Appendix Algebraic Specification of the Global CGE Model

This appendix provides a detailed mathematical specification of the ten-region, eleven-sector CGE model for world production and trade used in this paper.

Notation:

Regions are defined in set R and indexed by r or s;

Sectors are defined in set I and indexed by i or j;

Agricultural sectors are defined as a subset of I: IAG(I);

Natural Resource based sectors are defined as a subset of I: RES(I);

Primary factors are defined in set F and indexed by f;

Conventions:

Uppercase English letter indicates variables, unless they have a bar on top, in which case that variable always set exogenously. Greek letter or lower English letter refers to parameters, which need to be calibrated or supplied from exogenous sources. When multiple subscripts of a variable or parameter come from the same set, the first one represents the region or sector supplying goods; the next one represents the region or sector purchasing goods.

Price Equations

Equations 1-11 are price equations in the model. Equations 1 and 2 define the relationship between border (world) prices and internal prices, while equations 3, 4, 6 , 7, and 8 define price indices for aggregate imported goods, Arminton goods, composite value-added, and the firm's output with and without production taxes, respectively. In equations 3, 4, 6, and 7, the price indices are the unit cost functions, while in equation 8 they are unit revenue functions, all of which are dual to the corresponding unit quantity aggregator functions. For example, equation 7 is the result of cost minimization by the representative firm in each sector with respect to its aggregate factor and inputs, subject to a CES production function. Since CES functions are used as the basic building blocks of the model, and this quantity aggregator function is homogeneous of degree one, the total costs can be written as total quantity multiplied by unit cost (Varian, 1984, p28). This implies that the average cost, under cost minimization, is independent of the number of units produced or purchased. Thus, the unit cost function also stands for the price of the composed commodity. Equation 5 defines the unit price for aggregate inputs, which is the IO coefficient weighted sum of all the value of its contents. Equation 9 states the domestic consumer price is the Arminton goods price plus sales taxes. Equation 10 specifies an economy-wide consumer price index, which is used as price of household savings. Equation 11 defines the numeraire in the model. The relation among the various categories of prices in the model is illustrated in Figure A.1.

$$PWM_{isr} = (1 + trs_{isr}) \times PWE_{isr} \tag{1}$$

$$PWE_{isr} = (1 + te_{isr}) \times (\frac{1}{ER_r}) \times PE_{ir} \tag{2}$$

$$PM_{ir} = \frac{1}{\mu_{ir}} \times \{\sum_{s \in R} \xi_{irs}^{\sigma t_i} \times [(1 + tm_{irs} + tn_{irs}) \times ER_r \times PWM_{irs}]^{1-\sigma t_i}\}^{\frac{1}{1-\sigma t_i}} \tag{3}$$

$$PX_{ir} = \frac{1}{\Gamma_{ir}} \times \{\sum \alpha_{ir}^{\sigma m_i} \times PD_{ir}^{1-\sigma m_i} + (1 - \alpha_{ir})^{\sigma m_i} \times PM_{ir}^{1-\sigma m_i}\}^{\frac{1}{1-\sigma m_i}} \tag{4}$$

$$PN_{jr} = \sum_{i \in I} io_{ijr} \times PX_{ir} \tag{5}$$

$$PV_{ir} = \frac{1}{\Lambda_{ir} \times tfp_r \times ITFP_{ir}} \times \{\sum_{f \in F} \delta_{fir}^{\sigma v_i} \times PF_{fr}^{1-\sigma v_i}\}^{\frac{1}{1-\sigma v_i}} \tag{6}$$

$$PP_{ir} = \frac{1}{A_{ir}} \times \{\lambda_{ir}^{\sigma p_i} \times PN_{ir}^{1-\sigma p_i} + (1-\lambda_{ir})^{\sigma p_i} \times PV_{ir}^{1-\sigma p_i}\}^{\frac{1}{1-\sigma p_i}} \tag{7}$$

