The performance of Tunisia’s bank: evaluation by a DEA model

Boutheina BANNOUR
Haykel HADJ SALEM

Abstract

This paper consists in measuring the performance of some banks in Tunisia during the year 2004. To achieve this goal we used the Data Envelopment Analysis (DEA) method under a linear programming using GAMS software. Within this analysis, we presented two banking approaches: intermediate and productive; under two types of models: CCR (Charnes, Cooper and Rhodes (1981)) and BCC (Banker, Charnes and Cooper (1984)) through two types of orientations: input and output. With through this study, we noticed that the majority of the banks are efficient but their performances are summarized according to their classifications (class).

Introduction:

Before the installation of the reforms, the Tunisian financial system was "repressed". During many years, he knew a largely administrative management. Covered by multiple protections, the banks underwent also many constraints. Until the end of the Eighties, their room for laborer was narrow. Their methods of management, the products which they were authorized to offer, conditions of their marketing, rates, the appropriations which they could grant, the banking policy and rules of competition, the whole defined by the Central Bank of Tunisia (CBT) and the ministry for Finances.

The financial sector underwent, within the framework of a structural adjustment, transformations (controls by the laws of the market), based on the introduction of new instruments to improve the capacity of this system to mobilize the saving and to finance the productive investments.

This principal news banking regulations was installation since 1987, in sight particularly to guarantee a certain safety and stability with the banking structure. But the had aims are especially an increase in the efficiency of the banking environment, the creation of a more competing environment between the financial institutions and a reinforcement of the financial base of the banks.

The estimate of efficiency requires the application of the suitable econometric techniques. Indeed, the traditional methods consider only one function average, whereas the objective is to measure the distance from each observation compared to a function border. In their review of the microeconomic studies seeking to measure the efficiency of the banking environment, whose great majority is interested in industrialized countries, Berger and Humphrey (1997) count mainly five different techniques.

Among these five techniques, those of stochastic border (SFA) and envelopment of the data are used. Method SFA breaks up the error of the function studied into two terms: the first represents the random effects and the errors of measurement; the second represents the degree of productive effectiveness. By assumption, the traditional term of error follows a symmetrical normal distribution, while the term of effectiveness follows an asymmetrical distribution.
The DEA Method, used in our study, is an application of the techniques of linear programming. It considers the border by a convex polyhedron wrapping the whole of the observations, of which most effective are directly on the border. This method does not take account of the errors of measurement or the random effects. Any distance from the border is allotted to productive inefficiencies. The second limit of method DEA holds other than the allocative effectiveness of its field of analysis. Indeed, it does not integrate the variation of the prices of the factors.

In our paper we will devote our first section to the theoretical presentation of model DEA. A second section consists in defining the Tunisian bank on the one hand and on the other hand to present the concept of the Tunisian banking performance. Whereas the third section advances interpretations and remarks following an application of various extensions of DEA model in the case of some Tunisian banks.

I- Presentation of the model:

I-1 The DEA model: definition and extensions

It is in Farrell (1957), that returns the idea of measurement of efficiency. It focused in the first time to the concept of productive efficiency and its identification to model the production process. This way of research was generalized for the case of multi products and was reformulated like a mathematical problem of programming by Charnes, Cooper and Rhodos (1978, 1979, 1981), (CCR).

Indeed, these authors introduced the DEA method, to pass from the mono product simple case (studied initially by Farrell) to multi products general case by wrapping the data. The method DEA is a non-parametric method which uses the techniques of linear programming to determine the production border of an observation sample. Thereafter, while basing itself on work of Shephard (1953, 1970), Banker, Charnes and Cooper (1984), (BCC) which followed the same step that CCR, but they took account of the variability of the yield of scale, while preserving the same formulations of analyses as those of Shephard.

Admittedly, after the determination of the production border by the DEA method, this border rests on the principle consisting in estimating the function $y = F (x_1, x_2 \ldots, x_N)$ by an envelope curve formed of the segments of right-hand side joining the effective entities. Thus, positioning on the production border makes it possible to determine the most satisfactory level with regard to the quantity of outputs produced and inputs used. However, the distance between each observation and this border measures the degree of performance translating its relative efficacy (or inefficacy). In other words, a firm is not efficient in outputs if there are other companies or combinations of companies which, with equal endowment factors, produce more. In parallel, this same firm will be considered to be not efficient in inputs if other companies or combinations of companies can produce the same level of output by producing less quantities of one or several factors (inputs).

