USAGE-R51, a state-level multi-regional CGE model of the US economy

by

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This paper has two main parts. The first deals with sourcing assumptions underlying multi-regional models including GTAP and TERM (The Enormous Regional Model). By assuming that all users of a good in a particular region source that good from all regions in common proportions, the dimensions of a multi-regional CGE database are reduced many-fold. This greatly reduces the solution time in simulations. The paper details the derivation of formulae on which the common sourcing assumption is based and the necessary additional identities required to ensure that the assumption holds.

While the IMPLAN team has provided very detailed small region input-output tables for the US economy for many years, multi-regional modelling of the US economy is less common. USAGE-R51 is in the TERM school of multi-regional, sub-national models. Rather than be discouraged by the absence of interstate trade data, USAGE-R51 follows the TERM traditional of estimating such trade matrices so as to satisfy state level excess demands and distribute excess supplies. Exceptionally detailed regional employment numbers for the United States are available from census data. The master database of USAGE-R51 includes 497 sectors, including a split of electricity generation into different fuel types. It also includes 51 regions. Following the GTAP approach, the USAGE-R51 database is tailored for each application through appropriate aggregation.

1. Introduction

USAGE is a detailed, dynamic, CGE model of the U.S. economy. It has been created at the Centre of Policy Studies, Monash University, in collaboration with the U.S. International Trade Commission. The model has been used by, and on behalf of, the U.S. International Trade Commission, the U.S. Departments of Commerce, Agriculture, Energy and Homeland Security as well as private sector organizations such as the Cato Institute and the Mitre Corporation. Applications of the model involve preparation of baseline forecasts and analyses of a variety of issues including the effects of: trade policies; environmental regulations; carbon taxes; energy security; illegal immigration; Next-gen aviation infrastructure expenditures; the Obama stimulus package; and the National Export Initiative.

USAGE is essentially a national model, although it does have a facility for disaggregating national results in a top-down fashion to the 50 states and the District of Columbia. This facility is effective for working out the regional implications of national policies which are unlikely to have a significantly different effect on costs of production in one state compared with other states. A legitimate application of the top-down facility is the U.S. International Trade Commission's analysis of the effects on the state economies of changes in import restraints (tariffs and quotas).[1] However, a limitation of the top-down facility is that it is unsuitable for projecting the effects of policies that are initiated at the state level and affect costs in one state relative to those in other states.

To overcome this limitation, we have now developed a bottom-up regional version of USAGE, USAGE-R51. This version treats the 50 states plus the District of Columbia as 51

[1] See U.S. International Trade Commission (2004).

highly integrated economies connected by: trade; factor movements; and a common currency. In USAGE-R51, policies such as carbon taxes levied at the state level will cause changes in production costs in one state relative to those in others, and lead to changes in trade and factor flows. Thus USAGE-R51 will allow assessments of the costs and benefits of to states of state-initiated policies.

The rest of this paper is organized as follows. In Section 2 we set out a quite general specification of the demand side in a bottom-up multi-regional CGE model. It quickly becomes apparent that for a detailed model, this general specification raises two problems: (1) coefficient and parameter values need to be estimated at a level beyond what is normally available from statistical sources; and (2) variables with extremely high dimension need to be computed. In section 3 we show how both these problems are addressed in USAGE-R51 by procedures devised by our colleague Mark Horridge (see for example …). Subsection 3.1 sets out Horridge's data generating formulas. Subsection 3.2 shows that if data are generated by these formulas and if some other restrictions are imposed to guarantee that Horridge's data assumptions are maintained as the simulated economy moves away from its initial state, then the general system in section 2 reduces to a much lower dimension system. In subsection 3.3 we argue that Horridge's procedures allow us to manage data and dimensionality problems while not seriously sacrificing realism and policy-relevant detail. Section 4 describes the database for USAGE-R51. Section 5 is an illustrative application. Section 6 outlines next steps.

2. Bottom-multi-regional CGE modelling: a general demand specification

At a theoretical level, it is not a big step to go from a national CGE model to a bottom-up multi-regional model. In a national model, we assume: that industry i chooses inputs of primary factors and materials to minimize the costs of producing any given level of output; that industry i's output matches demand for that output; that demands reflect profit maximizing and utility maximizing decisions by national industries and final demanders; and that the price of industry i's output reflects i's unit costs. In a bottom-up multi-regional model, we assume: that industry i in region r chooses inputs of primary factors and materials to minimize the costs of producing any given level of output; that industry (i,r)'s output matches demand for that output; that demands reflect profit maximizing and utility maximizing decisions by regional industries and final demanders; and that the price of industry (i,r)'s output reflects (i,r)'s unit costs.

The extra dimensionality of bottom-up models relative to national models is most apparent in the specification of demand functions.

Direct demands

In USAGE-R51, a typical direct demand function in the theoretical specification is:

$$\begin{aligned} x(c,s,r,d,u) &= x(c,s,d,u) + a(c,s,r,d,u) - \sigma(c,s,d,u) * \left(pp(c,s,r,d,u) - pp(c,s,d,u) \right) \\ &-\sigma(c,s,d,u) * \left(a(c,s,r,d,u) - a\_avg(c,s,d,u) \right) \end{aligned} \tag{2.1}$$

$c \in COM,\ s \in SRC,\ r \in ORG,\ d \in DST,\ u \in USR.$

where

COM is the set of commodities

SRC is the set of sources (domestic and imported)

ORG is the set of origins of a flow (51 states)

DST is the set of destinations for a flow (51 states)

USR is the set of users of a flow. It consists of intermediate use by industry, investment use by industry and use by other final demanders.

All the variables in the equation, the x's and p's, are percentage changes (usually between adjacent years):

x(c,s,r,d,u) is the percentage change in the quantity of commodity c,s originating in r and flowing to d to be used by u;

pp(c,s,r,d,u) is the percentage change in the purchasers price of c,s originating in r and used by u in d;

x(c,s,d,u) the percentage change in the quantity of c,s used by u in d;

pp_r(c,s,d,u) the percentage change in the purchasers price of c,s used by u in d;

a(c,s,r,d,u) is (c,s,r) input-saving technical change or a taste change variable by user u in d; and

a_avg(c,s,d,u) is the weighted average of the a(c,s,r,d,u)s over r.

The substitution elasticity between the different origins r of c,s from the point of view of user u in destination d is given by the parameter σ(c,s,d,u).

The purchasers price of c,s to d,u and a_avg(c,s,d,u) are given by

$$pp(c,s,\bullet,d,u) = \sum_{rr} S(c,s,rr,d,u) * pp(c,s,rr,d,u) \tag{2.2}$$

and

$$a\_avg(c,s,d,u) = \sum_{rr} S(c,s,rr,d,u) * a(c,s,rr,d,u) \tag{2.3}$$

where S(c,s,r,d,u) is the share of origin r in the total purchasers value of c,s used by u in d:

$$S(c,s,r,d,u) = \frac{PUR(c,s,r,d,u)}{\sum_{rr} PUR(c,s,rr,d,u)} \tag{2.4}$$

and PUR(c,s,r,d,u) is the purchasers' value of c,s from r used by u in d:

$$PUR(c,s,r,d,u) = TRADE(c,s,r,d,u) + \sum_{p}\sum_{m} TRADMAR(c,s,r,d,u,p,m) + \sum_{r} TAX(c,s,r,d,u) \tag{2.5}$$

In (2.5), TRADE(c,s,r,d,u) is the basic value[2] of c,s,r flowing to d for use by u. TRADMAR(c,s,r,d,u,p,m) is the basic value of margin commodity m produced in p required to facilitate the flow of c,s originating in r and flowing to d to be used by u [e.g., trucking services (m) produced in Arizona (p) to move domestically-produced (s) beef (c) originating in Texas (r) to California (d) for use by households (u)]. TAX(c,s,r,d,u) is the collection of sales taxes on TRADE(c,s,r,d,u).

With these definitions it can be seen that the overall demand for c,s by u in d satisfies:

$$x(c,s,\bullet,d,u) = \sum_{r} S(c,s,r,d,u) * x(c,s,r,d,u) + a\_avg(c,s,d,u) \tag{2.5}$$

In interpreting (2.1) to (2.5) it is useful to note three conventions. First, imports (s= imported) are deemed to originate in the state of their entry into the U.S and correspondingly, exports (u= export) are deemed to be "used" in the state of their exit from the U.S. Second, the sets ORG and DST are the same (they contain the 51 regions of the U.S.). We find it

[2] The basic value of a domestic good (s=dom) is the factory-door or farm-gate price. The basic value of an imported good (s=imp) is the landed, duty-paid price.

convenient to use different nomenclature to distinguish origins from destinations: the alternative of using one set name for the regions would require careful tracking of the order of arguments in strings such as (c,s,r,d,u). Third, we assume that there are no margins on margins, thus TRADMAR(c,s,r,d,u,p,m) is a quantity of margins multiplied by a basic price.

Margin demands

Margin demands refer to the use of services to facilitate direct flows. The margin services in USAGE-R51 potentially[3] include: wholesale trade, retail trade, domestic air services, international air services, domestic water services, international water services, road transport, rail transport and pipelines. Margin demand functions in the theoretical specification of USAGE-R51 take the form:

$$\begin{aligned} \mathrm{xmar}(c,s,r,d,u,p,m) &= \mathrm{x}(c,s,r,d,u) + \mathrm{amar}(c,s,r,d,u,p,m) \\ &\quad - \sigma \mathrm{m}(c,s,r,d,u,m) * \big(\mathrm{p}(m,dom,p) - \mathrm{pmar}(c,s,r,d,u,m)\big) \\ &\quad - \sigma \mathrm{m}(c,s,r,d,u,m) * \big(\mathrm{amar}(c,s,r,d,u,p,m) - \mathrm{amar}(c,s,r,d,u,m)\big) \end{aligned} \qquad (2.6)$$

$c \in COM,\ s \in SRC,\ r \in ORG,\ d \in DST,\ u \in USR,\ p \in PRD,\ m \in MAR$

where

MAR is the set of margin commodities (9 in in the most detailed version of USAGE-R51)
PRD is the set of producing regions (51 states, the same as ORG and DST)
xmar(c,s,r,d,u,p,m) is the percentage change in the quantity of margin commodity m produced in p required to facilitate the flow of c,s originating in r and flowing to d to be used by u [e.g., trucking services (m) produced in Arizona (p) to move domestically-produced (s) beef (c) originating in Texas (r) to California (d) for use by households (u)];
amar(c,s,r,d,u,p,m) is technical change in the use of margin m produced in region p to facilitate the flow of c,s,r to d,u;
p(m,dom,p) is the percentage change in the basic price of margin m produced in p;
pmar(c,s,r,d,u,m) is the percentage change in the average over producing regions p of the price of margin m facilitating the flow of c,s from r to d for use by u;
amar(c,s,r,d,u,m) is the percentage change in the average over producing regions p of the technical change variables associated with margin m used in facilitating the flow of c,s from r to d,u;
σm(c,s,r,d,u,m) is the substitution elasticity between margin m from different regions used to facilitate flow c,s from r to d for use by u,

The variables pmar(c,s,r,d,u,m) and amar(c,s,r,d,u,m) are given by

$$\mathrm{pmar}(c,s,r,d,u,m) = \sum_p \mathrm{SMAR}(c,s,r,d,u,p,m) * \mathrm{p}(m,dom,p) \qquad (2.7)$$

$$\mathrm{amar}(c,s,r,d,u,m) = \sum_p \mathrm{SMAR}(c,s,r,d,u,p,m) * \mathrm{atradmar}(c,s,r,d,u,p,m) \qquad (2.8)$$

where SMAR(c,s,r,d,u,p,m) is the share, produced in p, of margin m used to facilitate the flow of c,s,r to d,u. It is given by

[3] As explained in section 6, USAGE-R51 can be run at various levels of aggregation for industries/commodities and regions. The margin services listed here reflect the maximum commodity disaggregation.

$$\mathrm{SMAR}(c,s,r,d,u,p,m)=\frac{\mathrm{TRADMAR}(c,s,r,d,u,p,m)}{\sum_{pp}\mathrm{TRADMAR}(c,s,r,d,u,pp,m)} \tag{2.9}$$

An assumption underlying (2.6) to (2.9) is that all margins are domestically produced (s= domestic) so that the basic price of margin m produced in p is the domestic price.

In practice, restrictions on the form of the technical change variable are imposed. Here we make the restriction:

$$\mathrm{amar}(c,s,r,d,u,p,m)=\mathrm{amar}(c,s,r,d,m) \tag{2.10}$$

which implies that the technical change in the use of margin m on the flow of c,s,r to d,u does not depend on the region p in which the margin was produced nor does it depend on the type of user u in d. Under this restriction, (2.5) reduces to

$$\begin{aligned}\mathrm{xmar}(c,s,r,d,u,p,m)&=x(c,s,r,d,u)+\mathrm{amar}(c,s,r,d,m)\\&\quad-\sigma m(c,s,r,d,u,m)*\left(p(m,\mathrm{dom},p)-\mathrm{pmar}(c,s,r,d,u,m)\right)\end{aligned} \tag{2.11}$$

3. Addressing data requirements and high dimensionality using the Horridge system

Potentially, variables such as x(c,s,r,d,u) and xmar(c,s,r,d,u,p,m) in (2.1) and (2.11) have very high dimensions. With 500 commodities, 2 sources, 51 regions of origin, 51 regions of destination, 9 margin commodities and around 1000 users (intermediate and investment use in 500 industries plus a handful of non-investment final demanders) the dimension of x(c,s,r,d,u) is approximately $2.6*10^9$ and the dimension of xmar(c,s,r,d,u,p,m) is $1.2*10^{12}$. Correspondingly, implemention of (2.1) to (2.11) requires high dimensional data matrices, TRADE(c,s,r,d,u), TAX(c,s,r,d,u) and TRADMAR(c,s,r,d,u,p,m) that are not available from statistical agencies.[4] The implementation also requires parameters, e.g. σ(c,s,r,d,u) and σm(c,s,r,d,u,m) at unavailable levels of detail.

