

# Green Accounting and Climate Change Problem: A New Evidence from the Turkish Economy

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## Abstract

The motivation of the paper is to include the environmental quality indicators in the national accounts of a country in high economic growth. We took as an example Turkey. Since the country has been rapidly growing both in terms of economy and population during the 1980s. During the last decade, its demand for energy increased about 4.4 percent per year, with electricity consumption growing at an average annual rate of about 8.5 percent. The use of the most common fuel (domestic lignite) has doubled between 1980 and 1997. Naturally, a growing energy consumption has negative impacts on the environmental quality. However, the determination of the optimal environmental policy without reducing the economic growth is still a challenge for the Turkish decision makers. In this paper, the level of environmental quality is defined by GHG emissions and the economic performance is determined by the Weitzman criteria. A dynamic applied general equilibrium model (DAGEM) is used in order to quantify the impact of taking into account of national environmental quality on the Turkish national accounts. We run the model under GAMS for the period 1980-2050 using Turkish data.

## 1 Introduction

Environmental accounting is an important tool for understanding the role played by the natural environment in the economy. Environmental accounts provide data that highlight both the contribution of natural resources to the economic well-being and the costs imposed to the economy by pollution or resource degradation.

Naturally, the pollution costs are much more important in a fast growing economy. We chose the Turkish economy as an example since the country has

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been rapidly growing both in terms of economy and population since the economic liberalization period of the 1980's. During the last decade the aggregate demand for energy increased about 4.4 percent per year, the electricity consumption grew by 8.5 percent per year and the use of the most commonly used fuel, domestic lignite, has doubled between 1980 and 1997.

Naturally, such an important increase in energy consumption has impact on the environmental quality. However the determination of the optimal environmental policy without reducing the economic growth performance is still a challenge for the Turkish governments.

In this paper we aim to define the economic growth and welfare for the Turkish economy in an alternative way within the concept of NNP. The definition of the green NNP appears as an important indicator. Therefore, in our article we first present some economic models on the estimation of the green NNP. On the theoretical aspect, we focus our analysis on the Weitzman model (1976). After comparing the results obtained by Weitzman model on a possible "sustainable" energy consumption path in Turkey and the real consumption figures, we suggest some improvements. We first add an indicator of an exhaustible resource stock and secondly an environmental sensitivity indicator to the Weitzman model.

Our choice of analyzing the case of lignite is due to the important role of lignite in the energy consumption figures of Turkish households as the "cheapest" and most "abundant" domestic fuel. We observe then a very high growth rate hence an "unsustainable" lignite consumption in Turkey that leads us to introduce economic measures in our model in order to slow down the lignite consumption. We conclude our paper by the comparison of two possible policy options ; that are the use of fiscal or sensibilisation policy measures in Turkey in order to reach a more "sustainable" consumption path.

This first analysis on the lignite as the most important energy source in Turkey can be expanded to the other exhaustible natural resources. This is also the first step towards our calculation of NNP for Turkey.

In this article, we constructed a simple Dynamic Applied General Equilibrium Model (DAGEM) that we ran under GAMS for the period 1980-2050 using Turkish data.

## 2 Overview of NNP definitions

In this section, we are interested in what the concept of Net National Product means, what it offers us as an economic indicator. Much of the current debate in the literature on the question of the sustainability of Net National Product (NNP) as an "indicator of social welfare" or as an "indicator of sustainability" goes back to the seminal work of Hicks (1946), Samuelson (1961) and Weitzman (1976). The discussion starts with the Hicks' definition of an individual's income is :

“the maximum value which he can consume during a week and still expect to be as well off at the end of the week as he was in the beginning” (op.cit.p.172).

If this concept is extended and applied to an economy as a whole, income would be a number representing the amount of welfare which can be enjoyed over a period of time and leave the economy with the capacity to enjoy that same amount of welfare for the next period of time. Clearly, the development of the economy over a period of time is then “sustainable”, if income, in the sense of the definition, is constant over that period of time.

We observe that the concept is old, since the early definitions appear in the above mentioned articles. Meanwhile, in the space of only a few years, the term “Green NNP” has gained much currency. Today that is a common place to say that in estimating NNP, the following points should be taken into consideration:

- depreciation of physical and human capital ought to be deducted from the Gross National Product (GNP),
- like the depreciation of natural capital and the social losses that are incurred due to increases in the stock of environmental pollution.

More precisely, on the literature, the *green* NNP has widely interpreted as “constant-equivalent consumption” of NNP in traditional terms. Citing again to Hicks (in *Value and Capital*, 1939):

“...The concept of income [is] one which the positive theoretical economist only employs in his arguments at his peril. For him, income is a very dangerous term, and it can be avoided;... a whole general theory of economic dynamics can be worked out without using it”.

In our paper, we expand the definition of economic well being from the concept of income to the social welfare including environmental quality. Before suggesting an alternative concept of welfare by including the environmental indicators in the social well being, it is also important to remember that according to Pigou (1932), economic welfare is defined as:

“that part of social welfare that can be bought directly or indirectly into relation with the measuring-rod of money”.

We focus our analysis on the contribution of the use of an exhaustible resource to the social well being and the integration of the disutility of pollution. This point is relevant since the environmental discussions in Turkey occupy an important place in the governments’ agenda. Some policy measures have already been taken in the country in order to slow down the use of the high sulfur content domestic lignite by the Turkish households. We now give an overview of the private consumption of energy in Turkey before presenting the theoretical modeling framework.

## 2.1 Weitzman's criteria of NNP

Weitzman (1976) shows that the welfare justification of Net National Product is based on the idea that in theory NNP is a proxy for the present discounted value of future consumption.

For simplicity, Weitzman assumed that there is just one consumption good. Hence, the consumption level in period  $t$  can be registered by the single number  $C(t)$ . The notion of capital good used in his paper is meant to be quite a bit more general than the usual equipment, structures and inventories. Strictly speaking, pools of exhaustible resources ought to qualify as capital.

For convenience, all non capital contributions to production are treated as fixed over time. In fact Weitzman is making the extreme abstraction that all resources of economic growth have been identified and attributed to one or another form of capital, broadly defined.