I-2 The aim of the DEA:

The DEA method is a method founded on the micro-economic theory. On the one hand, it makes it possible to compare all the similar units in a given population, by taking into account simultaneously several dimensions.

On the other hand, it makes it possible to determine the efficiency border from the point of view of best practical. It thus makes it possible to calculate the efficiency relating of each productive unit observed to the glance of the border. These units (services, organizations, institutions, etc.) are called DMU "Decision Making Unit" making it possible to transform the inputs which are the resources used in outputs which are the results obtained. In a simple way, the DEA method consists to wrap all the observations by means of another set called
reference set and to be used for of this set like border (the production border) to evaluate the efficiencies of each one of DMUs.

This fact, the measure of the efficacy of the observations results then from the calculation of the distance (in input and/or output) which separates each observation from the envelope. To rather build thus the reference starting from the data than by means of an external reference amounts identifying an efficacy precisely called that of the "Best Practice".

However, DEA "Data Envelopment Analysis" or envelopment of the data analyzes, can be defined as being the methodology based on a linear programming to identify the empirical production functions.

I-3 Methodology of the data envelopment:
The application of an analysis by envelopment of the data to the financial institutions and more specifically to the banks, dates from the middle of the years 1980, the article of Sherman and Gold (1985) is generally regarded as the first study. This analysis is a "method of the extreme point". In other words, as Seiford and Thrall (1990) point out it, it rather determines a border at the top of the observations than a plan of regression in their center. More precisely, it consists in seeking for each producer if there is another producer who outclasses it within the meaning of Koopmans: that is "better" than the initial producer to which it is compared, if it produces a greater quantity of outputs to quantity of inputs given or if, with quantity of outputs given, it uses a less quantity of inputs. This "better" producer, if there exists, is then characterized by quantities of inputs and outputs which are solutions of the mathematical program of optimization according to:

For each DMU k, we have:

\[
\begin{align*}
\text{Max } E_k \\
\text{Under constraints:} \\
E_k &\leq 1 \text{ for each DMU } k \\
V_1, \ldots, V_m &> 0 \\
W_1, \ldots, W_n &> 0
\end{align*}
\]

With: for each decisional unit k we have:

\[E_K = \frac{(W_1 \times \text{out}_1 + W_2 \times \text{out}_2 + \ldots + W_N \times \text{out}_N)}{(V_1 \times \text{inp}_1 + V_2 \times \text{inp}_2 + \ldots + V_m \times \text{inp}_m)}\]

With \(W_1, \ldots, W_N\) are weights of the outputs.
\(V_1, V_2, \ldots, V_m\) are weights of the inputs.
\(N\) numbers of the outputs.
\(m\) numbers of the inputs.
\(E_K\) is called "the ratio of efficiency" which varies according to values' of weights.

Therefore weights are those which give the best result for a unit considered.

And like the score of efficiency reveals each DMU as powerful as possible, then DMUs which are on the border have an score of efficiency equal to 1. And the inefficient units have a score of efficiency lower than 1.
And for better understanding this idea starting from the most powerful unit, we have recourse to a simple graph indicated below "the measurement of the performances".

And as in theory, the efficient units are on the production border indicating the maximum level of production which one can reach with various combinations of production factors for a given technology. Then, one can empirically know this production border only starting from set of units observed i.e..

![Graph showing results obtained and the measure of performances](image)

**Figure 1-1: The measure of performances**

According to this graph indicated above we can note certain remarks. First of all, we can notice that A, B, C and D are the DMUs on the empirical border and consequently are efficient. Then, we can notice that F, G and E are the DMUs which can be better wrapped: the question is about the input and output quantitative or qualitative.

Lastly, we can notice that the unit D is more powerful than the unit E since it uses less resources to obtain good results. Consequently, efficiency is measured under an empirical angle, since the most powerful unit is that which reaches a maximum production by using minimal resources. And this unit has a score of efficiency equal to 100%. Thus, the performances will be evaluated by scores of efficiency going from 0% to 100%.
II- The concept of the banking performance:

II-1 Definition of the Bank in Tunisia:

The Tunisian banking structure is a set of coherent and structured organizations which the parts are independent and related dialectically to the economic policy context.