Rather than using the high dimension equations (2.1) and (2.11) with their corresponding high dimension coefficients, the implemented version of USAGE-R51 incorporates lower dimension equations

$$x(c,s,r,d,\bullet)=x(c,s,\bullet,d,\bullet)-\sigma(c)*\left[\mathrm{pdelivrd}(c,s,r,d,\bullet)-\mathrm{pdelivrd}(c,s,\bullet,d,\bullet)\right], \tag{3.1}$$

$$\mathrm{xmar}(c,s,r,d,m)=x(c,s,r,d,\bullet)+\mathrm{amar}(c,s,r,d,m), \tag{3.2}$$

and

$$\mathrm{xmar}(r,d,p,m)=\mathrm{xmar}(r,d,m)-\sigma m(m)*\left[p(m,\mathrm{dom},p)-\mathrm{pmar}(r,d,m)\right] \tag{3.3}$$

where

- $x(c,s,r,d,\bullet)$ is the percentage change in the quantity of commodity c,s,r flowing to d;
- $x(c,s,\bullet,d,\bullet)$ is the percentage change in the overall demand for c,s in region d;
- $\mathrm{pdelivrd}(c,s,r,d,\bullet)$ is the percentage change in the delivered price of c,s,r to d;
- $\mathrm{pdelivrd}(c,s,\bullet,d,\bullet)$ is the percentage change in an index over region of origin r of the delivered prices of c,s,r to d;

[4] The tax matrix TAX(c,s,r,d,u) is also required. Under the assumption that the tax rate [RATE(c,s,d,u)] applying to c,s by user u in d does not depend on the origin of c,s, then the tax matrix can be generated by

$$\mathrm{TAX}(c,s,r,d,u)=\left[\mathrm{TRADE}(c,s,r,d,u)+\sum_p\sum_m\mathrm{TRADMAR}(c,s,r,d,p,u,m)\right]*\mathrm{RATE}(c,s,d,u).$$

σ(c) is the substitution elasticity between c,s from different origins from the point of view of users in d;

xmar(r,d,m) is the percentage change in composite margin m on flows from r to d; and

σm(m) is the substitution elasticity between margin m services produced by different regions.

Before we define the newly introduced concepts such as "overall demand" and "delivered price", we pause to interpret (3.1) to (3.3). The underlying story for (3.1) is that c,s from different origins r is delivered to a mixing agent in destination d. This mixing agent produces an overall quantity of c,s by combining c,s,r over all origins r in a CES production function. In making this combination, the mixing agent minimizes costs subject to creating enough c,s to satisfy demands by all agents u in d.

The underlying story for (3.2) is that the amount of margin m required to facilitate flow c,s,r to d depends on the quantity of that flow modified by technical change. Consistent with the idea that c,s,r is delivered to a mixing agent in region d, (3.3) implies that margins associated with c,s,r deliveries to d are independent of which agents in d use c,s,r. In (3.3), we visualize an agent who supplies all margin m required to facilitate flows from r to d. This agent chooses the regions p from which to buy units of m to minimize costs of satisfying demands for m to facilitate flows from r to d subject to a CES production function.

Reduced-dimension equations similar to (3.1) were first used in the GTAP model (Hertel *et al*., 1997). Subsequently, versions of (3.1) to (3.3) were used in TERM (Horridge *et al*., 2005). However, neither of these sources establishes precisely what simplifications and what losses of generality are involved in replacing (2.1) and (2.6) with (3.1) to (3.3). The purpose of the rest of this section is to fill this void.

In sub-section 3.1, we describe a system devised by Mark Horridge for generating the data items required in (2.1) and (2.6) from available data. We demonstrate that when Horridge's data-generating procedures are used and various simplifications are made concerning substitution elasticities and technical change, then (2.1) and (2.6) can be replaced by 3.1 to 3.3. However, before we do this we need to complete the definitions of the new concepts in (3.1) to (3.3).

In (3.1) the two price variables are defined by

$$\begin{aligned}\mathrm{DELIVRD}(c,s,r,d,\bullet)*\mathrm{pdelivrd}(c,s,r,d,\bullet) &= \mathrm{TRADE}(c,s,r,d,\bullet)*\mathrm{p}(c,s,r)\\ &+\sum_{m}\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m)*\left[\mathrm{pmar}(m,r,d)+\mathrm{amar}(c,s,r,d,m)\right]\end{aligned} \tag{3.4}$$

and

$$\mathrm{DELIVRD}(c,s,\bullet,d,\bullet)*\mathrm{pdelivrd}(c,s,\bullet,d,\bullet) = \sum_{r}\mathrm{DELIVRD}(c,s,r,d,\bullet)*\mathrm{pdelivrd}(c,s,r,d,\bullet) \tag{3.5}$$

where the coefficients DELIVRD(c,s,r,d,•) and $\mathrm{DELIVRD}(c,s,\bullet,d,\bullet)$ are given by

$$\mathrm{DELIVRD}(c,s,r,d,\bullet) = \mathrm{TRADE}(c,s,r,d,\bullet)+\sum_{m}\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) \tag{3.6}$$

$$\mathrm{DELIVRD}(c,s,\bullet,d,\bullet) = \sum_{r}\mathrm{DELIVRD}(c,s,r,d,\bullet) \tag{3.7}$$

Note that

$$\mathrm{DELIVRD}(c,s,r,d,\bullet) = \sum_{u} \mathrm{DELIVRD}(c,s,r,d,u) \quad (3.8)$$

3.1. Horridge's data generating procedures

The Horridge equations for generating the data matrices required in (2.1) to (2.11) from lower dimensional data are:

$$\mathrm{TRADE}(c,s,r,d,u) = \mathrm{TRADE}(c,s,r,d,\bullet) * \frac{\mathrm{USE}(c,s,d,u)}{\sum_{uu} \mathrm{USE}(c,s,d,uu)} \quad (3.1)$$

$$\begin{aligned} &\mathrm{TRADMAR}(c,s,r,d,u,p,m) \\ &\quad = \mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) * \frac{\mathrm{USE}(c,s,d,u)}{\sum_{uu} \mathrm{USE}(c,s,d,uu)} * \frac{\mathrm{SUPPMAR}(r,d,p,m)}{\sum_{pp} \mathrm{SUPPMAR}(r,d,pp,m)} \end{aligned} \quad (3.2)$$

and

$$\begin{aligned} &\mathrm{TAX}(c,s,r,d,u) \\ &\quad = \mathrm{TAX}(c,s,\bullet,d,u) * \frac{\mathrm{TRADE}(c,s,r,d,\bullet) + \sum_{m} \mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m)}{\sum_{rr} \left[\mathrm{TRADE}(c,s,rr,d,\bullet) + \sum_{m} \mathrm{TRADMAR}(c,s,rr,d,\bullet,\bullet,m) \right]} \end{aligned} \quad (3.3)$$

where the 5 matrices on the right hand sides of (3.1) to (3.3) [TRADE(c,s,r,d), TRADMAR(c,s,r,d,m), SUPPMAR(r,d,p,m), USE(c,s,d,u) and TAX(c,s,d,u)] are aggregations of our three required matrices on the left hand sides. These aggregations are defined by

$$\mathrm{TRADE}(c,s,r,d,\bullet) = \sum_{u} \mathrm{TRADE}(c,s,r,d,u) \quad (3.4)$$

$$\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) = \sum_{p} \sum_{u} \mathrm{TRADMAR}(c,s,r,d,u,p,m) \quad (3.5)$$

$$\mathrm{SUPPMAR}(r,d,p,m) = \sum_{c} \sum_{s} \sum_{u} \mathrm{TRADMAR}(c,s,r,d,u,p,m) \quad (3.6)$$

$$\mathrm{USE}(c,s,d,u) = \sum_{r} \left[\mathrm{TRADE}(c,s,r,d,u) + \sum_{m} \sum_{p} \mathrm{TRADMAR}(c,s,r,d,u,p,m) \right] \quad (3.7)$$

$$\mathrm{TAX}(c,s,\bullet,d,u) = \sum_{r} \mathrm{TAX}(c,s,r,d,u) \quad (3.8)$$

Estimation for USAGE-R51 of the five reduced-form matrices on the RHSs of (3.4) to (3.8) is described in section. Here we note that these reduced-form matrices must satisfy:

$$\sum_{u} \mathrm{USE}(c,s,d,u) = \sum_{r} \left[\mathrm{TRADE}(c,s,r,d,\bullet) + \sum_{m} \mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) \right] \quad (3.9)$$

$$\sum_{c} \sum_{s} \mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) = \sum_{p} \mathrm{SUPPMAR}(r,d,p,m) \quad (3.10)$$

Under the Horridge data-generating procedure implemented in (3.1) to (3.3) there are a number of built in assumptions.

In (3.1) it is assumed that the fraction of the total flow of c,s,r to d that is used by u does not depend on r. For example, if households in region d consume 80% of the flow of domestic vegetables to d then these households consume 80% of the flow of domestic vegetables from r to d for each region r. A convenient way of viewing this is that there is a mixing agent in region d that combines the domestic vegetables flowing from all regions r to d and distributes the mixture to the users in proportion to their demands.

In (3.2) two assumptions are made. First, it is assumed that the margin m required per unit of flow of c,s,r to user u in region d is independent of the user. For example, the amount of road transport per unit of domestic vegetables flowing from r to d is the same for households in d as for the restaurant industry in d. Second, it is assumed that the source of margin m to facilitate flows of c,s,r to d,u is independent of c,s and u: if region p supplies *x* per cent of margin m used to facilitate flows on the route from r to d then every flow on this route sources *x* per cent of it demand for margin m from p. This ensures that there is one price for margin m on a route. A way of viewing this is that on each route r to d there is a mixer combining the supplies across producing regions of each margin m used on this route and all users of margin m on the route use this mixture.

In (3.3) it is assumed that taxes are levied on delivered flows (direct flows plus margins) to region d with no regard to the origin of the flow, but the tax rate can vary by user: the power of the tax depends on the user u in d but not on the origin r of the flow to d. For example all domestic vegetables delivered to the restaurant industry in region d might incur a sales tax of 5% whereas the same delivery to households might be tax free. Moreover, domestic vegetables delivered from California to the restaurant industry in Nevada cannot be taxed at a different rate from domestic vegetables delivered from Oregon to the restaurant industry in Nevada. A convenient way of viewing this is that, as already mentioned, there is a mixing agent in region d that combines the domestic vegetables flowing from all regions r to d and then taxes are applied to the mixture according to the user.

Intuitively, the picture to have in mind for the Horridge system is as follows. At some central place in each region commodities c,s are produced (for s=imported, "produced" means arrived from a foreign country). There are direct flows from producing region (the region of origin) to using region (the region of destination). These flows can be thought of as being delivered for use to a central point in d. To facilitate any flow from r to d, margins are required. The margins on any route, r to d, can be sourced from all regions. On any route, there is a single mixer of margin m that combines margin m produced in all regions. All direct flows on the route requiring margin m use the single mixed margin m (same sourcing proportions for all users). Thus there is only one price for margin m on route r to d. Direct flows of c,s on route r to d are combined with the accompanying margin flows and delivered to the central point in region d where a single mixer combines the delivered c,s from r with the c,s's delivered from other origin regions and distributes to users (same sourcing proportions for all users). All users of c,s in d incur the same delivery price for c,s in d. But, different users incur different tax rates (applied to the delivered price) and so the users face different purchasers' prices.

Some useful notation to accompany this description of the Horridge system is the idea of the delivered value and purchasers' value of the flow of c,s,r to d,u: DELIVRD(c,s,r,d,u) and PUR(c,s,r,d,u). In terms of the matrices described in (3.1) to (3.3):

$$\mathrm{DELIVRD}(c,s,r,d,u) = \mathrm{TRADE}(c,s,r,d,u) + \sum_{m}\sum_{p}\mathrm{TRADMAR}(c,s,r,d,u,p,m) \qquad (3.11)$$

and

$$\begin{aligned}\mathrm{PUR}(c,s,r,d,u) &= \mathrm{DELIVRD}(c,s,r,d,u) + \mathrm{TAX}(c,s,r,d,u)\\ &= \mathrm{DELIVRD}(c,s,r,d,u) * \mathrm{POWT}(c,s,d,u)\end{aligned} \tag{3.12}$$

where POWT(c,s,d,u) is the power on the tax on c,s delivered to user u in d, given by

$$\mathrm{POWT}(c,s,d,u) = 1 + \frac{\mathrm{TAX}(c,s,d,u)}{\sum_{rr}\left[\mathrm{TRADE}(c,s,rr,d,\bullet) + \sum_{m}\mathrm{TRADMAR}(c,s,rr,d,\bullet,\bullet,m)\right]} \tag{3.13}$$

From (3.11), (3.1) and (3.2) we see that

$$\begin{aligned}&\mathrm{DELIVRD}(c,s,r,d,u)\\ &= \left[\mathrm{TRADE}(c,s,r,d,\bullet) + \sum_{m}\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m)\right] * \frac{\mathrm{USE}(c,s,d,u)}{\mathrm{USE}(c,s,d,\bullet)}\end{aligned} \tag{3.14}$$

so that

$$\mathrm{DELIVRD}(c,s,r,d,\bullet) = \mathrm{TRADE}(c,s,r,d,\bullet) + \sum_{m}\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) \tag{3.15}$$

The symbol $\bullet$ indicates summation over the missing argument, u in the case of $\mathrm{USE}(c,s,d,\bullet)$ and $\mathrm{DELIVRD}(c,s,r,d,\bullet)$.

Table 3.1. Useful identities

No.	Useful identities	Interpretation
I1	$\frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)} = \frac{\mathrm{USE}(c,s,d,u)}{\mathrm{USE}(c,s,d,\bullet)}$	The share of user u in delivered value is independent of the region of origin [see (3.14) & (3.15)]
I2	$\frac{\mathrm{PUR}(c,s,r,d,u)}{\mathrm{PUR}(c,s,\bullet,d,u)} = \frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,\bullet,d,u)}$	The share of origin region r in purchasers value is the same as the share of region r in delivery value [see (3.12)]
I3	$\frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,\bullet,d,u)} = \frac{\mathrm{DELIVRD}(c,s,r,d,\bullet)}{\mathrm{DELIVRD}(c,s,\bullet,d,\bullet)}$	The share of origin region r in delivery value is independent of the user [see I1 & (3.7)]
I4	$\frac{\mathrm{PUR}(c,s,r,d,u)}{\mathrm{PUR}(c,s,\bullet,d,u)} = \frac{\mathrm{PUR}(c,s,r,d,\bullet)}{\mathrm{PUR}(c,s,\bullet,d,\bullet)}$	The share of origin region r in purchasers value is independent of the user [see I2, I3 & (3.12)]
I5	$\frac{\mathrm{TRADE}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,r,d,u)} = \frac{\mathrm{TRADE}(c,s,r,d,\bullet)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)}$	The share of direct flows in delivery value is independent of the user [See I1 & (3.1)]
I6	$\frac{\mathrm{TRADMAR}(c,s,r,d,u,p,m)}{\mathrm{DELIVRD}(c,s,r,d,u)} = \frac{\mathrm{TRADMAR}(c,s,r,d,p,m)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)}$	The share of margin flows in delivery value is independent of the user [See (3.1)]
I7	$\frac{\mathrm{TRADMAR}(c,s,r,d,u,p,m)}{\mathrm{TRADMAR}(c,s,r,d,u,\bullet,m)} = \frac{\mathrm{TRADMAR}(c,s,r,d,\bullet,p,m)}{\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m)}$	The share of margin m produced in p to facilitate the flow of c,s,r to d,u is independent of the user u [See (3.1)]

3.2. Lowering dimensionality

The aim of this subsection is to show that with full-dimension data generated in the Horridge fashion described in the formulas (3.1) to (3.3), and with certain other restrictions to be described below, then it is possible to reduce the dimensionality burden of (2.1) and (2.10) by replacing these equations by equations of lower dimensionality. The replacement of (2.1) and

(2.10) by lower dimension equations is achieved by taking weighed sums to these equations. The weights are:

$$\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)} \quad \text{applied to (2.1) and summed over u} \tag{3.16}$$

$$\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)} * \frac{SUPMAR(r,d,p,m)}{SUPMAR(r,d,\bullet,m)} \quad \text{applied to (2.10) and summed over u,p} \tag{3.17}$$

$$\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)} * \frac{TRADMAR(c,s,r,d,p,m)}{TRADMAR(\bullet,\bullet,r,d,p,m)} \quad \text{applied to (2.10) and summed over u,c,s} \tag{3.18}$$

The rest of this subsection is organized as follows. In subsection 3.2.1 we introduce the lower dimension replacement equations for (2.1) and (2.10) under the Horridge system and identify the saving in dimension delivered by the replacement equations.. In subsection 3.2.2 we prove that a weighted summation of (2.1) with weights from (3.16) does in fact deliver the replacement equation given in 3.2.1. In subsection 3.2.3 we prove that weighted summations of (2.10) with weights from (3.17) and (3.18) deliver the two replacement equation given in 3.2.1.