He assumes that there are  $n$  capital goods. The stock of capital of type  $i$ , ( $1 \leq i \leq n$ ) in existence at time  $t$  is denoted by  $k_i(t)$ , hence the net investment is:

$$I_i(t) = \frac{dK_i}{dt}$$

- $K_i(t)$  stock of capital  $i$  in existence at time  $t$  ( $1 \leq i \leq n$ )
- $I_i(t)$  net investment flow
- $S(K(t))$  the production possibilities set at time  $t$
- $(C, I)$  the consumption-investment pair, producible at time  $t$  if and only if  $(C, I) \in S(k(t))$

A real Net National Product function could be defined as follows:

$$\left( C(t), \frac{dk}{dt}(t) \right) \in S(K(t)) \quad (1)$$

$K(0) = K_0$  where  $K_0$  is the original endowment of capital that is available at starting time  $t = 0$ .

The optimal control problem is then defined as following:

$$\begin{aligned} & \underset{c_t}{Max} \int_0^{\infty} C(t)e^{-rt} dt \\ & s.t. \left( C(t), \frac{dk}{dt}(t) \right) \in S(K(t)), K(0) = K_0 \end{aligned} \quad (2)$$

What Weitzman defined as the Net National Product is the Hamiltonian for a general optimization problem of the form above. We now apply the Weitzman model to the Turkish case.

## 2.2 Weitzman model applied to the Turkish data

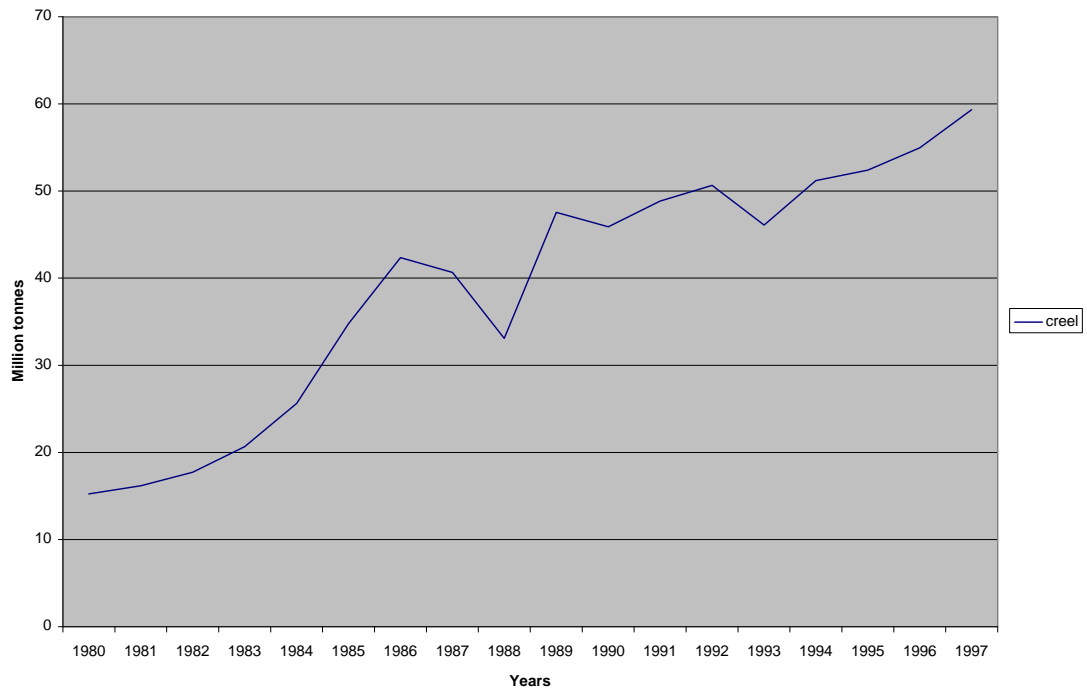
### 2.2.1 Evolution of lignite reserves and consumption in Turkey since 1980s

Turkey has been rapidly growing both in terms of economy and in terms of population since the 1980s. During the last decade its demand for domestic energy increased about 4.4 percent per year, with electricity consumption growing at an average annual rate of about 8.5 percent.

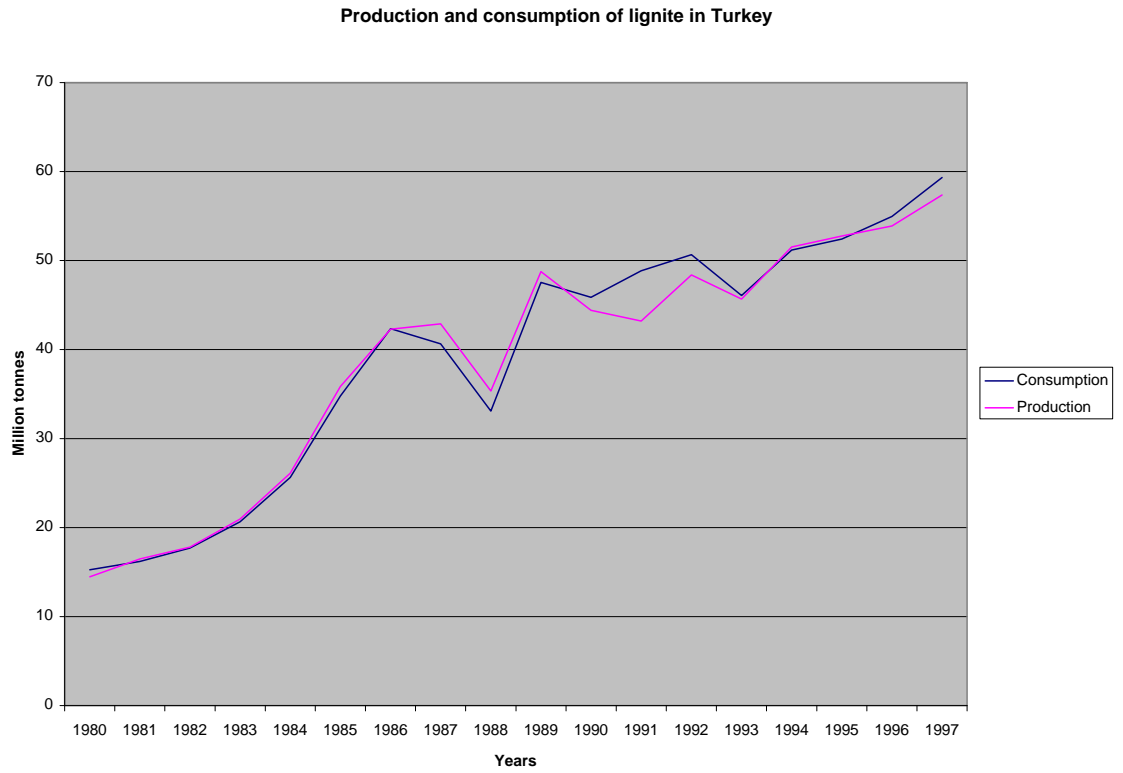
This important increase can be explained by the liberalization of the economy since the 1980. Turkish households' energy consumption increased due to the changing life styles ; an increasing purchase of new electrical equipment and cars.

