Evaluating the Robustness of Trade Restrictiveness Indices

by

Stephen Tokarick[1]

February 2007

Abstract

In a series of papers, Anderson and Neary have developed a trade restrictiveness index (TRI) that is firmly grounded in economic theory and that can be implemented empirically. In order to calculate TRIs in practice, one needs a model. This paper examines the robustness of TRI calculations to alternative model structures and it shows that the TRI is not very sensitive to changes in elasticities of substitution between factors of production, but it can be quite sensitive to alternative model structures (e.g. specific or mobile factor models). Also, the paper points out that in assessing trade restrictiveness over time, researchers need to be aware that changes in economic structure (e.g. factor accumulation and technological change) will alter the calculated values of the TRIs for unchanged trade policy. Therefore, researchers need to adjust the calculated values for TRIs for changes in economic structure if one wants a measure of changes in trade policy only.

JEL Codes: F13, C68

Keywords: trade restrictiveness, tariffs, general equilibrium, welfare cost.

---

[1] Senior economist, International Monetary Fund, Research Department, Trade and Investment Division, 700 19th Street N.W., Room 9-700A, Washington D.C., 20431. E-mail address: stokarick@imf.org. Phone: 202-623-7590. FAX: 202-623-7590. The views expressed in this paper are those of the author and should not be attributed to the International Monetary Fund, its Executive Board, or its Management.

I. Introduction

Over the last decade, there has been a great deal of interest in the relationship between a country's degree of openness to international trade and economic growth. A central issue is how to measure openness to trade in an economically meaningful way. In a series of papers and a book, Anderson and Neary (1996, 2005) develop a trade restrictiveness index (TRI) that has a firm foundation in economic theory and that can be implemented in practice. The TRI is defined as the uniform deflator, or scaling factor, applied to the prices of imported goods that would produce the same effect on real income as the country's differentiated structure of tariffs. Alternatively, trade restrictiveness is sometimes measured by computing the uniform tariff rate that is equivalent, in welfare terms, to the country's existing tariff structure—the uniform tariff equivalent (UTE). Anderson and Neary (1994), Lloyd and MacLaren (2002), and O'Rourke (1997) have all calculated TRIs using various types of computable general equilibrium (CGE) models, while Kee, Nicita, and Olarreaga (2004) calculated another type of trade restrictiveness index, the mercantilist trade restrictiveness index (MTRI) using a method developed by Feenstra (1995) that makes some simplifying assumptions, but does not require a CGE model.

Unfortunately, there has not been much work to date that examines how robust calculated TRIs are to alternative model structures and economic environments. This paper has two purposes. First, it explores the robustness of TRI calculations to different CGE model structures. Anderson and Neary (1994) calculated TRIs for twenty five countries using a CGE model for each country that was identical in structure, but different in data and parameter values. Their results showed that the resulting values for the TRIs, and the ranking of countries' restrictiveness, were generally insensitive to alternative elasticity values. So far, there has been very little exploration of how alternative model structures affect the calculation of TRIs and this paper addresses this issue. To date, O'Rourke (2002) is the only other paper to address this question, but he explored the sensitivity of TRI calculations to the specification of consumer demand.

The second purpose of this paper is to demonstrate that the calculated value of a TRI can change as a result of economic growth, i.e. factor accumulation, even though tariff rates remain unaltered. Typically, TRIs are estimated for two points in time and the values compared to reach a judgment regarding whether the country has become more or less open to international trade. In doing so, however, analysts typically do not take into account changes in the structure of the economy that may have taken place between the two time periods—changes that would affect the welfare cost of a country's tariffs and therefore, its TRI. It turns out that a country's TRI could rise or fall with factor accumulation, depending, among other things, on the bias of the accumulation. This paper points out that this may be a non-trivial issue for rapidly growing economies.

II. Robustness of TRI Calculations

This section reports the results of some sensitivity tests on calculated TRIs using a CGE model of a hypothetical economy. The objective is to determine how sensitive the calculated TRIs are to alternative values of the elasticity of substitution among factors of production, and to alternative model structures. Using a CGE model of Columbia, Anderson (1993) presented estimates that showed that changes in Columbia's TRI were relatively insensitive to alternative values of the elasticity of transformation, as well as the elasticities of final and intermediate demand. This conclusion was derived from a particular CGE model, one in which there is no local production of the importable good, no domestic consumption of the exportable, and no explicit factor markets. Instead of modeling factor markets explicitly, Anderson's model employed a transformation function between exportables and nontraded goods, and an elasticity of transformation governs how easy it is to shift production between the two types of goods.

O'Rourke (1997) used the same model structure as Anderson to assess how sensitive calculations of TRIs for Britain and France in the 1880s were to alternative specifications of consumer demand. He considered alternative nesting schemes for commodities in consumer demand and found the calculated TRIs to be quite sensitive to alternative commodity groupings and elasticities of substitution. Neither of these papers investigated the sensitivity of the TRI calculations to alternative production structures. This section provides the results from such an exercise.

A. Calculated TRIs and the Elasticity of Substitution Among Factors of Production

This section reports the results from using a CGE model to assess the sensitivity of calculated TRIs to alternative model structures and values for the elasticity of substitution among factors of production. The model consists of three goods: two imports and one exported good. Each good is produced using three factors of production using a constant elasticity of substitution (CES) production function; there are no intermediate inputs to keep the model as simple as possible. A representative consumer receives all factor income plus tariff revenue and is assumed to maximize a Cobb-Douglas utility function defined over the three goods. The terms of trade are exogenous and the price of the export good is taken as the numeraire. In a sense, this model is quite similar to the standard general equilibrium model used in international trade theory, except that there are three goods and factors instead of just two. Two variants of the model are used to conduct sensitivity tests: one with all factors of production mobile across sectors and one that assumes that one factor of production is sector-specific, i.e. immobile across sectors. This permits an evaluation of the sensitivity of the TRI to alternative values of the elasticity of substitution among factors, as well as with respect to model structure.

The model described above differs from the model used by Anderson (1993) and O'Rourke (1997) in several ways. First, the model used in this paper introduces factor markets explicitly, while the others do not. Second, the model allows for consumption of the country's export good, as well as domestic production of the two imported goods. In fact, the model assumes that domestic goods are perfect substitutes for imports, as is common in trade theory. Third, unlike Anderson and O'Rourke, the model has no nontraded goods. Nontraded goods are realistic features of many economies and should be included, but they are excluded here to keep the model as simple as possible and to create a structure that is significantly different from the one used by Anderson and O'Rourke.

B. The Trade Restrictiveness Index

Anderson and Neary derive the TRI using the balance-of-trade function for a small, open economy. The TRI, denoted by Δ, is the uniform deflator that when applied to the prices of imported goods *i* in situation 1 (p_{Mi}^{1}), would leave the consumer as well off as in situation 0, with prices of imports equal to p_{Mi}^{0} . For the simple case of two import goods, the TRI is given implicitly by:

$$B\left(\frac{p_{M1}^{1}}{\Delta},\frac{p_{M2}^{1}}{\Delta},p_{E}^{0},u^{0}\right)=B\left(p_{M1}^{0},p_{M2}^{0},p_{E}^{0},u^{0}\right) \tag{1}$$

where $B(\cdot)$ is the balance-of-trade function, p_{M1} and p_{M2} are the domestic prices of the two import goods, p_{E} is the price of exports, and u^{0} is the initial level of utility. The superscripts "0" denotes the initial situation, while "1" denotes the new situation.

