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Modern trade theory for CGE modelling: the Armington, Krugman and Melitz models

by

Peter B. Dixon, Michael Jerie and Maureen T. Rimmer*

Centre of Policy Studies, Victoria University

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Abstract

The Armington specification of international trade, based on product differentiation at the country level, has been at the heart of computable general equilibrium (CGE) modeling for 40 years. Starting in the 1980s with the work of Krugman and more recently Melitz, trade theorists have preferred specifications in which product differentiation is assumed at the firm level. We draw out the connections between the Armington, Krugman and Melitz models by deriving them as successively less restrictive special cases of an encompassing model.

We then investigate the optimality properties of the Melitz model, demonstrating that a Melitz general equilibrium is the solution to a global, cost-minimizing problem. This raises the possibility that envelope theorems can be used in interpreting results from a Melitz model.

Next we explain the Balistreri-Rutherford decomposition in which a Melitz general equilibrium model is broken into a set of Melitz sectoral models combined with an Armington general equilibrium model. Balistreri and Rutherford see their decomposition as a basis of an iterative approach for solving Melitz general equilibrium models. We see their decomposition as a means for interpreting Melitz results as the outcome of an Armington simulation with additional shocks to productivity and preferences variables.

The paper is written for CGE modelers and others who want to gain access to modern trade theory. This theory would not be of interest to CGE modellers if there we no prospect for empirically determining parameter values. Consequently we explain how parameters are being estimated for Melitz models.

Also with CGE modelers in mind, we describe our computational experience in solving a Melitz general equilibrium model using GEMPACK software. With GEMPACK, Melitz general equilibrium solutions can be computed directly without the Balistreri-Rutherford iterative process. However, the Balistreri-Rutherford decomposition plays a key role in our interpretation of welfare results. Their decomposition allows us to express the welfare result for the effects of a tariff change in a Melitz general equilibrium model as the sum of five components computed from an Armington model: the employment effect; the terms of trade effect; the tax-carrying-flow or efficiency effect; the production technology effect; and the conversion technology or preference effect. The first three of these components are familiar from Armington models. The last two factors are additions to Armington supplied by Melitz. A striking feature in our computations is that these last two components are approximately offsetting, leaving the Melitz welfare result close to that which could be obtained from an Armington model. We conjecture that the offsetting feature is an envelope implication.

That our computed welfare effects of a tariff change in a Melitz general equilibrium model depend almost entirely on Armington mechanisms (terms-of-trade and efficiency effects) suggests that results from a Melitz model might be more generally equivalent to those from an Armington model.

Initially we test this idea by comparing tariff results from Melitz and Armington models built with identical databases and with identical values for the inter-variety (or Armington) substitution parameter, σ. In this test, the Melitz results imply that tariff increases have much more restrictive effects on trade flows and larger welfare effects in absolute terms than the Armington results. It is tempting to interpret this as meaning that the Armington specification leads to under estimates of the restrictiveness and welfare effects of tariffs. However, we don't think that such an interpretation is legitimate. To us, it demonstrates that $\sigma = x$ in a Melitz model doesn't mean the same thing as $\sigma = x$ in an Armington model.

Potentially, it is possible to observe the response of trade flows to tariff changes. Let's assume for the sake of argument that a Melitz model with $\sigma = x$ correctly produces these responses. Can we build an Armington model on the same database as that of the Melitz model which also correctly produces the trade flow responses? Through a series of computations in an admittedly simple framework, we find that the answer is yes. There exists an Armington model with $\sigma > x$ that is closely equivalent in terms of trade responses and welfare effects to a Melitz model with $\sigma = x$.

Conditional on this result being substantiated in further research, we conclude that: (a) Melitz is really a micro-foundation story for Armington; and (b) that CGE modellers can embrace Melitz without throwing away their Armington-based models.

Key words: Armington, Krugman and Melitz; CGE modelling; international trade;

JEL codes: F12; D40; D58; C68.

1. Introduction

This paper is about modern trade theory. Our interest in this topic is from the point of view of computable general equilibrium (CGE) modellers working primarily on policy problems for governments. The paper was initially written just for us. We were trying to understand developments in trade theory over the last ten years and how they relate to the familiar Armington framework that CGE modellers have been using since the 1970s. However, in discussing what we have been doing with other CGE modellers it became apparent that the paper might have broader appeal. Modern trade theory is difficult for applied economists to absorb in a limited amount of time. While we would like to describe the paper as "modern trade theory made easy", that would set up false expectations. Rather, we can describe the paper as modern trade theory made accessible to CGE modellers who are prepared to struggle over some new concepts and follow the associated rather tedious algebra.

The currently dominant form of trade-oriented CGE modelling started with the ORANI model of Australia[1] which adopted the Armington (1969) idea of treating imported and domestic varieties of goods in the same classification as imperfect substitutes. The Armington specification now underlies almost all practical policy-oriented CGE modelling. However its theoretical basis is unattractive: it implies that Japan produces a single variety of cars which is an imperfect substitute for the single variety produced in Germany. Since the 1980s trade theorists have been working on models in which varieties are distinguished by firms rather than countries. Land-mark models in this literature are Krugman (1980) and Melitz (2003). This paper shows how the Armington, Krugman and Melitz models are all specialized versions of a basic model which we call the Armington-Krugman-Melitz Encompassing model or the AKME model. Our approach is inspired by Balistreri and Rutherford (2013) who set out stylized versions of the three models. In their exposition, Balsitreri and Rutherford develop each model separately. We draw out connections between the three models by developing them sequentially as special cases of the AKME model. The Armington model is derived by imposing strong assumptions on the AKME model. Some of these assumptions are relaxed to derive the Krugman model. Further relaxations are made to derive the Melitz model.

In the AKME model, widgets are produced in each country s by an industry containing N_s firms. Consumers in country d treat widgets from different firms around the world as imperfect substitutes. The widget industry in each country s earns zero pure profits. In producing and selling widgets, firms in country s incur three types of costs: variable costs that are proportional to output; fixed setup costs (H_s) ; and a fixed cost in selling to consumers in country d (F_{sd}). The fixed costs are the same for all firms in country s.

In the Armington model, the two types of fixed costs are zero. Armington's firms in country s have identical productivity and behave in a purely competitive manner: that is they perceive the elasticity of demand for their product as ∞. With competitive behaviour and with costs proportional to output, profits for each firm are automatically zero. The number of firms in country s is fixed exogenously. Output variations for the industry are accommodated by output variations for the firms.

In the Krugman model, there are non-zero setup costs, $H_s > 0$, but zero fixed costs on each trade link, $F_{sd} = 0$. Krugman's firms are monopolistically competitive: their perceived elasticity of demand for their product is the actual elasticity which is finite. All widget firms

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1 Dixon *et al.* (1977, 1982).

in country s have the same productivity. The number of firms in country s adjusts endogenously as part of the mechanism of achieving zero pure profits.

In the Melitz model, both types of fixed costs are non-zero. As for Krugman, firms are monopolistically competitive, correctly perceiving the elasticity of demand for their product. In a major departure from Armington and Krugman, Melitz allows for productivity variation across firms in country s. As in Krugman, the number of firms in country s adjusts endogenously to achieve industry-wide zero pure profits. Whereas in Armington and Krugman, all firms in country s sell on all trade links, in Melitz only high productivity firms can sell on trade links for which there are high fixed costs (large values for F_{sd}).

The paper is organized as follows. Section 2 sets out the AKME model and then derives the Armington, Krugman and Melitz models as special cases. Section 3 investigates the optimality properties of an equilibrium in the Melitz model. We demonstrate that in the absence of tariffs, the market equilibrium described by Melitz is cost minimizing, that is the world widget industry minimizes the costs of satisfying given widget demands in each country. Section 4 describes Balistreri and Rutherford's (2013) decomposition of a Melitz general equilibrium model into a set of Melitz single sector models and an Armington general equilibrium model. Balistreri and Rutherford see this decomposition as being valuable in computing solutions for Melitz general equilibrium models. We see it as being important for interpreting Melitz results. Section 5 shows how parameters for Melitz-style models can be estimated. Section 6 sets out an illustrative numerical general equilibrium model with Melitz sectors. We show how Melitz results can be interpreted and how Melitz solutions can be computed directly (without decomposition) via an off-the-shelf application with GEMPACK software. Concluding remarks are in section 7.

2. Armington, Krugman and Melitz as special cases of an encompassing model

2.1. An encompassing model of trade in 10 equations

We start by presenting an encompassing 10-equation system that describes production, pricing and trade for a particular commodity, say widgets. We refer to this as the AKME model: Armington, Krugman, Melitz Encompassing model.

In AKME, each country's widget industry is composed of monopolistically competitive firms. Each firm has the potential to produce its own variety of widget, distinct from widgets produced by other firms. To give itself this potential, a firm incurs a fixed setup cost. The firm then faces an additional fixed setup cost for every market in which it chooses to operate. The potential markets are the domestic market and the market in each other country. After explaining the 10-equation system in this subsection, we then show in subsection 2.2 that the Armington, Krugman and Melitz models are progressively less restrictive special cases.

The ten equations in the AKME model are:

$$P_{ksd} = \left(\frac{W_s T_{sd}}{\Phi_k}\right)\left(\frac{\eta}{1+\eta}\right) \qquad k \in S(s,d) \tag{2.1}$$

$$P_d = \left(\sum_s \sum_{k\in S(s,d)} N_s g_s\left(\Phi_k\right)\left(\delta_{sd}\gamma_{ksd}\right)^{\sigma} P_{ksd}^{1-\sigma}\right)^{1/(1-\sigma)} \tag{2.2}$$

$$Q_{ksd} = Q_d\left(\delta_{sd}\gamma_{ksd}\right)^{\sigma}\left(\frac{P_d}{P_{ksd}}\right)^{\sigma} \qquad k \in S(s,d) \tag{2.3}$$

$$Q_{sd} = \left(\sum_{k \in S(s,d)} N_s g_s\left(\Phi_k\right) \gamma_{ksd} Q_{ksd}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{2.4}$$

$$\Pi_{ksd} = P_{ksd} Q_{ksd} - \left(\frac{W_s T_{sd}}{\Phi_k} \right) Q_{ksd} - F_{sd} W_s \qquad k \in S(s,d) \tag{2.5}$$

$$\Pi_s = \sum_d \sum_{k \in S(s,d)} N_s g_s(\Phi_k) \Pi_{ksd} - N_s H_s W_s \tag{2.6}$$

$$L_s = \sum_d \sum_{k \in S(s,d)} N_s g_s(\Phi_k) \frac{Q_{ksd}}{\Phi_k} + \sum_d \sum_{k \in S(s,d)} N_s g_s\left(\Phi_k\right) F_{sd} + N_s H_s \tag{2.7}$$

$$S(s,d) = \left\{ k : \Phi_k \geq \Phi_{\min(s,d)} \right\} \tag{2.8}$$

$$\Pi tot_s = 0 \tag{2.9}$$

$$\Pi_{\min(s,d)} = 0 \tag{2.10}$$

In these equations,

N_s is the number of firms in country s and $g_s(\Phi_k)$ is the proportion of these firms that have productivity at level Φ_k. A firm's productivity level, assumed to be a given constant for each firm, is the number of additional units of output generated per additional unit of labour (for simplicity we assume that labor is the only input). When we refer to firms in class k in country s we mean the set of firms in s that have productivity Φ_k. The number of firms in this class is $N_s g_s(\Phi_k)$. By $\Phi_{\min(s,d)}$ we mean the minimum value of productivity Φ_k over all firms operating on the sd-link. Technically we do most of the mathematics in this paper as if the possible productivity levels are discrete. This is for ease of exposition.

P_{ksd} is the price in country d of widgets produced in country s by firms in productivity class k. We assume that each class-k firms operating on the sd-link charges the same price for its variety as each other such firm. This assumption is justified because, as we will see, all class-k firms in country s are assumed to be identical: they have the same costs and face the same demand conditions.

W_s is the cost of a unit of labor to widget makers in country s.

T_{sd} is the power[2] of the tariff or possibly transport costs associated with the sale of widgets from s to d. Following Melitz, we assume (rather strangely) that tariffs are charged on the value of the production-labor used in creating imports (excludes fixed costs).

η is the elasticity of demand (restricted to be <-1) perceived by producers in all countries on all their sales.

F_{sd} is the fixed cost (measured in units of labor) incurred by a firm in s to enable it to set up the export of its variety to d.

H_s is the fixed cost (measured in units of labor) for every firm in country s, even those that don't produce anything.

$S(s,d)$ is the set of k labels of firms that send widgets from s to d. With all firms in country s facing the same fixed costs, we can assume that if any class-k firm in country s

2 Power is one plus the rate.

operates on the sd-link then all firms in country s with productivity greater than or equal to Φ_k operate on the sd-link.

P_d is the average price paid by consumers in d for their widgets from all sources.

γ_{ksd} is a positive parameter reflecting d's preference for varieties produced by firms in class k in country s relative to other varieties from s.

δ_{sd} is a positive parameter reflecting d's preference for varieties in general from s relative to those from other countries.

σ (restricted to be >1) is the elasticity of substitution between varieties, assumed to be the same for all consumers in every country and for any pair of varieties wherever sourced.

Q_{ksd} is the quantity of widgets sent from country s to country d by each firm in class k (this includes the s-to-s flows).

Q_{sd} is the quantity of widgets of all varieties sent from s to d (a CES aggregate of the Q_{ksd}s).

Q_d is the total requirement for widgets in d. It can be shown via (2.2)-(2.4) to be a CES aggregate [defined in (2.13) below] of the Q_{sd}s.

Π_{ksd} is the contribution to the profits of a class-k producer in country s from its sales to d. In particular, $\Pi_{\min(s,d)}$ is the contribution of sd-sales to the profits of firms with the lowest productivity [$\Phi_{\min(s,d)}$] of those on the sd-link.

Πtot_s is total profits for firms in country s.

L_s is the employment in the widget industry in country s.

Equation (2.1) is an example of the Lerner mark-up rule. If a class-k firm in country s perceives that its sales to country d are proportional to P_{ksd}^{η} and that its variable cost per unit of sales in country d is $W_s T_{sd}/\Phi_k$, then to maximize its profits it will set its price to country d according to (2.1).[3] With η being less than -1, the mark-up factor on marginal costs $[\eta/(1+\eta)]$ is greater than 1. If firms perceive that they are in highly competitive markets [η approaches $-\infty$], then the mark-up factor is close to 1, that is prices are close to marginal costs. On the other hand, if firms perceive that they have significant market power [η close to -1], then the mark-up factor is large and prices will be considerably greater than marginal costs.

Equation (2.2) defines the average price (P_d) of widgets in country d as a CES average of the prices of the individual varieties sold in country d (P_{ksd}). Equation (2.3) determines the demand in country d for the product of each class-k firm in country s. This is proportional to the total demand for widgets in country d (Q_d) and to a price term which compares the price in d of class-k widgets from s with the average price of widgets in country d. The sensitivity of demand for widgets from a particular class and country to changes in relative prices is controlled by the substitution parameter, σ. Equation (2.4) defines the total quantity of widgets sent from s to d as a CES aggregate of the quantities of each variety sent from s to d. Underlying equations (2.2) to (2.4) is a nested CES optimization problem. People in country d are viewed as choosing Q_{sd} and Q_{ksd} to minimize

[3] Equation (2.1) applies to varieties that are actually sold from s to d, those in the set S(s,d). As to be discussed later, these are the varieties for which non-negative profits can be generated.

$$\sum_{s}\sum_{k\in S(s,d)} Q_{ksd}P_{ksd} \tag{2.11}$$

subject to

$$Q_{sd} = \left(\sum_{k\in S(s,d)} N_s g\left(\Phi_k\right)\gamma_{ksd} Q_{ksd}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{2.12}$$

and

$$Q_d = \left(\sum_{s} \delta_{sd} Q_{sd}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{2.13}$$

Equation (2.5) defines profits for a class-k firm in country s from its sales to country d as: revenue *less* variable costs *less* the fixed costs required to set up sales of a variety on the sd-link. Equation (2.6) defines total profits in the widget industry in country s as the sum of profits over all flows *less* fixed costs in developing the potential for producing varieties. Equation (2.7) defines total employment in the widget industry in country s as the sum of labor used as variable inputs and fixed inputs.

Equation (2.8) defines the set of firms on the sd-link. This is all the firms with productivity levels greater than or equal to $\Phi_{min(s,d)}$.

Equation (2.9) imposes zero profits in the widget industry in country s. Via equation (2.10) it is assumed that firms with the minimum productivity level on the sd-link [$\Phi_{min(s,d)}$] have zero profits on that link.

In considering the 10-equation system, (2.1) to (2.10), it is reasonable to think of W_s, Q_d and T_{sd} as exogenous. In a general equilibrium model, W_s and Q_d would be endogenous but determined largely independently of the widget industry, and T_{sd} can be thought of a naturally exogenous policy variable. We assume that the technology and demand parameters and the distribution of productivities [$g_s(\Phi_k)$] are given. If, initially, we also take as given the number of firms in each country (N_s) and the minimum productivities on each link [$\Phi_{min(s,d)}$] so that (T2.8) can be used to generate S(s,d), then (2.1) to (2.7) can be solved recursively: (2.1) generates P_{ksd}; (2.2) generates P_d; and so on through to (2.7) which generates L_s. The role of (2.9) and (2.10) is to determine N_s and $\Phi_{min(s,d)}$. It is assumed that the number of firms in country s adjusts so that the industry earns zero profits and that the number of firms on the sd-link adjusts so that the link contributes zero to the profits of the link's lowest productivity firm.

2.2. The special assumptions adopted by Armington, Krugman and Melitz

Equations (2.1) to (2.10) involve variables for individual firms. However, practical modelling is done at the industry level, with industries represented by aggregate variables (e.g. industry employment) and by variables for a representative firm (e.g. the price charged by the representative firm in the widget industry in country s). Table 1 shows assumptions adopted by Armington, Krugman and Melitz that assist in translating (2.1) to (2.10) into systems of equations connecting industry variables. These assumptions are largely implicit for Armington who did not start at the firm level but explicit for Krugman and Melitz who did start at the firm level.

Table 1. Assumptions in the Armington, Krugman and Melitz models

	Armington	Krugman	Melitz
Fixed costs for a firm to exist, H_s	0	+	+
Fixed costs for entering a trade link, F_{sd}	0	0	+
Perceived demand elasticity, η	$-\infty$	$-\sigma$	$-\sigma$
d's preference between varieties from s, γ_{ksd}	1 for all k,s,d	1 for all k,s,d	1 for all k,s,d
Productivity for firms in s	$\Phi_{\bullet s}$ $\forall$ firms	$\Phi_{\bullet s}$ $\forall$ firms	Pareto distribution
No. of firms (or potential varieties), N_s	1	endogenous	endogenous
Fraction of s firms on the sd-link, $\sum_{k\in S(s,d)} g_s(\Phi_k)$	1	1	endogenous

As shown in Table 1, there are no fixed costs in the Armington model. Krugman recognises a fixed cost for each firm but not an additional fixed cost for each trade link. Melitz recognises both types of fixed cost.

For Armington, firms operate as if they have no market power: they price at marginal cost. Both Krugman and Melitz assume that firms are aware of the elasticity of demand for their variety implied by (2.3). Consequently they set prices by marking up marginal costs by the factor $\sigma/(\sigma-1)$.[4]

In all three models, d's preferences for varieties from s are symmetric, implying that γ_{ksd} has the same value for all k. Without loss in generality, the γ's can be set at 1.

For Armington and Krugman all firms in country s have the same productivity. For Melitz, productivity varies across firms within a country. As explained in Appendix 1, Melitz sets $g_s(\Phi_k)$ so that productivities in country s form a Pareto distribution.

For Armington there is only one variety of widgets produced in each country. We can assume that this is produced by one firm.[5] For Krugman and Melitz the number of firms in country s (that is entities undertaking the setup cost H_s) is endogenous.

For Armington, the widget variety produced in country s is sold in every market. Similarly for Krugman, every widget variety produced in s is sold in every market. Neither an Armington nor a Krugman firm faces additional fixed costs from entering a market. Thus, with constant marginal costs in production and with the demand curve for its variety exhibiting a constant elasticity, these firms are able to find a price/quantity combination in each market that covers costs attributable to that market. By contrast, Melitz firms face an additional fixed cost for every market into which they sell. Consequently, they may sell into some markets but not others, depending on whether or not they can find a price/quantity combination that generates a sufficient margin over variable costs to cover the market-specific fixed costs. For some firms there may be no markets in which they can cover market-specific fixed costs. These firms will produce nothing. So why were they set up? Melitz assumes that entrepreneurs form firms (that is undertake setup costs H_s) before they

[4] This factor is greater than 1: recall that $\sigma > 1$. Also note that in using (2.3) to calculate country d's demand elasticity for a variety produced by a class-k firm in country s, we ignore the effect of changes in P_{ksd} on P_d.

[5] Our assumption that each variety is produced by only one firm means that for Armington there is only one widget firm in each country. This is not a limiting assumption. It would be acceptable in the Armington framework to assume that there are many firms in country s all producing the same variety. With no fixed costs, the number of firms involved in the production of country s's single variety is indeterminate.

know what productivity level their firm will be able to achieve. Zero production might then be the best they can do if their firm turns out to have low productivity.

For Armington and Krugman, the identity of the representative firm for the widget industry in country s is straightforward. Any firm will do because widget firms in country s are identical in all salient respects: they face identical demand conditions and have the same productivity. The first column of Table 2 sets out the AKME equations, renumbered as (T2.1) to (T2.10). Then the second and third columns show the results of applying the Armington and Krugman assumptions from Table 1. The dot subscript denotes the representative firm.

The Armington and Krugman industry versions of (T2.1) – (T2.8) differ in several ways. The most interesting is the role of N_s in the two versions of (T2.2) and (T2.4). With its value at one, N_s does not appear explicitly in the Armington versions but it does appear in the Krugman versions. For Krugman, the total quantity of widgets sent from s to d (Q_{sd}) is not simply the quantity sent by the representative firm ($Q_{\bullet sd}$) times the number of firms (N_s). Suppose for example that σ were 5. Then a 1 per cent increase in the *number* of firms in s with no change in the number of widgets sent from s to d per firm would generate a 1.25 per cent increase in the *quantity* of widgets sent from s to d even though the count of widgets on the sd-link has increased by only 1 per cent. How does this happen? Love of variety in country d means that the increase in Q_d generated by a 1 per cent increase in varieties from s is the same as that generated by a 1.25 per cent increase in d's consumption of all of the original varieties from s. Correspondingly, an increase in N_s reduces the cost per unit (P_d) to country d of satisfying any given widget requirement (Q_d) even without a change in the price of any variety. An increase in varieties allows d to fulfil its widget requirements with less physical units of widgets and therefore lower costs.

Other differences between Armington and Krugman brought out in Table 2 concern mark-ups and profits. Krugman's representative firm in country s sets prices by marking up marginal costs whereas Armington's representative firm prices at marginal cost in all markets. Profits of all firms in country s on all links and of the industry are automatically zero for Armington, implied by the pricing and technology assumptions. Consequently, we have marked (T2.9) and (T2.10) in the Armington column of Table 2 as not required. Zero industry profits is an additional assumption for Krugman, not implied by the Krugman versions of equations (T2.1) – (T2.8). For this reason, (T2.9) is explicitly included in the Krugman model and, as mentioned in subsection 2.1, can be thought of as determining N_s for all s. On the other hand, (T2.10) is omitted. It is not applicable in the Krugman model. With all firms in country s having the same productivity [$\Phi_{\bullet s}$], all firms receive a positive contribution to their profits from every link. These positive contributions are just sufficient to offset the fixed costs of setting up a firm, $W_s H_s$.

Before we can derive industry versions of (T2.1) – (T2.10) for Melitz, we need to provide an explicit definition for a firm to represent those that send widgets from s to d. Melitz adopts a rather abstract definition in which this is a firm that has the average productivity ($\Phi_{\bullet sd}$) over all firms on the sd-link. Average productivity is specified as a CES average of Φ_k over all $k \in S(s,d)$ with the "substitution" parameter being σ-1: why CES?, why σ-1? Here we provide more intuition.

