Is International Trade Guilty for an Enlarging Wage Differential?
A Dynamic Intertemporal General Equilibrium Model

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Abstract

There is much literature on the link between international trade, productivity growth and wage differential of skilled and unskilled labor. The theoretical and empirical research is mainly based on the Heckscher-Ohlin framework and on cases of large countries. More comprehensive theoretical models are needed to guide further empirical research. This paper contributes to the debate by constructing a dynamic intertemporal general equilibrium (DIGE) model incorporating endogenous skill formation. Both the short- and long-run transition mechanism with shocks can be illustrated. Also, it proposes education as an important factor in the determination of the wage differential.

The result supports the argument that trade has a responsibility for an increased wage differential. A cut in government education investment tends to enlarge the wage differential. In contrast to the existing literature, this paper suggests that productivity growth may not be the cause of an enlarged wage differential in the long run.

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1. **Introduction**

The relationship between international trade and wages has been a focus of an expanding literature. One viewpoint proposes that international trade is responsible for the enlarging wage differential between skilled and unskilled labor. For example Wood (1995) suggests that trade is the main cause of wage inequality. The other view is that trade is not the main cause of this phenomenon. For example Krugman and Lawrence (1994) point out that skill-biased productivity growth is the main cause of increasing wage inequality. This debate is ongoing in the OECD economies, and extensive research has contributed to large country cases. Only limited research has been based on small open economies. Whether for a large or small economy study, the theoretical and empirical research is mainly based on the Heckscher-Ohlin framework and the econometric methodology. Under the assumption of the Heckscher-Ohlin model, the endowments of skilled and unskilled labor are fixed. Therefore, an increased demand for skilled labor, moving into a situation that skilled labor become scarce relative to unskilled labor, increases the skilled-unskilled wage differential. Allowing numbers of labor endogenized in a general equilibrium system may raise a situation which fails the above concomitant of assuming fixed endowments, especially in the long run. In existing econometric models, either hinging on the Heckscher-Ohlin framework or using somewhat ad hoc basis to select variables. Therefore, more comprehensive theoretical models are needed to guide further empirical research. Since the empirical data come from a dynamic time frame, a theoretical model as playing a leading role in the empirical research should equip a dynamic essence. Based on this dynamic framework, both of the short- and long-run effects impelled by the investigated shocks can be explored.

This paper constructs a dynamic intertemporal general equilibrium (DIGE) model of a small open economy with three-sectors (exports, imports, and non-traded), two kinds of labor (skilled and unskilled), and three-agents (firms, households, and government). The process of skill formation is essential when considering both skilled and unskilled labor. Education is the fundamental input to skill formation. Hence, education production and consumption play important roles in this model. A general picture of the modeling methodology is presented in Section 2. Section 3 illustrates the theoretical framework. Section 4 frames the simulation results. Section 5 shows the sensitivity test of the model and Section 6 concludes.

2. **Modeling Methodology**

Firms employ physical capital, skilled and unskilled labor, to produce three types of goods, i.e. exports, import substitutes, and non-traded, and sell these goods to households for consumption, to government for education capital investment, and to themselves for physical capital investment. The export good is sold abroad based on foreign demand in exchange for imports. Households supply unskilled labor to firms and skilled labor to both firms and government in order to gain wages. Households also own the physical capital and earn financial dividends. Households purchase goods from firms and education from government. Households also consume leisure with an opportunity cost of not working, and

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1 Murphy and Welch (1991), Leamer (1992), and Sachs and Shatz (1994) support this strand.
2 Katz and Murphy (1992), Berman, Bound, and Griliches (1994), and Slaughter and Swagel (1997) support this point.
3 Krugman (1995) debating Leamer (1994) is an impressive example.
4 Chen and Hsu (2001) is a recent contribution for a case study of Taiwan.
5 Bhagwati and Srinivasan (1977) indicate education confers skills.
6 Three types of goods are also characterized as skilled labor intensive, unskilled labor intensive, and capital intensive.
maximize utility by distributing consumption optimally on both goods and leisure. To incorporate the issue of leisure enables this model to take account of the income effect and the substitution effect between working and not working, which is seldom discussed by the existing literature. Government buys goods from firms and transforms them into education capital, together with skilled labor, to produce education. The role of government as an education supplier is essential. This model captures the reality of government supplying education in consideration of the associated beneficial externalities. The government balances its budget by collecting labor income tax and selling education to households7. The accumulation of physical capital, skill formation, education capital and financial assets drives the dynamic evolution of the economy over time.