Totally differentiating the right-hand side of equation (1) and solving for du^{0} gives:

$$du^{0}=\frac{1}{B_{u}^{0}}\left[-B_{M1}^{0}dp_{M1}^{0}-B_{M2}^{0}dp_{M2}^{0}\right] \qquad (2).$$

Differentiating the left-hand side of equation (1) and using (2) gives the proportional change in the TRI:

$$\hat{\Delta}=\frac{-B_{u}^{\Delta}}{B_{u}^{0}}\left[\frac{B_{M1}^{0}dp_{M1}^{0}+B_{M2}^{0}dp_{M2}^{0}}{B_{M1}^{\Delta}p_{M1}^{\Delta}+B_{M2}^{\Delta}p_{M2}^{\Delta}}\right] \tag{3}$$

where B_k is the partial derivative of the balance-of-trade function and B^{Δ} denotes the derivative of the balance-of-trade function evaluated at prices $p_{Mi}^{\Delta} = \frac{p_{Mi}^{1}}{\Delta}$. The term $\left(\frac{B_u^{\Delta}}{B_u^{0}}\right)$ is a "conversion factor" in that the balance of trade function is evaluated at two different points. An alternative way to write equation (3) is:

$$\hat{\Delta} = \left[\frac{-B_u^{\Delta}}{B_u^{0}} \frac{B_{M1}^{0} p_{M1}^{0} + B_{M2}^{0} p_{M2}^{0}}{B_{M1}^{\Delta} p_{M1}^{\Delta} + B_{M2}^{\Delta} p_{M2}^{\Delta}}\right]\left[\frac{B_{M1}^{0} dp_{M1}^{0} + B_{M2}^{0} dp_{M2}^{0}}{B_{M1}^{0} p_{M1}^{0} + B_{M2}^{0} p_{M2}^{0}}\right] \tag{4}$$

where the first bracketed term on the right-hand side of (4) is an adjustment coefficient, since the balance-of-trade function is evaluated at two different points, p_{Mi}^{0} and p_{Mi}^{Δ}. For small changes, this bracketed term will be close to one.

Using equation (4), changes in model structure would have a significant impact on the calculation of the TRI. This is because a switch in model structure alters how outputs respond to changes in prices—the properties of each B_{Mi} term change. For example, in a model with specific factors, output supply functions are upward sloping, reflecting rising marginal costs of production. On the other hand, with all factors of production mobile, output supply curves are quite flat. Changes in the elasticity of substitution also affect output responses, but for a given model structure, these tend to alter both the numerator and denominator of equation (4) in similar ways and therefore, the effects tend to cancel out. Thus, changes in model structure would be expected to have more of an impact on calculated TRIs than changes in elasticities of substitution between factors of production. The next section uses a general equilibrium model to generate some numerical examples of how elasticities of substitution and model structure might affect the calculation of the "uniform tariff equivalent", UTE, which is related to the TRI through the relation: $\tau = \frac{1}{\Delta} - 1$, where τ is the UTE.

C. Sensitivity Results From a CGE Model

Table 1 presents the calculated UTEs from the model for alternative values of the elasticities of substitution among the three factors of production, σ_{M1}, σ_{M2}, and σ_E, in the two import sectors and the export sector. UTEs are calculated holding two of the elasticities of substitution constant, while varying the third only, from a value of 2.0 to 0.5. Estimates of the UTEs are presented for two cases: one where one of the factors is sector specific and the other where all three factors are mobile across all sectors.

Table 1 reveals that the calculated UTEs are generally insensitive to changes in the elasticity of substitution among factors for a given model structure, however, the UTEs can be quite sensitive to the choice of model structure. For example, in the specific-factors model, varying the elasticity of substitution among factors does have an impact on the calculated UTE, however, the magnitude of the change is relatively small. In table 1, altering the elasticity of substitution in the first import sector, σ_{M1}, by 75 percent (from 2.0 to 0.5) results in a decline in the UTE of only about 12 percent. Similarly, the same percentage reduction in σ_{M2} raises the UTE by about 13 percent. The largest impact comes in the export sector: reducing σ_X by 75 percent increases the UTE by 30 percent. In all cases, changes in the elasticity of substitution among factors translate into changes in the UTE that are far less than one-for-one. The responsiveness of the UTE to changes in the elasticity of substitution among factors of production is even smaller in the version of the model in which all factors of production are intersectorally mobile. The production possibilities frontier for the mobile-factors model is flatter than the frontier for the specific-factors model, since it is an "outer envelope" of the latter. This implies that changes in the elasticity of substitution between factors will have a smaller effect with all mobile factors, compared to a specific-factors model.

In contrast, calculated UTEs are quite sensitive to model structure. Reading across rows of Table 1, *for given elasticity values*, different assumptions regarding factor mobility significantly alter calculated values for the UTEs in most cases—there are only two cases in which the difference between the UTEs across the two model structures is less than 1 percent. On the other hand, the discrepancies between the values of the UTE for the two models can be substantial, as in the last row of table 1. This finding, although confined to the particular CGE model used here, reinforces the point made by O'Rourke (1997) that care should exhibited in choosing a particular model structure to calculate a UTE for a given country.

The results from this set of sensitivity tests demonstrates the importance of model choice in assessing trade restrictiveness. As the calculated UTEs can differ widely across models, the researcher should choose the type of model to answer the specific question at hand. In particular, for measuring trade restrictiveness over a short time period—defined as period of time for which some factor cannot adjust to new circumstances—then a specific-factors model would be appropriate. UTEs calculated from this type of model should then be thought of as "short-run" UTEs. In contrast, if the purpose is to measure how trade restrictiveness has changed over a long time period, e.g. several decades, then it would be appropriate to use a model in which all factors of production are mobile. UTEs calculated from this type of model can be thought of as "long-run UTEs". Thus, the choice of model structure for analyzing trade restrictiveness should depend on the purpose for which the UTE and TRI will be used. Model choice should also take into account the features of the economy during the period of change in trade policy. That is, was the period characterized by rapid investment, in which a mobile factor model would be appropriate, or by a period of time too short to alter the supply of the factor, in which case a specific-factors model would be appropriate.

Table 1. Sensitivity of the Uniform Tariff Equivalents to Changes in Values of the Elasticity of Substitution Between Labor and Capital

			Fixed Factor Model		All Factors Mobile		Percent Difference
σM1	σM2	σX	UTE	Percent Change 1/	UTE	Percent Change 1/	Between Two Models 2/
2.00	1.50	1.50	0.05367016		0.05152938		-3.99
1.50	1.50	1.50	0.05208624	-2.95	0.05115417	-0.73	-1.79
1.25	1.50	1.50	0.05114180	-1.81	0.05092261	-0.45	-0.43
0.75	1.50	1.50	0.04880535	-4.57	0.05033853	-1.15	3.14
0.50	1.50	1.50	0.04731791	-3.05	0.04996629	-0.74	5.60
1.50	2.00	1.50	0.05073410		0.05064263		-0.18
1.50	1.50	1.50	0.05208624	2.67	0.05115417	1.01	-1.79
1.50	1.25	1.50	0.05292666	1.61	0.05141959	0.52	-2.85
1.50	0.75	1.50	0.05535122	4.58	0.05197050	1.07	-6.11
1.50	0.50	1.50	0.05754775	3.97	0.05225622	0.55	-9.20
1.50	1.50	2.00	0.04936698		0.05077284		2.85
1.50	1.50	1.50	0.05208624	5.51	0.05115417	0.75	-1.79
1.50	1.50	1.25	0.05387572	3.44	0.05143039	0.54	-4.54
1.50	1.50	0.75	0.05926470	10.00	0.05222511	1.55	-11.88
1.50	1.50	0.50	0.06411704	8.19	0.05278760	1.08	-17.67

Source: Simulations with CGE model.

