Exchange Rates and CGE Models

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Does a CGE Model Have a Meaningful Exchange Rate?

• No
  – Whalley and Yeung: a “parameter”
  – Gunning and Keyzer: an “indicator” or “normalization” reflecting choice of numeraire
    • They refer to models with two numeraires
  – Srinivasan
  – GTAP (???)
Does a CGE Model Have a Meaningful Exchange Rate?

- Yes: single-country models
  - World Bank structural adjustment models
    - Dervis, de Melo, and Robinson
    - Tarr and associates
    - Devarajan, Lewis, and Robinson
      - 1-2-3 model and others
    - Branson, Bourguignon, and de Melo
    - Agenor: macro-CGE models
  - IFPRI standard model
  - Orani-type models
Does a CGE Model Have a Meaningful Exchange Rate?

- Yes: world models
  - World Bank world models
    - van der Mensbrugghe
  - OECD world models
    - Walras and descendents
    - Development Centre models
  - Michigan world model
  - McKibbin-Sachs-Wilcoxen models
  - IFPRI world models
If Yes, What Is It?

- A “real” variable?
- A price signal to agents?
- A “macro” variable?
- A “financial” variable?
- Does it correspond to “observed” exchange rates in data?
References


References

• V. Ginsburgh and M. Keyzer, *The Structure of Applied General Equilibrium Models.*

Elements of a CGE Model

• Actors: consumers and producers
• Motivation: utility maximization and profit maximization
• Institutional structure
  – Competitive markets
• Signals: prices and wages
Elements of a CGE Model

- Constraints on actors
  - Technology
  - Endowments (budget constraints)
- System constraints
  - Factor supplies
- Equilibrium conditions
  - Supply-demand balance on all markets
Important Distinctions

- Equilibrium conditions
- Equilibrating variables
- Equilibrating mechanisms
Negishi Formulation

$$\text{Max} \sum_{k} \alpha_k U_k(Q^k) \quad k \text{ "actors"}$$

$$\sum_k Q^k_i \leq f_i(F^D_{i,f}) \quad \text{technology} \quad \pi^Q_i$$

$$\sum_i F^D_{i,f} \leq \sum_k \bar{F}_f^k \quad \text{system constraints} \quad \pi^F_f$$

Find $\alpha$'s such that $\pi$'s act as equilibrium prices
Ginsburgh-Waelbroeck: Master Program

Add agent budget constraints:

\[ Y_k = \sum_{f} W_f \bar{F}_f^k = \sum_{i} P_i \bar{Q}_i^k \]

Find \( \alpha_k \) such that: \( W_f = \pi_f^F \)

and \( P_i = \pi_i^Q \)
CGE Formulation

Excess demand equations:

\[ e^Q_i (P, W) = 0 \]

\[ e^F_f (P, W) = 0 \]
CGE With Macro Constraints

\[ e_i^O (P, W, \mu) = 0 \]
\[ e_f^F (P, W, \mu) = 0 \]
\[ \psi_m (P, W, Q, F, \mu) = 0 \]

\( m \)  \hspace{1cm} \text{macro constraints}

\( \mu_m \)  \hspace{1cm} \text{macro equilibrating variables}
CGE Equilibrium

- “Flow” equilibrium in factor and product markets
- “Flow” macro equilibrium: “Closure”
  - $S = I$
  - $G = T + S^G$
  - $M = E + S^F$
- Define macro equilibrating variables
Other Equilibrium Concepts

• Asset market equilibrium
  – Portfolio behavior
  – “Financial” variables: “claims” on future delivery of goods or “title” to assets
  – Time enters the model

• Dynamic equilibrium
  – Recursive
  – “Forward looking” agents
  – Rational expectations
Trade-Focused CGE Models

• Add exports and imports to closed-economy model
• Add a new agent: “rest of world”
  – Budget constraint: trade balance
• New equilibrating variable: “parameter”
• Nature of traded and non-traded goods
  – Non-traded factors and commodities
  – “Tradable” commodities
Armington Insight