We define the representative sd-firm as one which employs the average number of production workers, $LPROD_{\bullet sd}$, to service the sd-link. This is given by

Table 2. Eliminating firms from the general equation system: deriving the Armington, Krugman and Melitz models

	AKME 10-equation system	Armington	Krugman	Melitz
(T2.1)	$P_{ksd} = \left(\frac{W_s T_{sd}}{\Phi_k}\right)\left(\frac{\eta}{1+\eta}\right) \quad k\in S(s,d)$	$P_{\bullet sd} = \left(\frac{W_s T_{sd}}{\Phi_{\bullet s}}\right)$	$P_{\bullet sd} = \left(\frac{W_s T_{sd}}{\Phi_{\bullet s}}\right)\left(\frac{\sigma}{\sigma-1}\right)$	$P_{\bullet sd} = \left(\frac{W_s T_{sd}}{\Phi_{\bullet sd}}\right)\left(\frac{\sigma}{\sigma-1}\right)$
(T2.2)	$P_d = \left(\sum_s \sum_{k\in S(s,d)} N_s g_s(\Phi_k)\left(\delta_{sd}\gamma_{ksd}\right)^{\sigma} P_{ksd}^{1-\sigma}\right)^{1/(1-\sigma)}$	$P_d = \left(\sum_s \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}\right)^{\frac{1}{(1-\sigma)}}$	$P_d = \left(\sum_s N_s \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}\right)^{\frac{1}{(1-\sigma)}}$	$P_d = \left(\sum_s N_{sd} \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}\right)^{\frac{1}{(1-\sigma)}}$
(T2.3)	$Q_{ksd} = Q_d\left(\delta_{sd}\gamma_{ksd}\right)^{\sigma}\left(\frac{P_d}{P_{ksd}}\right)^{\sigma} \quad k\in S(s,d)$	$Q_{\bullet sd} = Q_d \delta_{sd}^{\sigma}\left(\frac{P_d}{P_{\bullet sd}}\right)^{\sigma}$	$Q_{\bullet sd} = Q_d \delta_{sd}^{\sigma}\left(\frac{P_d}{P_{\bullet sd}}\right)^{\sigma}$	$Q_{\bullet sd} = Q_d \delta_{sd}^{\sigma}\left(\frac{P_d}{P_{\bullet sd}}\right)^{\sigma}$
(T2.4)	$Q_{sd} = \left(\sum_{k\in S(s,d)} N_s g_s(\Phi_k)\gamma_{ksd} Q_{ksd}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$	$Q_{sd} = Q_{\bullet sd}$	$Q_{sd} = N_s^{\sigma/(\sigma-1)} Q_{\bullet sd}$	$Q_{sd} = N_{sd}^{\sigma/(\sigma-1)} Q_{\bullet sd}$
(T2.5)	$\Pi_{ksd} = P_{ksd}Q_{ksd} - \left(\frac{W_s T_{sd}}{\Phi_k}\right)Q_{ksd} - F_{sd}W_s, k \in S(s,d)$	$\Pi_{\bullet sd} = \left(P_{\bullet sd} - \frac{W_s T_{sd}}{\Phi_{\bullet s}}\right)Q_{\bullet sd}$	$\Pi_{\bullet sd} = \left(P_{\bullet sd} - \frac{W_s T_{sd}}{\Phi_{\bullet s}}\right)Q_{\bullet sd}$	$\Pi_{\bullet sd} = \left(P_{\bullet sd} - \frac{W_s T_{sd}}{\Phi_{\bullet sd}}\right)Q_{\bullet sd} - F_{sd}W_s$
(T2.6)	$\Pi tot_s = \sum_d \sum_{k\in S(s,d)} N_s g_s(\Phi_k)\Pi_{ksd} - N_s H_s W_s$	$\Pi tot_s = \sum_d \Pi_{\bullet sd}$	$\Pi tot_s = \sum_d N_s \Pi_{\bullet sd} - N_s H_s W_s$	$\Pi tot_s = \sum_d N_{sd}\Pi_{\bullet sd} - N_s H_s W_s$
(T2.7)	$L_s = \sum_d \sum_{k\in S(s,d)} N_s g_s(\Phi_k)\frac{Q_{ksd}}{\Phi_k} + \sum_d \sum_{k\in S(s,d)} N_s g_s\left(\Phi_k\right)F_{sd} + N_s H_s$	$L_s = \sum_d \frac{Q_{sd}}{\Phi_{\bullet s}}$	$L_s = \sum_d \frac{N_s Q_{\bullet sd}}{\Phi_{\bullet s}} + N_s H_s$	$L_s = \sum_d \frac{N_{sd} Q_{\bullet sd}}{\Phi_{\bullet sd}} + \sum_d N_{sd} F_{sd} + N_s H_s$
(T2.8)	$S(s,d) = \left\{k : \Phi_k \ge \Phi_{min(s,d)}\right\}$	$S(s,d) =$ all firms	$S(s,d) =$ all firms	$N_{sd} = N_s * \left(\Phi_{min(s,d)}\right)^{-\alpha}$
(T2.9)	$\Pi tot_s = 0$	Not required	$\Pi tot_s = 0$	$\Pi tot_s = 0$
(T2.10)	$\Pi_{min(s,d)} = 0_s$	Not required	Not applicable	$\frac{1}{(\sigma-1)}\left(\frac{W_s T_{sd}}{\Phi_{min(s,d)}}\right)Q_{min(s,d)} - F_{sd}W_s = 0$
(T2.11)	**Additional equations to tie down $\Phi_{\bullet sd}$ and $Q_{min(s,d)}$ in the Melitz model**			$\Phi_{\bullet sd} = \beta\Phi_{min(s,d)}$
(T2.12)				$Q_{min(s,d)} = Q_{\bullet sd} / \beta^{\sigma}$

$$\text{LPROD}_{\bullet sd} = \frac{\sum_{k\in S(s,d)} N_s g_s(\Phi_k)\text{LPROD}_{ksd}}{N_{sd}} \tag{2.14}$$

where N_{sd} is the number of firms that operate on the sd-link and

$$\text{LPROD}_{ksd} = \frac{Q_{ksd}}{\Phi_k} \quad . \tag{2.15}$$

Under the assumption that all the γ's are one, equations (T2.1) and (T2.3) from the AKME model imply that

$$\frac{Q_{ksd}}{Q_{\ell sd}} = \left(\frac{\Phi_k}{\Phi_\ell}\right)^{\sigma} \quad , \tag{2.16}$$

where k and ℓ are any pair of firms in country s operating on the sd-link. Now from (2.15) we obtain

$$\frac{\text{LPROD}_{ksd}}{\text{LPROD}_{\ell sd}} = \left(\frac{\Phi_k}{\Phi_\ell}\right)^{\sigma-1} \quad , \tag{2.17}$$

In particular,

$$\frac{\text{LPROD}_{ksd}}{\text{LPROD}_{\bullet sd}} = \left(\frac{\Phi_k}{\Phi_{\bullet sd}}\right)^{\sigma-1} \quad , \tag{2.18}$$

where $\Phi_{\bullet sd}$ is the productivity of any sd-firm that employs $\text{LPROD}_{\bullet sd}$ production workers to service the sd-link. Finally, we substitute from (2.18) into (2.14). This gives

$$\Phi_{\bullet sd} = \left[\sum_{k\in S(s,d)} \frac{N_s g_s(\Phi_k)}{N_{sd}} \Phi_k^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{2.19}$$

which is Melitz' definition of the productivity of the representative firm on the sd link as a CES average of Φ_k over all $k\in S(s,d)$ with the "substitution" parameter being σ-1. Equation (2.19) establishes that our definition, (2.14), of the representative sd-firm identifies the same firm as Melitz' definition.

With the representative firm on the sd-link identified by (2.14) or equivalently by (2.19) we can derive the Melitz versions of (T2.1) – (T2.10). These are shown in the final column of Table 2.

On examining the Melitz versions of (T2.1) – (T2.7), it can be seen that they define relationships between industry variables as though every firm on the sd-link has the same productivity ($\Phi_{\bullet sd}$) as the representative firm. While this is obviously legitimate for Armington and Krugman, we cannot avoid a little algebra to show that it works for Melitz. This is set out in Appendix 1. Apart from the inclusion of link-specific fixed costs (F_{sd}) and the use of link-specific productivities ($\Phi_{\bullet sd}$ instead of $\Phi_{\bullet s}$) and link-specific numbers of firms (N_{sd} instead of N_s), the Melitz versions of (T2.1) to (T2.7) are the same as the Krugman versions.

The Melitz version of (T2.8) relies on Melitz' Pareto specification of the distribution of productivities. With this distribution, the fraction of firms whose productivity is greater than any given level Φ_{min} equals $\Phi_{min}^{-\alpha}$ where α is a positive parameter (details are in Appendix 1). Thus in the Melitz column of Table 2, we capture what we need to know about S(s,d) by recognizing that the proportion of productivities greater than $\Phi_{min(s,d)}$, which is the same as the proportion of country s firms on the sd-link, is given by

$$\frac{N_{sd}}{N_s} = \left(\Phi_{min(s,d)}\right)^{-\alpha} \quad . \tag{2.20}$$

As for Krugman, Melitz uses (T2.9) to tie down the number of firms (N_s) in country s. For the Melitz version of (T2.10) we have explicitly spelled out profits on the sd-link for the lowest productivity firm ($\Pi_{min(s,d)}$) and equated this to zero. As mentioned earlier, the role of this equation is to determine $\Phi_{min(s,d)}$. However, we still have two loose ends: $Q_{min(s,d)}$ introduced in the last equation as the volume of sales on the sd-link by the link's lowest productivity firm; and $\Phi_{\bullet sd}$ the average productivity of firms on the sd-link. These loose ends are tied up by (T2.11) and (T2.12). Equation (T2.11) uses a property of the Pareto distribution (discussed in Appendix 1) that the average over all productivities greater than any given level is proportional to that level. This leads to (T2.11) where β is a positive parameter. In (T2.12), $Q_{min(s,d)}$ is specified by using (T2.11) and (2.16) with k and ℓ being firms having average ($\Phi_{\bullet sd}$) and minimum ($\Phi_{min(s,d)}$) productivity on the sd-link.

2.3. Computational completeness of the Armington, Krugman and Melitz models in Table 2

In this subsection we briefly review the Armington, Krugman and Melitz models in Table 2 with a view to deciding whether they are likely to be sufficient for determining the widget sector's output, trade and prices for each country.

For Armington there is no difficulty. Under the most obvious closure (W_s, Q_d, T_{sd} exogenous and technology and demand parameters given), the solution of the Armington model in Table 2 can be computed recursively: (T2.1) gives $P_{\bullet sd}$; (T2.2) gives P_d; and so on.

If N_s is exogenous, the Krugman versions of (T2.1) – (T2.8) can also be solved recursively. However, Krugman's major innovation is to endogenize N_s. He does this via (T2.9). This condition has the right dimensions: an extra equation for each country s to determine an extra variable N_s. But the addition of (T2.9) doesn't guarantee a solution of the Krugman model. Nevertheless, in most empirical settings, we would expect a solution to exist and to be revealed by a simple algorithm in which we guess N_s for all s, solve the Krugman version of (T2.1) to (T2.7) recursively, check (T2.9), adjust N_s up (down) if Πtot_s is greater (less) than zero, recompute the recursive solution, and continue until (T2.9) is satisfied. The reason for expecting success with an algorithm of this nature is that in an empirical setting variations in N_s are likely to have a stronger effect on profits (Πtot_s) in country s than profits in other countries, that is we are likely to get a strong diagonal effect[6]. Thus variations in N_s can be assigned the role of guiding us to a situation in which Πtot_s is zero without unduly interfering with the path of Πtot_k towards zero for $k \neq s$.

For the Melitz model in Table 2 we can visualize an algorithmic search for a solution starting, as for Krugman, with a guess of N_s for all s. However we also need to guess $\Phi_{min(s,d)}$ for all s and d. Then, N_{sd}, $\Phi_{\bullet sd}$, $Q_{min(s,d)}$ and $Q_{\bullet sd}$ can be computed from (T2.8) and (T2.10) –

[6] This can be guaranteed if consumers in each country d have a strong preference for widgets produced in country d.

(T2.12). Using the guessed values of N_s and the computed values for $\Phi_{\bullet sd}$ and N_{sd} we solve (T2.1) to (T2.7) recursively. Then we check (T2.9), raising (lowering) our guess of N_s if Πtot_s is greater (less) than zero. Next we compare $Q_{\bullet sd}$ values implied by (T2.3) and (T2.12). If $Q_{\bullet sd}$ in (T2.3) is greater (less) than $Q_{\bullet sd}$ in (T2.12) then we lower (raise) our guess of $\Phi_{min(s,d)}$. We expect this adjustment to close the gap between the two values of $Q_{\bullet sd}$ because (T2.10) and (T2.12) imply that $Q_{\bullet sd}$ in (T2.12) is proportional to $\Phi_{min(s,d)}$ whereas (T2.11) and (T2.1) mean that $Q_{\bullet sd}$ in (T2.3) is approximately proportional to $\Phi^{\sigma}_{min(s,d)}$. Consequently, with $\sigma > 1$, an *x* per cent drop (rise) in $\Phi_{min(s,d)}$ reduces (increases) $Q_{\bullet sd}$ in (T2.3) by more than *x* per cent but reduces (increases) $Q_{\bullet sd}$ in (T2.12) by only *x* per cent. While the success of such an algorithm in a practical computational setting cannot be guaranteed, sketching it out is reassuring. It provides a *prima facie* case that the Melitz versions of (T2.1) – (T2.12) are adequate to determine a solution of the widget model. From a computational point of view, experience reported in Balistreri and Rutherford (2013) suggests to us that, at least for single sector, all of the equations (T2.1) – (T2.12) can be tackled simultaneously, obviating the need for an algorithmic approach at the sectoral level.

3. Optimality in the Armington, Krugman and Melitz models[7]

Krugman modifies Armington by including fixed setup costs for firms, monopolistic competition and prices that exceed marginal costs. Melitz adds intra-country variation across firms in productivity and endogenous determination of average productivity levels for the firms operating on each trade link. An important question is: in the absence of tariffs, do the Krugman and Melitz modifications imply that a market economy produces sub-optimal outcomes? Put another way, are tariffs the only distortions in the Krugman and Melitz specifications? To answer this question, we will work with the AKME model in Table 2. In common with Krugman and Melitz we assume that

$$\eta = -\sigma \text{ and } \gamma_{ksd} = 1 \quad \forall k, s, d. \tag{3.1}$$

With (3.1), the AKME model in Table 2 is a generalization of Melitz: we have not restricted the distribution function $g_s(\Phi_k)$ for productivity levels in country s.

3.1. The AKME model as a cost-minimizing problem

We consider a situation in which the worldwide widget industry is run by a planner whose objective is to satisfy given widget demands at minimum cost (labor costs in production and setup plus tariffs). The planner takes wage and tariff rates as given. We show that if widget technology is in line with AKME assumptions, then the planner will choose outputs and trade flows that could have been generated by a market economy of the type described by the AKME model [AKME equations (T2.1) to (T2.10)]. In short:

$$\text{Cost minimizing} \Rightarrow \text{AKME} \quad . \tag{3.2}$$

We can't go quite as strongly the other way round, but we can show that any AKME market equilibrium satisfies the first-order optimality conditions for the planner's cost minimizing problem:

$$\text{AKME} \Rightarrow \text{First-order optimality conditions for cost minimizing} \quad . \tag{3.3}$$

[7] For a more general presentation of the optimality results given here see Dhingra and Morrow (2012).

If there are no tariffs, then the objective for the planner is minimization of total resource (labor) costs. Consequently, proposition (3.2) creates a presumption that Armington, Krugman and Melitz are one distortion (tariffs) models: in the absence of tariffs we would expect these models to imply that the market generates a solution that meets worldwide widget requirements with minimum use of resources. We can't rule out the possibility *a priori* that an AKME model has multiple solutions some of which are suboptimal, although satisfying the first-order conditions. However, on the basis of the computational literature with which we are familiar (see sections 4 to 6) and on the basis of our own admittedly limited experience, we think that the problem of multiple solutions is more theoretical than practical.

The cost-minimizing planner's problem in (3.2) and (3.3) is:

choose Q_{ksd}, $\Phi_{min(s,d)}$, N_s to minimize

$$\sum_s W_s \left[\sum_d \sum_{k \in S(s,d)} N_s g_s(\Phi_k) * \left(\frac{T_{sd} Q_{ksd}}{\Phi_k} + F_{sd} \right) \right] + \sum_s W_s N_s H_s \tag{3.4}$$

subject to

$$Q_d^{(\sigma-1)/\sigma} = \sum_s \sum_{k \in S(s,d)} N_s g_s(\Phi_k) \delta_{sd} Q_{ksd}^{(\sigma-1)/\sigma} \qquad \forall\, d \tag{3.5}$$

where

$$S(s,d) = \left\{ k : \Phi_k \ge \Phi_{min(s,d)} \right\} \ . \tag{3.6}$$

Expression (3.4) gives the cost of worldwide widget production and distribution including the payment of tariffs. Equation (3.5), which is derived from (2.12), (2.13) and (3.1), requires that exogenous widget demands in country d (Q_d) are satisfied by a CES aggregate of widgets supplied to d from firms throughout the world. Implicit in (3.4) – (3.6) are the assumptions that in the cost-minimizing solution all class-k firms in country s have the same output and trade volumes and all firms in s with productivity greater than or equal to the endogenously determined level $\Phi_{min(s,d)}$ trade on the sd-link.[8]

The first-order conditions for a solution to (3.4) – (3.6) are that the constraint, (3.5) – (3.6), is satisfied and that there exist Λ_d (Lagrangian multipliers) such that

$$-W_s N_s g_s(\Phi_{min(s,d)}) * \left(\frac{T_{sd} Q_{min(s,d)}}{\Phi_{min(s,d)}} + F_{sd} \right) + \Lambda_d N_s g_s(\Phi_{min(s,d)}) \delta_{sd} Q_{min(s,d)}^{(\sigma-1)/\sigma} = 0 \quad \forall s,d \tag{3.7}$$ [9]

$$W_s \sum_d \sum_{k \in S(s,d)} g_s(\Phi_k) * \left(\frac{T_{sd} Q_{ksd}}{\Phi_k} + F_{sd} \right) + W_s H_s - \sum_d \Lambda_d \sum_{k \in S(s,d)} g_s(\Phi_k) \delta_{sd} Q_{ksd}^{(\sigma-1)/\sigma} = 0 \quad \forall s \tag{3.8}$$

$$W_s \left[N_s g_s(\Phi_k) \left(\frac{T_{sd}}{\Phi_k} \right) \right] - \Lambda_d N_s g_s(\Phi_k) \delta_{sd} \left(\frac{\sigma - 1}{\sigma} \right) Q_{ksd}^{-1/\sigma} = 0 \quad \forall s,d \quad \& \quad \forall k \in S(s,d) \tag{3.9}$$

8 If there is a firm in s that is not trading on the sd-link but has productivity greater than or equal to $\Phi_{min(s,d)}$, then it is easy to show that costs can be reduced by allowing this firm to trade on the sd-link and reducing the trade flow for a firm with equal or lower productivity.

9 In deriving this equation we treat Φ_k as a continuous variable.

Equations (3.5) to (3.9) are necessary conditions for a solution of the planners cost minimizing problem. To demonstrate proposition (3.2), we need to show that any set of variable values satisfying (3.5) to (3.9) is consistent with an AKME market equilibrium. To demonstrate proposition (3.3), we need to show that an AKME equilibrium satisfies (3.5) to (3.9).

Proving proposition (3.2)

Let $\Phi_{\min(s,d)}$, N_s, Q_{ksd} and Λ_d be a solution to (3.5) to (3.9) for given values of the exogenous variables W_s, Q_d and T_{sd}. Let P_d and P_{ksd} be defined by

$$\Lambda_d = P_d Q_d^{1/\sigma} \tag{3.10}$$

$$P_{ksd} = \frac{W_s T_{sd}}{\Phi_k}\left(\frac{\sigma}{\sigma-1}\right) \quad . \tag{3.11}$$

We also define Q_{sd}, Π_{ksd}, Πtot_s and L_s as in (T2.4) – (T2.7) of the AKME model. With these definitions, we show in Appendix 2 that $\Phi_{\min(s,d)}$, N_s, Q_{ksd}, P_d, P_{ksd}, Q_{sd}, Π_{ksd}, Πtot_s and L_s satisfies (T2.1) to (T2.10) and is therefore an AKME solution.

Proving proposition (3.3)

Let $\Phi_{\min(s,d)}$, N_s, Q_{ksd}, P_d, P_{ksd}, Q_{sd}, Π_{ksd}, Πtot_s and L_s satisfy (T2.1) to (T2.10) for given values of the exogenous variables W_s, Q_d and T_{sd}. Define Λ_d by (3.10). We show in Appendix 2 that $\Phi_{\min(s,d)}$, N_s, Q_{ksd} and Λ_d is a solution to (3.5) to (3.9).

3.2. Interpretation and significance

Classical presentations of the optimality of market economies generally rely on models in which there are constant or diminishing returns to scale in production and a predetermined or exogenous list of commodities that can be produced (see for example Debreu, 1959, chapter 6, and Negishi, 1960). The propositions outlined in subsection 3.1 show that market optimality can also apply in a model in which production processes exhibit increasing returns to scale and the range of commodities (varieties) produced is endogenous. Thus we have found that the phenomena introduced by Melitz do not necessarily provide a case for policy intervention in a market economy.

Apart from its theoretical and policy implications, we find the equivalence between the AKME model and cost minimization to be of interest for three reasons.

First, it implies that the envelope theorem is applicable. This is helpful in result interpretation. It means that if we start from a specification in the AKME family with zero tariffs, then small movements in exogenous variables will display the usual "envelope" effects. For example, small movements in tariffs will have zero welfare effects; and small movements in production parameters (such as H_s) will have welfare effects reflecting relevant cost shares (the share of $N_s W_s H_s$ in world widget costs). We illustrate this computationally in section 6.

Our second reason for being interested in the AKME cost-minimization equivalence is also related to result interpretation. In explaining the effects of changes in exogenous variables such as tariffs (T_{sd}) or fixed costs (H_s, F_{sd}), it is tempting to argue from the point of view of an all-encompassing agent. For example, if H_s goes up we would expect an all-encompassing agent to satisfy given widget demands (Q_d for all d) by reducing output in country s (in response to the cost increase) but substituting longer production runs for varieties in s (an increase in output per firm and a decrease in the number of firms). This would create a need to produce more in other countries particularly via greater variety. Thus, in other countries we would expect to see an increase in output with the percentage increase in the number of firms exceeding the percentage increase in output per firm. The cost minimizing problem (3.4) to (3.6) legitimizes such explanations based on the behaviour of an all-encompassing optimizing agent as a way of understanding results from AKME multi-agent market models.

Third, understanding the equivalence between the AKME model and cost minimization may be valuable in computations. Balistreri and Rutherford (2013) report that solving general equilibrium models with imperfect competition and increasing returns to scale can be challenging. [We review their computational approach in section 4.] A potential role for problem (3.2) to (3.4) is as a computational framework or at least as a tool for diagnosing computational difficulties. If direct solution of AKME equations proves difficult, then examination of the optimization problem (3.2) to (3.4) may reveal the reason.

4. Melitz sectors and Armington general equilibrium: a decomposition

The difficulty that Balistreri and Rutherford foresee in solving a large scale general equilibrium model with Melitz sectors is dimensionality. They point out that the Melitz model contains several endogenous country-by-country-by-sector variables (e.g. $\Phi_{\bullet sd}, N_{sd}, \Phi_{min(s,d)}$ in Table 2 for each Melitz sector) which are either absent or exogenous in an Armington model. They are also concerned that the increasing-returns-to-scale specification in Melitz (absent in Armington) can cause computational problems.

To overcome the computational problems that they perceive, Balistreri and Rutherford suggest a decomposition or “divide and conquer” approach. They start by solving each Melitz sector as an independent system of equations based on initial guesses of wage rates and overall demand for sectoral product (W_s and Q_d in Table 2). These Melitz computations generate estimates of sectoral productivity and other sectoral variables which are transferred into an Armington multi-sectoral general equilibrium model. The Armington model is solved to generate estimates of wage rates and overall demand for sectoral product which are fed back into the Melitz sectoral computations. A full solution of the general equilibrium model with Melitz sectors is obtained when wage rates and overall demand variables emerging from the Armington model coincide with those which were used in the Melitz sectoral computations.

Balistreri and Rutherford compute in levels using GAMS software.[10] As reported in section 6, we have carried out computations using a linear percentage-change representation of a Melitz model implemented in GEMPACK software.[11] On the basis of this experience, we conjecture that full-scale Melitz models can be solved relatively easily without resort to decomposition. Nevertheless, the Balistreri-Rutherford decomposition method is of theoretical interest: it casts light on the relationship between a traditional Armington model and a Melitz model. It is also of practical interest to CGE modellers who use GAMS. While Balistreri and Rutherford provide GAMS code for their decomposition method, they give only a sketchy account of how it works. In subsection 4.1 we fill in the details. Then in subsection 4.2 we focus on the theoretical relationship between Armington and Melitz exposed by Balistreri and Rutherford. We see this relationship as valuable in understanding simulation results from Melitz models.

[10] See Bisschop and Meeraus (1982), Brooke *et al.* (1992) and Horridge *et al.* (2013).

[11] See Pearson, K.R. (1988), Harrison *et al.* (2014) , and Horridge *et al.* (2013).

4.1. The Balistreri-Rutherford decomposition method for solving general equilibrium models with Melitz sectors

4.1.1. Completing the Melitz general equilibrium model

Imagine that an extra c subscript for c = 1, …, n is added to all of the variables in the Melitz panel of Table 2. These equations then refer to sector/commodity c in an n-commodity model. We complete the n-commodity Melitz model by adding the equations:

$$R_{sd,c} = \left(T_{sd,c} - 1\right)\frac{W_s}{\Phi_{\bullet sd,c}} N_{sd,c} Q_{\bullet sd,c} \qquad (4.1)$$

$$GDP_d = W_d * LTOT_d + \sum_c \sum_s R_{sd,c} \qquad (4.2)$$

$$LTOT_s = \sum_c L_{s,c} \qquad (4.3)$$

$$P_{d,c} Q_{d,c} = \mu_{d,c} * GDP_d \qquad (4.4)$$

Equation (4.1) defines tax revenue collected by country d on its purchases of c from country s. Equation (4.2) defines GDP in country d as the sum of factor income (the wagebill in this relatively simple model) plus indirect taxes collected by country d. Equation (4.3) defines aggregate employment in country s. Equation (4.4) is the consumer demand system in country d. In (4.4), $\mu_{d,c}$ is a non-negative parameter with $\sum_c \mu_{d,c} = 1$. Thus for simplicity we have assumed that the household in country d has a Cobb-Douglas utility function. We also assume that the trade balance for each country is zero: aggregate expenditure on consumption in d equals d’s GDP. With (4.1) – (4.4) added to the equations in the Melitz panel of Table 2, we have a complete general equilibrium model. With aggregate employment in each country ($LTOT_s$ for all s) treated as exogenous, our Melitz general equilibrium model can be solved in principle for all of the endogenous sectoral variables in the Melitz equations in Table 2 together with $R_{sd,c}$, W_s, GDP_d, and $Q_{d,s}$. In performing a solution we need a numeraire (e.g. $W_1 = 1$) and correspondingly we need to delete a component from (4.4), e.g. the component for the last sector in the last country (Walras law).

An obvious decomposition approach to solving the Melitz general equilibrium model is: guess values for $Q_{d,c}$ and W_d for all c and d; solve the Melitz sectoral models for each c, one at a time; use (4.1) to (4.4) to compute the values implied by the sectoral models for $R_{sd,c}$, GDP_d, $LTOT_s$ and $Q_{d,c}$; and then check for conflicts between the implied $LTOT_s$ values and the exogenously known values, and between the implied $Q_{d,c}$ values and those that were assumed in the Melitz sectoral models. If there are no conflicts then we have a solution to the Melitz general equilibrium model. If there are conflicts, then we must revise our guesses of $Q_{d,c}$ and W_d and resolve the Melitz sectoral models. The problem with this algorithm is that it does not offer a clear strategy for revising the guesses for $Q_{d,c}$ and W_d. The Balisteri-Rutherford algorithm overcomes this problem. As we will see, at the end of each iteration in their algorithm an Armington calculation suggests new values for $Q_{d,c}$ and W_d to be used as inputs to the Melitz sectoral models in the next iteration.

4.1.2. The Armington auxiliary model and the evaluation of its productivity, preference and tariff variables from the Melitz model

Table 3 sets out the Armington auxiliary model which can be used in the Balistreri-Rutherford decomposition algorithm to solve the Melitz general equilibrium model defined by the Melitz panel of Table 2 (with commodity subscripts added) plus (4.1) – (4.4). In Table 3 we use “A” to denote Armington variable. Thus, PA(s,d,c) is the Armington version of the price in country d of commodity c from country s.

Table 3. The Armington auxiliary model

Identifier	Equation	Dimension	Endogenous variable
(T3.1)	$PA(s,d,c) = \frac{WA(s)*TA(s,d,c)}{\Phi A(s,c)}$	r^2*n	$PA(s,d,c)$
(T3.2)	$PCA(d,c) = \left(\sum_s \delta A(s,d,c)^{\sigma} * PA(s,d,c)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$	r*n	$PCA(d,c)$
(T3.3)	$QA(s,d,c) = QCA(d,c)*\left(\delta A(s,d,c)*\frac{PCA(d,c)}{PA(s,d,c)}\right)^{\sigma}$	r^2*n	$QA(s,d,c)$
(T3.4)	$LTOTA(s) = \sum_{c,d}\left\{\frac{QA(s,d,c)}{\Phi A(s,c)}\right\}$	r	W(s)
(T3.5)	$RA(s,d,c) = (TA(s,d,c)-1)*\left(\frac{QA(s,d,c)*WA(s)}{\Phi A(s,c)}\right)$	r^2*n	$RA(s,d,c)$
(T3.6)	$GDPA(d) = WA(d)*LTOTA(d) + \sum_{c,s} RA(s,d,c)$	r	GDPA(d)
(T3.7)	$PCA(d,c)*QCA(d,c) = \mu_{d,c} * GDPA(d)$	r*n	$QCA(d,c)$
	Total	$3*r^2n+2*r*n+2*r$	

Notation:
PA(s,d,c) is the Armington version of the price in country d of commodity c from country s;
WA(s) is the Armington wage rate in country s;
TA(s,d,c) is the Armington power of the tariff in country d on sales of c from s;
ΦA(s,c) is the Armington productivity in country s in the production of c;
PCA(d,c) is the overall Armington price of c in d;
δA(s,d,c) is country d's preference variable for commodity c from s;
QA(s,d,c) is the Armington demand in country d for c from s;
QCA(d,c) is the Armington overall demand in country d for c;
σ is the elasticity of substitution between varieties of the same commodity;
LTOTA(s) is the Armington total employment in country s;
RA(s,d,c) is the Armington tariff revenue collected in d on c from s;
GDPA(d) is the Armington GDP in country d;
$\mu_{d,c}$ is the share of d's expenditure devoted to commodity c, $\mu_{d,c} > 0$ for all c and $\sum_c \mu_{d,c} = 1$.

