The marginal costs of funds in the VATTAGE model of Finland: a back of the envelope justification of the welfare effects of additional government revenue

by

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Abstract

This paper is part of a larger study on public finance aspects of population ageing in Finland.

We are using the VATTAGE model to calculate the welfare effects of various tax increases designed to improve the budgetary position of the Finnish government, that is we are calculating the marginal cost of funds (MCF) raised by different taxes. VATTAGE has been extended to include leisure and savings choice in the specification of household behaviour. This extension is necessary for useful MCF calculations.

The main contribution of this paper is the development of a back-of-the-envelope (BOTE) equation to guide the explanation of MCF results generated from a full-scale, dynamic CGE model.

1. Introduction

This paper is part of a larger study being carried out at VATT1 on public finance aspects of population ageing in Finland.

Coping with the effects of an ageing population is among the foremost fiscal challenges in most industrialised countries. The challenge is two-fold: the ageing of the population causes a rise in age-related expenses, which should be met with rising tax revenues; but it also leads to falling labour supply, which tends to have the effect of lowering tax revenues. However, as taxation also has an effect on the incentives to work and to invest, solving this two-fold problem calls for a comprehensive re-evaluation of the structure of the whole tax system.

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1 This the Government Institute of Economic Research located in Helsinki.
Finland provides an interesting case for studying the pressures of ageing on public finances, as the large post-war baby-boomer generations are some years older than in most other countries, and the country thus already has to cope with the effects of ageing. Currently, a large inter-departmental working group is considering alternatives for a major revision of the structure of taxation. Among the key targets of the working group is lowering the overall excess burden caused by taxation. The aim of the VATT project is to compare the costs of different taxes with the ultimate goal of providing measures that can be used in the evaluation of tax reforms and the effects of public spending.

We are using the VATTAGE model to calculate the welfare effects of various tax increases designed to improve the budgetary position of the Finnish government, that is we are calculating the marginal cost of funds (MCF) raised by different taxes. VATTAGE is a MONASH-style model\footnote{See Dixon and Rimmer (2002).} of Finland documented in Honkatukia (2009). However, unlike the original MONASH model, VATTAGE has been extended to include leisure and savings choice in the specification of household behaviour. This extension is necessary for useful MCF calculations because the essence of these calculations is tax-induced distortions in choices between consumption, leisure and savings. The VATTAGE database relies on disaggregated data on the economy and allows us to distinguish between commodity taxes and taxes on different types of factor incomes as well as taxes on income transfers.

The main contribution of this paper is the development of a back-of-the-envelope (BOTE) equation to guide the explanation of MCF results generated from a full-scale, dynamic CGE model. We apply the equation in a discussion of the welfare effects of raising extra revenue by an increase in the rate of taxation on labour income.

The paper is organised as follows. Section 2 introduces the concept of MCF and discusses its application in CGE modelling. Section 3 overviews the VATTAGE model and introduces the extensions necessary to deal with labour-supply and saving-aspects of taxation. Section 4 develops the back-of-the-envelope equation for explaining MCF results from a full-scale dynamic CGE model. Section 5 applies the back-of-the-envelope equation. Concluding remarks are in section 6.

2. The Concept of Marginal Cost of Funds

The large literature on the excess burden of taxation suggests that the costs of raising revenue differ across tax instruments. But as argued by Creedy (2000), measures of excess burden are often concerned with comparisons of distortionary and non-distortionary (such as lump-sum) tax systems. In practice, however, changes in the tax system involve collecting increased revenue with an existing, distorted tax system to finance increased public spending, for example. Whether the spending takes the form of
a public project or cash transfers, it is clear that it may affect the revenue collected from other taxes. The concept of marginal cost of funds, while related to excess burden considerations, is a broader concept that takes into account the type of effects that may arise when existing tax systems are modified.

MCF is usually defined as a money-metric measure of the loss of welfare resulting from the collection of extra revenue. Surprisingly, there seem to be several definitions for MCF. Here, we use the definition

\[ \text{MCF} = \left( \frac{-EV - \Delta R}{\Delta R} \right) \]

where EV is the equivalent variation resulting from the tax change and \( \Delta R \) is the change in revenue.