In the Tunisian legislation and according to article 2 of the banking law regulating the banking profession: "are regarded as " banks " and are subjected of this fact with approval in accordance with the terms of this law, all firms which are delivered, by way of activity usual with the following operations:

- to receive from the public of the deposits whatever are the duration and the form,
- to grant credit in all its forms,
- to carry out by way of intermediary of the exchange or stock exchange transactions,
- to ensure, for the customers of depositors, the payment or the covering of cheques, effects, coupons or any other title of payment or credit.

The banks can also carry out operations related to their activity such as the council and the assistance as regards management of inheritance, financial management, financial engineering and generally all the services intended to facilitate creation, the development and the reorganization of the companies ". (Official Journal of Tunisia Republic (1994)"

II-2 Structure of the banking system and specialized organizations:

- The central bank of Tunisia (CBT):

It arises from bearing article 2 of law 58/90 of the 19/09/1958 on the creation of the CBT, that this is a national publicly-owned establishment equipped with a financial autonomy. The CBT has, and since the law of 07/02/1994, a general mission to defend the value of the national currency (Tunisian Dinar) and to take care of its stability.

Consequently, it must control the money circulation and the distribution of the credit and must take care of the correct operation of the financial banking structure and of the maintenance of the evolution of the means of payment constantly compatible with that of the production.

- Deposit banks:

Article 5 of law 7/2.1994 introduced a modification into the activity of the deposit banks by widening their field of activity:
- to collect deposits near the various economic agents whatever are the duration and the form,
- to ensure the international commercial operations on behalf of the economic operators,
- to ensure a service of case and exchange.

- Development banks:

Article 6 of law 94-25 stipulates that the development banks are companies whose principal activity consists with:
- to grant appropriations in means and the long terms and under certain conditions of the short-term credits,
- to collect sight deposits of their personnel and companies of which they hold the majority of the capital.
• **Offshore banks:**
  These banks allow:
  - to collect deposits near non-residents, whatever are the form and the duration,
  - to grant any contest to the non-residents in particular in the form of acquisitions of a holding in borrow emitted by these last,
  - to ensure the exchange transactions manual the profit of the customers,
  - to carry out in quantity of intermediary approved the exchange transactions and trade foreign of their customers resident. They are subjected, for this reason, with the same obligations as the approved intermediaries resident,
  - to collect deposits, under certain conditions and to grant appropriations in dinars.

• **Investment banks:**
  This shape of bank was creates recently in Tunisia by law 94-25, its principal role is to ensure of the services, of council and assistance as regards management of inheritance, of financial management and financial engineering. Generally, all services intended to facilitate creation, the development and the reorganization of company.

• **Organizations of leasing:**
  These organizations allow first of all, to ensure the financing of acquisition of mobilized or immobilized material. Then, to place it in hiring for professional use at the disposal of an economic operator. This last has the possibility of acquiring it with a residual value in end of the contract.

• **Companies of factoring:**
  These companies consist:
  - to manage, by means of suitable financial management sciences the clients' accounts by acquiring their credits,
  - to ensure the covering of these credits for its own account.

**II-3 Definition of the performance:**

The idea of performance is not a simple concept and many reflexions followed on this subject. In its general direction, the performance can be defined like "the reach of the goals which we set" (Revue Française de Gestion Industrielle, Vol 17).

The definition of a powerful company is often limited to the simple analysis of its financial profitability. For C. Marmuse, the performance "clothe to multiple aspects, undoubtedly convergent, but which deserve to be approached in a most total logic than the only appreciation of profitability for the company or the shareholder" (C. Marmuse, art.118, "Performance"). It exposes indeed that the performance must be seen under 3 major angles: according to the total strategy of the company and in its interaction with its medium (competing performance), of its organizational structure which must be adapted to the social and economic evolution company and finally of its output and its financial profitability.

A first report can then be released: the performance cannot be defined like the only difference between anticipated results and results obtained. It is necessary to take into
account all that was implemented to reach the result obtained and to compare this result with those of the competitors and/or what was made in the past.

