The Effects of Federal Inter-Regional Transfers
With Optimizing Regional Governments

by

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1. Introduction
In certain federal systems, the federal government adopts an inter-regional redistributive role. In the case of the Australian federation, for example, the federal government taxes all six states (members of the federation) uniformly. From its tax collections it then makes an annual grant to each of the six state governments. These annual grants, however, are not uniform; the states which suffer most from revenue-raising and cost disabilities get the largest grants in per capita terms. The federal government’s system of annual grants is, therefore, redistributive in its effect, as between the six states.

An important question in the field of fiscal federalism concerns the economic and welfare consequences of federal inter-regional transfers of this type. The aim of the present paper is to develop a modelling approach suitable for examination of this question.

Several possible approaches are suggested by the fiscal-federalism literature. One would be to use a multi-regional computable general equilibrium (CGE) model as the analytical framework. The literature includes many studies in which fiscal-federalism issues have been approached along these lines. Examples are Jones and Whalley (1989), Dixon, Madden and Peter (1993), Madden (1993), Morgan, Mutti and Rickman (1996) and Nechyba (1997).

An alternative would be to exploit the approach developed by those game-theorists who have analysed questions in the fiscal-federalism area by viewing the regional governments of the federation as players in a non-cooperative, strategic-form game. Examples of this approach are the studies of Mintz and Tulkens (1986), Wildasin (1988), Hoyt (1993) and Laussel and Le Breton (1998).

1 The research reported in this paper was supported by a SPIRT Grant from the Australian Research Council. We are grateful to Peter Dixon for insightful comments and suggestions, particularly regarding the simulations.
A third possibility would be to combine these two approaches. As a first step this would mean setting up a multi-regional analytical framework of the CGE type in which fiscal decision-making by governments is exogenous (or endogenized in an *ad hoc* way), as is conventional in CGE modelling. In a second step decision-making by governments would be endogenized by making them behave as optimizers as in the game-theoretic approach. A combined approach along these lines is employed by Boadway and Keen (1997), but for a GE rather than a CGE model.

We exploit such a combined approach to examine the question about the regional consequences of federal inter-regional transfers, stated at the outset of the paper. Our particular interest is whether the presence of optimising regional governments undermines the effectiveness of the federal government’s transfers between the regions and, if so, to what extent this occurs.

In our modelling we build on the work of Pant (1997). Although Pant was not working in the fiscal-federalism field (his concern was with tariff-setting by national governments) we follow his two-stage approach. His first step was to set up a national CGE model in which tariff-setting by the national government was exogenous. Then, as a second step, he grafted on to this CGE model a relationship reflecting optimizing behaviour by the national government which served to make its tariff-setting decisions endogenous.

The present paper builds also on a recent paper by Groenewold, Hagger and Madden (2001). The model analysed here extends their model in that it relaxes the additive utility assumption and generalises the inter-regional migration condition. While, the model of Groenewold et al was purely analytical, and due to non-linearity was not readily amenable to analytical solution, the current paper develops a linearized numerical version of their model, calibrating it on the basis of Australian data and simulating the effects of a federal government transfer shock under alternative assumptions about the behaviour of regional governments.

The structure of the paper is as follows. The rest of the paper consists of five main sections. In section 2 we begin by building a small two-region general equilibrium (CGE) model which has optimizing private agents but not optimizing governments. Government fiscal decisions are simply treated as exogenous. We then extend this two-region CGE model by making the two regional governments behave in an optimizing way. Each of the two regional governments now makes its fiscal decisions so as to maximize the welfare of its citizens subject to an economic-structure constraint (captured by the CGE model) and with the decisions of the other regional government taken as given. We refer to the CGE model, thus extended, as the “Political-Economy CGE model” or PECGE model.

The model is linearized in section 3, calibrated in section 4 and put to work in section 5. We simulate the model by shocking the federal government’s transfer from one region to the other. We do this for each of the six states in turn, in each case treating the rest of the country as the second region. The results of these simulations are then used to generate conclusions about the effects (both direct and indirect) of inter-regional federal transfers in a regime of optimizing regional governments. These effects are then compared to those obtained from the same model but with exogenous regional governments.
The final section of the paper sets out the major conclusions, which are as follows. When regional governments are welfare optimizers the shock to federal government transfers has only trivial effects on per capita private consumption, on per capita consumption of the government good and hence on welfare. The main effect of the transfer shock is to induce migration of the labour force from the donor region to the recipient region. However, simulations of the model with exogenous regional governments show that the inability of the federal government to substantially influence welfare by changing transfers is the result of inter-regional migration rather than optimising governments. The principal effect of endogenising regional governments is to alter the balance between consumption of the private good and the government good.

