Deriving the Armington, Krugman and Melitz models of trade

by

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Abstract

This paper shows how the Armington, Krugman and Melitz models of international trade are all specialized versions of a basic model. It is inspired by Balistreri and Rutherford (2012) who set out stylized versions of three models: Armington (1969); Krugman (1980); and Melitz (2003). In their exposition, Balsitreri and Rutherford develop each model separately. This paper draws out connections between the three models by developing them sequentially as special cases of a common basic model. We derive the Armington model by imposing strong assumptions on the basic model. We relax some of these assumptions to derive the Krugman model and make further relaxations to derive the Melitz model.

Key words: Armington, Krugman and Melitz; CGE modelling; international trade;

JEL codes: F12; D40; D58; C68.

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1. Introduction

This paper shows how the Arminigton, Krugman and Melitz models of international trade are all specialized versions of a basic model.[1] In the basic model, widgets are produced in each country s by an industry containing N_s firms. Consumers in country j treat widgets from different firms around the world as imperfect substitutes. The widget industry in each country s earns zero pure profits. In producing and selling widgets, firms in country s incur three types of costs: variable costs that are proportional to output; fixed setting up costs (H_s) ; and a fixed cost in selling to consumers in country j (F_{sj}). The two types of fixed costs are the same for all firms in country s.

In the Armington model, the two types of fixed costs are zero. Armington's firms in country s have identical productivity and behave in a purely competitive manner: that is they perceive the elasticity of demand for their product as ∞. With competitive behaviour and with costs proportional to output, profits for each firm are automatically zero. The number of firms in country s is fixed exogenously. Output variations for the industry are accommodated by output variations for the firms.

In the Krugman model, there are non-zero setup costs, $H_s > 0$, but zero fixed costs on each trade link, $F_{sj} = 0$. Krugman's firms are monopolistically competitive: their perceived elasticity of demand for their product is the actual elasticity which is finite. All widget firms in country s have the same productivity. The number of firms in country s adjusts endogenously as part of the mechanism of achieving zero pure profits.

In the Melitz model, both types of fixed costs are non-zero. As for Krugman, firms are monopolistically competitive, correctly perceiving the elasticity of demand for their product. In a major departure from Armington and Krugman, Melitz allows for productivity variation across firms in country s. As in Krugman, the number of firms in country s adjusts endogenously to achieve industry-wide zero pure profits. Whereas in Armington and Krugman, all firms in country s sell on all trade links, in Melitz only high productivity firms can sell on trade links for which there are high fixed costs.

The paper is organized as follows. Section 2 sets out the basic model. Then sections 3, 4 and 5 introduce the special assumptions invoked by Armington, Krugman and Melitz and then derive their models. Concluding remarks are in section 6.

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[1] The paper is inspired by Balistreri and Rutherford (2012) who set out stylized versions of three models: Armington (1969); Krugman (1980); and Melitz (2003). In their exposition, Balsitreri and Rutherford develop each model separately. This paper draws out connections between the three models by developing them as special cases of a common basic model.

2. Basic model

People in country j choose Q_{sj} and Q_{ksj} to minimize

$$\sum_s \sum_{k\in S(s,j)} Q_{ksj}P_{ksj} \tag{2.1}$$

subject to

$$Q_{sj} = \left(\sum_{k\in S(s,j)} \delta_{ksj} Q_{ksj}^{-\rho} \right)^{-1/\rho} \tag{2.2}$$

and

$$Q_j = \left(\sum_s Q_{sj}^{-\rho} \right)^{-1/\rho} \tag{2.3}$$

where

Q_{ksj} is the quantity of widgets sent from firm k in country s to country j (this includes the ss flows);

Q_{sj} is the quantity of widgets sent from all firms in country s to country j (a CES aggregate of the Q_{ksj}s);

P_{ksj} is the price in j of widgets from firm k in s;

Q_j is the total requirement for widgets in j;

ρ is a parameter (-1< ρ, $\rho \neq 0$) related to the elasticity of substitution (σ) between widgets from different sources (firms and countries) by $\sigma = 1/(1+\rho)$;

$S(s, j)$ is the set of firms that send widgets from s to j; and

δ_{ksj} is a positive parameter reflecting j's preference for widgets from firm k in s.

