum(I,fd,(I,”hhld”))+lsr)
            I:pq(i)  Q:sub(i) les:
            I:pq(i)  Q:(fd(I,”hhld”)-sub(i)) cd:
            I:wanet Q:lsr
```