The numbers of workers in the two categories of skilled and unskilled labor can vary over time. In addition to government investment in education and training facilities, the motivation of households to pursue higher skills is an important factor when considering changes in the proportions of labor in different categories. The main incentive for acquiring skills is higher future income. This motivates candidates to forsake current unskilled wage, and to invest, by time consumption and direct cost (tuition), in upgrading their education level8. To model the transformation of unskilled into skilled labor, the method assumes a representative agent. A portion of the person serves as skilled labor and the remaining portion as unskilled labor. In each period of time, this representative person makes a choice to invest in education to gain skill, which is subject to an increasing convex cost of adjustment function. The skilled labor faces no transformation cost to work as unskilled labor. The equilibrium skill formation depends on the fixed skill formation, skill depreciation rate, and the skill stock as well as the demand for skilled labor. Due to the assumption of the mobility of labor internally, there is a single nominal wage for skilled workers and a single nominal wage for unskilled workers.

The elasticity of the skilled labor supply, with respect to the wage of skilled labor, is greater than zero, since the supply of skilled labor is not fixed. It is less than infinity in the short run, because the transformation from unskilled to skilled labor is not fully free. Some skills are specific or patented, and training facilities are not always established quickly. It is also not perfectly elastic in the long run. There are four main reasons proposed on this point by Wood (1994), but only two are appropriate in this model9. First, the imperfection of capital markets, which means the opportunity to acquire loans for schooling is not equal for all unskilled labor, emerges the issue of unequal availability of skill acquisition. While becoming a full time student means only a decrease in savings for some, it can mean starvation for others. Second, variations in trainability amongst unskilled labor arise from differences in innate ability and family background (Phelps 1977). These two factors provide the accuracy of embodying a positive and limited elasticity of skilled labor supply in the model.

3. The Model

This model follows the general approach of the G-Cubed model (McKibbin and Wilcoxen 1999) but with the first attempt to incorporate issues of education and of skill formation.

7 To avoid unnecessary complexities, a subsidy rate on investment and a tax rate on financial dividends is assumed to balance.
8 In the ensuing model, education is the necessary channel for acquiring skills. Learning by doing is treated as a skill improvement in the same category, not jumping into a high level. Spill over effect is embedded into the education investment function.
9 The other two reasons are that skill transformation is increasing returns, and external, which are proper to explain the enduring gap of skill levels between North and South.
3.1 Firms
There are three representative firms in the economy: an export sector, an import substituting sector, and a non-traded sector. A Cobb-Douglas production function is employed\(^{10}\),

\[
Q_i = A_{Qi} K_i^{\alpha_i} L_{i,i}^{\beta_i} L_{u,i}^{1-\alpha_i - \beta_i},
\]

where \(Q_i\) is the gross output, \(A_{Qi}\) is the sectoral technology variable, \(i\) is the sector index and \(i = 1, 2, 3\). The three inputs are capital \(K_i\), skilled labor \(L_{i,i}\), and unskilled labor \(L_{u,i}\), and \(\alpha_i, \beta_i, (1-\alpha_i - \beta_i)\) are, respectively, the shares of employment of capital, skilled labor, and unskilled labor in production.

The sector with a larger share of skilled labor is defined as a relatively skilled intensive sector. The sector with a larger share of unskilled labor is defined as a relatively unskilled intensive sector. And the sector with a larger share of capital is defined as relatively capital intensive. To initialize the model, it is assumed sector 1 is relatively skilled labor intensive, sector 2 is relatively unskilled labor intensive, and sector 3 is relatively capital intensive. It is also assumed this small open economy exports good 1, imports good 2 and good 3 is non-traded. That is, the representative firms export the relatively skilled intensive and import the relatively unskilled intensive goods. Table 1 illustrates this characteristic. The other combination of trade attributes and factor intensities are tested in Section 5 to see if the result from this designation is sensitive.

[Insert Table 1 Here]

Export demand \((X)\) is a function of foreign income \((Y^*)\) and the inverse of terms of trade as follows,

\[
X = \rho \left(\frac{P_2}{P_1}\right)^{\rho} \cdot Y^*,
\]

where \(\rho\) is a positive parameter. Equation (3.2) states that exports of the domestic country are boosted by an increase in foreign income, a decrease in export price \((P_1)\), and an increase in import price \((P_2)\). Exports decrease when the opposite is true. This model makes a case that the exported good is an imperfect substitute for foreign goods, whereas the imported good is a perfect substitute for the domestic good 2. To simplify, it is assumed that trade is balanced in all periods.