It is clear that optimization problem (2.1) to (2.3) can be simplified by substituting from (2.2) into (2.3): our only reason for including (2.2) is to provide a definition of the total flow of widgets from s to j. On performing the substitution, the optimization problem reduces to:

choose Q_{ksj} to minimize

$$\sum_s \sum_{k\in S(s,j)} Q_{ksj}P_{ksj} \tag{2.4}$$

subject to

$$Q_j = \left(\sum_s \sum_{k\in S(s,j)} \delta_{ksj} Q_{ksj}^{-\rho} \right)^{-1/\rho} . \tag{2.5}$$

Optimization problem (2.4) to (2.5) gives

$$Q_{ksj} = Q_j \delta_{ksj}^{\sigma} \left(\frac{P_j}{P_{ksj}} \right)^{\sigma} \quad \text{and} \tag{2.7}$$

$$P_j = \left(\sum_s \sum_{k\in S(s,j)} \delta_{ksj}^{\sigma} P_{ksj}^{1-\sigma} \right)^{1/(1-\sigma)} . \tag{2.8}$$

P_j can be interpreted as the average price paid by consumers in j for their widgets from all sources.

The contribution (Π_{ksj}) to profits of firm k in s from their sales to j is

$$\Pi_{ksj} = P_{ksj}Q_{ksj} - \left(\frac{W_s T_{sj}}{\Phi_{ks}}\right)Q_{ksj} - F_{sj}W_s \quad . \tag{2.9}$$

where

W_s is the cost of a unit of input (think labor) to widget making in country s;

Φ_{ks} is the output per unit of input (productivity) of widget maker k in country s;

T_{sj} is the power[2] of the tariff (or possibly transport costs) associated with the sale of one unit of widgets from s to j; and

F_{sj} is the fixed costs (measured in units of input) incurred by firms in s to enable them to export to j.

Assuming that firm k in s chooses its price and quantity for the sj link to maximize Π_{ksj}, we obtain

$$P_{ksj} = \left(\frac{W_s T_{sj}}{\Phi_{ks}}\right)\left(\frac{1}{1+\eta}\right) \text{ and} \tag{2.10}$$

$$\Pi_{ksj} = Q_j \delta_{ksj}^{\sigma} P_j^{\sigma}\left(\frac{W_s T_{sj}}{\Phi_{ks}}\right)^{1-\sigma}\left(\frac{1}{1+\eta}\right)^{1-\sigma}(-\eta) - F_{sj}W_s \tag{2.11}$$

where

η is the reciprocal of the elasticity of demand perceived by producers in all countries on all their sales.

We specify total profits for firms in country s as

$$\Pi_s = \sum_j \sum_{k\in S(s,j)} Q_j \delta_{ksj}^{\sigma} P_j^{\sigma}\left(\frac{W_s T_{sj}}{\Phi_{ks}}\right)^{1-\sigma}\left(\frac{1}{1+\eta}\right)^{1-\sigma}(-\eta) - \sum_j N_{sj}F_{sj}W_s - N_s H_s W_s \tag{2.12}$$

where

H_s is the fixed cost (measured in units of input) for every firm in country s (even those firms that don't produce anything);

N_{sj} is the number of firms that sent widgets from s to j, i.e. the number of firms in S(s,j); and

N_s is the number of firms in country s (including those that make an attempt to start up but don't actually produce anything).

Finally, we specify the number of inputs (employment) in the widget industry in country s as

$$L_s = \sum_j \sum_{k\in S(s,j)} \frac{Q_{ksj}}{\Phi_{ksj}} + \sum_j N_{sj}F_{sj} + N_s H_s \quad . \tag{2.13}$$

3. The Armington model

For Armington:

$F_{sj} = 0$ and $H_s = 0$ (no fixed costs); (3.1)

$\eta = 0$ (producers in country s are competitive); (3.2)

[2] Power is one plus the rate.

