PORTFOLIO ELASTICITIES AND THE EVOLUTION OF ENDOGENOUS CURRENT ACCOUNT BALANCES IN A CGE MODEL WITH INTERNATIONAL FINANCIAL ASSETS

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by

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Introduction

In this paper, we examine the role of portfolio elasticities in the PEP-w-t-F model (Lemelin *et al*. 2010, 2011), a worldwide recursive dynamic CGE model with international financial assets. Our model endogenizes current account balances by making explicit the international capital flows which balance the current account. Capital flows are constrained by supply-and-demand equilibrium conditions on the market for international debt securities, as determined by portfolio managers' optimizing behavior. Each country is a single agent, owning a portfolio of assets which constitutes its net wealth. Wealth consists of financial wealth and physical assets (ownership titles to productive capital or claims on the flow of income generated by it). Financial wealth is made up of international assets and liabilities (debt). The asset-liability structure of the financial portfolio is endogenous, and it is possible for a country-agent to have negative net financial assets. Borrowing is limited, however, by the willingness of other country-agents to lend, following their own portfolio choices, and by the competition from other borrowing countries.

The cumulative consequences of capital flows on the international investment positions (IIPs) of countries define the constraints under which portfolio choices are made. Interaction between the financial and the real economy may lead, for example, to endogenous sign reversals in current account balances, a feature which is absent in most models. Nonetheless, experiments with the model have shown that the real economy remains the principal driving force determining the simulated evolution of the world economy.

Obviously, simulation results depend on the choice of portfolio elasticities. Indeed, not all combinations of elasticity values allow for a feasible solution, and linear interpolations of feasible combinations are not necessarily feasible (the set of feasible combinations is not convex). The analyses presented aim to explore the range of feasible combinations of elasticities, to better understand their complex interactions, and to examine the sensitivity of results to variations in the elasticities.

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4 A preliminary version of this paper was presented under the title « Applied CGE modelling with financial assets » (with Véronique Robichaud and Bernard Decaluwé), at the conference « Applied General Equilibrium Modelling : the Walras Legacy 100 Years on », Université de Pau et des Pays de l'Adour, Pau, France, December 16-17, 2010.

1. Model

1.1 OUTLINE

Our model is the last-born of a family of standard models developed by our team for the Poverty and Economic Policy (PEP) Research Network. It is calibrated using GTAP data, and Lane and Milesi-Ferretti's (2006) « External Wealth of Nations, Mark II » data on country international investment positions (IIP).

In its current version, the model has four industries, each producing one commodity: the primary sector, industry, services, and public administration. There are two kinds of labor, skilled and unskilled, and three other production factors : capital, land, and natural resources. The countries of the world are aggregated into 14 regions:

Africa South of the Sahara (AfriSS)
China (incl. Hong Kong) (ChinaHK)
European Union Fifteen (EU15)[5]
Rest of the EU (before 2007) (EUplus)
India
Japan
Middle-East and North Africa (MENA)
Latin American developing countries (LAmDev)
Asia-Pacific developing countries (AsPaDev)
Rest of Latin America (RoLAm)
Rest of Asia (RoAsia)
Rest of the world (RoW)
Transition economies (Transit)
United States of America (USA)

The model is admittedly quite aggregated for the time being. The moderate size of result files allows a detailed examination and facilitates diagnostics during model development. The GAMS code would nonetheless allow to apply a finer classification, both in terms of industries/commodities and in terms of regions. However, the current level of aggregation is not so coarse as to render results uninteresting.

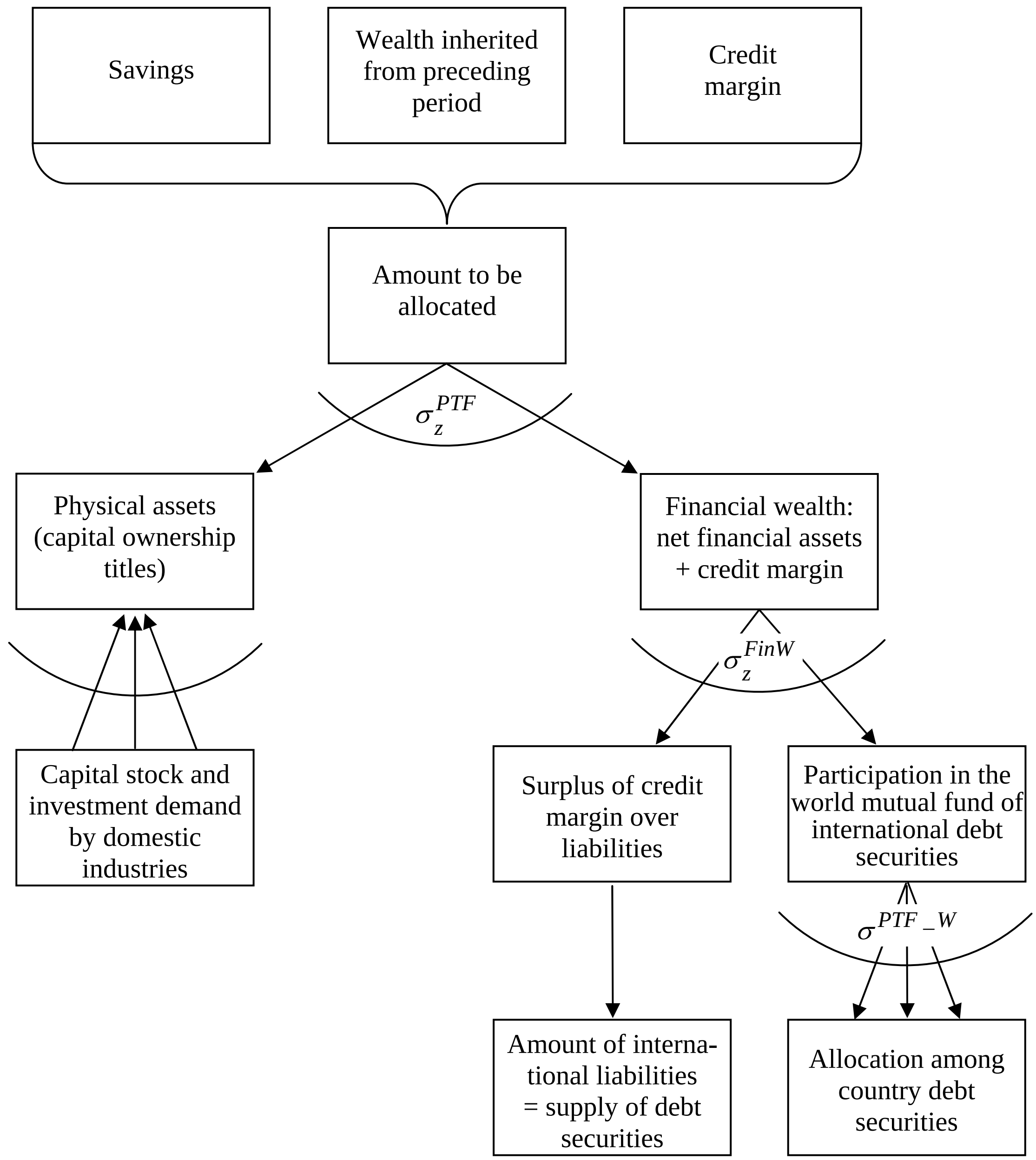
Most CGE trade models fix current account balances exogenously, in accordance with the widely accepted view that trade policy may influence trade flows, but that current accounts are constrained by symmetric capital and financial account balances, on which trade policy has little effect (see WTO, 2007). Our model was developed to make explicit the international capital flows which must take place to balance the current account implications of the simulated trade flows, and to compute the cumulative consequences of such capital flows on the international investment positions (IIP) of countries. In our model, current account balances and their capital and financial account counterparts are endogenous.

5 Member countries in 1995.

Each country or group of countries is modeled as a single agent. Every country-agent owns a portfolio of assets which constitutes its net wealth. There are two types of wealth : financial wealth, and physical assets. The latter are ownership titles to productive capital or, equivalently, claims on the flow of income generated by the capital. The financial component of the portfolio is made up of assets and liabilities (debt). The asset-liability structure of the financial portfolio is endogenous, and it is possible for a country-agent to have negative net financial assets (liabilities in excess of assets). The possibility of borrowing is limited, however, by the willingness of other country-agents to lend, which reflects their own portfolio choices, and by the competition from other borrowing countries. The allocation of capital among countries and industries is determined by an investment supply and demand equilibrating mechanism. Investment supply is the demand for new physical capital ownership titles resulting from the wealth allocation process, while investment demand is a constant elasticity function of Tobin's *q* in the Jung-Thorbecke (2001) style.