• Specify traded goods as imperfect substitutes for domestic goods with the same sector classification.
• Allow degrees of “tradability” rather than dichotomous classification.
• Armington approach provides a good theoretical and empirical framework for analyzing trade policy.
1-2-3 Model

- 1 country, 2 activities, 3 commodities
- 2 activities, producing D and E
  - E not consumed domestically
- Additional commodity, M, consumed domestically but not produced
- Generalization of the Salter-Swan model
1-2-3 Model

• Aggregate GDP (X) is fixed.
  – Full employment model.
• Trade balance set exogenously.
• World prices of M and E are fixed.
• Total absorption (Q) is endogenous.
Basic 1-2-3 CGE Model

**Flows**

1. \( \bar{X} = G\left(E, D^S; \Omega\right) \)
2. \( Q^S = F\left(M, D^D; \sigma\right) \)
3. \( Q^D = \frac{Y}{P^q} \)
4. \( \frac{E}{D^S} = g_2\left(P^e, P^d\right) \)
5. \( \frac{M}{D^D} = f_2\left(P^m, P^d\right) \)
6. \( Y = P^x \bar{X} + R\bar{B} \)

**Prices**

7. \( P^m = R\pi^m \)
8. \( P^e = R\pi^e \)
9. \( P^x = g_1\left(P^e, P^d\right) \)
10. \( P^q = f_1\left(P^m, P^d\right) \)
11. \( R\equiv1 \)

**Equilibrium Conditions**

12. \( D^D - D^S = 0 \)
13. \( Q^D - Q^S = 0 \)
14. \( \pi^m \bar{M} - \pi^e \bar{E} = B \)
Basic 1-2-3 CGE Model

Identities

15. $P^x X \equiv P^e E + P^d D^S$

16. $P^q Q^S \equiv P^m M + P^d D^D$

17. $Y \equiv P^q Q^D$
Basic 1-2-3 CGE Model

**Endogenous Variables**
- E: Export good
- M: Import good
- $D^S$: Supply of domestic good
- $D^D$: Demand for domestic good
- $Q^S$: Supply of composite good
- $Q^D$: Demand for composite good
- Y: Total income
- $P^e$: Domestic price of export good
- $P^m$: Domestic price of import good
- $P^d$: Domestic price of domestic good

**Exogenous Variables**
- $P^x$: Price of aggregate output
- $P^q$: Price of composite good
- R: Exchange rate

- $\pi^e$: World price of export good
- $\pi^m$: World price of import good
- B: Balance of trade
- $\sigma$: Import substitution elasticity
- $\Omega$: Export transformation elasticity
## SAM for 1-2-3 Model

<table>
<thead>
<tr>
<th></th>
<th>Activities</th>
<th>Commodities</th>
<th>Hshld</th>
<th>World</th>
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<tr>
<td>Activities</td>
<td>$P^d D^D$</td>
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<td>$P^e E$</td>
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<td>$P^q Q^D$</td>
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<td>$P^x X$</td>
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<td>$R B$</td>
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<td>World</td>
<td>$P^m M$</td>
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<tr>
<td><strong>Total</strong></td>
<td>$P^d D^S + P^e E$</td>
<td>$P^q Q^S$</td>
<td></td>
<td>$Y$</td>
</tr>
</tbody>
</table>
\[
E/D = k \left( \frac{P_E}{P_D} \right)^\Omega
\]

\[
P_E = R \cdot pwe
\]
\[ \frac{M}{D} = k'(\frac{P_D}{P_M})^\sigma \]

\[ P_M = R \cdot \pi^m \]
Maximize $Q = F(M, D; \sigma)$
with respect to: $M, E, D^D, D^S$
subject to:

1. $G(E, D^S; \Omega) \leq \bar{X}$ technology
2. $\pi^m \cdot M \leq \pi^e \cdot E + \bar{B}$ balance of trade
3. $D^D \leq D^S$ domestic market