$\Phi_{ks} = \Phi_s$ for all k (there is no difference in productivity across firms in country s); (3.3)

$\delta_{ksj} = \delta_{sj}$ for all k (preferences of j are symmetrical across firms on the sj link); (3.4)

Under these assumptions we obtain the Armington model:

$$P_{sj} = \frac{W_s T_{sj}}{\Phi_s} \tag{3.5}$$

$$P_j = \left(\sum_s N_s \delta_{sj}^{\sigma} P_{sj}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3.6}$$

$$Q_{sj} = N_s^{-1/\rho} Q_j \delta_{sj}^{\sigma} \left(\frac{P_j}{P_{sj}} \right)^{\sigma} \tag{3.7}$$

$$L_s = N_s^{\frac{1}{1-\sigma}} \sum_j \frac{Q_{sj}}{\Phi_s} \quad . \tag{3.8}$$

where

Q_{sj} is the total quantity of widgets made in country s that are sold in country j; and
P_{sj} is the price in j of all widgets from firms in s.

In deriving (3.5) through (3.8) we recognise that in the absence of fixed costs, all firms in s will sell in j. Hence,

$$N_{sj} = N_s \quad . \tag{3.9}$$

We also recognize that all firms on the sj link will sell the same quantity [$Q_j \delta_{sj}^{\sigma} \left(P_j / P_{sj} \right)^{\sigma}$] and charge the same price (P_{sj}).

In this stripped-down version of the Armington model, we can assume that

W_s, T_{sj}, Q_j, N_s and Φ_s are exogenous.

Then, (3.5) gives us P_{sj}, (3.6) gives us P_j, (3.7) gives us Q_{sj}, and (3.8) gives us L_s. Via (3.1) and (3.2), (2.11) and (2.12) imply that Π_{ksj} and Π_s are zero.

4. The Krugman model

Krugman wants to introduce variety as an explanatory variable for trade. If there is an increase in the number of varieties flowing from s to j then Krugman wants this to generate an increase in the quantity of widgets flowing from s to j. To accommodate love of variety Krugman endogenizes N_{sj}.

In common with Armington, Krugman adopts

$F_{sj} = 0$; (4.1)

$\Phi_{ks} = \Phi_s$ for all k; (4.2)

$\delta_{ksj} = \delta_{sj}$ for all k. (4.3)

However, Krugman replaces (3.2) with

$$\eta = \frac{-1}{\sigma} \quad \text{(perceived elasticities are equal to actual elasticities),} \tag{4.4}$$

and assumes that

$$H_s > 0 \quad . \tag{4.5}$$

These assumptions again imply that all firms in country s are identical and that all firms that produce anything will supply all markets. In fact, we can assume that all firms have positive production because if there is a firm that produces nothing then all firms in that country produce nothing (same productivities). Thus, as for Armington we have

$$N_{sj} = N_s \quad . \tag{4.6}$$

From here we obtain

$$P_{sj} = \frac{W_s T_{sj}}{\Phi_s}\left(\frac{\sigma}{\sigma-1}\right) \quad \text{, and} \tag{4.7}$$

$$P_j = \left(\sum_s N_s \delta_{sj}^{\sigma} P_{sj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{4.8}$$

$$Q_{sj} = N_s^{-1/\rho} Q_j \delta_{sj}^{\sigma}\left(\frac{P_j}{P_{sj}}\right)^{\sigma} \tag{4.9}$$

$$L_s = N_s^{\frac{1}{1-\sigma}} \sum_j \frac{Q_{sj}}{\Phi_s} + N_s H_s \quad . \tag{4.10}$$

How do we endogenize N_s? We start by assuming that the number of firms in country s adjusts so that there are zero pure profits in widget making. In the context of the other Krugman assumptions, (2.12) now becomes

$$\sum_j N_s Q_j \delta_{sj}^{\sigma} P_j^{\sigma}\left(\frac{W_s T_{sj}}{\Phi_s}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1}{\sigma}\right) - N_s H_s W_s = 0 \quad , \tag{4.11}$$

that is

$$\sum_j Q_j \delta_{sj}^{\sigma} P_j^{\sigma}\left(\frac{W_s T_{sj}}{\Phi_s}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1}{\sigma}\right) - H_s W_s = 0 \quad . \tag{4.12}$$