Capital accumulation in each sector depends on the rate of gross fixed capital formation \((J_i)\) and the rate of depreciation \((\delta_i)\),

\[
\frac{dK_i}{dt} = J_{i,i} - \delta_i K_{i,i}.
\]

Under the assumption of rising marginal costs of installation in the investment process, the total investment expenditure \((I_i)\) in sector \(i\) is

\(^{10}\) Every variable implicitly carries a subscript of time.
where \( \phi_i \) is a positive parameter, and \((\phi_i/2)(J_i/K_i)\) is the unit costs of adjustment, which is assumed to be a linear function of the rate of capital formation.

The firm maximizes intertemporal profit, and chooses \( L_s^e, L_u, \) and \( J \) subject to equations (3.3) and (3.4) by solving the current value Hamiltonian function with \( \lambda_i \) the shadow price of capital. The first order conditions state that the real return to factors is equal to its marginal productivity under perfect competition. The shadow price of capital is greater than one due to the adjustment cost. By solving the first-order differential equation and applying the transversality condition, the shadow price of capital becomes

\[
\lambda_i(t) = \int_t^\infty (Q_{Ki} + \Theta_i) e^{-sr_i} \cdot \delta \cdot ds,
\]

where \( \Theta_i = (\phi_i/2)(J_i/K_i)^2 \) and \( r \) is the discount rate. Equation (3.5) states that the shadow price of capital is equal to the present discounted value of future marginal products. It consists of two parts: \( Q_{Ki} \) is the marginal product of capital and \( \Theta_i \) is the marginal product of capital in reducing adjustment costs in investment at each point in time. Therefore, \( \lambda_i \) is the increment to the real value of the firm from one unit of installed capital at time \( t \).

### 3.2 Household

The household distributes after-tax labor income and dividends to consumption of the three goods, financial asset accumulation for future consumption and education investment, together with the choice of leisure, to maximize utility. The objective function of the household, the intertemporal budget constraint, a dynamic accumulation of skilled labor, the financial assets ownership, and education investment are shown as follows,

\[
\text{Max. } \int_0^\infty U(C_{1,t}, C_{2,t}, C_{3,t}, l_t) \cdot e^{-\theta \cdot dt}.
\]

Subject to

\[
\frac{df^e_t}{dt} = r_t \cdot F_t + \left(1 - \pi_t \right) \cdot \left( W_{s,t} \cdot L_{s,t} + W_{l,t} \cdot L_{l,t} \right) - \left[ \frac{P_{2,t} \cdot F_{2,t}}{P_{2,t} \cdot C_{2,t}} + C_{2,t} \right] \cdot C_{3,t} + \left( \frac{P_{E,t} \cdot S_{E,t}}{P_{E,t} \cdot C_{E,t}} \right) \cdot S_{E,t},
\]

\[
\frac{dl_{s,t}}{dt} = l_{s,t} - \delta \cdot L_{s,t},
\]

\[
F_t = \left( \frac{P_{2,t}}{P_{2,t}} \right) \cdot \lambda_{1,t} \cdot K_{1,t} + \lambda_{2,t} \cdot K_{2,t} + \left( \frac{P_{2,t}}{P_{2,t}} \right) \cdot \lambda_{3,t} \cdot K_{3,t},
\]

\[
S_{E,t} = l_{s,t} \cdot \left[ 1 + \frac{\Phi}{2} \cdot \left( \frac{J_{i,t}}{L_{s,t}} \right) \right],
\]
where $C_{i,t}$ is the consumption of goods, $l_t$ is leisure, $\theta$ is the rate of time preference, $F_t$ is financial assets, $\tau_t$ is the tax rate on labor income, $P_{E,t}$ is the price of one unit of education, $S_{E,t}$ is the amount of education buying, $J_{s,t}$ is the fixed skill formation, $\delta_t$ is the depreciation rate of skill, and $\Phi$ is the adjustment cost parameter.

Equation (3.7), the budget constraint, states that the accumulation of the household's financial assets depends on financial dividends, total after-tax labor income and total spending on goods and education. Equation (3.8) states that the net skill formation is the skill depreciation subtracted from the fixed skill formation. Equation (3.9) shows the contents of financial assets, which includes the value of capital in each sector, so-called equity. Equation (3.10) states that education investment depends on fixed skill formation and an adjustment cost. The adjustment cost reflects the foregone production and relies on the ratio of fixed skill formation to skilled labor. If skilled labor is increasing, the adjustment cost is decreasing. It is plausible due to the spillover effect among labor. Leisure is defined as the remaining time after deducting total labor hours.