An example of the application of the MCF concept in the context of CGE modelling to measuring the potential effects of a tax change is Go et al. (2005), which studies the effects of a value-added tax reform in South Africa. They compare the effects of an increase in VAT and income taxes on different households, finding marked differences in MCF across household types. Crucially, they assume no changes in factor supplies, government spending or the government’s budget balance. Revenue is re-distributed in a lump-sum fashion, whence, as they point out, MCF is more of a measure of the overall inefficiency of the economy than that of just the tax system\(^3\).

Working on Argentina, Cicowiez et al. (2007) study the impacts of the taxes under different regulatory systems. They adopt similar assumptions to Go et al. and find MCFs ranging from 0.1 to 0.5 for income taxes, -0.1 to 0.13 for capital income taxes, and 0.127 to 0.206 for value added taxes.

In the CGE studies cited above, the determination of labour supply and saving is not at the centre of the analysis. This is somewhat surprising, because the theoretical literature on MCF considers the effects on labour-leisure choice extensively. As Dahlby (2009) shows, labour markets affect the costs of taxation in several ways. First, the effects of income taxes depend on the supply and demand elasticities for labour. They also depend on labour/capital ratios, as part of the burden of labour taxes is actually borne by capital, depending on whether the price of labour inputs is affected. The latter, in turn, depends on the labour market specification, that is, whether wages are competitive, or whether there are wage rigidities.

\(^3\) The variation of MCF Go et al use is apparently \( \text{MCF} = -EV / \Delta R \), which is sometimes referred to as marginal welfare cost of taxation (Creedy, 2000).
The empirical literature on MCF also suggests that labour supply matters. A study by Kleven and Kreiner (2006) estimates MCFs from income taxes for several European countries, finding MCFs ranging from 0 to 0.32 for a proportional increase in income taxes, when they consider labour supply elasticities for the intensive margin (that is, for hours worked). Tax reforms have been also studied econometrically in several other European countries, but to our knowledge, the concept has not been introduced in recent European CGE applications.

Our study for Finland is similar in spirit to the studies of South Africa and Argentina in the sense that it aims to use a CGE model for evaluating the costs of different tax regimes. Where it differs from the earlier studies is that endogenous labour/leisure choice and endogenous saving decisions are introduced, allowing us to consider effects that are excluded from the earlier analyses by the model formulation.

3. An outline of VATTAGE

VATTAGE is a dynamic, computable general equilibrium (CGE) model of the Finnish economy. It can be applied to study the effects of a wide range of economic policies. The VATTAGE database contains detailed information about commodity and income taxes as well as the expenditures and transfers of the public sector and thus covers most policy instruments available to the government.

VATTAGE is based on the MONASH-model developed at the Centre of Policy Studies, Monash University. MONASH-style models are used in countries ranging from China and South Africa to the United States and Australia (Dixon and Rimmer, 2002). In Europe, models based on MONASH have been developed for Denmark, Finland, and the Netherlands.

In VATTAGE, there are three types of inter-temporal links connecting the consecutive periods in the model: (1) accumulation of fixed capital; (2) accumulation of financial claims; and (3) lagged adjustment mechanisms, notably in labour markets. Different fiscal rules for balancing public sector budgets can also be specified.

To study MCF, we have extended the basic VATTAGE model by allowing households to make endogenous choices between leisure (or equivalently labour-supply), consumption of commodities and savings. To do this we have adopted the simplest approach, treating leisure and saving (reserved consumption) as two more “commodities” in household choice. The household’s problem has been amended to allow for the treatment of full income, that is, income inclusive of the value of leisure.

The consumer’s maximization problem is then

\[
\max \sum_{i=1}^{n} \beta_i \ln (X_i - \gamma_i) \tag{3.1}
\]
subject to

\[ \sum_{i=1}^{n} P_i \cdot X_i = Z \]  

where \( Z \) is full income defined by

\[ Z = Y + P_0 \cdot H \]

with \( H \) being potential hours of work, \( P_0 \) being the after-tax wage rate, and \( Y \) being non-wage income. The resulting demand functions for consumption, leisure, and saving are of the form

\[ X_i = \gamma_i + \frac{P_i}{P} \left( Z - \sum_{j=1}^{n} P_j \cdot \gamma_j \right), \quad i = 1, \ldots, n, \]  

where good 0 is leisure and one of the goods is saving. It is readily apparent that changes in the taxation of commodities, for example, will have an effect on both saving and labour supply.