However lignite stayed as the most commonly used heating fuel in Turkey because of its abundance as a domestic resource and its cheaper price. Naturally, this fast growing energy consumption had negative impacts on the environmental quality. The emissions of greenhouse gases increased and some conflicts appeared during the official discussions with the international organizations (more specifically negotiations between the Turkish authorities and OECD on the Kyoto Protocol).

Lignite consumption of Turkey (million tonnes)

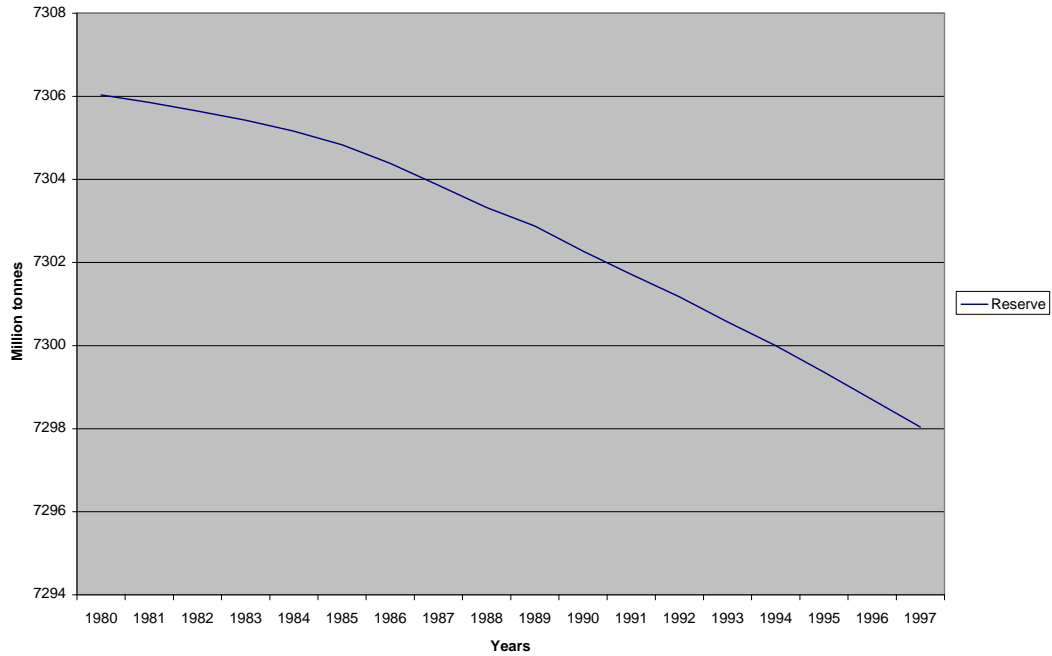


As we can see in the below figure, the domestic consumption absorbs the largest part of the lignite production in Turkey.



Given the rapid growth of the lignite consumption, the lignite reserve decrease represents a high rate. This consumption rate can not be qualified as “sustainable”.

Evolution of lignite reserves in Turkey

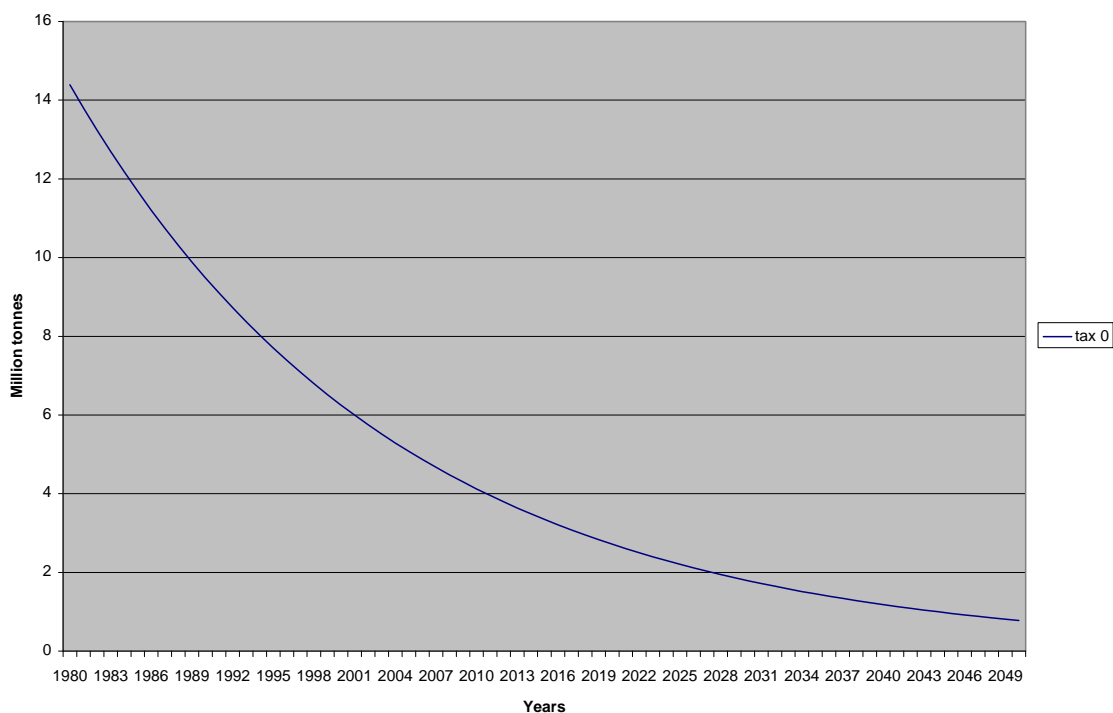


In order to suggest a “sustainable” policy of lignite consumption, we chose to apply the Weitzman model to the Turkish case. We continue by a presentation of the theoretical model that will be followed by the empirical results.