The problem with (4.11) is that it doesn't contain N_s, so alone it doesn't provide a suitable way to endogenize $N_{s.}$ However, by exogenizing L_s, and endogenizing W_s we obtain a system, (4.7) to (4.10) and (4.12) in which N_s is determined endogenously. To see how this works, we rearrange (4.12) as

$$W_s = \left(\left[\sum_j Q_j \delta_{sj}^{\sigma} P_j^{\sigma}\left(\frac{T_{sj}}{\Phi_s}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1}{\sigma}\right)\right]\Big/ H_s\right)^{\frac{1}{\sigma}} \quad , \tag{4.13}$$

and substitute (4.9) into (4.10) to obtain

$$L_s = N_s\left[\sum_j \frac{Q_j \delta_{sj}^{\sigma}}{\Phi_s}\left(\frac{P_j}{P_{sj}}\right)^{\sigma} + H_s\right] \quad . \tag{4.14}$$

At this stage the Krugman-style model can be solved as follows.

Guess P_j for all j

Compute W_s from (4.13)

Compute P_{sj} from (4.7)

Compute N_s from (4.14)

Compute P_j from (4.8)

Refine guess of P_j

When the P_js are right compute Q_{sj} from (4.9).

5. The Melitz model

In common with Armington and Krugman, Melitz assumes that

$$\delta_{ksj} = \delta_{sj} \text{ for all k,} \tag{5.1}$$

and in common with Krugman, Melitz assumes that

$$\eta = \frac{-1}{\sigma} \quad . \tag{5.2}$$

With regard to fixed costs, Melitz assumes that both

$$F_{sj} > 0 \text{ and } H_s > 0 \ . \tag{5.3}$$

The most important departure by Melitz from Armington and Krugman is in the treatment of technology. Rather than assuming all firms in country s have the same productivity, Φ_s, Melitz assumes that firms in country s have an array of productivities given by

$$g(\Phi) = \gamma\Phi^{(-\gamma-1)} \quad \text{for } 1<\Phi \tag{5.4}$$

where

γ is a positive parameter; and

$g(\Phi)$ is the frequency distribution describing the probability of a firm in country s having productivity Φ ($g(\Phi)$ can be thought of as the proportion of firms in country s with productivity Φ).

With productivity differing across firms, Melitz assumes that only firms with high productivity will service markets in which there are high fixed costs of entry (large values for F_{sj}). Thus the average productivity (Φ_{sj}^{ave}) of firms in country s that make non-zero sales on the sj link varies with j. With a clever definition (see Appendix) of what is meant by average productivity across firms on the sj link, Melitz is able to derive the following equations:

$$P_{sj} = \frac{W_s T_{sj}}{\Phi_{sj}^{ave}}\left(\frac{\sigma}{\sigma-1}\right) \ , \tag{5.5}$$

$$P_j = \left(\sum_s \left(1-G_{sj}\right) N_s \delta_{sj}^{\sigma} P_{sj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{5.6}$$

$$Q_{sj} = \left[(1-G_{sj})N_s\right]^{\frac{-1}{\rho}} Q_j \delta_{sj}^{\sigma}\left(\frac{P_j}{P_{sj}}\right)^{\sigma} \tag{5.7}$$

$$L_s = \sum_j \frac{\left[(1-G_{sj})N_s\right] Q_j \delta_{sj}^{\sigma}\left(\frac{P_j}{P_{sj}}\right)^{\sigma}}{\Phi_{sj}^{ave}} + \sum_j \left(1-G_{sj}\right) N_s F_{sj} + N_s H_s \tag{5.8}$$

and

$$\sum_j \left(1-G_{sj}\right) N_s Q_j \delta_{sj}^{\sigma} P_j^{\sigma} \left(\frac{W_s T_{sj}}{\Phi_{sj}^{ave}}\right)^{1-\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{1}{\sigma}\right) - \sum_j \left(1-G_{sj}\right) N_s F_{sj} W_s - N_s H_s W_s = 0 \quad , \tag{5.9}$$

where

G_{sj} is the proportion of firms in country s whose productivity is insufficient to allow them to make sales on the sj link. Thus 1-G_{sj} is the proportion of firms in s that make sales on the sj link.