III. Analyzing Trade Restrictiveness Over Time With the TRI

Researchers frequently want to know whether a particular country has become more or less open to international trade over some time period. One reason for this is that there is a great deal of interest in whether there is a causal relationship between greater openness to trade and economic growth. One way to determine whether an economy has become more open to trade would be to compute TRIs for the country in question for a number of years and observe how these TRIs changed over time. While this strategy sounds simple, it is actually complex. The reason for this is that the computed TRIs would be affected by factors other than just changes in trade policy—they would be affected by changes in the underlying structure of the economy, such as factor accumulation and technological change, which will alter the computed value of the TRI, even if trade policy remains unchanged. Thus, simply tracking changes in TRIs over time could give a misleading picture of whether an economy's *trade policy* has become more open. Therefore, if one wants to assess how a country's *trade policy* changed over time, it is necessary to decompose the calculated changes in the TRIs into a part that is due to changes in economic structure and a component that captures changes in trade policy alone.

Following a similar procedure to one in section II, the TRI can be defined as

$$B\left(\frac{p_{M1}^{*0}(1+t_{M1}^{1})}{\Delta},\frac{p_{M2}^{*0}(1+t_{M2}^{1})}{\Delta},p_E^0,u^0,v^0,Z^0\right)=B\left(p_{M1}^{*0}(1+t_{M1}^{0}),p_{M2}^{*0}(1+t_{M2}^{0}),p_E^0,u^0,v^0,Z^0\right) \quad (5)$$

where the only difference between equation (1) and (5) is that is the balance-of-trade function, $B(\cdot)$, is shown to depend explicitly on the state of technology, denoted Z^0, factor endowments v^0, and the terms of trade p_i^{*0}. This specification is very general—the precise form of the technological change (e.g. whether it is factor biased or sector biased) will determine the form for B_Z and this is discussed below in section B. The other parameters in equation (4) are defined as in equation (1) except that equation (5) now shows world prices, p_i^{*0}, explicitly. The specification in equation (5) could also include nontraded goods which operate in the background. As before, the superscripts "0" denotes the initial situation, while "1" denotes the post-change situation. In equation (4), the tariffs are assumed to be ad-valorem.

To see how changes in factor endowments, technological change, and changes in the terms of trade affect the TRI, totally differentiate the right-hand side of equation (5) and solve for du^0 gives, choosing p_E as numeraire, which gives:

$$du^0 = \frac{1}{B_u^0}\left[-B_{M1}^0\left[dp_{M1}^0(1+t_{M1}^0)+p_{M1}^{*0}dt_{M1}^0\right]-B_{M2}^0\left[dp_{M2}^0(1+t_{M2}^0)+p_{M2}^{*0}dt_{M2}^0\right]-B_v^0 dv^0-B_Z^0 dZ^0\right] \quad (6)$$

Totally differentiating the left-hand side of equation (5) and using (6) gives the proportional change in the TRI:

$$\hat{\Delta} = \frac{1}{(B_{M1}^{\Delta} p_{M1}^{\Delta} + B_{M2}^{\Delta} p_{M2}^{\Delta})} \left[\begin{array}{l} \left(B_v^{\Delta} - \left(\frac{B_u^{\Delta}}{B_u^{0}} \right) B_v^{0} \right) dv^{0} + \\ \left(B_Z^{\Delta} - \left(\frac{B_u^{\Delta}}{B_u^{0}} \right) B_Z^{0} \right) dZ^{0} + \\ \left(\frac{B_{M1}^{\Delta} p_{M1}^{\Delta}}{p_{M1}^{*0}} - \left(\frac{B_u^{\Delta}}{B_u^{0}} \right) B_{M1}^{0} (1 + t_{M1}^{0}) \right) dp_{M1}^{*0} + \\ \left(\frac{B_{M2}^{\Delta} p_{M2}^{\Delta}}{p_{M2}^{*0}} - \left(\frac{B_u^{\Delta}}{B_u^{0}} \right) B_{M2}^{0} (1 + t_{M2}^{0}) \right) dp_{M2}^{*0} + \\ - \left(\left(\frac{B_u^{\Delta}}{B_u^{0}} \right) B_{M1}^{0} p_{M1}^{*0} \right) dt_{M1}^{0} \\ - \left(\left(\frac{B_u^{\Delta}}{B_u^{0}} \right) B_{M2}^{0} p_{M2}^{*0} \right) dt_{M2}^{0} \end{array} \right] \quad (7).$$

Equation (7) shows how the TRI changes as a result of changes in economic structure (e.g. changes in factor endowments, technology, and the terms of trade), and changes in trade policy (e.g. changes in tariffs). The following sections discuss the components of equation (7).

A. TRIs and Factor Accumulation

Over time, an economy will typically accumulate factors of production and this will alter the calculated value of an economy's TRI and UTE for all else constant. The effect of factor accumulation on the TRI, for a given trade policy (i.e. unchanged tariffs) is given by:

$$\hat{\Delta} = \frac{1}{(B_{M1}^{\Delta} p_{M1}^{\Delta} + B_{M2}^{\Delta} p_{M2}^{\Delta})} \left[\left(B_v^{\Delta} - \left(\frac{B_u^{\Delta}}{B_u^0} \right) B_v^0 \right) dv^0 \right] \tag{8}$$

The effect of factor accumulation on the TRI therefore depends on the derivative of the balance-of-trade function, B_v, evaluated at two different price vectors, p^{Δ} and p^0.

For an economy with two imported goods, one exported good, and assuming that technology and the terms of trade are constant, the balance-of-trade function is given by:

$$\begin{aligned} B(p_{M1}, p_{M2}, p_E, t_1, t_2, u, z, v) &= E(p_E, p_{M1}, p_{M2}, u) - G(p_E, p_{M1}, p_{M2}, z, v) \\ &- t_1 P_{M1}^*(E_{M1} - G_{M1}) - t_2 P_{M2}^*(E_{M2} - G_{M2}) \end{aligned} \tag{9}$$

where p_E is the price of exports, p_{M1} is the price of the first import good, p_{M2} is the price of second import good, t_1 and t_2 are the corresponding tariff rates, u is the level of utility, v is the vector of factor endowments, z is the state of technology, $E(\cdot)$ is the expenditure function, and $G(\cdot)$ is the GDP function. Subscripts next to E or G denote a partial derivative with respect to that variable. For example, $E_{M1} = \frac{\partial E}{\partial P_{M1}}$, which is the compensated demand for imported good 1. The derivative of the balance-of-trade function with respect to factor endowments (v) is:

$$B_V dV = -\left[t_{M1} p^*_{M1} G_{M1V} + t_{M2} p^*_{M2} G_{M2V} - G_v \right] \tag{10}$$

where G_{M1V} and G_{M2V} capture how outputs of the two imported goods change as a result of factor accumulation at constant prices—the Rybczynski terms—and G_V equals the derivative of the GDP function with respect to factor endowments. The terms G_{M1V} and G_{M2V} can be positive or negative depending on the nature of the accumulation. Using the definition of G_V, equation (10) can be rewritten as

$$B_V dV = -\left[p^*_E G_{EV} + p^*_{M1} G_{M1V} + p^*_{M2} G_{M2V} \right] \qquad (11).$$

Equation (11) measures the change in the value of output at *world* prices. It can be shown that this term will be positive if the factor accumulation raises welfare and negative if it reduces welfare—the case of immiserizing growth.[2] Therefore, if growth is immiserizing, then $B_V > 0$, and if factor accumulation raises the value of output at world prices, then $B_V < 0$.