Applying the Weitzman model to the 1980-1997 energy data of the Turkish economy, we observe that the “sustainable” consumption path of the lignite resources should display a decreasing evolution. We anticipate in this way an extinction of lignite resources during the 2050s.

This prediction on the depletion of Turkish lignite in about 50 years, explains our motivation of constructing a theoretical model in order to predict a sustainability of the lignite consumption. According to Weitzman, a “sustainable” consumption path should behave as follows :

Consumption in Weitzman model



The above graphic is obtained by using the real consumption and reserve data concerning the Turkish lignite in 1980. The use of the lignite resources are anticipated for the 2050 and we draw the estimated consumption curve with the following parameters :

Interest rate	0.15
Elasticity of consumption	-1.5
Initial lignite consumption (1980)	10 million tones
Initial lignite reserves (1980)	7300 million tones

We therefore observe that this estimation does not correspond to the real evolution of lignite use by Turkish households - that is increasing rapidly (figure 1)-. The following section presents some economic measures in order to converge towards an “sustainable consumption” path.



### 3 Economic policy measures towards an sustainable lignite consumption in Turkey

As we observed in the previous section, an economic policy is necessary in order to slow down the consumption rate of the exhaustible lignite reserves. We suggest therefore two policies. The first one is commonly used by the developed economies ; the imposition of a tax on the lignite consumption. The second one is rather new, it consists on increasing the “environmental sensitivity” of consumers by various information campaigns.

#### 3.1 Introduction of a consumption tax in the Weitzman model

We introduce in the Weitzman model a consumption tax  $\tau$  that is constant over time in order to reduce the fast consumption rate of the exhaustible resources. The optimization program becomes :

$$\begin{aligned} & \underset{C_t}{Max} \int_0^{\infty} C(t)e^{-rt} dt \\ \text{s.t } \dot{K}_t &= -(1-\tau)C_t \end{aligned}$$

The current value of the Hamiltonian is defined as follows:

$$H^c = U(C_t) + \lambda_t(\tau - 1)C_t$$

Where  $\lambda_t$  is the utility shadow price of the resource stock in utility units. The first-order conditions for an optimal path are:

$$\frac{\partial H^c}{\partial C_t} = 0 \Leftrightarrow U'_1 = \lambda_t(1 - \tau) \quad (3)$$

$$\frac{\partial H^c}{\partial K_t} = r\lambda_t - \dot{\lambda} \Leftrightarrow \dot{\lambda}_t = r\lambda_t' \quad (4)$$

and the transversality condition:

$$\lim_{t \rightarrow \infty} \lambda_t^* K_t^* = 0 \quad (5)$$

In fact, along the optimal path, the consumption and stock levels declines exponentially over time at the constant rate of  $(1 - \tau)\frac{r}{\eta}$ . That means:

$$C_t^* = C_0 e^{(1-\tau)\frac{r}{\eta}t} \quad (6)$$

$$K_t^* = K_0 e^{(1-\tau)\frac{r}{\eta}t} \quad (7)$$

Where

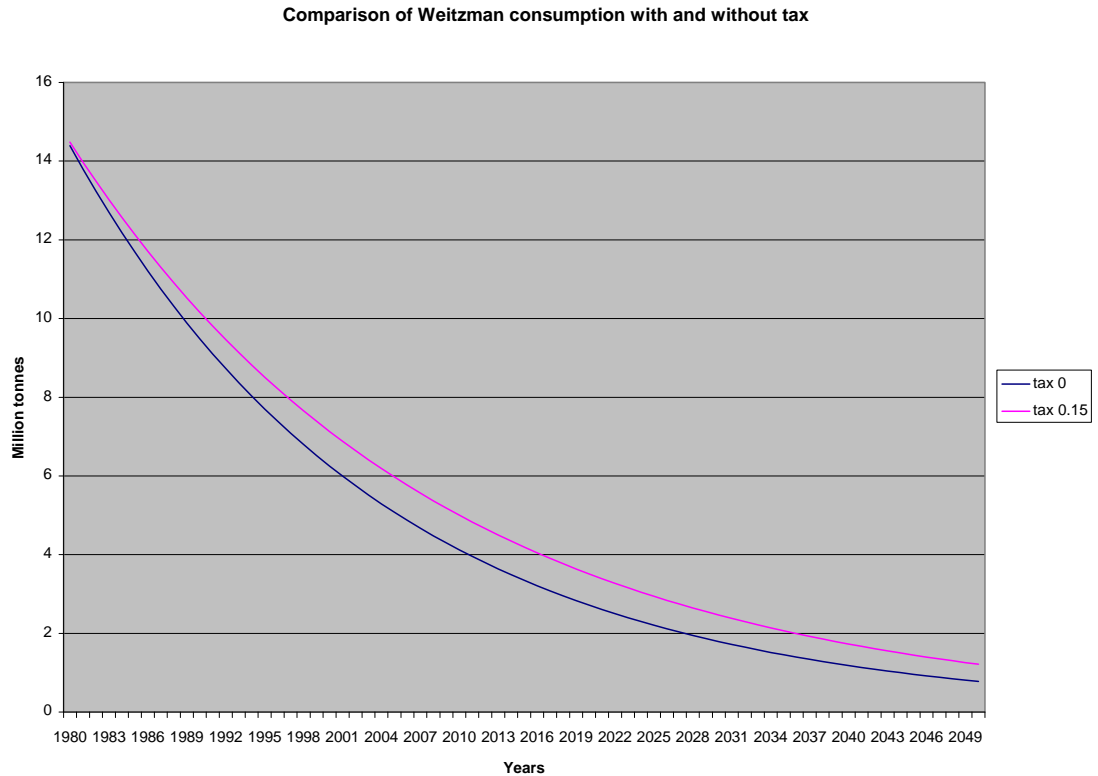
$$C_0 = \frac{r(1-\tau)}{\eta} K_0$$

### 3.2 Application to the Turkish case of the Weitzman model including a taxation policy

Considering again 1980 as the initial year, we draw the lignite consumption path following the Weitzman criteria with a 15% consumption tax.

