Developing a Cost of Capital Module for Computable General Equilibrium Modelling

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Motivation.

- Applied CGE models typically do not incorporate business taxes explicitly.
  - Example: Recent Review of Business Taxation in Australia
    - MONASH.
    - MM303.

- Investment outcomes are central in determining the behaviour of dynamic CGE models.
  - Changes in the value of the firm and the cost of capital, due to changes in tax rates and taxation regimes, are important for understanding investment outcomes.

- The ultimate aim is to enable CGE modelling of the economy-wide effects of reforms to business taxation.
Modelling Capital Gains Taxation.

- Theory and reality diverge.
  - Accrual systems dominate theoretical work.
  - Realisation systems dominate policy applications.

- Accrual vs. Realisation systems.
  - Timing issues.
  - Realisation-basis capital gains taxation drives a wedge between buyer and seller valuations.

- A solution.
  - Determine the rate of share realisations.
  - Accrual Equivalency.
The Value of the Firm.

- We are interested in the value of the firm to a prospective investor.
- We need to account for the dilution of pre-existing shareholder equity over time due to new share issues.

\[ V_t^j = VO_t^j - VN_t^j \]

- \( V \) = total value of the firm in industry \( j \).
- \( VO_t^j \) = value of pre-existing shareholders equity in a firm industry \( j \).
- \( VN_t^j \) = value of new share issues by firm in industry \( j \).

- We base the valuation on an arbitrage condition.
- Assume no taxes and perfect foresight,

\[ i_{t+1} V_t^j = E_{t+1}^j + (VO_{t+1}^j - V_t^j) \]

- \( E_t^j \) = shareholder earnings (cash flows) received on holding equity.
- \( i \) = discount factor.
The Value of the Firm...

- The value of the firm in period $t$ is the sum of all future income streams and its terminal value in period $T$, minus new share issues, appropriately discounted.

$$V_0^j = \sum_{t=1}^{T} \left[ \frac{D_t^j - VN_t^j}{\prod_{s=1}^{T} (1+i_s)} \right] + \frac{V_T^j}{\prod_{s=1}^{T} (1+i_s)}$$

- $D_t^j$ is the dividend paid to shareholders of a firm in industry $j$.

- A transversality condition allows us to assume away the firm’s terminal value.

Let $$\frac{V_T}{\prod_{s=1}^{T} (1+i_s)} \to 0 \text{ as } T \to \infty$$

- Assuming an infinite horizon...

$$V_0 = \sum_{t=1}^{\infty} \left[ \frac{D_t - VN_t}{\prod_{s=1}^{T} (1+i_s)} \right]$$
The Value of the Firm…

- Adding one generic income tax…

\[ V_{0}^{j} = \sum_{t=1}^{\infty} \left[ \frac{D_{t}^{j} (1 - \theta_{t}) - V N_{t}^{j}}{\prod_{s=1}^{t} (1 + i_{s} (1 - \theta_{t}))} \right] \]

- \( \theta \) is the rate of the income tax.

- Adding one generic accrual-basis capital gains tax…

\[ V_{0}^{j} = \sum_{t=1}^{\infty} \left[ \frac{D_{t}^{j} \left[ \frac{1 - \theta_{t}}{1 - c_{t}} \right] - V N_{t}^{j}}{\prod_{s=1}^{t} \left[ \frac{1 + i_{s} (1 - \theta_{t}) - c_{t}}{1 - c_{t}} \right]} \right] \]

- \( c \) is the rate of the capital gains tax.
The Value of the Firm…

- $V$ = total value of firm.
- $VN$ = value of new share issues.
- $D$ = unfranked dividends.
- $DF$ = franked dividends.
- $i$ = riskless interest rate.
- $\Omega$ = information set for determining expectations.

\[ V^j_0 = \sum_{t=1}^{\infty} \left[ \frac{\left[1 - \theta_t\right]}{1 - c_t} D_t^j \left[ \Omega_0^j \right] + \frac{\left[1 - \theta_t\right]\left[1 - (1 - \gamma_t)\tau_t\right]}{(1 - \tau_t)(1 - c_t)} DF_t^j \left[ \Omega_0^j \right] - VN_t^j \left[ \Omega_0^j \right] \right] \]

\[ \prod_{s=1}^{t} \frac{1 + i_s \left[ \Omega_0 \right](1 - \theta_s) - c_s \left[ 1 + \alpha \pi_s \left[ \Omega_0 \right] \right]}{1 - c_s} \]

and

\[ c_t = \varepsilon_t \psi_t c_t + \sum_{z=t}^{\infty} \frac{\varepsilon_z \prod_{k=t}^{z-1} (1 - \varepsilon_k) \psi_z c_z}{\prod_{s=t+1}^{z} \left[ 1 + i_s \left[ \Omega_{s-1} \right](1 - \theta_s) \right]} \] - is the effective, accrual equivalent rate of the capital gains tax.