In specifying the demand functions (4), we assume that the price for leisure is the nominal after tax wage rate. For saving, the choice of a price level is more complicated and is treated in section (4.6).

To implement the new formulation, we use demand elasticities with respect to full income for leisure and savings of 0.5 and 1.1.

The model also establishes a link from leisure demand to wage rigidities, in that the equation for wage adjustment can be formulated in the form that takes into account changes in labour supply.

As is apparent from the demand equations, consumption of commodities, leisure and saving are now interrelated, whereas in the original formulation of the VATTAGE model the labour market specification drives the reaction of employment to taxes, and saving is affected by taxation only to the extent that household disposable income changes.

4. A BOTE model for understanding VATTAGE results for the marginal cost of funds

4.1 Production

We assume that the BOTE economy produces one good with a production function given by
\begin{equation}
Y = F(K, N, M).
\end{equation}

where
Y is output; and
K, N and M are inputs of capital, labour and imports.

In the BOTE model we assume that all imports are used as intermediate inputs. This is a convenient assumption and not limiting.

Under cost-minimizing assumptions, K, N and M satisfy

\begin{equation}
P_Y * F_K = P_Q.
\end{equation}

\begin{equation}
P_Y * F_N = P_W.
\end{equation}

\begin{equation}
P_Y * F_M = P_M * T_M.
\end{equation}

where
F_K, F_N and F_M are the marginal products of capital, labour and imports (derivatives of F);
P_Y is the price of a unit of output;
P_Q is the pre-tax rental per unit of capital;
P_W is the c.i.f. price of a unit of imports; and
T_M is the power of the tariff on imports (one plus the rate).

Apart from the tariff on imports, we assume that there are no indirect taxes on inputs to production. This is realistic for Finland.

Totally differentiating (4.1) and substituting from (4.2) to (4.4) gives:

\begin{equation}
\frac{dY}{dY} = \frac{P_Q}{P_Y} * dK + \frac{P_W}{P_Y} * dN + \frac{P_M * T_M}{P_Y} * dM.
\end{equation}

where

\( dY, dK, dN \) and \( dM \) are the policy-induced deviations in output, capital, employment and imports.

4.2. Household behaviour in the BOTE model

We assume that in each year the household sector chooses its quantities of consumption (C), leisure (L) and reserved consumption (R) to maximize a utility function. By reserved consumption we mean the number of units of consumption that the household
sector expects to be able to finance in the future through this year’s saving. In VATTAGE the utility function is additive (Stone-Geary). Reflecting this, for the BOTE model we assume that households maximize:

\[ U_C(C) + U_L(L) + U_R(R) \]  
\[ \text{(4.6)} \]

subject to

\[ P_C C + P_R R = Z + \left( \frac{P_W}{T_W} \right) N \]  
\[ \text{(4.7)} \]

and

\[ L = H - B - N \]  
\[ \text{(4.8)} \]

where

- \( H \) is total hours available for work;
- \( B \) is hours in involuntary unemployment;
- \( N \) is hours of employment;
- \( P_W \) is the pre-tax wage rate;
- \( T_W \) is the power of the tax on labour income;
- \( Z \) is household non-labour income;
- \( P_R \) is the price of a unit of reserved consumption (to be discussed in subsection 2.6);
- \( P_C \) is the price of a unit of consumption.

In (4.8), we assume that involuntary unemployment is not leisure and consequently gives no utility.

The price of consumption is given by

\[ P_C = P_Y * T_C \]  
\[ \text{(4.9)} \]

where

- \( T_C \) is the power of the tax on consumption.

The first order conditions from problem (4.6) to (4.9) are:

\[ U'_C(C) = \lambda * P_Y * T_C \]  
\[ \text{(4.10)} \]

\[ U'_L(L) = \lambda * \frac{P_W}{T_W} \]  
\[ \text{(4.11)} \]

\[ U'_R(R) = \lambda * P_R \]  
\[ \text{(4.12)} \]
where
the superscript prime denotes derivative; and
λ is the Lagrangian multiplier which can be interpreted as the increase in utility that
the household would derive from an extra dollar of income (a unit increase in Z).

