



***Introducing Monopolistic Competition  
into the GTAP Model***

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**GTAP Technical Paper No. 6**

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## *Abstract*

This technical paper documents one approach to incorporating monopolistic competition into the GTAP model. In this framework, consumer preferences are heterogeneous, leading to an apparent “love of variety” in the aggregate utility function for each region. The more heterogeneous are preferences, the smaller the elasticity of substitution in the aggregate utility function, and the greater the value placed on the addition of new varieties. The same is true for firms, which experience lower unit costs for differentiated, intermediate inputs, as the number of varieties on offer increases. In order to meet the diverse needs of consumers, and firms, producers differentiate their products through research and development (R&D) as well as advertising activities. These costs are assumed to be invariant to the total volume of sales for a given variety of product. With production occurring at constant returns to scale, this gives rise to declining average total costs. A zero profits equilibrium in this model is characterized by firms marking up their price over marginal costs by an amount sufficient to cover the fixed costs associated with establishing a new variety in the marketplace. Since the optimal markup is itself determined by the elasticity of substitution among varieties, this establishes a direct relationship between fixed costs and the degree of preference heterogeneity.

The main differences between the monopolistically competitive sectors and the traditional GTAP sectors may be summarized as follows:

- Two new variables are introduced.  $n$ , the number of firms, and  $qof$ , output per firm.
- Minimum expenditure and unit costs are declining in  $n$ .
- Average total costs are declining in output per firm.
- Unlike the nested Armington specification, foreign and domestic firms compete directly in the representative consumer’s utility function.

We illustrate this framework with a 2 commodity/3 region example in which we eliminate US antidumping duties on the import of Japanese manufactured goods. This example demonstrates the role of changing varieties in determining aggregate utility. It also highlights the importance of the monopolistically competitive cost structure in determining the equilibrium change in output per firm. A comparison with the standard, perfectly competitive GTAP model is also provided.

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# *Introducing Monopolistic Competition into the GTAP Model*

## *1. Introduction*

### *1.1 Background*

Applied general equilibrium (AGE) models are ever more widely used in economic policy analyses. Major clients of these modeling efforts are policy makers dealing with issues of multilateral liberalization of international trade, regional integration of economies and the consequent implications for energy and environmental standards. Examples include the recently concluded Uruguay Round of GATT negotiations and the North American Free Trade (NAFTA) agreement. Simulations using AGE models have provided a vital input into these negotiations. However, the suppliers of this critical input are not homogeneous. Applied economists have used different model structures with very different underlying assumptions, different data sets, and various base years.

By way of example, of the five quantitative assessments of the Uruguay Round Agreement represented in the World Bank's recent conference on this topic (MARTIN and WINTERS), three different data bases were used. Analyses using the OECD's RUNS model (Goldin and VAN DER MENSBRUGGHE), and the University of Michigan model (BROWN *et al.*), both use their own data bases, while the other three studies (FRANCOIS *et al.* 1995, HARRISON *et al.*, and HERTEL *et al.*) all use the Global Trade Analysis Project (GTAP) data base. Given the tremendous effort involved in developing and maintaining a global data base, it appears likely that most such studies will gradually converge on a common data base, thereby eliminating this source of difference (the OECD has already begun to use the GTAP data base). However, there remains tremendous scope for differences in the treatment of economic behavior in these AGE models. The three GTAP-based studies cited above consider a variety of different assumptions regarding the treatment of investment as well as the nature of firm behavior. It is important that users of the GTAP data base have access to these alternative closures and behavioral assumptions, so that they can choose which is most appropriate for the problem which they seek to address. This is one of the primary goals of the GTAP technical paper series. FRANCOIS *et al.* (1996) have provided a GTAP technical on the macro closure (investment) issue. The purpose of this technical paper is to introduce GTAP users to a framework which permits them to treat selected sectors in the model as monopolistically competitive.

The standard implementation of the GTAP model assumes perfect competition in all industries. Hertel and Tsigas clearly recognize the need for a GTAP model with imperfect competition and endogenous product differentiation:

"In sum, while we are not particularly happy with the Armington specification, it does permit us to explain cross-hauling of similar products and to track bilateral trade flows. We believe that, in many sectors, an imperfect competition/endogenous product differentiation approach would be preferable. ... Clearly this is an important area for future work."

## 1.2 Motivation

A casual look at a modern market economy is sufficient to conclude that consumption and production are complex phenomena for the decision-makers involved. On any given visit to the supermarket, the typical consumer<sup>1</sup> wanting to buy, say a video camera, is confronted with a myriad of products to choose from. Note that the decision to buy *a* camera has already been made<sup>2</sup>, which is reflected from the kind of retail store or department within a super market the consumer has chosen. However, the actual type of camera chosen depends on the consumer's preferences.

With millions of consumers, it is natural to expect preferences to be rather diverse. Firms respond to this diversity by producing differentiated products of the same good. When viewed as an aggregate grouping, this is often termed a *love of variety*, even though each individual only purchases a single type of camera. Firms in the camera industry thus have an incentive to incur fixed costs related to R & D and marketing, in order to establish their product as differentiated -- in the eyes of the consumer. The existence of this fixed cost makes the market for this product imperfectly competitive on two counts: firms cannot adopt marginal cost pricing, and they do not produce a homogeneous good; both deviations from the perfectly competitive behavior. Assuming that production occurs at constant returns to scale, then firms experience *increasing returns to scale* in sales.

One of the important properties of this type of a market is that the share of total costs devoted to R & D and marketing activities will be directly related to the degree of preference heterogeneity. The more diverse are individual consumers, the greater the demand for variety, and the greater the scope for marking up the price of the differentiated product. It is this markup over marginal cost which allows

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1 The typical consumer differs from the idiosyncratic consumer, in that the former is likely to make different choices at different times, while the latter is loyal to the product they consume.

2 We distinguish between levels of decision-making and the implications for industry competition. The decision whether or not to buy a camera is made at a previous stage or higher level of decision-making. At this higher level, the group of firms in the camera industry compete with firms from a different industry, which is *inter-industry competition*. Once the decision to buy a camera has been made, the competition is among the firms within the camera industry, which is *intra-industry competition*.

firms to recoup the fixed costs associated with their marketing and R & D activities. Of course, in the extreme case where preferences are completely homogeneous, there is no reason to differentiate your product and the model collapses to one of homogeneous products/perfect competition.

While this theory is largely developed in the context of consumer preferences, the subsequent "love of variety" is also observed among firms, leading Ethier to apply the same approach to intermediate inputs. Since a large share of international trade is in intermediate goods, this extension is potentially very important. Therefore, we treat ALL sales (both intermediate and final) in a symmetric manner with respect to product differentiation.

Having settled on this general approach to monopolistic competition, the questions that naturally follow are then - *which* are the industries (sectors) that are more appropriately modeled by a differentiated products approach? and *when* is it appropriate to do so? Clearly, the answers to these are context-specific and we only make an attempt at them.

The answer to the 'which' question follows. Since preferences are unobservable, a more practical approach would be to go by the characteristics of the industries being modeled. This requires *both* a good knowledge of the industry and the availability of good data to support quantification. We propose to focus on industries where significant attempts at product differentiation are evidenced in high advertising and/or R & D expenditures. This suggests including industries producing processed foods, beverages, textiles, wearing apparel, automobiles, electricals, electronics, and other durables but not including primary industries producing chiefly homogenous intermediate inputs such as primary agriculture, natural resources, mining, and primary metals.

The answer to the 'when' question depends on the objective of the user. If the exercise is to address issues such as efficiency gains, inter- and intra-industry trade patterns, impacts of economic integration on the integrating countries, then a monopolistically competitive approach is superior, if only because it offers the user a wider spectrum of possibilities. Also it captures two important issues: the effects of variety on consumer utility and on potential scale economics. In a policy scenario, these have an important bearing on welfare and resource allocation matters. It is thus critical to embed this diversity in consumer preferences and examine its implications for trade policy effects.

This paper is laid out in the following manner. The next section explains the theoretical foundations for monopolistic competition. Section 3 reviews some of the existing applied trade models with imperfect competition and shows the link with our modeling efforts here. The structure of the Monopolistically Competitive GTAP model, GTAPMC, is laid out in section 4. (The GEMPACK<sup>3</sup> code for the model implementation, GTAPMC.TAB, is presented in the Appendix.) Section 5 outlines the data requirements for introducing monopolistic competition into the GTAP model. Section 6

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3. GEMPACK is the software suite used to implement the GTAP modeling framework. The equations in the model are written in algebraic form and read in the form of a TABLO code file as input to the program.

contains a simple example designed to illustrate a policy application using the monopolistically competitive GTAP model. This is followed by our summary and conclusions.

## ***2. Theoretical Foundations for Monopolistic Competition***

The theoretical literature on imperfect competition is characterized by a myriad of models, ranging from monopoly, to oligopoly, to monopolistic competition. Recent advances in industrial organization theory have proliferated the number of oligopoly models. Theoretically elegant and mathematically complex, these models are difficult to operationalize and are largely unsuitable for economy-wide AGE models, as they require information on the nature of strategic interaction between firms in the imperfectly competitive industry which is simply not available at the level of aggregation in most AGE models.

Chamberlin's monopolistic competition has been widely used in AGE models. What makes this market structure appealing, is the large group assumption which permits us to abstract from inter-firm rivalry. The variant of the Chamberlinian model which we use was first formulated by Spence, and Dixit and Stiglitz. They use a *representative consumer* as a simplifying construct whose utility embodies the preferences of the aggregate population of consumers. The utility of the representative consumer is defined over all existing (and potential) products. The representative consumer purchases some of *every* product available and is therefore at an interior solution. In these theoretical studies, preferences of the representative consumer are *symmetric*, that is no product can be ranked over another product, based on price. This form of product differentiation is known as the *non-address* approach since there is no notion of the most preferred or less preferred product.

There are two important criticisms of this approach to modeling heterogeneous preferences. First of all, individual consumers tend to buy only one, or at most a few, of the differentiated products offered. In the camera example alluded to earlier, an individual consumer buys only one camera and is at a corner solution for all of the other camera products available. Is the representative consumer a valid aggregate description of the underlying population of heterogeneous consumers characterized by discrete choices made by the individual? How are we to interpret this "love of variety"? What preference characteristics determine the all-important elasticity of substitution among varieties? In order to answer these questions, we draw on theoretical work by Anderson, de Palma, and Thisse.

Anderson *et al.* begin with a specific utility function for individual consumers in the economy. This utility function includes a quadratic penalty term for departures from the consumer's ideal variety. The authors then make the assumption that preferences in the population at large can be represented with the multinomial logit distribution function. In this case, the standard deviation associated with consumers' ideal variety is given by the parameter  $\mu$ . When  $\mu$  is large, then preferences are diverse,

when it is small, then preferences are homogeneous. Anderson *et al.* also permit the consumer to vary the quantity of their ideal variety purchased. In particular, they adopt a specification of the utility function which implies a unitary elasticity of demand for the chosen variety. With these assumptions, Anderson *et al.* are able to derive the consequences for aggregate, sectoral behavior of the sort we are attempting to model here. Interestingly, they obtain the result that the aggregate demand relationship implied by this specification is of the Constant Elasticity of Substitution (CES) form, with the elasticity of substitution between different varieties equal to the inverse of  $\mu$ . This provides us with a valuable interpretation of the CES demand function: the elasticity of substitution should be large in those cases where preferences are homogeneous, and small in those cases where preferences for the products in question are heterogeneous.

The second problem with the Spence-Dixit-Stiglitz approach to modeling preferences has to do with the symmetry assumption. In practice, market shares of firms supplying differentiated products vary in ways which this simple, aggregate representation of preferences cannot explain. From the point of view of international trade, the most striking illustration of this point is the predominance of domestic firms in consumer's consumption bundles. For example, in the GTAP framework, even after complete elimination of border distortions, domestic sales are still predominant in almost all regions (HERTEL and MCCORRISTON).