Country-agent wealth allocation behavior is represented in a three-tier portfolio management model (Lemelin, 2008, 2009), as illustrated in Figure 1 below. At every stage of the portfolio model, the manager maximizes a CES aggregate of the capitalized values of different assets, following an approach derived from the Decaluwé-Souissi portfolio model (Decaluwé *et al.*, 1993; Souissi, 1994; Souissi and Decaluwé, 1997; Lemelin, 2008 and 2009). The CES target function reflects the assumption that the capitalized values of different assets are not perfect substitutes in the eyes of the portfolio manager. This can be motivated by risk aversion and a desire to diversify the portfolio. It must be recognized, however, that this is an *ad hoc* representation of risk-averse behavior. A detailed presentation of the portfolio management model is found in Appendix B of Lemelin *et al.* (2011). There is one feature of the model, however, which must be understood for the rest of this paper to be comprehensible, the so-called « credit margins »; this is the object of the next section.

Figure 1. – Portfolio allocation in the PEP-w-t model

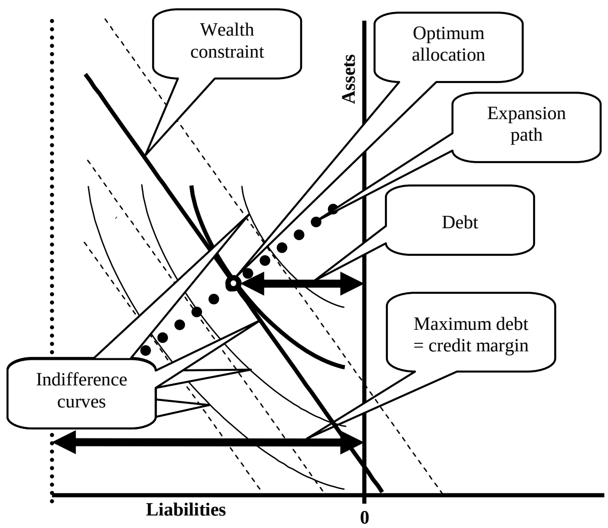


1.2 CREDIT MARGINS AND ASSET-LIABILITY STRUCTURES

Some countries have negative net international financial wealth (they are net international debtors). This can happen even if a country has positive savings: for example, if investment expenditures have been in excess of domestic savings (which is tantamount to saying the country has been running a current account deficit). But a portfolio model can hardly represent the allocation of a negative amount of net financial wealth.

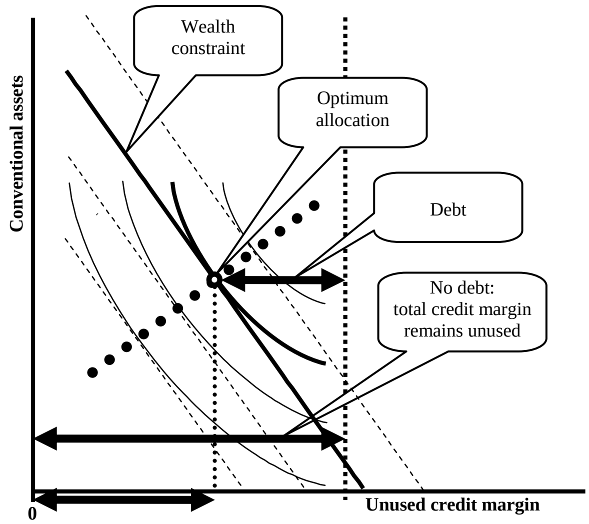
Moreover, a region's net financial position (assets, minus liabilities) is obviously far more volatile than the underlying stocks of assets and liabilities, making a net position variable often unstable, and therefore difficult to model. So it would seem desirable to model assets and liabilities as distinct variables. But, once again, how can the model accommodate negative asset values (liabilities)? A geometric solution is illustrated in Figure 2 below.

Figure 2 – Asset-liability structure



The credit margin is a device to adapt the portfolio model to handle liabilities. Negative liability variables are converted to positive variables by a simple shift of origin: rather than choosing the positive amount of assets and the negative amount of liabilities, subject to net financial wealth, the regional portfolio manager chooses the positive amount of assets, and the – also positive – amount of unused credit margin (i.e. borrowing possibilities), subject to a constraint on the positive total of net financial wealth redefined to include the credit margin[6]. This is represented in Figure 3.

Figure 3 – Unused credit margin as an asset



Financial wealth is allocated between (1) a composite asset, and (2) the surplus of the credit margin over liabilities, i.e. remaining borrowing capacity: debt reduction increases the maximum amount of new loans that could be contracted, and further borrowing reduces it.

$$FinW_{z,t} = Fasset_{z,t} + \left[CrdtMg_{z,t} - Debt_{z,t}\right] \quad [01]$$

where

$CrdtMg_{z,t}$ is region *z*'s international credit margin in period *t*, expressed in the international currency

$Debt_{z,t}$ is the value of region *z*'s international financial liabilities in period *t*, expressed in the international currency

$Fasset_{z,t}$ is the value of region *z*'s international financial assets in period *t*, expressed in the international currency

$FinW_{z,t}$ is the value of region *z*'s net financial wealth, including its credit margin, in period *t*, expressed in the international currency

6 This construct is analogous to the « full income » concept in consumer theory with endogenous labor supply, where labor time is what is left after subtracting leisure from the consumer's time budget, and the price of leisure is the opportunity cost of foregone labor income.

The rate of return on the composite asset is an aggregate of the interest rates on country debt securities, while the rate of return on debt reduction is the opportunity cost of debt, i.e. the interest rate on the region's own debt.

The credit margin has been arbitrarily set in the first period to equal the sum of assets and liabilities (in other words, each country is allowed to increase its debt by the amount of its assets).

$$CrdtMg_z^O = Fasset_z^O + Debt_z^O \qquad [02]$$

It is then assumed to grow slightly faster than the world sum of equity wealth (for an explanation, see Lemelin *et al*., 2011, Appendix B6). In spite of its simplicity, we believe that this formulation is not totally out of line with the reality of international financial markets: countries do have a total borrowing capacity, which usually exceeds their current level of debt. It is nonetheless recognized that, contrary to our specification, real total borrowing capacity is a fuzzy number, not an exact value. It is also acknowledged that the level of credit margins influences the values of the calibrated portfolio parameters, and consequently agents' behavior in the model.

1.3 LINKS OF THE FINANCIAL TO THE REAL ECONOMY MODEL

The portfolio allocation model can be viewed as a micro-founded bridge linking the current account, through the financial and capital account, to savings and wealth, and ultimately to investment.

We begin with the balance of payments identity, which states that the sum of the current account and capital and financial account balances is zero[7]. It follows that the counterpart of a current account *surplus* is a *negative* balance (a *deficit*) in the capital and financial account, which implies the *acquisition* of assets and/or a *reduction* in liabilities, and consequently an *increase* in the net international investment position of a country; symmetrically, the counterpart of a current account *deficit* is a *positive* balance in the capital and financial account, and a *decrease* in the net international investment position. Broadly speaking, the acquisition of assets can be assimilated to « lending », and the incurrence of liabilities to « borrowing ». Indeed, in the current version of the model, there is a single type of international asset, labeled « debt securities », which subsume all other assets, including portfolio equity and direct investments[8].

So net new international lending/borrowing by a region in each period must be equal to its current account surplus/deficit. And since the acquisition of assets and the incurrence of liabilities are determined

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7 In accordance with the *Balance of Payments and International Investment Position Manual* (International Monetary Fund, 2009a), the acquisition of an asset is registered as a *debit* in the capital and financial account, while incurring a liability is registered as a *credit*.

8 The next step in model development will be to introduce foreign direct investment.

in the portfolio allocation model, current accounts are endogenously constrained, regulated by regional agents' willingness to lend and borrow. It is noteworthy that, contrary to what characterizes many other specifications, current account sign reversals are possible in PEP-w-t-F.