The model in Table 3 is an Armington model for the special case, reflected in our simplified Melitz model, in which: labor is the only input to production; tariffs are the only indirect taxes; and households with Cobb-Douglas preferences are the only final demanders. Equation (T3.1) defines prices in terms of production costs and tariffs. Equation (T3.2) defines the average price of commodity c in country d as a CES function of the prices of commodity c from all sources. Equation (T3.3) is country d's demand function for c from s, derived from a CES cost-minimizing problem. Equation (T3.4) imposes market clearing for labor in country s. Equation (T3.5) defines tariff revenue collected by country d on imports[12] of c from s. Equation (T3.6) defines GDP in country d and (T3.7) determines overall demand for commodity c in country d under a Cobb-Douglas utility function.

If the values of the productivity, preference and tariff variables [$\Phi A(s,c)$, $\delta A(s,d,c)$ and $TA(s,d,c)$] are known and we treat $LTOTA_s$ as an exogenous variable, then the auxiliary model can be solved for the endogenous variables listed in the right hand panel of Table 3.[13]

[12] We assume TA(s,s,c) = 0 for all c and s.

[13] Of course, we would need a numeraire [e.g. WA(1)=1] and we would need to delete one equation (Walras law).

With the model in Table 3 being a standard Armington model the solution can be obtained relatively easily.

The model in Table 3 is the basis for Balistreri and Rutherford's Armington calculation mentioned at the end of subsection 4.1.1. However, before we can see how this works, we need to connect the Melitz general equilibrium model with the Armington model. To do this, we add to the Melitz general equilibrium model definitions of $\Phi A(s,c)$, $\delta A(s,d,c)$ and $TA(s,d,c)$. These definitions strip away complicating aspects of the Melitz model including multiple varieties and productivities in sector c in each country, fixed costs and imperfect competition. They define productivity, preferences, and the power of tariffs as seen through the eyes of an Armington modeller. The definitions do not change the Melitz general equilibrium model: they simply hang off the end using variable values generated in the Melitz model. The definitions are as follows:

$$\Phi A(s,c) = \frac{\sum_d Q_{\bullet sd,c} N_{sd,c}}{L_{s,c}} \tag{4.5}$$

[Productivity in sector c of country s defined as output divided by employment]

$$TA(s,d,c) = 1 + \frac{R_{sd,c}}{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)} \tag{4.6}$$

[Power of the tax on s,d,c sales. The power of the tax is 1 plus tax revenue divided by pre-tax s,d,c cost. We calculate the pre-tax cost of the s,d,c flow as the value of s,d,c sales less taxes on these sales.]

$$\delta A(s,d,c) = \left(\frac{\Phi A(s,c) * \frac{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)}{W_s}}{Q_{d,c}} \right)^{\frac{1}{\sigma}} * \left(\frac{\left(\frac{W_s * TA(s,d,c)}{\Phi A(s,c)} \right)}{\left(\frac{\sum_t P_{\bullet td,c} Q_{\bullet td,c} N_{td,c}}{Q_{d,c}} \right)} \right) \tag{4.7}$$

[Defines the preference variable for good c from country s in country d's CES composite for good c from all sources. Equation (4.7) can be understood as a rearrangement of the demand function for s,d,c set out in (T2.3) and (T2.4) of the Armington panel of Table 2. The numerator in the first fraction on the RHS of (4.7) is our Armington measure of the quantity of the s,d,c flow, i.e. labor productivity times labor input (which is the only input). The numerator in the second fraction is our Armington measure of the purchasers price in region d of commodity c from s, i.e. the wage rate in s inflated by the power of the tariff and deflated by productivity. The denominator in the second fraction is the average purchasers price of commodity c in country d, i.e. the total value of purchases of c in d divided by total quantity .]

4.1.3. The Balistreri-Rutherford algorithm

We now have enough apparatus to set out the Balistreri- utherford algorithm, as follows:

Step 1. Guess values for $Q_{d,c}$ and W_d for all d and c.

Step 2. Solve the Melitz sectoral models [Melitz panel of Table 2 plus (4.1)] for each c, one at a time.

Step 3. Evaluate the Armington productivity, tariff and preference variables recursively using (4.5), (4.6) and (4.7).

Step 4. Solve Armington auxiliary model in Table 3 with $\Phi A(s,c)$, TA(s,c) and δA(s,d,c) set according to the values found in step 3 and LTOTA(s) treated as an exogenous variable set at the level required in the Melitz general equilibrium.

Step 5. Compare the values for QCA(d,c) and WA(d) for all d and c generated at step 4 with the guesses of $Q_{d,c}$ and W_d at step 1.

Step 6. If there are differences at step 5, return to step 1 and revise the guesses. Possible revision rules include:

$$Q_{d,c}^{(1,n+1)} = Q_{d,c}^{(1,n)} + \varepsilon * \left[QCA(d,c)^{(4,n)} - Q_{d,c}^{(1,n)} \right] \text{ and}$$

$$W_{d}^{(1,n+1)} = W_{d}^{(1,n)} + \varepsilon * \left[WA(d)^{(4,n)} - W_{d}^{(1,n)} \right]$$

where the superscript (1,n) denotes guess used at step 1 in the nth iteration, the superscript (4,n) denotes value emerging from step 4 in the nth iteration, and ε is a parameter between 0 and 1.

If there are no differences at step 5 (or the differences are sufficiently small), then the algorithm terminates. In this case, as shown in Appendix 3, we have found a solution to the Melitz general equilibrium model. This consists of: (a) the values of the Melitz variables found at step 2; (b) the $Q_{d,c}$ and W_d values guessed in step 1 (and confirmed in step 5); and (c) the values for GDP_d that can be computed from (4.2).

4.2. The Armington auxiliary model: a tool for interpreting Melitz results

CGE modellers around the world have nearly 40 years experience in interpreting results from models with Armington specifications of international trade. This experience includes understanding the effects in an Armington framework of changes in tariffs [TA(s,c)], changes in productivity [$\Phi A(s,c)$] and changes in preferences [δA(s,d,c)]. The Balisteri-Rutherford decomposition makes this experience relevant in interpreting results from a Melitz general equilibrium model. Melitz results are equivalent to Armington results with extra shocks to productivity and preferences. For example, the effects of a tariff change under Melitz can be interpreted as the combined effects of three sets of shocks under Armington: the tariff shock and shocks to productivity and preferences. We illustrate this idea in section 6.

5. Calibration

Trade models with heterogeneous firms such as the Melitz model are attractive because they gel with findings from microeconomic studies. As explained by Balistreri and Rutherford (2013), micro studies show considerable diversity within industries in firm size and productivity. Consistent with the Melitz theory, micro studies typically show that only high-productivity, large firms have significant exports, and unlike models in which all firms in the country-s widget industry have equal productivity, models with heterogeneous firms offer the possibility of explaining trade-related changes in industry productivity via reallocation of resources between firms.

But how can we put worthwhile numbers to a heterogeneous-firm specification within a CGE model? In this section we explain the estimation/calibration method devised by Balistreri *et al.* (2011). Their method refers to sectors. However in explaining the method we will omit the sectoral/commodity subscript c.

The key to estimating/calibrating for a heterogeneous-firm CGE model is not to take the theory too literally. Consider the Melitz model. It relies on stark assumptions: the widget industry in each country is monopolistically competitive; each firm produces a single unique variety of widget; each widget firm throughout the world faces the same elasticity of demand, σ, in every market; σ is unresponsive to the number of available widget varieties – it is treated as a parameter implying potentially strong "love-of-variety" effects; in every country, the marginal productivities, Φ_k, of widget producers form a simple one-parameter distribution (a Pareto distribution); and every widget firm in country s faces the same fixed cost, W_sH_s to enter the widget industry and the same fixed cost, W_sF_{sd}, to set up trade with country d.

If we try to implement such a theory in a literal fashion with data on numbers of firms and firm-specific costs split into variable costs and different types of fixed costs, then we are likely to become lost in a maze of unsatisfactory data compromises. For example, how would we handle multi-product firms? How would we identify fixed costs specific to different trade links?

By treating the Melitz model as an underlying parable, Balistreri *et al.* (2011) devise a calibration method whereby Melitz sectors can be included in a CGE model in a way that is consistent with robust data and does not depend on impossible definitional conundrums like deciding how many varieties of chemical products are shipped from the U.S. to Japan.[14] Thus, it is possible to build CGE models that can be used to explore the implications of heterogeneous firm theory in the context of observed magnitudes at the industry and country level for trade, output, demands and employment.

5.1. Calibrating a Melitz sector in a CGE model: the Balistreri et al. (2011) method

Balistreri *et al.* (2011) calibrate a Melitz sector in a CGE model using readily available data on trade flows. Their technique starts by accepting a Melitz sectoral specification. If for example the accepted specification were the Melitz version of (T2.1) to (T2.12) in Table 2, then they would write

$$MV^{endo} = f\left(W, T, Q, F, H, \delta, \sigma, \alpha\right) \qquad (5.1)$$

where MV^{endo} is the vector of endogenous Melitz sectoral variables consisting of $P_{\bullet sd}$, $\Phi_{\bullet sd}$, N_{sd}, $Q_{\bullet sd}$, P_d, Q_{sd}, $\Pi_{\bullet sd}$, Πtot_s, N_s, L_s, $\Phi_{min(s,d)}$ and $Q_{min(s,d)}$. For a model with R countries this list contains $8R^2 + 4R$ variables. These can be determined from the corresponding number of Melitz equations provided that we have values for the arguments on the RHS of (5.1): wage rates (W) in each country; powers of tariffs & transport costs[15] (T); total requirements for widgets (Q); link-specific fixed costs (F); firm set-up costs (H); inter-country preferences (δ); the substitution elasticity (σ); and the Pareto parameter describing the distribution of productivity levels across firms (α).[16] Next, Balistreri *et al.* add equations determining trade flows:

$$V_{sd} = N_{sd} P_{\bullet sd} Q_{\bullet sd} \qquad (5.2)$$

[14] The work by Balistreri *et al.* described in this section is a leading example of what Costinot and Rodriguez-Clare (2013) have in mind when they say "… today's researchers try to use their own model to estimate the key structural parameters necessary for counterfactual analysis. Estimation and computation go hand in hand."

[15] In earlier sections we portrayed T as referring to only tariffs. For Balistreri *et al.*(2011), T also encompasses transport margins.

[16] We don't include β on the RHS of (5.1). As explained in Appendix 1, β can be determined from σ and α, see (A1.7).

where V_{sd} is the landed-duty-paid value of the flow of widgets on the sd-link.[17] With data on trade flows together with data on production costs and demands (W and Q in our simplified framework) Balistreri *et al.* have the basis for estimation. They choose values for a *selection* of the unknown variables and parameters (T, F, H, δ, σ and α) to minimize the gap between observed values for trade flows and simulated values from the system (5.1) – (5.2).

Why only a selection? With T, F, H, δ, σ and α we have $3R^2 + R + 2$ unknowns. Equation (5.2) offers only R^2 constraints on estimated values. Consequently, estimates can be obtained for no more than R^2 unknowns, and it is likely that meaningful estimates can be obtained for considerably less than R^2 unknowns. To deal with this problem Balistreri *et al.* adopt a two-prong strategy: they make assumptions concerning some unknowns and reduce the dimensions of others by imposing structures.

For δ, they assume a matrix of 1's. Thus they rule out inter-country preference biases. By contrast, inter-country preference biases play a dominant role in the Armington model in determining the pattern of trade flows. For Balistreri *et al.* (and Melitz), it is differences in link-specific fixed trade costs (the structure of the F matrix) that are used to fill in the explanation of trade patterns beyond what can be attributed to production costs, tariffs & transport costs and total requirements.

For σ, Balistreri *et al.* adopt a value from the literature. These elasticities have been the subject of econometric study since the pioneering work in Australia of Alaouze and colleagues in the 1970s.[18] Thus, in the context of estimating parameters for a Melitz model, it seemed reasonable to Balistreri *et al.* not to use a degree of freedom on σ.[19] Further, we suspect that Balistreri *et al.*'s data (focused mainly on *values* of trade flows) does not provide the sharp definition of differences across widget prices ($P_{\bullet sd}$) required for convincing estimation of substitution elasticities.

For H, Balistreri *et al.* adopt an arbitrary vector of equal values, H_s equals 2 for all s. The value 2 seems a little odd, but it is harmless. The scale of the H vector affects the scale that should be chosen for the F matrix but does not affect the implications of the Melitz model for anything that is potentially observable such as expenditure levels on widgets, values of trade flows, employment levels and the division of costs between fixed and variable. This can be checked by working through the Melitz versions of (T2.1) – (T2.12). Assume that we have an initial solution of these equations. Now double H_s and F_{sd} for all s and d. Then we can immediately generate a new solution in which: the essentially arbitrary numbers of firms (N_s and N_{sd}) are halved; the units for measuring widget requirements are changed so that average widget prices (P_d for all d) are multiplied by $2^{1/(\sigma-1)}$ while widget quantities (Q_d and Q_{sd} for all d and s) are multiplied by $2^{1/(1-\sigma)}$ leaving expenditure (P_dQ_d) on widgets unaffected; and output and profits ($Q_{\bullet sd}, Q_{min(s,d)}, \Pi_{\bullet sd}$) of representative firms are doubled but their productivity and prices ($\Phi_{\bullet sd}, \Phi_{min(s,d)}, P_{\bullet sd}$) are unaffected, as are industry profits and employment ($\Pi tot_s, L_s$ for all s). While an arbitrary choice for the scale of H is harmless, the assumption of

[17] Equation (5.2) is intuitively appealing. However, it needs to be justified. At the end of Appendix 1 we derive it under Melitz assumptions.

[18] See Alaouze (1976) & (1977) and Alaouze *et al.* (1977) which produced estimates of Armington elasticities (σ) for about 50 commodities. These papers are summarized in Dixon *et al.* (1982, section 29.1). Subsequent studies and surveys include Dimaranan and McDougall (2002), Head and Ries, (2001), Hertel *et al.* (2007), McDaniel and Balistreri (2003), Shomos (2005) and Zhang and Verikios (2003).

[19] Against this, the results in subsection 6.4 suggest that σ values appropriate in the context of an Armington model may not be appropriate in the context of a Melitz model.

uniformity across countries is restrictive. What the argument in this paragraph justifies is a free setting of the H for one country, but not the assumption that the H's are equal across countries.

For T, Balistreri *et al.* impose the structure

$$T_{sd} = \left(1+\tau_{sd}\right)D_{sd}^{\theta} \quad \text{for all s and d} \qquad (5.3)$$

where

τ_{sd} is the tariff rate applying to widget flows from s to d;

D_{sd} is a measure of distance between countries s and d, used to represent transport costs for widgets in international and intra-national trade[20]; and

θ is a parameter representing the elasticity of transport margins with respect to distance.

In the context of (T2.1), equation (5.3) implies that tariffs are charged on marginal production costs inflated by transport costs. This is probably not the right base for tariffs, and it is not clear that transport costs should be modelled as proportional to a value ($W_s Q_{\bullet sd} N_{sd} / \Phi_{\bullet sd}$) rather than a volume. However, these are only minor quibbles. With data on τ_{sd} and D_{sd} Balistreri *et al.* use (5.3) to reduce the problem of estimating the R^2 components of T to a problem of estimating a single parameter, θ.

For F, Balistreri *et al.* impose the structure:

$$F_{sd} = \begin{cases} Out_s + In_d & \text{for } s \neq d \\ Out_s & \text{for } s = d \end{cases} \quad . \qquad (5.4)$$

This structure disaggregates setup costs on the sd-link into two parts. First, there are costs (Out_s) required for firms in country s to setup in any market. Then there are additional setup costs (In_d) required only by foreign firms before they can make sales to country d. In part, these latter costs can be visualized as expenditures to overcome non-tariff trade barriers. While the theoretical validity of (5.4) may be questionable, the econometric payoff is clear. It reduces the dimensions of the F parameter space from R^2 to 2R.

The adoption of assumed values for δ, σ and H, and the imposition of structures for T and F gives Balistreri *et al.* a manageable econometric task. The initial list of $3R^2 + R + 2$ unknowns has been condensed to 2R +2: Out_s, In_d, θ and α. Balistreri *et al.* estimated these unknowns using manufacturing trade data for 2001 for the world divided into 12 regions. They obtained interpretable and impressively precise estimates for θ and α. Their estimates of Out_s and In_d seem problematic. However, econometric efforts in this area are in their infancy. Improvements can be expected as econometricians develop the Balistreri framework. Obvious directions for this work are: the use of time-series data rather than data for a single year; the use of data for a wider range of variables (e.g. prices and quantities for trade flows, not just values); refinement of the commodity dimension (e.g. 2- or 3-digit industries rather than a 1-digit sector such as manufacturing); refinement of the regional dimension (avoiding the use of aggregates such as Rest-of-Asia, Korea & Taiwan, etc); and the use of more compelling theoretical restrictions (e.g. relaxation of the assumption of no home bias in preferences).

[20] Normalization of D is required so that simulated total worldwide transport costs for trade in widgets is compatible with data on these costs.

6. Illustrative GEMPACK computations in a general equilibrium model with Melitz sectors

6.1. Setting up and solving a Melitz CGE model

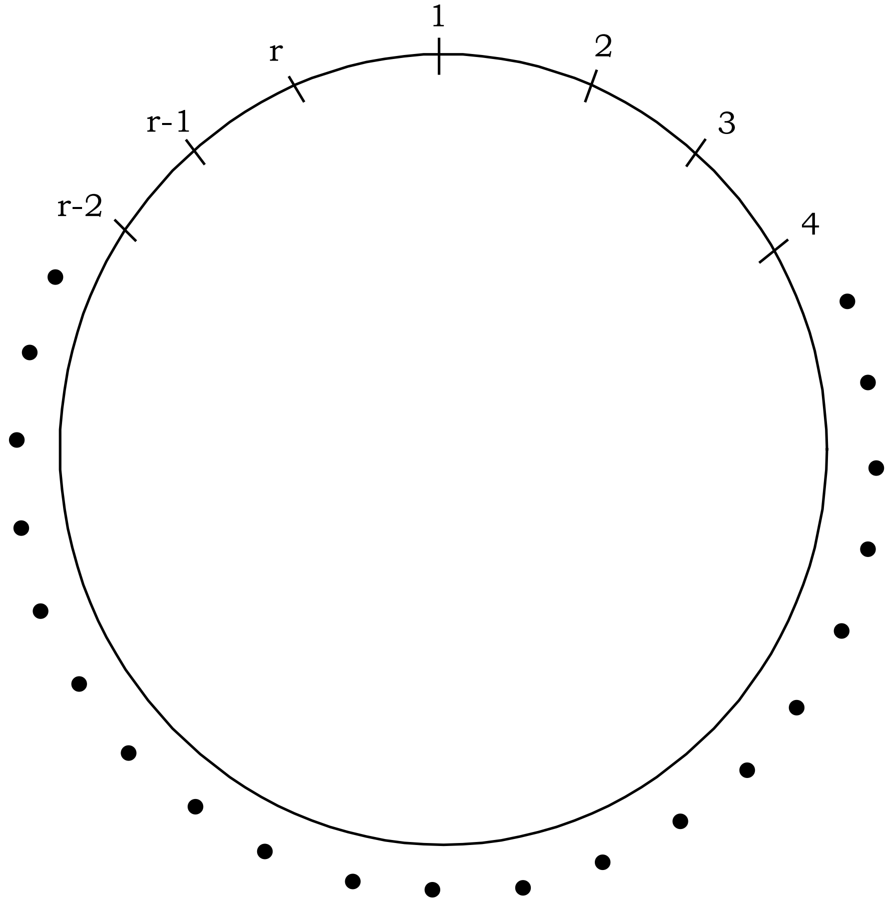
In this section we report results for simulations with an illustrative Melitz general equilibrium model (MelitzGE). The computations were performed using the GEMPACK code presented with annotations in Appendix 6. In computing solutions of an equation system that describes a general equilibrium, GEMPACK starts from an initial solution and then uses a system of linear equations in percentage changes or changes in variables to calculate the movements in the endogenous variables away from their initial values in response to movements in exogenous variables away from their initial values. To fully capture non-linearities in the equation system, GEMPACK computations are conducted in a series of steps. In the first step, the exogenous variables are moved a fraction of the way along the path from their initial values to their desired final values. This gives a new solution for the endogenous variables which is relatively free of linearization error provided that the step size (fraction) is not too big. In the second step GEMPACK calculates the effects on this new solution of another movement in the exogenous variables along the path towards the desired final values. With the movements in the exogenous variables broken into a sufficient number of steps, GEMPACK arrives at an accurate solution for the endogenous variables at the given final values of the exogenous variables.[21]

The code in Appendix 6 is for an n-sector, r-country version of the MelitzGE model specified by the Melitz versions of (T2.1) – (T2.12) and by (4.1) – (4.4). The code also includes linear percentage-change versions of: equations (4.5), (4.6) and (4.7) defining Armington variables for productivity, tariff powers and preferences; equations (T3.1) to (T3.7) specifying the Armington auxiliary model; and various other equations defining variables that will be helpful in analysing results. The code is set up for a special case in which the $n*r$ sectors are identical in the initial solution, facing identical demand and cost conditions. Initially, for all sectors/commodities (c) and countries (s or d): $W_s = 1$ (same wage rate in all countries); $T_{sd,c} = 1$ (zero tariffs); $H_{s,c} = H$ (same fixed setup costs in all sectors); $g_{s,c}(\Phi) = \alpha\Phi^{-\alpha-1}, \Phi \geq 1$ (same Pareto distribution of productivities in all sectors); $\delta_{sd,c} = 1$ (no country preference biases in any sector); $\mu_{d,c} = 1/n$ (equal expenditure shares on all commodities); the substitution elasticity σ is the same across all commodities; and $N_{s,c} = Q_{d,c} = 1$ (two harmless normalizations[22]). The countries can be thought of as located at equal distances on the circumference of a circle (Figure 1), with set up costs, $F_{sd,c}$, being determined by the shortest distance on the circle between s and d. Following Balsitreri and Rutherford (2013), we set α at 4.6 and σ at 3.8 giving $\beta = 1.398$ [see (A1.7)]. Then, in the initial situation, we assume that a firm k needs a productivity level of at least 1.1 ($\Phi_k \geq 1.1$) for it to operate in its own country (non-zero sales on the ss-link). At the other extreme, we assume that the

[21] References for GEMPACK software are given in footnote 11. The original description of the theory underlying the GEMPACK computing method is in Dixon *et al.* (1982, section 8 and chapter 5). For a more recent exposition see Dixon *et al.* (2013, section 2.4).

[22] Doubling the initial value of $Q_{d,c}$ affects the scale that should be chosen for the initial value of the vector $\delta_{sd,c}$ for all s to be consistent with observed values for trade flows, expenditure, etc, but does not affect the implications of the Melitz model for anything that is potentially observable. Following similar arguments to that in section 5.1, it can be shown that doubling the initial values of $N_{s,c}$ can be accommodated by scaling $H_{s,c}$ and $F_{sd.c}$ with no implications for anything that is potentially observable.

Figure 1. Location of countries 1 to r



minimum productivity level required for a firm to operate on all links is 2. With these assumptions and with the countries numbered from 1 to r, we compute initial values for $\Phi_{min(s,d,c)}$ according to:

$$\Phi_{min(s,d,c)} = 1.1 + \frac{(2.0-1.1)}{r} * 2 * MIN\{|s-d|, r-|s-d|\} \quad \text{for all s, d and c.} \qquad (6.1)$$

Under (6.1) the $\Phi_{min(s,d,c)}$'s for country s are spread evenly from 1.1 (for d equal to s) to 2 (for the country or countries furthest from s on the circle). With the initial values of Φ_{min} set in this way we determined the initial values recursively for: $\Phi_{\bullet sd,c}$ via (T2.11); $N_{sd,c}$ via (T2.8); $P_{\bullet sd,c}$ via (T2.1); $P_{d,c}$ via (T2.2); $Q_{\bullet sd,c}$ via (T2.3); $Q_{min(s,d,c)}$ via (T2.12); $F_{sd,c}$ via (T2.10); $Q_{sd,c}$ via (T2.4); $\Pi_{\bullet sd,c}$ via (T2.5); $H_{s,c}$ via (T2.6) and (T2.9);[23] and $L_{s,c}$ via (T2.7).

Identical sectors and countries is a special case. However, we do not use this feature to simplify or speed up our calculations. Thus we think that the GEMPACK experience reported later in the section is a reasonable guide to how the software would perform in an empirically specified model. While our computations refer to a special case, we think it is reasonably representative of a real world situation. In the two country n-commodity case, around which

[23] In the context of our other assumptions concerning the initial solution, $H_{s,c}$ computed from (T2.6) and (T2.9) is the same for all s and c.

most of our discussion is based, the initial solution that we have chosen implies for each country s that: exports (and imports) are 25.4 per cent of GDP; fixed setup costs ($\sum_c W_s H_{s,c} N_{s,c}$) are 16 per cent of GDP; and fixed costs on trade links ($\sum_c \sum_{d \neq s} W_s F_{sd,c} N_{sd,c}$) are 10 per cent of the fob value of exports.

In subsections 6.2 to 6.5 we report results from four sets of GEMPACK simulations with MelitzGE. The first set, reported in subsection 6.2, are test simulations designed mainly to check the validity of our coding. We also use these simulations to demonstrate a point from section 3: intuition gained from envelope theorems and from thinking of results as reflecting the behavior of a single optimizing agent can be useful in interpreting results. The second set, in subsection 6.3 , shows that Melitz tariff results can be interpreted as Armington tariff results with the addition of shocks to productivity and preferences. The third set, in subsection 6.4, investigates further the relationship between tariff results in Melitz and Armington models. We

find that Melitz results computed with the inter-variety substitution elasticity σ set at the value x, say, can be closely approximated in an Armington model built with the same data as the Melitz model but with the Armington elasticity set at a value greater than x. The fourth set, in subsection 6.5, demonstrates that GEMPACK solutions for Melitz models can be computed directly without decomposition in minimal time, even for models with large numbers of countries and Melitz sectors.

6.2. Test simulations and interpreting results

6.2.1. Test simulations

Table 4 contains results from four MelitzGE test simulations. These are simulations for which we know the correct results *a priori*. Test simulations are important in applied general equilibrium modeling because they offer the only reasonably foolproof way of checking the coding of a model. In addition, designing and thinking about test simulations is often a valuable part of understanding a model.

We conduct the test simulations with a two-country, two-commodity version of MelitzGE, that is r = n = 2. The closure (set of exogenous variables) is the same in all four simulations. The exogenous variables are: the average wage rate across countries, which acts as the numeraire; aggregate employment in each country; consumer preferences over sources of commodity c [$\delta_{sd,c}$]; tariff rates; setup costs for a firm in each country and for each commodity [$H_{s,c}$]; setup costs for trade on every link [$F_{sd,c}$]; and the Cobb-Douglas preference coefficients [$\mu_{d,c}$]. [24]

In the first test simulation we impose a 1 per cent increase in the numeraire, the average wage rate across countries. The expected result and the result shown in the first column of Table 4 is zero effect on all real variables (quantities) and a 1 per cent increase in all nominal variables (prices and values).

[24] For d= 2 we allowed uniform percentage endogenous adjustment in $\mu_{d,c}$ across c. This is equivalent to eliminating an equation in accordance with Walras law.