As we have seen previously, the “sustainable path” displays a decreasing consumption curve. We also observe that the imposition of a consumption tax leads us to a flatter (hence more sustainable) consumption trajectory.

We can therefore conclude that the imposition of a consumption tax lets us first to approach the real consumption figures in Turkey (the real consumption curve is increasing) and secondly to suggest a more sustainable consumption path.



The comparison of Weitzman’s model results with the real Turkish consumption figures 2.2.1 shows that there is still a big difference between the traditional lignite consumption patterns and the “sustainable” consumption paths. This is the reason why, in the next section, we suggest the extension of the Weitzman model in two ways.

## 4 The DAGEM model

Our Dynamic General Equilibrium Model (DAGEM) suggests some improvements to the Weitzman model. The first improvement consists in the integration of a stock variable representing the available reserves of the chosen depletable natural resources. In this objective, we use a separable utility function with two terms. The additional component of the utility function corresponds to the utility procured by the stock of the natural resource. Fixing a weight parameter for this component allows us to express the importance accorded to the future generations. The more altruist are the consumers, the more the weight parameter is important, the more resources are preserved for the future generations.

The second improvement we proposed is related to the sensibilisation of the consumers to the pollution problems. We look for an economic solution to increase their sensitivity to the GHG emissions.

We present the theoretical specifications of these two specificities and then we apply them to the Turkish case.

### 4.1 Management of an exhaustible resource integrating an environmental indicator

We summarized the Weitzman's idea in the previous section. Here we consider a purely exhaustible resource economy and assume that:

- (i) it has a fully known and fixed initial stock of the resource of size  $R_0 > 0$ ,
- (ii) no technical change,
- (iii) population size remains constant and
- (iv) citizen's preferences are identical. They are defined by the representative consumer's CES utility function,  $U(C_t, R_t)$ , which is a twice differentiable, increasing, and strictly concave function of the resource consumption and stock rate ( *i.e.*,  $U'(C) > 0, U'(R) > 0, U''(C) < 0, U''(R) < 0$  for all  $C \geq 0$  and  $R \geq 0$ ).  $U$  is defined as following:

$$U(C_t, R_t) = U_1(C_t) + \phi U_2(R_t) \quad (8)$$

Where

$U_1$ , utility procured from consumption of the resource

$$U_1 = \log C_t \quad (9)$$

$U_2$ , utility procured from the stock of the resource

$$U_2 = \log R_t \quad (10)$$

and,  $\phi$  a share parameter such that  $\phi \in [0, 1]$ .

The planner's optimization problem can be confused with the one using by Weitzman (1976) if and only if the share parameter  $\phi$  is equal to zero. Whenever  $\phi$  is different to zero, we resolve a dynamic program where a social

welfare function defined as the discounted sum of the representative consumer's utility flow given the resource stock constraint. The corresponding planner's optimization problem is:

$$\underset{C_t, R_t}{Max} \int_0^{\infty} U(C_t, R_t) e^{-rt} dt \quad (11)$$

$$s.t. \dot{R}_t = -C_t \geq 0, R_t \geq 0, R_0 \text{ (given)}$$

Where  $r > 0$  is the social time preference rate and it is supposed to be constant. Assuming the constraint  $R_t \geq 0$  holds, the current-value Hamiltonian ( $H^c$ ) of this problem becomes :

$$H^c(C_t, R_t, \lambda_t) = U(C_t, R_t) - \lambda_t C_t \quad (12)$$

Where  $\lambda_t$  is the utility shadow price of the resource stock in utility units. The first-order conditions for an optimal path are:

$$U'_1 = \lambda_t \quad (13)$$

$$\dot{\lambda}_t = r\lambda_t - \phi U'_2 \quad (14)$$

and the transversality condition:

$$\lim_{t \rightarrow \infty} \lambda_t^* R_t^* = 0 \quad (15)$$

Differentiating 13 with respect to time, using 14, we denote the elasticity of marginal utility of consumption by:

$$\eta(c) = -\frac{CU''(C)}{U'(C)} \quad (16)$$

the optimal consumption path is characterized by the familiar condition:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\eta} \left( r - \frac{\phi U'_2}{U'_1} \right) \quad (17)$$

The dynamic system is defined as following:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\eta} \left( r - \frac{\phi U'_2}{U'_1} \right) \quad (18)$$

$$\dot{R}_t = -C_t \quad (19)$$

In fact, along the optimal path, the consumption and stock levels decline exponentially over time at the constant rate of  $\frac{r}{\phi - \eta}$ .

That is:

$$C_t^* = C_0 e^{-\frac{r}{\phi-\eta}t} \quad (20)$$

$$R_t^* = R_0 \frac{1-\eta}{1-r} e^{-\frac{r}{\phi-\eta}t} \quad (21)$$

Where the level of initial consumption  $C_0$  is calculated by using the exogenous initial stock  $R_0$  :

$$C_0 = \frac{rR_0}{\phi-\eta} \quad (22)$$

So that, we can define the optimal path of consumption as follows:

$$C_t^* = \frac{rR_0}{\phi-\eta} e^{-\frac{r}{\phi-\eta}t}, \forall t \in [0, \infty) \quad (23)$$

## 4.2 Management of an exhaustible resource integrating a stock variable and an environmental indicator

In the preceding section, we have studied the effect of introduce the parameter  $\phi$  in the utility function, that translates the preference for the environment. In this section, we will introduce an other parameter  $\tau$ , which is a taxation rate of the consumption.