The current value Hamiltonian function is employed to solve the above autonomous two-state variables system with $\mu_1$ and $\mu_2$ the shadow prices respective to the financial assets and skill. The first order conditions imply $\mu_t = MU_1/P_1 = MU_2/P_2 = MU_3/P_3$, where $MU_i$ stands for the marginal utility of consuming good 1, good 2 and good 3, respectively. It demonstrates that marginal utility should be the same for each good to achieve optimality. The shadow price of skill is greater than the shadow price of the financial asset due to the adjustment cost of skill formation. If the shadow price of skill is not greater than that of the financial asset, the household would defer spending on skill formation instead of accumulating financial assets for future consumption.

Applying the transversality condition to the shadow price of skill, $\mu_2$, results in

$$
(3.11) \quad \mu_2(t) = \int_t^\infty \{\mu_1 \cdot [(1 - \tau_t) \cdot \frac{W_{s,t}}{P_{2,t}} + \frac{P_{E,t}}{P_{2,t}} \cdot \Phi \cdot \left(\frac{J_{s,t}}{L_{s,t}}\right)^2] + U_{L_{s,t}} \} \cdot e^{-(\theta + \delta) \cdot t} \cdot dt,
$$

where $U_{L_{s,t}}$ is the partial derivative of the utility function with respect to skilled labor. The transversality condition asserts that, in an infinite time horizon, the present value of the shadow price of one additional skilled labor formed is equal to zero. It eliminates the case of infinite accumulation of skilled labor. This assertion is plausible because the variation of the skilled wage initialized by the movement of the skilled labor supply and demand can tie down infinite accumulation of skilled labor. Equation (3.11) states that the shadow price of skill is equal to the present discounted value of future marginal utility. The first component of the shadow price of skill contains the marginal utility of consuming goods, the after-tax skilled wage, and the reduction of adjustment cost in education investment. It provides the gross increment of utility the household gains from supplying one additional unit of skilled labor. The second part is the marginal disutility of offering one unit of skilled labor. Combining these two ends with the net utility the household can achieve by supplying one unit of skilled labor, which substantiates the essence of $\mu_2$.

To derive consumption on goods and leisure, a Cobb-Douglas utility function is assumed, resulting in
\begin{align}
C_i &= \frac{\gamma_i \cdot U \cdot P_2}{\mu_i \cdot P_1}, \\
I &= \frac{(1 - \gamma_1 - \gamma_2 - \gamma_3) \cdot U \cdot P_2}{(1 - \tau) \cdot \mu_i \cdot W_g}.
\end{align}

Equations (3.12) and (3.13) state, ceteris paribus, that when the price $P_i$ or net unskilled earnings increases, the household decreases demand for goods or leisure. Also, when consumption goes up with fixed prices, the marginal utility $\mu_i$ decreases, i.e., the law of diminishing return. This is an outcome of the concave utility function. Equation (3.13) also shows that, if income tax increases, leisure increases. This corroborates that heavy taxation lessens the motivation for working in a partial equilibrium.

3.3 Government

The government collects income tax and sells education $(P_{E,i} \cdot S_{E,i})$ to finance total spending, and also has to buy good 1 $(I_{E,1}^G)$, good 2 $(I_{E,2}^G)$, and good 3 $(I_{E,3}^G)$ to produce education capital $(K_{E,J})$. Total government investment on education capital is represented by $I_E^G$. It is assumed that $I_E^G$ is exogenously controlled by the government. To produce education, the government employs skilled labor $(L_s^E)$ as well as using education capital,

\begin{equation}
S_{E,J} = K_E^L \cdot L_{s,E}^{G_{s,E}} ,
\end{equation}

where $\xi$ is the input share parameter.

The accumulation of education capital results from the total investment of government after subtracting depreciation,

\begin{equation}
\frac{dK_{E,J}}{dt} = I_{E,J}^G - \delta_E \cdot K_{E,J} ,
\end{equation}

where $\delta_E$ is the rate of depreciation and $I_E^G$ is defined as follow,

\begin{equation}
I_E^G = \frac{1}{P_E^G} \cdot (P_1 \cdot I_{E,1}^G + P_2 \cdot I_{E,2}^G + P_3 \cdot I_{E,3}^G) ,
\end{equation}

where $P_E^G$ is the weighted price index defined as below,

\begin{equation}
P_E^G = P_1^{\epsilon_1} \cdot P_2^{\epsilon_2} \cdot P_3^{\epsilon_3} ,
\end{equation}

where $\epsilon_i$ is the weight of this pooled index.
Government spending on each good for education investment follows \( I_{G,E}^G = \varepsilon_i \cdot \frac{P_{E}^G \cdot I_{E}^G}{P_i} \). To assure the model is consistent, as well as the economy is in equilibrium, the rule of demand equal to supply is applied on both sides of the goods and factor markets.