Of course, international lending and borrowing imply international income flows reflecting returns received on assets and paid on liabilities. To be consistent with the portfolio model, the current account balance in PEP-w-t-F includes income received on assets owned abroad, minus income paid on liabilities owned by foreigners:

$$CAB_{z,t} = TBAL_{z,t} + e_{z,t}\left(RFass_{z,t} - RDebt_{z,t}\right) \quad [03]$$

where

$CAB_{z,t}$: Current account balance of region *z*

$e_{z,t}$: Exchange rate; price of international currency in terms of region *z*'s local currency

$RDebt_{z,t}$: Income paid in period *t* by region *z* on its international financial liabilities, expressed in the international currency

$RFass_{z,t}$: Region *z*'s income in period *t* from international financial assets (shares in the world mutual fund of debt securities), expressed in the international currency

$TBAL_{z,t}$: Region *z*'s trade balance (exports, minus imports)

Net international financial income is not included, however, in household or government income. For lack of systematic information on the balance sheets of households and governments, there is no obvious way of allocating the income[9]. So each region's net international financial income is treated as an additional source of domestic savings. This implies, of course, that an increase in net international financial income has no direct effect on consumption.

We now turn to the savings-investment equilibrium. From the financial model perspective, funds dedicated to current investment expenditures are the difference between the desired value of physical assets (capital ownership) in the portfolio, given the optimal allocation of wealth, and the current value of capital inherited from the past. The latter is determined under an arbitrage-based pricing mechanism (Lemelin *et al*., 2011, Appendix B3). As demonstrated in Appendix B of Lemelin *et al*. (2011), this implies that gross investment expenditures be equal to gross savings (which include capital consumption

9 In an early version of the model, net international financial income was distributed in each region between households and the government according to their respective savings. But this distribution rule led to incongruities when the sum of household and government savings was not of the same sign as the region's net international financial income.

allowances, i.e. depreciation), plus net income from foreign assets, minus the current account balance (i.e. minus net new lending abroad) :

$$IT_{z,t} = SH_{z,t} + SG_{z,t} + DEP_{z,t} + e_{z,t}\left(RFass_{z,t} - RDebt_{z,t}\right) - CAB_{z,t} \qquad [04]$$

where

$DEP_{z,t}$: Amount of depreciation (capital consumption allowance) in region *z*

$IT_{z,t}$: Total investment expenditures in region *z*

$SG_{z,t}$: Government savings in region *z*

$SH_{z,t}$: Household savings in region *z*

Equation [04] is the traditional savings-investment equilibrium constraint, modified for the inclusion of net income from international financial assets. But here, it is not merely an *a priori* macroeconomic accounting identity; rather, it summarizes portfolio allocation behavior.

Alternatively, in view of equation [03], total investment expenditures are

$$IT_{z,t} = SH_{z,t} + SG_{z,t} + DEP_{z,t} - TBAL_{z,t} \qquad [05]$$

which is the formulation usually found in CGE models[10].

2. Reference scenario and macroeconomic closure

In the reference scenario, it is assumed that the economies of countries or groups of countries grow smoothly, based on demographic projections and the expected growth of GDP per capita. To take these trends into account, PEP-w-t-F defines a GDP index which is specific to each region and increases at a rate which may vary from one period to the next. This index is used to update the values of variables and parameters that are expected to grow with population and GDP per capita. These consist mostly of exogenously determined volume variables: labor supply, real government expenditures, and public investments.

In constructing the GDP index, we assume that demographic year-to-year projections are subject to less uncertainty than GDP projections, but we nonetheless consider the trend in GDP projections over several years to be acceptably reliable for the purposes of defining the reference scenario. So the GDP index

10 Matters would be slightly different if it were not assumed that net income from foreign assets is entirely dedicated to savings. In that case, equation [05] would be

$$IT_{z,t} = SH_{z,t} + SG_{z,t} + DEP_{z,t} - TBAL_{z,t} - \left(1 - \alpha_z\right)e_{z,t}\left(RFass_{z,t} - RDebt_{z,t}\right)$$

where α_z is the fraction of net income from foreign assets that goes to savings.

grows at a rate that combines the year-to-year rate of population growth with the average rate of GDP per capita growth over the simulation horizon.

More specifically, the growth rate from *t* to *t*+1 is $\left(1+g_{z,t}^{pop}\right)\left(1+\overline{g_{z}^{GDP.p.c}}\right)-1$, where $g_{z,t}^{pop}$ is the rate of growth of population in region *z* from *t* to *t*+1, and $\overline{g_{z}^{GDP.p.c}}$ is the average of year-to-year growth rates of per capita GDP over the simulation horizon[11].

The macroeconomic closure is neoclassical, with full employment of labor and capital, savings-driven investments, and fixed regional GDP deflators. The overall numeraire is the exchange rate of the reference region (USA). The regional GDP deflators can be interpreted as regional numeraires, and the endogenous exchange rates then become real exchange rates.

3. Portfolio elasticities

As was previously indicated, the portfolio allocation model is based on successive maximizations of CES aggregates of the capitalized values of different assets. But the parametrization of the model poses a challenge. The portfolio elasticities of substitution are free parameters, and it is unlikely that there exist any empirical evidence on which to base our choice of values. There are several reasons for that. One is that the assets that are combined at various stages of the portfolio model are aggregates of highly diverse components. Another is that the CES specification is an *ad hoc* representation of risk-averse diversification behavior, while no variable in the model actually represents risk. Finally, the model operates on some abstract constructs which do not exist in reality, such as the world mutual fund of debt securities, or the asset which consists of the surplus of the credit margin over liabilities (section 1.2).

So the elasticity parameters are more or less arbitrary, and the model specification makes some degree of arbitrariness inevitable. For lack of an alternative approach, the choice is between fixing current account balances exogenously (and having their cumulative effects on net international investment positions determined mechanically), or trying to select a set of elasticity values that will yield « realistic » results. The first step in that direction is to examine how the model's behavior changes with different elasticities. This is the course we take here. We shall explore a range of elasticity values, evaluate model sensitivity to changes in elasticities, and trace the transmission channels through which their effects are felt.

Of course, if elasticities didn't matter, things would be easier. But they do. First of all, model results vary according to the choice of elasticities. Moreover, in a simulation run of the reference scenario stretched

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[11] The authors thank David Laborde for having shared GDP forecast and population data used in the MIRAGE model. The demographic growth rates are from the ILO projections of population. The annual growth of GDP in each region is taken from the World Bank projections.

out to an indefinite horizon, PEP-w-t-F eventually crashes[12]. In a nutshell, there comes a point when there is no financial model solution that is compatible with the real model, and the model solver exits with a declaration of « Infeasible solution ». And how soon the model crashes depends critically on the choice of elasticities. Interactions are complex, however, and it is difficult to determine feasible combinations of elasticities.

3.1 EXPLORING FEASIBLE ELASTICITY COMBINATIONS

There are three [sets of] substitutions elasticities in the portfolio model:

σ_z^{PTF} : Elasticity of substitution between financial wealth and physical assets in regional wealth

σ_z^{FinW} : Elasticity of substitution between financial assets and the surplus of the credit margin over liabilities in the regional asset-liability structure

σ^{PTF_W} : Elasticity of substitution in the world portfolio between regional debt securities

Restricting our attention to combinations in which the first two elasticities are equal across regions *z*, we begin our investigation with a preliminary exploration of the feasible domain. But, as mentioned above, every set of elasticities we tried eventually makes the model crash. Therefore, « feasible » cannot be defined as feasible forever. The 2030 horizon is one landmark: while the reference scenario runs from 2004 to 2030 with some elasticity combinations, we have found none that can go beyond. Combinations that run to 2030 are in the neighborhood of $\sigma_z^{PTF} = 38$, $\sigma_z^{FinW} = 33$, and $\sigma^{PTF\_W} = 33$. But the feasible set of elasticity combinations that run beyond 2022 is quite narrow. So, for exploration purposes, our operational definition is that a combination of elasticities is said to be « feasible » if it allows the reference scenario to run from 2004 to 2022.

Before looking at numbers, to put things in proper perspective, a preliminary remark is in order: a high elasticity of substitution between assets does not imply such a high elasticity of demand for individual assets with respect to rates of return. The interested reader will find an illustration of this in the appendix.

It would have required rather enormous resources to conduct a systematic exploration of the three dimensional space of elasticities, in order to identify which combinations were feasible according to the above definition. Such an effort was beyond our means, and, moreover, was probably not appropriate at this stage of model development. So exploring feasible elasticity combinations was very much a matter of

[12] , We would conjecture that this is true of any recursive dynamic model following a path that is not a balanced growth path.

intuitive trial-and-error, and, even if it was not systematic, it nonetheless involved a lot of computation and looking through output to see for how many periods the model could solve with each combination.