2. The Two-Region PECGE Model

2.1 The Two-Region CGE Model

2.1.1 The Representative Household

We use the following explicit utility function for the representative household in region i:

\( U_i = \beta_i C_i^{\gamma_i} G_i^{\delta_i} \)

where

- \( U_i \) = utility, region i,
- \( C_i \) = real private consumption per household, region i,
- \( G_i \) = real government-provided consumption per household, region i.

\( \beta_i > 0 \)
\( 0 < \gamma_i < 1 \)
\( 0 < \delta_i < 1 \)

There is no saving in the model so that the constraint facing the household is:

\( P_i C_i = M_i = \pi_i + W_i \)

Where

- \( P_i \) = price of the (single) consumption good, region i,
- \( M_i \) = nominal income per household, region i,
- \( \pi_i \) = nominal profit distribution per household, region i,
- \( W_i \) = nominal wage, region i.

Equation (2) incorporates the assumption that each household supplies one unit of labour, so that labour income is \( W_i \). The household takes \( G_i \), \( \pi_i \), and \( W_i \) as given and has only a single choice-variable, \( C_i \). The utility-maximising level of \( C_i \) is:

\( C_i = M_i / P_i = (\pi_i + W_i) / P_i \)

We assume that there are \( L_i \) households in region i. Since each household supplies one unit of labour it follows that \( L_i \) is also the labour supply in region i. Total private consumption in region i must be \( L_i C_i \) and total consumption of the government-provided good, \( L_i G_i \).
2.1.2 The Representative Firm

We assume that there are $N_i$ firms in region $i$. $N_i$ is treated as exogenous. We assume that the production function has positive and declining marginal product of the single factor, labour. $L_i$ represents employment in region $i$ and, because of the decreasing returns to scale, each firm in region $i$ will be of the same size. Hence output, $Y_i$, for the representative firm in region $i$ is given by:

\[ Y_i = \left( \frac{L_i}{N_i} \right)^{\alpha_i} \quad i = 1, 2 \quad 0 < \alpha_i < 1 \]

The representative firm is assumed to operate in perfectly competitive output and labour markets and accordingly chooses employment to maximise profit:

\[ \Pi_i = P_i Y_i - W_i \left( \frac{L_i}{N_i} \right) (1 + T_i) \quad i = 1, 2 \]

subject to the production function (4) with $P_i$ and $W_i$ taken as given. In (5) $\Pi_i$ denotes profit per firm in region $i$ and $T_i$ the payroll tax rate imposed by region $i$’s government. Substituting (4) into (5) and maximising with respect to $L_i$ we get the single first-order condition:

\[ \alpha_i \left( \frac{L_i}{N_i} \right)^{\alpha_i-1} = \frac{W_i}{P_i} (1 + T_i) \quad i = 1, 2 \]

This is the standard marginal productivity condition adjusted for the presence of the payroll tax.

2.1.3 The Regional Government

The government of region $i$ purchases output from firms in region $i$ and receives revenue from the payroll tax levied in region $i$. The amount of output purchased is $GR_i$ per household or a total of $L_iGR_i$. Total tax revenue is $T_iW_iL_i$. We assume that the government of region $i$ balances its budget so that:

\[ L_iGR_i = T_iW_iL_i \]

or

\[ GR_i = T_iW_i \quad i = 1, 2 \]

2.1.4 The Federal Government

The federal government engages only in inter-regional transfers. In particular, it acquires part of the output purchased by the government of one region and supplies it to the households of the other region. It, too, balances its budget so that:

\[ L_1GF_1 + L_2GF_2 = 0 \]
where GF \(_i\) is the amount of output supplied per household to the residents of region \(i\).

The amount of the government good consumed per household in region \(i\), \(G_i\) (the variable which appears in the utility function), is given by:

\[
G_i = GR_i + GF_i \quad i = 1, 2
\]

where \(GR_i \geq 0\), \(GF_i\) may have either sign but \(G_i\) is assumed to be > 0.

### 2.1.5 Equilibrium

There are three equilibrium conditions. The first is that the national labour market clears:

\[
L_1 + L_2 = \bar{L}
\]

where \(\bar{L}\) is the national labour supply, treated as exogenous.

The second governs inter-regional migration. It is assumed that households move in response to inter-regional differences in utility and that equilibrium occurs when such differences have disappeared so that:

\[
U_1 = U_2
\]

or

\[
\beta_1C_1^iG_1^i = \beta_2C_2^iG_2^i
\]

Thirdly, we assume that the goods market clears in each region:

\[
N_iY_i = L_i(C_i + GR_i) \quad i = 1, 2
\]

Note that only regional governments purchase output and that the federal government simply transfers part of this from households in one region to households in the other.

The last equation of the CGE model is:

\[
L_i\pi_i = N_i\Pi_i \quad i = 1, 2
\]

which states that firms in region \(i\) distribute all of their profits to households in region \(i\).