### 3.4 Steady State

Table 2 summarizes this model in the steady state. It should be highlighted that the rate of time preference is equal to the rate of interest in the steady state. If the rate of time preference is greater than the interest rate, the household will decumulate financial assets instead of increasing consumption. This will raise the interest rate till it reaches the level of time preference. The adjustment reverses if the interest rate is greater than the rate of time preference. Hence, those two rates have to be equal in equilibrium. The shadow price of capital is well known as marginal Tobin’s \( q \), noted as \( q_i^M \). In the steady state, it follows

\[
q_i^M = 1 + \delta_i \cdot \phi_i .
\]

**[Insert Table 2 Here]**

Hence, without subsidy of investment, the marginal Tobin’s \( q \) is greater than 1 when an adjustment cost exists, and equal to 1 when the adjustment cost is zero. Similarly, the marginal Tobin’s \( q \) of skill, noted as \( q_s^M \), has a relationship with the shadow price \( \mu_2 \). That is \( q_s^M = \mu_2 \), and in the steady state,

\[
q_s^M = \mu_i \cdot P_e \cdot (1 + \delta_s \cdot \Phi) .
\]

Equation (3.19) states that, marginal Tobin’s \( q \) of skill is equal to the shadow price of financial assets, i.e., the marginal utility of consuming goods, times the total cost of one unit of skill formation. It asserts that skill generates higher wages, hence, more utility. The product of education price and the bracket is equivalent to the total price of upgrading one unit of skill. Hence, the multiple on the right-hand side is the marginal utility the household can have after gaining one unit of skilled labor. To maximize utility, the household will invest in skill formation until its generation of marginal utility is equal to its marginal cost.

### 3.5 Wage Differential

Due to the complexity of the above model, there is no reduced form that can be derived to present the wage differential. A possible simple expression of the wage differential in the steady state is as follows,

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11 This result is consistent with the argument in chapter two of Blanchard and Fischer (1989).
12 After the subsidy of investment is embedded, the marginal Tobin’s \( q \) is smaller when the rate of subsidy is higher. That is to say, if the government subsidizes investment with a higher rate, the investment cost is lower and will encourage firms to invest.
13 It can be regarded as human capital.
The expression of equation (3.20) is independent of the functional form of utility and production function\(^{14}\). It provides a rigorous theoretical result of the wage differential. The relationship between the skilled and unskilled wage depends on the rate of time preference, the depreciation rate of skill, the skill adjustment cost parameter, the tax rate, and the price of education. A higher skill adjustment cost and a higher skill depreciation rate tend to boost the cost of skill formation, therefore leading to a higher skilled wage. The rate of time preference counts because an investment in skill formation takes time to repay. A larger time preference involves a larger adjustment cost for skill formation, therefore a patient household will expect a higher skilled wage. The education price and the tax rate are endogenous in this model. Theoretically, each endogenous variable can be solved and substituted by the exogenous variables and parameters, with equation (3.20) becoming to the following,

\[
W_s = W_u + \frac{P_s}{(1 - r)} \cdot (\theta + \Theta \cdot \Phi \cdot \delta_s + \delta_s + \frac{1}{2} \cdot \Phi \cdot \delta_s^2).
\]

The wage differential equation (3.20) illustrates both the importance of, and the transmission channel of, education in the determination of the wage differential. This is the reason for the inclusion of education in the debate on the wage differential, in addition to the traditional arguments of trade and productivity growth. The government, as an education supplier and tax collector, has the ability to control the wage differential to a certain extent. What matters in a general equilibrium outcome is the interactive effect of the education price and the tax rate. Simulation becomes necessary to explore the short- and long-run transitions of each endogenous variable and so establish the policy implications.

4. Simulation Results

A number of simulations are considered in this section in order to explore the long run, defined as the steady state, and short run relationships between variables in the model. The exogenous variables are technology, time, government investment on education, and foreign income. All other variables are endogenous. The setting of the parameters and exogenous variables is presented in the Appendix \(^{15}\). The experiment is to investigate the transition of all endogenous variables between steady states in five cases: a 10% improvement in technology in each sector, a 10% increase in government education investment, and a 10% increase in foreign income. It should be noted that this model simultaneously provides the cases of total factor productivity (TFP) and input efficiency growth in each sector. The dynamic path of input efficiency shock is not shown because it is similar to the shocks of TFP\(^{16}\). An aggregation of three sectors leads to a case of factor biased productivity growth, a reason emphasized by Krugman and Laurence (1994) of causing enlargement of the wage differential. However, this