Figures 4-7 present a sample of results. The combinations in Figure 5 are made up of one high and two low elasticities; that shows that the elasticities needn't be close to one another. Figures 6 and 7 address the issue of convexity: are weighted averages of two feasible combinations of elasticities also feasible? Based on a sample of several points along the continuum between [1,0] and [0,1] weights, we are confident that yes, weighted averages of 5-19-19 and 45-100-100 are feasible (Figure 6). In contrast, the same technique confirmed that between combinations 5-19-19 and 127-100-100, there is a gap of infeasible combinations (Figure 7). Therefore, the set of feasible elasticity combinations is not convex in general.

Figure 4. – Feasible elasticity combinations (1)

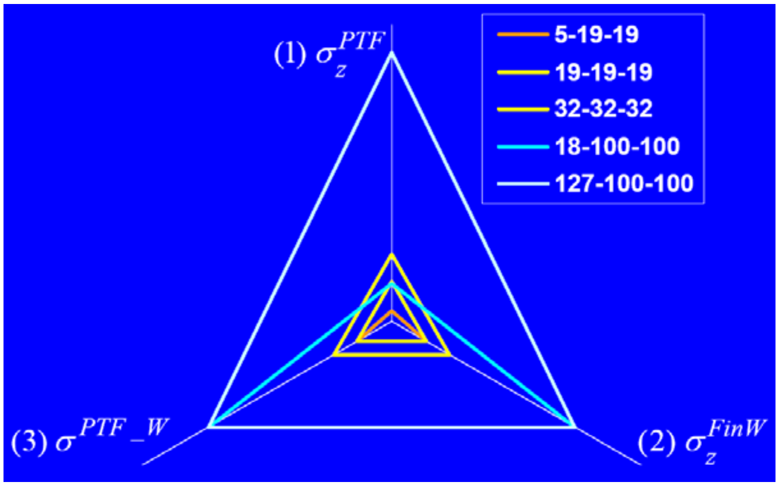


Figure 5. – Feasible elasticity combinations (2)

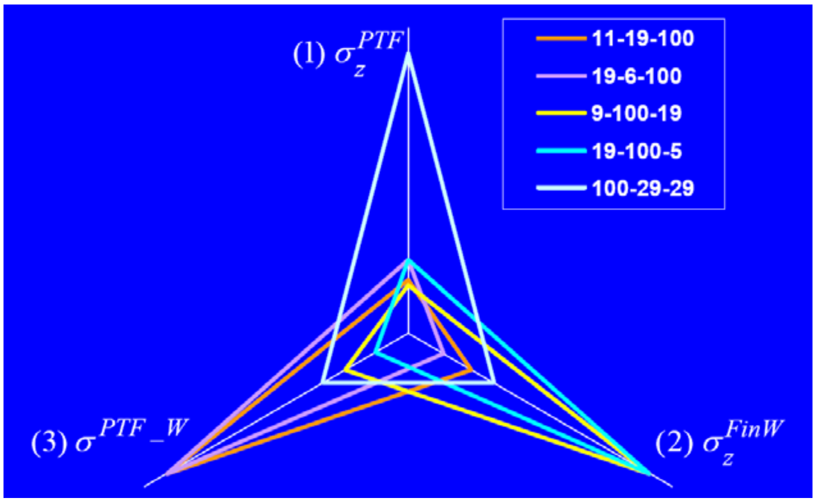


Figure 6. – Feasible elasticity combinations (3)

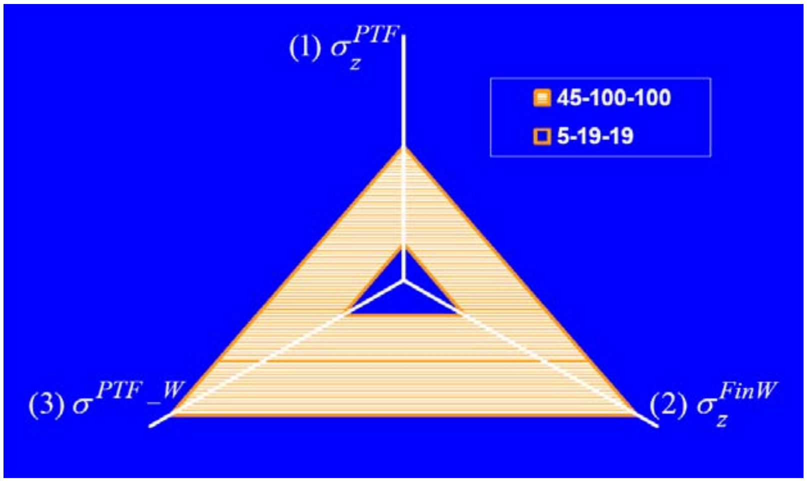
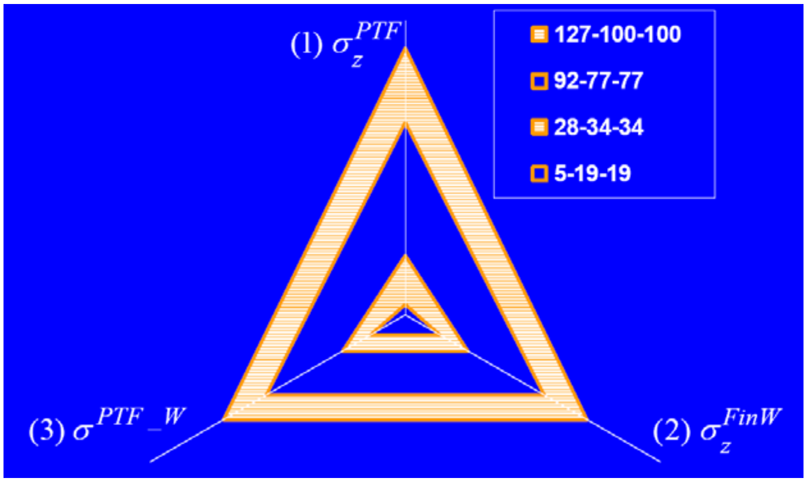


Figure 7. – Feasible elasticity combinations (4)



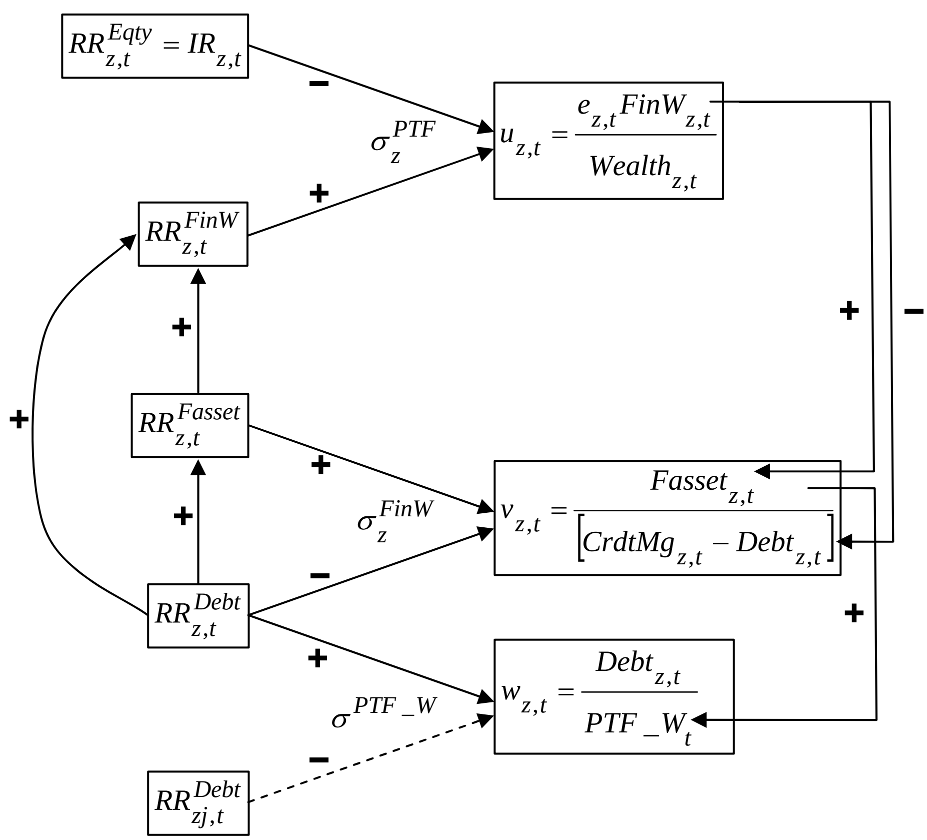
3.2 INTERACTIONS BETWEEN ELASTICITIES

We now turn to the longer horizon (2030), and try to understand what makes the model crash. More specifically, by examining key variables, we have identified some of the mechanisms that lead to a situation where there is no solution of the financial model that is compatible with the real model. This, we found out, has to do with the interactions between elasticities.