In equation (5.5), P_{sj} is the price charged by the average-productivity firm on the sj link, see (2.10). Equation (5.6) calculates the price of widgets in country j as though all j's purchases from s are at price P_{sj}, see (2.8). Equation (5.7) calculates the quantity of sales on the sj link by using (2.2) with each firm on the link treated as if it has average productivity. Equation (5.8) calculates the use of resources (employment) in the widget industry of country s as though all firms on the sj link have average productivity for that link. Equation (5.9) uses (2.12): in (5.9) we impose zero pure profits on the widget industry in country s, treating all firms on the sj link as having average productivity for that link.

To complete Melitz' model we need to specify Φ_{sj}^{ave} and (1-G_{sj}). Melitz does this by assuming that average productivity on a link is related to minimum productivity by

$$\Phi_{sj}^{ave} = \beta \Phi_{sj}^{min} \tag{5.10}$$

where β is a parameter with value greater than one and Φ_{sj}^{min} is the minimum productivity required for a firm to operate on the sj link. This minimum is determined by a zero-pure-profit condition [see (2.11)] for Π_{ksj}:

$$0 = Q_j \delta_{sj}^{\sigma} P_j^{\sigma} \left(\frac{W_s T_{sj}}{\Phi_{sj}^{min}}\right)^{1-\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{1}{\sigma}\right) - F_{sj} W_s \quad . \tag{5.11}$$

Finally, (1-G_{sj}) is determined as

$$\left(1-G_{sj}\right) = \int_{\Phi_{sj}^{min}}^{\infty} \gamma \Phi^{-\gamma-1} d\Phi = \left(\Phi_{sj}^{min}\right)^{-\gamma} \quad . \tag{5.12}$$

Equations (5.5) to (5.12) form our version of the Melitz model. We treat L_s, T_{sj} and Q_j as exogenous variables. To solve the model we start by substituting from (5.11) into (5.9) and use (5.10) and (5.12) to obtain

$$\sum_j \left(\Phi_{sj}^{min}\right)^{-\gamma} N_s F_{sj} W_s \left[\beta^{\sigma-1} - 1\right] \quad - \quad N_s H_s W_s = 0 \quad , \tag{5.13}$$

or

$$\sum_j \left(\Phi_{sj}^{min}\right)^{-\gamma} F_{sj} \left[\beta^{\sigma-1} - 1\right] \quad - \quad H_s = 0 \quad , \tag{5.14}$$

Next we rearrange (5.11) as

$$\Phi_{sj}^{min} = \left(Q_j \delta_{sj}^{\sigma} P_j^{\sigma}\right)^{\frac{-1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right) T_{sj} \left(\sigma\right)^{\frac{1}{\sigma-1}} F_{sj}^{\frac{1}{\sigma-1}} W_s^{\frac{\sigma}{\sigma-1}} \quad . \tag{5.15}$$

Combining (5.14) and (5.15) gives

$$\left(W_s^{\frac{-\gamma\sigma}{\sigma-1}}\right)\sum_j\left(\left(Q_j\delta_{sj}^{\sigma}P_j^{\sigma}\right)^{\frac{-1}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)T_{sj}\left(\sigma\right)^{\frac{1}{\sigma-1}}F_{sj}^{\frac{1}{\sigma-1}}\right)^{-\gamma}F_{sj}\left[\beta^{\sigma-1}-1\right] \quad - \quad H_s=0 \quad , \tag{5.16}$$

leading to

$$W_s=\left[\frac{H_s}{\sum_j\left(\left(Q_j\delta_{sj}^{\sigma}P_j^{\sigma}\right)^{\frac{-1}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)T_{sj}\left(\sigma\right)^{\frac{1}{\sigma-1}}F_{sj}^{\frac{1}{\sigma-1}}\right)^{-\gamma}F_{sj}\left[\beta^{\sigma-1}-1\right]}\right]^{\frac{1-\sigma}{\gamma\sigma}} \quad . \tag{5.17}$$