4.3. Economic welfare in the BOTE model

We assume that households in year t derive welfare from their consumption and leisure,
but not from their reserved consumption which will give them welfare in some future
year. Under this assumption, welfare (W) in any year is given by

\[ W = U_C(C) + U_L(L) \]  

(4.13)

and the deviation in welfare in the year caused by implementation of a policy is
calculated as

\[ \Delta W = U_C' \Delta C + U_L' \Delta L \]  

(4.14)

where
\( \Delta C \) and \( \Delta L \) are the policy-induced deviations in consumption and leisure.

Substituting from (4.10) and (4.11) into (4.14) gives

\[ \omega = P_C \Delta C + \frac{P_w}{T_w} \Delta L \]  

(4.15)

where
\[ \omega = \frac{dW}{\lambda} \]  

(4.16)

\( \omega \) is the monetary gift that would generate the same change in household welfare as that
generated by the policy.

4.4. National accounts identity and contribution of accumulated savings in the BOTE
model

We define national savings (S) by

\[ S = Y - \frac{P_M}{P_Y} M - (C + G) - V \]  

(4.17)

where
\( G \) is public consumption; and
\( V \) is net payments of interest and dividends to foreigners.
Equation (4.17) is expressed in real terms. To understand it we can think of output $Y$ as a number of bags of wheat. Similarly, $C$, $G$ and $V$ are numbers of bags of wheat that are: consumed by households; consumed by the government; and paid to foreign asset holders. $(P_M/P_Y)*M$ is the number of bags of wheat owed to foreigners as payment for imports. $S$ is then the increase in the nation’s assets (i.e. saving) calculated in bags of wheat.

In deviation form (4.17) can be written as

$$dS = dY - d\left(\frac{P_M}{P_Y}\right)*M - \frac{P_M}{P_Y}*dM - dC - dG - dV .$$

Then using (4.5) in (4.18) yields, after rearrangement

$$dS = \frac{P_W}{P_Y}*dN + \frac{P_M}{P_Y}*(T_m - 1)*dM - d\left(\frac{P_M}{P_Y}\right)*M - dC - dG + \left(\frac{P_Q}{P_Y}*dK - dV\right)$$

The last term in (4.19) is the contribution to national income in year $t$ of the policy-induced change in capital less the policy-induced increase in net dividends and interest payments to foreigners. If the policy is implemented in year 1 then this contribution in year $t$ arises from the accumulation ($dA$) of policy-induced increases in savings between years 1 and $t-1$. Under the simplifying assumption that investments, whether physical or financial, earn a constant exogenously given rate of return of $\rho$, we can write the last term in (4.19) as

$$\left(\frac{P_Q}{P_Y}*dK - dV\right) = \rho*dA$$

where

$dA$ for year $t$ is given by

$$dA = \sum_{\tau=1}^{t-1} dS_{\tau} \quad ,$$

or alternatively

$$dA_{\tau} = dA_{\tau-1} + dS_{\tau-1} \quad \text{for } \tau = 1, 2, \ldots, t$$

and

$$dA_0 = dS_0 = 0 \quad .$$
4.5. The effects of policies on welfare: an interpretable back of the envelope equation

Rearranging (4.19) and inserting (4.20) gives

\[
dC = \frac{P_w}{P_Y} dN + \frac{P_M}{P_Y} (T_m - 1) dM - d\left(\frac{P_M}{P_Y}\right) M - dG + \left(\rho dA - dS\right) .
\] (4.24)

From (4.8), with \(dH = 0\), we obtain

\[
dl = -dB - dN .
\] (4.25)

Now substituting from (4.24), (4.25) and (4.9) into (4.15) gives

\[
\omega = 0.01\left(\frac{P_w}{T_w} N\right) (T_w * T_c - 1) n \quad \text{employment/leisure effect}
\]
\[
+ 0.01 (P_M * M) * T_c * (T_m - 1) m \quad \text{import effect}
\]
\[
- 0.01 (P_M * M) * T_c * (P_m - P_Y) \quad \text{terms-of-trade effect}
\]
\[
- 0.01\left(\frac{P_w * B}{T_w}\right) b \quad \text{involuntary unemployment effect}
\]
\[
+ P_Y * T_c * (\rho dA - dS) \quad \text{inter-temporal trade-off effect}
\]
\[
- 0.01 P_Y * T_c * G * g \quad \text{government consumption effect}
\] (4.26)

where

\(n, m, p_M, p_Y, b\) and \(g\) are policy-induced percentage deviations in the variables denoted by the corresponding upper-case symbols.