For J. Pericat (Rhône-Poulenc) and in connection with the performance evaluation to the purchases, the performance is a "evaluation of advance and quality of the achievements compared to objectives lying within the scope of a strategy defined by the Directorate-General and leading to the development of action plans".

We can then add that the performance must also be "retroactive" (feedback). Indeed, into account the actions/modifications made following a measurement of the performance (dynamic of continuous progress must also be taken: identification of the cause of the variations and modification of the action plans).

II-4 the idea of measurement of the performance:

The idea of measurement of the performance is also a broad concept. It is then advisable to know on which bases measurement will be taken. Indeed, knowledge what is regarded as powerful or not requires the definition of precise criteria on which measurements will be based.

First of all, the measurement of performance is a relativistic concept. Indeed, when a performance is evaluated, it is necessary to know compared to what one does it and especially on which bases a comparison can be made.

Measurement can nevertheless be absolute or relative. Within the framework of a relative evaluation, the results obtained will be judged relative with performances last or external with the company. When the evaluation is absolute, they are judged without reference to precise data.

Moreover, the measure can be quantitative or qualitative i.e. be based on statistical data or more subjective characteristics. It is the mode of evaluation which will make it possible to determine this character.

The distinction between the concepts of efficacy and efficiency is also significant. Indeed, the efficacy is the fact of achieving the goal envisaged, whereas efficiency is the fact of reaching that point with a minimum of efforts and costs.

II-5 Why measure the performance?

The evaluation of the results as well as the comparison with other data requires a method of measurement of the performance. In other words, it is advisable to have an average objective and reliable of comparison and evaluation of the results obtained by a person, a service, a company...

The first goal of the measurement of the performance is to evaluate the contribution of the function purchases in the performance of the company. In other words, it is then a question of highlighting the added value or the bad management of the purchases in the company.

A performance evaluation, whatever its applicability, can also involve positive consequences as for the motivation of the actors and with the recognition of their work. Indeed, according to the indicators used and their results, this work of handing-over in question and evaluation can allow for example:

- a better recognition of the actors (allows to promote the successes), better credibility
- an encouragement "to be better done"
- to lay down basic rules for work
- to motivate the actors (to have a means of sanction and reward)
✓ to reorganize work when the results are insufficient
✓ better communication
✓ better decision-making
✓ to make the results transparent

These consequences can they even be at the origin of the creation of the system of performance evaluation. It is also on this level that will be located our research.

Indeed, it is significant to determine the elements which induced the installation of a system of performance evaluation. Those translate the real objectives of the step of measurement of the performance which can be not only the need for control but also a valorization of work and function in general.

II-6 the banking performance:

Few studies have until now tempted to measure the performances of the Tunisian banks. The majority of the publications relating to the recent period are satisfied with simple comparisons of accounting ratios. This comparative and one-dimensional approach presents the disadvantage of not making it possible to determine a clear hierarchy of the banks according to their performances, insofar as it does not offer any synthetic criterion of the banking performance. It is however possible to obtain this result by applying the recent techniques of estimate of borders of production, cost or profit. One then measures the performances of the banks in terms of their degree of efficiency. Such measurements largely developed at the international level since the Nineties, so much so that Berger and Humphrey listed their studies on the efficiency of the banks throughout the world.

To be powerful, a bank must at the same time make the good choices and do well what it chose to do. But if it is relatively easy to determine, in the theoretical plan, the conditions to fill so that it appears among the ‘best’, it is much more difficult, in the practical plan, to quantify the manner that point of reaching and of quantifying in a satisfactory way the variations of performance between the banks. Efficiency offers a solution to this problem of measurement of the performances.

Since the design of the DEA in 1978, the applications of this method developed in various sectors of the economy. The facility of application of the DEA in the fields where on the one hand there is no unquestionable conventional definition of the inputs and outputs and where other hand the prices are unknown or given partly by non tradable factors, supported very many applications in the public services. The diversity of the approaches of the banking production intended the DEA in the same way to be largely applied in the banking environment.