Shadow Prices

$\lambda^x = P^x / P^q$
$\lambda^b = R / P^q$
$\lambda^d = P^d / P^q$
1-2-3 Model

$P_D/P_M$

$P_M M = P_E E$

$P_D/P_E$
$Q = F(M,D)$

$X = G(E,D)$

Balance of Trade

Domestic Market
Q = F(M,D)

X = G(E,D)

Balance of Trade

Domestic Market
Simple 1-2-3 Model

(1) \[ \bar{X} = G(D, E; \Omega) \]

(2) \[ Q = F(D, M; \sigma) \]
1-2-3 Model

\[ \frac{E}{D} = k_2 \left( \frac{P^e}{P^d} \right)^\Omega \]

(3)

\[ \frac{M}{D} = k_1 \left( \frac{P^d}{P^m} \right)^\sigma \]

(4)
1-2-3 Model

(5) \[ P^m = R \cdot \pi^m \]

(6) \[ P^e = R \cdot \pi^e \]

(7) \[ \pi^m \cdot M = \lambda \cdot \pi^e \cdot E \]
Variables and Parameters

• Variables
  – $E, M, D, Q, P^d, P^e, P^m, R$

• Parameters or exogenous variables
  $\pi^e, \pi^m, \lambda, \sigma, \Omega, \bar{X}$

  $$B = (\lambda - 1)\pi^e E$$

  so $B = 0 \Rightarrow \lambda = 1$

• 8 variables, 7 equations
  – Choice of numeraire. Often $R \equiv 1$. 
Real Exchange Rate Change

\[ \hat{M} - \hat{D} = \sigma \left( \hat{P}^d - \hat{P}^m \right) \]

\[ \hat{E} - \hat{D} = \Omega \left( \hat{P}^e - \hat{P}^d \right) \]

\[ \hat{\pi}^m + \hat{M} = \hat{\lambda} + \hat{\pi}^e + \hat{E} \]

\[ \hat{P}^e = \hat{R} + \hat{\pi}^e \]

\[ \hat{P}^m = \hat{R} + \hat{\pi}^m \]
Equilibrium Domestic Price

\[ \hat{P}^d = \frac{1}{(\sigma + \Omega)} \left[ (\sigma - 1) \cdot \hat{\pi}^m + (1 + \Omega) \cdot \hat{\pi}^e + \hat{\lambda} \right] \]

\[ R \equiv 1 \Rightarrow \hat{R} = 0 \]
Equilibrium PLD EXR

\[ \hat{R} - \hat{P}^d = \left( \frac{\sigma \cdot \hat{\pi}^m + \Omega \hat{\pi}^e}{\sigma + \Omega} \right) \]

- PLD exchange rate

world inflation

\[ + \left( \frac{\hat{\pi}^m - \hat{\pi}^e}{\sigma + \Omega} \right) \]

terms of trade

\[ + \frac{\hat{\lambda}}{\sigma + \Omega} \]

trade balance
Choices of Numeraire

• R: exchange rate
• $P^X$: GDP deflator
• $P^Q$: consumer price index
• $P^D$: price index of non-traded goods
• “Natural” choice would be R or $P^D$, given equilibrating mechanism
  – But results are the same for all choices
Macro Trade Relationships

- Constraint: balance of trade
- Equilibrium condition: trade balance equation
- Equilibrating variable: exchange rate
- Equilibrating mechanism: relative prices of traded to non-traded commodities
  - Variation in “real” exchange rate
Macro Trade Relationships

- Generate “general equilibrium” import demand and export supply curves
  - Vary the trade balance exogenously and solve for:
    - Equilibrium exchange rate and
    - Aggregate exports and imports
- “Macro” flow relationships
E, M, Supply-Demand

R/P

M, E ($)

Import demand

Export supply

Trade balance
Trade Balance

• Units of trade balance: foreign currency
• Trade balance is a “claim” on foreign production; e.g., exports from rest of world
  – Odd notion, because there is no reason that any actor should want to hold such a claim in the CGE framework
  – Yet, must account for it in CGE model
Exchange Rate Variable