3.2.1. The lower dimension replacements of (2.1) and (2.10)

The Horridge replacement equation for (2.1) is

$$x(c,s,r,d,\bullet) = x(c,s,\bullet,d,\bullet) - \sigma(c) * \left[pdelivrd(c,s,r,d,\bullet) - pdelivrd(c,s,\bullet,d,\bullet)\right] \tag{3.19}$$

where

$x(c,s,r,d,\bullet)$ is the percentage change in the quantity of commodity c,s,r flowing to d;
$x(c,s,\bullet,d,\bullet)$ is the percentage change in an index over region of origin r of the quantities of commodity c,s,r flowing to d;
$pdelivrd(c,s,r,d,\bullet)$ is the percentage change in the delivered price of c,s,r to d;
$pdelivrd(c,s,\bullet,d,\bullet)$ is the percentage change in an index over region of origin r of the delivered prices of c,s,r to d; and
$\sigma(c)$ is the substitution elasticity between c,s from different origins from the point of view of users in d. This substitution elasticity is assumed to depend only on c.

In equation (3.19) the two price variables are defined by

$$\begin{aligned} DELIVRD(c,s,r,d,\bullet) * pdelivrd(c,s,r,d,\bullet) &= TRADE(c,s,r,d,\bullet) * p(c,s,r) \\ &+ \sum_m TRADMAR(c,s,r,d,m) * \left[pmar(m,r,d) + amar(c,s,r,d,m)\right] \end{aligned} \tag{3.20}$$

and

$$DELIVRD(c,s,\bullet,d,\bullet) * pdelivrd(c,s,\bullet,d,\bullet) = \sum_r DELIVRD(c,s,r,d,\bullet) * pdelivrd(c,s,r,d,\bullet) \tag{3.21}$$

where

$p(c,s,r)$ is the percentage change in the basic price of c,s,r; and

$pmar(m,r,d)$ is the percentage change in an index of basic prices of m over regions that supply margin m services on the route r to d, defined by

$$SUPMAR(m,r,d,\bullet) * pmar(m,r,d) = \sum_p SUPMAR(m,r,d,p) * p(m,dom,p) \tag{3.22}$$

As defined already in (3.6), $SUPPMAR(m,r,d,p)$ is the supply of margin m produced in region p to facilitate flows from r to d. The coefficients DELIVRD(c,s,r,d,$\bullet$) and $DELIVRD(c,s,\bullet,d,\bullet)$ are given by

$$DELIVRD(c,s,r,d,\bullet) = TRADE(c,s,r,d,\bullet) + \sum_m TRADMAR(c,s,r,d,m) \tag{3.23}$$

$$DELIVRD(c,s,\bullet,d,\bullet) = \sum_r DELIVRD(c,s,r,d,\bullet) \tag{3.24}$$

Note that

$$DELIVRD(c,s,r,d,\bullet) = \sum_u DELIVRD(c,s,r,d,u) \tag{3.25}$$

In the Horridge system of equations, (2.11) is replaced by two equations:

$$xmar(c,s,r,d,\bullet,\bullet,m) = x(c,s,r,d,\bullet) + amar(c,s,r,d,m) \tag{3.26}$$

and

$$xmar(\bullet,\bullet,r,d,\bullet,p,m) = xmar(\bullet,\bullet,r,d,\bullet,m) - \sigma m(m) * \left[p(m,dom,p) - pmar(r,d,m)\right] \tag{3.27}$$

where

xmar(r,d,m) is the percentage change in composite margin m on flows from r to d; and
σm(m) is the substitution elasticity between margin m services produced by different regions.

In equation (3.27)

$$\begin{aligned} &TRADMAR(\bullet,\bullet,r,d,\bullet,\bullet,m) * xmar(\bullet,\bullet,r,d,\bullet,\bullet,m) \\ &\quad = \sum_c \sum_s TRADMAR(c,s,r,d,\bullet,\bullet,m) * xmar(c,s,r,d,\bullet,\bullet,m) \end{aligned} \tag{3.28}$$

and

$$\begin{aligned} &SUPPMAR(r,d,\bullet,m) * pmar(r,d,m) \\ &\quad = \sum_p SUPPMAR(r,d,p,m) * p(m,dom,p) \end{aligned} \tag{3.29}$$

Compared with (2.1) and (2.10), equations (3.19), (3.26) and (3.27) considerably reduce dimensionality. With regard to direct flows, as mentioned earlier, in a full scale version of USAGE-R51, the variable x(c,s,r,d,u) on the left hand side of (2.1) has dimension $2.6*10^9$. By comparison the variable x(c,s,r,d,$\bullet$) on the left hand side of (3.19) has dimension $2.6*10^6$. For margin flows the dimensionality saving in the Horridge system is even more striking. The variables xmar(c,s,r,d,$\bullet$,$\bullet$,m) and xmar($\bullet$,$\bullet$,r,d,$\bullet$,p,m) on the left hand sides of (3.26) and (3.27) have dimensions $2.3*10^7$ and $1.2*10^6$ whereas xmar(c,s,r,d,u,p,m) on the left hand side of (2.10) has dimension $1.2*10^{12}$.

3.2.2. Showing that (3.19) can be a replacement of (2.1)

To motivate the replacement of (2.1) with (3.19) we start with a 3-equation system associated with equation (2.1) within the full theoretical model:

$$x(c,s,r,d,u) = x(c,s,d,u,\bullet) - \sigma(c,s,d,u) * \left(pp(c,s,r,d,u) - pp(c,s,d,u,\bullet)\right) \tag{3.30}$$

$$\sum_u \frac{DELIVRD(c,s,r,d,u)}{DELIVRD(c,s,r,d,\bullet)} * x(c,s,r,d,u) = x(c,s,r,d,\bullet) \tag{3.31}$$

$$\sum_{r}\frac{DELIVRD(c,s,r,d,\bullet)}{DELIVRD(c,s,\bullet,d,\bullet)}*x(c,s,r,d,\bullet)=x(c,s,\bullet,d,\bullet) \qquad (3.32)$$

Equation (3.30) is a repeat of (2.1). Equation (3.31) defines the quantity of c,s,r flowing to region d, x(c,s,r,d), as a weighted sum over u of x(c,s,r,d,u) and (3.32) defines the percentage change in the flow of c,s to d as a weighted sum over r of x(c,s,r,d,•). In this three equation system x (c,s,•,d,u), pp(c,s,r,d,u) and pp(c,s,•,d,u) can be thought of as being determined elsewhere in the model and the 3-equation system's role is to determine x(c,s,r,d,u), x(c,s,r,d,•) and x(c,s,•,d,•).

Our first step in showing that (3.19) can replace (2.1) is to derive from (3.30) to (3.32) another 3-equation system in which (3.31) and (3.32) are replaced by equations in which there is no appearance of the full-dimension variable x(c,s,r,d,u). In doing so we will be drawing on Horridge's data generating assumptions and restrictions.

Applying the weighted summation on the left hand side of (3.31) to equation (3.30) and using identity I1 in Table 3.1 we see that:

$$\begin{aligned} x(c,s,r,d,\bullet)=&\sum_{u}\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)}*x(c,s,\bullet,d,u)\\ &-\sum_{u}\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)}*\sigma(c,s,d,u)*\big(pp(c,s,r,d,u)-pp(c,s,\bullet,d,u)\big) \end{aligned} \qquad (3.33)$$

Now adopt the Horridge assumption that σ(c,s,d,u) = σ(c). This allows the elasticity σ to be taken out of the summation in the second term on the right hand side of (3.33) [5]:

$$\begin{aligned} x(c,s,r,d,\bullet)=&\sum_{u}\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)}*x(c,s,\bullet,d,u)\\ &-\sigma(c)*\sum_{u}\frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)}*\big(pp(c,s,r,d,u)-pp(c,s,\bullet,d,u)\big) \end{aligned} \qquad (3.34)$$

To get any further with simplifying the second term on the right hand side of (3.34) we need to recognize that under the Horridge data assumptions the purchasers value of c,s delivered from r to d for user u is the delivered price times the power of the tax, which does not depend on the origin of the flow [see (3.12)]:

$$PUR(c,s,r,d,u)=DELIVRD(c,s,r,d,u)*POWT(c,s,d,u) \qquad (3.35)$$

From this we can see that pp(c,s,r,d,u) is given by:

$$pp(c,s,r,d,u)=pdelivrd(c,s,r,d,u)+tuser(c,s,d,u) \qquad (3.36)$$

where

pdelivrd(c,s,r,d,u) is percentage change in the delivered price of c,s flowing from r to d for user u; and

tuser(c,s,d,u) is the power of the tax on c,s by user u in d.

From identity I2 in Table 3.1 we see that

$$pp(c,s,\bullet,d,u)=pdelivrd(c,s,\bullet,d,u)+tuser(c,s,d,u) \qquad (3.37)$$

where

[5] It would have been sufficient for just the u dimension to be removed from σ(c,s,d,u).

$$pp(c,s,\bullet,d,u) = \sum_r \frac{PUR(c,s,r,d,u)}{PUR(c,s,\bullet,d,u)} * pp(c,s,r,d,u) \tag{3.38}$$

and

$$pdelivrd(c,s,\bullet,d,u) = \sum_r \frac{DELIVRD(c,s,r,d,u)}{DELIVRD(c,s,\bullet,d,u)} * pdelivrd(c,s,r,d,u) \tag{3.39}$$

The delivered price of c,s,r to d,u is given by:

$$\begin{aligned} pdelivrd(c,s,r,d,u) &= \frac{TRADE(c,s,r,d,u)}{DELIVRD(c,s,r,d,u)} * p(c,s,r) \\ &+ \sum_m \sum_p \frac{TRADMAR(c,s,r,d,u,p,m)}{DELIVRD(c,s,r,d,u)} * \left[p(m,dom,p) + amar(c,s,r,d,m)\right] \end{aligned} \tag{3.40}$$

where

p(c,s,r) is the percentage change in the basic price of c,s produced in r; and

p(m,dom,p) of basic price of margin m produced in p (recall that by assumption margins are domestically produced).

From identities I5 and I6 in Table 3.1, we see that the delivered price of c,s,r to d, u does not depend on the user u:

$$\begin{aligned} pdelivrd(c,s,r,d,u) &= \frac{TRADE(c,s,\bullet,d,u)}{DELIVRD(c,s,\bullet,d,u)} * p(c,s,r) + \\ &\sum_m \sum_p \frac{TRADMAR(c,s,d,u,p,m)}{DELIVRD(c,s,\bullet,d,u)} * \left[p(m,dom,p) + amar(c,s,r,d,m)\right] = pdelivrd(c,s,r,d,\bullet) \end{aligned} \tag{3.41}$$

where pdelivrd(c,s,r,d,•) is the percentage change in the delivered price of c,s,r to all users in d.

Combining (3.39) with (3.42) we see in a similar fashion that the delivrd price of c,s to du does not depend on u:

$$pdelivrd(c,s,\bullet,d,u) = \sum_r \frac{DELIVRD(c,s,r,d,\bullet)}{DELIVRD(c,s,\bullet,d,u)} * pdelivrd(c,s,r,d,\bullet) = pdelivrd(c,s,\bullet,d,\bullet) \tag{3.42}$$

Combining (3.36) and (3.37) and using (3.41) and (3.42) into (3.40):

$$pp(c,s,r,d,u) - pp(c,s,\bullet,d,u) = pdelivrd(c,s,r,d,\bullet) - pdelivrd(c,s,\bullet,d,\bullet) \tag{3.43}$$

Substituting from (3.43) into the second term on the right hand side of (3.34)

$$\begin{aligned} x(c,s,r,d,\bullet) &= \sum_u \frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)} * x(c,s,\bullet,d,u) \\ &- \sigma(c) * \left(pdelivrd(c,s,r,d,\bullet) - pdelivrd(c,s,\bullet,d,\bullet)\right) \end{aligned} \tag{3.44}$$

Multiplying in (3.44) by $DELIVRD(c,s,r,d,\bullet)/DELIVRD(c,s,\bullet,d,\bullet)$ and summing over r, and using (3.22) and (3.43) gives:

$$x(c,s,\bullet,d,\bullet) = \sum_u \frac{USE(c,s,d,u)}{USE(c,s,d,\bullet)} * x(c,s,\bullet,d,u) \tag{3.45}$$

Substituting from (3.45) into (3.44) we obtain:

$$x(c,s,r,d,\bullet) = x(c,s,\bullet,d,\bullet) - \sigma(c) * \big(pdelivrd(c,s,r,d,\bullet) - pdelivrd(c,s,\bullet,d,\bullet)\big) \qquad (3.46)$$

The 3-equation system (3.30), (3.45) and (3.46) can replace the original 3-equation system (3.30), (3.31) and (3.32). This new set of equations has the property that the full dimension variable x(c,s,r,d,u) occurs only in one equation, (3.30). This means that equation (3.30) and x(c,s,r,d,u) can be omitted leaving a 2-equation system (3.45) and (3.46) which can be solved for the remaining variables x(c,s,d) and x(c,s,r,d). Note that (3.46) is the same as the required replacement (3.19) of (2.1)

