A practical method for explicitly modeling quotas and other complementarities

Jill Harrison, Mark Horridge, Ken Pearson and Glyn Wittwer
Centre of Policy Studies, Monash University, Melbourne, Australia

New GEMPACK features handle:

- **inequality constraints**
- **complementary conditions**
- **non-differentiable functions**

Arising from optimizing problem

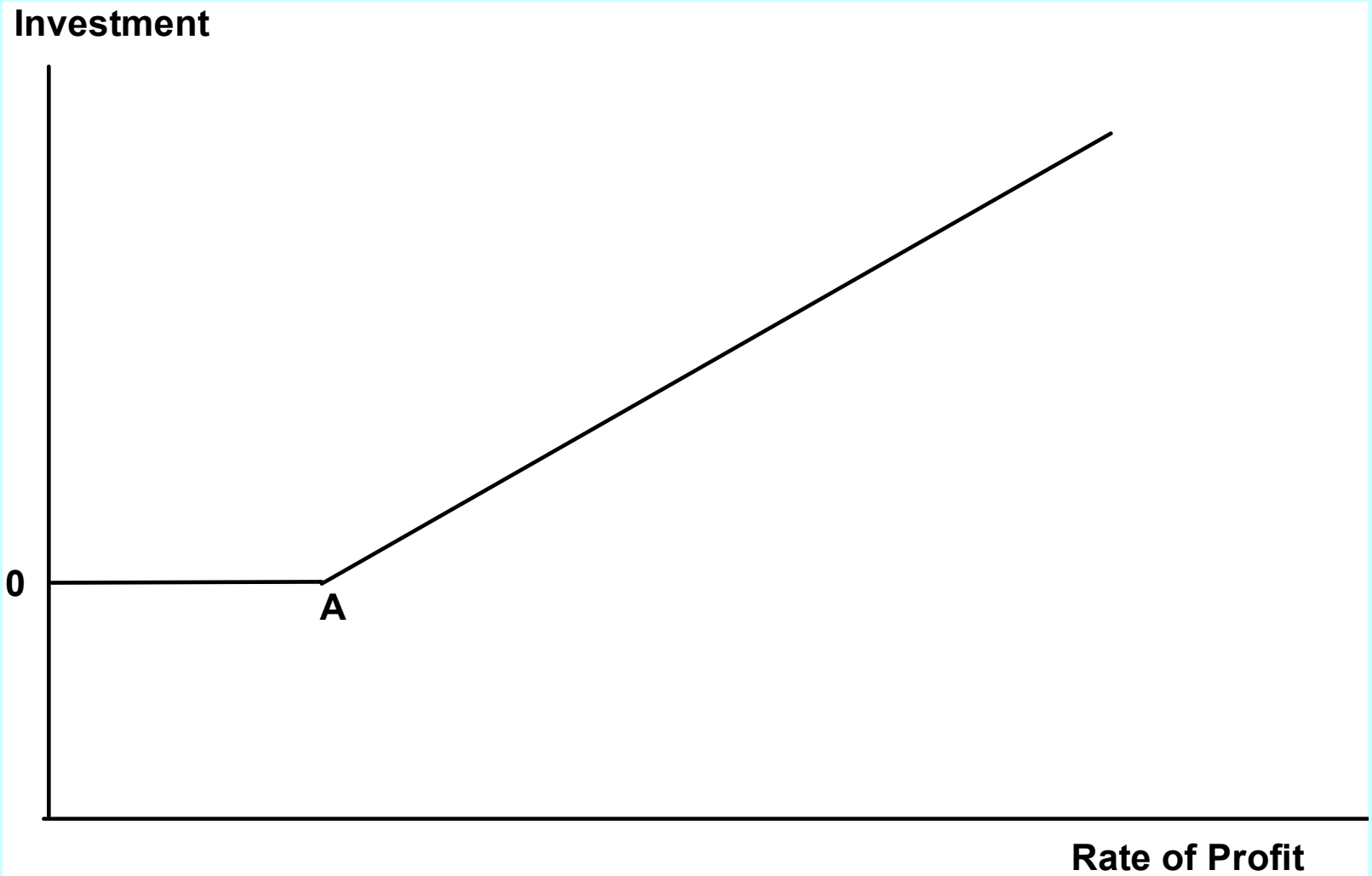
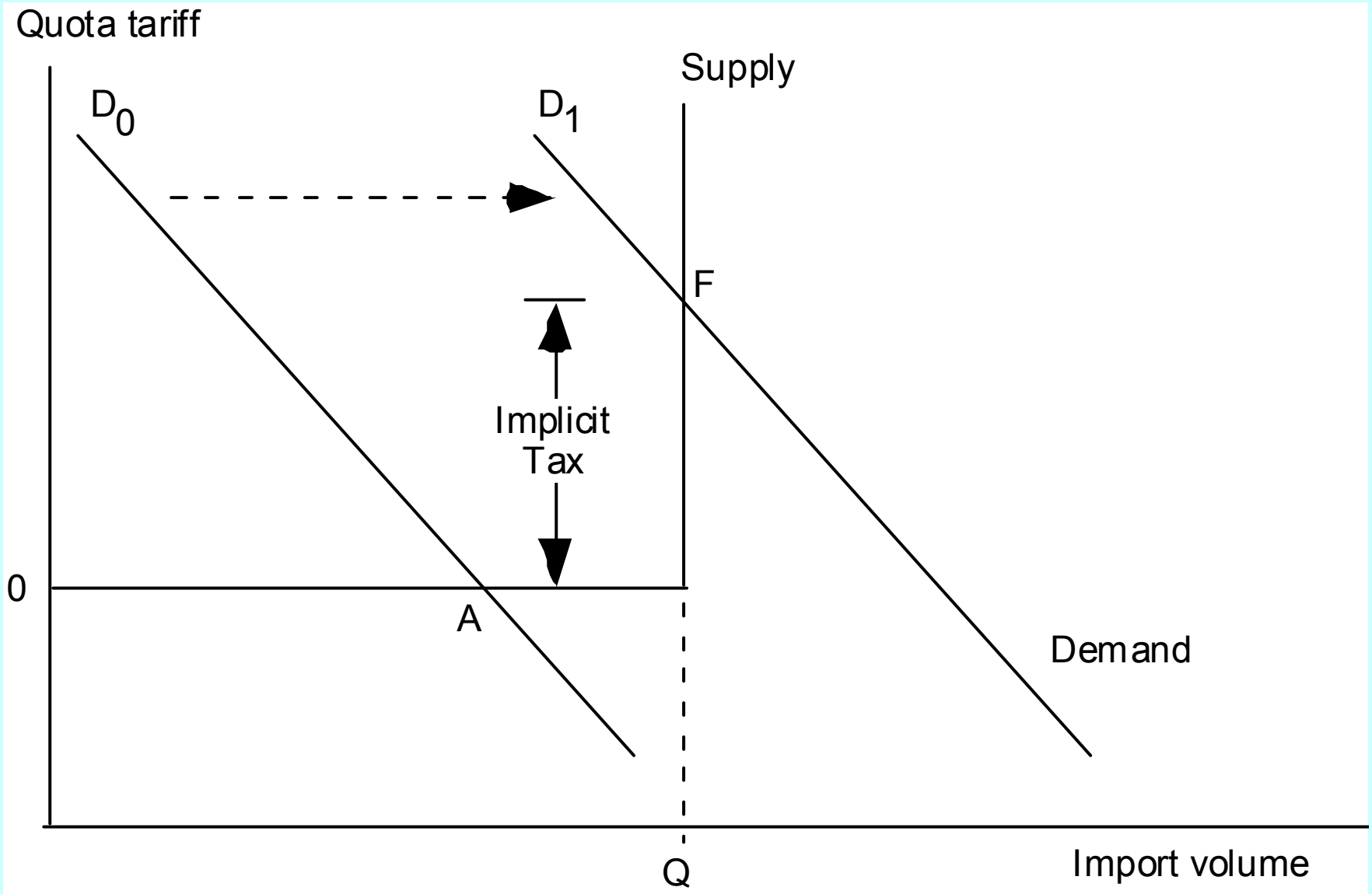


Figure 1: Investment related to profit rate

Arising from institutional quirk



If X is a variable and EXP is an expression, a simple complementarity is often written

$$X >= 0 \perp EXP$$

which is notation for:

Either $X > 0$ and $EXP = 0$

or $X = 0$ and $EXP >= 0$.