Now we can solve the model as follows:

Guess P_j for all j

Compute W_s from (5.17)

Compute Φ_{sj}^{min} from (5.15)

Compute Φ_{sj}^{ave} from (5.10)

Compute (1-G_{sj}) from (5.12)

Compute N_s from (5.8)

Compute P_j from (5.6)

Refine guess of P_j

When the P_js are right compute Q_{sj} from (5.7)

6. Concluding remarks

The choice between the Armington, Krugman and Melitz specifications of international trade is potentially important in policy-focused computable general equilibrium (CGE) modelling of trade issues.

Armington has been the standard specification in CGE models since its introduction via Australia's ORANI model in the 1970s [Dixon *et al.* (1977) and (1982)]. In earlier economy-wide trade-oriented models (e.g. Evans, 1972) imported and domestic varieties of a given commodity were treated as perfect substitutes. This led to 'flip-flop': import shares in domestic markets flipping between zero and one in response to seemingly minor changes in relative prices. The Armington specification dealt with this problem in a practical and empirically justified fashion.[3] Starting in the late 1980s, many modellers questioned the Armington specification. They were disappointed with Armington-based simulations which often show a welfare loss for a country that undertakes a unilateral reduction in tariffs, with the terms-of-trade loss outweighing the efficiency gain. Under the Krugman specification, there are two additional sources of welfare change from a tariff cut: cost reductions in the domestic economy through economies of scale and increased variety through extra imports (which may or may not be offset by a reduction in domestic varieties). Melitz adds another source of welfare change. In the Melitz model, tariff cuts can increase productivity by

[3] The ORANI model was supported by econometric estimates of Armington elasticities at a detailed level, see Alouze (1976) and (1977) and Alouze *et al.* (1977). These papers are summarized in Dixon *et al.* (1982, section 29.1).

weeding out inefficient domestic firms. As shown by Fan (2008) and Balistreri and Rutherford (2012), a CGE model with a Melitz specification can give considerably higher welfare gains from a given tariff cut than a model built with a similar database but with an Armington specification. So far Melitz simulations have been conducted in small models with stylized parameter values. A major challenge remains to establish the empirical significance of the mechanisms in the Melitz approach.

In this paper we have not dealt with either CGE modelling or empirical issues. Our aim has been to make the Armington, Krugman and Melitz theories more transparent by showing how they are related.

Appendix. Averages in the Melitz model: justifying (5.10) and the elimination of the firm dimension from (5.6) to (5.9)

Melitz defines average productivity on the sj link as

$$\Phi_{sj}^{ave} = \left[\int_{\Phi_{sj}^{min}}^{\infty} \Phi^{\sigma-1} \left(\frac{g(\Phi)}{1-G_{sj}} \right) d\Phi \right]^{\frac{1}{\sigma-1}} \quad . \tag{A1}$$

Under (5.4) and (5.12), (A1) gives

$$\Phi_{sj}^{ave} = \left(\frac{\gamma}{\gamma-(\sigma-1)} \right)^{1/(\sigma-1)} \Phi_{sj}^{min} \tag{A2}$$

justifying (5.10) with

$$\beta = \left(\frac{\gamma}{\gamma-(\sigma-1)} \right)^{1/(\sigma-1)} \quad . \tag{A3}$$

To make sense of this we must assume that $\gamma > \sigma$-1. This doesn't seem to have any economic interpretation.