Table 4. Test simulations with MelitzGE
(percentage changes)

Selected variables	Nominal homogeneity (1)	Scaling fixed costs (2)	Scaling consumption (3)	Increased scale (4)
Exogenous variables				
World average wage rate	1.0	0.0	0.0	0.0
Fixed costs , start up & links				
$H_{s,1}$ for all s	0.0	1.0	0.0	0.0
$H_{s,2}$ for all s	0.0	0.0	0.0	0.0
$F_{sd,1}$ for all s,d	0.0	1.0	0.0	0.0
$F_{sd,2}$ for all s,d	0.0	0.0	0.0	0.0
Preference variables				
$\delta_{s2,1}$ for all s	0.0	0.0	0.73588	0.0
All other δ's	0.0	0.0	0.0	0.0
Employment by country				
$LTOT_s$ for all s	0.0	0.0	0.0	1.0
Endogenous variables				
Price of composites, $P_{1,1}$	1.0	0.35601	0.0	-0.35475
$P_{1,2}$	1.0	0.0	0.0	-0.35475
$P_{2,1}$	1.0	0.35601	-0.99015	-0.35475
$P_{2,2}$	1.0	0.0	0.0	-0.35475
Typical link prices $P_{\bullet sd,c}$ for all s,d,c	1.0	0.0	0.0	0.0
Number of firms, $N_{d,1}$ for all d	0.0	-0.99015	0.0	1.0
$N_{d,2}$ for all d	0.0	0.0	0.0	1.0
No. firms on link, $N_{sd,1}$ for all s,d	0.0	-0.99015	0.0	1.0
$N_{sd,2}$ for all s,d	0.0	0.0	0.0	1.0
Employment by commodity $L_{s,c}$ for all s,c	0.0	0.0	0.0	1.0
Consumption by com & country $Q_{1,1}$	0.0	-0.35475	0.0	1.35955
$Q_{1,2}$	0.0	0.0	0.0	1.35955
$Q_{2,1}$	0.0	-0.35475	1.0	1.35955
$Q_{2,2}$	0.0	0.0	0.0	1.35955
Trade by typical firm, $Q_{\bullet sd,1}$ for all s,d	0.0	1.0	0.0	0.0
$Q_{\bullet sd,2}$ for all s,d	0.0	0.0	0.0	0.0
Cons. by com, src, & country $Q_{sd,1}$ for all s,d	0.0	-0.35475	0.0	1.35955
$Q_{sd,2}$ for all s,d	0.0	0.0	0.0	1.35955
Welfare by country welfare(1)	0.0	-0.17753	0.0	1.35955
welfare(2)	0.0	-0.17753	0.49876	1.35955

In the second test simulation we apply 1 per cent shocks to fixed setup costs for firms producing commodity 1 in both countries and to fixed costs for commodity 1 on all links ($H_{d,c}$ and $F_{sd,c}$ for c=1 and all s and d). As shown in column 2 of Table 4, a 1 per cent increase in the H's and F's for commodity 1 has no effect on observable quantities and values:

- employment by commodity and country shows zero effect;
- the price of composite commodity 1 in each country rises by 0.35601 per cent offset by a decline in consumption in each country of 0.35475 per cent leaving the

potentially observable *value* of consumption of commodity 1 in each country unchanged;[25] and

- the number of commodity-1 firms on each link decreases by 0.99015 per cent, the price charged by a typical firm on each link is unchanged and the quantity it ships increases by 1 per cent, implying zero effect on the potentially observable *values* of commodity-1 trade on each link.

These results confirm the argument in subsection 5.1 that in calibrating a Melitz model (setting parameter values) it is legitimate to assign for each commodity an arbitrary value to the H in one country: this merely affects the scaling of the H's for the other countries and all the F's. It doesn't affect the fit of the model to observable quantities and values.

As distinct from calibration, in simulation proportionate movements in the H's and F's matter. For example, column 2 of Table 4 shows that a 1 per cent increase in the H's and F's for commodity 1 reduces welfare in both countries. The percentage change in the welfare of country d arising from a shock is measured in MelitzGE by a weighted average of the percentage changes in d's consumption of each commodity ($Q_{d,c}$, for all c) with the weights being expenditure shares. We will return to the welfare effects of changes in H's and F's in the next subsection where we explain the quantitative result in column 2, a welfare reduction in each country of 0.17753 per cent.

The simulation in the third column of Table 4 confirms another calibration idea: that the initial consumption quantities of composite commodities ($Q_{d,c}$ for all d and c) are essentially arbitrary (see footnote 22). The simulation shows that scaling country 2's preference coefficients for commodity 1 from all sources ($\delta_{s2,1}$ for all s) increases country 2's consumption of composite commodity 1 ($Q_{2,1}$) with a corresponding reduction in its price ($P_{2,1}$) and no change in the potentially observable value ($P_{2,1}*Q_{2,1}$).[26] Again, calibration should not be confused with simulation. In simulation, a uniform percentage increase in $\delta_{s2,1}$ over all s represents an improved ability in country 2 to turn units of commodity 1 from different sources into units of composite commodity 1, and is thus welfare enhancing.

The final simulation in Table 4 shows the effects of a 1 per cent increase in employment in both countries. People imbued with constant-return-to-scale ideas would expect this simulation to generate a 1 per cent increase in all real variables with zero effect on prices. However, as can be seen from column 4 in Table 4, consumption of commodities identified by source ($Q_{sd,c}$ for all s, d and c), consumption of composite commodities ($Q_{d,c}$ for all d and c) and welfare in both countries increase by 1.35955 per cent, and the price of composite commodities falls by 0.35475 per cent.[27] With one per cent more resources (labor) in both countries, MelitzGE shows a 1 per cent increase in the number of firms for each commodity ($N_{s,c}$) and the number of firms on each trade link ($N_{sd,c}$). There is no change in the output of typical firms ($Q_{\bullet sd,c}$). Consequently the count (number of widgets) for each commodity on

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[25] These and other quantitative effects in column 2 of Table 4 can be traced out by following the argument in the paragraph before equation (5.3): an x% increase in the H's and F's for commodity 1 will: move $N_{s,1}$ and $N_{sd,1}$ to 1/(1+x/100) times their initial values, that is reduce them by 100*[1/(1+x/100) -1]% ; move $P_{d,1}$ to {1/(1+x/100)}^ 1/(σ-1)) times its initial values; etc.

[26] We simulated the effects of a 0.73588% increase in the $\delta_{s2,1}$'s. We chose this number in anticipation (confirmed in the simulation) that with σ equal to 3.8, $Q_{2,1}$ would increase by 1%. This can be worked out from (T2.2) and (T2.3): scaling the $\delta_{s2,1}$'s by 1.01^((σ-1)/ σ) multiplies $Q_{2,1}$ by the factor of 1.01, multiplies $P_{2,1}$ by the factor 1/1.01 and changes none of the other Melitz variables in Table 2.

[27] The key to this result is (T2.4). With a 1% increase in employment in all countries there is a 1% increase in the number of firms operating on every link. This multiplies the quantity of composite commodity c on the sd link by the factor 1.01^(σ/(σ-1)). With σ = 3.8, this factor is 1.0135955.

each link increases by 1 per cent. But more firms means more varieties, generating a "love-of-variety" benefit (see the discussion in subsection 2.1 of love of variety). In the Melitz world, even though country d's count for commodity c from country s increases by 1 per cent, the resulting consumption in d of c from s ($Q_{sd,c}$) increases by more than 1 per cent (1.35955 per cent), generating a similar percentage increase in d's consumption of composite c. With more varieties, any given demand for a composite commodity can be satisfied at lower cost. Thus, $P_{d,c}$ falls (by 0.35475 per cent) for all d and c.

6.2.2. Interpreting results: envelope theorems and an optimizing agent

In section 3 we demonstrated an equivalence between a Melitz general equilibrium model and a cost-minimizing problem and suggested that this may be useful in result interpretation. The equivalence indicates that envelope theorems and intuition based on single-agent behaviour may be applicable. In this subsection we return to simulation 2 in Table 4 to illustrate both these ideas.

The envelope theorem gives the expectation that a 1 per cent increase in the commodity-1 H's and F's (as in simulation 2 in Table 4) would reduce welfare by an amount equivalent to that from a loss of labor in each country of 1 per cent of its total fixed-cost labor for commodity 1. Referring to the data items for MelitzGE in Table 5, we see that total fixed-cost labor for commodity 1 in each country is 0.24457 units. The loss of 1 per cent of this fixed-cost labor represents a loss in total labor in each country of 0.131574 per cent (=100*0.0024457/1.85880). In simulation 4 in Table 4, we found that a 1 per cent increase in labor in both countries induces, through a variety effect, an increase in welfare of more than 1 per cent, 1.35955 per cent. Thus we would expect the welfare effect for each country in simulation 2 of Table 4 to be approximately -0.17888 per cent (= -0.131574*1.35955). This is close to the results shown for simulation 2 in the last two rows of Table 4.

Next, we think about the results in column 2 of Table 4 from the point of view of a single optimizing agent. With increases in fixed costs, we would expect a planner in charge of world-wide commodity-1 production to reduce the number of commodity-1 firms and increase output per firm. This is what we see in column 2 of Table 4. The number of commodity-1 firms in each country [$N_{d,1}$] and the number operating on each trade link [$N_{sd,1}$] fall by 0.99015 per cent. At the same time, the typical commodity-1 firm in each country increases its output [$Q_{\bullet sd,1}$] by 1 per cent. As with any increase in costs we would expect our planner to increase prices and for consumers to reduce demand. Again, this is what we see in column 2. The price to consumers per unit of composite commodity 1 [$P_{d,1}$] rises by 0.35601 per cent and demand [$Q_{d,1}$] falls by 0.35475 per cent.

6.3. The effects of a tariff increase in the MelitzGE model

In this subsection we analyse some MelitzGE results for the effects of increases in tariffs. We continue to use the two-country, two-commodity version of MelitzGE, with the same exogenous variables as in subsection 6.2: the average wage rate across countries, which acts as the numeraire; aggregate employment in each country; consumer preferences over sources of commodity c; tariff rates; setup costs for a firm in each country and for each commodity; setup costs for trade on every link; and the Cobb-Douglas preference coefficients.

Table 6 reports results for three experiments. In the first, country 2 increases its tariffs on all imports from an initial level of zero to 10 per cent, that is $T_{12,c}$ increases from 1 to 1.10 for all c. Experiments 2 and 3 give results for the effects of the imposition by country 2 of tariff rates of 19 per cent and 50 per cent. Following Melitz, in MelitzGE country d's tariffs on

Table 5. Selected items from the MelitzGE 2-commodity/2-country database (the data are the same for both commodities c and both countries s)

Data item	Value
Wage rate in country s, W_s	1
Labor requirement for setting up a c-producing firm in country s, $H_{s,c}$	0.14887
Labor requirements for setting up a c-firm on the sd link, $F_{sd,c}$, s = d	0.11065
$F_{sd,c}$, s ≠ d	0.59010
Number of c firms in country s, $N_{s,c}$	1
Number of c firms operating on the sd link, $N_{sd,c}$, s = d	0.64505
$N_{sd,c}$, s ≠ d	0.04123
Total employment in country s, $LTOT_s$	1.85880
Quantity of labor used in setting up c-firms in country s, $H_{s,c}*N_{s,c}$	0.14887
Quantity of labor used in establishing c-firms on sd link, $F_{sd,c}*N_{sd,c}$, s = d	0.07137
$F_{sd,c}*N_{sd,c}$, s ≠ d	0.02433
Fixed cost labor used by c-firms in country s, $H_{s,c} * N_{s,c} + \sum_{d=1}^{2} F_{sd,c} * N_{sd,c}$	0.24457
Value of GDP	1.85880
Value of consumption	1.85880
Value of exports	0.47259
Value of imports	0.47259

imports from country s are charged on production costs in s excluding fixed costs (see section 2). In the Armington auxiliary model in Table 3, tariffs are charged on total costs (there is no division of costs into production and fixed). The bigger base in Armington means that Armington tariff rates calculated in (4.6) are lower than the corresponding Melitz tariff rates. As can be seen from Table 6, the increases in the powers of country 2's Armington tariff rates in our three experiments are 7.180 per cent, 13.333 per cent and 32.558 per cent.

In explaining the results in Table 6 we focus mainly on the first experiment. The imposition of a 10 per cent tariff by country 2 (7.180 percent in Armington terms) causes a sharp contraction in trade. The volume of country 1's exports declines by 18.811 per cent and the volume of country 1's imports declines by 21.622 per cent. Despite the differences in volume movements between exports and imports, country 1's trade remains balanced: country 1 suffers a decline in its terms of trade with the price of its imports rising by 4.958 per cent and the price of its exports rising by 1.324 per cent. The terms of trade decline for country 1 explains why its consumption declines relative to GDP (-0.824 per cent relative to -0.006 per cent) and why its wage rate declines relative to the world-wide average wage rate (-2.011 per cent). The trade results for country 2 are the complement of those for country 1. As with country 1, GDP in country 2 declines, but unlike country 1, country 2's consumption rises relative to GDP (0.593 per cent compared with -0.208 per cent). At least at a qualitative level, none of these results owes anything to the Melitz aspects of MelitzGE. They will all be familiar to CGE modelers who work in the Armington tradition.

Table 6. MelitzGE results for the effects of tariffs imposed by country 2 (percentage changes)

Shocked exogenous variables,%Δ in: / Endogenous variables	$T_{12,c}$=10 for all c: Country d=1	Country d=2	$T_{12,c}$=19 for all c: Country d=1	Country d=2	$T_{12,c}$=50 for all c: Country d=1	Country d=2
Armington power of tariffs, TA(s,d,c)	0.000	7.180	0.000	13.333	0.000	32.558
Real GDP[1]	-0.006	-0.208	-0.011	-0.643	-0.078	-2.617
Real consumption[1]	-0.824	0.593	-1.436	0.726	-2.908	-0.046
Volume of exports[1]	-18.811	-21.622	-32.008	-36.364	-60.370	-66.389
Volume of imports[1]	-21.622	-18.811	-36.364	-32.008	-66.389	-60.370
Price of exports[1]	1.324	4.958	2.210	9.207	3.536	22.078
Price of imports[1]	4.958	1.324	9.207	2.210	22.078	3.536
Wage rate relative to average world rate[1]	-2.011	2.052	-3.678	3.819	-8.550	9.350
Number of firms and quantity flows of typical firms						
$N_{1d,c}$ for all c, number of c-firms on 1d link	5.471	-10.021	9.495	-18.231	18.796	-40.524
$N_{2d,c}$ for all c, number of c-firms on 2d link	-19.390	6.611	-33.062	11.271	-62.477	21.300
$Q_{\bullet 1d,c}$ for all c, typical c-firm flow on 1d link	-0.824	-6.672	-1.436	-11.745	-2.908	-24.767
$Q_{\bullet 2d,c}$ for all c, typical c-firm flow on 2d link	4.797	-1.382	9.118	-2.295	23.750	-4.111
$N_{d,c}$ for all c, c-firms set up in d	1.532	0.000	2.446	0.000	3.714	0.000
Productivity of typical c-firm on sd link						
$\Phi_{\bullet 1d,c}$ for all c, on 1d link	-0.824	2.661	-1.436	5.023	-2.908	12.849
$\Phi_{\bullet 2d,c}$ for all c, on 2d link	4.797	-1.382	9.118	-2.295	23.750	-4.111
Welfare decomposition						
Welfare(d)	-0.824	0.593	-1.436	0.726	-2.908	-0.046
made up of contributions from changes in:						
Employment	0.000	0.000	0.000	0.000	0.000	0.000
Tax-carrying flows	0.000	-0.164	0.000	-0.497	0.000	-1.994
Terms of trade	-0.818	0.802	-1.425	1.375	-2.832	2.617
Production technology or productivity	-3.332	-2.795	-5.890	-5.021	-12.229	-10.835
Conversion technology or preferences	3.327	2.750	5.879	4.869	12.152	10.165

1. All of these variables are calculated using Armington concepts. For example, the percentage change in real consumption for country d, which is the same as the percentage change in d's welfare, is calculated as an expenditure weighted average of percentage movements in d's consumption of composite commodities, QCA(d,c) over all c. The percentage change in the volume of imports for country d is a cif value-weighted average of the percentage changes in QA(s,d,c) over all c and s≠d. The percentage change in the price indexes for exports and imports are calculated from percentage changes in values deflated by percentage changes in volumes.

By contrast, the next three blocks of results in column 1 of Table 6 deliver Melitz insights. With the contraction in trade caused by country 2's imposition of tariffs, both countries increase production for home consumption. In count terms, country 1 increases its supply of commodity c to the domestic market by 4.6 per cent ($N_{11,c} * Q_{\bullet 11,c}$ increases by 4.6 per cent) and country 2 increases supplies to the domestic market by 5.1 per cent ($N_{22,c} * Q_{\bullet 22,c}$ increases by 5.1 per cent). With trade now being a less attractive means for providing variety, both countries increase the number of varieties of commodity c that they provide to the domestic market: $N_{11,c}$ increases by 5.471 per cent and $N_{22,c}$ increases by 6.611 per cent. In both countries, shipments on the domestic market by the typical firm decline slightly ($Q_{\bullet 11,c}$ declines by 0.824 per cent and $Q_{\bullet 22,c}$ declines by 1.382 per cent). The number of firms (includes non-producing start-ups as well as producing firms) in country 1 rises by 1.532 per cent. A rather curious result is that the number of c-firms in country 2 ($N_{2,c}$) is unaffected by country 2's tariffs. This result is derived in Appendix 4 using Table 2.

In count terms, both countries experience a contraction in their exports of about 16 per cent ($N_{12,c} * Q_{\bullet 12,c}$ and $N_{21,c} * Q_{\bullet 21,c}$ decline by about 16 per cent). Variety in both export bundles declines, meaning that export volumes decline by larger percentages than export counts (-18.811 and -21.622 per cent). The variety decline on the 2-to-1 link is particularly sharp. $N_{21,c}$ declines by 19.310 per cent whereas $N_{12,c}$ declines by 10.021 per cent. Correspondingly, the number of units of commodity c sent by the typical firm on the 2-to-1 link increases (4.797 per cent) whereas the number of units sent by the typical firm on the 1-to-2 link decreases (-6.672).

What explains the contrasting reactions of trading firms on the 2-to-1 and 1-to-2 links? That is, why does the trading sector in country 1 react to country 2's tariffs by cutting back on both variety (number of firms on the 1-to-2 link) and shipment per firm, whereas the trading sector in country 2 reacts by cutting back sharply on variety but increasing its shipment per firm. The key equation for understanding this result is (T2.10). With a 10 per cent increase in 1-to-2 tariffs, there must be a 10 per cent reduction in $Q_{min(1,2),c} / \Phi_{min(1,2),c}$, that is the lowest productivity firm on the 1-to-2 link will be one that has 10 per cent less production labor than the low productivity firm in the initial situation.[28] If $\Phi_{min(1,2),c}$ does not change, this would mean that the low-productivity firm on the 1-to-2 link ships 10 per cent less units of commodity c, with an approximately 10 per cent increase in price reflecting the 10 per cent increase in $T_{12,c}$. This would be compatible with a demand elasticity in country 2 of about -1. However in MelitzGE, demand elasticities are higher (in absolute terms) than this. Thus, if $\Phi_{min(1,2),c}$ did not change, $Q_{min(1,2),c} / \Phi_{min(1,2),c}$ would fall by more than 10 per cent. With a high demand elasticity, increases in $\Phi_{min(1,2),c}$ moderate the fall in $Q_{min(1,2),c} / \Phi_{min(1,2),c}$. By the time the number of firms on the1-to-2 link has fallen by 10.021 per cent, the productivity of the lowest productivity firm remaining on the link is sufficiently higher than that of the low productivity firm in the initial situation to ensure that the fall in $Q_{min(1,2),c} / \Phi_{min(1,2,c)}$ is restricted to 10 per cent. In the post-tariff situation, $\Phi_{min(1,2),c}$ is 2.661 per cent higher than in the initial situation and the fall in $Q_{min(1,2),c}$ is restricted to 6.672 per cent. These percentage changes apply also the typical firm on the 1-to-2 link.

Now consider the low productivity firm on the 2-to-1 link. With no change in the 2-to-1 tariffs, (T2.10) implies that the lowest productivity firm on the 2-to-1 link in the post-tariff situation will be one that has the same amount of production labor as the low productivity firm in the initial situation.[29] If $\Phi_{min(2,1),c}$ does not change, this would mean that the low-

[28] Eliminate W_s from (T2.10) and assume $F_{sd,c}$ constant. Then $Q_{min(s,d),c} / \Phi_{min(s,d),c}$, which is production labor used in the low productivity c-firm to service its trade on the sd link, must move to offset movements in $T_{sd,c}$.

[29] See previous footnote. Also note that with no change in $T_{21,c}$, $Q_{min(2,1),c} / \Phi_{min(2,1),c}$ is unchanged and hence $Q_{\bullet 21,c} / \Phi_{\bullet 21,c}$ is unchanged.

productivity firm on the 2-to-1 link ships the same number of units of commodity c in the post-tariff situation as in the initial situation. This is incompatible with the wage movements in the two countries. The wage increase in country 2 relative to that in country 1 means that in the absence of a productivity increase the low productivity firm on the 2-to-1 link in the post-tariff situation would not be sufficiently competitive to ship the same volume as the low productivity firm in the initial situation. $\Phi_{min(2,1),c}$ must be strongly raised in the post-tariff situation relative to the initial situation to allow the price charged by the low productivity 2-to-1 firm to fall sufficiently to be compatible with an increase in $Q_{min(2,1),c}$ that matches the increase in $\Phi_{min(2,1),c}$. As shown in Table 6, the required increase in $\Phi_{min(2,1),c}$ is not achieved until the number of firms on the 2-to-1 link is reduced by 19.390 per cent. By this time $\Phi_{min(2,1),c}$ is 4.797 per cent higher than in the initial situation with an equal percentage rise in $Q_{min(2,1),c}$. Again, these percentage changes apply also to the typical firm. Thus, while country 2's imposition of tariffs leads to an increase in productivity of the typical trading firm in both countries, the typical trading firm in country 1 reduces its shipments while the typical trading firm in country 2 increases its shipments.

6.3.1. Decomposing MelitzGE welfare results via an Armington model: theory

In section 4 we demonstrated that Melitz results are equivalent to Armington results with extra shocks to productivity and preferences. Using this idea we set out a decomposition equation for interpreting the welfare effects of a tariff change in MelitzGE.

As in the computations reported earlier in this section, we define the percentage change in welfare in country d in MelitzGE as a weighted average of the percentage changes in d's consumption of composite commodities:

$$\text{welfare}(d) = \sum_{c} Z(d,c) * q_{d,c} \quad \text{for all d} \ , \tag{6.2}$$

where

welfare(d) is the percentage change in d's welfare;

$q_{d,c}$ is the percentage change in d's consumption of composite commodity c (that is the percentage change in $Q_{d,c}$); and

Z(d,c) is the share of d's consumption expenditure devote to c, that is

$$Z(d,c) = \left(\frac{P_{d,c} Q_{d,c}}{\sum_{j} P_{d,j} Q_{d,j}} \right) \quad \text{for all c and d} \ . \tag{6.3}$$

Recognizing that MelitzGE results for welfare can be generated by the Armington auxiliary model with movements in productivity [$\Phi A(s,c)$], tariffs [TA(s,c)] and preferences [δA(s,d,c)] given by (4.5), (4.6) and (4.7), we can work with Table 3 to disaggregate the MelitzGE result for welfare(d) into five Armington components. These are shown for the two country case[30] in Figure 2 as the contributions to welfare of changes in: employment; tax-carrying flows; the terms of trade; production technology; and conversion technology or preferences. The algebra underlying Figure 2 is given Appendix 5. Here, we provide an intuitive explanation of the five components.

[30] The equation in Figure 2 is easily generalized to the r-country case.

Figure 2. Armington decomposition of Melitz welfare

$$\left(\sum_c \sum_s PA(s,d,c)*QA(s,d,c)\right)*\text{welfare}(d) =$$

$$WA(d)*LTOTA(d)*\text{ltota}(d)$$
$$\left[\textit{changes in employment}\right]$$
$$+\sum_c \sum_s \frac{PA(s,d,c)*QA(s,d,c)}{TA(s,d,c)}*\left(TA(s,d,c)-1\right)*qa(s,d,c)$$
$$\left[\textit{changes in tax-carrying flows}\right]$$
$$+\left\{\begin{array}{l}\sum_c \frac{PA(d,F,c)*QA(d,F,c)}{TA(d,F,c)}*\left(pa(d,F,c)-ta(d,F,c)\right)\\ -\sum_c \frac{PA(F,d,c)*QA(F,d,c)}{TA(F,d,c)}*\left(pa(F,d,c)-ta(F,d,c)\right)\end{array}\right\}$$
$$\left[\textit{changes in the terms of trade}\right]$$
$$+\sum_c \sum_j \frac{PA(d,j,c)*QA(d,j,c)}{TA(d,j,c)}*\phi a(d,c)$$
$$\left[\textit{changes in production technology or productivity}\right]$$
$$+\left(\frac{\sigma}{\sigma-1}\right)*\sum_c \sum_s PA(s,d,c)*QA(s,d,c)*\hat{\delta}a(s,d,c)$$
$$\left[\begin{array}{l}\textit{changes in preferences or the efficiency with which commodity c from different sources}\\ \textit{can be converted into composite units of c for consumption in d}\end{array}\right]$$

Notation: The decomposition refers to welfare for country d. Country F (foreign) is the other country. Uppercase symbols are defined in Table 3. Lowercase symbols are percentage changes in the variables denoted by the corresponding uppercase symbols, for example, pa(F,d,c) is the percentage change in PA(F,d,c). An exception is $\hat{\delta}a(s,d,c)$. This is the percentage change in $\delta a(s,d,c)$.

To start with, we interpret the decomposition equation as referring to small changes in variables. In this case we don't have to worry about movements in the levels variables, PA(s,d,c), QA(s,d,c) etcetera. We can imagine that these levels are fixed at their starting values. We will consider large changes later in this subsection.

The LHS of the decomposition equation is 100 times the change in welfare. The first term on the RHS is 100 times the contribution to the change in welfare of the change in employment. With labor in the Armington model being paid according to the value of its marginal product, an *x* per cent increase in employment in country d [ltota(d) = *x*] generates an expansion in the quantity of GDP (holding prices at their initial levels) of the wage rate [W(d)] times the increase in employment [0.01*LTOTA(d)*ltota(d)]. With prices held constant, this is also the contribution of the expansion in employment to welfare because, in our simplified model, the percentage change in the quantity of GDP is the same as that in the quantity of consumption which by definition is the same as the percentage change in welfare.

The second term on the RHS of the decomposition equation is the contribution to welfare in country d of changes in tax-carrying flows. This is the general equilibrium version of the consumer and producer surplus rectangles and triangles in familiar partial equilibrium demand and supply diagrams. Country d gains welfare if there is an expansion in its absorption of commodity c from source s [that is qa(s,d,c) > 0] and this flow is taxed by country d. The gain in welfare arises because d's users of c from s (commodity s,c) value an extra unit at the tax-inclusive price [PA(s,d,c)] but it costs country d only the tax-exclusive price to provide an extra unit of s,c. The welfare gain per unit of extra s,c is the gap between the tax-inclusive and tax-exclusive prices which, as reflected in the second term of the decomposition formula, is PA(s,d,c)*(TA(s,d,c)-1)/TA(s,d,c).

The third term is the terms-of-trade effect. A terms-of-trade improvement, that is an increase in fob export prices relative to cif import prices, enables country d to convert any given volume of exports into an increased volume of welfare-enhancing imports. The percentage movement in d's fob export price for commodity c is given by pa(d,F,c)-ta(d,F,c). In measuring d's welfare this percentage movement is weighted by the ratio of the fob value of the d,F,c flow to the value of d's total consumption. Similarly, the percentage movement in d's cif import price for commodity c is given by pa(F,d,c)-ta(F,d,c). In measuring d's welfare this is weighted by the ratio of the cif value of the F,d,c flow to the value of d's total consumption.

The fourth term is the contribution to d's welfare of changes in production technology. Country d's welfare is improved if it can produce more output per unit of labor input. If d's productivity in the production of commodity c improves by *x* per cent [ϕa(d,c) = *x*], then *x* per cent of the labor devoted to commodity c can be released to other productive uses without affecting d's production of c. From a welfare point of view, this is equivalent to an increase in employment. Quantitatively, the welfare effect is the value of *x* per cent of the labor devoted to c. With labor being the only input, this is *x* per cent of the tax-exclusive value of the output of c in country d.