III - DEA model applied to the Tunisian banking environment:

III-1 the choice of the inputs and the banking outputs:

On the question of the choice of the banking inputs and outputs, two approaches are opposed to determine the inputs and the outputs of the banks. Essentially, the opposition relates to the taking into account of the bank deposits like inputs (intermediate approach) or like outputs (production approach).

III-1-1 production approach

Sherman and Gold use for the first time the methodology of the DEA in the banking environment. They measure the technical efficiency of 14 agencies of the branch of an American Savings Bank. The choice of the inputs and the outputs is carried out according to
the approach of the production: the inputs are the Labour measured by the number of full-time employees, the physical Capital approached by the real surface of the agencies and the costs of the supplies as well as the other non-financial expenditure. The outputs are the banking services suggested by the agencies, measured by the number of transactions, the term deposits and the sight deposits as well as the various types of loans.

Thus, we consider that the banks transform non-financial inputs (for example: labour, the machines, the matters) in outputs and services financial. This approach is known under the name of "production approach" of banking. The application of DEA method enables us to use variables for all the inputs and the outputs measured in their units of origin without having to balance them or to transform them into monetary terms.

The banking inputs are:

- **INP1** general expenses (charges)
- **INP2** fixed assets
- **INP3** the authorized capital
- **INP4** loans and special resources
- **INP5** the other passive (expenses)

The banking outputs are:

- **OUT1** treated titles
- **OUT2** the treated capital
- **OUT3** deposits and assets of the banking and financial institutions
- **OUT4** deposits and assets of the customers
- **OUT5** commercial portfolio obligation
- **OUT6** investment portfolio obligation
- **OUT7** other credits
- **OUT8** credits on the banking and financial institutions

### III-1-2 Definition of the inputs and the outputs:

**General Expenses (charges):** the expenses of the personnel and the general loads of exploitation represent.

**Fixed assets:** the net amounts of the fixed assets at the end of the exercise define which still represent the gross amounts of the fixed assets at the beginning of exercise + acquisitions – the transfers and the regularizations – depreciation.

**The authorized capital:** defines by the whole of the contributions of the associates.

**Loans and special resources:** include the external resources, the budgetary resources, the bonds and the loads attached to the loans and special resources.

**The other passive (expenses):** defined by the accounts of regularization, the current liabilities and the provisions for contingencies and loads.

**Deposits and assets of the banking and financial institutions:** represent the loans from day to day and in the long term, the assets of the banks, correspondents foreign and specialized financial organizations and the attached debts.

**Deposits and assets of the customers:** gather the accounts at sight, terms, and cash vouchers, the debts attached to the accounts of the customers, the certificates of deposits and the savings accounts which are of two types, either separate accounts of saving open for the physical people, or open savings accounts investment for the people morals or physics to receive deposits for the subscription of securities having to carry out approved projects.
**Commercial Portfolio Obligation**: is composed of:

- Titles of transaction: in fact titles are characterized by their short duration from detention (lower than 3 months) and by their liquidity.
- Marketable securities: in fact the titles do not answer the criteria retained for the titles of transaction or investment.

**Investment Portfolio Obligation**: they are the titles acquired with the intention to hold them durably, they are entered at the date of acquisition at their cost of acquisition all expenses and loads excluded except for the fees for study and council engaged at the time of acquisition for titles for investment.

With the closing date of the accounts, is carried out to the evaluation of the titles as follows:

- titles of transaction: these titles are evaluated with the value of market (the balanced average stock exchange course). Consecutive variation of course to their evaluation with the value of market east carried in result.
- marketable securities: these titles are developed, for each title separately, with the value of market for the marketable quoted securities and with the right value for the non quoted titles. There cannot be compensation between the latent appreciations of some with the latent losses on other titles. Latent depreciation arising from the difference between the book value and the value of market gives place to the constitution of provisions, contrary to the latent appreciations which are not noted.

- titles of investment: the treatment of the latent appreciations on these titles is the same one as that planned for the marketable securities. Latent depreciations are the subject of provision only in the two following cases:
  - a strong probability that the establishment does not preserve these titles until the expiry;
  - the existence of default risks of the transmitter of the titles.

**Other credits**: - current credits: are regarded as current credits, the credits which the realization or integral covering within the deadlines appears assured.
- credits to be supervised: these are engagements of which the realization or integral covering within the deadlines is still assured and who are held by companies which are in a branch of industry which has problems or whose financial standing is degraded. The delays of payment of the interests or the main thing do not exceed the 90 days.