Equation (4.26) is a BOTE decomposition of the effect on welfare in year \(t\) of policy changes implemented in years 1 to \(t\).

Employment leisure effect

The first term on the right hand side of (4.26) shows the welfare effect of policy-induced changes in employment. To understand this term we should think of unemployment, \(B\), and total hours, \(H\), as fixed: the effects on welfare of movements in \(B\) are calculated in the fourth term on the right hand side of (4.26) and, in the derivation of (4.26), \(H\) was explicitly held constant (that is not affected by the policy). With \(B\) and \(H\) fixed, increases in employment, \(N\), are at the cost of leisure, \(L\). However, because of tax effects an extra hour of work generates more welfare than an extra hour of leisure. Specifically, an hour of work generates \(P_w/P_Y\) extra units of output, see (4.3). The value to households of this extra output is \(P_c * P_w/P_Y\), that is \(T_c * P_w\). The value of an extra hour of leisure to households is \(P_w/T_w\). Thus the net effect on welfare of an extra hour of labour with a
corresponding loss of one hour of leisure is $T_C * P_W - P_W / T_W$. From here we can quickly arrive at the employment/leisure effect given in (4.26).

**Import effect**

The second term on the right hand side of (4.26) shows the welfare effect of policy-induced changes in imports. To understand this term we should start by thinking of $K$ and $N$ as constant: the effects of movements in these variables are accounted for in the first and fifth terms. With $K$ and $N$ constant, an extra unit of imports generates an increase in output of $P_M * T_M / P_Y$ units, see (4.4). But not all of this increase in output can be consumed. We should think of $S$, $G$, $V$ and $P_M / P_Y$ as being constant, with the effects of their movements accounted for in other terms. With these variables constant, (4.18) implies that a unit increase in imports must be financed by a payment to foreigners of $P_M / P_Y$ units of output. Thus, a unit increase in imports allows an increase in consumption of $P_M * T_M / P_Y - P_M / P_Y$ units. To find the value of this to consumers we multiply through by $P_C$. This leads to the second term on the right hand side of (4.26).

**Terms-of-trade effect**

If the terms of trade decreases by 1 per cent, that is if the c.i.f. price of imports increases by 1 per cent relative to the price of domestic output ($p_M - p_Y = 1$), then foreigners will need to receive 1 per cent more units of domestic output to pay them for any given level of imports. With $S$, $K$, $N$ and $M$ held constant, this means that payments to foreigners must increase by $0.01 * P_M * M / P_Y$ units of output, with a corresponding reduction in consumption. The value of this reduction in consumption can be found by multiplying through by $P_C$, leading to the third term on the right hand side of (4.26).

**Involuntary unemployment effect**

With $H$ and $N$ held constant an increase of one hour in involuntary unemployment generates a reduction in leisure of one hour. The value of an hour’s leisure to households is $P_W / T_W$. Thus, the effect on welfare of a $b$ per cent increase in unemployment is given by the fourth term on the right hand side of (4.26).

**Inter-temporal trade-off effect**

The policy-induced deviation in accumulated saving up to year $t$ allows consumption to increase by $p * dA$ units of output in year $t$ whereas extra saving in year $t$ requires a reduction in consumption of $dS$ units in year $t$. On translating these two effects (one positive and one negative) into monetary units by multiplying through by $P_C$, we obtain the inter-temporal trade-off effect which is the fifth term on the right hand side of (4.26).