### 2.2 The Two-Region PECGE Model

Relationships (3) - (13) constitute the two-region CGE model. To move to the two-region PECGE model we add optimisation by the regional governments. It is assumed that each regional government chooses its payroll tax-rate so as to maximise the utility function of the representative household in its region. Thus regional government \(i\) chooses \(T_i\) to maximize:
subject to the economic structure of region $i$ as portrayed by the CGE model equations.

The first-order condition for this maximization problem is:

$$G_1^{\gamma-i} C_1^{\gamma-i} \frac{\partial C_1}{\partial T_i} + C_1^{\gamma-i} \delta_1 G_1^{\delta-i} \frac{\partial G_1}{\partial T_i} = 0 \quad i = 1, 2$$

The two partial derivatives which appear in (14) are multipliers from the two-region CGE model set out in section 2.1 which constitute the constraints which the regional government faces in its optimisation.

The CGE model is converted to the PECGE model by adding (14) and making $T_i$ endogenous. The federal government is assumed to choose one of the $G_F_i$ values (say $G_F_1$) with the second being determined via its budget constraint, equation (8). The PECGE model thus consists of 21 equations in the following 21 endogenous variables:

$$C_i, \pi_i, \Pi_i, W_i, P_i, O_i, L_i, T_i, GR_i, G_i, GF_2, \quad (i=1,2)$$

and the following four exogenous variables:

$$N_i, G_F_i, \bar{L} \quad (i = 1, 2)$$

We now write the model more compactly. First the endogenous variables are reduced to 19 by setting $P_i = 1 \quad (i = 1, 2)$, thus treating output in each region as the numeraire.

Next we use (13), (4) and (5) to write (3) as:

$$C_i = (L_i/N_i)^{\alpha-1} - W_i T_i \quad i = 1, 2$$

Equations (6), (7), (8) and (10) are reproduced as they stand:

$$\alpha \left( \frac{L_i}{N_i} \right)^{\alpha-1} = W_i (1 + T_i) \quad i = 1, 2$$

$$GR_i = T_i W_i \quad i = 1, 2$$

$$L_1 GF_1 + L_2 GF_2 = 0$$

$$L_1 + L_2 = \bar{L}$$

We next use (9) to substitute for $G_i$ in (11) and (14):

$$\beta_1 C_1^{\gamma-i} (GR_i + GF_1)^{\delta_i} = \beta_2 C_2^{\gamma-i} (GR_2 + GF_2)^{\delta_i}$$
Finally, we use (4) to write the product-market-clearing condition, (12), as:

\[(22) \quad \left(\frac{L_i}{N_i}\right)^{\alpha_i^{-1}} = C_i + GR_i \quad i = 1,2\]

This gives a total of 13 equations in 11 endogenous variables: $C_i$, $L_i$, $W_i$, $T_i$, $GR_i$, and $GF_1$ (say).

Two of the 13 equations are redundant, however. This can be seen by substituting (17) into (15) to obtain (22). We therefore drop (15) and as our final form of the PECGE model, take the eleven equations, (16)-(22) in the 11 endogenous variables listed above, exogenous variables $N_i$, $L$ and $GF_2$ and parameters, $\beta_i$, $\gamma_i$, $\delta_i$ and $\alpha_i$. We now proceed to linearise this model.

3. The Linearized Numerical Version of the Two-Region PECGE Model

The two-region PECGE model set out in the previous section is non-linear in the levels of the variables. For this reason it cannot easily be used to conduct comparative-static exercises which will throw light on the topic of the present paper - the regional effects of inter-regional federal transfers when regional governments behave as optimizing agents. We circumvent this problem by using a numerical linearized version of the model which we describe in this section.

3.1 Linearization of the PECGE Model

To linearize the PECGE model of section 2 we use a process of log differentiation. This converts the model from one which is non-linear in the levels of the model to one which is linear in the proportional rates of change of the variables. The resulting linearised versions of equations (11)-(22) are:

\[(16') \quad (\alpha_i - 1)l_i - w_i - \sigma_i t_i = (\alpha_i - 1)n_i \quad i = 1,2\]

where $x_i \equiv \frac{dX_i}{X_i}$ for all $X_i$ and $\sigma_i \equiv \frac{T_i}{1 + T_i}$.

\[(17') \quad gr_i = w_i + t_i \quad i = 1,2\]

\[(18') \quad l_1 + gf_1 = l_2 + gf_2\]

\[(19') \quad \sigma_{i1}l_1 + \sigma_{i2}l_2 = \bar{1}\]

where $\sigma_{ii} \equiv \frac{L_i}{\bar{L}} = L_i/(L_1 + L_2)$.