- $\theta$ = personal income tax rate.
- $\tau$ = company income tax rate.
- $\gamma$ = degree of dividend imputation.
- $c$ = capital gains tax rate.
- $\phi$ = proportion of capital gain taxable.
- $\varepsilon$ = rate of equity realisations.
- $\pi$ = inflation rate.
- $\alpha$ = switch on capital gains tax indexing.
  - (1=real CGT, 0=nominal CGT).
The Cost of Capital.

Two potential approaches to analysing the firm’s cost of capital:

1. Use changes in the value of the firm as a basis for analysis.
   - Firm behaviour given / solution to production problem assumed.
   - Analyse the effect of changes in tax provisions on the value of given income streams to shareholders.

2. Solve the firm’s production problem and analyse its financial policy.
   - The firm seeks to maximise its value to shareholders subject to a set of constraints.
   - Analyse how changes in tax provisions affect firm behaviour and the size and timing of income streams.
     - Most importantly for cost of capital:
       - Investment plan.
       - Financial policy.
The Cost of Capital – **Constraints.**

**Capital Accumulation.**

- Primary factors and intermediate goods enter the firm’s production function via CES nests.

  e.g. The firm’s stock of capital of type \( n \) evolves according to

\[
K_t^{n,j} = I_t^{n,j} + \left(1 - \delta^n\right) K_{t-1}^{n,j}
\]

...and augments the total effective capital stock via a CES nest

\[
K_t^j = \Lambda^j_t \left[ \sum_{n=1}^M S_t^{n,j} \left(K_t^{n,j}\right)^{-\sigma^j} \right]^{\frac{1}{\sigma^j}}
\]

Thus the firm’s total effective capital stock is

\[
K_t^j = \Lambda^j_t \left[ \sum_{n=1}^M S_t^{n,j} \left(I_t^{n,j} + \left(1 - \delta^n\right) K_{t-1}^{n,j}\right)^{-\sigma^j} \right]^{\frac{1}{\sigma^j}}
\]
Constraints…

- **Product Demand.**
  
  - The firm faces a finite elasticity of demand for its output.
    
    \[ p_t^i = (Y_t^{i,j})^{-\eta^j} \quad \text{with} \quad \text{ED}_t = -\frac{1}{\eta} \]

  Applied to the firm’s CES production function, the revenue function is

  \[ p_t^i Y_t^{i,j} = \left[ Y_t^j \right]^{-\eta^j} = \left[ A_t [ a_1^j (L_t^j)^{-\rho^j} + a_2^j (K_{t-1}^j)^{-\rho^j} + a_3^j (Q_t^j)^{-\rho^j} ]^{-\rho^j} \right]^{1-\eta^j} \]

  - \( K^j \) = capital stock.
  - \( I^{n,j} \) = real investment.
  - \( \delta^n \) = physical rate of depreciation.
  - \( \rho^j \) = producer price of firm’s output.
  - \( Y^{k,j} \) = firm’s level of output.
  - \( \eta^j \) = parameter for setting own-price elasticity of demand for firm’s product.
The nature of the firm’s sources and uses of funds are captured in the following expressions:

- **Cash Flow**

\[
D_t^j + DF_t^j = \left[ y_t^j \right]^{1-\eta^j} + B_t^j + VN_t^j - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - (1 + i_t) B_{t-1}^j - \sum_{n=1}^N q_t^n I_t^{n,j} - T_t
\]

- **Tax**

\[
T_t^j = \tau_t \left[ \left[ Y_t^j \right]^{1-\eta^j} - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - i_t B_{t-1}^j - \sum_{n=1}^N \Gamma_t^n q_t^n I_t^{n,j} - \sum_{s=-\infty}^t \sum_{n=1}^N \Delta^n_{s,t-s} q_s^n I_s^{n,j} \right]
\]

- \( q^n \) = asset price of capital of type \( n \).
- \( L^j \) = effective labour usage (CES nest).
- \( w \) = nominal wage rate.
- \( \nu \) = payroll tax rate.
- \( Q^i \) = effective total intermediate good usage (CES nest).
- \( P \) = purchaser’s price of intermediate goods.
- \( p^i \) = producer price of good \( i \) in period \( t \).
- \( A \) = efficiency parameter (all-primary factor technical change).
- \( a_i \) = distributional parameter.
- \( \rho \) = determines elasticity of substitution.
- \( I \) = real investment.
- \( \Delta \) = depreciation and investment allowances.
- \( \delta \) = physical rate of depreciation.
- \( B \) = debt issues.
Constraints…

- **Sign Constraints**

  - The firm is unable to issue “negative dividends”.
    \[
    D_t \geq 0 \\
    DF_t \geq 0
    \]

  - The firm’s investment is always non-negative (i.e. it cannot liquidate productive capital).
    \[
    I_t \geq 0
    \]

  - The firm is constrained to generate revenue through productive activities.
    \[
    B_t \geq 0
    \]

  - The firm is denied the ability to pursue share repurchases.
    \[
    VN_t \geq 0
    \]
The Cost of Capital – Theoretical basis.