\[^{14}\text{A detailed proof is available from the author.}\]
\[^{15}\text{The calibration is based on good empirical knowledge albeit subjective.}\]
\[^{16}\text{This model can easily pull in the input efficiency factor. The production function becomes } Q_i = A_0 (c_{ik} \cdot K_i)^{\alpha_i} \cdot (c_{is} \cdot L_i^{\beta_i})^{\beta_i} \cdot (c_{iu} \cdot L_u)^{1-\alpha_i-\beta_i}, \text{ where } c_{ij}, j = k, s, u, \text{ represents the relative efficiency of factor } K, L_u, L_u \text{ in sector } i. \text{ Obviously, the upgrade of input efficiency reaches a similar result, but in a different level of TFP growth.}\]
model allows the supply and demand of skilled labor to determine the skilled wage rather than the scenario in their paper, which asserts the demand in skill dominates the jump of skilled wage\textsuperscript{17}.

The simulation paths are presented in Figure 1. Table 3 summarizes the results. The main results from this model are that, in the long run, productivity growth and an increase in government education investment lessen the wage differential.\textsuperscript{18} Generally speaking, increased education investment also lessens the wage differential in the short run, albeit with a fluctuation at the early stage. (The fluctuation occurs because the adjustment process of skill formation takes time and households make optimal choices between working and leisure.) Productivity growth, at most, raises the wage differential only in the short run: it may reduce the wage differential in the short run if productivity growth is biased towards the sector which is intensive in unskilled labor. An increase in international trade via an increase in foreign income raises the wage differential to a larger extent in the short run than in the long run.\textsuperscript{19}

Intuitively, an increased demand for skilled labor resulting from a growth in productivity or in exports can eventually be filled in the long run as skill supply plays an important role in the wage determination. In the short run, the created demand cannot be filled immediately due to the time required for skill formation. By using equation (3.20), the transitions are as follows. Productivity growth pushes down goods prices. This reduces the costs of government purchases and motivates the government to cut the tax rate which, in turn, decreases the wage differential. If the government increases education investment, thereby decreasing the education price, it can cause diminution of the wage differential. While the international trade factor is not explicitly shown in equation (3.20), its effect is transmitted from production to wages through the education price. An increase in skill-intensive exports boosts the price of exports and the demand for skilled labor. This increases the demand for education and, hence, the price of education. Therefore, the wage differential rises.

[Insert Figure 1 Here]

[Insert Table 3 Here]

From a theoretical perspective it is unconvincing that productivity growth raises the wage differential in the long run since skill formation eventually catches up with the progress of technology as long as the adjustment cost is affordable for the unskilled labor.

5 Sensitivity Test

Since there are three sectors with a different intensity of each factor, and five different shocks—technological progress in sectors 1, 2 and 3, government education investment and foreign income,

\textsuperscript{17} To adapt this model to the scenario in Krugman and Laurence (1994), either to increase the adjustment cost of skill to a fairly high extent to make it very hard for households to transform skill, or to increase the shares of skilled labor in all sectors to force the skill demand dominating the skilled wage.

\textsuperscript{18} Productivity growth lessens the wage differential to a small but non-zero extent.

\textsuperscript{19} In the long run, the wage differential is enlarged to a small but non-zero extent.
there is a total of thirty cases within this framework. Sector 1 is designated the export sector, sector 2 the import sector, and sector 3 the non-traded sector. The key variable investigated is the change of the wage ratio in the steady state. The results, set out in Table 4, show that this model is fairly robust. In Table 4, the first column gives the combination of three sectors with different intensities of inputs. Numbers stand for sectors and letters stands for inputs, for example 1U2S3K represents sector 1 as unskilled-labor intensive, sector 2 as skilled-labor intensive and sector 3 as capital intensive. The second to sixth columns respectively stand for an improvement in technology in sectors 1, 2 and 3, an increase in government education investment and an increase in foreign income. A minus sign (-) means a decreased wage differential and a plus sign (+) means an increased wage differential.

To summarize, on the one hand, the effect on the wage differential of an improvement in technology in any of the sectors or of increased government investment in education is insensitive to different combinations of sectors, that is, in each case the wage differential decreases in the long run, whereas, on the other hand, the effect on the wage differential of a foreign income shock that raises exports is sensitive to different combinations of sectors. If the export sector is skilled intensive, an increase in export boosts the wage differential; if the export sector is unskilled intensive, an export expansion reduces the wage differential; if the export sector is capital intensive, an increase in exports may either boost or reduce the wage differential.