3.2.3. Showing that (3.26) and (3.27) can replace (2.10)

To motivate the replacement of (2.10) with (3.26) and (3.27) we start with a 5-equation system associated with equation (2.10) within the full theoretical model:

$$\begin{aligned} xmar(c,s,r,d,u,p,m) &= x(c,s,r,d,u) + amar(c,s,r,d,m) \\ &\quad - \sigma m(c,s,r,d,u,m) * \big(p(m,dom,p) - pmar(c,s,r,d,u,m)\big) \end{aligned} \qquad (3.47)$$

$$\sum_{u} \frac{DELIVRD(c,s,r,d,u)}{DELIVRD(c,s,r,d,\bullet)} * xmar(c,s,r,d,u,p,m) = xmar(c,s,r,d,\bullet,p,m) \qquad (3.48)$$

$$\sum_{p} \frac{SUPMAR(r,d,p,m)}{SUPMAR(r,d,\bullet,m)} * xmar(c,s,r,d,\bullet,p,m) = xmar(c,s,r,d,\bullet,\bullet,m) \qquad (3.49)$$

$$\sum_{c}\sum_{s} \frac{TRADMAR(c,s,r,d,p,m)}{TRADMAR(\bullet,\bullet,r,d,p,m)} * xmar(c,s,r,d,\bullet,p,m) = xmar(\bullet,\bullet,r,d,\bullet,p,m) \qquad (3.50)$$

$$\sum_{c}\sum_{s} \frac{TRADMAR(c,s,r,d,m)}{TRADMAR(\bullet,\bullet,r,d,m)} * xmar(c,s,r,d,\bullet,\bullet,m) = xmar(\bullet,\bullet,r,d,\bullet,\bullet,m) \qquad (3.51)$$

Equation (3.47) is a repeat of (2.10). Equation (3.48) defines the quantity of margin m from surplying region p used to facilitate the flow of c,s,r flowing to region d, xmar(c,s,r,d,p,m), as an aggregation over user u in region d, xmar(c,s,r,d,u,p,m). Equation (3.49) defines the quantity of margin m used to facilitate the flow of c,s,r flowing to region d, xmar(c,s,r,d,m), as an aggregation over supplying regions p of margin m, xmar(c,s,r,d,p,m). Equation (3.50) defines the percentage change in margin m from supplying region p required to facilitate the flow of all flows from r to d, xmar(r,d,p,m), as an aggregation over c,s of margin m from supplying regions p required to facilitate the flow of c,s,r to d, xmar(c,s,r,d,p,m). Equation (3.51) defines the percentage change in margin m required to facilitate all flows from r to d, xmar(r,d, ,m), as an aggregation over c,s of margin m required to facilitate the flow of c,s,r to d, xmar(c,s,r,d,m). In this 5-equation system x(c,s,r,d,u), p(m,dom,p), pmar(c,s,r,d,u,m) and amar(c,s,r,d,m) can be thought of as being determined elsewhere in the model and the 5-equation system's role is to determine xmar(c,s,r,d,u,p,m), xmar(c,s,r,d,•,p,m), xmar(c,s,r,d,•,• m), xmar(•,•,r,d,•,p,m) and xmar(•,•,r,d,•,•,m).

Note that from (2.6) and (2.8) and using (3.2), identity I7 in Table 3.1 and (3.22)

$$\mathrm{pmar}(c,s,r,d,u,\bullet,m)=\sum_p \frac{\mathrm{TRADMAR}(c,s,r,d,u,p,m)}{\mathrm{TRADMAR}(c,s,r,d,u,\bullet,m)} * \mathrm{p}(m,\mathrm{dom},p)$$
$$=\sum_p \frac{\mathrm{TRADMAR}(c,s,r,d,\bullet,p,m)}{\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m)} * \mathrm{p}(m,\mathrm{dom},p)$$
$$=\sum_p \frac{\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m) * \frac{\mathrm{SUPPMAR}(r,d,p,m)}{\mathrm{SUPPMAR}(r,d,\bullet,m)}}{\mathrm{TRADMAR}(c,s,r,d,\bullet,\bullet,m)} * \mathrm{p}(m,\mathrm{dom},p) \tag{3.52}$$
$$=\sum_p \frac{\mathrm{SUPPMAR}(r,d,p,m)}{\mathrm{SUPPMAR}(r,d,\bullet,m)} * \mathrm{p}(m,\mathrm{dom},p) = \mathrm{p}(r,d,m)$$

Thus we can replace (3.47) with

$$\begin{aligned}\mathrm{xmar}(c,s,r,d,u,p,m) &= \mathrm{x}(c,s,r,d,u)+\mathrm{amar}(c,s,r,d,m)\\ &-\sigma \mathrm{m}(c,s,r,d,u,m) * \big(\mathrm{p}(m,\mathrm{dom},p)-\mathrm{pmar}(r,d,m)\big)\end{aligned} \tag{3.53}$$

where

$$\mathrm{pmar}(\bullet,\bullet,r,d,\bullet,\bullet,m)=\sum_p \frac{\mathrm{SUPPMAR}(r,d,p,m)}{\mathrm{SUPPMAR}(r,d,\bullet,m)} * \mathrm{p}(m,\mathrm{dom},p) \tag{3.54}$$

Now we multiply in (3.53) by the ratio $\mathrm{DELIVRD}(c,s,r,d,u)/\mathrm{DELIVRD}(c,s,r,d,\bullet)$ and sum over u, and we note that from identity I1 in Table 3.1, the r's can be removed from this ratio. We obtain

$$\sum_u \frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)} * \mathrm{xmar}(c,s,r,d,u,p,m)$$
$$=\sum_u \frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)} * \mathrm{x}(c,s,r,d,u)+\mathrm{amar}(c,s,r,d,m) \tag{3.55}$$
$$-\sum_u \frac{\mathrm{USE}(c,s,d,u)}{\mathrm{USE}(c,s,d,\bullet)} * \sigma \mathrm{m}(c,s,r,d,u,m) * \big(\mathrm{p}(m,\mathrm{dom},p)-\mathrm{pmar}(r,d,m)\big)$$

Next we assume that σm(c,s,r,d,u,m) does not depend on u so that the σm term can be taken out of the share weight sum over u. Further we assume that σm depends only on m, that is

$$\sigma \mathrm{m}(c,s,r,d,m) = \sigma \mathrm{mm}(m) \tag{3.56}$$

This gives

$$\sum_u \frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)} * \mathrm{xmar}(c,s,r,d,u,p,m)$$
$$=\sum_u \frac{\mathrm{DELIVRD}(c,s,r,d,u)}{\mathrm{DELIVRD}(c,s,r,d,\bullet)} * \mathrm{x}(c,s,r,d,u)+\mathrm{amar}(c,s,r,d,m) \tag{3.57}$$
$$-\sigma \mathrm{mm}(m) * \big(\mathrm{p}(m,\mathrm{dom},p)-\mathrm{pmar}(r,d,m)\big)$$

From (3.48) and (3.31)

$$\begin{aligned}&\mathrm{xmar}(c,s,r,d,,p,m) = \mathrm{x}(c,s,r,d,)+\mathrm{amar}(c,s,r,d,m)\\ &-\sigma \mathrm{mm}(m) * \big(\mathrm{p}(m,\mathrm{dom},p)-\mathrm{pmar}(r,d,m)\big)\end{aligned} \tag{3.58}$$

Next we multiply in (3.58) by the ratio $\mathrm{SUPPMAR(r,d,p,m)/SUPPMAR(r,d,\bullet,m)}$ and sum over p. Using (3.49) this gives

$$\mathrm{xmar(c,s,r,d,\bullet,\bullet,m) = x(c,s,r,d,\bullet) + amar(c,s,r,d,m)} \\ \mathrm{-\sigma mm(m) * \sum_p \frac{SUPMAR(r,d,p,m)}{SUPMAR(r,d,\bullet,m)} * \big(p(m,dom,p) - pmar(r,d,m)\big)} \qquad (3.59)$$

and from (3.54) this becomes

$$\mathrm{xmar(c,s,r,d,\bullet,\bullet,m) = x(c,s,r,d,\bullet) + amar(c,s,r,d,m)} \qquad (3.60)$$

Again we multiply in (3.58), this time by the ratio $\mathrm{TRADMAR(c,s,r,d,p,m) / TRADMAR(\bullet,\bullet,r,d,p,m)}$. We sum over c and s and recognize from (3.2) that

$$\mathrm{\frac{TRADMAR(c,s,r,d,p,m)}{TRADMAR(\bullet,\bullet,r,d,p,m)} = \frac{TRADMAR(c,s,r,d,m)}{TRADMAR(\bullet,\bullet,r,d,m)} * \frac{\frac{SUPPMAR(r,d,p,m)}{SUPPMAR(r,d,\bullet,m)}}{\frac{SUPPMAR(r,d,p,m)}{SUPPMAR(r,d,\bullet,m)}}} \\ \mathrm{= \frac{TRADMAR(c,s,r,d,m)}{TRADMAR(\bullet,\bullet,r,d,m)}} \qquad (3.61)$$

to obtain:

$$\mathrm{\sum_c \sum_s \frac{TRADMAR(c,s,r,d,p,m)}{TRADMAR(\bullet,\bullet,r,d,p,m)} * xmar(c,s,r,d,p,m)} \\ \mathrm{= \sum_c \sum_s \frac{TRADMAR(c,s,r,d,m)}{TRADMAR(\bullet,\bullet,r,d,m)} * \big[x(c,s,r,d) + amar(c,s,r,d,m)\big]} \qquad (3.62) \\ \mathrm{-\sigma m(m) * \big(p(m,dom,p) - pmar(r,d,m)\big)}$$

Thus from (3.62) using (3.50), (3.51) and (3.60) give

$$\mathrm{xmar(r,d,p,m) = xmar(r,d,m) - \sigma m(m) * \big(p(m,dom,p) - pmar(r,d,m)\big)} \qquad (3.63)$$

Equations (3.53), (3.58), (3.60), (3.63) and (5.51), replace the 5-equation system (3.47) to (3.51). Note that in this new 5-equation system xmar(c,s,r,d,u,p,m), and xmar(c,s,r,d,•,p,m) each occur only once on the LHS of an equation: equation (3.53) in the case of xmar(c,s,r,d,u,p,m) and (3.58) in the case of xmar(c,s,r,d,•,p,m). Eliminating these two equations and variables reduces the new 5-equation system to a 3-equation system and adopting the simplification in (3.59) gives to

$$\mathrm{xmar(c,s,r,d,\bullet,\bullet,m) = x(c,s,r,d,\bullet) + amar(m)} \qquad (3.64)$$

and

$$\mathrm{xmar(\bullet,\bullet,r,d,\bullet,p,m) = xmar(\bullet,\bullet,r,d,\bullet,\bullet,m) - \sigma mm(m) * \big(p(m,dom,p) - pmar(r,d,m)\big)} \qquad (3.65)$$

$$\mathrm{\sum_c \sum_s \frac{TRADMAR(c,s,r,d,\bullet,\bullet,m)}{TRADMAR(c,\bullet,r,d,\bullet,\bullet,m)} * xmar(c,s,r,d,\bullet,\bullet,m) = xmar(\bullet,\bullet,r,d,\bullet,\bullet,m)} \qquad (3.66)$$

Equations (3.64) and (3.65) are the same as (3.26) and (3.27) .

3.2.4. Are the Horridge data restrictions maintained through a multistep simulation?

In subsections 3.2.2 and 3.2.3 we have established the legitimacy of the reduced dimension Horridge equations under the assumption that the initial data satisfy formulas (3.1) to (3.3). However, the use of these reduced dimension equations can only be justified if (3.1) to (3.3) apply not only to the initial data but also at every point throughout a simulation as the economy moves away from the initial state. In this subsection we establish that this is in fact the case.

We start by showing that (3.1) holds at each point in a simulation. To do this we must we must show that the percentage change version of (3.1) holds. This percentage change form is given by:

$$\begin{aligned} x(c,s,r,d,u)+p(c,s,r) &= x(c,s,r,d,\bullet)+p(c,s,r)+x(c,s,\bullet,d,u)+pdelivrd(c,s,\bullet,d,\bullet) \\ &-\sum_{uu}\frac{USE(c,s,d,uu)}{USE(c,s,d,\bullet)}*\left[x(c,s,\bullet,d,uu)+pdelivrd(c,s,\bullet,d,\bullet)\right] \end{aligned} \quad (3.67)$$

In (3.67) we acknowledge that TRADE is a matrix evaluated a basic prices (which don't depend on destination d or user u) and USE is a matrix evaluated at delivered prices (which don't depend on user u). Our aim is to show that (3.67) holds. Cancelling out common terms and using (3.45), (3.67) reduces to :

$$x(c,s,r,d,u)=x(c,s,r,d,\bullet)+x(c,s,\bullet,d,u)-x(c,s,\bullet,d,\bullet) \quad (3.68)$$

Equation (3.68) says that every user of c,s, in d sources the same quantity share from r. Substituting (3.46) into the RHS of (3.68) and using (3.43) gives

$$\text{RHS (3.68)}=x(c,s,\bullet,d,u)-\sigma(c)*\left(pp(c,s,r,d,u)-pp(c,s,\bullet,d,u)\right) \quad (3.69)$$

and then using (2.1) we obtain

$$\text{LHS (3.68)}=\text{RHS (3.68)} \quad (3.70)$$

as required.

In percentage change form (3.2) is given by:

$$\begin{aligned} xmar(c,s,r,d,u,p,m)+pdom(m,p) &= x(c,s,r,d,m)+p(m,r,d) \\ &+x(c,s,\bullet,d,u)-x(c,s,\bullet,d,\bullet)+xmar(\bullet,\bullet,r,d,\bullet,p,m)+pdom(m,p) \\ &-\sum_{pp}\frac{SUPMAR(r,d,pp,m)}{SUPMAR(r,d,\bullet,m)}*\left[xmar(\bullet,\bullet,r,d,\bullet,pp,m)+pdom(m,p)\right] \end{aligned} \quad (3.71)$$

Recalling (3.29) we see that the price terms cancel out of (3.71) and using (3.27) gives:

$$\begin{aligned} xmar(c,s,r,d,u,p,m) &= x(c,s,r,d,\bullet,\bullet,m)+x(c,s,\bullet,d,u)-x(c,s,\bullet,d,\bullet) \\ &+xmar(\bullet,\bullet,r,d,\bullet,p,m)-xmar(\bullet,\bullet,r,d,\bullet,\bullet,m) \end{aligned} \quad (3.72)$$

Our aim is to show that (3.72) holds. Starting with the LHS of (3.72) from (3.52) and replacing σm(c,s,r,d,u,m) by σmm(m) we see that:

$$\begin{aligned} \text{LHS (3.72)} &= x(c,s,r,d,u)+amar(c,s,r,d,m) \\ &-\sigma mm(m)*\left(p(m,dom,p)-pmar(r,d,m)\right) \end{aligned} \quad (3.73)$$

Substituting (3.68) into (3.73) we obtain

$$\begin{aligned} \text{LHS (3.72)} &= x(c,s,r,d,\bullet)+x(c,s,\bullet,d,u)-x(c,s,\bullet,d,\bullet)+amar(c,s,r,d,m) \\ &-\sigma mm(m)*\left(p(m,dom,p)-pmar(r,d,m)\right) \end{aligned} \quad (3.74)$$

Then substituting from (2.26) and (3.27):

$$\begin{aligned}\text{LHS (3.72)} &= \text{xmar}(c,s,r,d,\bullet,\bullet,m) + x(c,s,\bullet,d,u) - x(c,s,\bullet,d,\bullet) \\ &\quad + \text{xmar}(\bullet,\bullet,r,d,\bullet,p,m) - \text{xmar}(\bullet,\bullet,r,d,\bullet,\bullet,m) = \text{RHS (3.72)}\end{aligned} \quad (3.75)$$

as required.