The fifth term in the decomposition equation is the contribution to d's welfare of changes in conversion technology or preferences [changes in the δa(s,d,c)s]. This term is less familiar to CGE modellers than the previous terms. It appears in the welfare decomposition equation because increases in the δa(s,d,c)s improve the ability of country d to convert units of commodity c from different sources into welfare-carrying units of composite-commodity c. In looking at the fifth term, it is worth recalling that $\sigma > 1$, implying that the leading coefficient, $\sigma/(\sigma-1)$, is positive. To derive the fifth term a good starting point is (T3.2) in Table 3. In percentage-change form (T2.3) can be written as

$$\mathrm{pca}(d,c) = \frac{1}{1-\sigma}\sum_s \left(\frac{\delta A(s,d,c)^{\sigma} * PA(s,d,c)^{1-\sigma}}{PCA(d,c)^{1-\sigma}} \right) * \left(\sigma * \hat{\delta}a(s,d,c) + (1-\sigma) * pa(s,d,c) \right) . \quad (6.4)$$

Thus, a 1 per cent increase in $\delta A(s,d,c)$ has an impact percentage effect on the cost of creating a unit of composite c in country d given by

$$\mathrm{pca}(d,c)\Big|_{\hat{\delta}a(s,d,c)=1} = -\left(\frac{\sigma}{\sigma-1} \right) * \left(\frac{\delta A(s,d,c)^{\sigma} * PA(s,d,c)^{1-\sigma}}{PCA(d,c)^{1-\sigma}} \right) \quad (6.5)$$

Via (T3.3) this can be written as

$$\mathrm{pca}(d,c)\Big|_{\hat{\delta}a(s,d,c)=1} = -\left(\frac{\sigma}{\sigma-1} \right) * \left(\frac{PA(s,d,c) * QA(s,d,c)}{PCA(d,c) * QCA(d,c)} \right) \quad (6.6)$$

The significance of a reduction in the cost of creating units of composite c for country d's welfare depends on the share of c in d's total consumption. Combining this idea with (6.6) leads to the fifth term in the decomposition equation.

As mentioned earlier, with the levels variables fixed at their initial values, our decomposition equation accurately produces the change in welfare caused by small shocks to the exogenous variables in MelitzGE. With large shocks, we need to allow for changes in the levels variables. In GEMPACK computations this is done by applying the shocks to the exogenous variables in n steps. In the first step we apply 1/n-th of the required changes in the exogenous variables. If n is large we can work out accurately the change in welfare in the first step and the five contributions identified on the RHS of the decomposition equation. Then we update the levels variables according to the changes from the first step. In the second step we again apply 1/n-th of the required changes in the exogenous variables. We work out the welfare effects in this second step and the five contributions using the decomposition equation with updated levels variables. Proceeding in this way, we can use the decomposition equation to calculate accurately the welfare effect of the total shocks to the exogenous variables. The contribution of each of the five components is the sum of its contribution across the n steps.

6.3.2. Decomposing MelitzGE welfare results via an Armington model: results

In the bottom blocks in Table 6 we use the equation from Figure 2 to decompose the welfare effects of the increases in country 2's tariffs.

The most striking aspect of the welfare decomposition results is the offsetting nature of the production technology and conversion technology contributions (components 4 and 5). For both countries in the three tariff experiments, the production-technology contribution is negative, and is closely offset by a positive conversion-technology contribution. The production- and conversion-technology contributions are what Melitz adds to an Armington welfare calculation. Because these contributions offset, it appears that the Armington calculation of the welfare effects of a tariff change is not misleading, even in a world in which Melitz specifications are valid.

We suspect that this striking result is another implication of the envelope theorem. As demonstrated in section 3, with tariffs at zero, a Melitz model generates an optimal trade-off in the widgets market between keeping costs down through long-production runs and meeting consumer demand for variety. The envelope theorem suggests that marginal shifts in this trade-off (e.g. shorter production runs but more varieties) away from the optimum will have little effect on welfare. Thus, although the imposition of tariffs causes the cost/variety trade-off in each country to change, this change does not have a significant effect on welfare.

The cancelling out of the two technology effects leaves welfare in our MelitzGE tariff simulations determined by factors that have been familiar to trade economists since the 1950s or earlier[31]: the terms-of-trade effect and the efficiency or tax-carrying-flows effect.

For country 2, the welfare outcome of an increase in tariffs from zero to a low level, 7.180 per cent, is dominated by terms-of-trade effects: a 0.802 contribution to a total welfare effect of 0.593 per cent. By imposing a 7.180 per cent tariff, country 2 improves its terms of trade by 3.6 per cent (4.958 per cent increase in the price of its exports compared with 1.324 per cent increase in the price of its imports). With exports (and imports) being about 23 per

[31] See for example Corden (1957) and Johnson (1960).

cent of GDP[32], a 3.6 per cent improvement in the terms of trade is equivalent to a GDP gain of 0.83 per cent, close to the terms-of-trade welfare contribution shown for country 2 in the our first tariff simulation. The tax-carrying-flow effect or the familiar welfare triangle from textbook partial equilibrium diagrams provides a small offset, -0.164 per cent, to country 2's terms-of-trade gain. Again, the magnitude of this effect is easily understood via a simple calculation, see Figure 3 and the data in Table 5.

Consistent with the theory of the optimal tariff[33], as country 2 increases its tariffs, the negative welfare contribution from tax-carrying flows increases much more rapidly than the positive welfare contribution from the terms of trade. By the time country 2's tariffs in Armington terms have reached 32.558 per cent (third simulation in Table 6), the tax-carrying-flow effect has almost cancelled out the terms-of-trade effect. This, together with a small negative contribution from the combined technology components leaves the net welfare effect for country 2 slightly negative (-0.046 per cent). By conducting a series of simulations in which we varied the tariff imposed by country 2 we found that the optimal tariff for country 2 in the absence of retaliation by country 1 is 13.333 per cent in Armington terms (19 per cent in Melitz terms, second simulation in Table 6).

For country 1, the terms-of-trade movement accounts for almost the entire welfare effect in all three simulations: there are no employment effects because employment is held constant and there are no tax-carrying-flow effects because country 1 has no taxes. The terms-of-trade effects for country 1 are the opposite of those for country 2.

6.4. Is a Melitz model equivalent to an Armington model with a higher substitution elasticity?

That the welfare results computed in the previous subsection depend almost entirely on Armington mechanisms (terms-of-trade and efficiency effects) suggested to us that results from a Melitz model might be more generally equivalent to those from an Armington model.

Initially we tested this idea by comparing tariff results from Melitz and Armington models built with identical databases and with identical values for the substitution parameter σ, $\sigma = 3.8$. Table 7 gives the results for this exercise. The Melitz results in Table 7 are the same as those in Table 6: they refer to the effects of unilateral tariff increases by country 2 computed with the 2-country, 2-commodity version of MelitzGE. The Armington results were computed by the model set out in Table 3 with the shocks to TA(1,2,c) being the Armington equivalents [calculated in (4.6)] of the Melitz tariff shocks.

The results in Table 7 show much more restrictive effects on trade flows from tariff increases in the Melitz model than in the Armington model. For example, whereas the Melitz computation for $t_{12,c}=10$ [or ta(1,2,c)=7.18] gives reductions in country 2's exports and imports of 21.622 and 18.811 per cent, the Armington computation gives reductions of only 11.220 and 7.763 per cent. With less trade response (steeper implied export demand curves), the Armington model generates larger terms-of-trade gains for the country imposing the tariff and correspondingly larger terms-of-trade losses for the other country. At the same time, the Armington model generates smaller efficiencies losses than the Melitz model for the country imposing the tariff (a smaller triangle in Figure 3). Larger terms-of-trade gains and smaller

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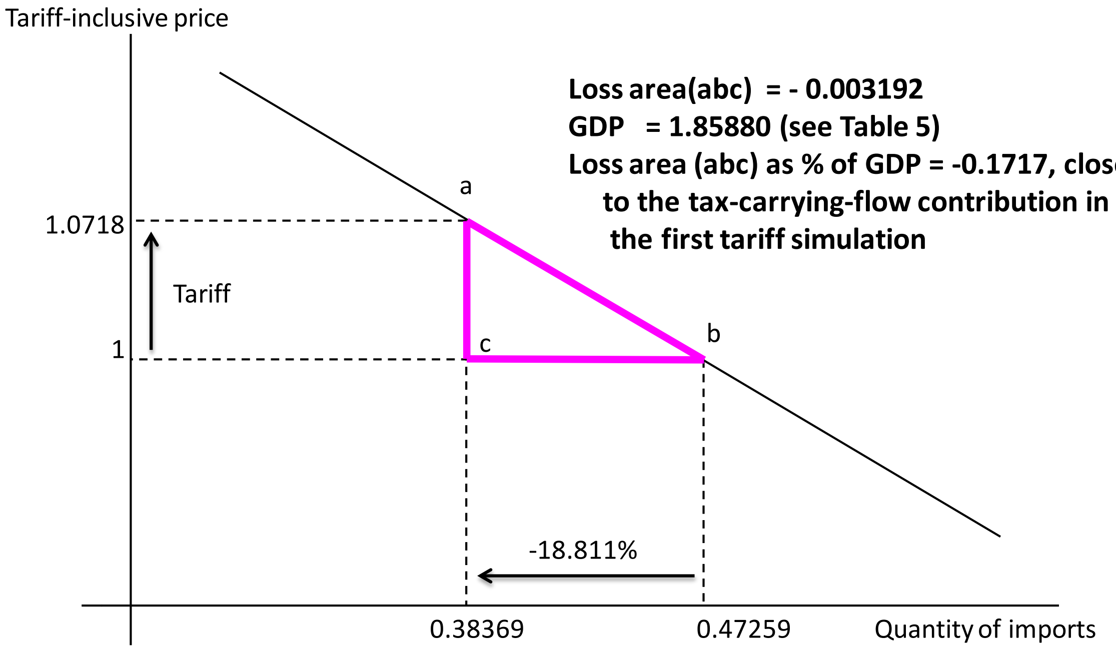
[32] With the tariffs at zero, exports are 25.42 per cent of country 2's GDP. With the imposition by country 2 of 10 per cent tariffs (Melitz basis), the export share for country 2 falls to 20.91 per cent. The average share as tariff rates move from zero to 10 is 23 per cent.

[33] See for example, Dixon and Rimmer (2010).

Figure 3. Country 2's demand for imports: back-of-the-envelope calculation of the welfare contribution of changes in tax-carrying flows in the first tariff simulation in Table 6



efficiency losses for the country imposing the tariff mean that the optimal tariff is much larger in the Armington model than the Melitz model. In computations not reported here we found that the optimal tariff rate for country 2 in the absence of retaliation by country 1 is about 42 per cent [TA(1,2,c) = 1.42]. As mentioned earlier, the optimal tariff for country 2 in the Melitz model is about 13 per cent [TA(1,2,c) = 1.13].

It is tempting to interpret the results in Table 7 as meaning that the Armington specification leads to under estimates of the restrictiveness of tariffs. However, we don't think that such an interpretation is legitimate. To us, Table 7 demonstrates that $\sigma = 3.8$ in a Melitz model doesn't mean the same thing as $\sigma = 3.8$ in an Armington model.

Potentially, it is possible to observe the response of trade flows to tariff changes. Let's assume for the sake of argument that MelitzGE with $\sigma = 3.8$ correctly produces these responses. Can we build an Armington model on the same database[34] as that of the Melitz model which also correctly produces the trade flow responses?

Table 8 repeats the Melitz results from Tables 6 and 7 with $\sigma = 3.8$ and compares them with Armington results computed with the same database but with $\sigma = 8.45$. The value 8.45 was chosen for the Armington model by trial and error with the objective of bringing the Armington trade responses into line with the Melitz responses. As can be seen from Table 8, this objective was achieved to a high level. The Melitz and Armington results for trade flows in Table 8 are close to identical. But what about the welfare results?

[34] By database we mean values of trade flows, outputs, wage rates and employment.

Table 7. Percentage effects of tariffs imposed by country 2: Melitz and Armington results with σ=3.8 in both models

	Melitz with σ=3.8		**Armington with σ = 3.8**		**Melitz with σ=3.8**		**Armington with σ = 3.8**		**Melitz with σ=3.8**		**Armington with σ = 3.8**	
Shocked exogenous variables	**$t_{12,c}$=10, all c**		**ta(1,2,c)=7.18, all c**		**$t_{12,c}$ =19, all c**		**ta(1,2,c)=13.33, all c**		**$t_{12,c}$ =50 for all c**		**ta(1,2,c)=32.56, all c**	
Endogenous variables	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**
Armington power of tariffs, TA(s,d,c)	0.000	7.180	0.000	7.180	0.000	13.333	0.000	13.333	0.000	32.558	0.000	32.558
Real consumption	-0.824	0.593	-0.929	0.845	-1.436	0.726	-1.624	1.360	-2.908	-0.046	-3.338	2.130
Volume of exports	-18.811	-21.622	-7.763	-11.220	-32.008	-36.364	-13.760	-19.530	-60.370	-66.389	-29.247	-39.558
Volume of imports	-21.622	-18.811	-11.220	-7.763	-36.364	-32.008	-19.530	-13.760	-66.389	-60.370	-39.558	-29.247
Welfare decomposition												
Welfare(d)	-0.824	0.593	-0.929	0.845	-1.436	0.726	-1.624	1.360	-2.908	-0.046	-3.338	2.130
made up of contributions from changes in:												
Employment	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Tax-carrying flows	0.000	-0.164	0.000	-0.067	0.000	-0.497	0.000	-0.213	0.000	-1.994	0.000	-0.983
Terms of trade	-0.818	0.802	-0.929	0.912	-1.425	1.375	-1.624	1.573	-2.832	2.617	-3.338	3.113
Production technology, productivity	-3.332	-2.795	0.0	0.0	-5.890	-5.021	0.0	0.0	-12.229	-10.835	0.0	0.0
Conversion technology, preferences	3.327	2.750	0.0	0.0	5.879	4.869	0.0	0.0	12.152	10.165	0.0	0.0

Table 8. Percentage effects of tariffs imposed by country 2: Melitz results with σ=3.8 compared with Armington results with σ=8.45

	Melitz with σ=3.8		Armington with σ = 8.45		Melitz with σ=3.8		Armington with σ = 8.45		Melitz with σ=3.8		Armington with σ = 8.45	
Shocked exogenous variables	**$t_{12,c}$=10, all c**		**ta(1,2,c)=7.18, all c**		**$t_{12,c}$ =19, all c**		**ta(1,2,c)=13.33, all c**		**$t_{12,c}$ =50 for all c**		**ta(1,2,c)=32.56, all c**	
Endogenous variables	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**
Armington power of tariffs, TA(s,d,c)	0.000	7.180	0.000	7.180	0.000	13.333	0.000	13.333	0.000	32.558	0.000	32.558
Real consumption	-0.824	0.593	-0.830	0.655	-1.436	0.726	-1.381	0.858	-2.908	-0.046	-2.476	0.460
Volume of exports	-18.811	-21.622	-18.789	-21.682	-32.008	-36.364	-32.009	-36.331	-60.370	-66.389	-60.226	-65.725
Volume of imports	-21.622	-18.811	-21.682	-18.789	-36.364	-32.008	-36.331	-32.009	-66.389	-60.370	-65.725	-60.226
Welfare decomposition												
Welfare(d)	-0.824	0.593	-0.830	0.655	-1.436	0.726	-1.381	0.858	-2.908	-0.046	-2.476	0.460
made up of contributions from changes in:												
Employment	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Tax-carrying flows	0.000	-0.164	0.000	-0.161	0.000	-0.497	0.000	-0.482	0.000	-1.994	0.000	-1.868
Terms of trade	-0.818	0.802	-0.830	0.816	-1.425	1.375	-1.381	1.340	-2.832	2.617	-2.476	2.329
Production technology, productivity	-3.332	-2.795	0.0	0.0	-5.890	-5.021	0.0	0.0	-12.229	-10.835	0.0	0.0
Conversion technology, preferences	3.327	2.750	0.0	0.0	5.879	4.869	0.0	0.0	12.152	10.165	0.0	0.0

These are also close. Why? With the trade responses in line, we would expect the efficiency and terms-of-trade effects in the two models to be similar. This is confirmed by the results for the welfare contributions in the tax-carrying-flows and terms-of-trade rows in Table 8. With the Melitz production-technology and conversion-technology effects largely cancelling out, the two model must produce similar welfare results.

Thus it appears in our computations that $\sigma = 3.8$ in MelitzGE means approximately the same thing as $\sigma = 8.45$ in the corresponding Armington model.

In Table 9 we try to discover a more general relationship between σ in MelitzGE and σ in the corresponding Armington model. We look for the Armington σ's that lead to similar results to those in MelitzGE for the effects of a 10 per cent tariff imposed by country 2 [$t_{12,c} = 10$ for all c] as we vary the Melitz σ's between 3 and 4.6. As can be seen from the table, the Melitz results for $\sigma = 3$ can be closely reproduced by Armington with $\sigma = 7.90$, the Melitz results for $\sigma = 4.6$ can be closely reproduced by Armington with $\sigma = 9.15$, and as we saw earlier in Table 8, the Melitz results for $\sigma = 3.8$ can be closely reproduced by Armington with $\sigma = 8.45$. The implied relationship between the Melitz and Armington σ's is illustrated in Figure 4.

In creating Table 9 we had to consider several technical issues. The first concerns the construction of the Melitz database, which is also the Armington database. We wanted to maintain the same initial data (e.g. export shares of 25.4 per cent of GDP) as we varied the Melitz σ. As will be recalled from subsection 6.1, we set up the initial database for MelitzGE by a recursive sequence of calculations starting from assumed values for σ and other parameters. The database emerging from these calculations depends on σ: a higher Melitz σ implies more trade. To counteract this effect and produce databases with identical initial trade shares, we varied not only σ across the three Melitz experiments in Table 9, but also α, the parameter in the Pareto distribution of productivities across firms (see Appendix 1). Increases in α reduce the proportion of firms that have productivity above 2, the minimum productivity for participation in international trade in the two-country version of MelitzGE [see (6.1)]. In this way, higher assumed values for α lead to lower export shares in GDP for each country in the initial database. Table 9 shows the α values we adopted.

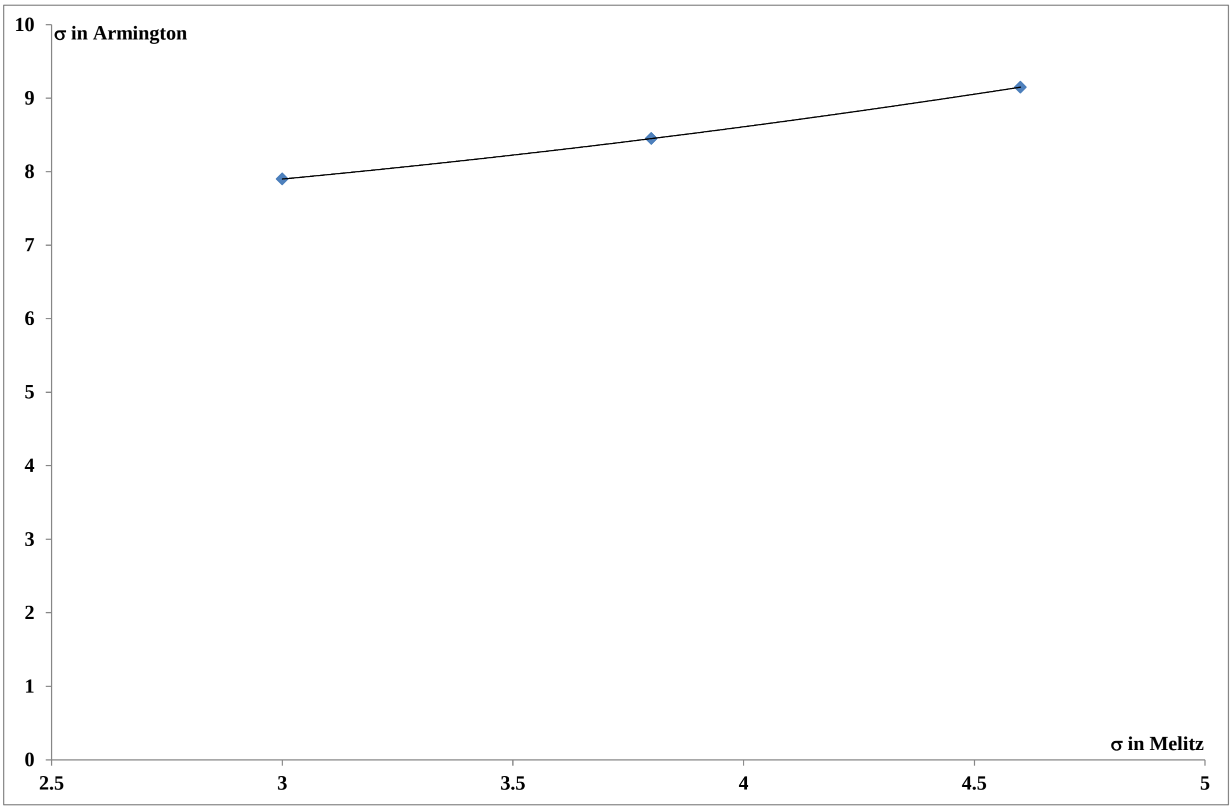
The second technical issue concerns the appropriate Armington tariff shock. While the Melitz tariff shock is the same across the three Melitz simulations in Table 9, the equivalent Armington tariff shock varies from 6.45 to 7.18 to 7.66. The reason can be traced back to the Melitz version of equation (T2.1) in Table 2. There we see that the markup factor on the variable costs of the sales of the typical firm on any link is $\sigma/(\sigma\text{-}1)$. As σ is moved from 3 to 4.6, this markup factor falls from 1.50 to 1.28 and the share of variable costs in total sales revenue increases from 67 per cent to 78 per cent. Thus, as σ is moved from 3 to 4.6, the Melitz tariff of 10 per cent is charged on a larger fraction of the value of country 2's imports. Consequently, the equivalent Armington tariff rate, which is charged on the entire cif value, rises.

The final technical issue concerns the range of values used for the Melitz σ, 3.0 to 4.6. Why did we restrict σ for the Melitz model to this range? With the Melitz σ less than 3.0, the implied mark-ups on variable costs are unrealistically large, greater than 50 per cent. With the Melitz σ greater than 4.6, we judged that the trade responses to tariff changes were unrealistically large: more than 24 per cent reductions in country 2's exports in response to its imposition of a 10 per cent Melitz tariff (a 7.66 per cent Armington tariff). In fact, a difficulty

Table 9. Percentage effects of tariffs imposed by country 2: Discovering the relationship between σ for Melitz and σ for Armington

	Melitz with σ=3 α=3.8		Armington with σ = 7.90		Melitz with σ=3.8 α=4.6		Armington with σ = 8.45		Melitz with σ=4.6 α=5.4		Armington with σ = 9.15	
Shocked exogenous variables	$t_{12,c}$=10, all c		ta(1,2,c)= 6.45, all c		$t_{12,c}$=10, all c		ta(1,2,c)= 7.18, all c		$t_{12,c}$=10, all c		ta(1,2,c)= 7.66, all c	
Endogenous variables	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**	**Country d=1**	**Country d=2**
Armington power of tariffs, TA(s,d,c)	0.000	6.452	0.000	6.452	0.000	7.180	0.000	7.180	0.000	7.659	0.000	7.659
Real consumption	-0.801	0.594	-0.763	0.628	-0.824	0.593	-0.830	0.655	-0.845	0.588	-0.866	0.655
Volume of exports	-16.074	-18.426	-15.916	-18.628	-18.811	-21.622	-18.789	-21.682	-21.524	-24.562	-21.557	-24.519
Volume of imports	-18.426	-16.074	-18.628	-15.916	-21.622	-18.811	-21.682	-18.789	-24.562	-21.524	-24.519	-21.557
Welfare decomposition												
Welfare(d)	-0.802	0.595	-0.763	0.628	-0.824	0.593	-0.830	0.655	-0.845	0.588	-0.866	0.655
made up of contributions from changes in:												
Employment	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Tax-carrying flows	0.000	-0.127	0.000	-0.123	0.000	-0.164	0.000	-0.161	0.000	-0.197	0.000	-0.196
Terms of trade	-0.670	0.658	-0.763	0.751	-0.818	0.802	-0.830	0.816	-0.903	0.885	-0.866	0.851
Production technology, productivity	-3.115	-2.247	0.0	0.0	-3.332	-2.795	0.0	0.0	-3.626	-3.307	0.0	0.0
Conversion technology, preferences	2.983	2.311	0.0	0.0	3.327	2.750	0.0	0.0	3.685	3.207	0.0	0.0

Figure 4. Melitz substitution elasticities and equivalent Armington elasticities in the siumulation of a 10 per cent tariff imposed by country 2



with the Melitz model is that the trade responses are large even when the Melitz σ is relatively low, 3.0. It appears that to generate trade responses consistent with econometrically observed Armington elasticities (values normally in the range 2 to 6) we would need Melitz σ of less than 3 with correspondingly enormous mark-ups.

6.4.1. Melitz/Armington welfare equivalence: some earlier literature

Consideration in earlier literature of the relationship between Melitz and Armington models has produced mixed results. Arkolakis *et al.* (2012, p.118) reached conclusions broadly compatible with the calculations in Table 9. They state that

> *"Within the class of trade models considered in this paper [which included Armington and Melitz], the number of sources of gains from trade varies, but conditional on observed trade data, the total size of the gains from trade does not."*

In other words, Arkolakis *et al.* are saying that over a fairly broad class of models if a shock gives the same trade response then it also gives the same welfare outcomes. This conclusion is disputed by Balistreri and Rutherford (2013, p. 1542):

> *"The strong equivalence results suggested by Arkolakis et al. (2008, 2012) are not supported in our empirical model. For us, this indicates that the real world complexities accommodated in CGE models are, indeed, important."*

However, it appears that Balistreri and Rutherford did not compare Armington and Melitz results across comparable experiments, that is experiments in which the Armington elasticities are adjusted so that the trade responses across models are the same. Put another way, we suspect that the Balistreri and Rutherford comparison is more like that in Table 7 and those in

Tables 8 and 9. Fan (2008, p. 593) reports an Armington/Melitz comparison in which he explicitly recognizes the need to equalize trade responses but finds significantly different welfare responses:

> *"To ensure the new model [a Melitz model] generates additional gains from trade expansion in comparison with the conventional model [an Armington model], I raise the Armington elasticies in the standard Armington CGE model by 33 per cent and run the tariff reduction simulation. Compared to the Melitz CGE model, the Armington CGE model with high elasticities predicts similar expansion in global real exports, but 23 per cent lower global welfare gains."*

Fan does not explain why his Melitz calculations give larger welfare effects than the comparable Armington calculations, but we note that he does not fully implement the Melitz model:

> *"I abstract from the dynamic parts of the Melitz model by assuming no entry and exit of firms …"* Fan (2008, p. 585).

We suspect that this introduces pure profits in Fan's version of the Melitz model that are not present in either the original Melitz model or Fan's version of the Armington model. An extra distortion, in the form of pure profits, in Fan's Melitz model but not in his Armington model would cast doubt on the legitimacy of the welfare comparison across the two models.