[2] See Caves and Jones (1974) and Johnson (1967) for an explanation of this result.

Thus, the effect of factor accumulation on the TRI depends on the sign of B_V and the sign of $\left[B^{\Delta}_{M1}p^{\Delta}_{M1}+B^{\Delta}_{M2}p^{\Delta}_{M2}\right]$, the sum of the derivatives of the balance-of-trade function with respect to the prices of the import good, evaluated at prices $p^{\Delta}_{Mi}=\dfrac{p^{1}_{Mi}}{\Delta}$:

$$\sum_i B^{\Delta}_{Mi}p^{\Delta}_{Mi}=\left[-t_1P^*_{M1}(E_{M1M1}-G_{M1M1})-t_2P^*_{M2}(E_{M2M1}-G_{M2M1})\right]p^*_{M1}dt_1$$
$$+\left[-t_1P^*_{M1}(E_{M1M2}-G_{M1M2})-t_2P^*_{M2}(E_{M2M2}-G_{M2M2})\right]p^*_{M2}dt_2 \quad (12).$$

Both $(E_{M1M1}-G_{M1M1})$ and $(E_{M2M2}-G_{M2M2})$ must be negative, but the cross-price terms could be positive or negative depending on whether the two goods are substitutes or complements. Thus, the bracketed term in (12) is ordinarily positive, since it captures the marginal cost of tariff changes. [3] Given these results, factor accumulation will raise the TRI if the bracketed terms in equation (12) and $\sum_i B^{\Delta}_{Mi}p^{\Delta}_{Mi}$ are both of the same sign and reduce it if they are of opposite sign.

The above results can be used to determine how factor accumulation affects the TRI and the uniform tariff equivalent (UTE), depending on how factor accumulation affects sectoral outputs. For example, in the context of the specific-factor's model, an increase in the

[3] Complementarity between import goods is possible, but all goods cannot be complements for each other if markets are stable.

quantity of the specific factor in the export sector raises output of that sector and reduces output of the two tariff-protected sectors. In this cases, welfare rises and $B_V < 0$. If there was an increase in the endowment of labor, outputs of all three goods would rise, so welfare rises and $B_V < 0$. Increases in the amount of the specific factor used in either of the two tariff-protected goods can either raise or lower welfare. As shown in equation (8), the effect of accumulation on the TRI depends on the extent to which B_v^Δ differs from B_v^0, adjusted by $\left(\frac{B_u^\Delta}{B_u^0}\right)$. For small changes, $\left(\frac{B_u^\Delta}{B_u^0}\right)$ will be close to one and the term $\left[B_v^\Delta - \left(\frac{B_u^\Delta}{B_v^0}\right)B_v^0\right]$ will be close to zero. Thus, the effect of accumulation on the TRI depends on the extent to which B_v^Δ diverges from B_v^0. Of course, in the case of balanced growth, the TRI would be unaffected. In sum, the extent to which factor accumulation affects the calculation of the TRI, at unchanged trade policy, depends on the degree of difference between the derivatives of the balance-of-trade function with respect to factor endowments evaluated at p_{Mi}^Δ and p_{Mi}^0: $\left[B_v^\Delta - \left(\frac{B_u^\Delta}{B_u^0}\right)B_v^0\right]dv^0$. The difference must lie with p_{Mi}^Δ and p_{Mi}^0, as B_v is computed holding prices constant. This suggests that the more the economy is distorted to start with, measured by a larger value of Δ, the TRI, or for τ^Δ, the uniform tariff equivalent equal to $\left(\frac{1}{\Delta}-1\right)$, the more likely it is that factor accumulation or structural change will alter the value for the TRI. For example, the larger the degree of dispersion in tariff rates, the more likely it is that factor accumulation could affect the calculation of the TRI and the UTE.

B. TRIs and Technological change

The effects of technical change on the TRI is very similar to the effects of changes in factor endowments, so this section will be brief. This section considers the impact of three types of technological change on an economy's TRI: (i) factor-specific technical change; (ii) sector-specific technical change;; and (iii) factor-specific technical change in a particular sector.[4] Assuming all else constant, except technological change, equation (7) simplifies to:

$$\hat{\Delta} = \frac{1}{(B^{\Delta}_{M1}p^{\Delta}_{M1} + B^{\Delta}_{M2}p^{\Delta}_{M2})}\left[\left(B^{\Delta}_{Z} - \left(\frac{B^{\Delta}_{u}}{B^{0}_{u}}\right)B^{0}_{Z}\right)dZ^{0}\right] \qquad (13).$$

The derivative of the balance of trade function with respect to Z is nearly identical to B_V, and is given by:

$$B_Z dZ = (t_1 P^{*}_{M1} G_{M1Z} + t_2 P^{*}_{M2} G_{M2Z} - G_Z) dZ \qquad (14)$$

where G_{EZ}, G_{M1Z}, and G_{M1Z} measure how outputs of the exported and two imported goods are affected by a change in technology. The form of these derivatives depends on the precise nature of the technological change.

[4] The appendix contains expressions for how each of these three types of technical change alter factor prices and sectoral outputs following the treatment in Jones (1965).

1. Factor-Specific Technical Change

The effects of factor-specific technical change are equivalent to an increase in the supply of that factor. Thus, the effects of technological change of this type are the same as the analysis in section II.A above. For this type of technical change, the GDP function can be written

$$G(p,zv) = G(p, z_{K1}v_{K1}, z_{K2}v_{K2}, z_{KE}v_E, v_L) \tag{15}$$

so a change in z_i has the same effect on the GDP function as a change in a factor endowment v_i. Following a similar procedure to the one above (see Dixit and Norman (1980)):

$$z_i\left(\frac{\partial G}{\partial z_i}\right) = v_i\left(\frac{\partial G}{\partial v_i}\right) \text{, so:} \tag{16}$$

$$B_Z dZ = B_V dV = \left[t_{M1}P^*_{M1}G_{M1V} + t_{M2}P^*_{M2}G_{M2V} - G_V\right]dV \tag{17}$$

which is similar to equation (10). The effects of factor-biased technical change on the TRI is similar to those of sector-specific technical change: factor-specific technical change can raise or lower the TRI depending whether it increases or reduces the outputs of the tariff-protected goods. Technical change specific to the export sector will reduce the TRI, because output of the export sector will rise and outputs of the two import goods will fall.