Venables has explored the theoretical implications of this preference for domestic varieties in a model of monopolistic competition. He shows that this asymmetry translates into additional welfare losses when protection for differentiated products is cut and some domestic firms/varieties exit from the industry. Thus, in order to make an accurate assessment of the welfare effects of trade policy, it is important to take account of asymmetries in preferences. (There is also a practical motivation for introducing these preference biases, since it is impossible to calibrate the model to observed data in their absence!)

### ***3. Applied GE Models with Imperfect Competition***

In the last ten years, many AGE models have introduced imperfect competition, economies of scale, and product differentiation. Harris is credited as being the pioneer in this effort. He implemented versions of imperfectly competitive models with and without product differentiation for Canada, a *small open economy*. Norman argues that the presence of imperfect competition has significant implications for inter-industry trade patterns and welfare effects of trade liberalization. He goes on to show that the Armington product differentiation, used to explain intra-industry trade in perfectly competitive AGE models, does a bad job in explaining the inter-industry trade patterns and welfare effects of trade policy in the presence of imperfect competition.

Abayasiri-Silva and Horridge have adopted a Harris-type implementation for Australia, assuming monopolistic competition for all sectors. However both models have assumed *competitive foreign product differentiation* for the behavior of the imperfectly competitive sectors in the residual rest of world, though recognizing the need for modeling foreign importers. Moreover, the former study treats *all* sectors as monopolistically competitive for purposes of comparing different assumptions on technology and pricing on model results. Harris treated 20 of the 29 industries as imperfectly competitive.

A number of recent studies of the Uruguay Round have employed model variants with imperfect competition. The main purpose of these studies was to compare the welfare gains using the model with increasing returns to scale (IRTS) with the constant returns to scale model. Harrison *et al.* reported *relatively small increases in global welfare* of only 3% due to incorporating scale economies internal to the firm. In order to make a pure comparison between the two model variants, they retain the Armington nesting structure, but introduce an additional level of nesting for all regions at the bottom level in the IRTS model. Thus, there are no benefits due to additional varieties.

The values of the cost disadvantage ratio (CDR), otherwise known as markups, are important for the results generated by Harrison, *et al.* Of the 13 industries that are modeled with firm level economies out of a total of 22, the CDR ranges from a low of 3% for minerals and energy sectors to a high of 13% for processed rice. The small increases in welfare are attributed to these two factors - relatively low CDRs and maintaining a strict IRTS regime in the imperfect competition model. (It is not clear how they have chosen to treat the composition of fixed costs, since in a pure IRTS model with a standard markup pricing equation, this might have a key role to play.)

Harrison *et al.* choose their treatment of monopolistic competition in order to facilitate easy comparison with existing, perfect competition, Armington-based analyses of the Uruguay Round. This is useful in terms of understanding what these studies might be missing, but it may not be the best way to construct a model based on the theory of monopolistic competition. We observe this tension between maintaining consistency with earlier studies, and implementing a new theory, in a number of other studies.<sup>4</sup> In the approach outlined below, we have opted for a sharper break with the Armington-based models, such as GTAP.

FRANCOIS *et al.* (1995) provide a critical review of a number of alternative approaches to modeling the impact of the Uruguay Round, analyzing the effects of theoretical and data structure on model outcomes. Thirteen of the nineteen sectors are modeled as monopolistically competitive, with CDRs ranging from a low of 8% in the mining industry to a high of 15% in the processed food, chemicals, transport equipment and machinery industries. In their assessment of the Round, FRANCOIS *et al.* use

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4. We find it useful to model a plausible monopolistically competitive model and then proceed to make a perfectly competitive variant of this to allow for a straight forward comparison between the two. We do this by eliminating the variety effect on utility/cost functions as well as the scale effect. The former is achieved by rewriting the derived demand equations to resemble the perfectly competitive model and the latter by fixing the output per firm.

two variants of monopolistic competition - the non-nested (global) monopolistic competition, where firms from different regions compete directly and the nested (regional or national) monopolistic competition where firms in each region compete with those in other regions according to an Armington specification. The non-nested specification yields greater welfare effects than the nested version.

Hertel and Lancelos emphasize the importance of changes in the cost structure of monopolistically competitive firms (see also LANCLOS and HERTEL). They point out that when tariffs are cut on intermediate inputs, there is a tendency for average variable costs to fall, relative to scale-constant, average total costs. When production is subject to constant returns to scale, average variable costs equal marginal costs. With a constant markup, the change in marginal cost dictates price. Yet for zero profits to apply, price must also equal average total cost. Therefore, in order to simultaneously satisfy the markup pricing and zero profit conditions, output per firm must increase, thereby permitting average variable costs and average total costs to move in concert. In their analysis of trade liberalization, they find that these cost-driven changes in output per firm account for the majority of the welfare gains attributable to the monopolistic competition formulation.

Modeling the production structure of the imperfectly competitive industries is a rich area offering a number of modeling choices in terms of the following issues: Are all or only a few sectors monopolistically competitive? Is the sector characterized by only scale economies or both scale economies and love of variety? What is the structure of nesting in the monopolistically competitive sectors? Is the product differentiation firm-based or firm-based and national? What is the nature of the markup pricing equations? What is the composition of the fixed costs? And finally, are both inputs and outputs differentiated?

Further progress in the application of this theory will hinge on finding appropriate answers to each of the questions for the particular industry under consideration.

#### ***4. Structure of the Monopolistically Competitive GTAP Model, GTAPMC***

This section provides a detailed description of the monopolistically competitive structure underlying the model. We refer to version 2.2a of the GTAP model (GTAP94.TAB) as the standard model, which is characterized by perfectly competitive industries. We build on the standard model and therefore assume a fair amount of familiarity with its structure on part of the user. This allows us to focus on features that have been added and on those that have been modified. The changes to notation, to accommodate the new data and model features, have been kept to a minimum.

Adopting the Chamberlinian theory of monopolistic competition introduces two new concepts to the model - economies of scale and differentiated products. Modeling these involve substantive

modifications to the behavioral assumptions made for all agents in the economy - firms in the different industries, private household, and government.

#### ***4.1 Demand Structure***

We retain the Cobb-Douglas functional form for the regional household's utility function and the government's utility function as well as the Constant Difference Elasticity of substitution (CDE) utility structure of the private household. The sub-utility function for composite commodities is the most appropriate level in the utility tree to incorporate the differentiated products.

Before we describe the specification of the agent-specific demand functions, some discussion on the treatment of "foreign" versus "domestic" commodities is warranted. In the *standard model*, homogeneous commodities are produced by perfectly competitive industries in each region. Due to the Armington assumption, an *ad hoc* product differentiation scheme is made possible based on the geographical origin of the homogeneous commodity (national product differentiation).

The same physical commodity produced in different regions are imperfect substitutes in the demand for a composite imported commodity. The quantity index resulting from the consumption/usage of imports from different sources is termed the composite import. In turn, the composite import, devoid of its geographical origins, imperfectly substitutes for the domestically produced commodity. Thus, sourced imports do not directly compete with the domestic commodity. Also, imports are not sourced directly to the agents. The composite import commodity, formed at the border, is demanded by all the agents in the economy.

When firms within the same region produce differentiated products, the geographic origin of a product becomes an issue of lesser importance. In fact, the characteristics of the product that differentiate it from another product produced in the *same* region is what is striking. Consumers are often unaware of where a product originates, though they are aware of the 'brand name'. In many instances, even if the educated consumer cares about the geographic origin of the commodity they are buying, there is no distinct national identity for a vast number of cases. Individual components are often sourced from many different countries, and the site of assembly for a given product may change from one year to the next.

Consider the case of a personal computer - with the mouse made in Malaysia, the keyboard assembled in Mexico using USA-made components, the monitor made in Taiwan, and the Intel chip itself made in the USA. One is hard put to identify the national origin of such a good! Rather, a consumer finds herself choosing between a Compaq, a Dell, or a Packard Bell personal computer. This is also the case with textiles and clothing, software products, processed foods, pharmaceuticals, and the list goes on. Therefore, in our opinion, it makes little sense to leave the import-domestic distinction for monopolistically competitive industries producing differentiated products. We thus let imported varieties compete directly with domestic varieties in a non-nested fashion.

This has many implications. *First*, we cannot afford to lose the geographic origin of the imported product at the border because this corresponds to a particular set of firms in the exporting region. If the number of firms in this region increases, so too will the varietal index associated with that flow. We must retain the origin of the foreign product until we source it to the agent. We must *source imports to agents* just as we source domestic products to the various agents in the economy. This involves a fundamental change in data structure. We explain in the data requirements section the procedure used to obtain this new structure from the standard GTAP data base.

One important aspect of this new structure is reflected in the price linkages. The modified price linkages, due to the sourcing of imports by agents, are defined for *all* tradeable commodities and shown in figure 1. The price linkages relating to the endowment commodities remain the same as in the standard model. Note that the price definitions do not discriminate between domestic prices or import prices. For example,  $PPS(i,r,s)$ , the private household price for the sourced commodity, in GTAPMC, replaces  $PPD(i,r)$  - private household price for domestics and  $PPM(i,r)$  - private household price for composite imports, in the standard model. The market price of commodity  $i$  in region  $r$ ,  $PM(i,r)$ , would thus equal  $PPS(i,r,s)$ , for  $r = s$ , when  $r$  is a single country region, and tax on private household demand is zero.<sup>5</sup> Similarly,  $PGS(i,r,s)$  and  $PFS(i,r,j,s)$  replace  $PGD(i,r)$  and  $PGM(i,r)$  and  $PFD(i,j,r)$  and  $PFM(i,j,s)$  respectively. It is useful to introduce the modified price-quantity-value notation here. Figure 2 tracks the disposition of sales, indicating the associated price concepts and the corresponding value flows.

The *second*, and more serious implication, is a very practical one in terms of modeling. Sourcing of imports has introduced *an extra dimension* into the demands of all agents, as evidenced from the price concepts discussed above. The extra dimensionality causes a dramatic increase in model size<sup>6</sup> - the number of variables to be solved for and the number of equations to determine them - as the number of regions increases.

We first explain the modifications made to the private household utility/expenditure structure. We will then briefly explain changes made to the specification of government demand and intermediate input demands by industries.

#### **4.1.1 Private Household Utility**

In the representative consumer approach, an aggregate utility function is used to represent the consumption of all varieties<sup>7</sup>. To capture the effect of variety on utility, the average consumer's utility

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5. This equality between  $PM(i,r)$  and  $PPS(i,r,s)$  may not hold for intra-regional sales when  $r$  is a composite region. The data requirements section contains more discussion on this.

6. The data requirements section gives some hints on how model size may be reduced before attempting to solve it.

7. Varieties and products are used interchangeably.

is defined over the quantities of the different varieties consumed *and* the total number of varieties consumed. The sub-utility function in the levels<sup>8</sup> is thus given by,

$$Y_{i,s} = \text{sum}\{r, N_{i,r} * Q_{i,r,s}^{(\sigma(i)-1)/\sigma(i)}\}^{\sigma(i)/(\sigma(i)-1)} \quad (1)$$

where,  $Y_{i,s}$  is the sub-utility derived by the representative consumer in region  $s$  from the consumption of commodity  $i$ ,  $Q_{i,r,s}$  is the quantity consumed by the representative consumer in region  $s$  of a *representative product* of commodity  $i$  produced in region  $r$ ,  $N_{i,r}$  is the number of varieties of  $i$  produced in  $r$ , and  $\sigma(i) > 1$  is the positive, constant elasticity of substitution between the differentiated products of  $i$ . To relate to the bilateral trade data among regions, we assume that all firms in any given region,  $r$ ,

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8. For purposes of clarity, we derive new or modified equations using generic notation in the 'Demand' sub- section. The generic notation is applicable to all agents. We begin the derivation from the levels form. In the final step of the derivation, we translate the generic notation to agent-specific, GTAP notation and write it in linearized form as found in the implementation of the TAB file.