6 Conclusion

This paper discusses the relationship between productivity growth, international trade, education, and the wage differential by constructing a DIGE open economy model. The main insight of this model is the choice of the household to undertake skill formation. This varies the endowment of skilled labor from short to long run. The result provides a comparison to a standard result from the Heckscher-Ohlin model in which skill endowments are fixed. Education is shown to be quite important yet its effect on the wage differential is seldom discussed in the literature. It would be biased to only focus on the external trade effect in this issue. At the same time, an internal education policy should be recruited into the debate. The education issue is especially important when a developing country is investigated, due to the scarcity of skilled labor relative to a developed country. It is crucial to embed the dynamic adjustment process for guiding future econometric modeling in which data is used from a dynamic adjustment period. This model also shows the steady state relation which underlies the literature, can be misleading in the short run.

The main results from this model are that, in the long run, productivity growth and/or an increase in government education investment decrease the wage differential. At most productivity growth raises the wage differential in the short run. An increase in trade creates the wage differential. Furthermore, in the long run, an increase in government education investment has a relatively larger effect on the wage differential than the shocks of productivity growth and international trade. This model shows that a 10% increase in government education investment reduces the wage differential by about 0.76%. A 10% improvement in technology reduces the wage differential by around 0.02 to 0.03% and a 10% increase in foreign income enlarges the wage differential by about 0.02%. To summarize the results and policy implications from thirty cases by combining factor intensity in each sector with trade characteristic and shocks, the wage differential tends to decrease with a productivity upgrade. It is also
found that if the export sector is skilled labor intensive, an increase in export boosts the wage differential; if the export sector is unskilled labor intensive, an export expansion reduces the wage differential; if the export sector is capital intensive, an increase in exports may either boost or reduce the wage differential. This small-open-economy model implies that a developed country’s trade with a developing country enlarges the wage differential in the developed country and reduces the wage differential in the developing country. The result also tends to substantiate the argument that trade has a responsibility for the wage differential. From a theoretical perspective it is unclear how productivity growth raises the wage differential in the long run due to skill formation eventually catching up with the progress of technology.

A couple of extensions of this model can be undertaken in future research to explore other interesting issues. For example, the effect of tariffs can be included. A test of the impact of unskilled labor immigration would be important for some countries. The model could also be extended to the case of an open capital account which may have important implications for the adjustment process to a range of shocks.
### Tables

#### Table 1 Sector Characteristic

<table>
<thead>
<tr>
<th>Sector</th>
<th>Factor Intensive</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Skilled</td>
<td>Export</td>
</tr>
<tr>
<td>2</td>
<td>Unskilled</td>
<td>Import substitute</td>
</tr>
<tr>
<td>3</td>
<td>Capital</td>
<td>Non-traded</td>
</tr>
</tbody>
</table>