• Units of an exchange rate
• Standard macro relationships
  – Export supply and import demand curves
• Exchange rate equilibrating mechanism
  – Salter-Swan and 1-2-3 models
  – Standard relative price adjustment mechanism
Old Saying

• If it quacks like a duck,
• And it walks like a duck,
• And it looks like a duck,
• Then it must be a duck.
Macro Relationship

• Model specifies a monotonic relationship between the real exchange rate and the trade balance
  – Fix B, real exchange rate is endogenous
  – Fix real exchange rate, B is endogenous

• Three variables: R, B, and P
  – One variable, R or P, set by choice of numeraire
  – Can fix one other variable: B or either P or R.
  – Definition of real exchange rate variable is open
Other Macro Relationships

- **R as a “signal” in financial markets**
  - Equalization of interest rate minus inflation minus devaluation
  - Adjustment in asset markets affects trade balances, which are then endogenous
  - Time is inherently part of the analysis

- **Different agents, different model, same variable**
  - These financial models need to honor the relationship in commodity markets between R and the trade balance
Is R a “Macro” Variable?

• Yes
• Equilibrating variable defining a flow macro equilibrium in the trade balance
  – Trade balance “constraint”
  – Not “merely” the budget constraint of an “agent”.
Is R a “Financial” Variable?

• No.
• No financial assets in the model.
  – No money, domestic or foreign
  – Trade balance is odd: an implicit “claim” but no accounting of the stock of claims.
    • Similar to treatment of $S=I$ in macro closure
• No intertemporal equilibrium concept
Is R a “signal” to Agents?

• No
• No agent sees R as a signal in his/her decision making
  – It is **not** the price of foreign exchange
• There is an implicit relationship between R and the trade balance
  – Works through the real exchange rate
  – Relative prices of traded to nontraded commodities
Is R “Observable” in data?

• Yes
• Can compute real exchange rate as defined, say, in the IMF *International Financial Statistics*
  – Either through choice of numeraire or by post-solution calculation
• R works exactly as it should in flow macro models
  – Can compute “equilibrium” R after a shock
“Cash in Advance” Model

• Money in utility or production functions
  – Credit requirements for production

• Money/credit in these models need not be treated as a “financial” variable
  – An “input” that provides a flow of services
    – Standard flow equilibrium

• Long CGE tradition
  – Taylor and others
Rest of World as an “Agent”

• In a single-country model, the rest of the world is treated as an agent with simple behavior
  – E.g., small-country assumption
  – Demand curves for some exports
Rest of World as an “Agent”

• Treatment is inconsistent with viewing the “agent” as “country” with its own CGE model
  – Whalley and Yeung
  – Gunning and Keyzer

• So what?
  – “Representative” country may be empirically a very bad specification
  – Agent approach supports econometric estimates
  – Must still honor trade balance constraint
    • No problem with Walras’ Law
Multi-Country Models

- “Rest of world” is now one or more countries rather than a single “agent”
- Numeraire issue
  - Country numeraires
  - Additional exchange rate numeraire
    - Reference country or composite
  - Units of trade balances
    - Numeraire “matters”
Trade Balances

• “Units” of trade balances matter
  – Must add up on global scale
  – “Claims” against what?

• Choices and numeraires
  – U.S. dollar: many models
  – OECD basket: van der Mensbrugghe
  – “Wage-rental” units: GTAP (?)

• Numeraire does not matter if you “deflate” trade balances explicitly in the world model
Conclusion

• Single and multi-country CGE models contain exchange rates
• The models are theoretically clean generalizations of the Salter-Swan model
• Exchange rates are “macro” variables that equilibrate trade balance constraints
  – No financial variables in the model
• They models define only flow equilibria, and macro equilibria must be defined in flow terms
Conclusions

• The treatment of exchange rates in CGE models is consistent with treatment in many macro models
  – CGE models are commonly being used as the “supply side” in macro models

• Extensions to assets, asset markets, and dynamics is challenging