Figure 8 attempts to summarize the interactions between elasticities. Reading from bottom to top, the share $w_{z,t}$ of region z's debt securities in the world portfolio increases with rate of return $RR_{z,t}^{Debt}$ it pays, and falls with the rate of return $RR_{zj,t}^{Debt}$ offered by other regions; and the higher is elasticity of substitution σ^{PTF_W}, the more the share changes in response to a change in the rate of return. But

$RR_{z,t}^{Debt}$ also intervenes in the asset-liability structure of the region: the optimal asset-liability ratio of a region increases with the rate of return offered on its debt securities, and with the rate of return it receives on its international financial assets $RR_{z,t}^{Fasset}$; and the higher is elasticity of substitution σ_z^{FinW}, the more sensitive the asset-liability ratio to changes in the rates of return. The (notional) rate of return $RR_{z,t}^{FinW}$ on financial wealth increases both with the rate of return $RR_{z,t}^{Fasset}$ on international assets, and with the implicit rate of return $RR_{z,t}^{Debt}$ on debt reduction (see section 1.2)[13].

Figure 8. – Interaction between elasticities



13 It may seem paradoxical that $RR_{z,t}^{Fasset}$ increases with $RR_{z,t}^{Debt}$. This is due to the fact that the international financial assets owned by a region are shares in the world mutual fund of debt securities, which include its own debt. In the case of the U.S., whose liabilities represent 21%, and assets 17% of the world portfolio, this feature reduces the moderating impact that would be expected of a rise in the rate of return on its liabilities. The same is true of the EU15, but its current account is closer to balance.

Finally, the share of financial wealth in total wealth increases with the rate of return $RR_{z,t}^{FinW}$ on financial wealth, and falls with the rate of return $RR_{z,t}^{Eqty}$ on physical assets (capital), which is equal to interest rate $IR_{z,t}$ (for details, see Appendix B in Lemelin *et al*., 2011).

Now let's see how these interactions come into play in simulations. For instance, China maintains a large positive trade balance, which implies it accumulates international financial wealth. Its debt tends to decrease. In order for the world portfolio of debt securities to adjust, the rate of return on Chinese debt must fall[14], and it must fall all the more if elasticity σ^{PTF_W} is weak. If σ^{PTF_W} is too small, then the rate of return on Chinese debt threatens to turn negative, and the model crashes. But the crash may be prevented if, rather than paying off most of its debt, China chooses to apply leverage: reduce its debt less to acquire more assets. This will happen as the cost of debt $RR_{z,t}^{Debt}$ falls for China, provided elasticity σ_{z}^{FinW} is sufficiently high. So, in the presence of an accumulating country like China, there is a trade-off between the elasticity of substitution between international debt securities σ^{PTF_W} and the elasticity of substitution σ_{z}^{FinW} of the asset-liability structure. In contrast to China, the USA runs a large trade deficit and accumulates debt. In order for the world portfolio of debt securities to adjust, the rate of return on U.S. debt must rise, and it must rise all the more if elasticity σ^{PTF_W} is weak. But that does not conflict with nonnegativity constraints on return rates. So, with a proper choice of elasticities, the model can handle the dwindling Chinese debt and, even more easily, the ballooning U.S. debt.

But the « EUplus »[15] region poses a more difficult problem. As a group, these countries run a trade deficit that is much deeper proportionally than the USA's. As the region accumulates debt, the share of the world portfolio allocated to it grows, and so the rate of return on its debt must increase relative to others, and more so if σ^{PTF_W} is weak. Now, recall that the rate of return on international assets is the same for all regions participating in the world mutual fund of debt securities, and it is not much affected by EUplus debt. This implies that the rate of return on EUplus debt increases relative to the rate of return on its international assets. Consequently, the optimal EUplus $Fasset_{z,t} / \left[CrdtMg_{z,t} - Debt_{z,t} \right]$ ratio of

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[14] The way the model is written, a rise in the price of a financial asset takes the form of a fall in its rate of return.

[15] Countries which joined the European Union after 1995, but before 2007: Eastern and Central European countries (Bulgaria, the Czech Republic, Hungary, Poland, Rumania, Slovakia, and Slovenia), Baltic countries (Estonia, Latvia, and Lithuania), and the Mediterranean islands of Cyprus and Malta.

assets to the surplus of credit margin over debt falls while the region keeps on borrowing to cover its current account deficit, and debt accumulation tends to increase that same ratio. There are two ways to restore portfolio equilibrium: a rise in the rate of return on EUplus international financial assets or a reduction of its assets. The first possibility is ruled out, as we have seen, by the fact that the aggregate rate of return on debt securities is determined on world markets, in which EUplus is a relatively small player. So the region must shed assets, and it must do so at an accelerated pace as debt begins to increase faster than the credit margin. This implies a continuing reduction in financial wealth, both in absolute terms and relative to physical assets (capital) in the region's total wealth. For the portfolio to remain in equilibrium, the rate of return on financial wealth must fall relative to that on capital. Since the former is largely determined on world markets, it is the latter which must rise. Indeed, the EUplus return on capital reaches 23.5% in the low-elasticity scenario, and a more modest 8% in the alternate scenario, which is nonetheless much more than the 2.9% rate on financial wealth. The end comes around year 2030: the value of international financial assets held by EUplus approaches zero, while the amount of debt becomes nearly equal to its credit margin; consequently, financial wealth virtually disappears, the rate of return on capital explodes, and the model crashes.

Thank God, this is just a model!

Indeed, this story says more about the model than about reality. It points to two aspects of the model that are in urgent need of improvement: (1) the way the credit margin evolves, at the same rate for all regions, regardless of each one's net international investment position relative to its GDP; (2) the world portfolio of debt securities is allocated without taking account of default risk, such as might be indicated, for instance, by the ratio of net IIP to GDP.

3.3 Sensitivity analysis

To complete our examination of the role of portfolio elasticities, we proceed to compare results obtained with different sets of values.

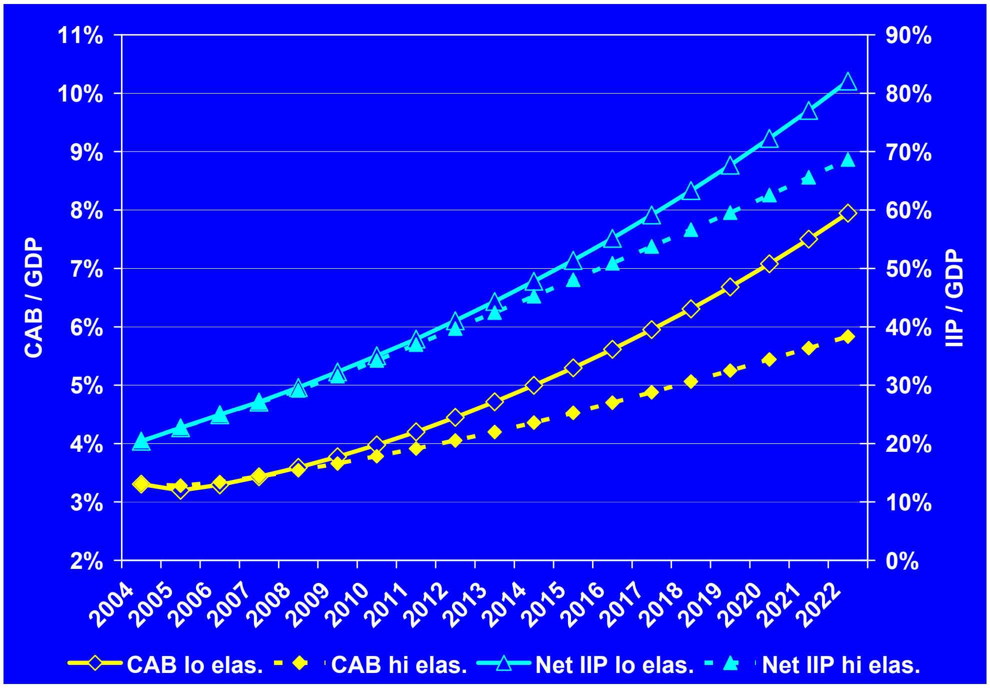
We first restrict our attention to combinations of elasticities that allow the model to run until 2030. As metioned earlier, the range of feasible combinations is relatively narrow, so there are no significant differences between the results they generate.