From (3.12) we can rewrite (3.3) as

$$\begin{aligned}&\text{DELIVRD}(c,s,r,d,u)*\left[\text{POWT}(c,s,d,u)-1\right] \\ &= \text{DELIVRD}(c,s,\bullet,d,u)*\left[\text{POWT}(c,s,d,u)-1\right]*\frac{\text{DELIVRD}(c,s,r,d,\bullet)}{\text{DELIVRD}(c,s,\bullet,d,\bullet)}\end{aligned} \quad (3.76)$$

Simplifying and rearranging this becomes

$$\frac{\text{DELIVRD}(c,s,r,d,u)}{\text{DELIVRD}(c,s,r,d,\bullet)} = \frac{\text{DELIVRD}(c,s,\bullet,d,u)}{\text{DELIVRD}(c,s,\bullet,d,\bullet)} \quad (3.77)$$

In percentage change form (3.77) is:

$$\begin{aligned}&x(c,s,r,d,u) + \text{pdelivrd}(c,s,r,d,u) - x(c,s,r,d,\bullet) - \text{pdelivrd}(c,s,r,d,\bullet) = \\ &x(c,s,\bullet,d,u) + \text{pdelivrd}(c,s,\bullet,d,u) - x(c,s,\bullet,d,\bullet) - \text{pdelivrd}(c,s,\bullet,d,\bullet)\end{aligned} \quad (3.78)$$

Our aim is to show that (3.78) holds. From (3.41) and (3.41) the price terms can be eliminated to give:

$$x(c,s,r,d,u) - x(c,s,r,d) = x(c,s,d,u) - x(c,s,d) \quad (3.79)$$

Equation (3.79) holds since we have established above that (3.68) holds.

4. Database for USAGE-R51

In this section we describe how the five matrices, USE(c,s,d,u), TAX(c,s,d,u), TRADE(c,s,r,d,•), TRADMAR(c,s,r,d,•,•,m) and SUPPMAR(r,d,p,m), required for implementation of USAGE-51 are estimated. The estimation is conducted at a 497 sector, 51 region level.

Adjustments to the national database

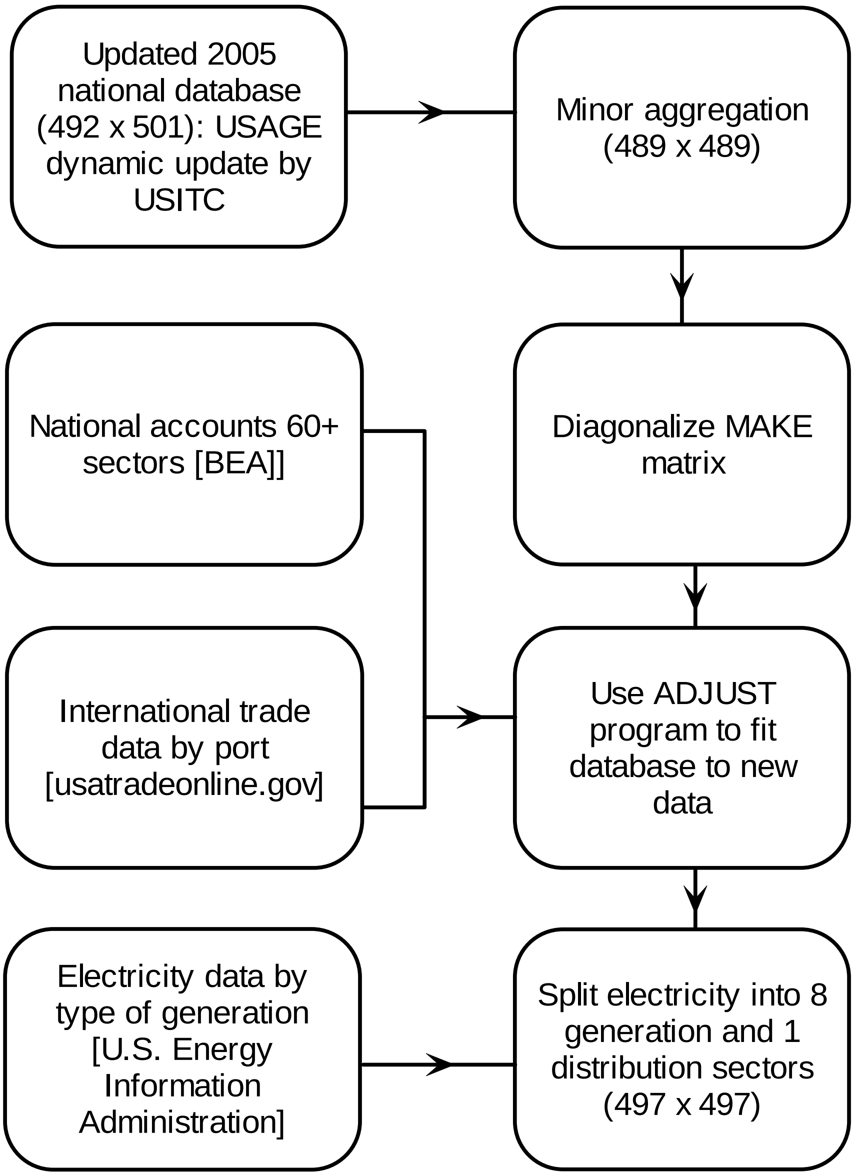
A preliminary task in preparing a regional master database is to make adjustments to an existing national CGE database. The database chosen was for 2005, updated from 1997 at the USITC.

Figure 1 outlines the process by which we devised a suitable 2005 national CGE database. The MAKE matrix, which details the commodity outputs of each industry, was diagonalized (i.e., each industry produces a single, unique commodity) in order to simplify the task of creating a regional database. We have two main sets of data for adjusting the national database. First, we have national accounts data for 60 sectors provided by the Bureau of Economic Analysis. These include employee compensation (wages bills), gross operating surplus and production taxes. Second, U.S.A. Trade Online provide international merchandise export and import value data by port. We use these two sets of data twice, first in the national database adjustment process, and later in estimating regional activities used to create the USA-TERM master database.[6]

[6] A program downloadable from www.monash.edu.au/policy/archivep.htm item TPMH0058 is used to make national database adjustments.

In addition, using data from U.S. Energy Information Administration, the national electricity sector was split into eight types of generation and electricity distribution.[7] The available data on type of generation are also used to infer state shares of national activity. The share of the national electricity distribution sector ("ElecDist") is estimated as the sum of generated electricity in each state divided by the national sum.

Figure 1: National database amendments



[7] Data were downloaded in November 2010 from the following site (no longer available): http://factfinder.census.gov/servlet/MetadataBrowserServlet?type=dataItem&id=ESTAB_F&dsspName=ECN_2007&dataset=EC0700A1&count=0&back=update&survey=2007+Economic+Census&sector=2007

Gathering data to estimate the USE and TAX matrices

The task of estimating the USE matrix at the 51 region state level requires production shares (R001), investment shares (R002), household consumption shares (R003), international trade shares (R004 and R00M) and government expenditure shares (R005) by state.

The primary source for state production shares by sector is the 2007 census. The census employment details are contained in a single text file that includes 2.5 million lines of data. We extracted three different groups of data from this file. First, we extracted the annual payroll bill. The coverage of this is far from comprehensive due to confidential cells. Next, we extracted the number of employed. These two items of information give us wages per employee that we can use in inferring wages bills for confidential cells provided employment numbers are available. Finally, using median employment numbers inferred by the employment flags, we extracted employment numbers for confidentialized cells.[8]

Where no wages per employee were available for confidential cells in the census data, we use the wages of closely related industries. In sectors where this strategy fails, we resort to broad sector wages (i.e., primary, manufacturing and services) to provide wages for which there is no more specific estimate. So far, including electricity sectors, this method is sufficient to obtain state shares for 454 of the 497 sectors in USAGE-R51.

U.S. Department of Agriculture data provide state share estimates for 13 agricultural sectors. Health data give estimates for two public sectors and education data a third public sector, thereby bringing the number of sectors for which there are regional estimates to 470 sectors. Data on public health spending by state are downloadable from the website of the U.S. Department of Health and Human Services.[9] Medicaid data are used to split the sector "SLGOther" into state production shares. Medicare data are used to split "NonDefG" into state shares in the absence of better data. In addition, government expenditure shares are based on Medicaid and Medicare data. State education sector shares are based on data from the National Center for Education Statistics.[10] Satellite accounts provided employment shares for the sectors Holidays, Foreign holidays, Tourism Exports and Education Exports.

However, before using these shares, we compare the national wages bill inferred by our processing of census data with the wages bill in the national 2005 CGE database. Sectors in which the wages bill in the national CGE database aligned poorly with the national bill inferred by the sum of our estimates were excluded, dropping the number of sectors to 450. For the remaining 47 sectors, we used default shares, based on data for per capita income and population in each state.

So far, we have outlined sources of estimates of production (R001) and government spending (R005) shares at the state level. Remaining users for whom we need state shares are investors (R002), households (R003) and exporters (R004). State investment shares by industry are set equal to state production shares. We were not able to find state level household consumption data. Our estimates of state shares of national household expenditure are based on state

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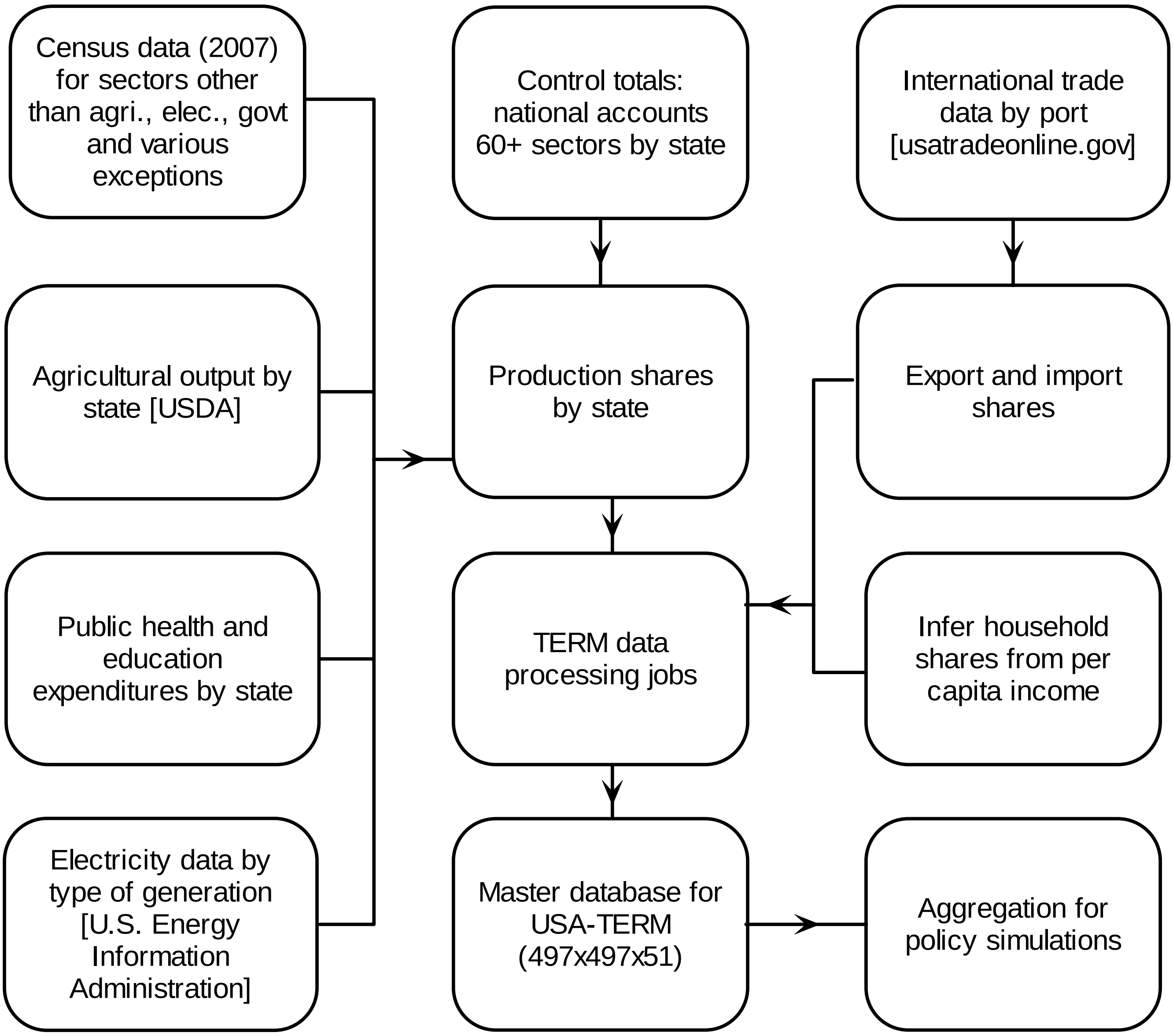
[8] The authors are grateful for Michael Jerie's assistance in extracting census data.

[9] The website is
http://www.cms.gov/NationalHealthExpendData/05_NationalHealthAccountsStateHealthAccountsResidence.asp
(accessed November 2010).

[10] http://nces.ed.gov/quicktables/result.asp?SrchKeyword=state+level+expenditure&topic=All&Year=2005
(accessed November 2010).

aggregate income shares. The international trade data used in the national database preparation are used again to obtain state export and import shares.

Figure 2: USA-TERM database jobs



We assume in the regional data estimation process that the technologies or cost structures of each industry are identical across all regions. An important strategy in making this assumption defensible is that we work at a high level of commodity/industry disaggregation. For example, we split the electricity sector into nine, with eight technologies in generation, namely coal, gas, biomass, hydroelectric, nuclear, geothermal, wind and other renewables. We would expect two different states to have different electricity generation technologies. Splitting the national sector enables us to capture such differences at the state level while retaining our assumption of identical technologies for a specific type of electricity generation across states.

The data gathered at the state level are sufficient to estimate the USE matrix, plus the state level MAKE and primary factor usage matrices. We distribute the indirect taxes included in the national CGE database according to state shares of national usage, thereby estimating the regional TAX matrix.

The TRADE matrix

In the TRADE(c,s,r,d) matrix, c includes 497 commodities either domestic or imported (s), with 51 regions of origin (r) and 51 destination regions (d). Diagonal elements of the (r,d) sub-matrix show production which is locally consumed. We already know the supply by commodity and region from the regional MAKE matrix, and demand by commodity and region from the USE matrix. Since there are no customs posts at state borders, we are never going to be able to collect comprehensive interstate trade data. Any vaguely relevant data tend to be based on volumes rather than values, reflecting freight movements, limited to quite coarse commodity categories. Rather than search for such data that are neither comprehensive nor adequate, we estimate excess supplies and excess demands based on the USE and MAKE matrices. We used the gravity formula (trade volumes follow an inverse power of distance) to construct trade matrices consistent with the regional USE and MAKE matrices. In defence of this procedure, we note that wherever production (or, more rarely, consumption) of a particular commodity is concentrated in one or a few regions, the gravity hypothesis is called upon to do very little work. Because our sectoral classification is so detailed, this situation occurs frequently.