$$P_{ir} = \frac{1}{\chi_{ir}} \times \{\kappa_{ir}^{\sigma e_{ir}} \times PD_{ir}^{1-\sigma e_i} + (1-\kappa_{ir})^{\sigma e_i} \times PE_{ir}^{1-\sigma e_i}\}^{\frac{1}{1-\sigma e_i}} \tag{8}$$

$$PC_{ir} = (1 + tc_{ir}) \times PX_{ir} \tag{9}$$

$$CPI_r = \frac{\sum_{i \in I} PC_{ir} \times C_{ir}}{\sum_{i \in I} PCO_{ir} \times C_{ir}} \tag{10}$$

$$PID_r = \prod_{i \in I} PC_{ir}^{\beta_{ir}} \times CPI_r^{mps_r} \tag{11}$$

(Insert Figure A.1 here)

Factor Demand and Firms' Supply Equations

Equation 12 and 13 specify the demand functions for aggregate factor and intermediate inputs, while equation 14 gives demand functions of each primary factor. They equal unit demand function multiplied by the quantities of total output, and the unit demand functions are obtained by taking partial derivatives of the unit cost functions (equation 6 and 7) with respect to the relevant factor prices, according to Shephard's lemma.

$$NX_{ir} = (\frac{1}{A_{ir}})^{1-\sigma p_i} \times (\lambda_{ir} \times \frac{PP_{ir}}{PN_{ir}})^{\sigma p_i} \times Q_{ir} \tag{12}$$

$$VA_{ir} = (\frac{1}{A_{ir}})^{1-\sigma p_i} \times [(1-\lambda_{ir}) \times \frac{PP_{ir}}{PV_{ir}}]^{\sigma p_i} \times Q_{ir} \tag{13}$$

$$DF_{fir} = (\frac{1}{\Lambda_{ir} \times tfp_r \times ITFP_{ir}})^{1-\sigma v_i} \times (\delta_{fir} \times \frac{PV_{ir}}{PF_{fr}})^{\sigma v_i} \times VA_{ir} \qquad \sum_{f \in F} \delta_{fir} = 1 \tag{14}$$

Equations 15-18 are the domestic and export supply functions corresponding to the constant elasticity of transformation (CET) function commonly used in today's CGE models. They are derived from revenue maximization, subject to the CET function, in a way similar to the derivation of factor demand functions. Where sv $\subset$ I, represents trade and transport sector. A portion of its output is used in international shipping to transports products from region to region. Equation 19 aggregates exports by the representative firm in each region and determines average export price for each region (PE_{ir}), which implies that producers only differentiate output sold in domestic and foreign markets, but do not differentiate exports by destination (foreign markets are perfect substitutes). Equations 15-18 can be partially or entirely turn off in the model, in such case, $PD_{ir} = PE_{ir} = P_{ir}$ will be enforced and exports and domestic sales become perfect substitutes in the model.

$$DX_{ir} = (\frac{1}{\chi_{ir}})^{1-\sigma e_i} \times (\kappa_{ir} \times \frac{P_{ir}}{PD_{ir}})^{\sigma e_i} \times Q_{ir} \qquad \text{for i} \neq \text{sv} \tag{15}$$

$$DX_{sv,r} = (\frac{1}{\chi_{sv,r}})^{1-\sigma e_{sv}} \times (\kappa_{sv,r} \times \frac{P_{sv,r}}{PD_{sv,r}})^{\sigma e_{sv}} \times (Q_{sv,r} - TRQS_r) \tag{16}$$

$$EX_{ir} = (\frac{1}{\chi_{ir}})^{1-\sigma e_i} \times \{(1-\kappa_{ir}) \times \frac{P_{ir}}{PE_{ir}}\}^{\sigma e_i} \times Q_{ir} \qquad \text{for i} \neq \text{sv} \tag{17}$$

$$EX_{sv,r} = (\frac{1}{\chi_{sv,r}})^{1-\sigma e_{sv}} \times \{(1-\kappa_{sv,r}) \times \frac{P_{sv,r}}{PE_{sv,r}})^{\sigma e_{sv}} \times (Q_{sv,r} - TRQS_r)$$