We turn next to the shorter 2022 horizon. As shown in section 3.1 above, there is much more variety among possible combinations. Consequently, results differ significantly. To illustrate, we choose two contrasted combinations, one with relatively low values ($\sigma_z^{PTF} = 15$, $\sigma_z^{FinW} = 19$, and $\sigma^{PTF\_W} = 19$), and the other with relatively high values ($\sigma_z^{PTF} = 45$, $\sigma_z^{FinW} = 100$, and $\sigma^{PTF\_W} = 100$).

With high elasticity values, the proportions of assets in the three types of portfolios involved are more flexible, requiring less variation in rates of return to assimilate changes in relative asset stocks. In short, one would expect that, with higher elasticities, the financial system would more readily accommodate imbalances; in particular, one might anticipate current accounts that would deviate more from zero. But that's not what the results show.

In Figure 9, the sum of absolute values of regional current account balances increases as a ratio of world real GDP[16], but *less* with high elasticities. The sum of absolute values of international investment positions follows the same pattern, since IIPs are the cumulative results of current account balances.

Figure 9. – Sum of absolute values of current account balances (CAB, left axis) and net international investment positions (IIP, right axis), as percentages of world real GDP[17]



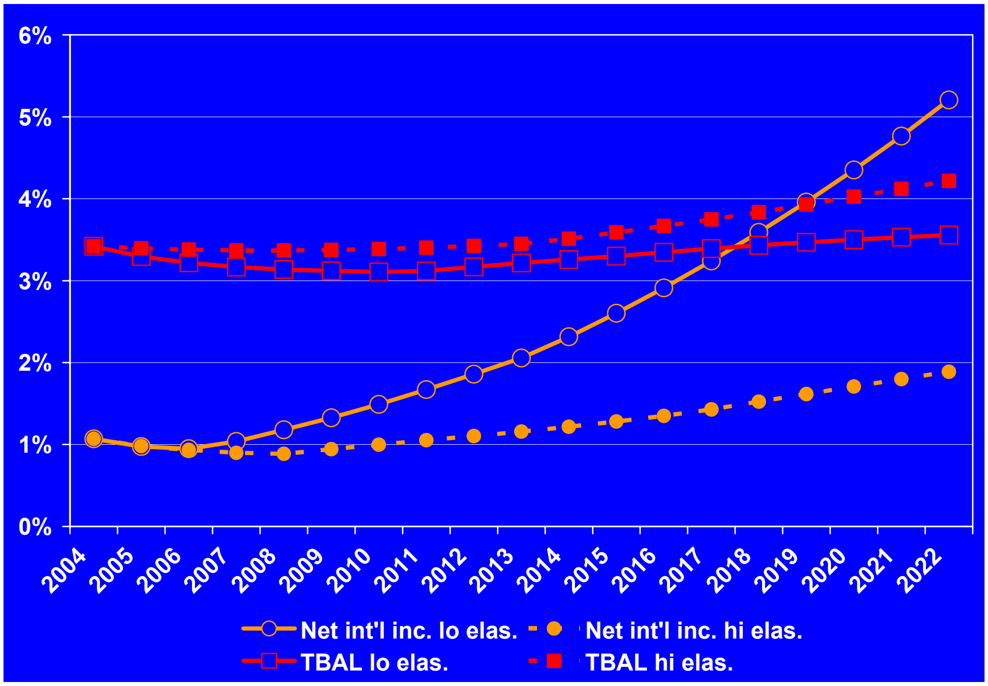
But the paradox is resolved when one looks at the two components of the current account: the trade balance, and net income from abroad. In Figure 10, both increase as ratios of the world real GDP, at least after a few periods of slight decline. But, comparing the low- and the high-elasticity scenarios, trade balances deviate *more* from zero in the latter, while the net incomes from abroad deviate *less*. What

16 It also increases in absolute terms.

17 Remember that in the model closure applied here, regional GDP deflators play the part of regional numeraires, and exchange rates are real exchange rates, the U.S. exchange rate being the global numeraire. Therefore, in comparing current account balances, trade balances or regional GDPs, all values are divided by e_z, the region's exchange rate (more accurately, the price of the international currency in terms of the regional currency).

happens is now clear: with high elasticities, rates of return remain lower; so, as international assets and debts accumulate, incomes paid and received do not increase so much. That leaves room for larger trade deficits and counterpart surpluses.

Figure 10. – Sum of absolute values of trade balances (TBAL) and net income from abroad, as percentages of world real GDP



What we see in the aggregate is also observed in general for individual regions, with some exceptions. In Table 1, only three regions have 2022 simulated current account balances that are greater in the high- than in the low-elasticity scenario. In the case of India, both values are above the negative base-year value, but one remains negative, and the other turns positive. In the case of the Rest-of-Latin-America (RoLAm), there is a bifurcation between scenarios: with low elasticities, the current account turns slightly negative, while it improves with high elasticities. As for the case of Asia-Pacific developing countries (AsPaDev). we shall return to it shortly.

Table 1 – Current account balance (10 G$)

		Elasticities		
		5-19-19	45-100-100	Ratio (%)
	2004	**2022**		
AfriSS	-0.81	0.11	-0.02	-14.3%
AsPaDev	5.77	13.77	19.49	141.5%
ChinaHK	11.29	157.15	108.70	69.2%
EU15	2.32	-79.86	-51.02	63.9%
EUplus	-7.28	-20.05	-13.38	66.7%
India	-2.63	1.37	-1.88	-137.1%
Japan	20.75	14.37	4.05	28.2%
LAmDev	-2.08	-2.39	-2.13	89.4%
MENA	1.30	14.48	6.25	43.2%
RoAsia	14.63	61.07	41.88	68.6%
RoLAm	0.91	-0.63	3.17	-501.5%
RoW	1.37	-5.12	-4.28	83.6%
Transit	4.34	24.02	16.18	67.4%
USA	-49.88	-178.30	-127.01	71.2%

In Table 2, the absolute size of net income from abroad (income received on assets owned abroad, minus income paid on domestic assets owned by foreigners) is less in the high-elasticity scenario, except for India and the aggregate of countries not included elsewhere (RoW), which experience a bifurcation between scenarios: an improvement with low elasticities, and a deterioration with high elasticities.

Table 2 – Net income from abroad (10 G$)

		Elasticities		
		5-19-19	45-100-100	Ratio (%)
	2004	**2022**		
AfriSS	-1.66	-1.81	-1.77	97.6%
AsPaDev	-2.68	6.01	3.51	58.4%
ChinaHK	-0.61	82.40	27.85	33.8%
EU15	4.23	-24.19	-5.87	24.3%
EUplus	-3.49	-19.07	-9.09	47.6%
India	-0.32	0.08	-1.32	-1748.6%
Japan	9.12	34.12	16.08	47.1%
LAmDev	-2.17	-4.26	-3.12	73.4%
MENA	-0.08	9.52	1.90	20.0%
RoAsia	-0.58	40.98	14.50	35.4%
RoLAm	-4.92	-5.56	-3.64	65.4%
RoW	-1.87	-1.14	-1.91	167.7%
Transit	-1.89	13.87	3.63	26.1%
USA	6.91	-130.94	-40.75	31.1%

In Table 3, the picture regarding trade balances is not quite so clear. Although globally, as we have seen, the sum of absolute deviations from zero is somewhat larger with high than with low elasticities, there is more variance in the evolution of individual regions. As a matter of fact, only seven out of the fourteen regions have a 2022 trade balance that is closer to zero with low elasticities. But these include the biggest players in the game: the USA, whose trade deficit in 2004 represents nearly 90% of the total of trade deficits among the 14 regions of the model, and China and the Rest-of-Asia (RoAsia), the two leading regions with positive trade balances. Note also that, for both sets of elasticities, most regions end up with a 2022 trade balance of greater magnitude, but of the same size as in base year 2004. One exception is Japan, whose growing net income from abroad (Table 2) makes it possible for its imports to grow faster than its imports to the point of eventually running a trade deficit. Another exception is the RoW region, an aggregate of rather diverse countries. Regarding India, its trade balance drives the current account improvement in the low elasticity scenario, while it remains negative, but of a smaller magnitude, in the high elasticity scenario.