For a particular commodity the traditional gravity formula may be written:

$$V(r,d) = \lambda(r).\mu(d).V(r,*).V(*,d) / D(r,d)^2 \qquad r \neq d \qquad (4.1)$$

where

$V(r,d)$ = value of flow from r to d (TRADE)

$V(r,*)$ = production in r (MAKE)

$V(*,d)$ = demand in d (USE)

$D(r,d)$ = distance from r to d

The $\lambda(r)$ and $\mu(d)$ are constants chosen to satisfy:

$$\Sigma_r V(r,d)= V(*,d) \text{ and } \Sigma_d V(r,d)= V(r,*). \qquad (4.2)$$

For TERM, the formula above gave rather implausible results, especially for service commodities. Instead we set:

$$V(r,d)/V(*,d) \propto \sqrt{V(r,*)}/D(r,d)^k \qquad r \neq d \qquad (4.3)$$

where K is a commodity-specific parameter valued between 0.5 and 2, with higher values for commodities not readily tradable. Diagonal cells of the trade matrices were set according to:

$V(d,d)/V(d,*)$= locally-supplied demand in d as share of local production

$$= MIN\{ V(d,*)/V(*,d),1\} \times F \qquad (4.4)$$

where F is a commodity-specific parameter valued between 0.5 and 1, with a value close to 1 if the commodity is not readily tradable.

The initial estimates of $V(r,d)$ were then scaled (using a RAS procedure) so that:

$$\Sigma_r V(r,d)= V(*,d) \text{ and } \Sigma_d V(r,d)= V(r,*). \qquad (4.5)$$

Transport costs as a share of trade flows were set to increase with distance:

$$T(r,d)/V(r,d) \propto \sqrt{D(r,d)} \qquad (4.6)$$

where T(r,d) corresponds to the matrix TRADMAR. Again, the constant of proportionality is chosen to satisfy constraints derived from the initial national IO table.

All these estimates are made with the fully-disaggregated database. In many cases, zero trade flows can be known *a priori*. For example, Kentucky and North Carolina together produce about three-quarters of the national tobacco crop, with a handful of southern states accounting for the remainder.

The TRADMAR and SUPPMAR matrices

The supply of margins is estimated once the TRADE and TRADMAR (i.e., demand for margins) matrices are known. The TRADMAR matrix estimation procedure starts with a regional estimate of MARGINS(c,s,u,m,d) usage based on NATMARGINS from the national CGE database and shares of national usage USHR for all users. Users include industries (i.e., USHR(u=industry,d)=R001(u,d)), investment (R002), households (R003), exports (R004) and government consumption (R005):

$$\text{MARGINS(c,s,u,m,d)} = \text{NATMARGINS(c,s,u,m)*USHR(u,d)} \tag{4.7}$$

TRADMAR does not identify the user of each margin, so the estimation procedure aggregates across the user dimension:

$$\text{MARGINS\_U(c,s,m,d)} = \sum_u \text{MARGINS(c,s,u,m,d)} \tag{4.8}$$

The gravity assumption used in (4.1), and in the context of margins (4.6), provides estimates of source shares (SRCSHR(c,s,r,d)). In addition, a parameter MARWGT(d,m) applies, giving a higher weight to shipping and a lower weight to rail transport in the case of an island state such as Hawaii:

$$\text{TRADMAR(c,s,m,r,d)=MARWGT(d,m)*SRCSHR(c,s,r,d)*MARGINS\_U(c,s,m,d)} \tag{4.9}$$

For each margin m, total supply in region of origin p for direct and margins use of the margins commodity is given by

$$\text{MAKE\_I(m,p)} = \sum_i \text{MAKE(m,i,p)}\ , \tag{4.10}$$

We define total direct usage of margins commodities as

$$\text{TRADE\_D(m,"dom",p)} = \sum_d \text{TRADE(c,"dom",p,d)} \tag{4.11}$$

and the total supply of margins for margins usage

$$\text{SUPPMAR\_RD(m,p)} = \sum_r \sum_d \text{SUPPMAR(m,r,d,p)}\ . \tag{4.12}$$

We set the supply of margins for all uses equal to the supply of margins for margins use minus direct demand for margins:

$$\text{MAKE\_I(m,p)} = \text{SUPPMAR\_RD(m,p)} - \text{TRADE\_D(m,"dom",p)} \tag{4.13}$$

Next, we need to set the supply of margin commodities for margin activities equal to the demand for margins:

$$\text{SUPPMAR\_P(m,r,d)} = \text{TRADMAR\_CS(m,r,d)} \tag{4.14}$$

where

$$\text{SUPPMAR\_P(m,r,d)} = \sum_p \text{SUPPMAR(m,r,d,p)} \tag{4.15}$$

and

$$\text{TRADMAR\_CS(m,r,d)} = \sum_{c}\sum_{s}\text{TRADMAR(c,s,m,r,d)}\,. \qquad (4.16)$$

Since the margins matrices TRADMAR and SUPPMAR have many dimensions, the context in which they appear changes in identities such as (4.13) and (4.14). TRADMAR is based on demands and hence depends on the TRADE matrix. SUPPMAR, on the other hand, reflects the supply of margins and may apply to TRADE flows in which the origin and destination differ from the region of margins supply. Consequently, enforcing identities (4.13) and (4.14) requires a substantial RAS adjustment.

5. Test simulations

In this section we report results from a series of simulations that we ran primarily to test the coding of USAGE-R51. In all of these simulations we adopted a long-run closure with the following properties:

- *Technology and tax variables*. All exogenous.
- *Employment*. Exogenous at the national level, endogenous for all industries and regions.
- *Wage rates*. Industrial wage rate relativities exogenous within regions (i.e. one wage movement per region). Labour supply to each region (set equal to regional employment) responds positively to regional wage rate.
- *Capital and rates of return*. Rates of return for all industries in all regions exogenous, and capital endogenous.
- *Investment*. Investment/capital ratios for all industries and all regions exogenous, and investment endogenous.
- *Household consumption*. Ratio of nominal household consumption to nominal GDP exogenous at the national level. Nominal household consumption linked to nominal wagebill at the region level.
- *Public consumption*. Exogenized via shifters for every commodity c,s in every demanding region d.
- *International exports*. Exports for each commodity from each region of exit are endogenously determined via price-sensitive demand functions. Shift variables in these functions are exogenous. Model currently incorporates unsatisfactory independence of foreign demand curves for same product from different regions.
- *International imports*. Determined via Armington specification as described in section 3.
- *Exchange rate*. This is exogenous. When there is zero shock to the exchange rate, movements in the domestic price level indicate movements in the real exchange rate.

The particular aggregation used in these tests identifies 54 commodities (COM has 54 elements) and 5 regions (REG has 5 elements). The regions are California, Nevada, Oregon, Arizona, and Rest of U.S.). USAGE-R51 allows for flexible aggregation of both commodities and regions.

We conduct four test simulations. The first, AR1, is a test simulation in which the correct solution is known *a priori*. In the remaining three simulations, AR2 to AR4, the results are not known *a priori*, they are dependent on data and theory. In the case of these simulations we will use back-of-the-envelope (BOTE) explanations to check the validity of the model solutions.

The shocks in the test simulations are as follows:

AR1, 10% increase in the nominal exchange rate

AR2, -10% all-input-saving technical change in all industries in California

AR3, -10% primary-factor-saving technical change in all industries in California

AR4, -10% primary-factor-saving technical change in all industries in Oregon

In analysing the results of the first three simulations we will restrict attention to the national macro results. For the final simulation, AR4, we will extend our analysis to regional macro results and to industry outputs at the regional level. Table 5.1 shows national macro results from the four test experiments. Tables 5.2 reports regional macro results for AR4 and Table 5.3 presents industry output by region.

In analysing the macro results in Table 5.1 we concentrate on equations for GDP from the income side and from the expenditure side.

$$Y = A * F(K, L) \qquad (5.1)$$

and

$$Y = C + I + G + X - M \qquad (5.2)$$

Equation (5.1) describes GDP (Y) in terms of a technology variable (A) and a function (F) of capital (K) and labour (L). Equation (5.2) describes GDP as consumption (C) plus investment (I) plus government expenditure (G) plus exports (X) less imports (M).

Simulation AR1

The first test simulation, AR1, is conducted to check that the model has been properly implemented. It is a nominal homogeneity test. If a CGE model is set up with no nominal rigidities[11], then a 10 per cent shock to all of the exogenous nominal variables should increase all endogenous nominal variables by 10 per cent, but leave all real variables unchanged. USAGE-R51 is a one-country model in which the exchange rate and foreign-currency prices of imports are exogenous. In this case the exchange rate is a nominal variable that should be shocked: it is the reciprocal of the *domestic* dollar price of a foreign dollar. The foreign currency prices of imports are “real” variables which should not be shocked: they do not involve domestic dollars in their definition.

Column 1 of Table 5.1 confirms that the 10 per cent nominal homogeneity test (implemented via a 10 per cent increase in the nominal exchange rate) increases nominal variables by 10 per cent and leaves real variables unchanged, as required.

Simulation AR2

In simulation AR2 all inputs to current production in California suffer a 10 per cent decrease in their ability to produce output (-10 per cent all-input-saving technical change). This is a negative shock to A in (5.1). What is the size of the shock to A? The share of Californian current production costs in national GDP is 23.2 per cent (= 2.9/12.5t where 2.9t is Californian costs and 12.5t is U.S. GDP). The impact effect of the -10% all-input-saving technical change in California is then a reduction in GDP of 2.4 per cent (-0.24*10). This is

[11] By nominal rigidities, we mean relationships that prevent some prices from adjusting fully over the time horizon of interest to an increase in the overall price level.

broadly consistent with the technology term (-2.13) in row 9, column 2 of Table 5.1. As can be seen from row 6, column 2 of Table 5.1, the actual decrease in real GDP is greater than this, 2.90 per cent. This is due to an induced reduction on K. In percentage change form (5.1) can be written as:

$$y = a + S_L * \ell + S_K * k \qquad (5.3)$$

where

y, a, ℓ and k are percentage changes in Y, A, L and K; and

S_L and S_K are the shares of labour and capital in GDP.

With rates of return in all industries and all regions fixed and L fixed, the technology deterioration in technology in California induces a fall in K of -1.51 per cent (row 8, column 2). With the share of capital in GDP being 44 per cent, the reduction in capital contributes -0.66 per cent (=-1.51*0.44) to GDP. Combining the technology and capital effects (5.3) suggests a reduction in GDP of 2.97 per cent (=2.32+0.66), which is close the simulated reduction of 2.90 per cent.

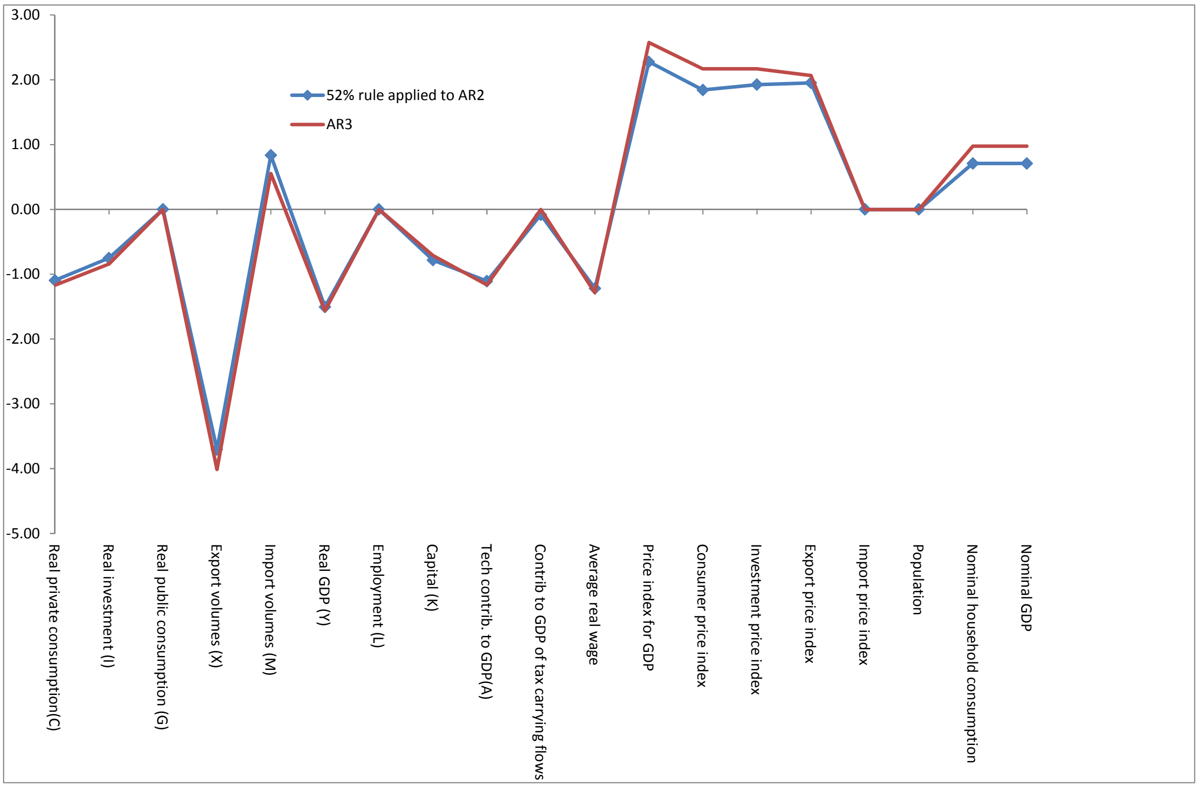
On the income side of GDP, with investment/capital ratios for all industries and all regions exogenous, the reduction in capital should be approximately matched by the reduction in investment. As can be seen in column 2 of Table 5.1, this is approximately true: investment falls by 1.45 per cent (row 2) whereas capital falls by 1.51 per cent (row 8). The slight discrepancy is accounted for by differences between shares of invest by industry and region in total investment and the shares of capital by industry and region in total capital. With nominal household consumption linked to nominal GDP (rows 17 and 18, column 2), real household consumption falls by 2.11 per cent in AR2, somewhat less than the fall in real GDP. The reason that the fall in real consumption is less than the fall in GDP is that the price of GDP rises by 4.38 per cent (row 12, column 2) whereas the price of household consumption rises by only 3.54 per cent (row 15, column 2). The reason that the price of GDP rises relative to the price of private consumption is that the price of exports increases and the price of imports is fixed: the price of GDP includes the price of exports, but not the price of imports, and the price of private consumption includes the price of imports but not the price of exports. But what causes the increase in the price of exports?