To represent the import quota of Figure 2 we could set:

X = Quota Tariff

EXP = Q - Import Volume

GEMPACK's more general specification of a complementarity

$$L <= X <= U \perp EXP$$

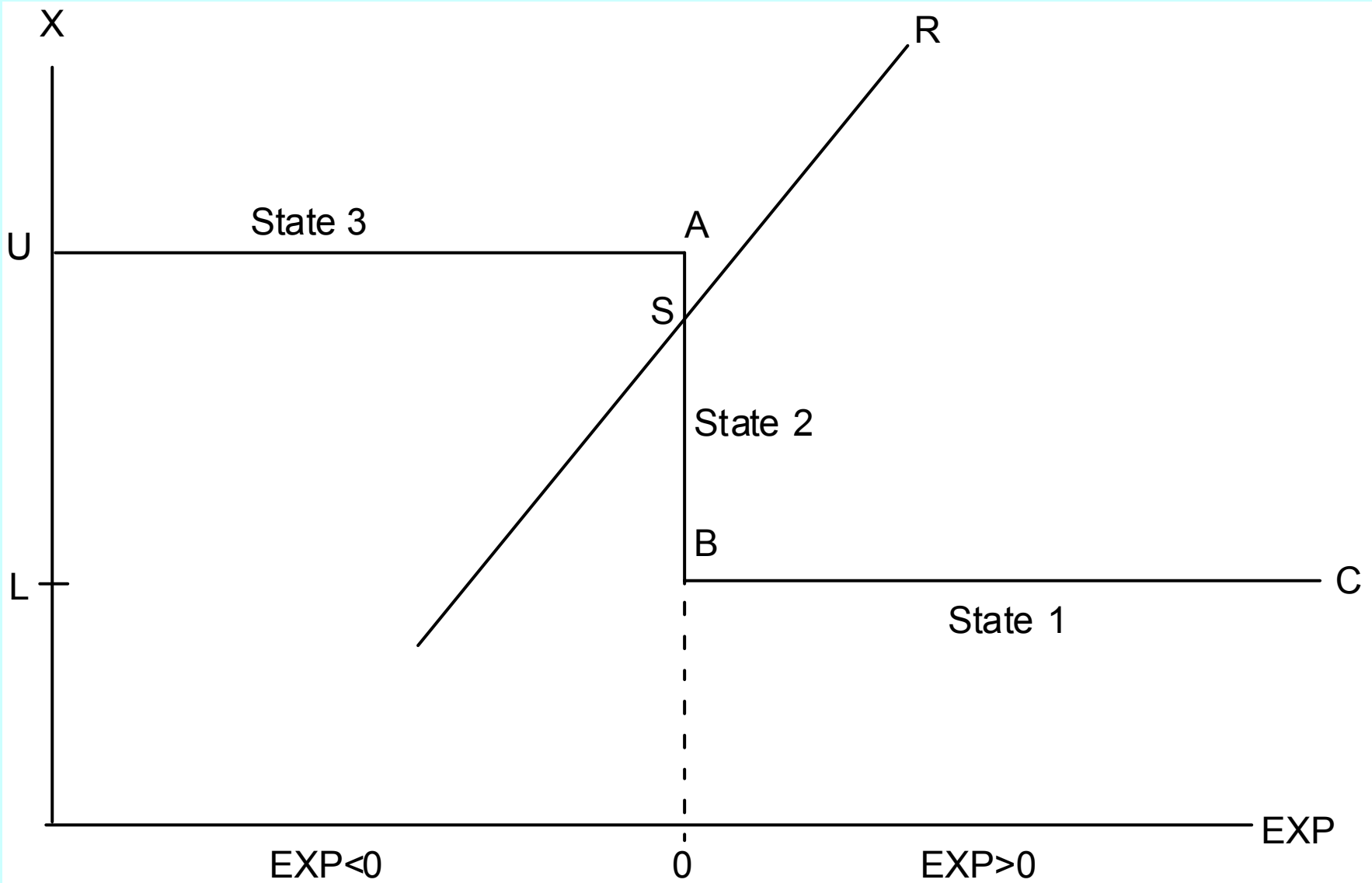
meaning that:

(1) Either $X = L$ and $EXP > 0$

(2) or $L <= X <= U$ and $EXP = 0$

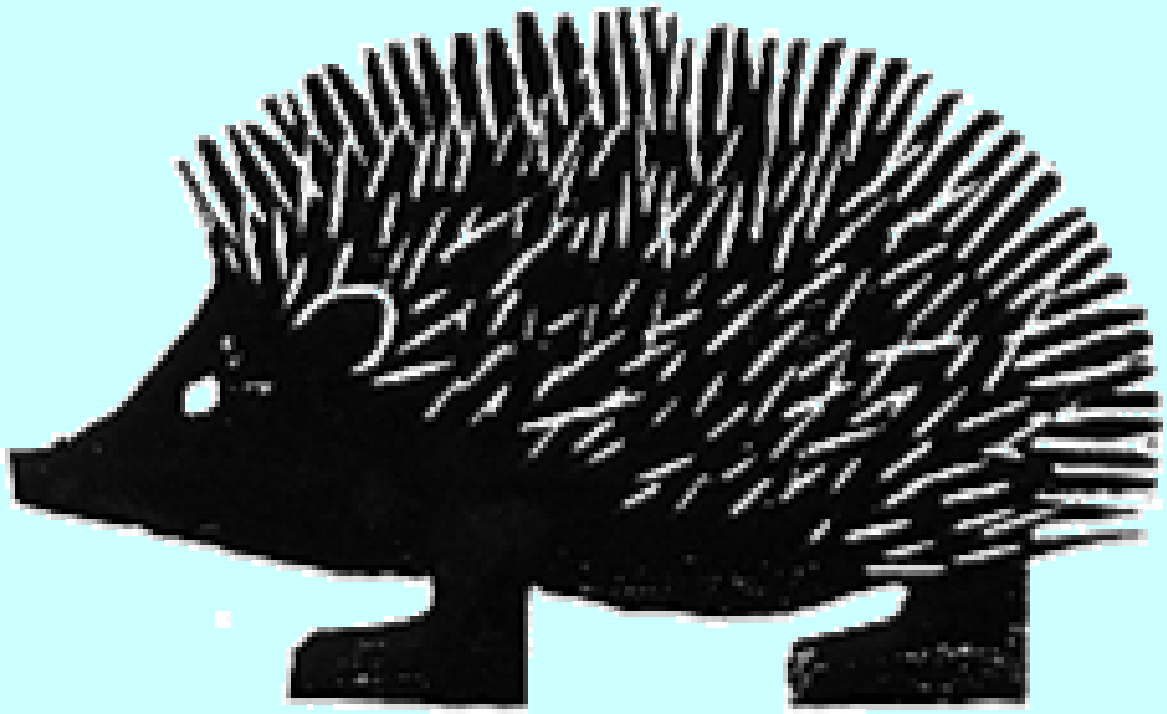
(3) or $X = U$ and $EXP < 0$

Figure 3: A generalized complementarity



Old Greek saying:

Hedgehog only knows one thing,



but he knows that thing very well.