**Credits on the banking and financial institutions**: defined by the loans from day to day and in the long term in the banks, the loans at the specialized financial organizations, the placements in currencies, the debtor ordinary accounts of the banks in dinars, the interests rediscounts ready on the money market and the interests rediscounts accounts banks and correspondents.

**III-1-3 the intermediate approach**:

Rangan, Grabowski, Aly and Pasurka improve the application of DEA method to the banking environment by widening the sample size of the analysis and by developing the programming. The intermediate approach is adopted for the selection of the inputs and the outputs. Concerning the inputs are: labour, buildings and other fixed credits, the funds borrowed of which term deposits and of saving, other types of deposits as well as the financial and non-financial expenditure. The selected outputs are: various categories of loans (real, commercial and industrial, with consumption, the other loans), incomes of interest, incomes except interests as well as the sight deposits.
The banking inputs are:

- **INP1** general expenses (charges)
- **INP2** fixed assets
- **INP3** the authorized capital
- **INP4** loans and special resources
- **INP5** the other passive
- **INP6** deposits and assets of the banking and financial institutions
- **INP7** deposits and assets of the customers

The banking outputs are:

- **OUT1** treated titles
- **OUT2** the treated capital
- **OUT3** commercial portfolio obligation
- **OUT4** investment portfolio obligation
- **OUT5** other credits
- **OUT6** credits on the banking and financial institutions.

However, the DEA method makes it possible to classify the agencies of the various banks, in term of efficiency, by defining a whole strategy standardized for each agency. And for the agencies detailed on their specific weaknesses.

Thus, this analysis applied to the banking agencies provides a broad vision of performance and can be directly used to reorganize the network.

### III-2 DEA model applied to the Tunisian banks:

For the case of the Tunisian banks, we used two standard models DEA: the CCR model and the BCC model for the two approaches.

#### III-2-1 the CCR model:

The ratio form of the CCR model is characterized by the reduction of the multiple outputs and the multiple inputs for each DMU in only one "virtual" output and only one "virtual" input.

And this ratio makes it possible to provide a measurement of efficiency for each DMU, which is a function of the multipliers.

Indeed, we maximize this ratio by forming an objective function for a particular DMU, DMU₀ which is evaluated, so that it has symbolically:

\[
\text{Max } h_0 (u,v) = \frac{u^T.y_0}{v^T.x_0}
\]

\[
\text{also: Max } h_0 (u,v)= \frac{\sum_{r=1}^{S} u_r y_{r0}}{\sum_{i=1}^{M} v_i x_{i0}}
\]

With: **u, v**: are respectively the vectors of weights of the outputs and inputs for an evaluated DMU₀.
And $X_0, Y_0$ indicate respectively the vectors of inputs and outputs for a DMU$_0$.

It is about an unbounded equation, i.e. without additional constraint.

- the CCR model input oriented:

To obtain optimal weights, Charnes, Cooper and Rhodes (1978) specified the problem of mathematical programming in the following form:

\[
\begin{align*}
\text{Max} & \quad \frac{U^t Y_0}{V^t X_0} \\
\text{U,V} & \quad \frac{U^t Y_j}{V^t X_j} \leq 1 \quad ; \quad j=1, \ldots, N \\
& \quad \frac{U}{V^t X_0} \geq \epsilon I \\
& \quad \frac{V}{V^t X_0} \geq \epsilon I
\end{align*}
\]

This program provides values for $U$ and $V$, from which the efficiency of the DMU$_0$ can be increased. However, this ratio form produces an infinite number of optimal solutions. To fill this problem, Charnes and Cooper (1962) developed a transformation for the linear fractional programming by selecting a representative solution (i.e. the solution $(U, V)$ for which $V^t X_0 = 1$) from where, the constraint of standardization $V^t X_0 = 1$ is added to the program above. We will have then:

\[
\begin{align*}
\text{Max} & \quad \mu^t Y_0 \\
\mu, V & \quad S/c \quad V^t X_0 = 1 \\
& \quad \mu^t Y_j - V^t X_j \leq 0 \quad ; \quad j = 1, \ldots, N \\
& \quad \mu^t \geq \epsilon I \\
& \quad V^t \geq \epsilon I
\end{align*}
\]
the CCR model output oriented:

Alternatively, we could have left with the side output and have considered the ratio of the virtual input to the virtual output, such as it is given by the following problem:

\[
\begin{align*}
\text{Min } & \quad \frac{V^t X_0}{U^t Y_0} \\
& \quad U, V \\
\text{S/c } & \quad \frac{V^t X_j}{U^t Y_j} \geq 1 \quad ; \quad j = 1, \ldots, N \\
& \quad \frac{U}{U^t Y_0} \geq \varepsilon I \\
& \quad \frac{V}{U^t Y_0} \geq \varepsilon I
\end{align*}
\]

Again, the transformation of Charnes and Cooper (1962) for this linear fractional programming, by adding the constraint of normality \( \mu^T Y_0 = 1 \) to the program above, produces this model with its following Dual problem:

\[
\begin{align*}
\text{Min } & \quad V^t X_0 \\
& \quad \mu, V \\
\text{S/c } & \quad \mu^T Y_0 = 1 \\
& \quad -\mu^T Y + V^t X \geq 0 \\
& \quad \mu^t \geq \varepsilon I \\
& \quad V^t \geq \varepsilon I
\end{align*}
\]
III-2-2 the BCC model:

This ratio form of the BCC model (after standardization) is differed from the preceding form only by the addition of a variable $U_0$ in the orientation input and of another $V_0$ in the orientation output.

- The BCC model input oriented:

$$\begin{align*}
\text{Max } & \quad \frac{U'Y_0 + U_0}{V'X_0} \\
& \quad U, V, U_0 \\
\text{S/c } & \quad \frac{U'Y_j + U_0}{V'X_j} \leq 1 \quad ; \quad j = 1, \ldots, N \\
\frac{U}{V'X_0} & \geq \varepsilon I \\
\frac{V}{V'X_0} & \geq \varepsilon I
\end{align*}$$

- The BCC model output oriented:

$$\begin{align*}
\text{Min } & \quad \frac{V'X_0 + V_0}{U'Y_0} \\
& \quad U, V, V_0 \\
\text{S/c } & \quad \frac{V'X_j + V_0}{U'Y_j} \geq 1 \quad ; \quad j = 1, \ldots, N \\
\frac{U}{U'Y_0} & \geq \varepsilon I \\
\frac{V}{U'Y_0} & \geq \varepsilon I
\end{align*}$$
III-2-3 Interpretations

We used the GAMS software for the resolution of our DEA models. The latter were solved under linear programs of optimization, inspired of the modeling of Erwin Kalvelagen (2002). We have the results in the form of two following tables for each type of model.

### The CCR Model

<table>
<thead>
<tr>
<th>The production Approach</th>
<th>input oriented</th>
<th>output oriented</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank 1: <strong>UBCI</strong></td>
<td></td>
<td>Rank 3: <strong>BIAT</strong></td>
</tr>
<tr>
<td>Rank 2: <strong>BTE</strong></td>
<td>Rank 3: <strong>BTE</strong></td>
<td></td>
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<tr>
<td>Rank 3: <strong>BNA</strong></td>
<td>Rank 3: <strong>STB</strong></td>
<td></td>
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<tr>
<td>Rank 5: <strong>UBI</strong></td>
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<th>The intermediate Approach</th>
<th>Input oriented</th>
<th>Output oriented</th>
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Source: calculated by the authors using the GAMS software.
### The BCC Model

#### The production Approach

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#### The intermediate Approach

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Source: calculated by the authors using the GAMS software.
From the results obtained, by the two approaches (production and intermediate), we notice well that all the banks are completely efficient since their scores of efficiency are equal to 1 (i.e. 100% efficient), that it is for the CCR model and BCC model with two orientations (input and output). Thus, all these banks are located at the efficacy border. On the other hand, which is impressive, although all the banks are well wrapped within the meaning of the DEA, they are not classified on the same level.