With government expenditure fixed by assumption and with real consumption and real investment falling by less than real GDP we see from (5.2) that X-M must decrease. We see from column 2 of Table 5.1 that the decrease in X-M is achieved with a decrease in X (row 4) and an increase in M (row 5). Was this inevitable? Suppose for a moment that contrary to column 2, X increases. Then with a fixed exchange rate and fixed positions of export demand curves, the price of exports would have to decrease. With U.S. exports broadly spread across industries this would have meant a decrease in the price of GDP. But with a decrease in the domestic price level and no change in the price of imports, the Armington assumption implies that imports would decline more that GDP (if there were no relative price movements it could be expected that imports would move in line with GDP). Hence X-M increases, contradiction. Hence our supposition that X increases is incorrect. With X decreasing and with a fixed exchange rate and fixed positions of export demand curves, the price of exports must increase.

Simulation AR3

In simulation AR3 all primary factor inputs to current production in California suffer a 10 per cent decrease in their ability to produce output (-10 per cent primary-factor-saving technical change). The effects of the technology shocks in AR3 and AR2 are readily comparable. In AR2 the 10 per cent technological deterioration impacts on all inputs costs in California whereas in AR3 the deterioration is restricted to primary factors. In California primary factors are worth \$1.5b whereas all costs account for \$2.9b. Thus the shock is AR3 is about 52 per cent (=100*1.5/2.9) of the size of the AR2 shock. As shown in Figure 5.1, the 52%-rule AR2 results are closely in line with the AR3 results. To get any further with the comparison between AR2 and AR3 we would need to take into account differences between industries. Consumption in AR3 fares slightly worse that would be expected on the basis of the 52%-rule applied to AR2. Consumption is intensive in Ownership of dwellings. This industry does relatively poorly in AR3 because it accounts for 7.4 per cent of primary factors but only 5.3 per cent of total costs.

Figure 5.1. Comparing AR3 national macro results with the corresponding AR2 results scaled to 52 per cent of their original size



Simulation AR4

In simulation AR4 there is -10 per cent primary-factor-saving technical change in Oregon rather than in California. Whereas in California primary factors are worth \$1.5b, in Oregon primary factors are worth \$0.14b. Thus we would expect the results in AR4 to be about 9% (=100*0.14/1.5) of those AR3. We see from Figure 5.2 that the 9% AR3 results are broadly in line with the AR4 results, but there are differences. Consider for example, the capital (K) results. On the basis of the 9%-rule applied to Californian primary-factor-saving technical change experiment in AR3 we would expect the Oregon primary-factor-saving technical change experiment in AR4 to reduce K by 0.06 per cent (=0.71*0.09, see row 8, column 3, Table 5.1) but in fact it reduced K by 0.12 per cent (row 8, column 4). What explains this discrepancy?

To answer this question we start by assuming that capital earns the value of its marginal product:

$$Q = P_g * A * f_K\left(\frac{K}{L}\right) \tag{5.4}$$

where
Q is the rental on capital;
P_g is the price of GDP;
f_K is the derivative of F with respect to K: f_K is a decreasing function of K/L.

With the rate of return (ROR) on capital being approximated by the rental price divided by the asset price (P_I) we see from (5.4) that

$$ROR = \frac{P_g}{P_I} * A * f_K\left(\frac{K}{L}\right) \tag{5.5}$$

We will use equation (5.5) as a back-of-the-envelope (BOTE) model to explain why the Oregon simulation in AR4 gives the larger fall in K than the 9%-rule applied to the AR3 Californian simulation would suggest. In the long-run closure adopted in AR3 and AR4, ROR is fixed. With a 9%-rule version of AR3, the fall in A is the same as in AR4. In both simulations L is fixed. We notice in Table 5.1 that P_g/P_I in AR4 rises by 0.01 per cent [=100*(1.0015/1.0014-1), see rows 10 &14, column 4] and in AR3 it rises by 0.391 per cent [=100*(1.0258/1.0218-1), see rows 10 &14, column 3]. In a 9%-AR3 simulation we would expect P_g/P_I to rise by 0.04per cent (=0.391*0.09). We think that most of the reason for the higher P_g/P_I in 9%-AR3 compared with AR4 is that the share of Oregon's output in national investment is approximately proportional to the share of Oregon in GDP whereas California accounts for a disproportionately low share of national investment. The under representation in production of investment goods in California means that the deterioration in Californian technology would induce a smaller increase in the investment price index in a 9%-AR3 simulation than the corresponding deterioration in Oregon technology in the AR4 simulation. Thus we would expect

$$\left(\frac{P_g}{P_I} * A\right)_{\text{AR4 sim}} < \left(\frac{P_g}{P_I} * A\right)_{\text{9\%-rule AR3 sim}} \tag{5.6}$$

With P_g/P_I rising by 0.01 per cent in AR4 and by 0.04 in 9%-AR3 and with A falling in both simulations by 0.01 per cent we see that

$$0.00 \approx \%\Delta\left(\frac{P_g}{P_I} * A\right)_{\text{AR4 sim}} < \%\Delta\left(\frac{P_g}{P_I} * A\right)_{\text{9\%-rule AR3 sim}} \approx 0.03 \tag{5.7}$$

where %Δ indicates percentage change.

With ROR we would expect from (5.7) and (5.5) that

$$0.00 \approx \%\Delta f_K\left(\frac{K}{L}\right)_{\text{AR4 sim}} > \%\Delta f_K\left(\frac{K}{L}\right)_{\text{9\%-rule AR3 sim}} \approx -0.03 \tag{5.8}$$

Now with f_K a decreasing function of K/L and L fixed it follows from (5.8) that K in AR4 should fall more than a 9%-rule AR3. Can we get a quantitative handle in the gap between K results in AR4 and 9%-AR3?

To explain the K results in a quantitative fashion we build another BOTE model. Production can be thought of as a combination of a technology variable and a CES function of K and L:

$$Y = A * F(K,L) = A * \left[\delta * K^{-\rho} + (1-\delta) * L^{-\rho}\right]^{-1/\rho} \tag{5.9}$$

where δ and ρ are positive parameters. The marginal product of capital is then given by:

$$\begin{aligned} MPK = \frac{\partial Y}{\partial K} &= A * \left(\frac{-1}{\rho}\right) * \left[\delta * K^{-\rho} + (1-\delta) * L^{-\rho}\right]^{\frac{-(1+\rho)}{\rho}} * (-\rho) * \delta * K^{-(1+\rho)} \\ &= A * \delta * \left(\frac{Y}{A}\right)^{(1+\rho)} * K^{-(1+\rho)} = A^{1-\frac{1}{\sigma}} * \delta * Y^{\frac{1}{\sigma}} * K^{\frac{-1}{\sigma}} \end{aligned} \tag{5.10}$$

where σ [=1/(1+ρ)] is the substitution elasticity between capital and labour. In percentage change form (5.10) reduces to

$$mpk = \left(1 - \frac{1}{\sigma}\right) a + \frac{1}{\sigma}(y - k) \tag{5.11}$$

and using (5.3)

$$mpk = a - \frac{S_\ell}{\sigma}(k - \ell) \tag{5.12}$$

so that from (5.1)

$$\%\Delta f_K\left(\frac{K}{L}\right) = -\frac{S_\ell}{\sigma}(k - \ell) \tag{5.13}$$

The share of labour in GDP is approximately 0.56 and σ=0.5 so that in the AR4 and 9%-AR3 simulations with common a and $\ell = 0$ we see that

$$\%\Delta f_K\left(\frac{K}{L}\right)_{9\%\text{-AR3}} - \%\Delta f_K\left(\frac{K}{L}\right)_{AR4} \approx k_{AR4} - k_{9\%\text{-AR3}} \tag{5.14}$$

Combining (5.8) with (5.14) we would expect

$$k_{AR4} - k_{9\%\text{-AR3}} \approx -0.03 \tag{5.15}$$

In fact we find from Figure 5.2 that

$$k_{AR4} - k_{9\%\text{-AR3}} = -0.06 \tag{5.16}$$

So the BOTE model described above has explained half the observed difference in capital growth seen in Figure 5.2. We will return to this discrepancy later.

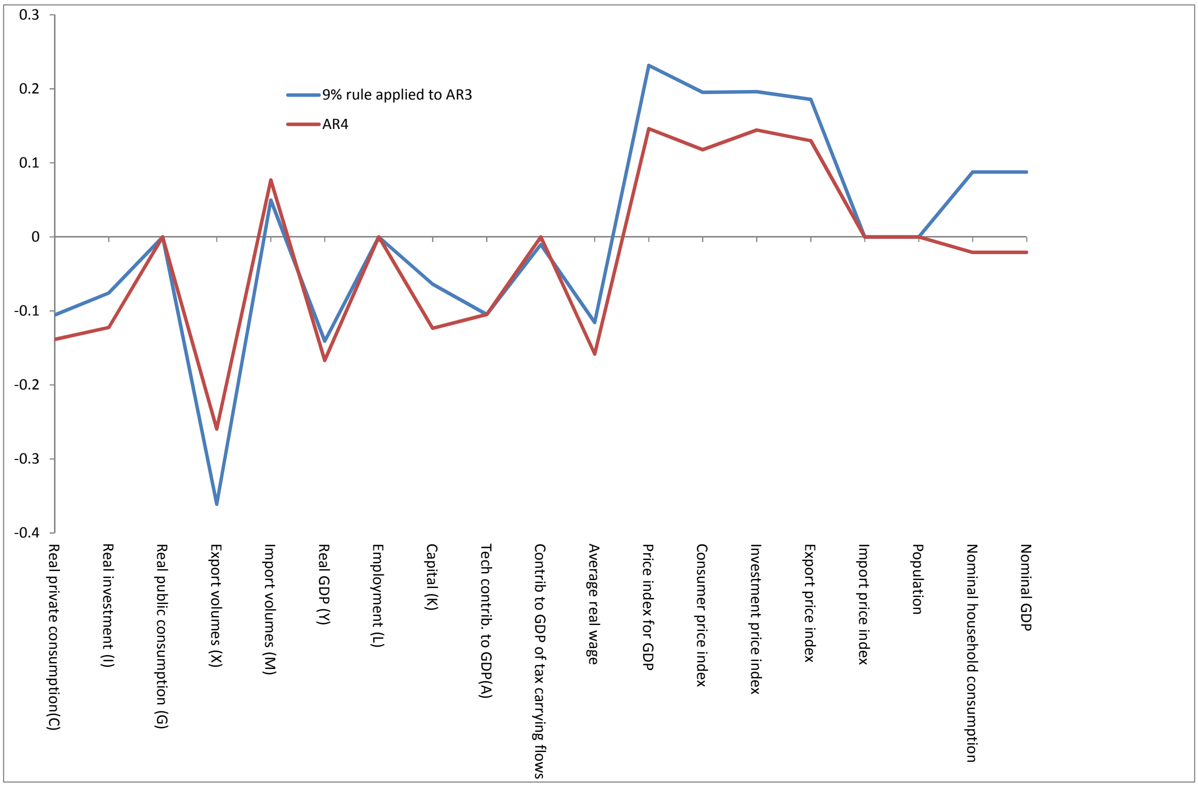
With the same level of deterioration in technology but less K in AR4 compared with 9%-AR3, GDP is lower in AR4 than in 9%-AR3(see Figure 5.2). From (5.3) we would expect that the gap between the GDP points in Figure 5.2 to be about 44 % of that between the capital points. This is closely confirmed in Figure 5.2.

On the expenditure side of GDP, since investment/capital ratios are fixed at the industry and region level the investment points in Figure 5.2 are approximately the same as the capital points. Thus we see a narrowing of the gap between the growth in GDP and investment as we move from AR4 to 9%-AR3:

$$y_{9\%\text{-AR3}} - i_{9\%\text{-AR3}} < y_{AR4} - i_{AR4} < 0 \qquad (5.17)$$

Ignoring for the moments terms-of-trade effects we would expect private consumption to move with GDP and with government expenditure fixed by assumption it follows from (5.17) that the gap between GDP and absorption widens as we move from AR4 to 9%-AR3. Thus there must be a compensating more favourable movement in exports in AR4 compared with 9%-AR3. This can be seen in Figure 5.2 where exports decline less in AR4 than in 9%-AR3. With fixed positions in export demand curves and fixed prices for imports, reduced exports leads to higher prices for exports and consequently improved terms of trade, especially in 9%-AR3. With exports spread broadly across industries the prices of GDP, C and I all rise with again, the rises in 9%-AR3 being greater than those in AR4. The price rises in consumption are less than those in GDP because of positive terms-of-trade effects. With nominal household consumption linked to GDP this means that in both 9%-AR3 and AR4, C declines by less than GDP. The improved private consumption result relative to GDP in 9%-AR3 compared with AR4 helps to explain the gap between the growth in capital anticipated in our BOTE model [see (5.15)] and the actual gap [see (5.16)]. This is because private consumption is a capital-intensive type of expenditure, due to the highly capital intensive consumption item Ownership of dwellings.

Figure 5.2. Comparing AR4 national macro results with the corresponding AR3 results scaled to 9 per cent of their original size



We now turn our attention to regional macro results for AR4. We start with the macro results for Oregon in column (3) of Table 5.2. With -10 per cent primary-factor-saving technical change in Oregon and with primary factors in Oregon worth$140b and national GDP worth $12495b we would expect the contribution of technology to GSP growth in Oregon to contribute -0.11 per cent (=-10*140/12495) close to the result of -10 in row 9 column 3 of Table 5.2.

References

U. S. International Trade Commission (2004), *The Economic Effects of Significant U.S. Import Restraints: Fourth Update*, Investigation No. 332-325, Publication 3701.

Horridge, J.M., J.R. Madden and G. Wittwer (2005), "Impact of the 2002-03 Drought on Australia", *Journal of Policy Modeling*, 27(3), pp. 285-308.