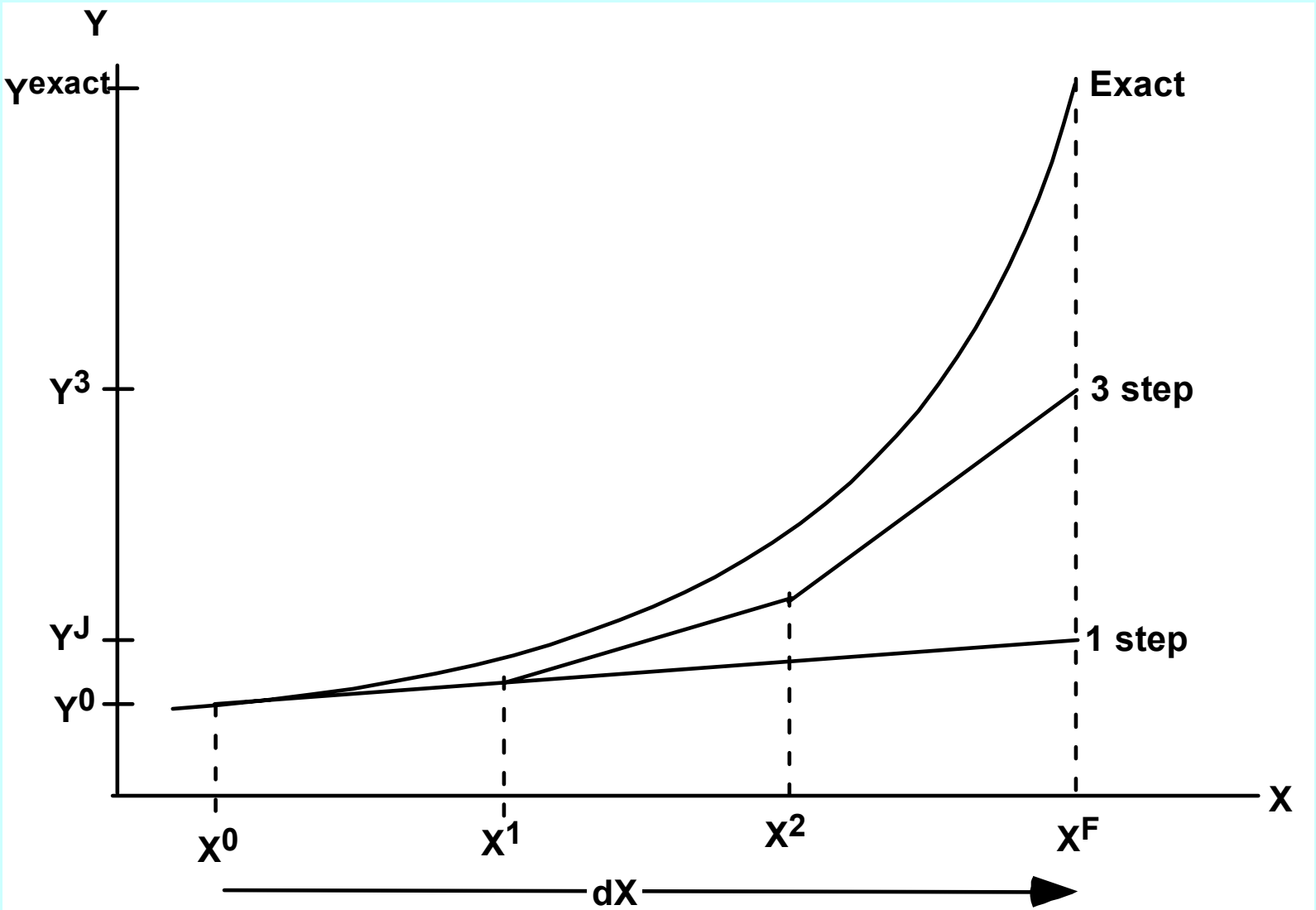
Euler small-change approach

How can it be adapted to solve complementarities?