2. Sector-Specific Technological Change

Technological change that is sector specific is mathematically equivalent to a change in the price of the product since the value of output can increase. For example, the GDP function can be written as:

$$G(zp, v) = G(z_1 p_{M1}, z_2 p_{M2}, z_E p_E, v) \tag{18}$$

so a change in z_j has the same effect on the GDP function as a change in p_j. Following the treatment in Dixit and Norman (1980):

$$z_i \left(\frac{\partial G}{\partial z_i} \right) = p_i \left(\frac{\partial G}{\partial p_i} \right) \qquad (19).$$

For example, technical progress confined to the export sector is given by the following expression for $B_Z dZ$:

$$B_{Z_E} dZ = -\left[p_E^* G_{EE} + p_{M1}^* G_{M1E} + p_{M2}^* G_{M2E} \right] \qquad (20).$$

As before, the first bracketed term is positive. In the second bracketed term, both G_{M1E} and G_{M2E} are negative, since an increase in the price of the export good causes the output of the two imported goods to fall, and output of the export good to rise.

3. Factor-Specific Technical Change Confined to a Particular Sector

Technical change that is specific to a factor of production in a particular sector alters the demand for that factor. Following the treatment in Jones (1965), the amount of input i needed to produce a unit of good j depends on factor prices and the state of technology. Jones (1965) models technical change by writing the proportional change in each a_{ij} as:

$$\hat{a}_{ij} = \hat{b}_{ij} - \hat{c}_{ij} \tag{21}$$

where each $\hat{b}_{ij}$ captures the proportional change in the factor demands as a consequence of changes in factor prices, holding technology constant, while $\hat{c}_{ij}$ measures technological change in due to factor i used in sector j. Each $\hat{c}_{ij}$ equals:

$$\hat{c}_{ij} = -\frac{\partial a_{ij}}{\partial z}\frac{1}{a_{ij}} \tag{22}$$

so, a technological improvement reduces the amount of factor i needed to produce a unit of good j. Using equation (21), equations relating the effects of changes in commodity prices, factor supplies, and technology on the endogenous variables (factor prices and sectoral outputs) can be derived. These are reproduced in equations (32a-34a) in the appendix.

As shown in the appendix, a technological improvement associated with the specific factor used in a sector unambiguously raises the output of that sector and reduces the output of the other sectors. Furthermore, a technological improvement associated with labor used in a particular sector also raises output of that sector. The only ambiguous cases involve the impact of technological change associated with labor in a given sector on outputs of the other sectors. In particular:

$$\hat{X}_{M1} = \left[\frac{\theta_{L1}\sigma_1\theta_{KE}\lambda_{L2}(\theta_{K2} - \sigma_2\theta_{L2})}{\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}} \right] \hat{c}_{L2}$$

$$\hat{X}_{M1} = \left[\frac{\theta_{L1}\sigma_1\theta_{K2}\lambda_{LE}(\theta_{KE} - \sigma_E\theta_{LE})}{\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}} \right] \hat{c}_{LE}$$

$$\hat{X}_{M2} = \left[\frac{\theta_{L2}\sigma_2\theta_{KE}\lambda_{L1}(\theta_{K1} - \sigma_1\theta_{L1})}{\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}} \right] \hat{c}_{L1} \tag{23}$$

$$\hat{X}_{M2} = \left[\frac{\theta_{L2}\sigma_2\theta_{K1}\lambda_{LE}(\theta_{KE} - \sigma_E\theta_{LE})}{\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}} \right] \hat{c}_{LE}$$

$$\hat{X}_{E} = \left[\frac{\theta_{LE}\sigma_E\theta_{K2}\lambda_{L1}(\theta_{K1} - \sigma_1\theta_{L1})}{\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}} \right] \hat{c}_{L1}$$

$$\hat{X}_{E} = \left[\frac{\theta_{LE}\sigma_E\theta_{K1}\lambda_{L2}(\theta_{K2} - \sigma_2\theta_{L2})}{\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}} \right] \hat{c}_{L2}$$

In general, the impact of technological change associated with labor used in a particular sector on outputs of the other sectors depends on the capital-labor ratio in the sector experiencing the technological change and the elasticity of substitution between factors in that sector. For example, the impact of technical change associated with the use of labor in the second import sector $(\hat{c}_{L2})$ on output of good 1 $(\hat{X}_{M1})$ depends on the sign of $(\theta_{K2} - \sigma_2\theta_{L2})$. And, technical change will affect the TRI according to how it affects sectoral outputs: technical change that tends to lead to an expansion in output of the tariff-protected sectors will tend to raise the TRI, while technical change favoring the output of the export sector will tend to reduce the TRI.

C. TRIs and the terms of trade

If tariffs are ad-valorem, then changes in the terms of trade will affect the TRI as follows, using equation (7):

$$\hat{\Delta} = \frac{1}{(B_{M1}^{\Delta} p_{M1}^{\Delta} + B_{M2}^{\Delta} p_{M2}^{\Delta})} \begin{bmatrix} \left(\dfrac{B_{M1}^{\Delta} p_{M1}^{\Delta}}{p_{M1}^{*0}} - \left(\dfrac{B_u^{\Delta}}{B_u^0} \right) B_{M1}^0 (1 + t_{M1}^0) \right) dp_{M1}^{*0} \\ + \\ \left(\dfrac{B_{M2}^{\Delta} p_{M2}^{\Delta}}{p_{M2}^{*0}} - \left(\dfrac{B_u^{\Delta}}{B_u^0} \right) B_{M2}^0 (1 + t_{M2}^0) \right) dp_{M2}^{*0} \end{bmatrix} \quad (24).$$

As with the other types of exogenous changes, changes in the terms of trade will alter the TRI according to how sectoral outputs are affected. In general, terms-of-trade improvements ($dp_{M1}^{*0} < 0$ or $dp_{M2}^{*0} < 0$) will raise welfare and therefore reduce the TRI, but it depends on the derivatives of the balance-of-trade function evaluated at the two different price vectors noted in equation (24).

IV. Simulations Using a CGE Model

This section presents the results of some simulations using a simple general equilibrium model to demonstrate how changes in exogenous variables (e.g. factor accumulation, technological change, and changes in the terms of trade) might affect the calculated values of TRIs.

Briefly, the model consists of four sectors (an export good, two import goods, and a nontraded good). Output of each sector is produced using labor, which is mobile across all sectors and a sector-specific factor. Thus, the wage rate is the same in every sector, but the return to capital differs. The country is taken to be "small", and thus, unable to influence its terms of trade through changes in tariff rates.

The model is used to demonstrate that in assessing whether an economy has become more open to trade over time, it is very important to recognize that the underlying structure of the economy is changing and this will affect the calculated values of the TRIs independently of changes in trade policy. The simulation results are presented in Table 2.

Table 2. The Importance of Changes in Economic Structure in Calculating TRIs

Calculated Uniform Tariff Equivalents (in percent)

	Year 1	Year 2	Year 3	Year 4
	$t_{M1} = 10$ $t_{M2} = 5$	$t_{M1} = 15$ $t_{M2} = 5$	$t_{M1} = 20$ $t_{M2} = 5$	$t_{M1} = 25$ $t_{M2} = 5$
Changes in Tariffs and Economic structure	7.918	11.347	14.697	17.916
Changes in Tariffs Only	8.058	11.952	16.009	20.078
Percent deviation from UTEs above (row 1)	1.8	5.3	8.9	12.1
	$t_{M1} = 10$ $t_{M2} = 5$	$t_{M1} = 10$ $t_{M2} = 5$	$t_{M1} = 10$ $t_{M2} = 5$	$t_{M1} = 10$ $t_{M2} = 5$
Changes in Economic Structure only (tariffs constant)				
(increase in capital used in the export sector (5%), terms of trade improvement (5%), and labor specific technical change in the Export sector (5%))	7.918	7.803	7.713	7.646

Source: Model simulations

The first row of table 2 reports the UTEs for a hypothetical economy in four different years. The tariff rates in year 1 are 10 and 5 percent and only the top rate is assumed to change over time, rising to 25 percent in year 4. At the same time, the economy is assumed to undergo structural changes. In particular, the amount of the sector-specific factor used in the export sector is expected to grow by 5 percent a year, the terms of trade are assumed to improve by 5 percent a year, and there is labor-saving technical improvement in the export sector of 5 percent a year. Taking into account the change in the highest tariff rate and structural changes, the UTE for each year is given in row 1 of table 2. The UTE ranges from 7.9 percent in year 1 to 17.9 percent in year 4.