Appendix. Note on regional GDP

In national income accounting GDP is defined by

$$Y = C + I + G + X - M \tag{A1}$$

where C, I and G are the purchasers' values of private consumption, investment and public consumption, X is the f.o.b. value of exports and M is the c.i.f. value of imports. In this note we set out an analogous GDP identity for a region in the USAGE-R51 model. We start by defining GDP in region q as

$$\begin{aligned} Y(q) &= C(q) + I(q) + G(q) + (XIB(q) + XRB(q)) - (MIB(q) + MRB(q)) \\ &+ \left[\sum_{m\in MAR} \sum_{r\in ORG} \sum_{\substack{d\in DST \\ d\neq q}} SUPMAR(m,r,d,q) + \sum_{m\in MAR} \sum_{r\in ORG} SUPMAR(m,r,(q,EXP),q) \right] \\ &- \left[\begin{aligned} &\sum_{m\in MAR} \sum_{r\in ORG} \sum_{\substack{p\in PRD \\ p\neq q}} SUPMAR(m,r,q,p) \\ &\quad - \sum_{m\in MAR} \sum_{r\in ORG} \sum_{\substack{p\in PRD \\ p\neq q}} SUPMAR(m,r,(q,EXP),p) \end{aligned} \right] \\ &+ \sum_{c\in COM} \sum_{s\in SRC} TAX(c,s,(q,EXP)) \end{aligned} \tag{A2}$$

In (A2), the first three terms, C(q), I(q) and G(q), are the purchasers' values of private consumption, investment and public consumption in region q. XIB(q) is the basic values of international export that leave the country from q. Similarly MIB(q) is the basic values of international imports that arrive in q from a foreign country. XIB(q) and MIB(q) are defined by:

$$XIB(q) = \sum_{c\in COM} \sum_{s\in SRC} \sum_{r\in ORG} TRAD(c,s,r,(q,EXP)) \tag{A3}$$

$$MIB(q) = \sum_{c\in COM} \sum_{d\in DST} TRAD(c,\text{"imp"},q,d) \tag{A4}$$

XRB(q) and MRB(q) are the basic values of q's regional exports and imports, defined by

$$XRB(q) = \sum_{c\in COM} \sum_{s\in SRC} \sum_{\substack{d\in DST \\ d\neq q}} TRAD(c,s,q,d) \tag{A5}$$

$$MRB(q) = \sum_{c\in COM} \sum_{s\in SRC} \sum_{\substack{r\in ORG \\ r\neq q}} TRAD(c,s,r,q) \tag{A6}$$

The next term in (A2) is the basic value of region q's exports of margin services. This is calculated as the sum of all margin services supplied by region q to facilitate commodity flows for any purpose to domestic regions outside q plus margin services supplied by q to facilitate international exports that leave the country from q. The basic value of region q's imports of margin services is calculated in (A2) as the sum of all margin services supplied from outside q to facilitate flows to region q but excluding flows that come to q to be exported internationally. The final term in (A2) is taxes collected in q on international exports that leave from q. In USAGE-R51 we assume that all indirect taxes are collected in the destination region.

Equation (A2) provides an adequate definition of GDP in region q. However the terms covering direct and margin exports and imports involve offsetting items. For example, $\mathrm{TRAD}(c,\text{"imp"},q,d)$ for $d \neq q$ appears in (A2) both as an international import for region q and as a regional export for q. This raises the possibility of eliminating terms from the definition of GDP in q. In thinking about this issue, we decided that what should be left in the definition as q's exports is a measure of the value of goods and services produced in q and sold outside q (internationally or to other regions). Similarly what should be left in a definition of q's imports is a measure of the value of goods and services produced outside q but absorbed in q. What we are looking for is definitions of q's imports and exports that are analogous to the definitions used at the national level. Thus, we want to exclude from q's exports and imports values of goods and services that merely pass through q on their way to another destination. To achieve this objective we rearrange (A2) as:

$$\mathrm{Y}(q) = \mathrm{C}(q) + \mathrm{I}(q) + \mathrm{G}(q) + \mathrm{X}(q) - \mathrm{M}(q) \tag{A7}$$

where

$$\begin{aligned}
\mathrm{X}(q) = &\sum_{c\in COM}\sum_{\substack{d\in DST\\ d\neq q}} \mathrm{TRAD}(c,\text{"dom"},q,d) + \sum_{c\in COM} \mathrm{TRAD}(c,\text{"dom"},q,(q,EXP)) \\
&+ \sum_{c\in COM}\sum_{s\in SRC} \mathrm{TAX}(c,\text{"dom"},(q,EXP)) \\
&+ \sum_{m\in MAR}\sum_{\substack{d\in DST\\ d\neq q}} \mathrm{SUPMAR}(m,q,d,q) + \sum_{m\in MAR} \mathrm{SUPMAR}(m,q,(q,EXP),q) \\
&+ \sum_{m\in MAR}\sum_{\substack{r\in ORG\\ r\neq q}}\sum_{\substack{d\in DST\\ d\neq q}} \mathrm{SUPMAR}(m,r,d,q) + \sum_{m\in MAR}\sum_{\substack{r\in ORG\\ r\neq q}} \mathrm{SUPMAR}(m,r,(q,EXP),q)
\end{aligned} \tag{A8}$$

$$\begin{aligned}
\mathrm{M}(q) = &\sum_{c\in COM}\sum_{s\in SRC}\sum_{\substack{r\in ORG\\ r\neq q}} \mathrm{TRAD}(c,s,r,q) + \sum_{c\in COM} \mathrm{TRAD}(c,\text{"imp"},q,q) \\
&- \sum_{c\in COM}\sum_{\substack{r\in ORG\\ r\neq q}} \mathrm{TRAD}(c,\text{"dom"},r,(q,EXP)) \\
&+ \sum_{m\in MAR}\sum_{\substack{r\in ORG\\ r\neq q}}\sum_{\substack{p\in PRD\\ p\neq q}} \mathrm{SUPMAR}(m,r,q,p) - \sum_{m\in MAR}\sum_{\substack{r\in ORG\\ r\neq q}}\sum_{\substack{p\in PRD\\ p\neq q}} \mathrm{SUPMAR}(m,r,(q,EXP),p) \\
&+ \sum_{m\in MAR}\sum_{\substack{p\in PRD\\ p\neq q}} \mathrm{SUPMAR}(m,q,q,p) - \sum_{m\in MAR}\sum_{\substack{p\in PRD\\ p\neq q}} \mathrm{SUPMAR}(m,q,(q,EXP),p)
\end{aligned} \tag{A9}$$

In deriving (A7)-(A9) from (A2) we assume that

$$TRAD(c,"imp",r,(q,EXP)) = 0 \quad \text{for } c \in COM,\ r \in ORG,\ q \in DST \tag{A10}$$

This is consistent with our data and means that internationally imported commodities are never exported internationally without undergoing processing in a domestic industry.

In (A8) our aim is to produce a definition of q's exports that is as closely analogous as possible with the f.o.b. value of exports used in the national measure of GDP. Thus we include on the right hand side of (A8) the basic value of commodities produced in q and sent out of q either to another domestic region or directly to international exports together with associated taxes and margins. The tax term on the right hand side of (A8) is taxes collected in region q on international exports sent from q. This tax term is not ideal: potentially it includes some taxes collected on international exports not produced in q. However, these taxes are negligible. In calculating margins associated with q's exports [the fourth and fifth terms on the right hand side of (A8)] we include only margin services produced in q to facilitate flows from q to other regions or to international exports. We could also have included margins used to facilitate q's exports but produced outside q. However, we decided to treat these margins as outside q's economy. The last two terms on the right hand side of (A8) is what we can think of as the value of q's direct exports of margin services. It covers margins produced in q but used to facilitate flows originating outside q and not *absorbed* in q (excludes deliveries to q that are exported). While margins produced in q and used to facilitate flows between two other regions are clearly exports, the situation is not so clear with respect to margins produced in q and used to facilitate flows from outside q to absorption in q. In excluding these from q's exports we made the judgement that these margin services are delivered mainly within the borders of q. That is, we assumed that they are not part of the c.i.f. value of q's imports (national or international).

In (A9) our aim is to produce a definition of q's imports that is as closely analogous as possible with the c.i.f. value of imports used in the national measure of GDP. The first three terms on the right hand side of (A9) cover the basic value of commodities produced outside q and *absorbed* in q (excludes deliveries to q that are exported). The fourth and fifth terms on the right hand side of (A9) cover margins produced outside q and involved in the delivery of commodities to q for absorption. [As already mentioned we judge that margins produced in q for facilitating q's absorption of goods from outside q are not part of the c.i.f value of q's imports.] Altogether, the first five terms on the right hand side of (A9) are the c.i.f. value of q's imports apart from direct imports of margin services. By direct imports of margin services [the sixth and seventh terms on the RHS of (A9)] we mean services produced outside q to facilitate flows originating in q and absorbed in q (note again that exports from q are not absorbed from q).

Table 5.1 National macro results from four test simulations (percentage changes)

		AR1	AR2	AR3	AR4
1	Real private consumption(C)	0.00	-2.11	-1.17	-0.14
2	Real investment (I)	0.00	-1.45	-0.84	-0.12
3	Real public consumption (G)	0.00	0.00	0.00	0.00
4	Export volumes (X)	0.00	-7.13	-4.01	-0.26
5	Import volumes (M)	0.00	1.61	0.55	0.08
6	Real GDP (Y)	0.00	-2.90	-1.56	-0.17
7	Employment (L)	0.00	0.00	0.00	0.00
8	Capital (K)	0.00	-1.51	-0.71	-0.12
9	Tech contrib. to GDP(A)	0.00	-2.13	-1.16	-0.10
10	Contrib to GDP of tax carrying flows	0.00	-0.16	-0.12	-0.01
11	Average real wage	0.00	-2.35	-1.28	-0.16
12	Price index for GDP	10.00	4.38	2.58	0.15
13	Consumer price index	10.00	3.54	2.17	0.12
14	Export price index	10.00	3.75	2.06	0.13
15	Import price index	10.00	0.00	0.00	0.00
16	Population	0.00	0.00	0.00	0.00
17	Nominal household consumption	10.00	1.36	0.97	-0.02
18	Nominal GDP	10.00	1.36	0.97	-0.02

Table 5.2 Regional macro results from test simulation AR4 (percentage changes)

		California	Nevada	Oregon	Arizona	RoUSA
1	Real private consumption(C)	-0.15	-0.50	-9.31	-0.06	0.00
2	Real investment (I)	-0.13	-0.40	-8.47	-0.06	0.01
3	Real public consumption (G)	0.00	0.00	0.00	0.00	0.00
4	Export volumes (X)	-0.29	-0.18	-8.16	-0.34	-0.20
5	Import volumes (M)	0.00	-0.15	-4.91	-0.05	0.11
6	Real GDP (Y)	-0.05	-0.28	-15.39	0.00	0.05
7	Employment (L)	-0.01	-0.18	-4.70	0.04	0.07
8	Capital (K)	-0.11	-0.38	-9.48	-0.03	0.02
9	Tech contrib. to GDP(A)	0.00	0.00	-0.10	0.00	0.00
10	Contrib to GDP of tax carrying flows	0.00	0.00	-0.01	0.00	0.00
11	Average real wage	-0.16	-0.34	-4.85	-0.12	-0.09
12	Price index for GDP	0.00	-0.08	12.45	0.03	0.01
13	Consumer price index	0.08	0.08	6.07	0.08	0.05
14	Export price index	0.15	0.09	4.33	0.17	0.10
15	Import price index	0.00	0.00	0.00	0.00	0.00
16	Population	0.00	0.00	0.00	0.00	0.00
17	Nominal household consumption	-0.07	-0.42	-3.77	0.02	0.04
18	Nominal GDP	-0.05	-0.36	-4.74	0.03	0.06

Table 5.3 Industry output by region results in simulation AR4 (percentage changes)

	California	Nevada	Oregon	Arizona	RoUSA
1 CropsOthAg	0.22	0.18	-21.93	0.22	0.28
2 Livestock	0.13	-0.14	-25.52	-0.13	0.14
3 ForestFish	1.24	1.25	-35.65	1.04	1.41
4 EnergyMins	-0.37	0.00	-32.21	-0.46	-0.13
5 OresRocks	-0.02	0.13	-23.47	-0.07	0.06
6 Utilities	-0.03	-0.14	-22.31	-0.01	0.05
7 Construction	-0.19	-0.45	-5.53	-0.10	-0.01
8 FoodDrnkTob	0.08	0.13	-24.31	0.08	0.10
9 FabTextClth	-0.22	0.02	-27.44	-0.15	-0.02
10 WoodProds	4.63	4.41	-22.47	3.52	1.87
11 PulpPapPrnt	0.76	0.80	-26.81	0.40	0.23
12 PetrolProds	-0.26	-0.41	-27.26	-0.41	-0.06
13 Chemicals	-0.25	-0.18	-30.15	-0.23	-0.02
14 Plastics	-0.45	-0.43	-26.96	-0.48	-0.04
15 SoapClenPant	-0.16	-0.15	-25.47	-0.14	-0.01
16 TireRubPrds	0.02	0.16	-25.65	-0.05	0.02
17 CemTilePorc	0.11	0.13	-26.96	0.00	0.08
18 GlassPrd	0.66	0.76	-29.36	0.49	0.28
19 OthCemPrds	-0.13	-0.55	-32.90	-0.07	0.04
20 MiscMinPrds	0.10	0.07	-32.26	0.09	0.16
21 SteelPrds	0.87	1.06	-29.70	0.58	0.40
22 NonFePrds	0.07	-0.19	-29.53	0.07	0.16
23 HeatPlmbEtc	0.54	0.54	-23.90	0.30	0.22
24 OthFabrMetl	0.22	0.18	-32.19	0.11	0.15
25 MachineEqp	-0.24	-0.09	-31.20	-0.12	0.04
26 IndElecEqp	0.40	0.46	-28.27	0.22	0.13
27 ITEqp	0.83	1.03	-19.84	0.75	0.67
28 HhldApplElec	0.40	0.48	-23.44	0.58	0.56
29 EltrncOthEle	1.58	1.89	-31.96	1.69	1.90
30 CarsTrksMtBk	-0.15	-0.10	-23.43	-0.15	-0.05
31 MiscManuf	0.24	0.36	-24.49	0.04	0.17
32 WholesleTrde	-0.02	-0.56	-18.76	-0.07	0.08
33 OthTransport	-0.08	-0.19	-17.76	-0.17	-0.03
34 RailRoad	-0.03	-0.01	-16.54	-0.08	0.06
35 Communicat	-0.19	-0.37	-12.36	-0.11	-0.02
36 RetailTr	0.00	-0.22	-16.07	0.05	0.08
37 Publish	-0.32	-0.42	-4.35	-0.22	-0.09
38 MovieSftwr	-0.15	-0.31	-7.14	-0.07	-0.08
39 BusinessSrv	-0.08	-0.24	-19.00	-0.09	0.02
40 ProfServ	-0.23	-0.46	-11.38	-0.14	-0.02
41 WasteMgmt	-0.09	-0.32	-17.93	-0.06	0.04
42 Education	0.08	0.29	-22.21	0.06	0.08
43 Health	-0.13	-0.43	-10.46	-0.05	0.01
44 CommuniCare	-0.17	-0.42	-8.02	-0.08	-0.01
45 Entertain	-0.02	-0.13	-19.27	-0.02	0.04
46 HotelRestrnt	-0.18	-0.35	-8.59	-0.11	-0.02
47 OthService	-0.19	-0.40	-9.13	-0.10	-0.02
48 StatLocGov	-0.01	-0.03	-1.08	-0.01	0.00
49 OwnOccDwell	-0.14	-0.48	-11.88	-0.05	0.01
50 Holiday	-0.14	-0.31	-15.77	-0.09	0.01
51 FgnHol	-0.03	-0.04	-1.23	-0.04	-0.02
52 ExpTour	-0.14	-0.20	-4.44	-0.17	-0.09
53 ExpEdu	-0.23	-0.29	-6.54	-0.21	-0.11
54 OthNonRes	-0.20	-0.36	-7.74	-0.28	-0.13