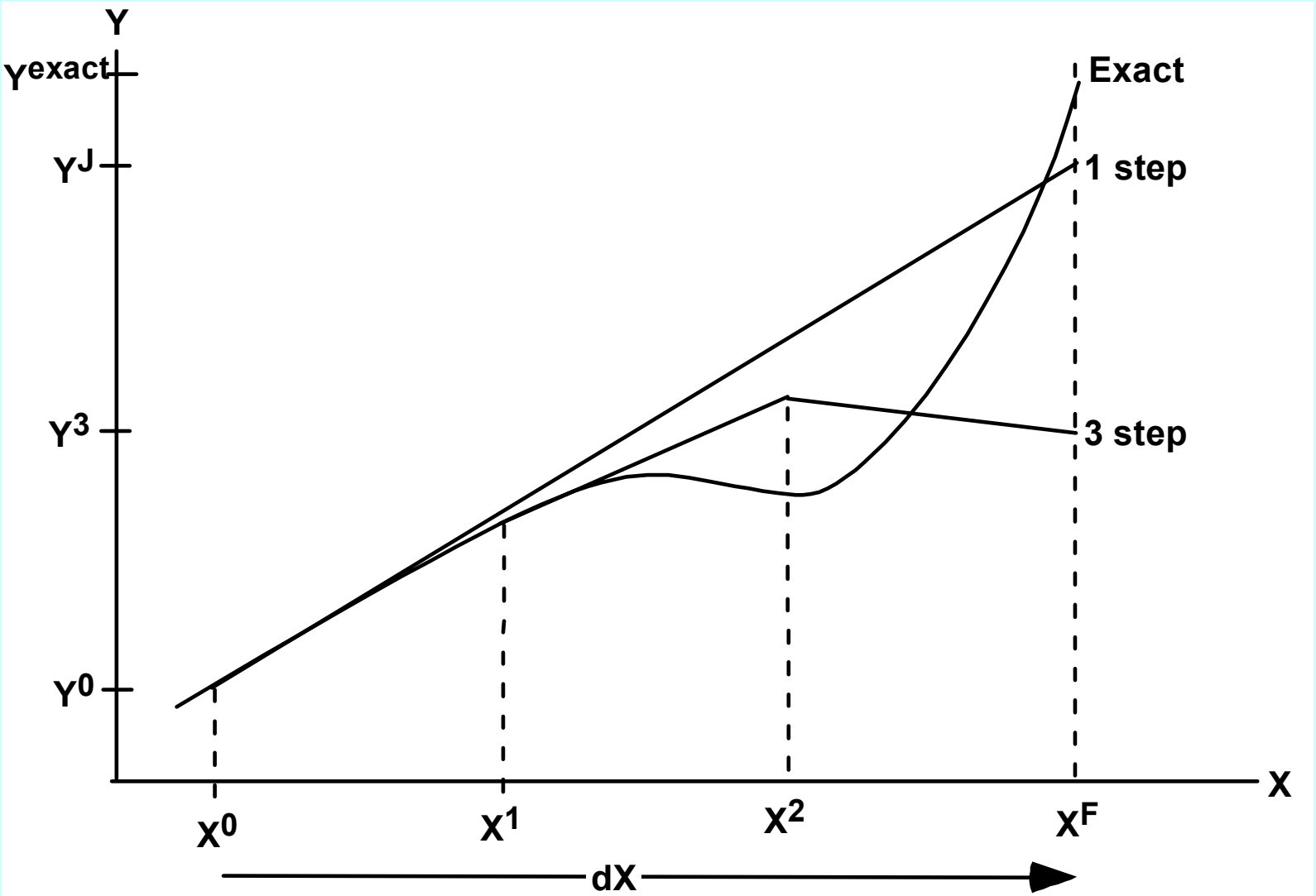
Euler small-change approach

- **Linearized system:**
  $$F_Y(Y,X)dY + F_X(Y,X)dX = 0$$
- **Changes divided into several steps**
- **More, smaller, steps give more accuracy**
- **From 3 simulations of say 3, 5, and 7 steps, we can extrapolate to estimate solution with infinitely many steps (true solution)**
- **The extrapolation is an estimate with error bounds**

Euler small change approach



More steps not always more accurate



but "enough" steps will be accurate

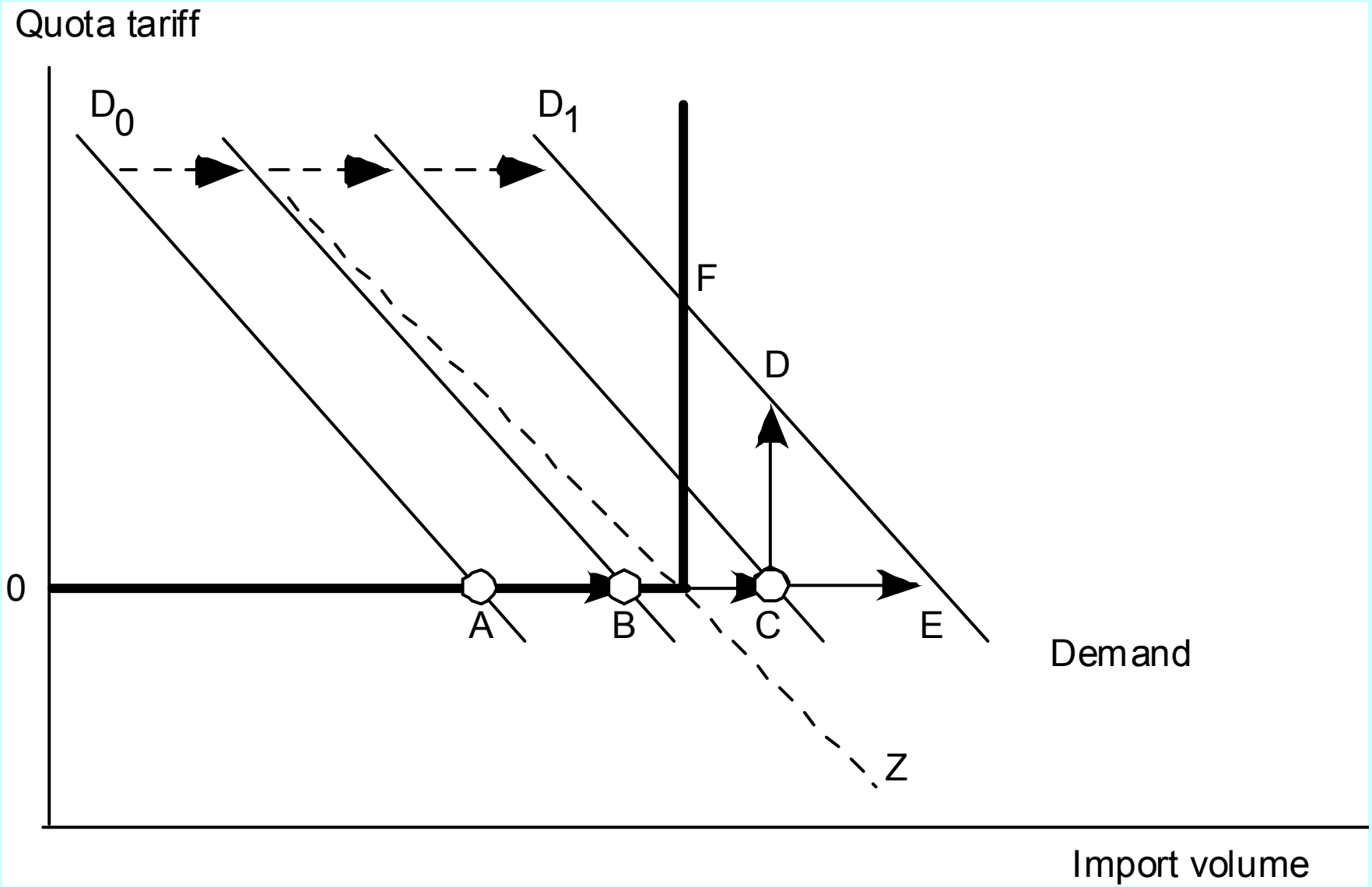
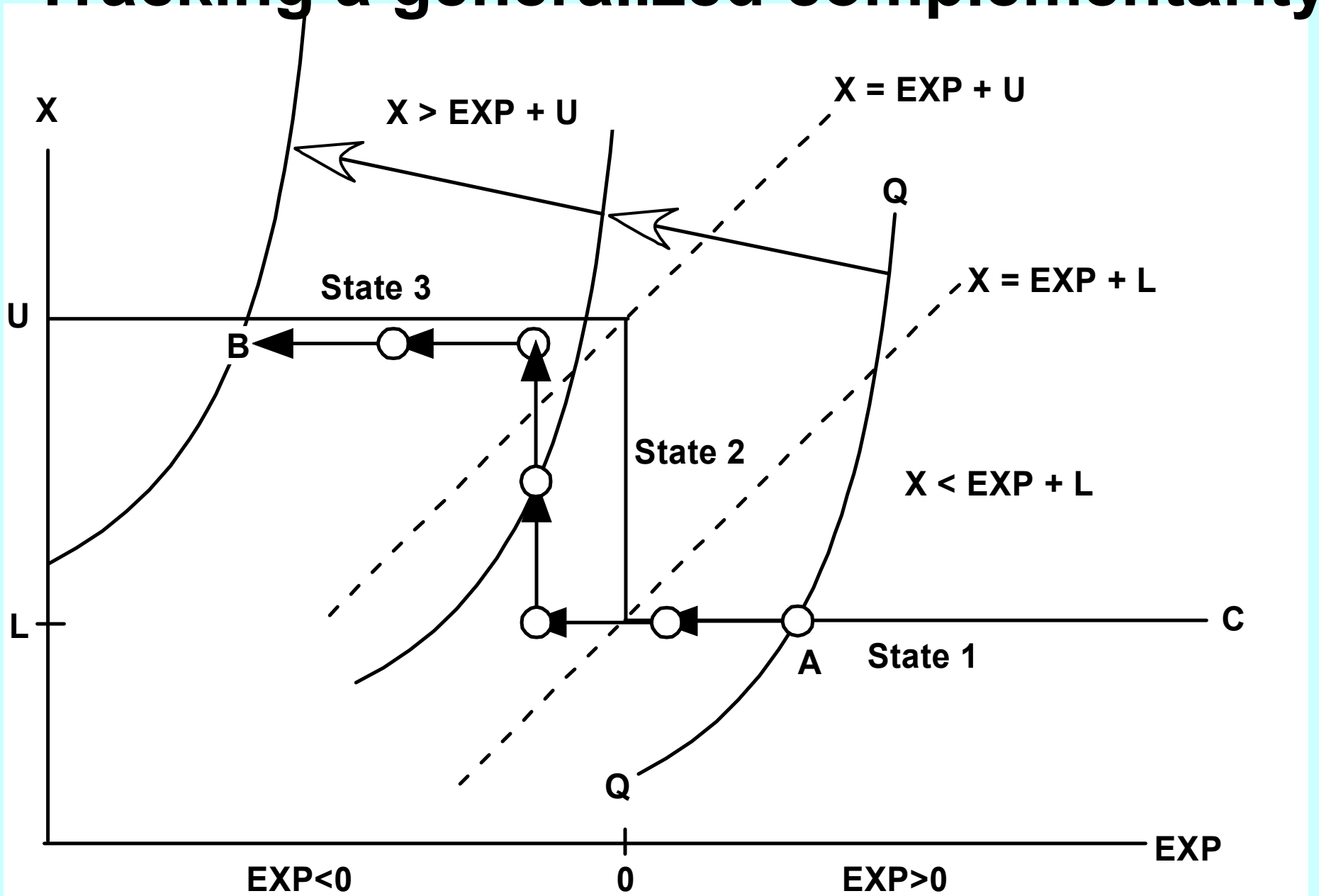