Given $\beta_i$ is a constant,
\[\gamma_i c_i + \delta_i \left( \sigma_{gr_i}^{g} + \sigma_{gf_i}^{g} \right) = \gamma_i c_2 + \delta_i \left( \sigma_{gr_2}^{g} + \sigma_{gf_2}^{g} \right)\]

where
\[\sigma_{gr_i}^{g} = \frac{GR_i}{GR_i + GF_i} \quad \text{and} \quad \sigma_{gf_i}^{g} = \frac{GF_i}{G_i}\]

\[c_i - \sigma_{gr_i}^{g} gr_i - \sigma_{gf_i}^{g} gf_i = 0 \quad i = 1, 2\]

\[\sigma_{ci}^{y} c_i + \sigma_{gr_i}^{y} gr_i - (\alpha_i - 1) l_i = -(\alpha_i - 1) n_i \quad i = 1, 2\]

where
\[\sigma_{ci}^{y} = \frac{C_i}{C_i + GR_i} \quad \text{and} \quad \sigma_{gr_i}^{y} = \frac{GR_i}{C_i + GR_i}\]

Equations (16')-(22') constitute a linear system in the eleven endogenous variables: \(c_i, l_i, w_i, t_i, gr_i\) and \(gf_2\) and the 4 exogenous variables: \(n_i, gf_i\) and \(\bar{I}\).

### 3.2 Numerical Versions of the Linearized PECGE Model

We now put the linearized PECGE model, (16')-(22'), into numerical form by evaluating the various coefficient which appear there.

Six numerical versions are constructed. Australia has six states. The states are New South Wales (NSW), Victoria (Vic), Queensland (Qld), South Australia (SA), Western Australia (WA), Tasmania (Tas). One of the six numerical versions has NSW as region 1 and the rest of the country (ROC) as region 2, a second has Vic as region 1 and ROC as region 2 and so on for each of the other four states. We therefore simulate six alternative versions of the model.

The model, (16')-(22') contains seven parameters which have to be evaluated: \(\alpha_i, \gamma_i, \delta_i, \sigma_{si}, \sigma_{ri}, \sigma_{gr_i}^g, \sigma_{gf_i}^g, \sigma_{ci}^y\) and \(\sigma_{gr_i}^y\). These seven parameters fall into two groups. The first three appear in model relationships; \(\gamma_i\) and \(\delta_i\) appear in the utility function (1) and \(\alpha_i\) in the production function (4). The last six, on the other hand, are linearization parameters.

The model parameters can be evaluated with the help of model restrictions and appropriate past information on model aggregates. Start with \(\alpha_i\). Using (16) and (22) we get:

\[\alpha_i = \frac{W_i (1 + T_i)}{C_i + GR_i}\]
This expression can be used to evaluate $\alpha_i$ for NSW as region 1 and ROC as region 2 given a figure for each $W_i$, $T_i$, $C_i$ and $GR_i$ for NSW and each of the other five states, i.e. given these figures for all six states; and similarly for the other five versions of the linearized PECGE model.

Turn now to $\gamma_i$ and $\delta_i$. Using (21) it can be shown that:

$$\frac{\gamma_i}{\delta_i} = -\frac{g_{i}^{GE}}{c_{i}^{GE}}$$

$i = 1, 2$

where $g_{i}^{GE}$ and $c_{i}^{GE}$ are derived from the CGE part of the model and are, respectively, the proportional changes in $G_i$ and $C_i$ consequent on a one per cent change in $T_i$, with all exogenous variables held fixed. Given an $\alpha$ value for each of the states, both $g_i$ and $c_i$ can be computed from the numerical version of the CGE part of the PECGE model. Having determined the $\gamma_i/\delta_i$ ratio for each of the six states in this way the levels of the two parameters were fixed by requiring that they sum to unity.

The linearization parameters can be evaluated directly from their definitions, as presented in section 3, given appropriate base values on model aggregates for each of the six states. To evaluate all the linearization parameters we need values for $T_i$, $GR_i$, $GF_i$, $G_i$, $C_i$, and $L_i$. We use the model constraints to calculate $T_i = GR_i/W_i$ and $G_i = GR_i + GF_i$, thus ensuring that the parameters values are consistent with the constraints. In each case the value we use for the aggregate is the average value for the years 1994-95 to 1995-99. The values used for each of the model simulations can be found in Appendix A.

4. Simulations With Numerical Versions of the Linearized PECGE Model

In this section we discuss six comparative-static exercises. In each simulation we choose one of the six states to be region 1 and the rest of the country to be region 2 and examine the effects of an increase in the federal government’s transfer from the rest of the country to region 1. In this way we throw light on the topic of the present paper - the regional effects of inter-regional federal transfers when regional governments behave as optimizing agents.