(18)

$$EX_{ir} = \frac{1}{PE_{ir}} \times \sum_{s \in R} \frac{ER_r}{(1+te_{irs})} \times PWE_{irs} \times X_{irs}$$

(19)

Trade and Final demand Equations

Trade and final demand equations are listed in equations 20-26. Equation 20 is the consumer demand function, which is the Extended Linear Expenditure System derived from maximizing a Stone-Geary utility function subject to household disposable income, which is specified in equation 31. Equation 21 defines household supernumerary income, which is disposal income less total expenditure on the subsistence minimum. Equations 22 and 23 give government and investment demands. Equations 24-26 are demand functions for domestic goods, for aggregate imported goods, and for imported goods by source, respectively. They describe the cost-minimizing choice of domestic and import purchases, as well as import sources. They are derived from corresponding cost functions according to Shephard's lemma in a way similar to the derivation of factor demand functions (taking partial derivatives of the cost function with respect to the relevant component prices). Because of the linear homogeneity of the CES function, the cost function that is dual to the commodity aggregator can be represented by its unit cost function (equations 3 and 4) multiplied by total quantity demanded.

$$C_{ir} = \gamma_{ir} + \frac{\beta_{ir}}{PC_{ir}} \times SY_r$$

(20)

$$SY_r = HDI_r - \sum_{j \in I} PC_{jr} \times \gamma_{jr}$$

$$GC_{ir} = \frac{\theta_{ir}}{PC_{ir}} \times GSP_r \tag{21}$$

$$ID_{ir} = \frac{kio_{ir}}{PC_{ir}} \times INV_r \tag{22}$$

$$DX_{ir} = (\frac{1}{\Gamma_{ir}})^{1-\sigma m_i} \times (\alpha_{ir} \times \frac{PX_{ir}}{PD_{ir}})^{\sigma m_i} \times TX_{ir} \tag{23}$$

$$MX_{ir} = (\frac{1}{\Gamma_{ir}})^{1-\sigma m_i} \times \{(1-\alpha_{ir}) \times \frac{PX_{ir}}{PM_{ir}}\}^{\sigma m_i} \times TX_{ir} \tag{24}$$

$$X_{isr} = (\frac{1}{\mu_{ir}})^{1-\sigma t_i} \times \{\xi_{isr} \times (\frac{PM_{ir}}{(1+tm_{isr}+tn_{irs}) \times ER_r \times PWM_{isr}})^{\sigma t_i} \times MX_{ir} \tag{25}$$

$$\sum_{s \in R} \xi_{isr} = 1 \quad \text{for } s \neq r \tag{26}$$

International Shipping Equations

Equations 27-30 describe international shipping industry in the model. Equations 27 and 28 describe the supply side of the international shipping industry. Equation 27 states that at equilibrium, the returns from shipping activity must cover its cost. Like other industries in the model, it also earns zero profit. Equation 28 describes the demand for each region's service sector exports to the international shipping industry, which is generated by the assumed Cobb-Douglas technology in this industry. The next two equations (29 and 30), refer to the demand side of the international shipping industry. The demand for shipping services associated with commodity i in region r is generated by a fixed proportion input requirement (Leontif) coefficient trs_{isr}, which is routine/commodity specific (equation 29). In equilibrium, the total demand of shipping service must equal its total supply (equation 30).

$$TRQ = \frac{1}{PTR} \times \sum_{r \in R} \frac{P_{sv,r}}{ER_r} \times TRQS_r \tag{27}$$

$$TRQ = \sum_{r \in R} \sum_{i \in I} TRQD_{ir} \tag{28}$$

$$TRQS_r = \frac{\tau_r \times ER_r}{P_{sv,r}} \times PTR \times TRQ \tag{29}$$

$$TRQD_{ir} = \frac{1}{PRT} \times (\sum_{s \in R} trs_{isr} \times PWE_{isr} \times X_{isr}) \tag{30}$$