The Melitz/Armington comparisons by Arkolakis *et al.* (2012) and us are based on special assumptions. The formal analysis in Arkolakis *et al.* is confined to the effects on country j's welfare of shocks to the price of imports from country i in 1-sector, 1-factor-of-production, n-country models with iceberg trade costs. We also assume that there is only one factor of production in each of n countries but we allow for multiple sectors, revenue-generating tariffs and focus on the effects on country j's welfare not only of shocks emanating from other countries but also from j's own trade policy. In these circumstances, strong statements are not warranted concerning the empirical relevance of welfare equivalence between Armington and Melitz models. On the other hand, strong non-equivalence statements are equally unwarranted. For example it is premature to accept uncritically that:

> *"… Balistreri, Hillberry and Rutherford (2011) show that adding firm heterogeneity to standard computable general equilibrium models of trade raises the gains from trade liberalization by a factor of four. Empirical confirmation of the gains from trade predicted by models with heterogeneous firms represents one of the truly significant advances in the field of international economics."* Melitz and Trefler (2013, p. 114)

6.5. Experience with GEMPACK solutions of high dimension versions of MelitzGE

As explained in subsection 6.1, in computing solutions for MelitzGE we use GEMPACK software applied to a log-linear representation of the equations. Linearization errors are effectively eliminated by imposing the shocks to exogenous variables in a series of steps. While all of the solutions discussed in subsections 6.1 to 6.4 were based on a tiny version of MelitzGE (2 commodities and 2 countries), we foreshadowed that GEMPACK would be a suitable platform for solving large Melitz models directly without necessitating decomposition algorithms of the type described in section 4. Supporting evidence for this idea is given in Table 10.

Table 10. Computational times for solving MelitzGE in GEMPACK (seconds)

No. of Commodities	No. of countries		
	2	**10**	**100**
2	1	1	34
10	1	2	198
57	1	8	5887
100	1	15	24312

The cells in Table 10 show GEMPACK solution times[35] for versions of MelitzGE with different numbers of commodities and countries. In all cases, we computed the effects of a 10 per cent increase in the power of the tariffs imposed by country r, the last country, on imports from all other countries: $t_{sr,c}$=10 for all commodities c and all regions s ≠ r.[36] The computations were carried out with the GEMPACK code in Appendix 6 implemented in a standard 64-bit computing environment[37]. Highly accurate solutions were computed with the steps along the path of the exogenous variables set according to the 4-8-16 Gragg method[38]. No special effort was made to minimize times beyond condensation/backsolving of the type routinely carried out by non-expert GEMPACK users. Condensation/backsolving is the process, automated in GEMPACK, of substituting out high dimension variables using their defining equations and recovering their values post simulation. For example, we asked GEMPACK to substitute out $P_{\bullet sd,c}$ (the price charged by the typical c-producing firm on the sd-link) using the linearized version of the defining Melitz equation (T2.1) in Table 2. Then post simulation equation (T2.1) can be used to backsolve for $P_{\bullet sd,c}$.

The times shown in Table 10 are trivial for versions of MelitzGE with up to 100 commodities and up to 10 countries. Even with large numbers of commodities and countries the computational times are comfortably moderate. For example, with 57 commodities and 100 countries (about the size of the full-dimension GTAP database[39]), GEMPACK accurately solved the MelitzGE model in 5887 seconds or about an hour and a half. With 100 commodities and 100 countries the solution time begins to blow out, 24312 seconds or about 6.7 hours. If in a practical situation we were tackling a giant model, then we would seek help from GEMPACK experts who can often suggest time-minimizing options.

While Table 10 shows GEMPACK in a favorable light, it should be emphasized that MelitzGE is a very simple model. There are no intermediate inputs and only one scarce primary factor in each country. Introduction of intermediate inputs and multiple primary

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[35] The version of GEMPACK was: source-code GEMPACK (64-bit) using Intel Fortran 13.1 targeting 64-bit executables.

[36] As we varied the number of countries, r, we also reset the minimum productivity level required for a firm to operate on all links: from 2 when r = 2 [see equation (6.1)], to 15 when r = 10, to 172 when r = 100. These resets were necessary to maintain the export shares in each country's GDP in the initial database at 0.254. No resetting was required to accommodate variations in the number of commodities, n.

[37] Operating system, Windows 7 64-bit; CPU, Intel i7-4770; Memory, 32GB; HDD, 500GB SSD. An implementatation of MelitzGE using 64-bit GEMPACK is necessary to meet the memory requirements of the 57-commodity by 100-country and 100 by 100 simulations in Table 10.

[38] See section 3.12.2 in Harrison *et al.* (2014).

[39] See https://www.gtap.agecon.purdue.edu/databases/v7/ .

factors would certainly increase computational times. Other simplifying features of MelitzGE are country symmetry (see Figure 1) and identical industries. However we did not take advantage of these features in the GEMPACK computations and we don't think that they materially affected computational times. A reasonable interpretation of Table 10 is that it establishes an expectation, but not a certainty, that GEMPACK would be a highly effective platform for solving empirically-based Melitz models of the size and complexity that could be supported by available multi-country data on industries and trade flows.

7. Concluding remarks

In this paper we derived the Armington, Krugman and Melitz trade models as special cases of a more general model. We showed that the special assumptions leading to Melitz are less restrictive than those leading to Krugman, which in turn are less restrictive than those leading to Armington. The main objective of these derivations was to increase the accessibility of Melitz' work to CGE modellers who have Armington as their main frame of reference.

Armington has been the standard trade specification in CGE models since its introduction via Australia's ORANI model in the 1970s. In earlier economy-wide trade-oriented models (e.g. Evans, 1972) imported and domestic varieties of a given commodity were treated as perfect substitutes. This led to 'flip-flop': import shares in domestic markets flipping between zero and one in response to seemingly minor changes in relative prices. The Armington specification dealt with this problem in a practical and empirically justified fashion. Starting in the 1980s, many modellers questioned the Armington specification. They were disappointed with Armington-based simulations which often show a welfare loss for a country that undertakes a unilateral reduction in tariffs, with the terms-of-trade loss outweighing the tax-carrying-flow or efficiency gain. Under the Krugman specification, there are two additional sources of welfare change from a tariff cut: cost changes in the domestic economy through economies/diseconomies of scale and increases in variety through extra imports which may or may not be offset by a reduction in domestic varieties. Melitz adds another source of welfare change. In the Melitz model, tariff cuts can increase productivity by weeding out inefficient domestic firms.

Fan (2008) and Balistreri and Rutherford (2013) find that a CGE model with a Melitz specification can give considerably higher welfare gains from a tariff cut than a model built with a similar database but with an Armington specification. This was not our experience. We found in a Melitz simulation of the effects of a tariff change that the extra welfare effects added to Armington by Melitz tended to be offsetting. This left our Melitz welfare results much the same as those in an Armington model.

We described this result as an envelope effect. Despite the introduction of economies of scale, imperfect competition and technology differences across firms, the Melitz model describes an optimizing world. The Melitz market outcome is the same as that which would be achieved by a cost-minimizing world-wide planner. In particular, the market and cost-minimizing outcomes show the same numbers of varieties of each commodity being supplied to consumers and the same lengths of production runs by firms. While the imposition of tariffs causes the market economy to adjust the number of varieties and the lengths of production runs, in total these adjustments carry minimal welfare effects. This is in accordance with the envelope theorem concerning the welfare implications of adjustments away from an optimal situation.

As in Armington-based CGE models, the welfare effects in Melitz models of changes in tariffs from contemporary low levels are dominated by terms-of-trade effects. We do not see Melitz specifications as offering a panacea to those who would like to use general equilibrium modelling to support *unilateral* tariff reductions. In a Melitz world, as in an Armington world, tariff reductions make most economic sense when carried out on a multi-lateral or bi-lateral basis. With Melitz and Armington welfare results being very similar, Melitz modelling will not provide support for people who see large gains from free trade. It is difficult to obtain large welfare numbers for the effects of changes in low tariffs in models such as Melitz in which agents are fully informed profit and utility maximizers. The most likely arguments to support large welfare numbers are still those associated with X-efficiency (Leibenstein, 1966), rent seeking (Krueger, 1974), technology transfer (Tarr, 2013) and pro-competitive or cold-shower effects (Chand, 1999).

In analysing the North American Free Trade Agreement, Kehoe (2005), Shikher (2012) and others have argued that Armington CGE models underestimate the extent to which tariff cuts create trade. They argue that other specifications including that of Melitz give larger and more realistic trade and welfare responses. However, as illustrated in subsection 6.4, MelitzGE results for the effects of tariff cuts on trade and welfare can be reproduced in an Armington model simply by running the Armington model with an inter-country substitution elasticity (Armington elasticity) higher than the inter-variety substitution elasticity used in the corresponding Melitz model (e.g. 8.45 for Armington versus 3.8 for Melitz, Figure 4). On this basis, it is reasonable to interpret Melitz as providing a micro-theoretic foundation for an Armington implementation, but not a reason for abandoning Armington-based CGE modeling. What the MelitzGE results underline is the importance for model-based policy analysis of empirical effort devoted to the estimation of elasticities describing trade responses. These elasticities should be estimated in frameworks consistent with the CGE models in which they are to be used: elasticities estimated under Armington assumptions are not appropriate for use as variety elasticities in a Melitz model.

In solving Melitz models, we used GEMPACK software which works with linear equations expressed in percentage changes of variables. Previous Melitz computations have been carried out with GAMS software relying on non-linear levels representations of equations. The GEMPACK approach proved highly efficient and simplified the solution of Melitz models. Using GEMPACK we are able to avoid Balisterri and Rutherford's iterative decomposition approach which generates Melitz solutions by iterating between Melitz and Armington models.

Nevertheless, the idea underlying Balistreri and Rutherford's decomposition approach is highly suggestive. Using their idea we were able to decompose Melitz results for the welfare effects of a tariff change into five components computed via an Armington model. Our welfare decomposition allowed us to identify the offsetting nature of the contributions to welfare that Melitz adds to Armington.

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Appendix 1. Mathematical details of the Melitz model in Table 2

This appendix provides the mathematical details necessary to understand fully the elimination of the firm dimension in the derivation of the Melitz versions of (T2.2), (T2.4), (T2.6) and (T2.7). We also derive the Melitz equations (T2.8) and (T2.11). At the end of this appendix we justify (5.2).

In setting out the mathematics, it will sometimes be convenient to assume that the possible productivity values across firms form a continuous variable, rather than discrete. Following Melitz, we assume that productivity values in country s form a Pareto distribution:

$$g_s(\Phi) = \alpha\Phi^{-\alpha-1},\ \Phi \geq 1 \qquad \text{(A1.1)}$$

where α is a positive parameter. Under (A1.1), we assume that the lowest potential productivity value is 1. This assumption can be made without loss of generality through a suitable choice of units for labour.

From (A1.1) we obtain

$$\int_{\Phi_{min}}^{\infty} g_s(\Phi)\, d\Phi = \Phi_{min}^{-\alpha} \qquad \text{(A1.2)}$$

(A1.2) means that the proportion of productivity values in country s that are greater than any given level, Φ_{min}, is $\Phi_{min}^{-\alpha}$. Thus the proportion of firms in country s with productivity of at least $\Phi_{min(s,d)}$, i.e. the proportion of firms (N_{sd}/N_s) operating on the sd-link is $\Phi_{min(s,d)}^{-\alpha}$. This justifies the Melitz version of (T2.8).

Next, we apply (A1.1) and (T2.8) in a continuous version of (2.19). This gives

$$\Phi_{\bullet sd}^{\sigma-1} = \int_{\Phi_{min(s,d)}}^{\infty} \Phi_{min(s,d)}^{\alpha}\alpha\, \Phi^{-\alpha-1}\Phi^{\sigma-1}d\Phi \qquad \text{(A1.3)}$$

that is

$$\Phi_{\bullet sd}^{\sigma-1} = \left(\frac{\alpha}{\alpha-(\sigma-1)}\right)\Phi_{min(s,d)}^{\sigma-1} \quad . \qquad \text{(A1.4)}$$

In deriving (A1.4), we assume that

$$\alpha > (\sigma\text{-}1) \qquad \text{(A1.5)}$$

This doesn't have any obvious economic interpretation. However, without it, the integral on the RHS of (A1.3) is unbounded. From (A1.4), we get

$$\Phi_{\bullet sd} = \beta\Phi_{min(s,d)} \quad . \qquad \text{(A1.6)}$$

where

$$\beta = \left(\frac{\alpha}{\alpha-(\sigma-1)}\right)^{1/(\sigma-1)} \quad . \qquad \text{(A1.7)}$$

This justifies the Melitz version of (T2.11).

Now we turn to the derivation of the Melitz version of (T2.2). From the AKME version of (T2.1), we can see that the ratio of prices for any two firms on the sd-link is the ratio of their productivities raised to the power -1. Applying this idea in the AKME version of (T2.2) gives[40]

$$P_d^{1-\sigma} = \sum_s \sum_{k \in S(s,d)} N_s \alpha \Phi_k^{-\alpha-1} \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma} \left(\frac{\Phi_k}{\Phi_{\bullet sd}} \right)^{\sigma-1} \quad , \tag{A1.8}$$

which can be rewritten in continuous format as

$$P_d^{1-\sigma} = \sum_s \left(N_s \alpha \Phi_{\bullet sd}^{1-\sigma} P_{\bullet sd}^{1-\sigma} \delta_{sd}^{\sigma} \int_{\Phi_{\min(s,d)}}^{\infty} \Phi^{-\alpha-1} \Phi^{\sigma-1} \, d\Phi \right) \quad . \tag{A1.9}$$

Applying (A1.3) gives

$$P_d^{1-\sigma} = \sum_s \left(N_s \Phi_{\bullet sd}^{1-\sigma} P_{\bullet sd}^{1-\sigma} \delta_{sd}^{\sigma} \Phi_{\min(s,d)}^{-\alpha} \Phi_{\bullet sd}^{\sigma-1} \right) \quad . \tag{A1.10}$$

Via the Melitz version of (T2.8), (A1.10) reduces to

$$P_d^{1-\sigma} = \sum_s \left(N_{sd} \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma} \right) \quad . \tag{A1.11}$$

which leads to the Melitz version of (T2.2).

The starting point for deriving the Melitz version of (T2.4) is the AKME version. With γ_{ksd} equal to 1, we write this as:

$$Q_{sd}^{(\sigma-1)/\sigma} = \sum_{k \in S(s,d)} N_s \, g_s(\Phi_k) \, Q_{ksd}^{(\sigma-1)/\sigma} \quad . \tag{A1.12}$$

Then using the AKME versions of (T2.3) and (T2.1) together with (A1.1) we obtain

$$Q_{sd}^{(\sigma-1)/\sigma} = \sum_{k \in S(s,d)} N_s \, \alpha \Phi_k^{-\alpha-1} \left(\frac{\Phi_k}{\Phi_{\bullet sd}} \right)^{\sigma-1} Q_{\bullet sd}^{(\sigma-1)/\sigma} \quad , \tag{A1.13}$$

that is,

$$Q_{sd}^{(\sigma-1)/\sigma} = N_s \Phi_{\min(s,d)}^{-\alpha} Q_{\bullet sd}^{(\sigma-1)/\sigma} \Phi_{\bullet sd}^{1-\sigma} \sum_{k \in S(s,d)} \Phi_{\min(s,d)}^{\alpha} \, \alpha \Phi_k^{-\alpha-1} \, \Phi_k^{\sigma-1} \quad . \tag{A1.14}$$

Via (A1.3), (A1.14) simplifies to

$$Q_{sd}^{(\sigma-1)/\sigma} = N_s \Phi_{\min(s,d)}^{-\alpha} Q_{\bullet sd}^{(\sigma-1)/\sigma} \quad . \tag{A1.15}$$

The Melitz version of (T2.8) gives a further simplification, leading to the Melitz version of (T2.4).

To derive the Melitz version of (T2.6), we start by writing the AKME version as

$$\Pi tot_s = \frac{1}{\sigma} \sum_d \sum_{k \in S(s,d)} N_s \alpha \Phi_k^{-\alpha-1} P_{ksd} Q_{ksd} - \sum_d N_{sd} F_{sd} W_s - N_s H_s W_s \quad . \tag{A1.16}$$

In deriving (A1.16), we used the AKME versions of (T2.5) and (T2.1) with η set at $-\sigma$. Via the AKME version of (T2.3), (A1.16) becomes

[40] As in Table 1, we assume that γ_{ksd} =1 for all k.

$$\Pi tot_s = \frac{1}{\sigma}\sum_d \sum_{k\in S(s,d)} N_s \alpha \Phi_k^{-\alpha-1}\Phi_k^{\sigma-1}\Phi_{\bullet sd}^{1-\sigma} P_{\bullet sd} Q_{\bullet sd} - \sum_d N_{sd} F_{sd} W_s - N_s H_s W_s \quad . \tag{A1.17}$$

Applying (A1.3) and (T2.8), we simplify (A1.17) to

$$\Pi tot_s = \frac{1}{\sigma}\sum_d N_{sd} P_{\bullet sd} Q_{\bullet sd} - \sum_d N_{sd} F_{sd} W_s - N_s H_s W_s \quad , \tag{A1.18}$$

which quickly leads to the Melitz version of (T2.6).

The final task in justifying the elimination of the firm dimension from the Melitz equations is to derive the Melitz version of (T2.7). Applying (T2.3) and (T2.1) in the first term on the RHS of the AKME version of (T2.7) and simplifying the second term gives

$$L_s = \sum_d \sum_{k\in S(s,d)} N_s \alpha \Phi_k^{-\alpha-1}\Phi_k^{\sigma-1}\frac{Q_{\bullet sd}}{\Phi_{\bullet sd}^{\sigma}} + \sum_d N_{sd} F_{sd} + N_s H_s \quad . \tag{A1.19}$$

Then via (A1.3) and (T2.8), (A1.19) reduces to the Melitz version of (T2.7).

Deriving equation (5.2)

In (5.2) we assume that the landed-duty-paid value of trade on the sd-link, V_{sd}, is the value for the typical firm, $P_{\bullet sd} Q_{\bullet sd}$, times the number of trading firms, N_{sd}. To derive (5.2) we start from

$$V_{sd} = \sum_{k\in S(s,d)} N_s g(\Phi_k) P_{ksd} Q_{ksd} \quad , \tag{A1.20}$$

that is, the landed-duty-paid value of widgets sent from s to d is the value, $P_{ksd} Q_{ksd}$, sent by a k-class firm times the number of such firms, $N_s g(\Phi_k)$, aggregated over all k in S(s,d). Using the AKME versions of (T2.1) and (T2.3) and continuing to assume that γ_{ksd} =1 for all k, we obtain

$$V_{sd} = \sum_{k\in S(s,d)} N_s g(\Phi_k) P_{\bullet sd} \frac{\Phi_{\bullet sd}}{\Phi_k} Q_{\bullet sd} \left(\frac{\Phi_k}{\Phi_{\bullet sd}}\right)^{\sigma} \quad . \tag{A1.21}$$

Simplifying and using (2.19) leads to (5.2).

Appendix 2. Equivalence between worldwide cost minimizing and the AKME model

Proof of proposition (3.2): Cost minimizing $\Rightarrow$ AKME

Let $\Phi_{min(s,d)}$, N_s, Q_{ksd} and Λ_d be a solution to (3.5) to (3.9) for given values of the exogenous variables W_s, Q_d and T_{sd}. Let P_d and P_{ksd} be defined by (3.10) and (3.11) and define Q_{sd}, Π_{ksd}, Πtot_s and L_s as in (T2.4) – (T2.7) of the AKME model. We show that $\Phi_{min(s,d)}$, N_s, Q_{ksd}, P_d, P_{ksd}, Q_{sd}, Π_{ksd}, Πtot_s and L_s then satisfy the remaining AKME equations, (T2.1) to (T2.3) and (T2.8) to (T2.10), and is therefore an AKME solution.

Equations (T2.8) and (T2.1) are satisfied: (T2.8) is the same as (3.6) and under (3.1), (T2.1) is the same as (3.11).

From (3.9) – (3.11) we have

$$P_{ksd} = P_d Q_d^{1/\sigma} \delta_{sd} Q_{ksd}^{-1/\sigma} \quad , k \in S(s,d) \quad . \tag{A2.1}$$

Hence

$$Q_{ksd} = \delta_{sd}^{\sigma} Q_d \left(\frac{P_d}{P_{ksd}} \right)^{\sigma} \quad k \in S(s,d) \quad . \tag{A2.2}$$

Under (3.1) this establishes (T2.3).

From (3.7)

$$-W_s \left(\frac{T_{sd} Q_{min(s,d)}}{\Phi_{min(s,d)}} + F_{sd} \right) + \Lambda_d \delta_{sd} Q_{min(s,d)}^{(\sigma-1)/\sigma} = 0 \quad . \tag{A2.3}$$

Combining (3.10) and (A2.2) gives

$$P_{ksd} = \Lambda_d \delta_{sd} Q_{ksd}^{-1/\sigma} \quad . \tag{A2.4}$$

In particular

$$P_{min(s,d)} = \Lambda_d \delta_{sd} Q_{min(s,d)}^{-1/\sigma} \quad . \tag{A2.5}$$

Putting (A2.5) into (A2.3) gives

$$-W_s \left(\frac{T_{sd} Q_{min(s,d)}}{\Phi_{min(s,d)}} + F_{sd} \right) + P_{min(s,d)} Q_{min(s,d)} = 0 \quad , \tag{A2.6}$$

establishing (T2.10).

From (3.8) and (A2.4) we obtain

$$\left[W_s \sum_d \sum_{k \in S(s,d)} g_s(\Phi_k) * \left(\frac{T_{sd} Q_{ksd}}{\Phi_k} + F_{sd} \right) \right] + W_s H_s - \sum_d \sum_{k \in S(s,d)} \frac{P_{ksd} Q_{ksd}^{1/\sigma}}{\delta_{sd}} g_s(\Phi_k) \delta_{sd} Q_{ksd}^{(\sigma-1)/\sigma} = 0. \tag{A2.7}$$

Simplifying, rearranging and multiplying through by N_s gives

$$\left[W_s \sum_d \sum_{k \in S(s,d)} N_s g_s(\Phi_k) * \left(\frac{T_{sd} Q_{ksd}}{\Phi_k} + F_{sd} \right) \right] + N_s W_s H_s = \sum_d \sum_{k \in S(s,d)} N_s g_s(\Phi_k) P_{ksd} Q_{ksd} \, . \tag{A2.8}$$

Via (T2.5) and (T2.6), (A2.8) leads to (T2.9).

Now all that remains is to establish (T2.2). We start by rearranging (A2.2) as

$$\delta_{sd}^{\sigma} P_{ksd}^{1-\sigma} = Q_{ksd}^{(\sigma-1)/\sigma} \delta_{sd} Q_d^{(1-\sigma)/\sigma} P_d^{1-\sigma} \quad . \tag{A2.9}$$

Then multiplying through by $N_s g_s(\Phi_k)$, aggregating over s and k, and using (3.5) we obtain (T2.2) under assumption (3.1).

Proof of proposition (3.3): AKME $\Rightarrow$ First-order optimality conditions for cost minimizing

Let $\Phi_{min(s,d)}$, N_s, Q_{ksd}, P_d, P_{ksd}, Q_{sd}, Π_{ksd}, Πtot_s and L_s satisfy (T2.1) to (T2.10) for given values of the exogenous variables W_s, Q_d and T_{sd}. Define Λ_d by (3.10). We show that $\Phi_{min(s,d)}$, N_s, Q_{ksd} and Λ_d is a solution to (3.5) to (3.9).

Condition (3.6) is the same as (T2.8).

Under (3.1), (T2.3) gives

$$\delta_{sd} Q_{ksd}^{(\sigma-1)/\sigma} = Q_d^{(\sigma-1)/\sigma} \delta_{sd}^{\sigma} P_d^{\sigma-1} P_{ksd}^{1-\sigma} \quad . \tag{A2.10}$$

Multiplying through by $N_s g_s(\Phi_k)$, summing over all s and all $k \in S(s,d)$ and using (T2.2) and (3.1) gives (3.5).

Equations (T2.5) and (T2.10) give

$$P_{min(s,d)} Q_{min(s,d)} - \left(\frac{W_s T_{sd}}{\Phi_{min(s,d)}} \right) Q_{min(s,d)} - F_{sd} W_s = 0 \,. \tag{A2.11}$$

To establish (3.7) we need to eliminate $P_{min(s,d)}$ and introduce Λ_d. We do this via (T2.3), (3.1) and (3.10) which give

$$P_{ksd} = \delta_{sd} \Lambda_d Q_{ksd}^{-1/\sigma} \tag{A2.12}$$

and, in particular

$$P_{min(s,d)} = \delta_{sd} \Lambda_d Q_{min(s,d)}^{-1/\sigma} \quad . \tag{A2.13}$$

Multiplying (A2.11) through by $N_s g_s(\Phi_k)$ and using (A2.13) quickly leads to (3.7).

From (T2.5), (T2.6) and (T2.9) we obtain

$$\sum_d \sum_{k \in S(s,d)} g_s(\Phi_k) \left[P_{ksd} Q_{ksd} - \left(\frac{W_s T_{sd}}{\Phi_k} \right) Q_{ksd} - F_{sd} W_s \right] - H_s W_s = 0 \quad . \tag{A2.14}$$

Then, substituting from (A2.12) gives (3.8).

To obtain (3.9), we start from (A2.12) and then use (T2.1).

Appendix 3. Establishing the validity of the Balistreri-Rutherford decomposition algorithm

We define a Balistreri-Rutherford (BR) solution as a list of values of Melitz and Armington variables that satisfy the Melitz versions of (T2.1) to (T2.12) together with (4.1), (4.5) to (4.7) and (T3.1) to (T3.7). That is a BR solution is what appears after implementation of steps 1 to 4 of the algorithm set out in subsection 4.1.3.

A converged BR solution is one that also satisfies

$$QCA(d,c) = Q_{d,c} \text{ for all c and d} \tag{A3.1}$$

and

$$WA(d) = W_d \text{ for all d} \quad . \tag{A3.2}$$

We prove that a converged BR solution reveals a Melitz GE solution.

Proof

Assume that we have a converged BR solution. Then the Melitz variables in this solution satisfy the Melitz versions of (T2.1) to (T2.12) and (4.1). We can compute the Melitz value for total employment in country s, $LTOT_s$, from (4.3) and then the Melitz value for GDP_d from (4.2). To prove the proposition it will be sufficient to demonstrate that under (A3.1) and (A3.2) the variables in our BR solution satisfy (4.4).

To do this, we will demonstrate that

$$R_{sd,c} = RA(s,d,c), \tag{A3.3}$$

$$LTOT_s = LTOTA(s) \tag{A3.4}$$

and

$$P_{d,c} = PCA(d,c) \quad . \tag{A3.5}$$

In combination with (4.2), (A3.2) and (T3.6), equations (A3.3) and (A3.4) are enough to prove that

$$GDPA(d) = GDP_d \tag{A3.6}$$

and then (T3.7), (A3.5) and (A3.1) lead to (4.4).