Row 2 of table 2 reports the calculated UTEs for the assumed changes in tariff rates, but assumes that the structure of the economy remains unchanged. This calculation generates UTEs that are uniformly higher than the UTEs in row 1, with the deviation rising from 2 to 12 percent by year 4. The UTEs calculated assuming that the structure of the economy remains unchanged are higher because the assumed structural changes all work to reduce output of the tariff-protected sectors and expand output of the export sector and these tend to raise welfare. Thus, these structural changes act to reduce the calculated UTEs, independently of changes in tariff rates. This is shown in row 3 of table 2. This exercise is simply intended to demonstrate that underlying structural changes in an economy can significantly influence the calculated values of the TRI and UTE, and thus, affect the degree of measured trade restrictiveness over time. In order to gain a perspective on how trade policy alone changes over time, it is necessary to adjust calculated UTEs for structural change.

IV. Conclusions

This paper has made two principal contributions. First, it evaluates how sensitive calculated measures of trade restrictiveness (UTEs) are to alternative productions structures. The simulations show that UTEs are relatively insensitive to changes in the elasticity of substitution between factors of production, but quite sensitive to alternative model structures. For example, the simulations compared the calculated UTEs under two types of production structures—mobile factors and specific factors—and found that the calculated UTEs can differ markedly. This finding highlights the importance of the model structure used to assess trade restrictiveness. In particular, short-run UTEs can differ significantly from long-run UTEs.

Second, there is a great deal of interest in knowing whether a country has become more open to trade over time. The TRI and UTE could be used to answer this question, but with an important caveat. Typically, the structure of an economy changes over time and this will affect the calculated values of the TRI and UTE, even in trade policy does not change. If one wants a measure of changes in trade policy, then it would be incorrect to allow changes in economic structure to affect the calculated values of the TRI and UTE. The second part of the paper demonstrated how changes in economic structure could affect the calculated values of the UTEs. In general, one needs to adjust calculated UTEs for changes in structure to obtain an accurate measure of how trade policy changed over time.

References

Anderson, James, and J. Peter Neary, 1994, "Measuring the Restrictiveness of Trade Policy," *World Bank Economic Review*, vol. 8, no. 2, pp. 151-169, (May).

_______, 1996, "A New Approach to Evaluating Trade Policy," *Review of Economic Studies*, vol. 63, no. 1, pp. 107-125, (January).

_______, 2005, *Measuring The Restrictiveness of International Trade Policy*, Cambridge: MIT Press.

Feenstra, Robert, 1995, "Estimating the Effects of Trade Policy," in Gene Grossman and Kenneth Rogoff eds., *Handbook of International Economics*, Amsterdam: Elsevier, vol. 3.

Johnson, Harry, 1967, "The Possibility of Income Losses From Increased Efficiency or Factor Accumulation in the Presence of Tariffs," *Economic Journal*, vol. 77, pp. 151-54.

Jones, Ronald, W., 1965, "The Structure of Simple General Equilibrium Models," *Journal of Political Economy*, vol. 73, pp. 557-572

Kee, Hiau Looi, Alassandro Nicita, and Marcelo Olarreaga, 2004, "Estimating Mercantilist Trade Restrictiveness Indices," Washington: World Bank, Working Paper 3452, (March).

Lloyd, Peter, and Donald MacLaren, 2002, "Measures of Trade Openness Using CGE Analysis," *Journal of Policy Modeling*, vol. 24, pp. 67-81.

O'Rourke, Kevin, 1997, "Measuring Protection: A Cautionary Tale," *Journal of Development Economics*, vol. 53, pp. 169-183.

Appendix: Structure of the Applied General Equilibrium Model

Model Structure

This paper uses an applied general equilibrium model of the Egyptian economy that consists of six sectors (oil, service exports, manufactured exports, agriculture, imported manufactures, and a nontraded good) and eight factors of production (labor, capital, and a sector-specific factor). Labor and capital are mobile across all sectors. A representative household receives all factor income, as well as all revenue collected from taxation. Egypt is assumed to be a small country, so the terms of trade are exogenous. The price of nontraded goods adjusts to bring about equilibrium in the goods market.

Production Structure

Value added in each sector VA_j is produced by combining a labor input L_j, with capital K_j and a specific factor F_j according to a constant elasticity of substitution (CES) production function:

$$X_j = A_j[\alpha_j L_j^{-\rho_j} + \beta_j K_j^{-\rho_j} + (1-\alpha_j-\beta_j)F_j^{-\rho_j}]^{(-1/\rho_j)} \tag{1}$$

where A_j, α_j, and β_j, are constants, and $\rho_j = \frac{(1-\sigma_j)}{\sigma_j}$ where σ_j is the elasticity of substitution between factors in sector j. Note that this specification assumes that the elasticity of substitution among all three factors is the same within a given sector. The allocation of the mobile factors—labor and capital—across sectors is determined by equating the value of the marginal product of each factor with its factor price. For labor, this is where the value of the marginal product of labor equals the aggregate wage rate:

$$W = \frac{\partial X_j}{\partial L_j} PD_j \tag{2}$$

where PD_j is the consumption price of the jth good and W is the wage rate. Similarly for capital:

$$R = \frac{\partial X_j}{\partial K_j} PD_j \tag{3}$$

where R is the rental rate on capital. Each factor must be fully employed, so

$$\sum_j L_j = \bar{L} \tag{4}$$

and

$$\sum_j K_j = \bar{K} \tag{5}$$

The return to the specific factor in each sector, f_j, is determined as a residual (since F_j is fixed) so as to satisfy a zero-profit condition:

$$PS_j VA_j = WL_j + RK_j + f_j F_j \qquad (6),$$

where PS_j is the producer price of good j.

Aggregate income and demand

Aggregate income available for spending by the representative consumer (Y) equals the sum of factor income, government revenue, and foreign borrowing, B, which is assumed to be fixed in terms of the numeraire:

$$Y = W\bar{L} + R\bar{K} + \sum_j f_j F_j + GR + B \qquad (7).$$

Government revenue equals indirect tax revenue plus tariff revenue:

$$GR = \sum_j tx_j PS_j X_j + \sum_j tm_j PW_j MD_j \tag{8}$$

where tx_j is the indirect tax (or subsidy rate if negative) on good j, tm_j is the tariff rate on good j, PW_j is the international price of good j, and MD_j are imports of good j. As imports are treated as perfect substitutes for domestically produced goods, imports equal the difference between domestic demand and production.