Table 3 – Trade balance (exports, minus imports) (10 G$)

		Elasticities		
		5-19-19	45-100-100	Ratio (%)
	2004	**2022**		
AfriSS	0.85	1.92	1.75	91.3%
AsPaDev	8.45	7.76	15.98	205.8%
ChinaHK	11.90	74.76	80.86	108.2%
EU15	-1.91	-55.67	-45.15	81.1%
EUplus	-3.79	-0.98	-4.29	439.7%
India	-2.31	1.30	-0.56	-43.2%
Japan	11.63	-19.75	-12.03	60.9%
LAmDev	0.09	1.87	0.99	53.0%
MENA	1.38	4.96	4.35	87.7%
RoAsia	15.21	20.08	27.38	136.3%
RoLAm	5.83	4.93	6.81	138.1%
RoW	3.24	-3.98	-2.37	59.6%
Transit	6.23	10.16	12.56	123.6%
USA	-56.80	-47.36	-86.26	182.2%

Finally, net international investment positions (Table 4) reflect the cumulative effect of current account balances. Just like current account balances, net international investment positions are generally closer to zero with high elasticities, and of the same sign as in base year 2004. Exceptions are regions with particular patterns in the evolution of their current accounts (Table 1).

Table 4 – Net international investment position (10 G$)

		Elasticities		
		5-19-19	45-100-100	Ratio (%)
	2004	**2022**		
AfriSS	-14.27	-18.79	-20.77	110.5%
AsPaDev	-27.24	94.80	169.36	178.7%
ChinaHK	60.69	1 299.58	992.97	76.4%
EU15	11.71	-580.01	-370.09	63.8%
EUplus	-33.15	-215.49	-203.16	94.3%
India	-6.41	-2.72	-38.08	1399.2%
Japan	192.72	494.51	431.69	87.3%
LAmDev	-14.92	-43.77	-47.31	108.1%
MENA	40.31	163.37	106.45	65.2%
RoAsia	64.92	657.60	557.99	84.9%
RoLAm	-67.46	-83.74	-41.48	49.5%
RoW	0.27	-20.97	-10.71	51.1%
Transit	16.40	237.41	194.10	81.8%
USA	-223.56	-1 981.77	-1 720.96	86.8%

How do the developments just described in trade and international finance reflect on the evolution of real GDP? Table 5 shows the value of real GDP for 2004 (base year) and for 2022 (horizon), under both the low- and the high-elasticity scenario. For most regions, real GDP increases less with high than with low elasticities, and that is also true for the world as a whole. Our conjecture is that, since high elasticities are more permissive of trade imbalances (Table 3), savings are diverted from more productive exporting regions to less productive importing regions (see equation [05]). Three regions fare better with high elasticities: India and the U.S. benefit marginally, and the EUplus region is substantially better off. Conversely, the group of Asia-Pacific developing countries (AsPaDev) is markedly less well-off in the high elasticity scenario.

Table 5 – Real GDP (10 G$)

		Elasticities		
		5-19-19	45-100-100	Ratio (%)
	2004	2022		
AfriSS	48.3	114.3	110.9	97.0%
AsPaDev	140.6	503.6	429.4	85.3%
ChinaHK	179.6	776.9	727.8	93.7%
EU15	1 052.8	1 799.5	1 690.7	94.0%
EUplus	59.8	99.4	113.1	113.8%
India	59.0	187.6	188.6	100.6%
Japan	445.9	804.5	738.2	91.8%
LAmDev	55.2	114.6	114.4	99.9%
MENA	40.3	100.7	98.6	97.9%
RoAsia	115.1	319.2	287.2	90.0%
RoLAm	139.7	303.8	281.9	92.8%
RoW	250.5	464.7	440.6	94.8%
Transit	66.8	197.4	180.9	91.6%
USA	1 137.5	1 794.0	1 815.7	101.2%
World	3 790.8	7 580.2	7 218.2	95.2%

We shall now examine the two more « deviant » cases, EUplus and AsPaDev, more closely.

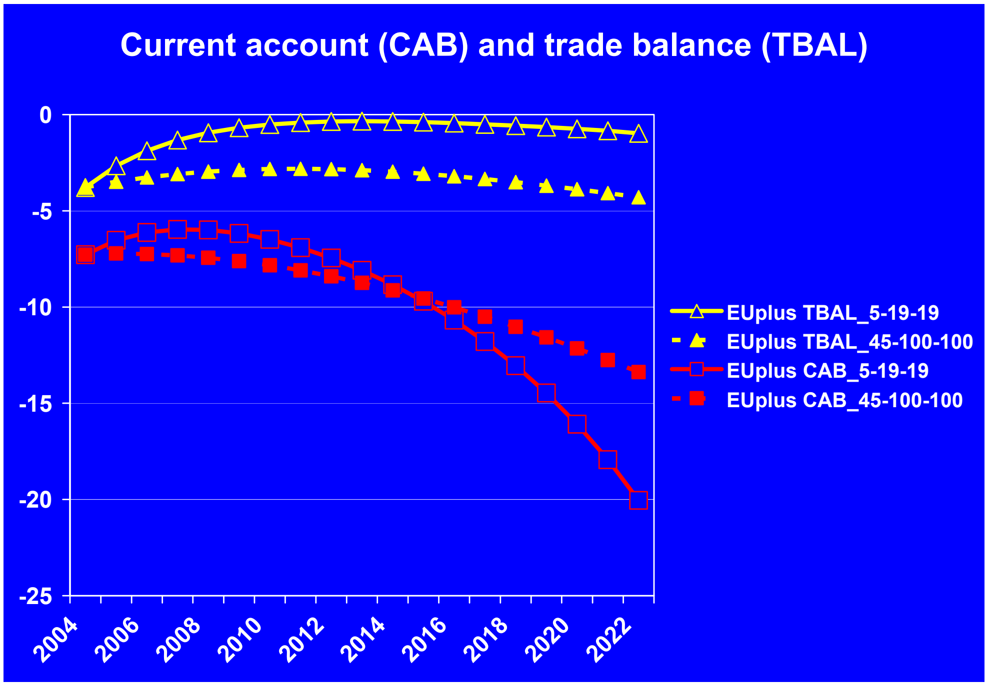
The EUplus region GDP ends up almost 14% greater than with low elasticities, while the world GDP is 5% less. Our interpretation of this is that the recurring trade deficit incurred by the region and the growing indebtedness exert less upward pressure on the rate of return paid on debt with high elasticities than otherwise. With a low world debt security portfolio elasticity, the steep rise in the rate of return paid on EUplus debt pressures the region into reducing its trade balance deficit. Indeed, its debt grows more slowly than in the case of high elasticity. At first, this improves the current account in the low- relative to the high-elasticity scenario, but, as debt accumulates, albeit more slowly, the heavier rates of return increase investment income paid abroad, and, four years into the simulation, the current account begins to deteriorate again; by 2016, the current account deficit with low elasticities becomes deeper than with high elasticities. Figure 11 illustrates these developments.

The consequence of the reduction in the trade deficit with low elasticities is that the contribution of foreign savings to investment financing is less. This is compounded by household dissaving[18]. Moreover, through the mechanism just described in 3.2, high elasticities allow the rate of return on capital to increase less than otherwise: with low elasticities, the rate of return on capital reaches 23.5% in 2022, while it is a more manageable 8.0% with high elasticities. As a result, with weak portfolio elasticities,

[18] The intercept of the household savings function for the EUplus is negative, and it increases in absolute terms with the reference scenario GDP index (see secton 2 above). In both simulation scenarios, EUplus GDP grows less than implied by the GDP index, and the gap is more pronounced in the low elasticity scenario, so the burden of household dissaving is greater.

investment demand is crushed. Overall, when elasticities are low, real growth is stifled by the combination of a high discount rate weighting on investment demand, and less foreign and domestic savings to drive investment, due to a forced reduction in the trade deficit.