Concerning the CCR model (in constant yield of scale) for an production approach of, the UBCI bank is in the first class (for an orientation input) but it has a bad quality of output, since it is in the 3rd class (7th rank) of the orientation output. Thus, the UBCI does not allocate optimal manner its resources to obtain good outputs. On the other hand, although the BIAT has a bad quality of input (classified in the 5th class (9th rank)), it is located in the 1st class (3rd rank) for the CCR model output oriented.

We can consider BT as a bad pupil (or DMU) of our group of bank, in the CCR model with two orientations input and output. Indeed, it does not use better resources making it possible to produce good results (9th rank). It is regarded as bad producer.

By contrast, the BTE is the best producer in the case of the CCR model. It has well wrapped data what locates them at a level of production and optimal efficacy. The BTE is regarded as good an intermediate also for the same CCR model, since it is located in the 1st class for the quality of its inputs and the 2nd class for the quality of its outputs.

To be more profitable, all the banks must reduce their inputs to approach more reference bank (DMU) or to belong to the 1st class, for the CCR model input oriented. In other words, they must have a better reference technology in accordance with best DMUs (i.e. located at the 1st class).

**How is necessary it to be caught there? (for the CCR model)**

Our investigations reveal that, for example, the STB bank (bad element as an intermediate and producer in the model input oriented) must use a volume of input comparable with that of the UBCI by reducing their fixed assets of 69 533 thousands of dinars (TD) to 30 391 TD, into 2004 and to reduce the number of the loans and the special resources of 824 570 TD with 96 658 TD. Thus, these effective banks classified with the last class will be better wrapped and will approach thus more at the best banks classified with the first class the tradable banks; if they adopt reduced volumes of the inputs of the last selected banks.

Concerning the BCC model (in variable yield scale) for an production approach, the BTE is in the second class (for an input oriented) but it has a good quality of output, since it is in the 1st class (23rd rank) of the output oriented. Thus, the BTE allocates in an optimal way its resources to obtain good outputs. On the other hand, although the STB and the UIB have a good quality of input (classified respectively in the 1st class (23rd rank) for the input oriented and before last class (the last class) for the output oriented).

We can consider the BIAT as a bad pupil (or DMU) of our group of bank, in the BCC model with two orientations input and output. Indeed, it does not use better resources making it possible to produce good results (3rd rank). It is regarded as bad producer.

Although the BTE is the best producer in this model, it is not regarded as best the intermediate particularly for the output oriented. Whereas BT and the UBCI are regarded as good examples to follow as an intermediate, since they are classified in the 1st class for the quality of their inputs and the 23rd class for the quality of their outputs.
How is necessary it to be caught there? (for the BCC model)

Our investigations reveal that, for example, the BH bank (bad element as an intermediate in the model input oriented) must use a volume of input comparable with that of the UBCI by reducing their fixed assets of 31,802,665 thousands of dinars with 30,391 TD, into 2004 and to reduce the number of the loans and the special resources of 517,073, 230 TD with 96,658 TD. Thus, these effective banks classified with the last class will be better wrapped and will approach thus more at the best banks classified with the first class the tradable banks; if they adopt reduced volumes of the inputs of the last selected banks.

Conclusion:

From the point of view of analysis of the efficacy, the production function is not only interpreted as being a relation which binds the inputs and the outputs, but it is regarded as a border which wraps a set of the possibilities of production. For that, the "DEA" analysis is still called the analysis of the efficacy by the border which is done by means of the mathematical programming.

And as our objective was to measure the performance of some tradable banks in Tunisia, then we used the "DEA" method under a linear programming allowing to detect the differential efficacy of the various banks.

On the level of the efficient banks, are well wrapped and located at the production border, the "DEA" model makes it possible to classify them according to the quality of their inputs or their outputs (according to the orientation (input or output) and according to the model selected (CCR or BCC)).

In a preoccupation with an efficacy, the supplier of formation may find it beneficial to act on the alternative combinations of inputs or outputs until arriving at a solution considered to be optimal compared to the objectives that the community tends to realize.

In short, this "DEA" analysis applied to the banking environment in Tunisia provides a broad vision of performance and can be directly used to reorganize the network. With through our study, the people can have an idea on the banking performance in Tunisia and they can choose in a rational way their bank.

Within the framework to look further into our study, we can put the following question: Can we identify within the efficient banks, the agencies inefficient, using the DEA method?

References