Aggregate demand

Absent information on elasticities of demand in Egypt, we assume that a representative consumer maximizes a Cobb-Douglass utility function defined over the six goods. The resulting demand functions are:

$$DD_j = \frac{s_j Y}{PD_j} \tag{9}$$

where PD_j is the consumer price (inclusive of taxes or tariffs), DD_j is the demand for good j, and s_j is the budget share of good j. Of course, with this demand structure, the own-price elasticity of demand is -1, the cross-price elasticities are zero, and the income elasticity of demand is 1.

The prices paid by the consumer differ from the prices received by the producer, due to indirect taxes. Furthermore, for the traded goods, prices paid by the consumer and received by the producer differ from world prices as a result of tariffs on imports. For imported goods:

$$PS_j = PW_j(1+tm_j) \tag{10}$$

while for exported goods, the producer price equals the world price, since there are no export taxes or subsidies:

$$PS_j = PW_j \qquad (11).$$

For commodities subject to a consumption tax, the price paid by the consumer differs from the price received by the producer according to:

$$PD_j = PS_j(1+tx_j) \qquad (12).$$

Equilibrium

Equilibrium in the model is achieved when a set of factor prices is found that generates zero profits in each sector and is consistent with full employment of each factor. In this model, the terms of trade are given exogenously, so the price of the nontraded good adjusts to achieve equilibrium. In the nontraded sector, demand must equal supply:

$$DD_N = X_N \qquad (13).$$

For the imported good:

$$DD_M = X_M + MD_M \qquad (14),$$

while for the exported good:

$$DD_X + E_X = X_X \tag{15}$$

where E_j are exports of good j.

Appendix II

This appendix presents an analysis of how factor accumulation affects real income in the presence of protection. The complete model used in the second section of the paper is given here. The model contains an export good, two import goods, and a nontraded good. Each good is produced using labor and sector-specific capital. The zero-profit conditions are:

$$wa_{LE} + r_E a_{KE} = p_E^* \tag{1a}$$

$$wa_{LM1} + r_{M1} a_{KM1} = p_{M1}^*(1+t_1) \tag{2a}$$

$$wa_{LM2} + r_{M2} a_{KM2} = p_{M2}^*(1+t_2) \tag{3a}$$

where p_E^* is the world price of exports, p_{M1}^* and p_{M2}^* are the world prices of imports, t_1 and t_2 are the ad-valorem tariff rates applied to imports, a_{ij} is the amount of factor i used per unit of good j, w is the wage rate, and r_j is the return to capital in sector j. The full-employment conditions are:

$$a_{KE} X_E = K_E \tag{4a}$$

$$a_{KM1} X_{M1} = K_{M1} \tag{5a}$$

$$a_{KM2} X_{M2} = K_{M2} \tag{6a}$$

$$a_{LE} X_E + a_{LM1} X_{M1} + a_{LM2} X_{M2} = L \tag{7a}$$

where K_j is the amount of capital used in sector j, L is the endowment of labor, and X_j is output of good j.

Totally differentiating equations (1a) through (7a) and putting them in proportional change form gives:

$$\hat{w}\theta_{L1} + \hat{r}_1\theta_{K1} = \hat{p}_{M1} - (\theta_{L1}\hat{a}_{L1} + \theta_{K1}\hat{a}_{K1}) \tag{8a}$$

$$\hat{w}\theta_{L2} + \hat{r}_2\theta_{K2} = \hat{p}_{M2} - (\theta_{L2}\hat{a}_{L2} + \theta_{K2}\hat{a}_{K2}) \tag{9a}$$

$$\hat{w}\theta_{LE} + \hat{r}_E\theta_{KE} = \hat{p}_E - (\theta_{LE}\hat{a}_{LE} + \theta_{KE}\hat{a}_{KE}) \tag{10a}$$

$$\lambda_{K1}\hat{X}_1 = \hat{K}_1 - \lambda_{K1}\hat{a}_{K1} \tag{11a}$$

$$\lambda_{K2}\hat{X}_2 = \hat{K}_2 - \lambda_{K2}\hat{a}_{K2} \tag{12a}$$

$$\lambda_{KE}\hat{X}_E = \hat{K}_E - \lambda_{KE}\hat{a}_{KE} \tag{13a}$$

$$\lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2 + \lambda_{LE}\hat{X}_E = \hat{L} - \hat{a}_{L1}\lambda_{L1} - \hat{a}_{L2}\lambda_{L2} - \hat{a}_{LE}\lambda_{LE} \qquad (14a).$$

In these equations, θ_{ij} is the share of good j's cost accounted for by factor i, λ_{ij} is the proportion of the supply of factor i used by industry j, and a "^" denotes proportional change.

Each a_{ij} is the amount of factor i used to produce a unit of good j and depends on the factor prices and the state of technology, z:

$$a_{ij} = a_{ij}(w, r_j, z).$$

As in Jones (1965), technological change can be introduced in the following way:

$$\hat{a}_{ij} = \hat{b}_{ij} - \hat{c}_{ij} \tag{15a}$$

where $\hat{b}_{ij}$ is the proportional change in each $\hat{a}_{ij}$ due to changes in factor prices and $\hat{c}_{ij}$ is a measure of technological change—the change in each $\hat{a}_{ij}$ at constant factor prices.

Each $\hat{b}_{ij}$ is given through the relation:

$$\sigma_j = \frac{\hat{b}_{Kj} - \hat{b}_{Lj}}{\hat{w} - \hat{r}_j}, \quad \text{or} \quad \sigma_j(\hat{w} - \hat{r}_j) = \hat{b}_{Kj} - \hat{b}_{Lj} \qquad (16a).$$

Cost minimization requires that:

$$\theta_{L1}\hat{b}_{L1} + \theta_{K1}\hat{b}_{K1} = 0 \tag{17a}$$

$$\theta_{L2}\hat{b}_{L2} + \theta_{K2}\hat{b}_{K2} = 0 \tag{18a}$$

$$\theta_{LE}\hat{b}_{LE} + \theta_{KE}\hat{b}_{KE} = 0 \tag{19a}$$

So using 16a through 19a, the following expressions for $\hat{b}_{ij}$ can be derived (see Jones (1965)):

$$\hat{b}_{L1} = -\theta_{K1}\sigma_1(\hat{w} - \hat{r}_1) \quad (20a)$$

$$\hat{b}_{L2} = -\theta_{K2}\sigma_2(\hat{w} - \hat{r}_2) \quad (21a)$$

$$\hat{b}_{LE} = -\theta_{KE}\sigma_E(\hat{w} - \hat{r}_E) \quad (22a)$$

$$\hat{b}_{K1} = \theta_{L1}\sigma_1(\hat{w} - \hat{r}_1) \quad (23a)$$

$$\hat{b}_{K2} = \theta_{L2}\sigma_2(\hat{w} - \hat{r}_2) \quad (24a)$$

$$\hat{b}_{KE} = \theta_{LE}\sigma_E(\hat{w} - \hat{r}_E) \quad (25a).$$

Substituting (15a) and (20a) through (25a) into (8a) through (14a) gives:

$$\hat{w}\theta_{L1} + \hat{r}_1\theta_{K1} = \hat{p}_{M1} + \pi_1 \quad (26a)$$

$$\hat{w}\theta_{L2} + \hat{r}_2\theta_{K2} = \hat{p}_{M2} + \pi_2 \quad (27a)$$

$$\hat{w}\theta_{LE} + \hat{r}_E\theta_{KE} = \hat{p}_E + \pi_E \quad (28a)$$

$$\lambda_{K1}\hat{X}_1 = \hat{K}_1 - \lambda_{K1}\theta_{L1}\sigma_1(\hat{w} - \hat{r}_1) + \pi_{K1} \quad (29a)$$

$$\lambda_{K2}\hat{X}_2 = \hat{K}_2 - \lambda_{K2}\theta_{L2}\sigma_2(\hat{w} - \hat{r}_2) + \pi_{K2} \quad (30a)$$

$$\lambda_{KE}\hat{X}_E = \hat{K}_E - \lambda_{KE}\theta_{LE}\sigma_E(\hat{w} - \hat{r}_E) + \pi_{KE} \quad (31a)$$

$$\lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2 + \lambda_{LE}\hat{X}_E = \hat{L} + \theta_{K1}\sigma_1(\hat{w} - \hat{r}_1)\lambda_{L1} + \theta_{K2}\sigma_2(\hat{w} - \hat{r}_2)\lambda_{L2} + \theta_{KE}\sigma_E(\hat{w} - \hat{r}_E)\lambda_{LE} + \pi_L$$