We start on (A3.3). From (4.6) we have

$$TA(s,d,c) - 1 = \frac{R_{sd,c}}{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)} \quad . \tag{A3.7}$$

Then substituting from (A3.7) and (T3.3) into (T3.5) gives

$$RA(s,d,c) = \left(\frac{R_{sd,c}}{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)}\right) * \left(\frac{QCA(d,c) * \left(\delta A(s,d,c) * \frac{PCA(d,c)}{PA(s,d,c)}\right)^{\sigma} * WA(s)}{\Phi A(s,c)}\right) \tag{A3.8}$$

Then using (4.7) we obtain

$$RA(s,d,c) = \left(\frac{R_{sd,c}}{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)}\right) * QCA(d,c) * \frac{WA(s)}{\Phi A(s,c)} *$$

$$\left(\left(\frac{\Phi A(s,c) * \frac{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd} - R_{sd,c}\right)}{W_s}}{Q_{d,c}} \right)^{\frac{1}{\sigma}} * \left(\frac{\left(\frac{W_s * TA(s,d,c)}{\Phi A(s,c)}\right)}{\left(\frac{\sum_t P_{\bullet td,c} Q_{\bullet td,c} N_{td,c}}{Q_{d,c}}\right)} \right) * \frac{PCA(d,c)}{PA(s,d,c)} \right)^{\sigma} \tag{A3.9}$$

Using (T3.1) and the properties of a converged solution, (A3.1) and (A3.2), (A3.9) can be simplified to

$$RA(s,d,c) = R_{sd,c} * \left(\frac{PCA(d,c)}{\frac{\sum_t P_{\bullet td,c} Q_{\bullet td,c} N_{td,c}}{Q_{d,c}}} \right)^{\sigma} \quad . \tag{A3.10}$$

From (T3.2), (4.7) and (T3.1), we have

$$PCA(d,c)^{1-\sigma} =$$

$$\sum_s \left(\frac{\Phi A(s,c) * \frac{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)}{W_s}}{Q_{d,c}} \right) * \left(\frac{\left(\frac{W_s * TA(s,d,c)}{\Phi A(s,c)} \right)}{\left(\frac{\sum_t P_{\bullet td,c} Q_{\bullet td,c} N_{td,c}}{Q_{d,c}} \right)} \right)^{\sigma} * \left(\frac{WA(s) * TA(s,d,c)}{\Phi A(s,c)} \right)^{1-\sigma}$$

(A3.11)

Then using (A3.2) and (4.6) and simplifying gives

$$PCA(d,c) = \sum_s \frac{P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c}}{Q_{d,c}} \quad . \qquad \text{(A3.12)}$$

Now from (A3.10) and (A3.12) we get (A3.3).

Next we move to the derivation of (A3.5). Using (T2.3) to eliminate $\delta^{\sigma}_{sd,c}$ from (T2.2) gives

$$P_{d,c} = \left(\sum_s N_{sd,c} \frac{Q_{\bullet sd,c}}{Q_{d,c}} \left(\frac{P_{\bullet sd,c}}{P_{d,c}} \right)^{\sigma} P^{1-\sigma}_{\bullet sd,c} \right)^{\frac{1}{(1-\sigma)}} \qquad \text{(A3.13)}$$

which simplifies to

$$P_{d,c} = \sum_s \frac{N_{sd,c} Q_{\bullet sd,c} P_{\bullet sd,c}}{Q_{d,c}} \quad . \qquad \text{(A3.14)}$$

Comparing (A3.14) and (A3.12) establishes (A3.5).

Finally we work on (A3.4). Substituting from (T3.3) and (4.7) into (T3.4) gives

$$LTOTA(s) =$$

$$\sum_{c,d} \frac{QCA(d,c)}{\Phi A(s,c)} * \left(\left(\frac{\Phi A(s,c) * \frac{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)}{W_s}}{Q_{d,c}} \right)^{\frac{1}{\sigma}} * \frac{\left(\frac{W_s * TA(s,d,c)}{\Phi A(s,c)} \right)}{\left(\frac{\sum_t P_{\bullet td,c} Q_{\bullet td,c} N_{td,c}}{Q_{d,c}} \right)} * \frac{PCA(d,c)}{PA(s,d,c)} \right)^{\sigma}$$

(A3.15)

Using (T3.1), (A3.2), (A3.12) and (A3.1) and simplifying gives

$$LTOTA(s) = \sum_{c,d} \frac{\left(P_{\bullet sd,s} Q_{\bullet sd,c} N_{sd,c} - R_{sd,c}\right)}{W_s} \quad . \qquad \text{(A3.16)}$$

Now we eliminate $R_{sd,c}$ via (4.1):

$$\text{LTOTA}(s) = \sum_{c,d} \frac{\left(P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c} - \left(T_{sd,c} - 1 \right) \frac{W_s}{\Phi_{\bullet sd,c}} N_{sd,c} Q_{\bullet sd,c} \right)}{W_s} \quad . \qquad \text{(A3.17)}$$

Through (T2.1) we obtain

$$\text{LTOTA}(s) = \sum_{c,d} \frac{P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c}}{W_s T_{sd,c}} \left(\frac{T_{sd,c}}{\sigma} + 1 - \frac{1}{\sigma} \right) \quad . \qquad \text{(A3.18)}$$

From the Melitz versions of (T2.9), (T2.5) and (T2.6), we have

$$0 = \sum_d N_{sd,c} \left(P_{\bullet sd,c} - \frac{W_s T_{sd,c}}{\Phi_{\bullet sd,c}} \right) Q_{\bullet sd,c} - \sum_d N_{sd,c} F_{sd,c} W_s - N_{s,c} H_{s,c} W_s \quad . \qquad \text{(A3.19)}$$

Using (A3.19) we can eliminate the F and H terms from (T2.7). Then, adding over c, we obtain

$$\sum_c L_{s,c} = \sum_{c,d} \frac{N_{sd,c} Q_{\bullet sd,c}}{\Phi_{\bullet sd,c}} + \sum_{c,d} N_{sd,c} \left(\frac{P_{\bullet sd,c}}{W_s} - \frac{T_{sd,c}}{\Phi_{\bullet sd,c}} \right) Q_{\bullet sd,c} \quad . \qquad \text{(A3.20)}$$

Using (T2.1) gives

$$\sum_c L_{s,c} = \sum_{c,d} \frac{N_{sd,c} Q_{\bullet sd,c}}{\Phi_{\bullet sd,c}} + \sum_{c,d} N_{sd,c} \left(\frac{P_{\bullet sd,c}}{\sigma W_s} \right) Q_{\bullet sd,c} \qquad \text{(A3.21)}$$

which can be rearranged via (T2.1) as

$$\sum_c L_{s,c} = \sum_{c,d} \frac{P_{\bullet sd,c} Q_{\bullet sd,c} N_{sd,c}}{W_s T_{sd,c}} \left(\frac{T_{sd,c}}{\sigma} + 1 - \frac{1}{\sigma} \right) \quad . \qquad \text{(A3.22)}$$

Comparing (A3.18) and (A3.22) and using (4.3)we see that (A3.4) holds.

Appendix 4. Showing that an increase in country 2's tariffs doesn't affect the number of firms in country 2

In the tariff simulations reported in subsection 6.3 we increased $T_{12,c}$ for all c by the same percentage. This resulted in: changes in the number of c-firms in country 1, $N_{1,c}$; changes in the number of c-firms operating on all international links, $N_{12,c}$ and $N_{21,c}$; but curiously no change that the number of c-firms in country 2, $N_{2,c}$. In this appendix we show why $N_{2,c}$ is constant. As it turns out this is not a fundamental or robust result. It depends on a series of special assumptions.

In demonstrating that $N_{2,c}$ is constant under the conditions imposed in subsection 6.3, we start by combining the Melitz versions of (T2.6) and (T2.9) for country 2:

$$0 = \sum_d N_{2d,c} \Pi_{\bullet 2d,c} - N_{2,c} H_{2,c} W_{2,c} \quad . \qquad \text{(A4.1)}$$

Now using (T2.1) and (T2.5) we obtain

$$0 = \sum_d N_{2d,c} \left[W_2 \left(\frac{T_{2d,c}}{(\sigma - 1)} \right) \frac{Q_{\bullet 2d,c}}{\Phi_{\bullet 2d,c}} - W_2 * F_{2d,c} \right] - N_{2,c} H_{2,c} W_2 \qquad \text{(A4.2)}$$

Next we note that production labor in the typical firm in country 2 producing c for sale on the 2-to-d link, $Q_{\bullet 2d,c}/\Phi_{\bullet 2d,c}$, is constant. This result can be derived as follows. From (T2.11) and (T2.12) we see that $Q_{\bullet 2d,c}/\Phi_{\bullet 2d,c}$ equals $\beta^{\sigma-1} * Q_{min(2,d),c}/\Phi_{min(2,d),c}$. With $F_{2d,c}$ and $T_{2d,c}$ fixed, (T2.10) implies that $Q_{min(2,d),c}/\Phi_{min(2,d),c}$ is fixed and hence $Q_{\bullet 2d,c}/\Phi_{\bullet 2d,c}$ is fixed.

Eliminating W_2 in (A4.2) and using the fixity of $Q_{\bullet 2d,c}/\Phi_{\bullet 2d,c}$, $F_{2d,c}$, $T_{2d,c}$ and $H_{2,c}$, we create a changes version:

$$0 = \sum_d N_{2d,c} \left[\left(\frac{T_{2d,c}}{(\sigma-1)} \right) \frac{Q_{\bullet 2d,c}}{\Phi_{\bullet 2d,c}} * n_{2d,c} - F_{2d,c} * n_{2d,c} \right] - N_{2,c} H_{2,c} * n_{2,c} \qquad \text{(A4.3)}$$

where $n_{2d,c}$ and $n_{2,c}$ are percentage change in $N_{2d,c}$ and $N_{2,c}$. With aggregate employment fixed in country 2, the symmetry of industries 1 and 2 implies that employment in each industry is fixed. Thus, from (T2.7) we obtain

$$0 = \sum_d \frac{N_{2d,c} Q_{\bullet 2d,c}}{\Phi_{\bullet 2d,c}} * n_{2d,c} + \sum_d N_{2d,c} F_{2d,c} * n_{2d,c} + N_{2,c} H_{2,c} * n_{2,c} \quad . \qquad \text{(A4.4)}$$

In setting up MelitzGE we assumed that $T_{2d,c} = T_{2,c}$ for all d. In fact we assumed that $T_{2d,c}$ is initially 1 for all c. But this is not important. The important thing is that country 2 faces the same tax/tariff rate on all trade links. With $T_{2d,c}$ the same for all d, we can move the T term in (A4.3) to the outside of the summation in which it occurs. Under this assumption, adding (A4.3) and (A4.4) yields

$$0 = \sum_d \frac{N_{2d,c} Q_{\bullet 2d,c}}{\Phi_{\bullet 2d,c}} * n_{2d,c} \quad . \qquad \text{(A4.5)}$$

Combining (A4.5) and (A4.4) implies that

$$N_{2,c} H_{2,c} * n_{2,c} = -\sum_d N_{2d,c} F_{2d,c} * n_{2d,c} \quad . \qquad \text{(A4.6)}$$

Substituting from (T2.10), (T2.11) and (T2.12) into (A4.6) gives

$$N_{2,c} H_{2,c} * n_{2,c} = -\frac{T_{2,c}}{(\sigma-1) * \beta^{\sigma-1}} * \sum_d N_{2d,c} * \frac{Q_{\bullet 2d,c}}{\Phi_{\bullet 2d,c}} n_{2d,c} \qquad \text{(A4.7)}$$

and combining (A4.5) and (A4.7) gives

$$n_{2,c} = 0 \text{ for all c} \quad . \qquad \text{(A4.8)}$$

This result is an artefact of the particular data setup of MelitzGE. It depends on the tariff/tax rates applying to country 2's c-firms on the 2-to-1 link being the same as those on the domestic 2-to-2 link. It also depends on the identical data setup in country 2 for industries/commodities 1 and 2. It was this assumption, combined with the constancy of aggregate employment in country 2 and the uniformity of the tariff shocks imposed by country 1 that led to the constancy of employment in each industry in country 2, enabling us to derive (A4.4).

Appendix 5. Deriving the Armington decomposition of Melitz welfare

Section 4 and Appendix 3 explain that a solution to the Melitz general equilibrium model specified by the Melitz versions of (T2.1) – (T2.12) and by (4.1) – (4.4) can be computed via the Armington model specified by (T3.1) – (T3.7). This requires that we: (a) adopt the same numeraire in the Armington model as in the Melitz model; (b) set the Armington production technology coefficients, tariff rates and conversion technology coefficients according to (4.5) – (4.7); and (c) assume the same level of aggregate employment in each country in the two models, that is

$$\mathrm{LTOTA(d)} = \mathrm{LTOT_d} \quad \text{for all d} \quad . \tag{A5.1}$$

Under (a) to (c),

$$\mathrm{QCA(d,c)} = \mathrm{Q_{d,c}} \quad \text{for all d and c} \tag{A5.2}$$

and

$$\mathrm{PCA(d,c)} = \mathrm{P_{d,c}} \quad \text{for all d and c} \tag{A5.3}$$

where

QCA(d,c) and $Q_{d,c}$ are the Armington and Melitz levels of consumption of composite commodity c in country d; and

PCA(d,c) and $P_{d,c}$ are the Armington and Melitz prices of composite commodity c in country d.

Via (A5.2) and (A5.3) we can rewrite (6.2) as

$$\mathrm{welfare(d)} = \sum_{c} \mathrm{ZA(d,c)} * \mathrm{qca(d,c)} \quad \text{for all d} \quad , \tag{A5.4}$$

where

qca(d,c) is the percentage change in QCA(d,c) computed in the Armington model satisfying (a) to (c) and ZA(d,c) is the Armington share of d’s expenditure devoted to commodity c. With Cobb-Douglas preferences, ZA(d,c) is a parameter and is the same as $\mu_{d,c}$ in Table 3.

Continuing to assume that (a) to (c) are satisfied, we work with the Armington model in Table 3 to derive the decomposition equation in Figure 2. Using the notational conventions explained at the foot of Figure 2, we start by writing Table 3 in percentage change and change form as:

$$\mathrm{pa(s,d,c)} = \mathrm{wa(s)} + \mathrm{ta(s,d,c)} - \phi\mathrm{a(s,c)} \tag{A5.5}$$

$$\mathrm{pca(d,c)} = \sum_{s} \mathrm{SA(s,d,c)} * \mathrm{pa(s,d,c)} + \frac{\sigma}{1-\sigma} * \sum_{s} \mathrm{SA(s,d,c)} * \hat{\delta}\mathrm{a(s,d,c)} \tag{A5.6}$$

$$\mathrm{qa(s,d,c)} = \mathrm{qca(d,c)} + \sigma * \hat{\delta}\mathrm{a(s,d,c)} + \sigma\left(\mathrm{pca(d,c)} - \mathrm{pa(s,d,c)}\right) \tag{A5.7}$$

$$\mathrm{LTOTA(s)} * \mathrm{ltota(s)} = \sum_{c,d} \left\{ \frac{\mathrm{QA(s,d,c)}}{\Phi\mathrm{A(s,c)}} * \left(\mathrm{qa(s,d,c)} - \phi\mathrm{a(s,c)}\right) \right\} \tag{A5.8}$$

$$\begin{aligned} &100 * \Delta\mathrm{RA(s,d,c)} = \\ &\quad \frac{\mathrm{TA(s,d,c)} * \mathrm{QA(s,d,c)} * \mathrm{WA(s)}}{\Phi\mathrm{A(s,c)}} * \left(\mathrm{ta(s,d,c)} + \mathrm{qa(s,d,c)} + \mathrm{wa(s)} - \phi\mathrm{a(s)}\right) \\ &- \frac{\mathrm{QA(s,d,c)} * \mathrm{WA(s)}}{\Phi\mathrm{A(s,c)}} * \left(\mathrm{qa(s,d,c)} + \mathrm{wa(s)} - \phi\mathrm{a(s)}\right) \end{aligned} \tag{A5.9}$$

$$GDPA(d)*gdpa(d) = WA(d)*LTOTA(d)*\left(wa(d)+ltota(d)\right) + 100*\sum_{c,s}\Delta RA\left(s,d,c\right) \quad (A5.10)$$

$$pca(d,c)+qca(d,c) = gdpa(d) \quad . \quad (A5.11)$$

The only new notation in these equations is SA(s,d,c) and ΔRA(s,d,c). SA(s,d,c) is the share of d's expenditure on c that is devoted to source s. It is given by[41]:

$$SA(s,d,c) = \frac{PA(s,d,c)*QA(s,d,c)}{\sum_j PA(j,d,c)*QA(j,d,c)} \quad (A5.12)$$

or equivalently by[42]

$$SA(s,d,c) = \frac{PA(s,d,c)*QA(s,d,c)}{PCA(d,c)*QCA(d,c)} \quad . \quad (A5.13)$$

ΔRA(s,d,c) is the change in RA(s,d,c). Because tariff collection can be zero, we use the change rather than the percentage change in RA(s,d,c).

Our first step in deriving the decomposition equation is to substitute from (A5.11) and (A5.6) into (A5.4). This gives

$$welfare(d) = gdpa(d) - \sum_c\sum_s ZA(d,c)*SA(s,d,c)*pa(s,d,c) - \frac{\sigma}{1-\sigma}\sum_c\sum_s ZA(d,c)*SA(s,d,c)*\hat{\delta}a(s,d,c) \quad (A5.14)$$

Now we work on gdpa(d). We substitute from (A5.9) into (A5.10) to obtain

$$GDPA(d)*gdpa(d) = WA(d)*LTOTA(d)*\left(wa(d)+ltota(d)\right) + \sum_{c,s}\frac{TA(s,d,c)*QA(s,d,c)*WA(s)}{\Phi A(s,c)}*\left(ta(s,d,c)+qa(s,d,c)+wa(s)-\phi a(s)\right) - \sum_{c,s}\frac{QA(s,d,c)*WA(s)}{\Phi A(s,c)}*\left(qa(s,d,c)+wa(s)-\phi a(s)\right)$$

(A5.15)

Substituting from (A5.5) into (A5.15) and using (T3.1) gives

$$GDPA(d)*gdpa(d) = WA(d)*LTOTA(d)*\left(wa(d)+ltota(d)\right) + \sum_{c,s}PA(s,d,c)*QA(s,d,c)*\left(pa(s,d,c)+qa(s,d,c)\right) - \sum_{c,s}\frac{QA(s,d,c)*PA(s,d,c)}{TA(s,d,c)}*\left(qa(s,d,c)+pa(s,d,c)-ta(s,d,c)\right)$$

(A5.16)

We rearrange (A5.16) as

---

[41] In deriving (A5.6), we use (T3.3) to obtain $SA(s,d,c) = \frac{\delta A(s,d,c)^{\sigma}*PA(s,d,c)^{1-\sigma}}{\sum_j \delta A(j,d,c)^{\sigma}*PA(j,d,c)^{1-\sigma}}$

[42] (T3.2) and (T3.3) imply that $PCA(d,c)*QCA(d,c) = \sum_j PA(j,d,c)*QA(j,d,c)$.

$$
\begin{aligned}
GDPA(d)*gdpa(d) = {} & WA(d)*LTOTA(d)*ltota(d) + WA(d)*LTOTA(d)*wa(d) \\
& + \sum_c \sum_s \frac{PA(s,d,c)*QA(s,d,c)}{TA(s,d,c)} * \left(TA(s,d,c) - 1\right) * qa(s,d,c) \\
& - \sum_c \frac{PA(F,d,c)*QA(F,d,c)}{TA(F,d,c)} * \left(pa(F,d,c) - ta(F,d,c)\right) \\
& + \sum_c \frac{PA(d,F,c)*QA(d,F,c)}{TA(d,F,c)} * \left(pa(d,F,c) - ta(d,F,c)\right) \\
& + \sum_c \frac{PA(d,d,c)*QA(d,d,c)}{TA(d,d,c)} * ta(d,d,c) + \sum_c \frac{PA(d,F,c)*QA(d,F,c)}{TA(d,F,c)} * ta(d,F,c) \\
& + \sum_c \sum_s PA(s,d,c)*QA(s,d,c)*pa(s,d,c) \\
& - \sum_c \frac{PA(d,d,c)*QA(d,d,c)}{TA(d,d,c)} * pa(d,d,c) \\
& - \sum_c \frac{PA(d,F,c)*QA(d,F,c)}{TA(d,F,c)} * pa(d,F,c)
\end{aligned}
\tag{A5.17}
$$

In (A5.17), as in Figure 2, we use the argument F to denote foreign country (not d). From here, we: use (A5.5) to substitute out pa in the last two terms on the RHS of (A5.17); cancel out some ta terms; separate newly introduced wa and ϕa terms; and use (T3.4) and (T3.1) to eliminate wa terms. These operations give

$$
\begin{aligned}
GDPA(d)*gdpa(d) = {} & WA(d)*LTOTA(d)*ltota(d) \\
& + \sum_c \sum_s \frac{PA(s,d,c)*QA(s,d,c)}{TA(s,d,c)} * \left(TA(s,d,c) - 1\right) * qa(s,d,c) \\
& - \sum_c \frac{PA(F,d,c)*QA(F,d,c)}{TA(F,d,c)} * \left(pa(F,d,c) - ta(F,d,c)\right) \\
& + \sum_c \frac{PA(d,F,c)*QA(d,F,c)}{TA(d,F,c)} * \left(pa(d,F,c) - ta(d,F,c)\right) \\
& + \sum_c \sum_s PA(s,d,c)*QA(s,d,c)*pa(s,d,c) \\
& + \sum_{c,j} \frac{PA(d,j,c)*QA(d,j,c)}{TA(d,j,c)} * \phi a(d,c)
\end{aligned}
\tag{A5.18}
$$

Now we return to (A5.14). Substituting from (A5.18) into (A5.14) gives

$$
\begin{aligned}
\mathrm{GDPA(d)*welfare(d)} &= \mathrm{WA(d)*LTOTA(d)*ltota(d)} \\
&+\sum_{c}\sum_{s}\frac{\mathrm{PA(s,d,c)*QA(s,d,c)}}{\mathrm{TA(s,d,c)}}*\left(\mathrm{TA(s,d,c)}-1\right)*\mathrm{qa(s,d,c)} \\
&-\sum_{c}\frac{\mathrm{PA(F,d,c)*QA(F,d,c)}}{\mathrm{TA(F,d,c)}}*\left(\mathrm{pa(F,d,c)}-\mathrm{ta(F,d,c)}\right) \\
&+\sum_{c}\frac{\mathrm{PA(d,F,c)*QA(d,F,c)}}{\mathrm{TA(d,F,c)}}*\left(\mathrm{pa(d,F,c)}-\mathrm{ta(d,F,c)}\right) \\
&+\sum_{c,j}\frac{\mathrm{PA(d,j,c)*QA(d,j,c)}}{\mathrm{TA(d,j,c)}}*\phi\mathrm{a(d,c)} \\
&+\sum_{c}\sum_{s}\mathrm{PA(s,d,c)*QA(s,d,c)*pa(s,d,c)} \\
&-\mathrm{GDPA(d)}*\sum_{c,s}\mathrm{ZA(d,c)*SA(s,d,c)*pa(s,d,c)} \\
&-\mathrm{GDPA(d)}*\frac{\sigma}{1-\sigma}\sum_{c,s}\mathrm{ZA(d,c)*SA(s,d,c)}*\hat{\delta}\mathrm{a(s,d,c)}
\end{aligned}
\tag{A5.19}
$$

Recalling that ZA(d,c) is the same as $\mu_{d,c}$ and using (T3.7) and (A5.13) we see that

$$
\mathrm{GDPA(d)*ZA(d,c)*SA(s,d,c)} = \mathrm{PA(s,d,c)*QA(s,d,c)} \quad . \tag{A5.20}
$$

This allows us to cancel the second-last and third-last terms in (A5.19). Equation (A5.20) also implies that

$$
\mathrm{GDPA(d)} = \sum_{c,s}\mathrm{PA(s,d,c)*QA(s,d,c)} \tag{A5.21}
$$

Using (A5.20) and (A5.21) in (A5.19) then gives us the decomposition equation in Figure 2.

Appendix 6. GEMPACK code for MelitzGE and a closure file for running Melitz and Armington in linked mode

This appendix sets out the GEMPACK code for MelitzGE (subsection A6.1) together with a closure file suitable for running Melitz and Armington in linked mode (subsection A6.2). Annotations referencing relevant equations and sections of this paper are provided. Readers who would like to work with the code can download it from the CoPS website (http://www.copsmodels.com/archivep/tpmj0140.zip). Enquires about GEMPACK licences can be made by contacting GEMPACK staff via sales@gempack.com.