4.1 Determination of Shocks

For each simulation we shocked $GF_1$ by choosing a non-zero value for $gf_1$ and setting the changes in the remaining exogenous variables at zero. In each case we chose a shock large enough to ensure perceptible results but not so large as to be implausible from an historical perspective. The assumed increase in the per capita transfer to region 1 was set at 10% of the average per capita transfer for all regions over the five-year base period. The average per capita transfer was calculated at $\$3226.20$ so that $gf_1$ was shocked by an amount calculated to ensure a rise in $GF_1$ of $\$322.62$ in each simulation.

We presume that, for whatever reason, the federal government undertakes this policy in order to improve the welfare of residents of region 1, if necessary at the expense of the welfare of those living in region 2.
4.2 Results of Simulations

We now turn to the simulation results which are set out in Table 1. The initial effect of the increase in GF$_1$ is to increase the consumption of the government good in region 1 and decrease it in region 2. Both individuals and regional governments react to this shock.

Initially the residents of region 1 find that they are better-off and those living in region 2 find that they are worse-off. This is clear from the numbers in the row for “initial-u” in Table 1 which gives the effect of the shock on utility before either the regional governments or individuals themselves have responded. Individuals in region 2, therefore, find that they could improve their welfare by moving to region 1 and inter-regional migration occurs until the equality between utility in the two regions is re-established.

In the process of migration the labour force in region 1 expands but it contracts in region 2. Since the total labour force is fixed, the increase in $L_1$ is exactly offset by the fall in $L_2$ as is evident from the results in Table 1.

Output also increases in region 1 and falls in region 2 although output per capita moves in the opposite direction reflecting the diminishing marginal product of labour in each region which ensures that average product falls as employment rises.

Since output in region 1 rises and output in region 2 falls, the effect of the federal government’s policy on national output is of ambiguous sign. The results reported in Table 2 show that the percentage change in national output is positive for some simulations and negative for others.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Output Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSW as Region 1</td>
<td>0.0661</td>
</tr>
<tr>
<td>Vic as Region 1</td>
<td>0.0172</td>
</tr>
<tr>
<td>Qld as Region 1</td>
<td>-0.0366</td>
</tr>
<tr>
<td>SA as Region 1</td>
<td>-0.0106</td>
</tr>
<tr>
<td>WA as Region 1</td>
<td>-0.0120</td>
</tr>
<tr>
<td>Tas as Region 1</td>
<td>-0.0039</td>
</tr>
</tbody>
</table>

Whether national output falls or rises depends on the relative magnitudes of the regional marginal products of labour. To see this, it can be shown that the sign of $y$, the percentage change in per capita national output (which equals the percentage change in national output given that total population is fixed), depends on the relative magnitudes of regional wage costs including the payroll tax. Since in equilibrium the gross wage is equal to the marginal product of labour, it follows that national output increases if the labour reallocation is from the region with the lower marginal product of labour to the region with the higher marginal product.
Table 1
Results of Simulation with gf Shock