Figure 6: Path-following around a kink

Figure 8:
Tracking a generalized complementarity



The Problem

- **With enough steps we can get a solution as accurate as we want (could take time)**
- **But we can never be sure that taking more, smaller steps will yield more accuracy**
- **So we cannot use extrapolation to get accurate solutions with error bounds at reasonable cost.**

Relation between number of steps and accuracy with smooth function

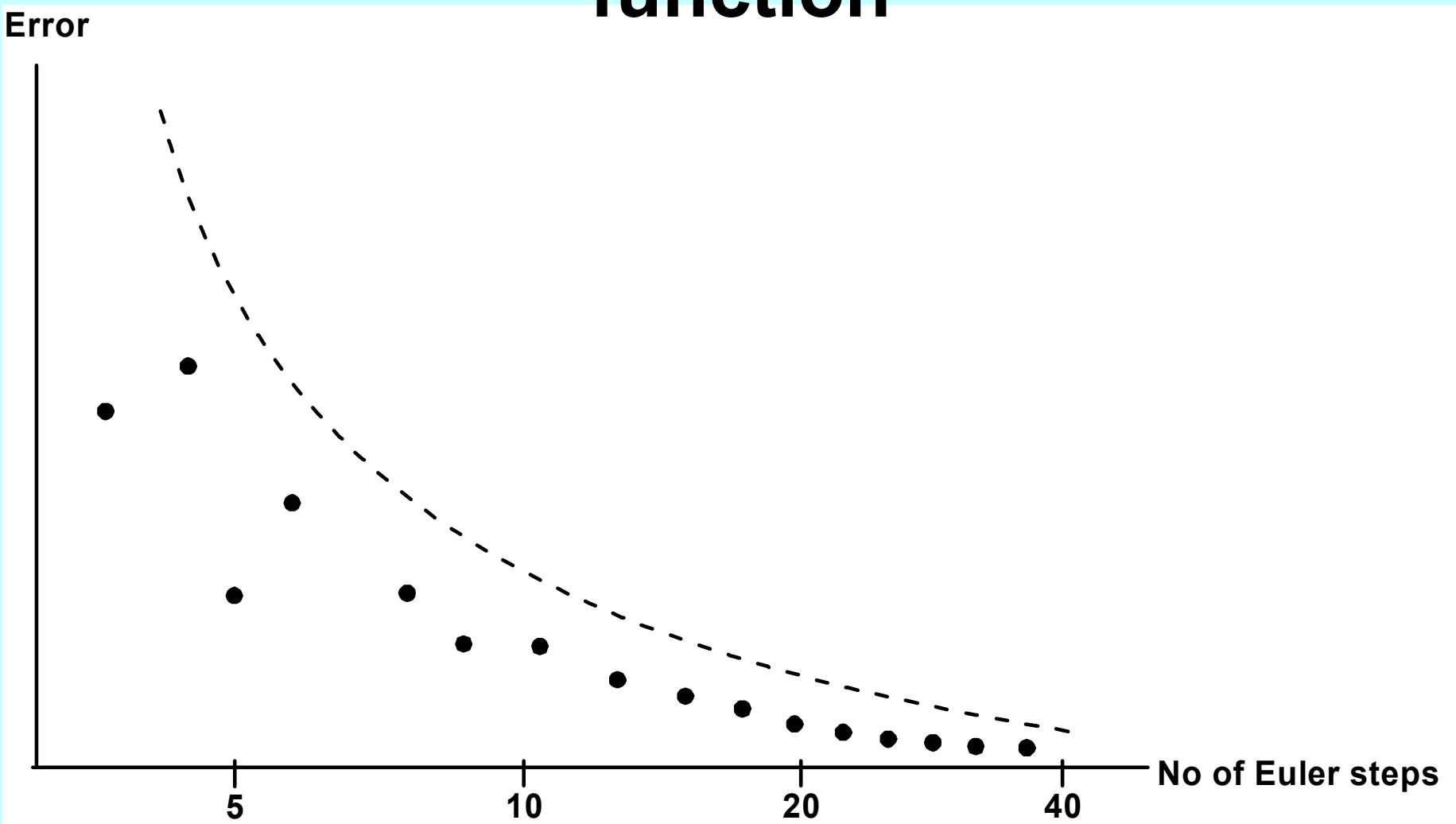
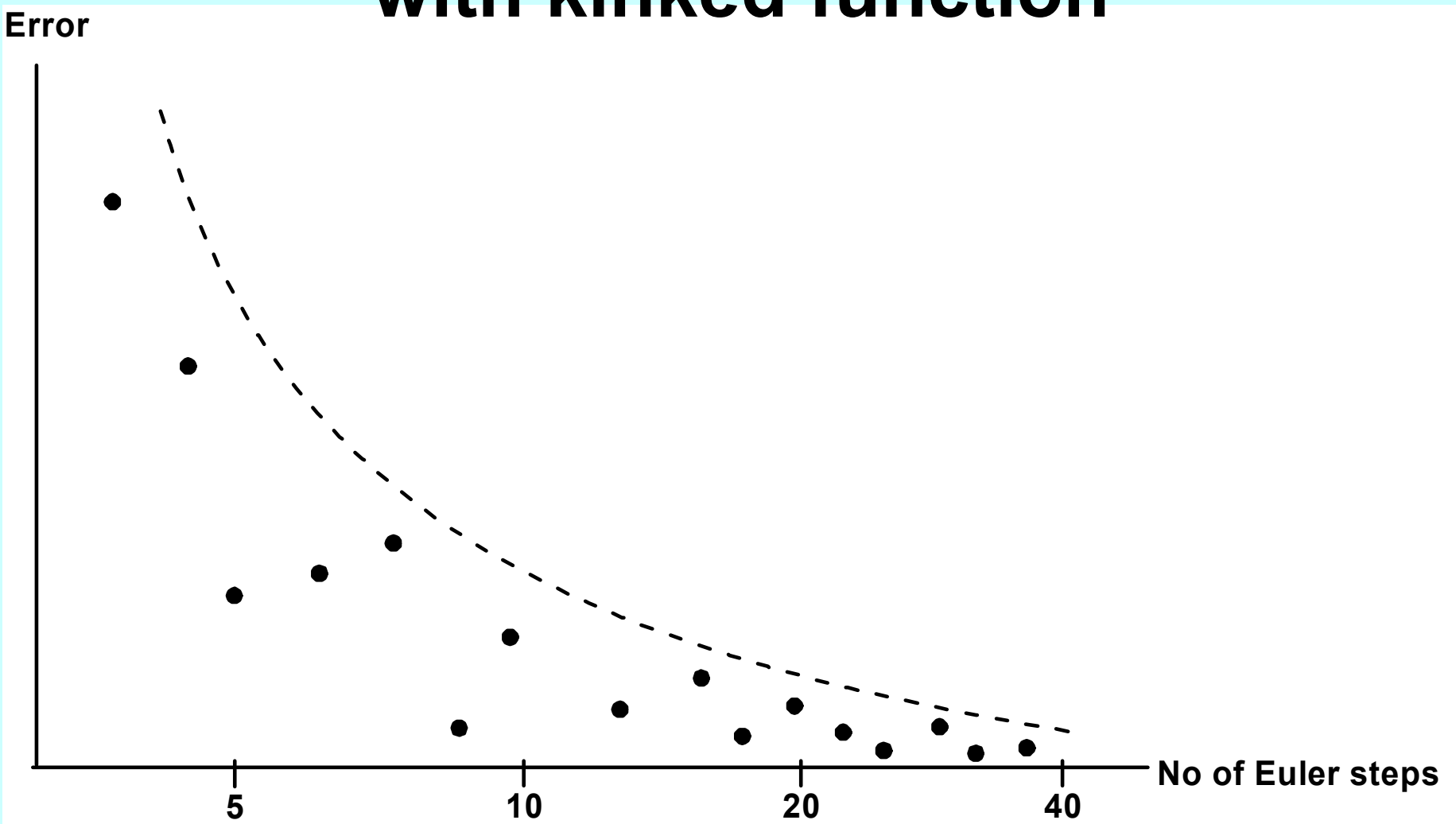