Figure 1. Price Linkages,  $i \in \text{TRAD\_COMM}$ ,  $r, s \in \text{REG}$

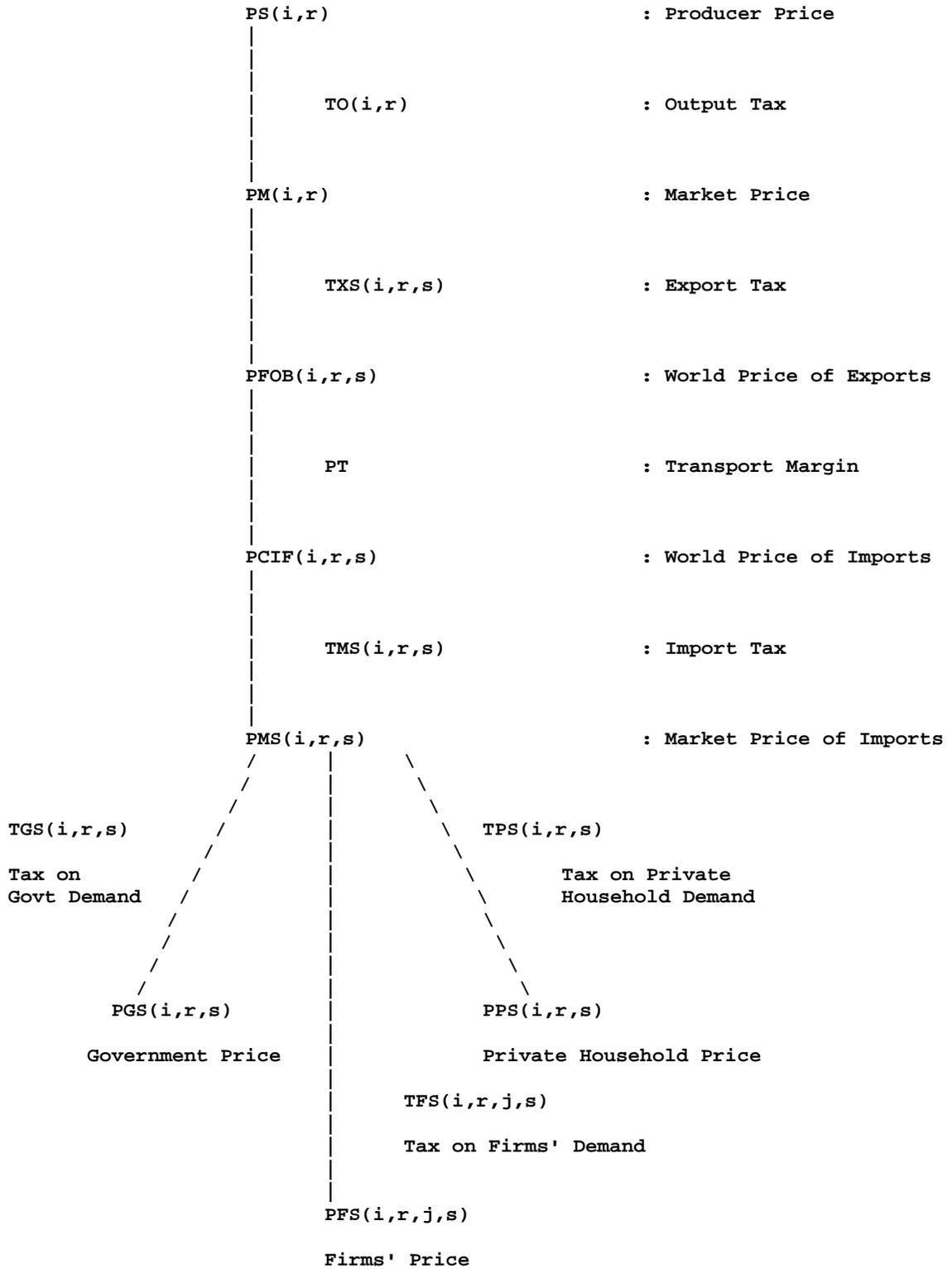
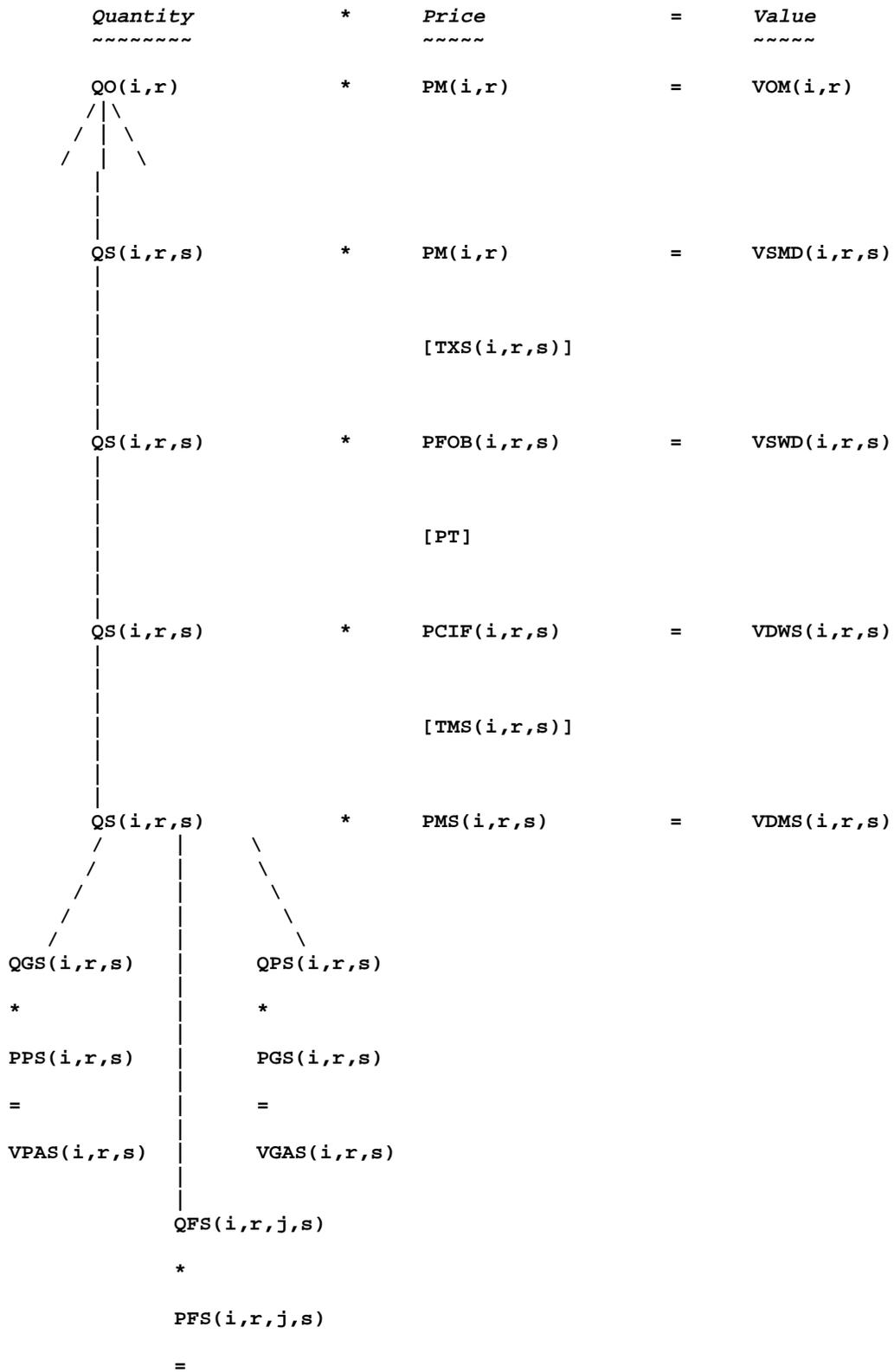




Figure 2. Disposition of Sales,  $i \in \text{TRAD\_COMM}$ ,  $r, s \in \text{REG}$



charge the same price, and furthermore, *that firms are active in any region s, where sales from r appear.*

Maximizing the sub-utility subject to the budget constraint yields the *derived demand for a product of i* from source *r*,

$$Q_{i,r,s} = Y_{i,s} / [ P_{i,r,s} / Z_{i,s} ]^{\sigma(i)} \quad (2)$$

where,  $Z_{i,s}$ , the expenditure spent on all varieties of commodity *i* in *s*, is given by

$$Z_{i,s} = \text{sum}\{r, N_{i,r} * P_{i,r,s}^{(1-\sigma(i))}\}^{(1/\sigma(i))} \quad (3)$$

and  $P_{i,r,s}$  is the unit price of *a product of i* in *s* sourced from *r*.

Equation (2) must be modified so as to define the aggregate derived demand for differentiated products originating from source *r*. Lacking information on the number, size, composition, and sales data of firms in the monopolistically competitive industry, we are forced to impose symmetry here. Total derived demand for all products of *i* sourced from *r* is thus given by

$$Q_{i,r,s} * N_{i,r} = N_{i,r} * Y_{i,s} / [ P_{i,r,s} / Z_{i,s} ]^{\sigma(i)} \quad (4)$$

Totally differentiating the aggregate derived demand function given in (4) we have,<sup>9</sup>

$$q_{i,r,s} + n_{i,r} = n_{i,r} + y_{i,s} - \sigma(i) * [ p_{i,r,s} - z_{i,s} ] \quad (4')$$

(4') is the linearized representation of the aggregate derived demand equation in generic notation. To derive the linearized representation of the price index equation, we begin by totally differentiating (3),

$$z_{i,s} = \text{sum}\{r, \theta_{i,r,s} * p_{i,r,s}\} - [1 / [\sigma(i) - 1]] * \text{sum}\{r, \theta_{i,r,s} * n_{i,r}\} \quad (3')$$

where  $\theta_{i,r,s}$ , the expenditure share of all varieties of commodity *i* originating from source *r* in total expenditure on all varieties from all sources, in region *s*, is given by

$$\theta_{i,r,s} = N_{i,r} * [ P_{i,r,s} / Z_{i,s} ]^{(1-\sigma(i))} \quad (5)$$

The expenditure shares,  $\theta_{i,r,s}$ , can be expressed in an alternative manner that is more transparent. Rewriting (2),

$$Q_{i,r,s} / Y_{i,s} = [ P_{i,r,s} / Z_{i,s} ]^{-\sigma(i)}$$

Multiplying both sides by the ratio of  $P_{i,r,s}$  to  $Z_{i,s}$ ,

$$[ Q_{i,r,s} / Y_{i,s} ] * [ P_{i,r,s} / Z_{i,s} ] = [ P_{i,r,s} / Z_{i,s} ]^{-\sigma(i)} * [ P_{i,r,s} / Z_{i,s} ]$$

Rearranging terms,

---

99. Lower case letters denote percentage changes in the corresponding upper case variables. Thus,  $x = [ dX / X ] * 100$  gives the percentage change in *X*.

$$[ Q_{i,r,s} * P_{i,r,s} ] / [ Y_{i,s} * Z_{i,s} ] = [ P_{i,r,s} / Z_{i,s} ]^{(1-\sigma(i))}$$

Multiplying both sides by  $N_{i,r}$ ,

$$N_{i,r} * [ Q_{i,r,s} * P_{i,r,s} ] / [ Y_{i,s} * Z_{i,s} ] = N_{i,r} * [ P_{i,r,s} / Z_{i,s} ]^{(1-\sigma(i))}$$

$$\theta_{i,r,s} = [ N_{i,r} * Q_{i,r,s} * P_{i,r,s} ] / [ Y_{i,s} * Z_{i,s} ] \quad (6)$$

We can thus express  $\theta_{i,r,s}$  in terms of the data base, where the numerator is the expenditure on *all* varieties of  $i$  originating from source  $r$ , and the denominator is the aggregate expenditure on  $i$  in region  $s$  (total expenditure on all varieties of  $i$  originating from all sources consumed/used). The actual computation of  $\theta_{i,r,s}$  for the different agents is as follows (see also figure 2):

Private Household:

$$PTHETA(i, r, s) = VPAS(i, r, s) / \text{sum}\{t, REG, VPAS(i, t, s)\}$$

Government:

$$GTHETA(i, r, s) = VGAS(i, r, s) / \text{sum}\{t, REG, VGAS(i, t, s)\}$$

Firms in industry  $j$ :

$$FTHETA(i, r, j, s) = VFAS(i, r, j, s) / \text{sum}\{t, REG, VFAS(i, t, j, s)\}$$

$VPAS(i,r,s)$  is the Value of Private household expenditure in region  $s$ , at Agents' prices by Source. Note that  $VPAS(i,r,s)$  gives the total expenditure on all products of  $i$  originating from  $r$ , but no breakdown per product per region is available. From (1), it can be seen that the effect of additional variety on utility is positive. As the number of varieties of a differentiated commodity increases, the sub-utility derived from their consumption increases, even if the quantity consumed remains the same. Also from (3), we see that additional variety has a negative effect on unit expenditure. At constant prices, an increase in the number of varieties lowers the amount of expenditure necessary to attain a unit of utility.