In a simulation in which there is a sustained policy-induced increase in saving, the inter-temporal trade-off effect is negative in early years but positive in later years.
Government consumption effect

In deriving (4.26) we assumed that government expenditure does not generate welfare. Under this assumption an increase in government expenditure of one unit, with a corresponding reduction in consumption of one unit, generates a welfare loss with a monetary value of $P_C$. This leads to the last term on the right hand side of (4.26). It would be a relatively simple matter to modify VATTAGE and the BOTE model to allow public consumption to be welfare generating.

4.6. The price $P_R$ of reserved consumption

When households in year $t$ save a unit of output worth $P_Y$ dollars, we assume that they expect this to be transformed into a unit of capital. [For simplicity we assume that a unit of capital is created from one unit of output.] They expect the unit of capital to generate extra units of output in the future in line with the current marginal product of capital ($P_Q/P_Y$). They take into account the power of the tax on capital income ($T_K$) and recognize that their entitlement to the extra output is limited to $P_Q/(P_Y*T_K)$. They assume that the extra output that they are entitled to is translated into extra units of consumption (an increase in reserved consumption $R$) by multiplying by the ratio of output prices to consumption prices ($P_Y/P_C$). Putting all of this together, we have

$$P_Y \rightarrow \left( \frac{P_Q}{P_Y*T_K} \right) \ast \left\{ \frac{P_Y}{P_C} \right\} , \quad \text{(4.27)}$$

that is, saving of $P_Y$ in year $t$ yields a quantity of reserved consumption that is proportional to the expression on the right hand side of (4.27). Where the factor of proportionality is $\alpha$, (4.27) gives us the price of a unit of reserved consumption as

$$P_R = \left( \frac{1}{\alpha} \right) \ast \left\{ \frac{P_Y}{P_Q/(P_Y*T_K)} \ast (1/T_c) \right\} , \quad \text{(4.28)}$$

that is,

$$P_R = \left( \frac{1}{\alpha} \right) \ast \left\{ \frac{P_Y*T_c*T_K}{P_Q/P_Y} \right\} , \quad \text{(4.29)}$$

The role of $P_R$ in VATTAGE is clear from section 4.2. If $P_R$ increases relative to the prices of consumption ($P_C$) and leisure ($P_W/T_W$), then the household sector allocates more of its full income to consumption and leisure and less to reserved consumption, that is less to saving. Thus, under (4.29) saving is inhibited by increases in taxes on capital income and stimulated by increases in the marginal product of capital ($P_Q/P_Y$). An increase in taxes on consumption ($T_c$) generates an increase in $P_R$ but it also generates an
increase in PC. Thus it is not clear a priori how an increase in consumption taxes affects the savings/consumption ratio. However, it is clear that an increase in consumption taxes will increase leisure relative to both savings and consumption.

4.7. Notes on implementing (4.26) in the GEMPACK representation of VATTAGE

We added (4.26) to the GEMPACK code for VATTAGE to allow us to calculate BOTE estimates (ω) of the welfare effects of policy changes. By comparing these BOTE estimates with the VATTAGE results for equivalent and compensating variations (EV and CV) we can assess the adequacy of (4.26) as a stylized representation of how VATTAGE behaves.

This section explains how (4.26) was added to the GEMPACK code.

4.7.1. Variables in (4.26)

The variables in (4.26) are n, m, pM, g, b, pY, dS and dA. All of these variables should be thought of “up-and-down-the-page” deviations, that is differences or percentage differences in year t between policy and baseline values. The first five of these variables correspond directly to existing variables in VATTAGE.

The variable pY is the percentage change in the cost of a unit of output in a setting in which all imports are treated as intermediate inputs. Translating this setting to VATTAGE, we assume that total costs are given by:

\[ \text{COSTS} = \text{V1PUR(dom)} + V0IMP + V1PRIM \quad . \tag{4.30} \]

where

\[ \text{V1PUR(dom)} \text{ is the cost to industries of domestically produced intermediate inputs;} \]

\[ \text{V0IMP is the landed-duty-paid value of imports;} \text{ and} \]

\[ \text{V1PRIM is the value of primary factors}. \]

On the assumption that the movement in the price of domestically produced intermediate inputs is the same as the movement in the price of output, we can represent pY as:

\[ \text{COSTS} \cdot p_Y = \text{V1PUR(dom)} \cdot p_Y + V0IMP \cdot p_{slp} + V1PRIM \cdot plprim \quad , \tag{4.31} \]

that is,

\[ p_Y = \left( \frac{V0IMP}{V1PRIM + V0IMP} \right) \cdot p_{slp} + \left( \frac{V1PRIM}{V1PRIM + V0IMP} \right) \cdot plprim \tag{4.32} \]
where

\[ p_{ldp} \] is the percentage change in the index of landed-duty-paid prices of imports; and

\[ p_{1prim} \] is the percentage change in the average price of primary factors.