Now we justify (5.6). From (2.10) we can see that the ratio of prices for any two firms on the sj link is the ratio of their productivities raised to the power -1. Applying this idea in (2.8) gives

$$P_j = \left(\sum_s \sum_{k \in S(s,j)} \delta_{sj}^{\sigma} P_{sj}^{1-\sigma} \left[\Phi_{sj}^{ave} / \Phi_{ks} \right]^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad . \tag{A4}$$

Treating Φ as a continuous rather than discrete variable gives us

$$P_j = \left[\sum_s \delta_{sj}^{\sigma} P_{sj}^{1-\sigma} \left(\Phi_{sj}^{ave} \right)^{1-\sigma} N_s \int_{\Phi_{sj}^{min}}^{\infty} \Phi^{\sigma-1} g(\Phi) d\Phi \right]^{\frac{1}{1-\sigma}} \quad . \tag{A5}$$

(A5) reduces to (5.6).

Next we justify (5.7). From (2.7), (2.10) and (5.1) we can see that the ratio of sales on the sj link for any two firms operating on the link is the ratio of their productivities raised to the power σ. Applying this idea in (2.2) with one of the firms being the average-productivity firm, we obtain

$$Q_{sj} = \left[\sum_{k \in S(s,j)} \delta_{sj} \left(\frac{Q_{sj}^{ave}}{\left(\Phi_{sj}^{ave}\right)^{\sigma}} \left(\Phi_{ksj}\right)^{\sigma} \right)^{-\rho} \right]^{\frac{-1}{\rho}} \quad . \tag{A6}$$

that is

$$Q_{sj} = \left[\delta_{sj} \left(\frac{Q_{sj}^{ave}}{\left(\Phi_{sj}^{ave}\right)^{\sigma}} \right)^{-\rho} \sum_{k \in S(s,j)} \Phi_{ksj}{}^{\sigma-1} \right]^{\frac{-1}{\rho}} \quad . \tag{A7}$$

Treating Φ as a continuous variable, we obtain

$$Q_{sj} = \left[\delta_{sj} \left(\frac{Q_{sj}^{ave}}{\left(\Phi_{sj}^{ave}\right)^{\sigma}} \right)^{-\rho} N_s \int_{\Phi_{sj}^{min}}^{\infty} \Phi^{\sigma-1} g(\Phi) d\Phi \right]^{\frac{-1}{\rho}} \quad , \tag{A8}$$

leading to

$$Q_{sj} = \left[\delta_{sj} \left(\frac{Q_{sj}^{ave}}{\left(\Phi_{sj}^{ave}\right)^{\sigma}} \right)^{-\rho} N_s \left(1 - G_{sj}\right) \left(\Phi_{sj}^{ave}\right)^{\sigma-1} \right]^{\frac{-1}{\rho}} \quad . \tag{A9}$$

which gives (5.7).

To justify (5.8), we start with (2.13). Under Melitz' assumptions, this can be written as

$$L_s = \sum_j \sum_{k \in S(s,j)} \frac{Q_{sj}^{ave}}{\Phi_{ksj}} \left(\frac{\Phi_{ksj}}{\Phi_{sj}^{ave}} \right)^{\sigma} + \sum_j N_{sj} F_{sj} + N_s H_s \quad . \tag{A10}$$

Hence

$$L_s = \sum_j \frac{Q_{sj}^{ave}}{\left(\Phi_{sj}^{ave}\right)^{\sigma}} \sum_{k \in S(s,j)} \Phi_{ksj}^{\sigma-1} + \sum_j N_{sj} F_{sj} + N_s H_s \quad , \tag{A11}$$

leading to

$$L_s = \sum_j \frac{Q_{sj}^{ave}}{\Phi_{sj}^{ave}} \left(1 - G_{sj}\right) N_s + \sum_j N_{sj} F_{sj} + N_s H_s \quad , \tag{A12}$$

justifying (5.8).

Finally, to justify (5.9), we start with (2.12). Using (5.1) and (5.2) we obtain

$$\Pi_s = \sum_j Q_j \delta_{sj}^{\sigma} P_j^{\sigma} \left(W_s T_{sj}\right)^{1-\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left(\frac{1}{\sigma} \right) \sum_{k \in S(s,j)} \Phi_{ks}{}^{\sigma-1} - \sum_j N_{sj} F_{sj} W_s - N_s H_s W_s \quad . \tag{A13}$$

Treating Φ as a continuous variable quickly leads to (5.9).

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