The linearized forms of the derived demand and price index equations now have to be related to the agent-specific equations in GTAPMC.TAB. The private household's demand for differentiated products of commodity  $i$  are given by a set of three linearized equations.

#### *Private Household's Demand for Differentiated Products*

The derived demand by the private household in region  $s$  for the differentiated products of commodity  $i$  sourced from  $r$ ,  $qdfps(i,r,s)$ , is determined by

EQUATION PHLDSRCDF

```
! Private HousehoLD demand for SouRCed DiFFerentiated commodity !
(all, i, MCOMP_COMM) (all, r, REG) (all, s, REG)
  qdfps(i, r, s) = qp(i, s) + n(i, r) - SIGMA(i) * [pps(i, r, s) - pp(i, s)] ;
```

Equation PHLDSRCDF relates to (4') above.  $qdfps(i,r,s)$  thus depends on the sub-utility derived by the private household in  $s$  from the consumption of differentiated products,  $qp(i,s)$ , number of products of  $i$  produced in  $r$ ,  $n(i,r)$ , private household's price of the sourced differentiated product of  $i$  produced in  $r$ ,  $pps(i,r,s)$ , and the unit expenditure of the private household in region  $s$  on commodity  $i$ ,  $pp(i,s)$ .



### *Private Household's Composite Price Index for Differentiated Product*

The composite price index for differentiated products facing the private household in region  $s$ ,  $pp(i,s)$ , is computed by

EQUATION PHLDDFCOMPR

```
! Private Household Price for Differentiated Composite commodity. !
(all,i,MCOMP_COMM)(all,s,REG)
pp(i,s) = sum{r,REG, PTHETA(i,r,s) * pps(i,r,s)} - {1/[SIGMA(i) - 1]} * vp(i,s)
```

Equation PHLDDFCOMPR relates to (3') above.  $pp(i,s)$  thus depends on the price to the private household in  $s$  for differentiated product  $i$  sourced from  $r$ ,  $pps(i,r,s)$ , the number of varieties of  $i$  produced in  $r$ ,  $n(i,r)$ , and the budget share of differentiated products from source  $r$  in the total expenditure on commodity  $i$ ,  $PTHETA(i,r,s)$ .

### *Private Household Varietal Index for Differentiated Products*

The private household's index measuring the availability of varieties of commodity  $i$ ,  $vp(i,s)$ , is given by

EQUATION PHLDVARIN

```
! Private Household Variety Index. !
(all,i,MCOMP_COMM)(all,s,REG)
vp(i,s) = sum{r,REG, PTHETA(i,r,s) * n(i,r)} ;
```

Equation PHLDVARIN shows that the varietal index rises with an increase in the number of varieties produced in a region or with an increase in the budget share of a region.

The utility structure for the consumption of differentiated products by the private household is shown in panel A of figure 3. The sub-utility derived from the consumption of commodity  $i$  by the private household in region  $s$ ,  $QP(i,s)$ , is a function of the quantities demanded by the private household of differentiated products of  $i$  sourced from region  $r$ ,  $QDFPS(i,r,s)$ , the number of differentiated products of  $i$  produced in  $r$ ,  $N(i,r)$ , and the elasticity of substitution between differentiated products of  $i$ ,  $SIGMA(i)$ . This substitution parameter is also the perceived demand elasticity of the firm producing the differentiated product. More about this parameter in the section on production structure.

### *Private Household's Derived Demand Undifferentiated Commodity*

The derived demand by the private household in region  $s$  for the undifferentiated commodity  $i$  sourced from  $r$ ,  $qhmps(i,r,s)$ , is given by

EQUATION PHLDSRCPC

```
! Private Household demand for Sourced Perfectly Competitive commodity !
(all,i,PCOMP_COMM)(all,r,REG)(all,s,REG)
qhmps(i,r,s) = DELTA(r,s) * {qp(i,s) + ESUBD(i) * [pp(i,s) - pps(i,s,s)]}
+ [1 - DELTA(r,s)] * {qpm(i,s) + ESUBM(i)
* [ppm(i,s) - pps(i,r,s)]} ;
```

Figure 3.A Utility Structure,  $i \in \text{MCOMP\_COMM}$ ,  $r, s \in \text{REG}$

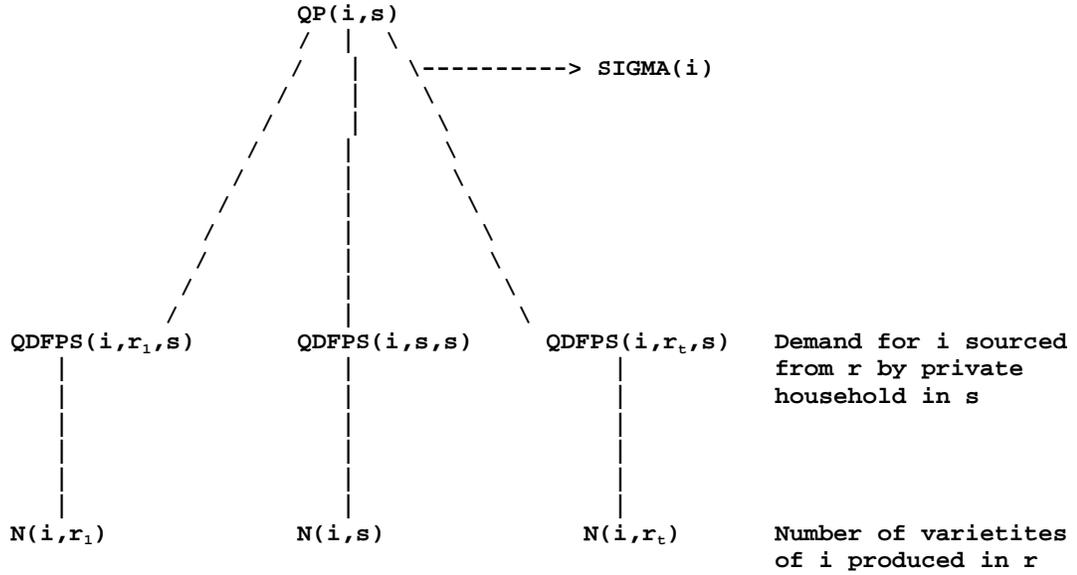
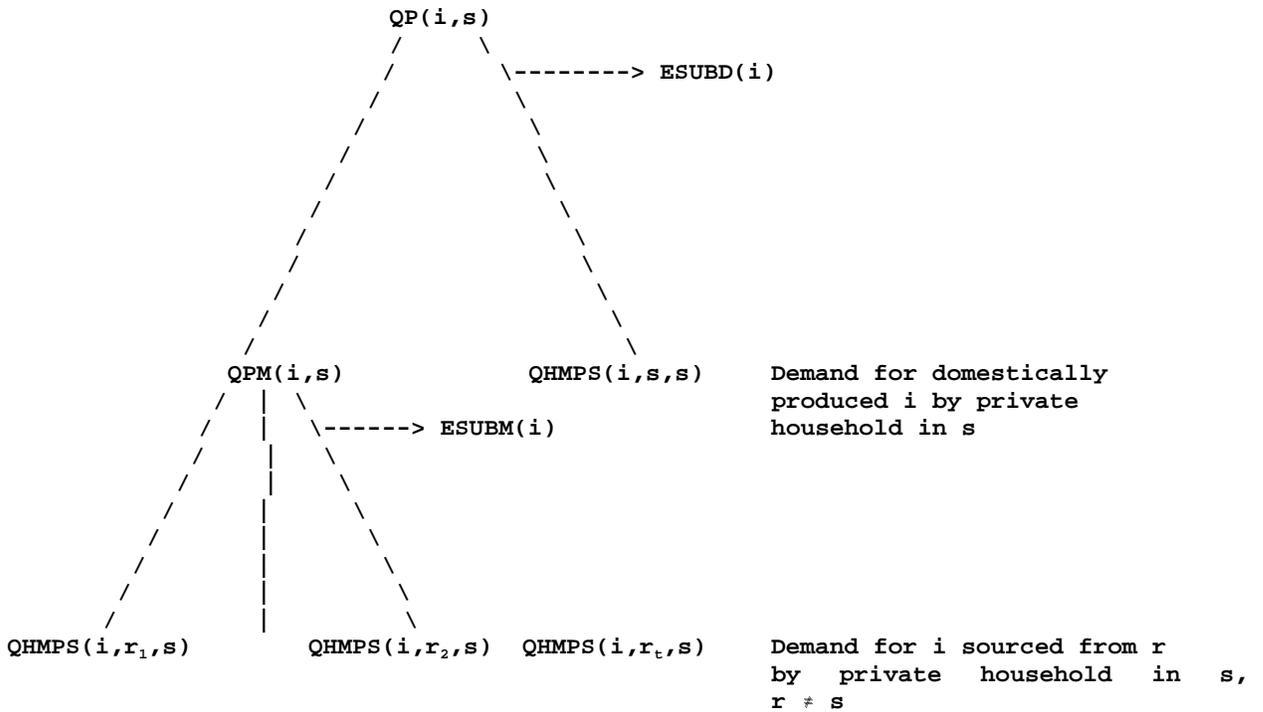


Figure 3.B Utility Structure,  $i \in \text{PCOMP\_COMM}$ ,  $r, s \in \text{REG}$



Equation PHLDSRCPC combines two different equations - IMPORTDEMAND( $i,r,s$ ) and PHHLDDOM( $i,s$ ) - in the standard model due to the sourcing of imports by agents and the consequent change in notation. This is made possible using the Kronecker delta,  $DELTA(r,s) = 1$  for  $r = s$ , and 0 otherwise, to specify  $qhmps(i,r,s)$ . The first term in this equation computes the derived demand for the commodity sourced from within the same region (which includes domestically produced commodity and intra-regional imports)<sup>10</sup>, and the second term computes the derived demand for sourced imports of commodity  $i$ .

Panel B of figure 3 shows the utility structure for the consumption of undifferentiated products by the private household. The sub-utility structure retains the Armington nesting and the import-domestic distinction is also maintained. Note that the sourced imports are assembled into a composite import by each agent in the economy, here the private household.

#### **4.1.2 Government Utility**

The derivation of government consumption equations is analogous to that for the private household. New equations - GOVSRCDF, GOVDFCOMPR, and GOVVARIN - are introduced to represent the demand for differentiated products analogous to the private household.

