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COBB-DOUGLAS UTILITY — EVENTUALLY!

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The Centre of Policy Studies (COPS) is a research centre at Monash University devoted to quantitative analysis of issues relevant to Australian economic policy.
Cobb-Douglas-Eventually!

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TOPICS COVERED IN PAPER

- historical review of directly additive preferences (LES)
- Engel flexibility, [Engel] regularity, convergence to Cobb-Douglas — differing opinions about desirability of the latter
- implicit indirect additivity — AIDADS

An Implicitly Directly Additive Demand System
TOPICS [2]

• **problems in CGE sims where there are v. large changes in income**

• **insertion of AIDADS into a CGE model & its calibration — ORANI-G → ORANAIDAD**

• **demonstration of ORANAIDAD’s ability to maintain regularity throughout a large (esp. negative) change in per capita incomes**
TOPICS [3]

• **effectively global regularity of AIDADS** [in the Appendix]

Main ideas to be covered in this presentation:

Engel Flexibility, [Engel] Regularity, Convergence to what as income → ∞?
Relevance for CGE?

If large changes in per capita income are involved, Engel flexibility and regularity are needed to keep the consumer demand system valid over the course of a simulation.
... but AIDS is much more flexible than CD

AIDS - a luxury good
Irregular and inflexible
AIDS - a necessity

Regular but inflexible

Constant budget shares (Cobb-Douglas)

Figure 2.1(a) Engel curves which show globally constant unit elasticity or which are irregular in certain regions (indicated by shading)
Linear Expenditure System

Engel curve for a luxury

\[ p_i \gamma_i < \beta_i p' \gamma \]

\[ W_i = \beta_i + \{ p_i \gamma_i - \beta_i p' \gamma \}/M \]

Linear Expenditure System
LES - a necessity

Why not?

Regular but not very flexible

Figure 2.1(b) Engel Curve in the Linear Expenditure System for a necessity. The irregular region of the LES is indicated by shading.
In the LES, budget shares and expenditure elasticities are monotonic in total expenditure.
Typical monotonic convergence of expenditure elasticities to unity in Engel rank 2 demand systems.
Engel Flexibility—Definitions

• Extremely inflexible

All budget shares are invariant to changes in total expenditure and to changes in relative prices; all total expenditure elasticities are equal to one, and all own price elasticities are equal to minus one; all cross price elasticities are equal to zero—**COBB-DOUGLAS** utility function
Engel Flexibility [2]

- Very inflexible

All expenditure elasticities are globally constant and equal to unity, but with budget shares varying as functions of relative prices — all homothetic utility functions, including CES.
Engel Flexibility [3]

- Somewhat inflexible

Budget shares vary with changes in total expenditure at any given setting of relative prices, while expenditure elasticities vary among commodities and change with changes in total expenditure and changes in relative prices. Total expenditure elasticities are MONOTONIC in real total expenditure —

all demand systems of Engel rank 2 (which means virtually all demand systems commonly in use, including the LES).
Engel Flexibility [4]

• Flexible

Engel elasticities & budget shares are not necessarily monotonic in total expenditure at any given setting of relative prices —

Engel rank 3 demand systems, including AIDADS

An Implicitly Directly Additive Demand System
Monotonic
Non-monotonic
Possible Engel Curves in AIDADS for necessities
Non-parametric empirical work with household survey data establishes that such non-monotonic behavior is a factual attribute of the data: Lewbel (1991), Rimmer & Powell (1994)
Convergence to Cobb-Douglas with increasing per capita income — 1st opinion

(a) As the real income of a consumer becomes indefinitely large, re-mixing the consumption bundle becomes irrelevant: having chosen the ultimately satisfying budget shares at any given set of relative prices, the superlatively wealthy continue to allocate additional income in the same proportions. With very large and increasing per capita income, ultimately the utility function becomes indistinguishable from Cobb-Douglas.
2nd opinion

(b) Consumer demand systems in which the income elasticities monotonically approach one (from above, in the case of luxuries; from below, in the case of necessities) are unsatisfactory both theoretically and empirically. For instance, a necessity with a low (< 1) income elasticity may very well become less elastic with further increases in income.
Figure 3: Behaviour of food’s budget share under AIDADS estimated from 1975 cross sectional international comparisons data and from Australian national accounts time series data. The lowest income country in the data is India and the highest the USA. (Source: Rimmer and Powell (1992).
AIDADS does converge to CD

AIDADS

AIDADS can accommodate inferiority

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Figure 1, page 7D-11, Vol. 3
AIDADS again

Figure 2, page 7D-11, Vol. 3
The graph illustrates the relationship between the natural logarithm of per capita real expenditure and the natural logarithm of real per capita total expenditure. The graph is labeled with three different shares: 1, 2, and 3. Each share corresponds to a different line on the graph, represented by the labels: Engel elas1, Engel elas2, and Engel elas3. The x-axis represents the natural logarithm of real per capita total expenditure, while the y-axis represents the natural logarithm of per capita real expenditure. The graph shows how the expenditure shares change as the expenditure levels increase.
The end