Income and Saving Equations

Equations 31-39 are income and saving equations in the model. Equations 31 and 32 define household disposal income and savings. Equations 33-37 determine government revenue from production taxes, consumption taxes, tariffs and export taxes (its negative equals a subsidy), respectively, while equations 38-39 define government transfer to household and the balance of trade (foreign savings) in each region.

$$HDI_r = \sum_{f \in F} PF_{fr} \times \overline{FS}_{fr} - dk_r \times \overline{FS}_{KAr} + GTRANS_r \tag{31}$$

$$SAV_r = \frac{HDI_r - \sum_{i \in I} PC_{ir} \times C_{ir}}{CPI_r} \tag{32}$$

$$GR_r = PTAX_r + CTAX_r + TARRIF_r + ETAX_r \tag{33}$$

$$PTAX_r = \sum_{i \in I} tp_{ir} \times P_{ir} \times Q_{ir} \tag{34}$$

$$CTAX_r = \sum_{i \in I} tc_{ir} \times PX_{ir} (C_{ir} + GC_{ir} + ID_{ir}) \tag{35}$$

$$TARRIF_r = \sum_{s \in R} \sum_{i \in I} (tm_{isr} + tn_{irs}) \times ER_r \times PWM_{isr} \times X_{isr} \tag{36}$$

$$ETAX_s = \sum_{r \in R} \sum_{i \in I} te_{isr} \times PE_{is} \times X_{isr} \tag{37}$$

$$GTRANS_r = GR_r - GSP_r - GSVA_r \tag{38}$$

$$BOT_r = \sum_{s \in R} \sum_{i \in I} PWE_{irs} X_{irs} + \frac{P_{sv,r}}{ER_r} \times TRQS_r - \sum_{s \in R} \sum_{i \in I} PWM_{isr} \times X_{isr} \tag{39}$$

General Equilibrium Conditions

Equations 40-43 define general equilibrium conditions of the model, which are system constraints that the model economy must satisfy. For every sector in each region, the supply of the composite goods must equal total demand (equation 40), which is the sum of household consumption (C_{ir}), government purchases (GC_{ir}), investment (ID_{ir}) and the firm's intermediate demand. Similarly, the demand for each factor in every region must equal the exogenously fixed supply (equation 41). In this dual formulation, output in each region is determined by demand. Sectoral equilibrium is determined in equation 42, unit output price equals average cost, which is also the zero profit condition. Equation 43 describes the macroeconomic equilibrium identity in each region, which is also the budget constraint for the investor. Since all agents in each region (households, government, investor, and firms) satisfy their respective budget constraints, it is well known that the sum of the excess demand for all goods is zero; that is, Walras's law holds for each region. Therefore, there is a functional dependence among the equations of the model. One equation is redundant in each region and thus can be dropped.

$$TX_{ir} = C_{ir} + GC_{ir} + ID_{ir} + \sum_{j \in I} io_{ijr} \times NX_{jr} \tag{40}$$

$$\sum_{i \in I} DF_{fir} = \overline{FS_{fr}} \tag{41}$$

$$P_{ir} = \frac{PN_{ir} \times NX_{ir} + PV_{ir} \times VA_{ir} + tp_{ir} \times P_{ir} \times Q_{ir}}{Q_{ir}} \tag{42}$$

$$INV_r = dr_r \times \overline{FS_{k,r}} + CPI_r \times SAV_r + GSAV_r - ER_r \times BOT_r \tag{43}$$

There are 5,712 equations and 5,792 variables in the model. Since the 50 factor endowment variables (FS_r) are determined by initial stock, three additional sets of variables (30) have to be set exogenously as macro closures in order to make the model fully determinate. They are chosen from following variables for alternative closures: (1) gross investment or government transfer (INV_r or $GTRANS_r$), (2) balance of trade or exchange rate (BOT_r or ER_r), (3) government spending or surplus (deficit) (GSP_r or $GSAV_r$).