#### **4.1.3 Intermediate Input Demands**

Industry derived demand equations for intermediate inputs are four dimensional. The extra dimension for this agent indexes the industry making the input purchases. Firms also have heterogeneous needs. Production costs are lowered if the number of input varieties increases, at constant input prices. Considering the huge volume of trade in intermediate inputs, this approach is potentially important. Firms are thus treated symmetrically and the derivation of industry demand equations for intermediate inputs - INDSRCDF, INDDFCOMPR, and INDVARIN - is analogous to the derivation of the demand equations for the private household. The demand for sourced differentiated products of intermediate inputs in the production structure is shown in panel A of figure 4. Panel B in the same figure shows the demand for undifferentiated sourced intermediate inputs in the production structure. Note that it is the nature of the intermediate input that determines the specification of the derived demand equations regardless of the industry or agent demanding it.

## **4.2 Production Structure**

The production structure characterizing the firms in the monopolistically competitive industry is discussed below. We assume that the modeler can specify *a priori* which industries are perfectly competitive and which are monopolistically competitive. The monopolistically competitive industry is characterized by scale economies internal to the firm and firms producing differentiated products.

---

10. The treatment of intra-regional imports and domestic sales is discussed in the data requirements section.

Products are said to be *differentiated* when they are imperfect substitutes in demand, although produced by firms within the same industry. A point to note here is that if an industry  $j$  is monopolistically competitive in a region  $r$ , then so it is in all other regions modeled. Again, due to the lack of data on firms' sales, we are forced to assume that firms producing differentiated products in any region are symmetric. This allows us to use industry level data, cost shares, quantities etc. to describe the behavior of the representative firm. The representative firm is thus a scaled down version of the industry as a whole. The firm specific concepts include output per firm and fixed costs. For convenience, we will use the notation used in the model in deriving the markup, cost, and firm output equations.

#### 4.2.1 Firms and Varieties

A firm producing in the monopolistically competitive industry incurs fixed costs to produce a differentiated product. It is appealing to treat the fixed component of costs as arising due to research and development expenditures incurred by the firm to produce a new product as well as marketing and advertisement costs incurred on artificially differentiating an otherwise homogeneous product. There is a one-to-one mapping between varieties and firms. Due to this simplifying assumption, the number of varieties produced is equal to the number of firms in the industry. This falls out of our assumptions that (a) each firm produces only one product (the complexity of the model and calibration procedures is vastly increased if we have firms producing multiple varieties necessitating a different pricing rule for each variety) (SMITH and VENABLES), and (b) any given product is produced by no more than one firm (a firm about to enter the industry would always do better by producing a new product rather than duplicating an existing one). The key thing to note here is that firms producing differentiated products are trying to capture a "niche" in the product space.

#### 4.2.2 Fixed Costs, Markups and Output per Firm

Firms seeking to enter the monopolistically competitive industry require a set of fixed inputs comprising marketing and R&D outlays. In addition, variable inputs are required to produce the differentiated product. Variable inputs are combined according to a constant returns to scale technology. Average variable cost, AVC, is thus equal to the *constant* marginal cost of production, MC. Economies of scale (increasing returns to scale) in sales result due to the fixed component of total costs which is itself unaffected by production level. With free entry, unit output price,  $P$ , is equal to the total unit cost of production or the average total cost, ATC.

$$P = ATC = AVC + AFC \tag{7}$$

#### Markup Pricing

From (2), the own price elasticity of demand for a firm producing differentiated products is  $-\sigma(j)$  which is the negative of the elasticity of substitution between differentiated products of commodity  $j$ . Due to the Chamberlinian large group assumption, the cross price elasticity of demand is zero.  $-\sigma(j)$  is thus the perceived demand elasticity faced by a firm in the monopolistically competitive industry  $j$ .



Figure 4.A Intermediate Input Nest,  $j \in \text{PROD\_COMM}$ ,  $i \in \text{MCOMP\_COMM}$ ,  $r, s \in \text{REG}$

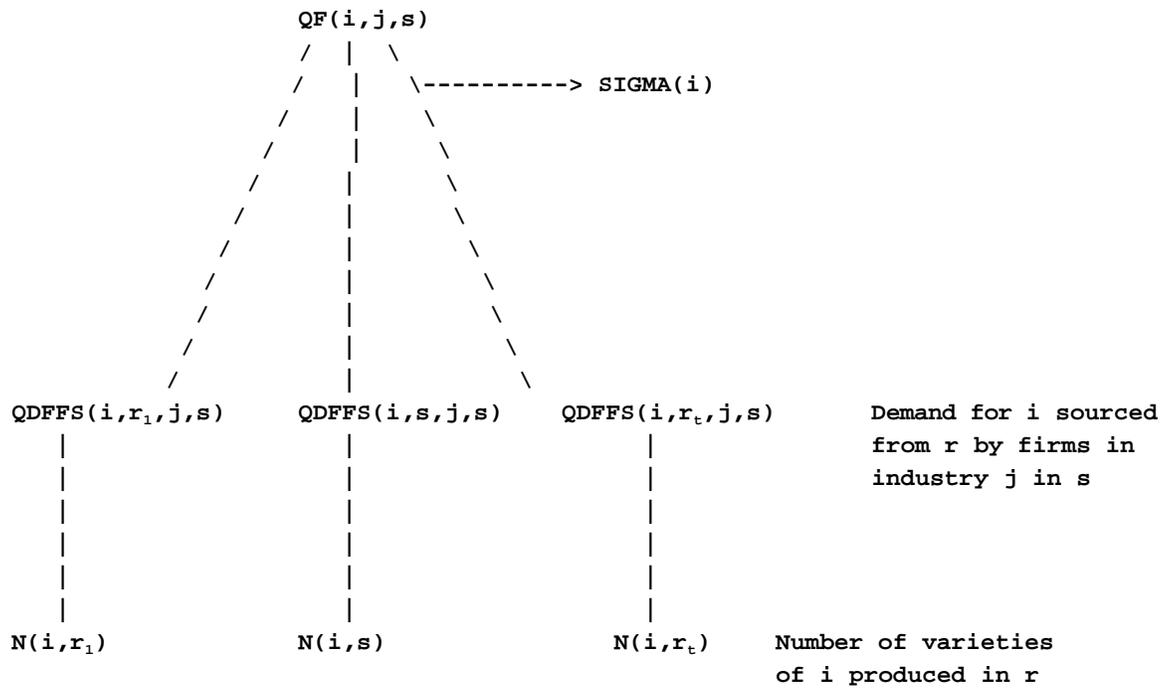
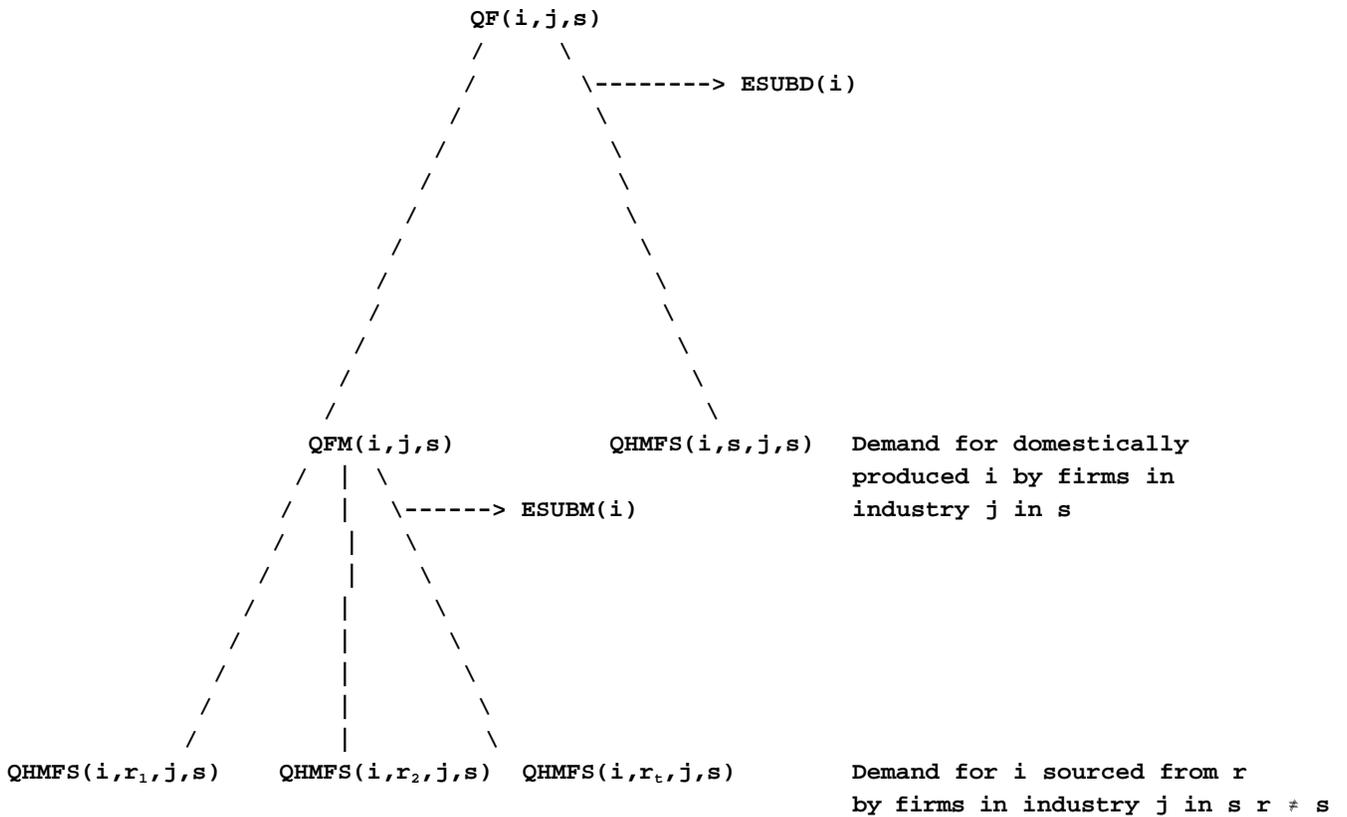


Figure 4.B Intermediate Input Nest,  $j \in \text{PROD\_COMM}$ ,  $i \in \text{PCOMP\_COMM}$ ,  $r, s \in \text{REG}$



A profit maximizing firm equalizes marginal revenue, MR, with marginal cost. The following equation gives the marginal revenue on the left hand side, where  $PS(j,s)$  gives the output price of a firm producing in the monopolistically competitive industry  $j$  in region  $s$ .

$$PS(j,s) * [1 - [1 / \sigma(j)]] = MC(j,s) = AVC(j,s) \quad (8)$$

$$MARKUP(j,s) = PS(j,s) / AVC(j,s) = [\sigma(j)/[\sigma(j) - 1]] > 1, \text{ for } \sigma(j) > 1 \quad (9)$$

The monopolistically competitive firm thus marks up its output price above average variable cost, by the amount of the markup,  $MARKUP(j,s)$ . The markup decreases with the elasticity of substitution,  $\sigma(j)$ . Note that the *markup is a constant* due to the assumption of a constant elasticity of substitution between varieties in consumption/usage, in the Dixit-Stiglitz tradition. Totally differentiating (9) yields,

$$\text{markup}(j,s) = ps(j,s) - avc(j,s) = 0 \quad (9')$$

According to (9'),  $PS(j,s)$  rises proportionately with  $AVC(j,s)$ . The markup equation determines the output level of the firm,  $QOF(j,s)$ , which, when multiplied by  $W(j,s)$ , yields industry output,  $QO(j,s)$ .  $QOF(j,s)$  is therefore, the complementary variable for the markup pricing equation. In order to be able to fix the firm's output level, we introduce a slack variable,  $mkupslack(j,s)$ , which absorbs the difference between price and variable costs. The associated MKUPPRICE equation is:

EQUATION MKUPPRICE

```
! Markup pricing (with constant markup). !
(all, j, MCOMP_COMM) (all, s, REG)
  ps(j, s) = avc(j, s) + mkupslack(j, s) ;
```

*Average Variable Cost, and Scale Constant Average Total Cost*

Owing to entry/exit, industry total revenue equals total costs in this model. Total costs are composed of intermediate and primary input costs. Value added output,  $VA(j,s)$ , is equal to the sum of primary input costs (see also figure 5).

$$VOA(j,s) = \text{sum}\{i, \text{TRAD\_COMM}, \text{VFA}(i,j,s)\} \quad (10)$$

$$\begin{aligned} & + \text{sum}\{i, \text{ENDW\_COMM}, \text{VFA}(i,j,s)\} \\ VOA(j,s) & = \text{sum}\{i, \text{TRAD\_COMM}, \text{VFA}(i,j,s)\} + VA(j,s) \end{aligned} \quad (11)$$

We make an important assumption that all fixed costs are made up of primary factor costs and that variable costs are composed of both primary factor and intermediate input costs. The fixed overhead associated with producing new products such as the salaries of engineers engaged in R & D activities

and of marketing staff involved in advertising and the capital required to build the R & D labs are thus primary factor costs.<sup>11</sup> Furthermore, we assume that fixed and variable value-added components have

---

1111. Brown studied the effects on firm output of tariffs in a monopolistically competitive industry, using different assumptions on factor intensity in this industry. She discusses the importance of this effect on average total costs.