Figure 7: Relation between number of steps and accuracy with kinked function



What if we knew in advance the final (post-solution) state of each complementarity ?

We could replace kinked, non-differentiable functions with smooth functions, and recover good numerical properties, such as efficient, extrapolated solutions.

Figure 8:

Tracking a generalized complementarity

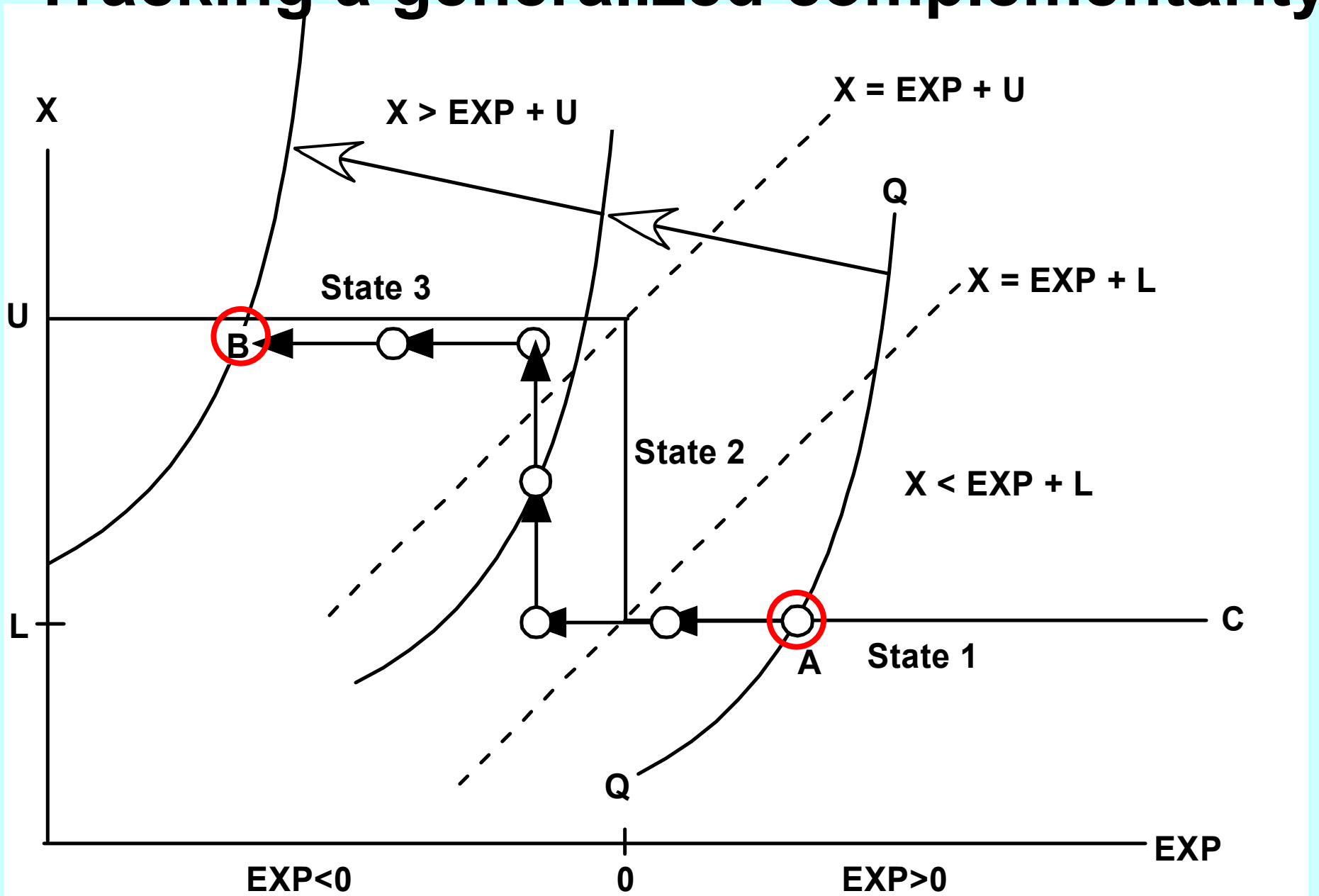
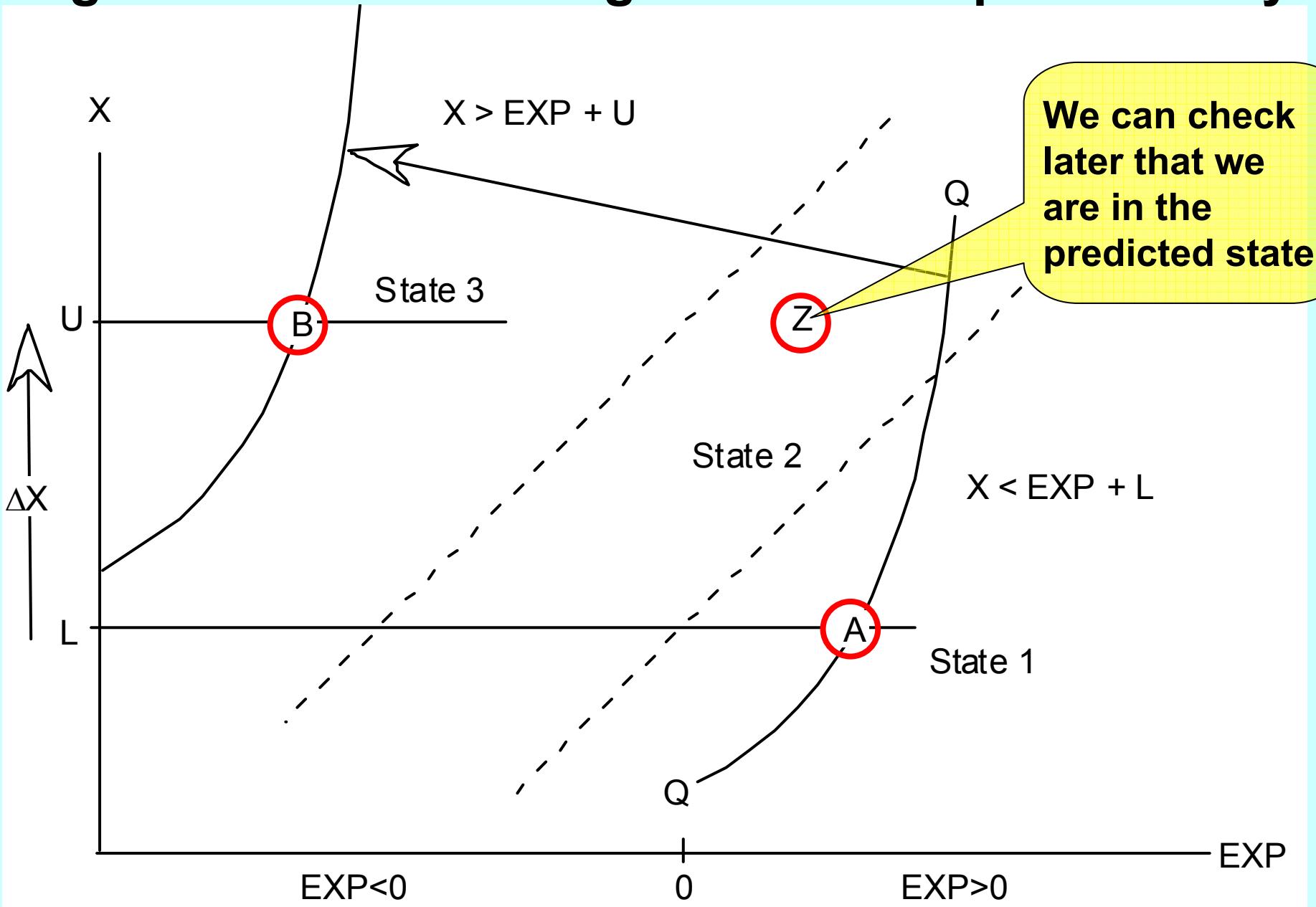