Both \( p_{ldp} \) and \( p_{1prim} \) are standard VATTAGE variables and so we were able to add (4.32) to the VATTAGE code to define \( p_Y \) in (4.26).

The variable \( dS \) is the BOTE concept of the policy-induced deviation in real national savings. We calculated this in VATTAGE as

\[
dS = 0.01 * \left( \frac{\text{NAT}_S\text{AV}}{P_Y} \right) * (\text{natsav} - p_Y)
\]  

(4.33)

where

\( \text{NAT}_S\text{AV} \) and \( \text{natsav} \) are a coefficient and variable already included in VATTAGE for national saving (private plus public saving); and

\( P_Y \) is a coefficient with a starting value of 1. It represents the level of the cost of a unit of output. The percentage movement in \( P_Y \) is \( p_Y \).

The variable \( dA \) is the policy-induced deviation in real accumulated national savings. This can be read initially at zero and then calculated according to

\[
dA = dA_B + dS_B
\]  

(4.34)

where

\( dA_B \) and \( dS_B \) in the year \( t \) computation are the year \( t-1 \) values for \( dA \) and \( dS \).

4.7.2. Coefficients in (4.26)

\( T_w \) can be evaluated according to

\[
T_w = \frac{\text{AGG}_LAB}{\text{AGG}_LAB - \text{TAX}_LAB - \text{PAYRTOT}}
\]  

(4.35)

where

\( \text{AGG}_LAB \) is the aggregate cost of employing labour;

\( \text{TAX}_LAB \) is income taxes paid by labour; and
PAYRTOT is total payroll taxes.

P\textsubscript{w}*N can be evaluated as AGGLAB.

T\textsubscript{C} can be evaluated according to:

$$T\textsubscript{C} = \frac{\text{AGGCON}}{\text{AGGCON} - \text{V3TAX}}.$$  \hspace{1cm} (4.36)

where

AGGCON is the aggregate private consumption expenditure; and

V3TAX is sales taxes on consumption.

P\textsubscript{M}*M can be evaluated as V0CIF, the c.i.f. value of imports.

T\textsubscript{M} can be evaluated according to:

$$T\textsubscript{M} = \frac{\text{V0IMP}}{\text{V0CIF}}.$$  \hspace{1cm} (4.37)

P\textsubscript{w}*B can be evaluated according to

$$P\textsubscript{w}*B = \left( \frac{\text{AGGLAB}}{\text{EMPLOY}} \right)^* \left( \text{LAB\_SUP} - \text{EMPLOY} \right)$$ \hspace{1cm} (4.38)

where

LAB\_SUP is the supply of labour [H-L or B+N in equation (4.8)]; and

EMPLOY is aggregate employment [ N in equation (4.8)].

As mentioned earlier P\textsubscript{Y} has an initial value of 1 in the base period and moves with p\textsubscript{Y}.

G can be evaluated as aggregate public expenditure, V5TOT in VATTAGE.

\(\rho\) can be set as the rental per unit of capital net of depreciation, that is,

$$\rho = \left( \frac{\text{V1CAP}}{\text{VCAP}} - \text{DEP\_AVE} \right)$$ \hspace{1cm} (4.39)

where

V1CAP is the aggregate rental on capital;
VCAP is the asset value of capital; and

DEP_AVE is the average across industries of the depreciation rates

We treated $\rho$ as a parameter evaluated from the base period data.

5. Test of equation (4.26)

We have run two simulations of the effects of an increase in income tax rates (taxes on labour income) designed to move the budget towards surplus by €1 billion after 4 years. In one simulation we assume that wages are flexible so that the tighter fiscal policy causes no change in the unemployment rate (and barely any change in employment). In the other simulation we assume that wage rates adjust in a sticky manner so that tighter fiscal policy causes a short-run increase in the unemployment rate and a corresponding reduction in employment.