Trade-productivity Linkages

Equation 44 links import embodied technology transfer (via imports of capital goods, intermediate inputs, and professional services) and total factor productivity. Where $X0_{isr}$ is the base year real trade flows, IM is a subset of I, including those products embodied with advanced technology. It operates through share parameter and elasticities. An elasticity (ip_{ir}) of 0.1 implies that a ten percent increase in real imports of capital and technology intensive goods and services would result a non more than 1 percent increase in total factor productivity in that sector depending the share of intermediate inputs in the sector's total imports.

$$ITFP_{ir} = 1 + ims_{ir} \times \left\{ \frac{NX_{ir}}{NX_{ir} + VA_{ir}} \times \left[\frac{\sum_{j \in IM} \sum_{s \in R} x_{jsr}}{\sum_{j \in IM} \sum_{s \in R} xo_{jsr}} \right]^{\sigma ip_{ir}} + \frac{VX_{ir}}{NX_{ir} + VA_{ir}} - 1 \right\} \tag{44}$$

The model is implemented by the General Algebraic Modeling system (GAMS; Brooke, et. al. 1988) and solved in levels. Readers who are interested in the computer code and related data files may contact the author. Definitions of variables and parameters are list in tables A.1 and A.2.

Welfare Measure

We measure the change in welfare induced by trade liberalization in each period by the Hicksian equivalent variation (EV), with changes in government consumption and investment spending valued according to private household's preference and play the same weight in the regional utility function. The regional investment spending represents future consumption for household and government in the region, which equal the sum of household, government, and foreign savings. Variables ending with a "0" represents base years values of the corresponding variables.

$$\begin{aligned} EV_r = & (HDI_r - CPI_r \times SAV_r) \prod_{i \in I} (\frac{PC0_{ir}}{PC_{ir}})^{\beta_{ir}} - (HDI0_r - SAV0_r) \\ & + GSP_r \prod_{i \in I} (\frac{PC0_{ir}}{PC_{ir}})^{\theta_{ir}} - GSP0_r + INV_r \prod_{i \in I} (\frac{PC0_{ir}}{PC_{ir}})^{kio_{ir}} - INV0_r \end{aligned} \tag{45}$$

Table A.1--Definitions of variables

Variable	Definition	No. of variables
PWE_{isr}	World f.o.b. price for goods from region s to region r s r	I × R(R-1) (990)
PWM_{isr}	World c. i.f. price for goods from region s to region r s r	I × R(R-1) (990)
PM_{ir}	Price of aggregate imported goods in region r	I × R (110)
PX_{ir}	Price of composite goods in region r	I × R (110)
PD_{ir}	Price of domestic products sold at domestic market in region r	I × R (110)
PE_{ir}	Price of domestic goods for exports in region r	I × R (110)
PC_{ir}	Domestic consumer price in region r	I × R (110)
PP_{ir}	Average output price before production tax in region r	I × R (110)
P_{ir}	Average output price after production tax in region r	I × R (110)
PF_{fr}	Factor price in region r	F ×R (102)
PV_{ir}	Price of value added in region r	I × R (110)
PN_{ir}	Price of aggregate intermediate inputs in region r	I × R (110)
CPI_r	Price of savings in region r (consumer price index)	R(10)
ER_r	Exchange rate of region r	R (10)
PID_r	Price index in region r	R (10)
Q_{ir}	Sector output in region r	I × R (110)
VA_{ir}	Variable sector production cost in region r	I × R (110)
NX_{ir}	Aggregate sector intermediate input in region r	I × R (110)
DF_{fir}	Sector factor demand in region r	(F-3) × I × R +(IAG +RES) × R (240)
DX_{ir}	Sector domestic sales in region r	I × R (110)
EX_{ir}	Domestic goods for exports in region r	I × R (110)
C_{ir}	Household consumption in region r	I × R (110)
GC_{ir}	Government spending in region r	I × R (110)
ID_{ir}	Investment demand in region r	I × R (110)
TX_{ir}	Composite goods demand (supply) in region r	I × R (110)
MX_{ir}	Sector composite goods imports in region r	I × R (110)
X_{isr}	Trade flows from region s to region r s r	I × R(R-1) (990)
TRQ	Total international transportation supply	1
PTR	Price of international shipping service	1
$TRQD_{ir}$	International shipping demand by region r	I × R (110)
$TRQS_r$	International shipping service supply by region r	R (10)