Figure 5.A Total Output and Value-Added Nests,  $j \in \text{PCGDS\_COMM}$ ,  $s \in \text{REG}$

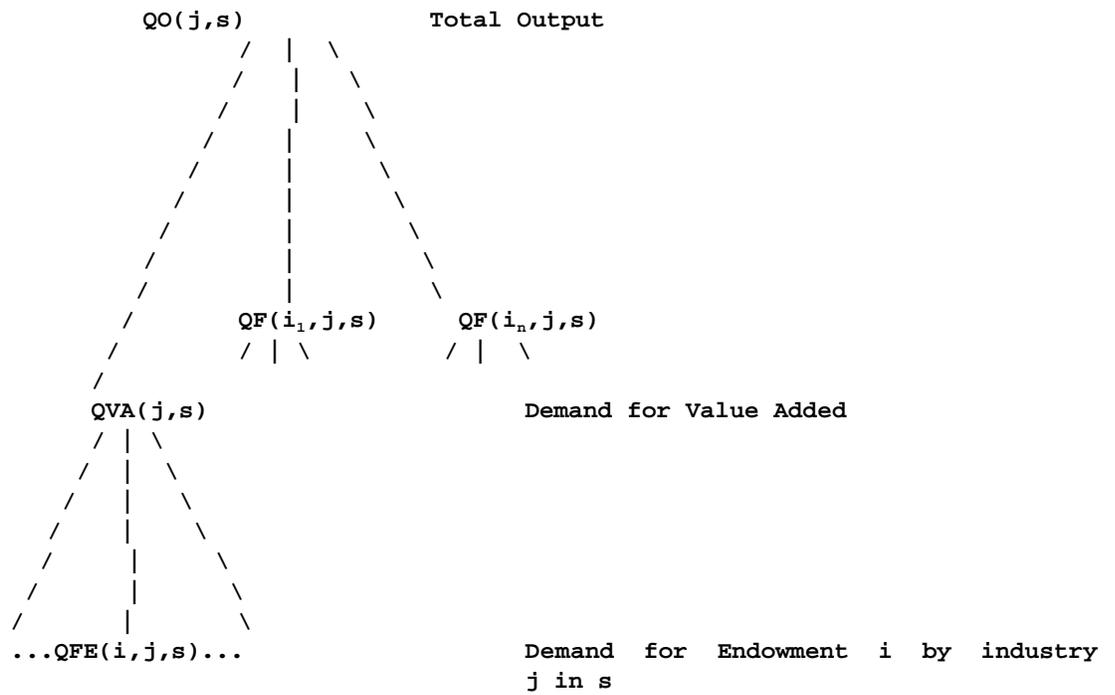
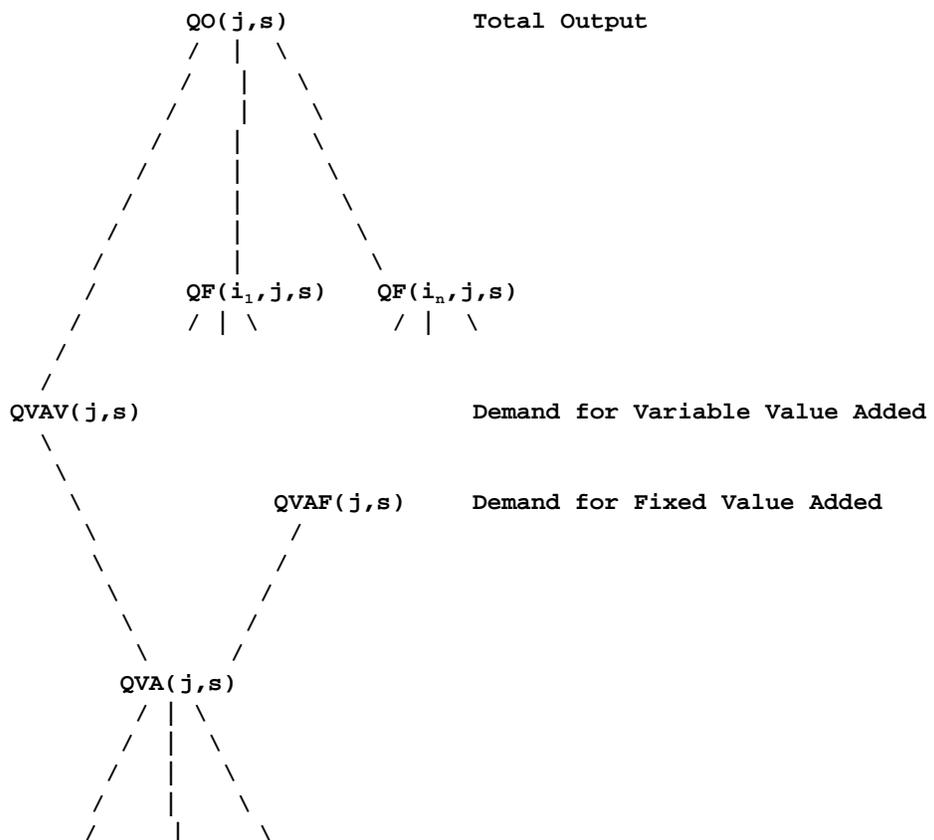


Figure 5.B Total Output and Value-Added Nests,  $j \in \text{MCOMP\_COMM}$ ,  $s \in \text{REG}$



...QFE(i,j,s)...

Demand for Endowment i by industry  
j in s

the same primary factor intensities. These two assumptions are open to criticism. Our contention is that the composition of fixed costs must be dictated by a knowledge of the industries in question where data is available. Varying capital-labor intensities across the fixed and variable cost components requires more industry-specific data to do the actual split. In other cases, it should be the discretion of the modeler to adopt some sensible *ad hoc* rule that is transparent and lends itself to easy analysis and simple interpretation.

Total value added costs are therefore split into two components - variable value added,  $VAV(j,s)$ , and fixed value added,  $VAF(j,s)$ . (See also figure 5.).

$$VOA(j,s) = \text{sum}\{i, \text{TRAD\_COMM}, VFA(i,j,s)\} + VAV(j,s) + VAF(j,s) \quad (12)$$

Variable costs,  $VC(j,s)$  and fixed costs,  $FC(j,s)$  are thus equal to

$$VC(j,s) = \text{sum}\{i, \text{TRAD\_COMM}, VFA(i,j,s)\} + VAV(j,s) \quad (13)$$

$$\begin{aligned} &= \text{sum}\{i, \text{TRAD\_COMM}, PF(i,j,s)*QF(i,j,s)\} + PVA(j,s)*QVAV(j,s) \\ FC(j,s) = VAF(j,s) &= PVA(j,s)*QVAF(j,s) \end{aligned} \quad (14)$$

$AVC(j,s)$  is determined by equation AVERAGEVC. This is derived by total differentiation of (13) and using the envelope theorem (when all quantity changes are zero in the neighborhood of an optimum).

$$VC(j,s)*avc(j,s) = \text{sum}\{i, \text{TRAD\_COMM}, VFA(i,j,s)*pf(i,j,s)\} + VAV(j,s)*pva(j,s) \quad (13')$$

EQUATION AVERAGEVC

```
! AVERAGE Variable Cost !
(all, j, MCOMP_COMM)(all, s, REG)
VC(j, s) * avc(j, s) = sum(i, TRAD_COMM, VFA(i, j, s) * pf(i, j, s)) +
VAV(j, s) * pva(j, s)
```

Fixed costs are invariant to firm output level and variable costs rise linearly with output, at constant input prices. In other words, average variable cost is invariant to output level but average fixed cost varies with output and with fixed input prices. From (13') we see that average variable cost is just a function of the variable input prices. However, changes in a firm's average total cost (which is output price as well, under zero profits) can arise from two sources: a.) change in firm's output,  $qof(j,s)$ , given constant prices of all inputs, and b.) change in one or more of the input prices, at constant firm output level.

We calculate the change in average total cost that is attributable only to changes in all input prices as the *scale constant average total cost*,  $scatc(j,s)$ , by holding the level of output per firm (scale) constant. To derive scale constant average total costs, we begin from the total cost equation in (11). Totally differentiating it and using the envelope theorem,

$$\begin{aligned} \text{VOA}(j,s)*\text{scatc}(j,s) &= \text{sum}\{i,\text{TRAD\_COMM}, \text{VFA}(i,j,s)*\text{pf}(i,j,s)\} \\ &+ \text{VA}(j,s)*\text{pva}(j,s) \end{aligned} \quad (11')$$

The variable  $\text{scatc}(j,s)$  is determined by equation SCLCONATC.

EQUATION SCLCONATC

```
! Average Total Cost at CONstant SCAle. !
(all, j, MCOMP_COMM)(all, s, REG)
  VOA(j,s) * scatc(j,s) = sum(i, TRAD_COMM, VFA(i, j, s) * pf(i, j, s))
                        + VA(j, s) * pva(j, s) ;
```

### 4.2.3 Industry Profits and Number of Firms

Perhaps the most distinguishing feature of the monopolistically competitive model is the *endogeneity* of the number of firms owing to the possibility of entry and exit of firms into/from the industry, in order to satisfy the zero profit condition. We follow the more common approach first taken by Harris in allowing the number of firms to be near continuous<sup>12</sup>.

#### Zero Profits

The change in output price must equal the change in average total cost to earn zero profits in equilibrium. The change in average total cost now includes changes in *all* arguments. To derive the zero profit equation we start from equation (11).

$$\begin{aligned} \text{VOA}(j,s) &= \text{sum}\{i,\text{TRAD\_COMM}, \text{VFA}(i,j,s)\} + \text{VA}(j,s) \\ \text{PS}(j,s)*\text{QO}(j,s) &= \text{sum}\{i,\text{TRAD\_COMM}, \text{PF}(i,j,s)*\text{QF}(i,j,s)\} + \text{PVA}(j,s)*\text{QVA}(j,s) \end{aligned}$$

Totally differentiating this (using the envelope theorem result) we have,

$$\begin{aligned} \text{VOA}(j,s)*\text{ps}(j,s) &= \text{sum}\{i,\text{TRAD\_COMM}, \text{VFA}(i,j,s)*\text{pf}(i,j,s)\} \\ &+ \text{VA}(j,s)*\text{pva}(j,s) - \text{VAF}(j,s)*\text{qof}(j,s) \end{aligned}$$

Substituting (11') on the right hand side, we have,

$$\text{VOA}(j,s)*\text{ps}(j,s) = \text{VOA}(j,s)*\text{scatc}(j,s) - \text{VAF}(j,s)*\text{qof}(j,s)$$

A slack variable,  $\text{zpislack}(j,s)$  is introduced in this equation to let firms earn positive profits in the short run<sup>13</sup>. Under certain closure assumptions we fix the number of firms in an industry by allowing  $\text{zpislack}(j,s)$  to be non-zero. This allows existing firms to earn positive profits when output expansion occurs.