Figure 9: It's easier to ignore the complementarity



TWO-PASS PROCEDURE

- **A single Euler computation, of limited accuracy, is used to discover which constraints will finally bind.**
- **Using this information the equations are recast into an equivalent smooth system which is then solved accurately, using extrapolation.**

Two tricks to improve efficiency of the first, approximate solution

- Newton corrections
- Variable step length

Figure 10: Newton corrections

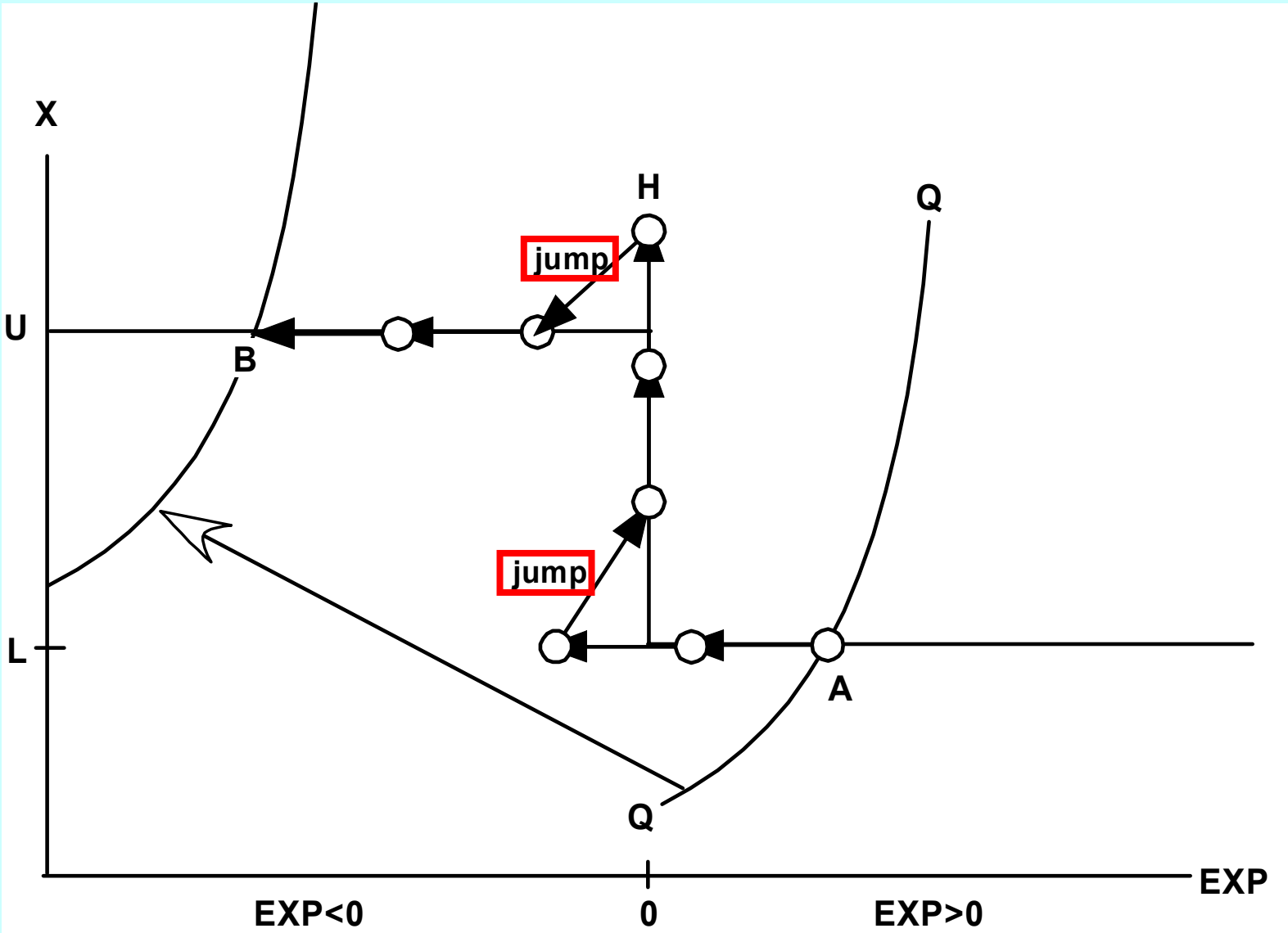
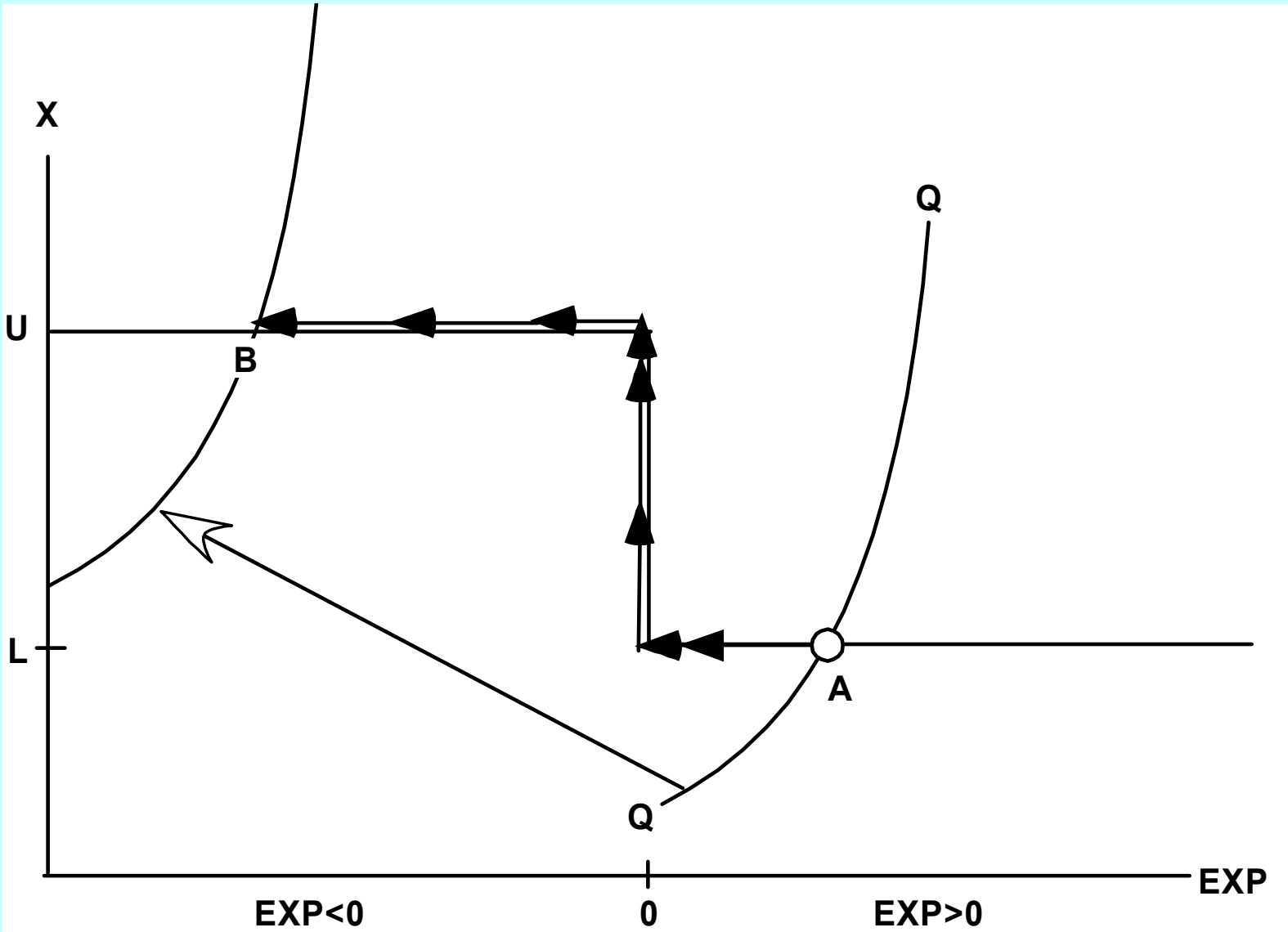