HDI_r	Household disposable income in region r	R (10)
SY_r	Household supernumerary income in region r	R (10)
GR_r	Total government revenue in region r	R (10)
GSP_r	Total government spending in region r	R (10)
$TARRIF_r$	Total tariff revenue in region r	R (10)
$ETAX_r$	Total export tax revenue (subsidy expenditure) in region r	R (10)
$PTAX_r$	Total production tax revenue in region r	R (10)
$CTAX_r$	Total consumer sale tax in region r	R (10)
SAV_r	Household savings in region r	R (10)
$GSAV_r$	Government saving (deficit) in region r	R (10)
$GTRNS_r$	Government transfer in region r	R (10)
BOT_r	Balance of trade in region r (net capital inflow)	R (10)
INV_r	Gross investment by region r	R (10)
$ITFP_{ir}$	Import embodied TFP shifter by sector in region r	I × R (110)
FS_{fr}	Factor endowment by region r	F × R (50)
	Total number of variables:	
	17×R+(2×F+IAG+RES)×R+21×I×R+3×I×R(R-1)+(F-3)×I×R+2 (5,792)	

Table A.2--Definitions of parameters

Parameter	Definition
te_{isr}	Sector export tax (subsidy) rate for goods to region r from region s
tm_{isr}	Sector tariff rate for goods from region s in region r
tn_{isr}	Sector NTB for goods from region s in region r
tp_{ir}	Sector indirect tax rate in region r
tc_{ir}	Consumer sale tax rate in region r
trc_{isr}	International transportation cost margin as percent value of f.o.b.
io_{ijr}	Input/output coefficients for region r
kio_{ir}	Sector share of total investment in region r
dk_r	Depreciation rate of capital stock in region r
τ_r	Regional share of international shipping service supply
Γ_{ir}	Unit coefficients in first level Arminton aggregation function
μ_{ir}	Unit coefficients in second level Arminton aggregation function of region r
α_{ir}	Share parameters in the first level Arminton aggregation function of region r
ξ_{ir}	Share parameters in the second level Arminton aggregation function of region r

σm_i	Substitution elasticities between domestic and import goods
σt_i	Substitution elasticities among import goods from different regions
χ_{ir}	Unit coefficients in CET function of region r
κ_{Ir}	Share parameters in CET function of region r
σe_i	Elasticities of transformation between domestic sales and exports
A_{ir}	Unit parameter in aggregate cost function
λ_{Ir}	Intermediate input share in aggregate cost function
σp_{ir}	Elasticities of substitution between aggregate factor and intermediate input
Λ_{ir}	Unit parameter in value added function
δ_{fir}	Factor share in value added function
σv_{ir}	Elasticities of substitution among primary factors in value added
γ_{ir}	Sector minimum subsistence requirements for private households in region r
$ß_{ir}$	Marginal propensity to consume for private households in region r
mps_r	Marginal propensity to savings for private households in region r
θ_{ir}	Sector share of government spending in region r
tfp_r	General TFP shifter in region r
ims_{ir}	The share of of intermediate inputs in sector's total imports
σip_{ir}	Elasticity between intermediate goods import growth with TFP growth

Figure 1 Price system in the model