The dynamic program is defined as following:

$$Max \int_0^{\infty} U(C_t, R_t) e^{-rt} dt \quad (24)$$

under the constraint:

$$\dot{R}_t = -(1-\tau)C_t, R_t \geq 0, R_0 \text{ (given)} \quad (25)$$

Where  $U(C_t, R_t)$ , is a CES utility function (as defined in section 4) with the form below:

$$U(C_t, R_t) = U_1(C_t) + \phi U_2(R_t) \quad (26)$$

The current-value of the Hamiltonian is defined as follow:

$$H^c = U_1(C_t) + \phi U_2(R_t) + \lambda_t(\tau - 1)C_t \quad (27)$$

The first order conditions are:

$$\frac{\partial H^c}{\partial C_t} = 0 \Leftrightarrow U_1' = \lambda_t(1-\tau) \quad (28)$$

$$\frac{\partial H^c}{\partial R_t} = r\lambda_t - \dot{\lambda} \Leftrightarrow \dot{\lambda}_t = r\lambda_t - \phi U_2' \quad (29)$$

And the transversality condition is

$$\lim_{t \rightarrow \infty} \lambda_t^* R_t^* = 0 \quad (30)$$

The dynamic system is defined the optimal consumption path  $\left(\frac{\dot{C}_t}{C_t}\right)$  and the intertemporal constraint of the dynamic problem:

$$\frac{\dot{C}_t}{C_t} = \frac{(1-\tau)}{\eta} \left( r - \phi(1-\tau) \frac{U'_2(R)}{U'_1(C)} \right) \quad (31)$$

$$\dot{R}_t = (\tau - 1)C_t \quad (32)$$

Let

$$X = \frac{C}{R} \quad (33)$$

and

$$\frac{\dot{X}}{X} = \frac{\dot{C}}{C} - \frac{\dot{R}}{R} \quad (34)$$

Therefore, we resolve the dynamic problem bellow

$$\frac{\dot{X}}{X} = \frac{1-\tau}{\eta} \left[ r - \phi(1-\tau) \frac{U'_2}{U'_1} + (1-\tau) \right] X \quad (35)$$

$$\frac{\dot{R}}{R} = (\tau - 1)X \quad (36)$$

At the steady state, we have

$$\frac{\dot{X}}{X} = 0 \Leftrightarrow \frac{1-\tau}{\eta} \left[ r - \phi(1-\tau) \frac{U'_2}{U'_1} + (1-\tau) \right] X = 0 \quad (37)$$

Then

$$X^* = \frac{r}{\phi^2(1-\tau) - \eta} \quad (38)$$

In fact, a long the optimal path, the consumption and stock levels decrease following an exponential function over time at the constant rate  $-\frac{r(\tau-1)}{\phi^2(1-r)-\eta}$ :

$$C_t^* = C_0 e^{\frac{r(\tau-1)}{\phi^2(1-r)-\eta} \cdot t} \quad (39)$$

and the optimum value of the stock is as follows:

$$R_t^* = C_0 \frac{\phi^2(1-\tau) - \eta}{r} e^{\frac{r(\tau-1)}{\phi^2(1-r)-\eta} \cdot t} \quad (40)$$