<table>
<thead>
<tr>
<th>Variable</th>
<th>Version</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>c (%)</td>
<td>-0.0006</td>
<td>-0.0006</td>
<td>0.0237</td>
<td>0.0237</td>
<td>-0.0038</td>
<td>-0.0038</td>
<td>-0.0106</td>
</tr>
<tr>
<td>l (%)</td>
<td>2.2475</td>
<td>-1.1796</td>
<td>2.3465</td>
<td>-0.8136</td>
<td>2.2119</td>
<td>-0.5176</td>
<td>2.0986</td>
</tr>
<tr>
<td>L (number)</td>
<td>63031</td>
<td>-63031</td>
<td>49226</td>
<td>-49226</td>
<td>34177</td>
<td>-34177</td>
<td>13666</td>
</tr>
<tr>
<td>w (%)</td>
<td>0.1484</td>
<td>-0.1005</td>
<td>0.1819</td>
<td>-0.0373</td>
<td>0.2342</td>
<td>-0.0418</td>
<td>0.2054</td>
</tr>
<tr>
<td>t (%)</td>
<td>-4.6304</td>
<td>2.5882</td>
<td>-5.1561</td>
<td>1.5941</td>
<td>-5.3968</td>
<td>1.1023</td>
<td>-4.4994</td>
</tr>
<tr>
<td>gr (%)</td>
<td>-4.4820</td>
<td>2.4877</td>
<td>-4.9742</td>
<td>1.5569</td>
<td>-5.1626</td>
<td>1.0605</td>
<td>-4.2940</td>
</tr>
<tr>
<td>GR(Spc)</td>
<td>-322.66</td>
<td>168.05</td>
<td>-321.17</td>
<td>109.98</td>
<td>-322.86</td>
<td>74.88</td>
<td>-323.55</td>
</tr>
<tr>
<td>g (%)</td>
<td>-0.0006</td>
<td>-0.0006</td>
<td>0.0237</td>
<td>0.0237</td>
<td>-0.0038</td>
<td>-0.0038</td>
<td>-0.0106</td>
</tr>
<tr>
<td>G(Spc)</td>
<td>-0.04</td>
<td>-0.04</td>
<td>1.45</td>
<td>1.70</td>
<td>-0.24</td>
<td>-0.27</td>
<td>-0.93</td>
</tr>
<tr>
<td>y (%)</td>
<td>-0.6837</td>
<td>0.3878</td>
<td>0.6977</td>
<td>0.2663</td>
<td>-0.7744</td>
<td>0.1621</td>
<td>-0.7547</td>
</tr>
<tr>
<td>ytot (%)</td>
<td>1.5638</td>
<td>-0.7918</td>
<td>1.6468</td>
<td>-0.5473</td>
<td>1.4376</td>
<td>-0.3555</td>
<td>1.3440</td>
</tr>
<tr>
<td>initial-u (%)</td>
<td>0.6861</td>
<td>-0.3870</td>
<td>0.7380</td>
<td>-0.2396</td>
<td>0.7725</td>
<td>-0.1657</td>
<td>0.7166</td>
</tr>
<tr>
<td>u (%)</td>
<td>-0.0006</td>
<td>-0.0006</td>
<td>0.0237</td>
<td>0.0237</td>
<td>-0.0038</td>
<td>-0.0038</td>
<td>-0.0106</td>
</tr>
</tbody>
</table>
Note that national output may rise even though both per capita consumption and per capita government expenditure fall in both regions as is the case when NSW is region 1. An alternative decomposition of the growth in national output shows that this is possible when migration is from the region with the lower per capita output to the one with the higher per capita output. Using the data in Appendix A, it is straightforward to calculate the per capita output for each of the regions when NSW is region 1 and show that it is, indeed, considerably higher for region 1 than for region 2.

Table 1 shows that the wage rate rises in region 1 and falls in region 2 and the question arises whether this is consistent with the fall in the marginal product of labour in region 1 and the rise of region 2’s marginal product of labour. To see that it is, recall that in equilibrium the marginal product is equal to the wage including the payroll tax so that a higher wage is consistent with a lower marginal product if the tax rate falls which it does in region 1 as we will see when we consider the reaction of the regional governments to the federal government policy shock, a matter to which we turn now.

An important part of the PECGE model is that each regional government chooses its tax and expenditure to maximise the utility of the representative household in its region (subject to its budget constraint). We can see the regional governments as maintaining a balance between private consumption and consumption of the government good in their region. After the initial shock to the federal government transfer, \( G \) will be too large relative to \( C \) in region 1 and too small relative to \( C \) in region 2 so that region 1’s government will reduce its expenditure and region 2’s government will increase its expenditure. Since regional governments are assumed to balance their budgets, this will require a tax cut in region 1 and a tax hike in region 2. The numerical results show that in all cases the reaction by the regional governments is such as to almost exactly offset the effect on \( G \) of the shock to \( GF \) so that the final effect on \( G \) is very small. This is particularly clear from the numbers for \( GR(\text{Sp}) \) and \( G(\text{Sp}) \) which show the change in \( GR \) and \( G \) in terms of dollars per capita – in each simulation the region-1 value for \( GR(\text{Sp}) \) is almost exactly equal to the negative of the per capita federal government shock of $322.62 so that the overall change in \( G \) is close to zero.

The results in Table 1 show that this negligible final effect on \( G \) flows through into per capita consumption and, ultimately, utility: \( c, g \) and \( u \) all have the same value for both regions. This follows directly from the equations of the model since equation \( (21') \) implies that \( c_1 = g_1 \), the inter-regional migration equilibrium condition requires equality of utility across the two regions, i.e. \( u_1 = \gamma_1 c_1 + \delta_1 g_1 = u_2 = \gamma_2 c_2 + \delta_2 g_2 \) and, finally, \( \gamma_1 + \delta_1 = 1 \) implies that \( u_1 = u_2 = c_1 = c_2 = g_1 = g_2 \).