#### Table 2 Model in the Steady State

<table>
<thead>
<tr>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_i = A_{Qi} \cdot K_i^{\alpha_i} \cdot L_{s,i}^{\beta_i} \cdot L_{u,i}^{1-\alpha_i-\beta_i}$</td>
</tr>
<tr>
<td>$J_{t,i} = \delta_i K_{t,i}$</td>
</tr>
<tr>
<td>$I_t = J_t \cdot (1 + \phi \delta_t/2)$</td>
</tr>
<tr>
<td>$Q_{l,s} = W_s/P_i$</td>
</tr>
<tr>
<td>$Q_{l,u} = W_u/P_i$</td>
</tr>
<tr>
<td>$\lambda_t = 1 + \phi_t \delta_t$</td>
</tr>
<tr>
<td>$Q_{K_t} = (r + \delta_t) \lambda_t - \phi_t \delta_t^2/2$</td>
</tr>
<tr>
<td>$P_{2,t} \cdot M_{t} = P_{1,t} \cdot X_t$</td>
</tr>
<tr>
<td>$X = (P_2/P_1)^\rho \cdot Y^*$</td>
</tr>
<tr>
<td>$\theta = r \cdot F_t + (1-\tau_t) \cdot [(W_s/P_2) \cdot L_{s,t} + (W_u/P_2) \cdot L_{u,t}] - [(P_1/P_2) \cdot C_{1,t} + C_{2,t} + (P_2/P_2) \cdot C_{3,t} + (P_E/P_2) \cdot S_{E,t}]$</td>
</tr>
<tr>
<td>$J_{s,t} = \delta_s L_{s,t}$</td>
</tr>
<tr>
<td>$F_t = (P_1/P_2) \cdot \lambda_{t,t} \cdot K_{1,t} + (P_2/P_2) \cdot \lambda_{2,t} \cdot K_{2,t} + (P_3/P_2) \cdot \lambda_{3,t} \cdot K_{3,t}$</td>
</tr>
<tr>
<td>$l_{t} = T - L_{s,t} - L_{u,t}$</td>
</tr>
<tr>
<td>$U_{C_t} = (P_2/P_2) \cdot \mu_t$</td>
</tr>
<tr>
<td>$U_{L_{s,t}} = \mu_t \cdot (1 - \tau) \cdot W_s/P_2$</td>
</tr>
<tr>
<td>$\mu_2 = \mu_1 \cdot (1 + \Phi \cdot \delta_t)/P_2$</td>
</tr>
<tr>
<td>$r_t = \theta$</td>
</tr>
<tr>
<td>$U_{L_{s,t}} = (\theta + \delta_t) \cdot \mu_2 - \mu_1 \cdot [(1 - \tau) \cdot W_s + P_E \cdot (\Phi \cdot \delta_t^2)/2]/P_2$</td>
</tr>
<tr>
<td>$S_{E} = K_{E} \cdot L_{s,t}^{G_{s,t}}$</td>
</tr>
<tr>
<td>$I_{E}^{G} = \delta_E \cdot K_{E}$</td>
</tr>
<tr>
<td>$I_{E}^{G} = (P_{1} \cdot I_{E,1}^{G} + P_{2} \cdot I_{E,2}^{G} + P_{3} \cdot I_{E,3}^{G}) / P_{E}^{G}$</td>
</tr>
<tr>
<td>$P_{E}^{G} = P_{1}^{G} \cdot P_{2}^{G} \cdot P_{3}^{G}$</td>
</tr>
<tr>
<td>$P_{E}^{G} \cdot I_{E}^{G} + W_s \cdot L_{s}^{G} = \tau \cdot (W_s \cdot L_s + W_u \cdot L_u) + P_E \cdot S_{E}$</td>
</tr>
<tr>
<td>$Q_{l,s} = C_t + I_{E,1}^{G} + I_1$</td>
</tr>
<tr>
<td>$Q_{l,u} + M_{t} = C_2 + I_{E,2}^{G} + I_2$</td>
</tr>
<tr>
<td>$Q_{3,t} = C_3 + I_{E,3}^{G} + I_3$</td>
</tr>
</tbody>
</table>
### Table 3: The Results of the Wage Differential to Each Shock

<table>
<thead>
<tr>
<th>Wage Differential</th>
<th>Tech 1</th>
<th>Tech 2</th>
<th>Tech 3</th>
<th>Education</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Run</td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
</tr>
<tr>
<td>Long Run</td>
<td>Decrease</td>
<td>Decrease</td>
<td>Decrease</td>
<td>Decrease</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.02%</td>
<td>0.76%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Note: Each shock is 10% up.

### Table 4: Sensitivity Test: The Long-run Effect of Shocks on the Wage Differential

<table>
<thead>
<tr>
<th></th>
<th>Tech 1</th>
<th>Tech 2</th>
<th>Tech 3</th>
<th>Government</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Education</td>
<td>Invest</td>
<td>Income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1S2U3K</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>1S2K3U</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>1U2S3K</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1U2K3S</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1K2S3U</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1K2U3S</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

*When the share of education capital in education production is equal to or greater than 0.5, the sign becomes +.
Figures

Figure 1. Simulation Results: Wage Ratio (%)
Appendix 1

The parameters and exogenous variables settings are as follows:

\[
\begin{align*}
\alpha_1 &= 0.3; & \beta_1 &= 0.5; & \delta_1 &= 0.1; & \phi_1 &= 5; \\
\alpha_2 &= 0.3; & \beta_2 &= 0.2; & \delta_2 &= 0.1; & \phi_2 &= 5; \\
\alpha_3 &= 0.5; & \beta_3 &= 0.3; & \delta_3 &= 0.1; & \phi_3 &= 5; \\
\delta_s &= 0.05; & \gamma_1 &= 0.3; & \gamma_2 &= 0.2; & \gamma_3 &= 0.2; \\
\theta &= 0.1; & \xi &= 0.5; & \delta_E &= 0.1; & \Phi &= 10; \\
\varepsilon_1 &= 1/3; & \varepsilon_2 &= 1/3; & \rho &= 0.5; & T &= 8760; \\
A_{Qh} &= 1; & I_E^C &= 100; & Y^* &= 100; \\
\end{align*}
\]
References