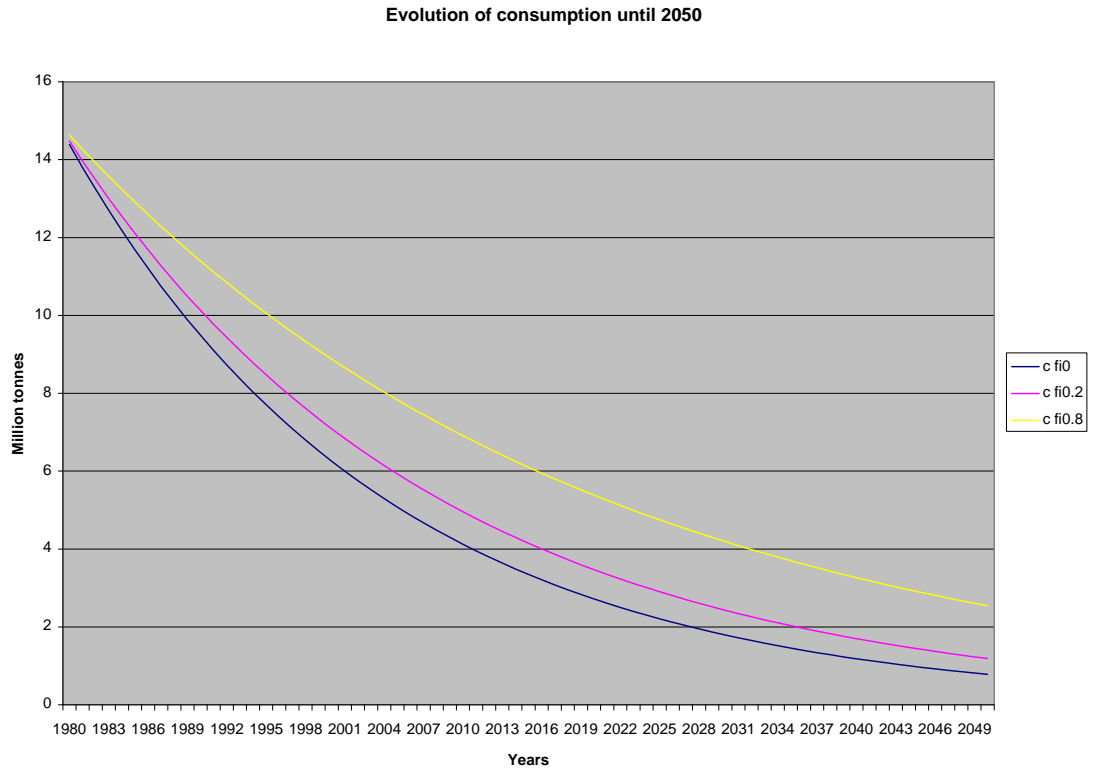
So that

$$C_0 = \frac{rR_0}{\phi^2(1-\tau) - \eta} \quad (41)$$

### 4.3 Application to Turkey

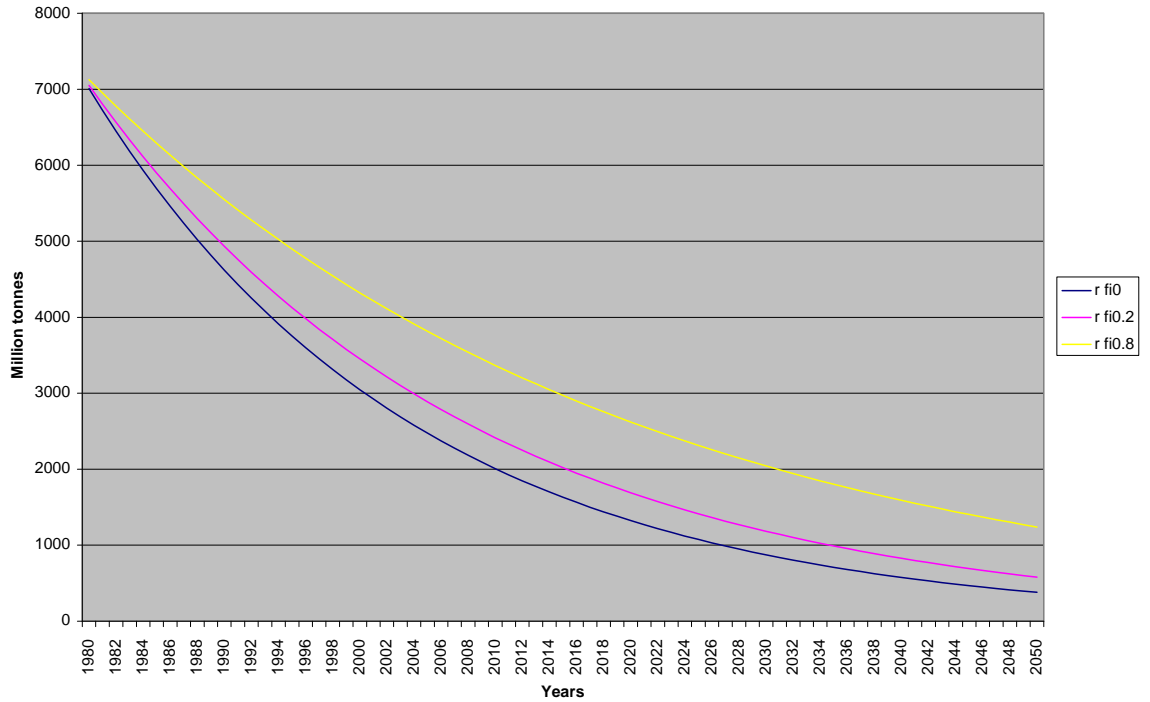
We present in this section the application of the “environmental sensitivity” parameter to the Turkish case. It is also important to note that the actual value of this parameter should be extremely low since the lignite consumption curve is strictly increasing.

Here we test the impact of a “sensibilisation” campaign of the Turkish consumers to the environmental problems in terms of sustainability. We observe in the following figure that the introduction of the “sensitivity” parameter pushes up the consumption trajectory. Hence the consumption curve becomes flatter and more “sustainable”.



Following the same logic, we observe that the lignite reserves are conserved in a sustainable pattern. In the previous section we have talked about the 2050s as the depletion years of lignite reserves in Turkey. The below figure shows us that this limit can be postponed if the Turkish consumers adopt a more “altruist” and “sustainable” consumption behavior.

Evolution of lignite reserves until 2050



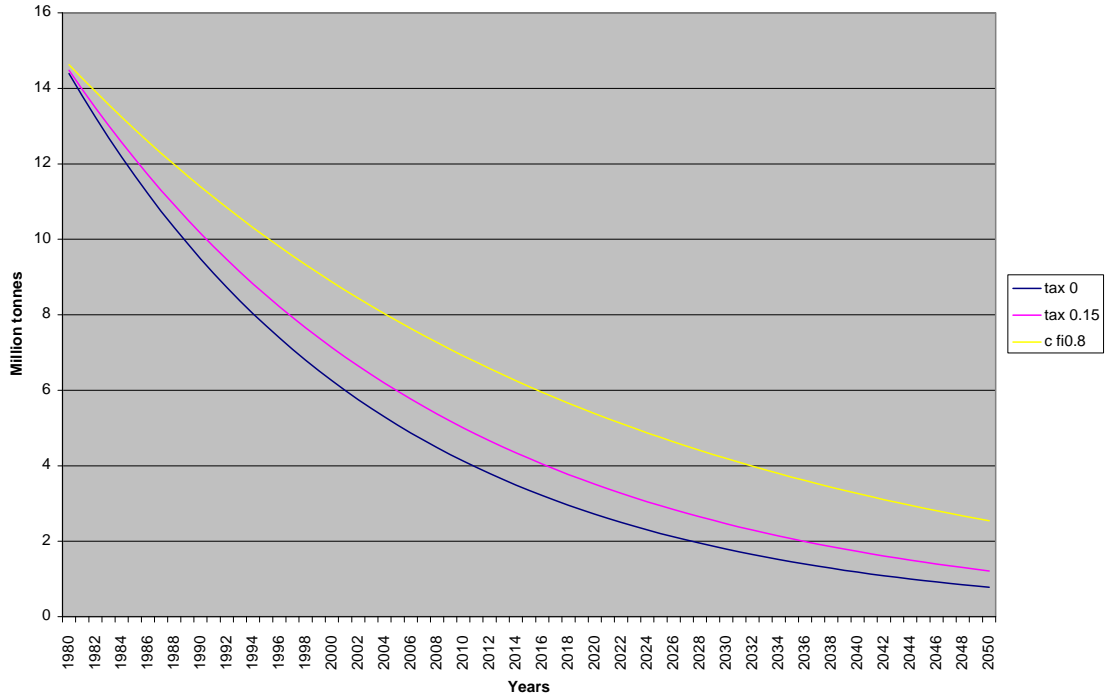
We now propose to compare the potential impacts of a fiscal policy in the Weitzman model with those of the DAGEM model with a high “sensitivity” assumption.