A final noteworthy feature of the results reported in Table 1 is that, although the final change in utility is always small, in four of the six cases it is negative so that the attempt by the federal government to influence the inter-regional distribution of resources actually makes residents of both regions worse-off even though in all cases the initial effect is to make the citizens of region 1 better-off and those of region 2 worse-off. Whether the final effect is detrimental or beneficial is shown in Appendix B to depend on the sign of \( K = [(\Pi_1 + GF_1) - (\Pi_2 + GF_2)] \). \( u_1 \) and \( u_2 \) are positive if \( K < 0 \) and vice versa. This condition is consistent with Petchey’s (1995) result. Petchey shows in a model of a federation consisting of two states that if there is a region-specific factor, the income to which is distributed on an equal-per-capita basis to the residents of the region, then inter-regional migration in response to inter-regional income differentials results in too many residents
in the region which has the larger endowment of the fixed factor. Hence inter-regional grants (which induce inter-regional migration) can be efficiency-enhancing. In our model we can interpret $\Pi + GF$ as region-specific income which is distributed on an equal-per-capita basis. Our results show that welfare is improved if inter-governmental grants are made to the region with the smaller value of $\Pi + GF$, i.e. to the region with the population which is too small and that efficiency gains have been exhausted when $\Pi_1 + GF_1 = \Pi_2 + GF_2$.

It is clear from the results in Table 1, that under the influence of inter-regional migration and regional government reaction, the effects of the federal government’s attempt to shift the inter-regional resource allocation in favour of region 1 has very little effect on welfare. This observation raises the question of how much of the offsetting of the federal government initiative results from inter-regional migration and how much results from the endogenous policy response by the regional governments. We address this question with the help of the information reported in Table 3. The table has two rows for each variable, the first of which replicates the relevant figures from Table 1 and the second shows the corresponding figure for the case when the regional governments are assumed not to react, i.e. they are treated as exogenous as in the CGE model.

Two variables are relatively unaffected by keeping the regional governments exogenous – the percentage changes in employment and utility. Irrespective of whether regional governments optimise or not, substantial inter-regional migration follows the federal government’s shock and this movement of labour from region 2 to region 1 serves to largely wipe out the welfare effects of the inter-regional transfer. Thus, in our model regional government optimisation does not materially affect welfare. The results in the table make clear that, on the other hand, endogenising the regional governments does substantially affect consumption, the wage rate and total government expenditure.

We conclude that the federal government’s increase in transfers to region 1 from region 2 initially improve welfare in region 1 at the costs of welfare in region 2. However, this effect is largely undone by the process of inter-regional migration in which residents move to the high-utility region from the low-utility region. The effect of endogenous regional governments is largely to restore the balance between government and private consumption which was upset by the federal government’s initiative but this re-balancing has only minor effects on welfare. The only noticeable long-term effect of the federal government’s action is to shift population from region 2 to region 1.

5. Conclusion
In this paper we have set out to analyse the regional effects of inter-governmental transfers by a federal government. We have done so in a model in which regional governments determine their tax and expenditure policies so as to maximise the utility of the representative household in their region subject to a budget constraint consisting of a CGE model describing the regional economy. Each regional government was assumed to take the other regional government’s actions as given. The model used was a two-region linear numerical model which extends the model in Groenewold, Hagger and Madden (2001). The model was calibrated using Australian data.

We conducted a series of six simulations of an increase in the federal government’s transfer to one region matched by a decrease in the transfer payment to the other. In each
simulation one of the six Australian states was taken as region 1 and the rest of the country as region 2.

We found that substantial changes in the amount transferred by the federal government from one region to the other had little effect on welfare, per capita consumption and wages. In fact, the main effect of the shock to transfers was on to re-distribute labour from the losing region to the gaining region. Inter-regional migration ensured that the welfare effects were very small and equal across the regions and the optimising behaviour of the regional governments ensured that the effect on per capita consumption and wages were negligible.
Table 3
Results of Simulation with gf Shock: CGE Model

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<tr>
<th>Variable</th>
<th>Version</th>
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<th>3</th>
<th>4</th>
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<td>-0.0006</td>
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<td>0.0237</td>
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<td>0.3881</td>
<td>-0.7047</td>
<td>0.2682</td>
<td>-0.7750</td>
<td>0.1623</td>
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<tr>
<td>l (%)</td>
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<td></td>
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<tr>
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<td>-0.1005</td>
<td>0.1819</td>
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<td>0.3881</td>
<td>-0.7047</td>
<td>0.2682</td>
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<td>0.1623</td>
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<tr>
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<td>-0.0006</td>
<td>0.0237</td>
<td>0.0237</td>
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<tr>
<td>u (%)</td>
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<td>0.0280</td>
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## Appendix A

### Table A1

**Data-Base**

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<tr>
<th>Region</th>
<th>NSW</th>
<th>ROC</th>
<th>1000</th>
<th>LW</th>
<th>Y/L</th>
<th>L</th>
<th>W</th>
<th>Y</th>
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<tr>
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<td>112265.4</td>
<td>195152.8</td>
<td>307418.2</td>
<td>80040.8</td>
<td>227377.4</td>
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<td>LW ($m)</td>
<td>92160.8</td>
<td>15227.8</td>
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<td>13545.4</td>
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<td>2097.9</td>
<td>6050.1</td>
<td>8148</td>
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<td>6602.9</td>
</tr>
<tr>
<td>W ($’000)</td>
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<td>53.2635</td>
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<td>63.2635</td>
<td>53.2635</td>
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<td>53.2635</td>
</tr>
<tr>
<td>Y/L ($’000)</td>
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</table>