---

1212. Mercenier and Schmitt reported extreme complexity in the computation of an integer number of firms in equilibrium.

1313. The medium and long run cases are characterized by entry/exit and firms make zero profits. However, the long run is not very meaningful in a comparative static model in the absence of factor accumulation.

EQUATION MZEROPROFITS

```
!ZERO pure PROFITS condition for firms in the Monopolistically
competitive industry!
(all, j, MCOMP_COMM)(all, s, REG)
  VOA(j, s) * ps(j, s) = VOA(j, s) * scatc(j, s) - VAF(j, s) * qof(j, s)
+ zpislack(j, s);
```

Note that both equations SCLCONATC and MZEROPROFITS are derived from the same total cost equation given in (11). There lies a critical difference between the two. In getting to SCLCONATC, we allow only the prices of all inputs to vary, holding the scale of the firm's operation constant, whereas in getting to MZEROPROFITS, we allow *all* arguments to vary.

The presence of fixed costs creates a wedge between price and marginal costs, which is equal to the markup. However, the subtle point to note is that price and average total cost will not move in tandem since average variable cost and average total cost are separated by a wedge that decreases (increases) with increase (decrease) in output per firm.

#### 4.2.4 Industry Output

Industry output in sector  $j$  of region  $s$ ,  $QO(j, s)$ , is the product of a number of symmetric firms,  $N(j, s)$ , and the output of the representative firm in the industry,  $QOF(j, s)$ .

$$QO(j, s) = N(j, s) * QOF(j, s)$$

Totally differentiating this we have,

$$qo(j, s) = n(j, s) + qof(j, s)$$

Industry output is computed by equation INDOUTPUT.

EQUATION INDOUTPUT

```
! INDUSTRY OUTPUT in the monopolistically competitive industry !
(all, j, MCOMP_COMM)(all, s, REG)
  qo(j, s) = qof(j, s) + n(j, s) ;
```

If industry output were to increase purely due to an increase in firm output level with no change in the number of firms, we have a decline in average total cost of production. On the other hand, if industry output does not change but there is exit of firms from the industry, output per firm rises and we have a *rationalization* of the industry due to the decline in the fixed costs and the average cost of production.

#### 4.2.5 Industry Derived Demands

The industry exhibits derived demands for both primary factors and intermediate inputs. The derived demand for intermediate inputs has already been covered in the sub-section on 'Intermediate Input

Demands'. The derived demand equations for primary factors in the output of composite value-added, ENDWDEMAND(j,s) are left unmodified. However, unlike the perfectly competitive industry, the monopolistically competitive firms now have derived demands for both fixed and variable inputs.

*Fixed, Variable, and Total Value Added*

The demand for fixed value-added is directly proportional to the number of firms. The more differentiated an industry is, the more resources are devoted to fixed costs. The existence of fixed costs in the monopolistically competitive industry is viewed as the "economic cost" of differentiating products. The economic benefit stems from the demand side due to a negative effect on the unit expenditure/cost function, as discussed in the sub-section on 'Demand Structure'. This is key to industry rationalization effects that will be explained in the results section.

EQUATION VAFDEMAND

```
! Monopolistically competitive industry DEMAND for Fixed
Value-Added!
(all,j,MCOMP_COMM)(all,s,REG)
  qvaf(j,s) = n(j,s) ;
```

The derived demand for the variable value-added (and intermediate) inputs is directly proportional to the industry output since we retain the Leontief production structure for the total output nest.

EQUATION VAVDEMAND

```
! monopolistically competitive industry DEMAND for Variable
Value-Added. !
(all,j,MCOMP_COMM)(all,s,REG)
  qvav(j,s) = qo(j,s) ;
```

The demand for total value-added in the industry producing differentiated products is a sum of the fixed and variable components.

EQUATION VATOT

```
! monopolistically competitive industry demand for total
Value-Added !
(all,j,MCOMP_COMM)(all,s,REG)
  qva(j,s) = [VAV(j,s)/VA(j,s)] * qvav(j,s) + [VAF(j,s)/VA(j,s)]
* qvaf(j,s) ;
```

## 5. *Data Requirements*

To implement the monopolistically competitive GTAP model, we have to work with a data base whose structure is fundamentally different than the standard GTAP data base. This requires that the new GTAP data base - the sets (GTAPSETS), the data (GTAPDATA), and the parameters (GTAPPARAM) files - be compatible with the monopolistically competitive model, GTAPMC, outlined in the previous section. We explain in this section the procedures used in transforming the standard GTAP data base. The associated files, in electronic form, are in the ZIP file associated with the paper (MCOMP.ZIP)

### 5.1 *Transformation of the Sets File*

Three new sets have to be added to the standard GTAPSETS file. We implement this by modifying the standard sets file, using the GEMPACK program MODHAR<sup>14</sup>. The new sets are:

MCOMP_COMM	Monopolistically <b>competitive commodities</b>
PCOMP_COMM	Perfectly <b>competitive commodities</b>
PCGDS_COMM	Perfectly competitive and <b>capital goods commodities</b>

All the elements of the set of tradable commodities, TRAD\_COMM, in the standard sets file are classified as either monopolistically competitive or perfectly competitive, such that MCOMP\_COMM and PCOMP\_COMM are *mutually exclusive sets*. All the perfectly competitive commodities and capital goods (investment), CGDS\_COMM, are classified as PCGDS\_COMM. It follows that,

$$\begin{aligned} \text{MCOMP\_COMM} \cup \text{PCOMP\_COMM} &= \text{TRAD\_COMM}, \text{ and} \\ \text{PCOMP\_COMM} \cup \text{CGDS\_COMM} &= \text{PCGDS\_COMM} \end{aligned}$$

### 5.2 *Transformation of the Parameters File*

In the parameters file, ESUBD(i) and ESUBM(i) should now index only the set PCOMP\_COMM. The array elements corresponding to the elements that are not in PCOMP\_COMM should be deleted. Care should be exercised to preserve the ordering of the elements in each array. The ordering should match the order in which the elements are defined. We introduce SIGMA(i), indexed over MCOMP\_COMM, as a new array in this file. The modified parameters file is best created by hand editing the standard parameters file.

### 5.3 *Transformation of the Data File*

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1414. An alternative to MODHAR would be to directly define the new sets in the model file. This requires hard coding the elements of these new sets in the TAB file. We prefer the MODHAR approach to keep track of changes independent of the model file.

The data program, DATMCV3.TAB, transforms the standard GTAP data base to a monopolistically competitive GTAP data base. The three main steps in this process are - sourcing of imports valued at market prices by agents, deriving the value of these flows at agents' prices, and deriving trade data. The data program copies the following data (stored under headers with the same name in the data file) from the input data set to the output data set without any changes - *EVOA*, *EVFA*, *VFM*, *VST*, *SAVE*, *VDEP*, *VKB*, *VAD*, *VTA*, *VMFA*, *VPU*, *VVER*, *VXT*, and *URVT*.

### **5.3.1 Sourcing of Imports Valued at Importer's Market Prices by Agents, $VPMS(i,r,s)$ , $VGMS(i,r,s)$ , and $VFMS(i,r,j,s)$**

The consumption of aggregate imports of  $i$  by the private household in  $s$  is given by  $VIPM(i,s)$  in the standard data base. We define the market share of source  $r$  in the total imports of  $i$  by region  $s$ ,  $MSHRS(i,r,s)$ , and use it to source out the aggregate imports consumed by the agent. This yields the consumption of imports of  $i$  from source  $r$  by the private household in region  $s$ ,  $VPMS(i,r,s)$ , valued at market prices.

When the source region,  $r$ , is the same as the destination region,  $s$ , we have the case of intra-regional imports, usually associated with a margin. But  $MSHRS(i,r,s)$  could be zero if  $r$  is a single-country GTAP region. When sourcing out to the same region, we must include domestic sales that are already sourced. Thus  $VPMS(i,s,s)$  includes  $VDPM(i,s)$ . Therefore, when  $r = s$ , domestic sales and intra-regional imports get blended. This procedure is extended to industry and government demands to yield  $VFMS(i,r,j,s)$  and  $VGMS(i,r,s)$ .

As a result of this transformation, agents' domestic and import demands are replaced by sourced demands, all valued at market prices. Thus,  $VPMS(i,r,s)$  replaces  $VDPM(i,s)$  and  $VIPM(i,s)$ ,  $VGMS(i,r,s)$  replaces  $VDGM(i,s)$  and  $VIGM(i,s)$ , and  $VFMS(i,r,j,s)$  replaces  $VDFM(i,j,s)$  and  $VIFM(i,j,s)$ .

### **5.3.2 Deriving the Value of the Sourced Demands at Agents' Prices, $VPAS(i,r,s)$ , $VGAS(i,r,s)$ , and $VFAS(i,r,j,s)$**

We define the power of the average (ad valorem) tax on total demand by an agent as the ratio of total demand valued at agents' prices to the total demand valued at market prices. Thus, the power of the average tax on the demand for  $i$  by the private household in  $s$ ,  $TP(i,s)$ , is the ratio of the sum of  $VIPA(i,s)$  and  $VDPA(i,s)$  to the sum of  $VIPM(i,s)$  and  $VDPM(i,s)$ . This average tax,  $TP(i,s)$ , multiplied by the sourced demand at market prices,  $VPMS(i,r,s)$ , yields the value of the sourced demand at agents' prices,  $VPAS(i,r,s)$ . This procedure is extended to government and industry demands to yield  $VGAS(i,r,s)$  and  $VFAS(i,r,j,s)$ .

Consequently, agents' domestic and import demands are replaced by sourced demands, all valued at agents' prices. Thus,  $VPAS(i,r,s)$  replaces  $VDPA(i,s)$  and  $VIPA(i,s)$ ,  $VGAS(i,r,s)$  replaces  $VDGA(i,s)$  and  $VIGA(i,s)$ , and  $VFAS(i,r,j,s)$  replaces  $VDFA(i,j,s)$  and  $VIFA(i,j,s)$ .

### 5.3.3 Deriving Trade Data

Basically, the bilateral trade matrices (fob and cif) do not undergo much transformation. "Exports" and "imports" are renamed as "sales" and "demands" respectively and the notation is changed to reflect this. Therefore,  $VXMD(i,r,s)$ ,  $VXWD(i,r,s)$ ,  $VIWS(i,r,s)$ , and  $VIMS(i,r,s)$  are renamed as  $VSMD(i,r,s)$ ,  $VSWD(i,r,s)$ ,  $VDWS(i,r,s)$ , and  $VDMS(i,r,s)$  respectively. For  $r = s$ , we need to take account of domestic sales as well, to ensure market equilibrium. The sum of domestic sales to all agents of commodity  $i$  in region  $s$ ,  $VDM(i,s)$ , evaluated at market prices, is therefore added to  $VSMD(i,s,s)$  and is carried forward through the system. This procedure is justified on the grounds that there are hardly any consumption/usage taxes on domestic demands. Where domestic sales are very large relative to intra regional trade, we note that this will dilute the distortion, if any, on intra regional trade. The protection data (export or import tax revenues/subsidy costs) by instrument do not undergo any change either.