## 5 Comparison of the Weitzman model and DAGEM results

The below figure represents the comparison between the Weitzman model with a 15% tax, this without tax and the DAGEM sensibilisation policy. We have already noted an amelioration of the lignite consumption trajectory following the introduction of the consumption tax in Weitzman’s framework. The result that we reach by the implementation of a sensibilisation policy is even better (more sustainable). This is also the one that approach the most of the real lignite consumption path of Turkey (increasing between 1980 and 1997).

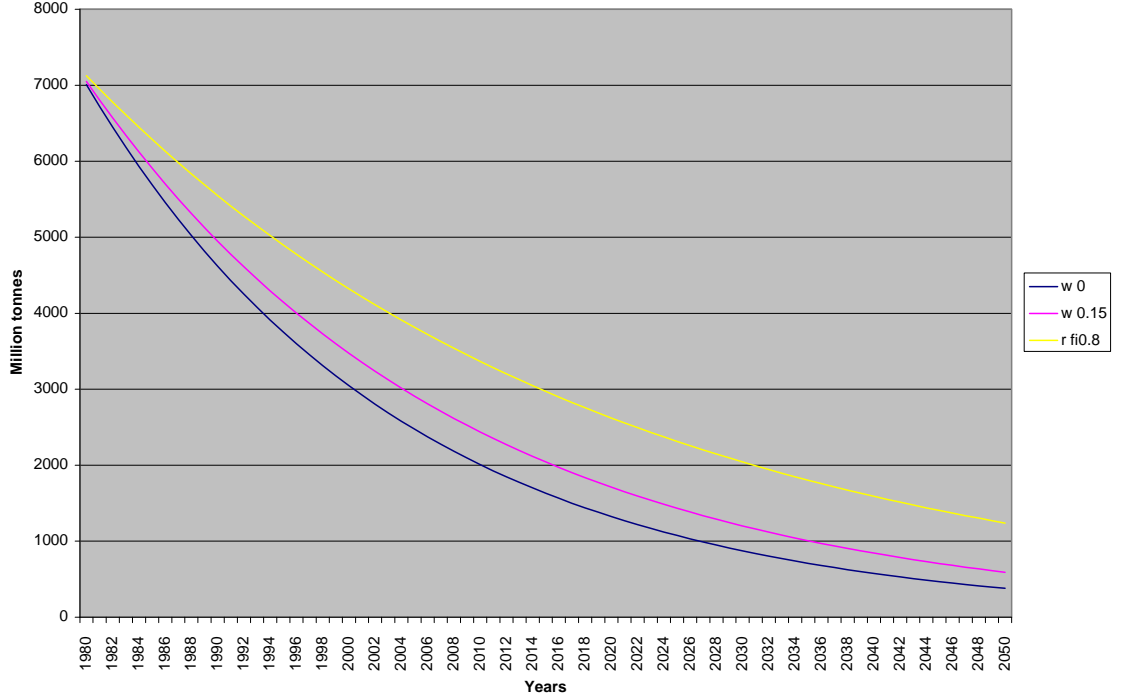


Comparison of consumption paths



The sensibilisation policy allows also a more “sustainable” path for the lignite depletion compared to the Weitzman’s model with a consumption tax.

Comparison of the evolution of lignite reserves



The last section describes the NNP definition following Weitzman’s point of view and suggests a methodology for application to the Turkish case.

## 6 Weitzman’s NNP concept

According to Weitzman (1976), even granted that consumption is the ultimate end of economic activity, the national income statistician’s practice of adding in investment goods to the value of consumption by weighing them with prices measuring their marginal rates of transformation might still be defended as a measure of the economy’s power to consume at a constant rate. After all, a standard welfare interpretation of NNP is that it is the largest permanently maintainable value of consumption.

If all investment was convertible into consumption at the price-transformation rates, the maximum attainable level of consumption that could be maintained forever without running down capital stocks would appear to be NNP as conventionally measured by  $C^* + p \frac{dK^*}{dt}$

It turns out that the maximum welfare actually attainable from time  $t$  on along a competitive trajectory :

$$\int_t^\infty C^*(s)e^{-r(s-t)}ds \quad (42)$$

This is exactly the same as what would be obtained from the *hypothetical* constant consumption level :

$$C^*(t) + p(t)\frac{dK^*}{dt} \quad (43)$$

In this sense, the naive interpretation of the current power to consume at a constant rate idea gives the right answer, although for the wrong reason. Net National Product is what might be called the *stationary equivalent* of future consumption, and this is its primary welfare interpretation (see Appendix I for proof).

## 6.1 Measuring Turkey's NNP

To be completed.

## 7 Appendix I

Weitzman shows that along a competitive trajectory

$$\int_t^\infty \left[ C^*(t) + p(t) \frac{dK^*}{dt}(t) \right] e^{-r(s-t)} ds = \int_t^\infty C^*(s) e^{-r(s-t)} ds \quad (44)$$

or that

$$Y^*(t) \approx r \int_t^\infty C^*(s) e^{-r(s-t)} ds \quad (45)$$

Where:

$$Y^*(t) = Y(K^*(t), p(t)) \quad (46)$$

$$= C^*(t) + p(t) \frac{dK^*}{dt}(t) \quad (47)$$

## 8 Appendix II

In order to solve this system we define an additional variable  $X$  that corresponds to the ratio between the consumption and the stock at the steady state.

$$X = \frac{C}{R} \quad (48)$$

Therefore the system can be specified as follows:

$$\begin{aligned} \frac{\dot{X}}{X} &= \frac{1}{\eta} \left( r - \frac{\phi U'_2}{U'_1} \right) + X \\ \frac{\dot{R}}{R} &= -X \end{aligned}$$

At the steady state, we have:

$$\frac{\dot{X}}{X} = 0 \quad (49)$$

Therefore we get the optimal value of  $X^*$ :

$$X^* = \frac{r}{\phi - \eta} \quad (50)$$

With  $X^*$  positive (hence  $\eta$  is positive)

For the existence of  $X^*$ , the following conditions can be satisfied:

$$\phi \neq \eta \quad (51)$$

$$\phi \in [0, 1] \quad (52)$$

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