Figure 11: Variable step length



GEMPACK [Levels] Syntax

```
Set COM # commodities #(C1-C27);
File MDATA # data file #;
Coefficient
(all,c,COM) V0CIF(c)  # Ex-duty value of imports of good c #;
Read V0CIF from file MDATA header "POQU";
Variable
(All,c,COM) XIMP_QUOTA(c) # import volume quotas #;
(All,c,COM) XIMP_RATIO(c) # ratios of import volume to import volume quota #;
(All,c,COM) TIMP_QUOTA(c) # EXTRA power of import tariff due to import quota
(All,c,COM) X0CIF_L(c) # Level of import quantity #;
Read TIMP_QUOTA from file MDATA header "POQU";
Read XIMP_RATIO from file MDATA header "MVQ";
Formula
(Initial)(All,c,COM) X0CIF_L(c)= V0CIF(c);
(Initial)(All,c,COM) XIMP_QUOTA(c) = X0CIF_L(c)/XIMP_RATIO(c);

 Equation E_XIMP_RATIO
 (All,c,COM) XIMP_RATIO(c)*XIMP_QUOTA(c) = X0CIF_L(c);
```

```
Complementarity (Variable = TIMP_QUOTA, Lower_Bound = 1)
  IMPQUOTA (All,c,COM) 1 - XIMP_RATIO(c);
```

Questions ?

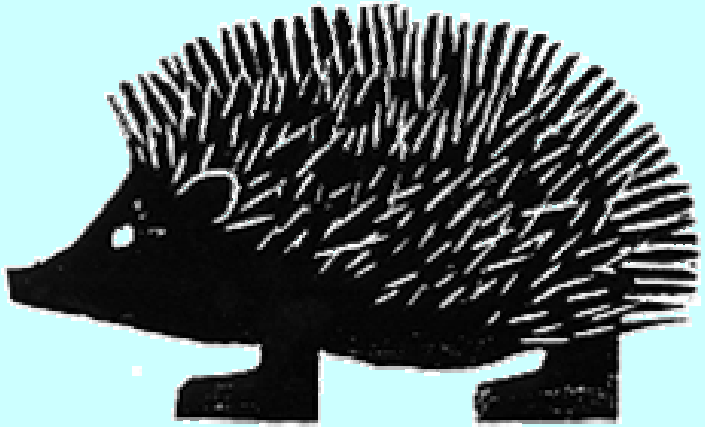


Table 1: Real aggregate household consumption
(% change relative to the base case)

	Rural1	Rural2	Rural3	Rural4	Rural5	Rural6	Rural7	Urban1	Urban2	Urban3
x3tot_hh	-1.042	0.401	-0.237	0.140	-1.425	0.553	-0.137	0.142	-0.705	0.690
w3tot_hh	-3.994	-2.592	-3.223	-2.876	-4.350	-2.429	-3.156	-2.871	-3.683	-2.366
p3tot_hh	-2.982	-2.980	-2.992	-3.011	-2.967	-2.966	-3.023	-3.009	-2.999	-3.035
w0hhinc	-3.229	-1.816	-2.452	-2.103	-3.588	-1.653	-2.385	-2.098	-2.916	-1.588
Decomposition of w0hhinc: rows below add up to row above										
Land	-0.104	-0.222	-1.410	-1.392	-1.001	-0.323	-1.905	-0.584	-0.376	-0.565
Labour	-2.850	-1.587	-0.855	-0.479	-2.412	-0.833	-0.370	-1.485	-2.120	-0.171
Variable capital	-0.109	-0.209	-0.212	-0.227	-0.105	-0.135	-0.171	-0.178	-0.079	-0.214
Fixed capital	-0.043	-0.001	-0.127	-0.124	-0.095	-0.020	-0.184	-0.043	-0.031	-0.037
Taxes & transfers	-0.123	0.203	0.152	0.119	0.025	-0.341	0.245	0.193	-0.310	0.168
Quota rents	0	0	0	0	0	0	0	0	0	-0.769