## 5.4 Checks on the Transformed Data Base

DATMCV3 performs two checks on the new data set it creates - non-negativity of variable value added costs and balancing conditions.

Non negativity of Variable Value Added Costs

Fixed costs are made up of primary factor costs as alluded to earlier in the discussion on production structure. Value added costs,  $VA(j,s)$ , are split into variable,  $VAV(j,s)$ , and fixed value added,  $VAF(j,s)$  [see (12) above]. Fixed costs are thus equal to  $VAF(j,s)$  [see (14) above]. The perceived demand elasticity determines the share of fixed costs in total costs, in the Dixit-Stiglitz tradition. From (8) it follows that fixed costs are comprised of  $[1 / \sigma(j)]$  portion of total costs.

$$(all, i, TRAD\_COMM) (all, r, REG)$$

$$VAF(i, r) = VOA(i, r) * \{1 - [1 / MARKUP(i, r)]\} \quad ;$$

Once fixed costs are calibrated in this manner, variable value added is a residual in total value added.

$$(all, i, TRAD\_COMM) (all, r, REG)$$

$$VAV(i, r) = VA(i, r) - VAF(i, r) \quad ;$$

$$(all, i, TRAD\_COMM) (all, r, REG)$$

$$CHK\_VAVPOS(i, r) = IF\{ [VA(i, r) - VAF(i, r)] > 0, 1 \} \quad ;$$

The coefficient  $CHK\_VAVPOS(i,r)$  checks that fixed value added does not exceed total value added costs, for a given  $SIGMA(i)$ . This is a check to warn of those industries that might be characterized by an extremely low expenditure on value-added relative to total output. If this coefficient returns a "0" (which is its default) for a given  $SIGMA(i)$ , then it is problematic to classify that industry  $i$  in region  $r$ , as monopolistically competitive. Either it must be reclassified, or the value of  $SIGMA(i)$  must be increased. The data program thus serves as a useful precursor to the model.

### *Check Balancing Conditions of the New Data Set*

We also check the balancing conditions on the newly created data set after it gets written. These include the standard accounting relationships that are checked for the standard data base in the program GTAPCHK.TAB.

## **5.5 *Implementation of the GTAPMC Model***

Due to the increased size of the model, it becomes necessary to perform a condensation of the model before implementing it. This procedure effectively reduces the model size and reduces the computation time. The condensation operation must be modified depending on the closure chosen by the user.

## **6. *Policy Application***

The objective of this policy application using the monopolistically competitive formulation outlined above is to show the effects of introducing economies of scale and product differentiation into the GTAP model. We explain the strategic aggregation used, the experimental design adopted, and the results of the simulations conducted.

### **6.1 *Strategic Aggregation***

We illustrate the monopolistically competitive GTAP model with a simple, unilateral trade policy liberalization experiment. For this purpose, we have aggregated the version 3 GTAP global data base into 2 tradable commodities and 3 regions (2x3). We have kept the size of the model small to aid in exposition of the results.

The three regions are Japan (JPN), United States (USA), and the Rest of World (ROW). The two tradable commodities are manufacturing (MNFG) and non-manufacturing (nonMNFG). This aggregation is referred to as "3P-MC", where 3P denotes version 3 prerelease and MC denotes monopolistic competition. This aggregated 2x3 data set is then transformed into a form that is compatible with the monopolistically competitive model using the data transformation program, DATMCV3.TAB, discussed in the previous section and outlined in Appendix B. The sets file is also modified to include the new sets. We treat manufacturing as the monopolistically competitive sector and non-manufacturing as the perfectly competitive sector. Thus, MNFG  $\in$  MCOMP\_COMM and nonMNFG  $\in$  PCOMP\_COMM.

### **6.2 *Experimental Design***

We carry out a trade policy liberalization experiment whereby the USA region eliminates the antidumping duty on its manufacturing imports from JPN, leaving the tariffs in place. This policy change is implemented as a shock to the variable denoting the (percentage change in the) power of the overall import tariff on U.S. imports of Japanese manufacturing products,  $tms(MNFG,JPN,USA)$ . The initial wedge between the border price of imports,  $PCIF(MNFG,JPN,USA)$  and the import market price,  $PMS(MNFG,JPN,USA)$  is such that  $TMS(MNFG,JPN,USA) = 1.2189$ . This comprises a 17.8% antidumping duty and a 4.09% tariff. Therefore, elimination of the antidumping duties amount to a shock of  $tms(MNFG,JPN,USA) = [-0.178/1.2189] * 100 \% = -14.6\%$ . This same shock is used in all experiments.

In the base experiment we adopt the standard closure. In the remaining three experiments we adopt different closures. In the second experiment, we drop the markup pricing equation and fix the output per firm,  $qof(MNFG,r)$ . This is designed to isolate the industry rationalization effects occurring due to output expansion, also termed the *scale effect*. In the third experiment we drop the zero profit condition and fix the number of varieties,  $n(MNFG,r)$ . This isolates the industry rationalization effects occurring due to exit of firms. This is known as the *variety effect*. The final experiment is designed to mimic a perfectly competitive model and hence uses a slightly different model file. It eliminates both the variety and scale effects by fixing  $qof$  and eliminating  $n$ . However, the industry still makes zero profits. This is made possible by restoring the perfectly competitive structure to the derived demand and price index equations.

### 6.3 Results

The results of the four experiments are presented in table 1. The direct effect of the policy change is seen as a 12.58% reduction in the USA market price of manufacturing products sourced from Japan,  $pms(MNFG,JPN,USA)$ . This reduction is *smaller* than the reduction in the power of the import tariff because of a 2.37% rise in the *cif* price of imports. The market price of Japanese manufacturing products sold in ROW rise by 2.34%. The benefits from a decline in the price of global shipping services by 0.87% accrue to the US and ROW and not to Japan which experiences the full impact of the rise in the price of manufacturing products,  $pm(MNFG,JPN)$  by 2.53%. The change in the market price in the region of origin mirrors the change in the supply price, which will be explained via changes in the industry cost structure.

In order to meet the increased demand for Japanese manufactures, industry output of manufacturing products,  $qo(MNFG,JPN)$  increases by 0.53%. Of course, this displaces demand for US and ROW products in the US market, leading to a 0.39% decline in total US output and a 0.07% decline in the ROW region. The increased output in Japan is sold to the US, with the quantity of bilateral sales increasing by 89.07%. This dramatic increase causes the price of Japanese manufactures to rise, as seen above. This permits some of the increase in net supply to the US to be met by diverting sales from

the domestic and ROW markets, where the quantity of Japanese manufactures sold actually declines. Interestingly enough, despite decreased overall output, the US actually *increases its sales to Japan* and ROW by 32.02% and 9.53% respectively. This is due to the cost reduction experienced by US exporters relying on imported Japanese intermediate inputs. As expected, however, US firms sell less domestically, and this effect dominates, leading to an overall decline in output. In sum, the removal of US antidumping duties on Japanese imports does a great deal to stimulate intra-industry trade. Aggregate exports from Japan rising by  $qxw(MNFG,JPN) = 13.18\%$ , and US exports of manufactures increasing by 11.69%. Overall, the volume of global trade rises by 4.5%, with a 5.9% increase in manufactures and a 2% increase in world non-manufactures trade.

Up to this point, we have only discussed industry-wide variables. These are common across all sectors, whether or not they are monopolistically competitive. However, the key feature of the monopolistically competitive sectors is that they also incorporate information about changes in the number of varieties on offer, as well as changes in output per firm. Together, these explain the change in industry output. However, there is no reason why all three of these variables must move in the same direction. Indeed, we find this not to be the case in the present example. Our strategy for analysis will be as follows. We first explain the change in output per firm, as dictated by the changes in cost structure flowing from the removal of antidumping duties. We then observe the change in number of varieties/firms as the difference between the change in industry output and the change in output per firm. If output per firm rises by more than total output, then some firms must exit the industry in order to satisfy the equilibrium conditions.

In the standard closure,  $mkupslack(j,s)$  is exogenous, so that firms in the industry mark up marginal cost by to determine their optimal price. Hence, price and average variable cost move by the same proportion.

$$ps(j,s) = avc(j,s)$$

Also, we have from the zero profits equation,

$$ps(j,s) = scatc(j,s) - [ VAF(j,s) / VOA(j,s) ] * qof(j,s)$$

Thus, we can solve for  $qof(j,s)$  as,

$$qof(j,s) = [ VAF(j,s) / VOA(j,s) ]^{-1} * \{scatc(j,s) - avc(j,s)\}$$

Thus, per firm output changes as scale constant average total cost and average variable cost change at differing rates. An increase in the price of a primary factor or an intermediate input increases both  $scatc(.)$  and  $avc(.)$ , but *not* at the same rate.

In the case of Japan, removal of the antidumping duties faced by its products causes a general appreciation in the relative price of its non-tradeable, primary factors. This has the effect of bringing the economy back into external balance, following the increase in demand for its exports. It does so

by raising the cost of Japanese products. Of course, the extent to which various cost indices go up will depend on their primary factor intensity. Since fixed costs are assumed to be entirely made up of primary factors, and since firms have the option of importing intermediate inputs, average total costs tend to rise more than average variable costs. Specifically, in the case of Japanese manufactures, the variable  $scatc(.)$  rises faster than  $avc(.)$  because price index of value-added increases by 3.19% but that of intermediates by less than 2.5%. *Value-added matters more for  $scatc$  than  $avc$* . Thus,  $avc$  rises by 2.53% and  $scatc$  rises more, by 2.66%. As a consequence,  $qof(.)$  must increase. As it happens, the increase of  $qof = 0.74\%$  exceeds the increase in total Japanese manufacturing output, so firms exit the industry, and  $n = -0.22\%$ .

In the case of ROW,  $qof(.)$  falls for the opposite reason of that in Japan. For this region, there is a decline in demand for the manufacturing exports, as they are displaced by Japan in the US market. For this reason, primary factor prices must fall, relative to imported inputs. This causes  $scatc$  to fall more rapidly than  $avc$  so that  $qof$  also falls, in this case by  $-0.023\%$ . This reinforces a decline in the number of firms, so that overall output falls by  $-0.07\%$ .

In the US, the forces operating on  $qof(.)$  are more complex. The decline in demand for US products lowers the relative price of US primary factors, relative to tradeables *in general*. However, US manufacturers also import a significant share of their inputs from Japan. The cost of these inputs falls when antidumping duties are removed. As we can see from the results, this latter effect is strong enough to outweigh the real exchange rate effect and  $avc$  falls more than does  $scatc$  and  $qof$  rises. This is a somewhat striking effect, in light of the overall contraction in output in this sector. Therefore the number of firms must fall more than does aggregate output.

In order to understand the welfare effects of this experiment, it is useful to begin with the change in real GDP,  $qgdp$ . With fixed endowments, the only way to produce a greater volume of goods and services is to allocate existing resources more efficiently. This could involve moving factors from relatively lightly to heavily taxed sectors, or it could involve increasing output per firm in industries with scale economies. The latter effect is dominant in this experiment, with  $qgdp$  increasing in USA and JPN, and decreasing in ROW. Another determinant of welfare is what happens to the variety of goods available for consumption. Since the number of firms falls in all regions, this is clearly negative. Finally, it is useful to consider the impact which removal of antidumping duties has on the regional terms of trade. Here, there is a significant shift in favor of Japan, with the TOT declining for USA and ROW. The combination of these three factors gives rise to a decline in utility in USA and ROW, with utility increasing substantially in Japan ( $EV = \$41,585\text{mill.}$ ).

Finally, it is useful to consider the impact of fixing output per firm and varieties. These results are also reported in table 1. It is clear that the largest impact is generated by the change in output per firm. When this component is fixed (and the markup equation eliminated), we break the link between cost structure and optimal scale. Consequently, aggregate output in Japanese manufacturing rises by much less, and the welfare gain is moderated.