The dashed lines in Figure 1 show results from the first simulation (flexible wages) for equivalent variation (EV) and compensating variation (CV) expressed as a percentage of the value of private consumption plus leisure. These results are almost indistinguishable. The figure also shows as a dashed line the BOTE result for the welfare deviation calculated via equation (4.26). The BOTE result is a reasonable approximation to the VATTAGE result. However, it overestimates the VATTAGE result for the reduction in welfare in all years.

What does VATTAGE know that is left out of the BOTE calculation?

We noticed that VATTAGE implies that tighter fiscal policy would reduce consumer prices (P3TOT) quite sharply relative to what we have called the price of output ($P_Y$). The explanation is that the rental on housing falls sharply relative to the prices of all other products. This reflects the capital intensity of the provision of housing services. In the short run there can be almost no change in the provision of these services. Thus a reduction in demand must be counteracted by a reduction in relative price. What is the relevance of the reduction in consumer prices relative to output prices ($p3tot-p_Y < 0$)? An assumption in the BOTE calculation is that there is only one good. This is equivalent to assuming that there are no changes in relative prices. We suspected that the reduction in consumer prices relative to $P_Y$ implied by VATTAGE was the relevant ingredient that VATTAGE knows about but is left out of BOTE.

To check this we repeated the first simulation with one change: we exogenized the price of housing services [$p3_s(C_{701_2})$] so that it was not affected by the policy. To do this we endogenized a taste-change variable [$a3con(C_{701_2})$]. Without a reduction in the price of housing services, the policy-induced deviations in P3TOT and $P_Y$ were similar. In accordance with our suspicion, elimination of the change in relative prices allowed
BOTE to closely reproduce CV and EV: notice the three solid lines (marked fixed housing price) in Figure 1 are close together.

Figure 2 repeats the exercise for the second simulation, the sticky wage case. Again we find that once the reduction in the price of housing services is eliminated, the BOTE calculation gives a close approximation to the CV and EV results from VATTAGE.

Why does (4.26) overestimate the welfare loss in simulations in which the price of housing services falls sharply. The answer is that when the price of housing services falls sharply, the decline in \( P_Y \) (which includes the price of housing services) is greater than the decline in the price of exports (which doesn’t include the price of housing services). In terms of (4.26), \( p_M - p_Y \) overestimates the decline in the terms of trade.

Figures 3 to 6 show BOTE decompositions of welfare results. In all cases, the inter-temporal effect is the most important contributor to the welfare deviations associated with fiscal tightening undertaken by a tax on labour. The terms of trade effect is the second most important contributor, although as indicated above it is somewhat overestimated. With sticky wages (Figures 5 and 6), there are short run reductions in employment and increases in unemployment. Consequently, Figure 5 and 6 show significant negative employment/leisure and unemployment effects in the short run.

### 6. Conclusions

We have extended the VATTAGE model to include labour/leisure and savings/consumption choice. We have done this to facilitate the application of the model in the estimation of the marginal cost of raising funds from different types of taxes. So far we have applied the extended version of VATTAGE to look at the marginal cost of funds raised from an increase in taxes on labour income. We are planning to carry out similar calculations for taxes on capital income and taxes on consumption.

The main focus of this paper was the development of a back-of-the-envelope equation for explaining results from a full-scale dynamic CGE model on the welfare effects of the imposition of different taxes. Back-of-the-envelope equations are necessary for figuring out how a CGE model works and what has been taken into account in any set of results and what has been left out.

### References


Figure 1. Percentage welfare deviations: flexible wages

Figure 2. Percentage welfare deviations: sticky wages
**Figure 3. Decomposition of percentage welfare deviations: flexible wages and price for housing services**

**Figure 4. Decomposition of percentage welfare deviations: flexible wages and fixed price for housing services**
Figure 5. Decomposition of percentage welfare deviations: sticky wages and flexible price for housing services

Figure 6. Decomposition of percentage welfare deviations: sticky wages and fixed price for housing services