Sources: C_i, L_i, LW_i, and GR_i are from ABS times series averaged over the period 1994/95 - 1998/99. GF_i is computed as L_i(MGF/L_i - MGF/L) where MGF_i is final consumption expenditure by the federal government plus grants to state i. All other data is calculated from these figures to ensure that the model constraints hold: L = L_1 + L_2, W_i = W_i\cdot L_i/L_i, Y_i = GR_i + C_i, G_i = GR_i + GF_i, T_i = GR_i/W_i. It should be noted that, as the model excludes investment and net interstate and overseas exports, Y_i will not conform with official figures.
APPENDIX B

Proof that \( c_1 > 0 \Rightarrow (\Pi_1 + GF_1) < (\Pi_2 + GF_2) \)

We begin by solving the linearized PECGE model for \( c_1 \). The equations defining the solution are:

1. \( \sigma_{i}^x c_i + \sigma_{gi}^x gr_i = (\alpha_i - 1)\ell_i \)
2. \( \sigma_{i_1} \ell_1 + \sigma_{i_2} \ell_2 = 0 \)
3. \( L_1(GF_1 \ell_1 + \Delta GF_1) = -L_2(GF_2 \ell_2 + \Delta GF_2) \)
4. \( c_1 = g_1 = c_2 = g_2 \)
5. \( \sigma_{gi}^x gr_i + \sigma_{gi}^x \frac{\Delta GF_i}{GF_i} = g_i \)

where \( \Delta GF_i = gf_i GF_i \). Equation (1) is \((22')\) with \( n_i \) put at zero while (2) is \((19')\) with \( \bar{\ell} \) put at zero. Equation (3) is \((18')\) after manipulation based on \((18)\). (4) is the equation derived in section 5.2 from (14) while (5) is derived from (9).

We use (5) and the definitions of \( g_{gi}^x \) and \( \sigma_{gi}^x \) to write:

6. \( gr_i = \frac{G_i}{GR_i} g_i - \frac{\Delta GF_i}{GR_i} \)

This expression is substituted for \( gr_i \) in (1) and \( g_i = c_i \) and the definition of \( \sigma_{c_i}^x \) used to write:

7. \( c_i = \frac{Y_i}{C_i + G_i} (\alpha_i - 1)\ell_i + \frac{\Delta GF_i}{C_i + G_i} \)

Use \( c_1 = c_2 \) and simplify notation by writing \( Q_i \equiv C_i + G_i \). Then:

8. \( \frac{Y_1}{Q_1} (\alpha_1 - 1)\ell_1 + \frac{\Delta GF_1}{Q_1} = \frac{Y_2}{Q_2} (\alpha_2 - 1)\ell_2 + \frac{\Delta GF_2}{Q_2} \)

Use (3) to substitute out for \( \Delta GF_2 \) and (2) to substitute out for \( \ell_2 \) to get:
\[
\ell_1 = \frac{-\left[ \frac{1}{Q_1} + \frac{L_1}{Q_2L_2} \right]}{\left[ \frac{Y_1}{Q_1} (\alpha_1 - 1) + \frac{Y_2}{Q_2} (\alpha_2 - 1) \frac{L_1}{L_2} + \frac{L_1}{Q_2 L_2} (GF_1 - GF_2) \right]} \Delta GF_1
\]

Now the numerator of (9) < 0 since \( Y_1, Y_2, Q_1, Q_2, L_1, L_2 > 0 \). But since \( \ell_1 > 0 \) if \( \Delta GF_1 > 0 \) (i.e. labour flows \textit{into} the region receiving the positive transfer) the denominator of (7) must be < 0 which we assume.

Now substitute the solution for \( \ell_1 \) into (7) and use \( \Pi_i = Y_i (1 - \alpha_i) \) to get the following:

\[
c_1 = \frac{\frac{L_1}{Q_2 L_1 Q_2} \left[ (\Pi_1 + GF_1) - (\Pi_2 + GF_2) \right]}{\left( \frac{Y_1}{Q_1} (\alpha_1 - 1) + \frac{Y_2}{Q_2} \frac{L_1}{L_2} (\alpha_2 - 1) + \frac{L_1}{Q_2 L_2} (GF_1 - GF_2) \right)} \Delta GF_1
\]

Since the denominator of (10) is < 0 it follows that for \( GF_1 > 0 \):

\[
c_1 > 0 \iff \Pi_1 + GF_1 < \Pi_2 + GF_2
\]
References