(32a)

where:

$\pi_1 = \theta_{L1}\hat{c}_{L1} + \theta_{K1}\hat{c}_{K1}$ (measures technical change in sector 1)

$\pi_2 = \theta_{L2}\hat{c}_{L2} + \theta_{K2}\hat{c}_{K2}$ (measures technical change in sector 2)

$\pi_E = \theta_{LE}\hat{c}_{LE} + \theta_{KE}\hat{c}_{KE}$ (measures technical change in export sector)

$\pi_L = \hat{c}_{L1}\lambda_{L1} + \hat{c}_{L2}\lambda_{L2} + \hat{c}_{LE}\lambda_{LE}$ (labor-saving technical progress)

$\pi_{K1} = \hat{c}_{K1}\lambda_{K1}$ (capital-saving technical progress in sector 1)

$\pi_{K2} = \hat{c}_{K2}\lambda_{K2}$ (capital-saving technical progress in sector 2)

$\pi_{KE} = \hat{c}_{KE}\lambda_{KE}$ (capital-saving technical progress in the export sector)

Equations (26a) through (32a) can be solved for $\hat{X}_1, \hat{X}_2, \hat{X}_E, \hat{w}, \hat{r}_1, \hat{r}_2, \hat{r}_E$. The solutions for the proportional changes in outputs of each good are given by:

$$
\begin{aligned}
\hat{X}_{M1} = &\left[\frac{\lambda_{L1}\sigma_1\theta_{KE}\theta_{K1}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}}{\lambda_{K1}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{K}_1 + \pi_{K1}) \\
&+\left[\frac{-\theta_{L1}\sigma_1\theta_{KE}\theta_{K2}\lambda_{L2}}{\lambda_{K2}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K2}\theta_{K1}\right]}\right](\hat{K}_2 + \pi_{K2}) \\
&+\left[\frac{-\theta_{L1}\sigma_1\theta_{KE}\theta_{K2}\lambda_{LE}}{\lambda_{KE}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{K}_E + \pi_{KE}) \\
&+\left[\frac{\theta_{L1}\sigma_1(\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2})}{\theta_{K1}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{P}_1 + \pi_1) \\
&+\left[\frac{-\theta_{L1}\sigma_1\theta_{KE}\lambda_{L2}\sigma_2}{\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{P}_2 + \pi_2) \\
&+\left[\frac{-\theta_{L1}\sigma_1\theta_{K2}\lambda_{LE}\sigma_E}{\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{P}_E + \pi_E) \qquad \text{(32a)} \\
&+\left[\frac{\theta_{L1}\sigma_1\theta_{K2}\theta_{KE}}{\left[\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{L} + \pi_L)
\end{aligned}
$$

$$\hat{X}_2 = \left[\frac{-\theta_{L2}\sigma_2\theta_{KE}\theta_{K1}\lambda_{L1}}{\lambda_{K1}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]} \right](\hat{K}_1 + \pi_{K1})$$

$$+ \left[\frac{\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}\theta_{K2} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}}{\lambda_{K2}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K2}\theta_{K1}\right]} \right](\hat{K}_2 + \pi_{K2})$$

$$+ \left[\frac{-\theta_{L2}\sigma_2\theta_{KE}\theta_{K1}\lambda_{LE}}{\lambda_{KE}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]} \right](\hat{K}_E + \pi_{KE})$$

$$+ \left[\frac{-\theta_{L2}\sigma_2\theta_{KE}\lambda_{L1}\sigma_1}{\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]} \right](\hat{P}_1 + \pi_1)$$

$$+ \left[\frac{\theta_{L2}\sigma_2(\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2})}{\theta_{K2}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]} \right](\hat{P}_2 + \pi_2)$$

$$+ \left[\frac{-\theta_{L2}\sigma_2\theta_{K1}\lambda_{LE}\sigma_E}{\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2} + \lambda_{L2}\sigma_2\theta_{KE}\theta_{K1} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]} \right](\hat{P}_E + \pi_E) \qquad (33a)$$

$$+ \left[\frac{\theta_{L2}\sigma_2\theta_{K1}\theta_{KE}}{\left[\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE} + \lambda_{L2}\sigma_2\theta_{K1}\theta_{KE} + \lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]} \right](\hat{L} + \pi_L)$$

$$
\begin{aligned}
\hat{X}_E = & \left[\frac{-\theta_{LE}\sigma_E\theta_{K2}\theta_{K1}\lambda_{L1}}{\lambda_{K1}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{K}_1+\pi_{K1}) \\
& +\left[\frac{-\theta_{LE}\sigma_E\theta_{K2}\theta_{K1}\lambda_{L2}}{\lambda_{K2}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K2}\theta_{K1}\right]}\right](\hat{K}_2+\pi_{K2}) \\
& +\left[\frac{\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\theta_{KE}}{\lambda_{KE}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{K}_E+\pi_{KE}) \\
& +\left[\frac{-\theta_{LE}\sigma_E\theta_{K2}\lambda_{L1}\sigma_1}{\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{P}_1+\pi_1) \\
& +\left[\frac{-\theta_{LE}\sigma_E\theta_{K1}\lambda_{L2}\sigma_2}{\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{P}_2+\pi_2) \\
& +\left[\frac{\theta_{LE}\sigma_E(\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{K1}\theta_{KE})}{\theta_{KE}\left[\lambda_{L1}\sigma_1\theta_{KE}\theta_{K2}+\lambda_{L2}\sigma_2\theta_{KE}\theta_{K1}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{P}_E+\pi_E) \qquad (34a) \\
& +\left[\frac{\theta_{LE}\sigma_E\theta_{K1}\theta_{K2}}{\left[\lambda_{L1}\sigma_1\theta_{K2}\theta_{KE}+\lambda_{L2}\sigma_2\theta_{K1}\theta_{KE}+\lambda_{LE}\sigma_E\theta_{K1}\theta_{K2}\right]}\right](\hat{L}+\pi_L)
\end{aligned}
$$

Equations (32a) through (34a) demonstrate that: (i) technical change that is sector specific (π_1, π_2, π_E) have the same effects on sectoral outputs as a price change; and (ii) factor-specific technical change ($\pi_{K1}, \pi_{K2}, \pi_{KE}$, and π_L) have effects on outputs that are equivalent to changes in factor endowments. The third type of technical change—factor specific that is confined to a particular sector—can by accommodated through a change in the relevant $\hat{c}_{ij}$.