- An optimising firm invests up to the point where $M_R P^K_t = V M P^K_t$, or, in terms of our model,

$$k_t^{n'} = \frac{(1-\eta)a_2}{A_{t+1}^\rho} \left[ Y_{t+1} \right]^{-\eta(1+\rho)} - \delta^n = \frac{\lambda_3^3 k_t^{n'} \phi_{t+1} - \lambda_1^3 k_t^{n'} \left(1-\delta^n\right)}{\left[ \lambda^1_{t+1} - \lambda^2_{t+1} \tau_{t+1} \right]} - \delta^n$$

- $k_t^{n'}$ = change in total capital stock due to a change in stock of capital of type $n$.
- $\delta^n$ = physical rate of depreciation on a unit of capital of type $n$.
- $\tau$ = company tax rate.
- $\lambda^1_t, \lambda^2_t, \lambda^3_t$ = Lagrangian multipliers on cash flow, tax and capital stock constraints respectively.

- Our task now is to define the Lagrangian multipliers in terms of the parameters of the model – most importantly, tax-related parameters.
- This involves an analysis of the financial incentives facing the firm to determine its financial policy – specifically, the source of the marginal unit of finance.
The Cost of Capital – Some Examples.

Financing at the Margin with Retained Unfrankable Earnings - a \textit{SLACK} franking account constraint.

- $K_{j'} = \text{change in stock of capital of type } n \text{ due to a change in investment in capital of type } n$.
- $k_{n'} = \text{change in total capital stock due to a change in stock of capital of type } n$.
- $q_n = \text{asset price of capital of type } n$.
- $I_n = \text{real investment in capital of type } n$.
- $\Xi_n = \text{present value of depreciation and investment allowances on a unit of capital of type } n$.
- $\Gamma_n = \text{investment expenditure allowances on capital of type } n$.
- $\delta^n = \text{physical rate of depreciation on a unit of capital of type } n$.
- $\tau = \text{company tax rate}$.
- $\theta = \text{personal income tax rate}$.
- $\alpha = \text{switch on capital gains tax indexing}$.
- $\pi = \text{inflation rate}$.

\[
\text{COC}_t^j = \left[ \frac{1 - \tau \left( \Gamma^n + \Xi^n \right)}{1 - \tau} \right] \left[ \frac{1}{1 - c} \left[ 1 + i_{t+1} \left[ \Omega_t \right] (1 - \theta) - c \left[ 1 + \alpha \pi_{t+1} \left[ \Omega_t \right] \right] \right] \right] \frac{k_t^n}{K_t^j} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) - \delta^n
\]
The Cost of Capital – Some Examples

Financing at the Margin with Retained Unfrankable Earnings - a **BINDING** franking account constraint

- $K_j$ = change in stock of capital of type $n$ due to a change in investment in capital of type $n$.
- $k^{n'}$ = change in total capital stock due to a change in stock of capital of type $n$.
- $q^n$ = asset price of capital of type $n$.
- $I^n$ = real investment in capital of type $n$.
- $\Xi^n$ = present value of depreciation and investment allowances on a unit of capital of type $n$.
- $\Gamma^n$ = investment expenditure allowances on capital of type $n$.
- $\delta^n$ = physical rate of depreciation on a unit of capital of type $n$.
- $\tau$ = company tax rate.
- $\theta$ = personal income tax rate.
- $\gamma$ = degree of dividend imputation ($1 = $full$)$.
- $\alpha$ = switch on capital gains tax indexing.
- $\pi$ = inflation rate.

$$ COC_i^j = \left[ \frac{1-(1-\gamma)\tau(\Gamma^n + \Xi^n)}{1-(1-\gamma)\tau} \right] \left[ \frac{1+i_{t+1}[\Omega_t](1-\theta)c[1+\omega \Omega_{t+1}[\Omega_t]]}{1-c} \right] \left[ \frac{k_{t+1}^{n'}}{K_{t+1}'(1-\delta^n)} - \frac{q_t^n}{q_t^n} K_{t+1}'(1-\delta^n) \right] - \delta^n $$
Further Areas of.

- Further complicate the modelling of firm financial policy
  - New equity issues: Transaction costs, underwriting fees?
  - Retained earnings: Do dividends carry any non-cash benefits?

- Relaxing the assumption of the representative shareholder

- CGE Modelling tasks
  - Linearise the equations around a solution
  - Data set
  - Programming / Calibration tasks