Figure 11 – EUplus region: current account and trade balance (10 G$)



While high elasticities favor growth in the EUplus region – in effect, borrowed growth –, they have the opposite effect on the group of Asia-Pacific Developing countries (AsPaDev)[19]. Together, these countries run a trade surplus. Although their IIP is initially negative, and their net income from assets abroad is negative, their current account balance is positive and they accumulate international financial assets. This is illustrated in Figure 12.

The effect of the larger trade surplus that results from higher elasticities is to dampen investment somewhat, which slows growth, because, in effect, the region is exporting more savings to finance investment abroad. The financial counterpart of these developments is that, in both scenarios, the ratio of financial wealth to physical assets (capital) increases, but more so with high elasticities. The very opposite happens to the EUplus region: its ratio of financial wealth to physical assets falls as its debt accumulates, and it falls faster in the high elasticity scenario.

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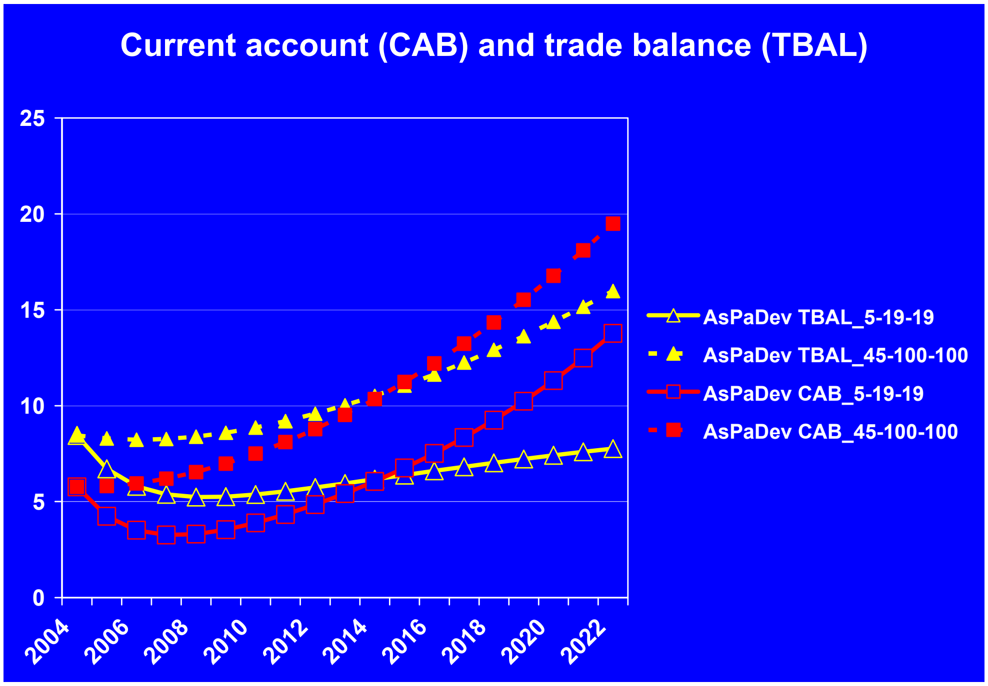
19 The group includes all of South-East Asia, and, among others, Bengladesh and Pakistan in South Asia, and the Pacific nations of Indonesia and the Philippines.

Figure 12 – AsPaDev region: current account and trade balance (10 G$)



Concluding remarks: a progress report on the model's development

The model presented here should still be considered a prototype. The innovation we are implementing is to introduce international financial assets in a trade model, in order to take into account the cumulative financial implications of trade flows, and to do it in such a way as to make current account balances endogenous, regulated by the willingness of surplus regions to lend to deficit regions.

Our exploratory work showed that there was a wide variety of feasible elasticity combinations, some with high values, others with lower values, and yet others with both high and low elasticities. We also examined the issue of convexity: are weighted averages of two feasible combinations of elasticities also feasible? In general, no, although we found some pairs of combinations that formed a convex set of feasible elasticities.

To evaluate model sensitivity to portfolio elasticities, we compared results obtained with different sets of values. Not surprisingly, the longer the time horizon required for feasibility (2022 or 2030), the more restrictive the definition, the narrower the range of feasible combinations, and the weaker the differences between the results they generate. But it must be recognized that our tests do not even come close to a systematic sensitivity analysis.

Finally, by examining key variables, we have identified some of the mechanisms that lead to a situation where there is no solution of the financial model that is compatible with the real model. This, we found out, has to do with the interactions between elasticities. Our main conclusion is that in most instances, the model crash diagnostics have an economic interpretation. Two situations are most common. The first is related to the supply of debt securities issued by regions which recurringly run current account surpluses (China, the group of Asia-Pacific developing countries...): as the stock of these securities vanishes, the corresponding rate of return falls; in the end, it threatens to turn negative, at which point the model becomes infeasible. The second situation concerns regions with recurring current account deficits (most notably, the EUplus region): as they run deeper and deeper into debt, their international financial wealth dwindles, while their physical assets continue to grow; in order for the portfolio manager to accept such a lopsided allocation, the rate of return on equity must increase; in the end, it blows up, and the model is infeasible. There are also cases, however, where the crash diagnostic is « Pivot too small », indicating that the Jacobian is nearly singular. The reason for this is still not clearly identified, but it seems to be related to the existence of tiny international trade flows (Japanese agricultural exports, in particular).

In short, we have acquired a better understanding of the adjustment mechanisms at work and of the channels of transmission that link the financial model and the real model.

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APPENDIX: ELASTICITY OF SUBSTITUTION BETWEEN ASSETS AND ELASTICITY OF DEMAND FOR INDIVIDUAL ASSETS

A high elasticity of substitution between assets does not imply such a high elasticity of demand for individual assets with respect to rates of return. We illustrate this property refering to the world mutual fund of international debt securities. In our model, every region owns international financial assets and, simultaneously, has international financial liabilities. For simplicity, each region issues a single international debt security. Individual region financial assets are pooled in what could be called a world mutual fund of international debt securities. The world fund is allocated maximizing a CES aggregate of the capitalized values of debt securities issued by the regions. The portfolio manager's problem is

$$\text{Maximize } \left\{ \sum_{z} \beta_{z}^{PTF\_W} \left[\left(1 + RR_{z,t}^{Debt}\right) Debt_{z,t} \right]^{-\rho^{PTF\_W}} \right\}^{-\frac{1}{\rho^{PTF\_W}}} \quad [06]$$

$$\text{subject to } \sum_{z} Debt_{z,t} = PTF\_W_{t} \quad [07]$$

where

$Debt_{z,t}$ is the value of region z's international financial liabilities in period t, expressed in the international currency

PTF_W_t is the world portfolio of debt securities

$RR_{z,t}^{Debt}$ is the rate of interest paid on international financial liabilities

with elasticity of substitution

$$\sigma^{PTF\_W} = \frac{1}{\rho^{PTF\_W} + 1}, \; 0 < \sigma^{PTF\_W} < \infty$$

Solving the problem leads to optimal portfolio shares

$$w_{z,t} = \frac{Debt_{z,t}}{PTF\_W_{t}} = \frac{\left(\beta_{z}^{PTF\_W}\right)^{\sigma^{PTF\_W}} \left(1 + RR_{z,t}^{Debt}\right)^{\sigma^{PTF\_W} - 1}}{\sum_{zj} \left(\beta_{zj}^{PTF\_W}\right)^{\sigma^{PTF\_W}} \left(1 + RR_{zj,t}^{Debt}\right)^{\sigma^{PTF\_W} - 1}} \quad [08]$$

It follows that the elasticity of share $w_{z,t}$ with respect to rate of return $RR_{z,t}^{Debt}$ is

$$\frac{RR_{z,t}^{Debt}}{w_{z,t}}\frac{\partial w_{z,t}}{\partial RR_{z,t}^{Debt}} = \left(1 - w_{t,z}\right)\frac{RR_{z,t}^{Debt}}{\left(1 + RR_{z,t}^{Debt}\right)}\left(\sigma^{PTF\_W} - 1\right) \qquad [09]$$

It can be seen that even a high value of σ^{PTF_W} will result in a moderate elasticity, because it is multiplied by $\frac{RR_{z,t}^{Debt}}{\left(1 + RR_{z,t}^{Debt}\right)}$. For example, if an asset represents 1% of the portfolio, a substitution elasticity of 100, with a rate of return of 3%, yields a share elasticity of less than 2.9.