A6.1. GEMPACK code for MelitzGE

```
!**************************************************************************!
! GEMPACK program for solving MelitzGE and Armington Auxiliary
  model                                                                   !
!                                                                       . !
!**************************************************************************!

File    SETS  #  Commodities and countries # ;
File    DATA  #  Other data e.g. parameter values #;

Set CNT # Set of regions # read elements from file SETS header "CNT";
Set COM # Set of commodities # read elements from file SETS header "COM";

Coefficient SIZECNT # Size of set CNT #;
Formula SIZECNT=0;
Formula SIZECNT = sum(c,CNT, 1);
Set CNTL = (all,c,CNT: $Pos(c) < SIZECNT);! All countries excluding last !

Coefficient (Parameter)
SIGMA # Substitution elasticity between varieties #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_F(c,s,d) # Units of labor required to setup a c-firm for trade on the sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_T(c,s,d) # Power of tariff on c imposed by d on flows from s #;
        (All,s,CNT)
C_W(s) # Wage rate in region s #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PIT(c,s,d) # Profits earned on the sd-link by the typical c-firm on sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_DELTA(c,s,d) # d's preference for c from s #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PHI_MIN(c,s,d) # Productivity (marginal output/worker) of minimum prod'ty c firm on sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
```

```
C_N(c,s,d) # Number of firms in region s sending c on the sd-link #;
        (All,c,COM)(All,s,CNT)
C_ND(c,s) # Number of c-producing firms in region s #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PHIT(c,s,d)# Productivity (marginal output/worker) of a typical c firm on sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PT(c,s,d) # Price of c charged by the typical c-producing firm on sd-link #;
        (All,c,COM)(All,d,CNT)
C_P(c,d) # Price of the c-composite in region d #;
        (Parameter)
ALPHA # Parameter in Pareto distribution of firm productivities #;
        (Parameter)
BETA # Ratio typical prod'ty on link to minimum, determined by sigma and alpha #;
        (All,c,COM)(All,d,CNT)
C_QD(c,d) # Demand for composite c in region d # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_Q(c,s,d) # The CES aggregate quantity of c sent on the sd-link # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_QT(c,s,d) # Quantity sent by the typical c-producing firm on the sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_Q_MIN(c,s,d) # Quantity sent by lowest productivity c-firm operating on the sd-link #;
        (All,c,COM)(All,s,CNT)
C_H(c,s) # Fixed setup cost for a c-firm in region s # ;
        (All,c,COM)(All,s,CNT)
C_L(c,s) # Employment in the c-industry in region s # ;
        (parameter)
LB_C_PHI_MIN  # Minimum productivity for a firm to produce # ;
        (parameter)
UB_C_PHI_MIN # Minimum productivity of firms that trade on all links #;
        (integer, parameter)
NUMREG #  Number of regions #;

Read SIGMA from file DATA Header "SGMA";! 3.8 is value used by Balistreri and Rutherford (2013)!
Read ALPHA from file DATA Header "ALFA";! 4.6 is value used by Balistreri and Rutherford (2013)!
Read  UB_C_PHI_MIN from file DATA Header "UBMN";
```

```
Formula (initial) NUMREG = sum(c,CNT, 1) ;
Formula (initial) (All,s,CNT) C_W(s) = 1.0 ;
Formula (initial) (All,c,COM)(All,s,CNT)(All,d,CNT) C_T(c,s,d) = 1.0 ;
Formula (initial) (All,c,COM)(All,s,CNT)(All,d,CNT) C_DELTA(c,s,d) = 1.0 ;
Formula (initial) LB_C_PHI_MIN = 1.1 ;

! Here we calculate the minimum productivity that enables source region s to
  trade with destination region d. (Explained in section 6, see 6.1) !
Formula (initial)
(All,c,COM)(All,s,CNT)(All,d,CNT)
  C_PHI_MIN(c,s,d) = LB_C_PHI_MIN
  + (UB_C_PHI_MIN-LB_C_PHI_MIN)*2*(1.0/NUMREG)*
  min{ABS($Pos(s,CNT)-$Pos(d,CNT)), NUMREG-ABS[$Pos(s,CNT)-$Pos(d,CNT)]} ;

Formula (initial) (All,c,COM)(All,s,CNT) C_ND(c,s) = 1.0 ; ! Normalization, see section 5 !

Formula (initial) (All,c,COM)(All,d,CNT) C_QD(c,d) = 1.0 ; ! Normalization, see section 6 !

!     Starting from values for SIGMA, C_W, C_T, C_DELTA, C_PHI_MIN, C_ND,      !
!     ALPHA and C_QD, we compute values for the other parameters and           !
!     coefficients in Table 2 in a sequence.  The sequence is recursive in     !
!     the sense that the coefficients on the RHS of each formula are known     !
!     from earlier formulas.                                         !

Formula
! Equation (A1.7) !
(Initial)  BETA = [ALPHA/(ALPHA - SIGMA + 1)]^(1/(SIGMA-1));
! Metlitz (T2.11) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PHIT(c,s,d)= BETA*C_PHI_MIN(c,s,d);
! Melitz (T2.8) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT)
C_N(c,s,d)=  [C_PHI_MIN(c,s,d)^(-ALPHA)]*C_ND(c,s);
```

```
! Melitz (T2.1) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PT(c,s,d) = [C_W(s)*C_T(c,s,d)/C_PHIT(c,s,d)]*SIGMA/(SIGMA-1);
! Melitz (T2.2) !
(Initial) (All,c,COM)(All,d,CNT)
C_P(c,d) = sum(s,CNT,
      C_N(c,s,d)*(C_DELTA(c,s,d)^SIGMA)*C_PT(c,s,d)^(1-SIGMA) )^(1/(1-SIGMA));
! Meltiz (T2.3) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT)
C_QT(c,s,d) = C_QD(c,d)*(C_DELTA(c,s,d)^SIGMA)*[C_P(c,d)/C_PT(c,s,d)]^SIGMA ;
! Melitz (T2.12) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT)
C_Q_MIN(c,s,d) = C_QT(c,s,d)/(BETA^SIGMA);
! Melitz (T2.10) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT) C_F(c,s,d)
= (1/(SIGMA-1))*[C_T(c,s,d)/C_PHI_MIN(c,s,d)]*C_Q_MIN(c,s,d);
! Melitz (T2.4) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT) C_Q(c,s,d)
= C_QT(c,s,d)*C_N(c,s,d)^[SIGMA/(SIGMA-1)];
! Melitz (T2.5) !
(Initial) (All,c,COM)(All,s,CNT)(All,d,CNT) C_PIT(c,s,d) =
(C_PT(c,s,d) - C_W(s)*C_T(c,s,d)/C_PHIT(c,s,d))*C_QT(c,s,d) - C_F(c,s,d)*C_W(s);
! Melitz (T2.6 and T2.9)!
(Initial) (All,c,COM)(All,s,CNT)
  C_H(c,s) = Sum(d,CNT, C_N(c,s,d)*C_PIT(c,s,d))/[C_ND(c,s)*C_W(s)];
! Melitz (T2.7) !
(Initial) (All,c,COM)(All,s,CNT)
  C_L(c,s) = Sum(d,CNT, C_N(c,s,d)*C_QT(c,s,d)/C_PHIT(c,s,d))
    + Sum(d,CNT, C_N(c,s,d)*C_F(c,s,d)) +C_ND(c,s)*C_H(c,s);

Variable ! These are all percentage changes !
        (All,c,COM)(All,s,CNT)(All,d,CNT)
p_tsd(c,s,d) # Price of c charged by the typical c-producing firm on sd-link #;
        (All,s,CNT)
w(s) # Wage rate in region s #;
```

```
        (All,c,COM)(All,s,CNT)(All,d,CNT)
t(c,s,d) # Power of tariff on c imposed by d on flows from s #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
phi_tsd(c,s,d)# Productivity (marginal output/worker) of a typical c firm on sd-link #;
        (All,c,COM)(All,d,CNT)
p_d(c,d) # Price of the c-composite in region d # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
q_tsd(c,s,d) # Quantity sent by the typical c-producing firm on the sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
n_sd(c,s,d) # Number of firms in region s sending c on the sd-link #;
        (All,c,COM)(All,d,CNT)
q_d(c,d) # Quantity of composite c consumed region d # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
q_sd(c,s,d) # The CES aggregate quantity of c sent on the sd-link # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
pi_tsd(c,s,d) # Profits earned on the sd-link by the typical c-firm on sd-link #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
delta(c,s,d);
        (All,c,COM)(All,s,CNT)(All,d,CNT)
f_sd(c,s,d) # Units of labor required to setup a c-firm for trade on the sd-link #;
        (All,c,COM)(All,s,CNT)
nd(c,s);
        (All,c,COM)(All,s,CNT)
h(c,s) # Fixed setup cost for a c-firm in region s # ;
        (All,c,COM)(All,s,CNT)
l(c,s) # Employment in the c-industry in region s #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
phi_min(c,s,d);
        (All,c,COM)(All,s,CNT)(All,d,CNT)
q_min(c,s,d) # Quantity sent by lowest productivity c-firm operating on the sd-link #;

Update
(All,c,COM)(All,s,CNT)(All,d,CNT) C_T(c,s,d) = t(c,s,d);
(All,s,CNT) C_W(s) = w(s);
(All,c,COM)(All,s,CNT)(All,d,CNT) C_DELTA(c,s,d)= delta(c,s,d);
```

```
(All,c,COM)(All,s,CNT)(All,d,CNT) C_PHI_MIN(c,s,d)= phi_min(c,s,d);
(All,c,COM)(All,s,CNT) C_ND(c,s) = nd(c,s);
(All,c,COM)(All,d,CNT) C_QD(c,d) = q_d(c,d);
(All,c,COM)(All,s,CNT)(All,d,CNT) C_F(c,s,d) = f_sd(c,s,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_PIT(c,s,d) = pi_tsd(c,s,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_N(c,s,d) = n_sd(c,s,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_PHIT(c,s,d) = phi_tsd(c,s,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_PT(c,s,d) = p_tsd(c,s,d) ;
(All,c,COM)(All,d,CNT) C_P(c,d) = p_d(c,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_Q(c,s,d) = q_sd(c,s,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_QT(c,s,d) = q_tsd(c,s,d) ;
(All,c,COM)(All,s,CNT)(All,d,CNT) C_Q_MIN(c,s,d) = q_min(c,s,d) ;
(All,c,COM)(All,s,CNT) C_H(c,s) = h(c,s) ;
(All,c,COM)(All,s,CNT) C_L(c,s) = l(c,s) ;

! ************************************************************************ !
!    Percentage change version of the Melitz sectoral model, in Table 2.     !
! ************************************************************************ !

! We start by evaluating C_R(c,s,d).  This is the region d's share of its
  expenditure on c that is sourced from region s.  This coefficient appears in
  the percentage change version of Melitz equation (T2.2). !
Coefficient
        (all,c,COM)(All,s,CNT)(All,d,CNT)
C_R(c,s,d) # Region d's share of expenditure on c that is sourced from region s #;
        (All,c,COM)(All,d,CNT)
C_RBot(c,d);
Formula (All,c,COM)(All,d,CNT) C_RBot(c,d)
        = Sum(s,CNT,  C_N(c,s,d)*(C_DELTA(c,s,d)^SIGMA)*(C_PT(c,s,d)^(1-SIGMA)) );
(All,c,COM)(All,s,CNT)(All,d,CNT) C_R(c,s,d)
        = [ C_N(c,s,d)*(C_DELTA(c,s,d)^SIGMA)*(C_PT(c,s,d)^(1-SIGMA)) ]/C_Rbot(c,d);

! Percentage change forms for the Melitz equations from table 2. !

Equation E_p_tsd # Melitz equation (T2.1) #
```

```
(All,c,COM)(All,s,CNT)(All,d,CNT)
p_tsd(c,s,d) = w(s) + t(c,s,d)-phi_tsd(c,s,d);

Equation E_p_d # Melitz equation (T2.2) #
(All,c,COM)(All,d,CNT)
p_d(c,d) = (1/(1-SIGMA))* Sum(s, CNT, C_R(c,s,d)*[n_sd(c,s,d)+SIGMA*delta(c,s,d)
   +(1-SIGMA)*p_tsd(c,s,d) ]);

Equation E_q_tsd # Melitz equation (T2.3) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
q_tsd(c,s,d) = [q_d(c,d) + SIGMA*(p_d(c,d) + delta(c,s,d) - p_tsd(c,s,d))] ;

Equation E_q_sd # Melitz equation (T2.4) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
q_sd(c,s,d) = (SIGMA/(SIGMA - 1))*n_sd(c,s,d) + q_tsd(c,s,d) ;

Equation E_pi_tsd # Melitz equation (T2.5) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
C_PIT(c,s,d)*pi_tsd(c,s,d) =
  C_PT(c,s,d)*C_QT(c,s,d)*[p_tsd(c,s,d) + q_tsd(c,s,d)]
  - C_W(s)*C_T(c,s,d)*C_QT(c,s,d)/C_PHIT(c,s,d)*[w(s) + t(c,s,d) + q_tsd(c,s,d)
                                                              - phi_tsd(c,s,d) ]
  - [C_F(c,s,d)*C_W(s)]*(f_sd(c,s,d)+w(s));

Equation E_nd # Melitz equation (T2.6 and T2.9) #
(All,c,COM)(All,s,CNT)
C_ND(c,s)*C_H(c,s)*C_W(s)*[nd(c,s) + h(c,s)+ w(s)] =
Sum(d,CNT, C_N(c,s,d)*C_PIT(c,s,d)*[n_sd(c,s,d)+pi_tsd(c,s,d) ]);

Equation E_l # Melitz equation (T2.7) #
(All,c,COM)(All,s,CNT) C_L(c,s)*l(c,s) = sum(d,CNT,
[C_N(c,s,d)*C_QT(c,s,d)/C_PHIT(c,s,d)]
*[n_sd(c,s,d)+q_tsd(c,s,d) -phi_tsd(c,s,d)])
     + sum(d,CNT, C_N(c,s,d)*C_F(c,s,d)*[n_sd(c,s,d)+f_sd(c,s,d)])
     + C_ND(c,s)*C_H(c,s)*[nd(c,s) + h(c,s)];
```

```
Equation E_n_sd # Melitz equation (T2.8) #
(All,c,COM)(All,s,CNT)(All,d,CNT) n_sd(c,s,d) = nd(c,s) -ALPHA*phi_min(c,s,d);

Equation E_phi_min # Meltiz equation (T2.10) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
 phi_min(c,s,d)+f_sd(c,s,d)  = t(c,s,d) +q_min(c,s,d);

Equation E_phi_tsd # Melitz equation (T2.11) #
(All,c,COM)(All,s,CNT)(All,d,CNT) phi_tsd(c,s,d) =phi_min(c,s,d);

Equation E_q_min # Meltiz equation (T2.12) #
(All,c,COM)(All,s,CNT)(All,d,CNT) q_min(c,s,d) = q_tsd(c,s,d);

! *************************************************************************** !
! Completing the Melitz general equilibrium model & adding useful definitions  !
! *************************************************************************** !

Coefficient
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_REV(c,s,d) # Tariff revenue on sd-link # ;
        (All,d,CNT)
C_LTOT(d) # Aggregate employment in region d # ;
        (All,d,CNT)
C_GDP(d) #  Nominal GDP in region d #;
        (All,c,COM)(All,d,CNT)
C_MU(c,d) # share of d's expenditure devoted to commodity c # ;
        (All,d,CNT)
C_BTS(d) #  Balance of trade surplus calculated as exports (fob) minus imports (cif) #;
        (All,d,CNT)
C_BTS_CHK(d) # Balance of trade surplus calculated as GDP minus absorption #;
        (All,c,COM)
C_LS(c) # World-wide employment in the c-industry # ;
```

```
Formula
! Equation (4.1) !
(initial) (All,c,COM)(All,s,CNT)(All,d,CNT)
C_REV(c,s,d) = (C_T(c,s,d)-1)*(C_W(s)/C_PHIT(c,s,d))*C_N(c,s,d)*C_QT(c,s,d) ;
! Equation (4.3) !
(initial) (All,d,CNT) C_LTOT(d) = sum(c,COM, C_L(c,d)) ;

! Equation (4.2) !
(initial) (All,d,CNT) C_GDP(d) = sum(c,COM, C_W(d)*C_L(c,d))
  + sum(c,COM, sum(s,CNT, C_REV(c,s,d)));
! Equation (4.4) !
(initial) (All,c,COM)(All,d,CNT) C_MU(c,d) = C_QD(c,d)*C_P(c,d)/C_GDP(d) ;

! Other useful coefficients !
(initial) (All,d,CNT) C_BTS(d) = C_GDP(d) - sum(c,COM, C_P(c,d)*C_QD(c,d));
(initial) (All,d,CNT) C_BTS_CHK(d) =
  sum(c,COM, sum(r,CNT, C_PT(c,d,r)*C_QT(c,d,r)*C_N(c,d,r)))
  - sum(c,COM, sum(r,CNT,
             (C_T(c,d,r)-1)*(C_W(d)/C_PHIT(c,d,r))*C_N(c,d,r)*C_QT(c,d,r)))
  - sum(c,COM, sum(s,CNT, C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d)))
  + sum(c,COM, sum(s,CNT,
             (C_T(c,s,d)-1)*(C_W(s)/C_PHIT(c,s,d))*C_N(c,s,d)*C_QT(c,s,d)));

(initial) (All,c,COM)  C_LS(c) = Sum(s,CNT, C_L(c,s));

Variable
        (All,d,CNT)
gdp(d) # GDP for region d  # ;
        (Change)(All,d,CNT)
d_bts(d) # Balance of trade #;
        (Change)(All,d,CNT)
d_bts_chk(d) # Balance of trade check # ;
        (All,c,COM)
ls(c) # worldwide employment in industry c # ;
```

```
        (All,d,CNT)
ltot(d) # Aggregate employment in region d # ;
        (change)(All,c,COM)(All,s,CNT)(All,d,CNT)
d_rev(c,s,d) # Tariff revenue on sd-link # ;
        (All,c,COM)(All,d,CNT)
mu(c,d) # share of d's expenditure devoted to commodity c # ;
        (All,c,COM)(All,d,CNT)
f_mu(c,d) # Matrix shifter on mu # ;
        (All,d,CNT)
ff_mu(d) # Vector shifter on mu # ;
ave_wage # Average worldwide wage rate #;
        (All,s,CNT)
welfare(s) #  Welfare, calculated in Melitz as real consumption #;
wld_welfare  #  World welfare, calculated in Melitz as real consumption #;

Update
(change) (All,c,COM)(All,s,CNT)(All,d,CNT) C_REV(c,s,d) = d_rev(c,s,d) ;
(All,d,CNT) C_LTOT(d) = ltot(d) ;
(All,d,CNT) C_GDP(d) = gdp(d) ;
(All,c,COM)(All,d,CNT) C_MU(c,d) = mu(c,d) ;
(change) (All,d,CNT) C_BTS(d) = d_bts(d) ;
(change) (All,d,CNT) C_BTS_CHK(d) = d_bts_chk(d) ;
(All,c,COM) C_LS(c) = ls(c) ;

! Percentage change versions of  equations that complete the Melitz           !
! general equilibrium model (section 4.1.1) and other useful macro equations   !

Equation E_rev # Equation (4.1) #
(All,c,COM)(All,d,CNT)(All,r,CNT)
100*d_rev(c,d,r)
 =  C_T(c,d,r)*(C_W(d)/C_PHIT(c,d,r))*C_N(c,d,r)*C_QT(c,d,r)
               *[t(c,d,r) +w(d) +n_sd(c,d,r)+q_tsd(c,d,r) -phi_tsd(c,d,r)]
- (C_W(d)/C_PHIT(c,d,r))*C_N(c,d,r)*C_QT(c,d,r)
               *[w(d) +n_sd(c,d,r)+q_tsd(c,d,r) -phi_tsd(c,d,r)]  ;
```

```
Equation E_gdp # Equation (4.2) #
(All,d,CNT) C_GDP(d)*gdp(d) = sum{c,COM, C_W(d)*C_L(c,d)*(w(d)+l(c,d))}
+sum(c,COM, sum(s,CNT,  100*d_rev(c,s,d) )) ;

Equation E_w # Equation (4.3)  #
(All,s,CNT) C_LTOT(s)*ltot(s) = sum(c,COM, C_L(c,s)*l(c,s) ) ;

Equation E_q_d # Equation (4.4)  #
(All,c,COM)(All,d,CNT) p_d(c,d) + q_d(c,d) = mu(c,d) + gdp(d) ;

!  Other useful equations for the Melitz model !

Equation E_bts # Balance of trade surplus: GDP - Absorption #
(All,d,CNT) 100*d_bts(d) = C_GDP(d)*gdp(d) -
    sum{c,COM, (C_P(c,d)*C_QD(c,d))*[p_d(c,d) + q_d(c,d)] };

Equation E_bts_chk # Balance of trade: exports - imports #
(All,d,CNT) 100*d_bts_chk(d) = sum{c,COM,
  Sum(r,CNT, C_PT(c,d,r)*C_QT(c,d,r)*C_N(c,d,r)
                *[p_tsd(c,d,r)+ q_tsd(c,d,r)+n_sd(c,d,r)]
 - 100*d_rev(c,d,r))
 - Sum(s,CNT, C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d)
                *[p_tsd(c,s,d)+ q_tsd(c,s,d)+n_sd(c,s,d)]
 - 100*d_rev(c,s,d))       };

Equation E_ls # Worldwide employment in industry c #
(All,c,COM) C_LS(c)*ls(c) = sum(s,CNT, C_L(c,s)*l(c,s));

Equation E_ave_wage # average world-wide wage rate#
 Sum(tt,CNT, C_LTOT(tt))*ave_wage = sum(s,CNT, C_LTOT(s)*w(s));

Equation E_mu2 # Allows movements in total consumption to GDP ratio in d: useful for Walras' law #
(All,c,COM)(All,d,CNT)
mu(c,d) =ff_mu(d) + f_mu(c,d);
```

```
Equation E_welfare  #  Welfare, calculated in Melitz as real consumption #
(All,d,CNT)
welfare(d) = (1/sum{cc,COM, C_P(cc,d)*C_QD(cc,d)})
                                  *sum{c,COM, C_P(c,d)*C_QD(c,d)*q_d(c,d)};

Equation E_wld_welfare  #  World welfare, calculated in Melitz as real consumption #
wld_welfare = (1/sum(d,CNT, sum(cc,COM, C_P(cc,d)*C_QD(cc,d))))
                                  *sum(dd,CNT, sum{c,COM, C_P(c,dd)*C_QD(c,dd)*q_d(c,dd)});

Variable ag_ltot # Total employment, world #;
Variable (All,s,CNT) rel_wage(s)  # Wage in s relative to world average # ;

Equation E_ag_ltot # Total employment, world #
 Sum(tt,CNT, C_LTOT(tt))*ag_ltot = sum(s,CNT, C_LTOT(s)*ltot(s));

Equation E_rel_wage # Wage in s relative to world average #
(All,s,CNT) w(s) = ave_wage +rel_wage(s);

! *************************************************************************** !
!     The Armington Auxiliary model                                           !
! *************************************************************************** !

! *************************************************************************** !
! Setting parameter values and finding an initial solution for the            !
! Armington auxiliary model consistent with initial solution for Melitz       !
! *************************************************************************** !
Coefficient
        (Parameter)
SIGMAA # Substitution elasticity between varieties, for Armington #;
! connecting the Armington and Melitz models is legitimate only if SIGMA and
SIGMAA are the same  !
        (All,d,CNT)
C_WA(d) # Wage rate in region d, Armington #;
```

```
        (All,c,COM)(All,d,CNT)
C_QDA(c,d) # Demand in d for composite c, Armington #;
        (All,d,CNT)
C_LTOTA(d) # Aggregate employment in region d, Armington # ;
        (All,c,COM)(All,s,CNT)
C_PHIA(c,s) # Productivity, industry c region s, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_QA(c,s,d) # Quantity of c sent from s to d, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_TA(c,s,d) # Power of tariff on c sent from s to d, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_PA(c,s,d) # Price to consumers in d of c sent from s, Armington model # ;
        (All,c,COM)(All,d,CNT)
C_PCA(c,d) # Price of composite c to consumers in d, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_DELTAA(c,s,d) # Regions d's preference coefficient for c from s, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
C_REVA(c,s,d) # Tariff revenue on c sent from s to d, Armington model # ;
        (All,d,CNT)
C_GDPA(d) # Income side GDP in region d, Armington model # ;
        (all,c,COM)(All,s,CNT)(All,d,CNT)
C_SHA(c,s,d) # Used in % form of T3.2: region d's share of expenditure on c sourced from s # ;
        (All,c,COM)(All,d,CNT)
C_SHBotA(c,d) # Used in forming C_SHA #;

Read SIGMAA from file DATA Header "SGAA";

! Aligns initial solution for Armington with that for Melitz !
Formula
(Initial) (All,d,CNT) C_WA(d) =C_W(d);
(initial) (All,c,COM)(All,d,CNT) C_QDA(c,d) = C_QD(c,d) ;
(initial) (All,d,CNT) C_LTOTA(d) = C_LTOT(d) ;
```

```
! Sets Armington values for productivity, tariffs and tastes consistent with Melitz !
Formula
! Equation (4.5) !
(Initial)(All,c,COM)(All,s,CNT)
  C_PHIA(c,s) = sum(d,CNT, C_QT(c,s,d)*C_N(c,s,d))/C_L(c,s) ;
! Equation (4.6) !
(Initial)(All,c,COM)(All,s,CNT)(All,d,CNT)
  C_TA(c,s,d) = 1 + [C_REV(c,s,d)]/[C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d) - C_REV(c,s,d)] ;

! Equation (4.7). In section 4 we assume that SIGMAA = SIGMA.  This is essential
if we are linking the Armington and Melitz models. However we sometimes want to
delink them as in Tables 8 and 9 and assume different substitution elasticities
in the two models.  The code below is legitimate even when we want to calculate
Armington solutions starting from the same database (value flows)
as Melitz but using a different substitution elasticity !
(Initial)(All,c,COM)(All,s,CNT)(All,d,CNT)
 C_DELTAA(c,s,d) = {[ C_PHIA(c,s)
     *{[C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d) - C_REV(c,s,d)]/C_W(s)}/C_QD(c,d)]^(1/SIGMAA)}
     *{[C_W(s)*C_TA(c,s,d)/C_PHIA(c,s)]/[sum{r,CNT, C_PT(c,r,d)*C_QT(c,r,d)*C_N(c,r,d)}/C_QD(c,d)]};

! Completing the initial solution for the Armington model  !
 ! Equation (T3.1) !
(Initial)(All,c,COM)(All,s,CNT)(All,d,CNT)
  C_PA(c,s,d) = C_WA(s)*C_TA(c,s,d)/C_PHIA(c,s) ;
! Equation (T3.2) !
(Initial)(All,c,COM)(All,d,CNT)
 C_PCA(c,d) = [sum{s,CNT, (C_DELTAA(c,s,d)^SIGMAA)*(C_PA(c,s,d)^(1-SIGMAA))}]^(1/(1-SIGMAA));
! Equation (T3.3) !
(Initial)(All,c,COM)(All,s,CNT)(All,d,CNT)
  C_QA(c,s,d) =  C_QDA(c,d)*[C_DELTAA(c,s,d)*C_PCA(c,d)/C_PA(c,s,d)]^SIGMAA;
! Equation (T3.5) !
(Initial)(All,c,COM)(All,s,CNT)(All,d,CNT)
  C_REVA(c,s,d) = {C_TA(c,s,d)-1}*{C_QA(c,s,d)*C_WA(s)/C_PHIA(c,s)};
```

```
! Equation (T3.6) !
(Initial)(All,d,CNT)
  C_GDPA(d) = C_W(d)*C_LTOT(d) + sum(c,COM, sum(s,CNT, C_REVA(c,s,d)));

! Evaluating C_SHA(c,s,d): region d's share of expenditure on c sourced from s.
This is used in the % change form of T3.2.  We rely on C_SHA(c,s,d) being
C_DELTAA(c,s,d)^(SIGMAA)*C_PA(c,s,d)^(1-SIGMAA) divided by the sum over s of these terms !

Formula (All,c,COM)(All,d,CNT) C_SHBotA(c,d) = C_PCA(c,d)*C_QDA(c,d);

(All,c,COM)(All,s,CNT)(All,d,CNT) C_SHA(c,s,d)
=  C_PA(c,s,d)*C_QA(c,s,d)/ C_SHBotA(c,d);

Variable
        (All,d,CNT)
wa(d) # Wage rate in region d, Armington #;
        (All,c,COM)(All,d,CNT)
q_da(c,d) # Demand in d for composite c, Armington #;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
qa(c,s,d)  # Quantity of c sent from s to d, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
pa(c,s,d) # Price to consumers in d of c sent from s, Armington model # ;
        (All,c,COM)(All,d,CNT)
pca(c,d) # Price of composite c to consumers in d, Armington model # ;
        (change)(All,c,COM)(All,s,CNT)(All,d,CNT)
d_reva(c,s,d) # Tariff revenue on c sent from s to d, Armington model # ;
        (All,d,CNT)
gdpa(d) # GDP in region d, Armington model # ;
        (All,c,COM)(All,s,CNT)
phia(c,s) # Productivity, industry c region s, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
ta(c,s,d) # Power of tariff on c sent from s to d, Armington model # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
deltaa(c,s,d) # Region d's preference coefficient for c from s, Armington model # ;
```

```
        (All,d,CNT)
ltota(d) # Employment in d, Armington #;
        (All,c,COM)(All,d,CNT)
mua(c,d) # Share of d's expenditure devoted to c, Armington # ;
        (All,c,COM)(All,s,CNT)
slack_phia(c,s) # Endogenize to set phia independently of Melitz # ;
        (change)(All,c,COM)(All,s,CNT)(All,d,CNT)
d_slack_ta(c,s,d) # Endogenize to set ta independently of Melitz # ;
        (All,c,COM)(All,s,CNT)(All,d,CNT)
sl_deltaa(c,s,d) # Endogenize to set deltaa independently of Melitz # ;
        (All,d,CNT)
f_wa(d) # Exogenize for one country to equalize Armington & Melitz numeraires # ;
        (All,d,CNT)
f_ltota(d) # Exogenize to equalize Armington & Melitz aggregate employment # ;
        (All,c,COM)(All,d,CNT)
f_mua(c,d) # Exogenize to equalize d's expend. share devoted to c in Armington & Melitz  # ;
        (All,c,COM)(All,d,CNT)
f_muan(c,d) # Matrix shifter on mua # ;
        (All,d,CNT)
ff_mua(d) # Vector shifter on mua # ;

Update
(All,d,CNT) C_WA(d)=wa(d);
(All,c,COM)(All,d,CNT) C_QDA(c,d) = q_da(c,d);
(All,c,COM)(All,s,CNT)   C_PHIA(c,s)= phia(c,s);
(All,c,COM)(All,s,CNT)(All,d,CNT)  C_QA(c,s,d) = qa(c,s,d);
(All,c,COM)(All,s,CNT)(All,d,CNT)   C_TA(c,s,d) =ta(c,s,d);
(All,c,COM)(All,s,CNT)(All,d,CNT)   C_PA(c,s,d) = pa(c,s,d);
(All,c,COM)(All,d,CNT) C_PCA(c,d) = pca(c,d);
(All,c,COM)(All,s,CNT)(All,d,CNT) C_DELTAA(c,s,d) = deltaa(c,s,d);
(Change) (All,c,COM)(All,s,CNT)(All,d,CNT)  C_REVA(c,s,d)=d_reva(c,s,d);
(All,d,CNT)  C_GDPA(d) = gdpa(d);
(All,s,CNT)  C_LTOTA(s)= ltota(s);
```

```
! ************************************************************************ !
! Equations for transfering results from Melitz to Armington               !
! ************************************************************************ !

! Determination of movements in Armington productivity, taste and tariff
  variables consistent with Melitz !

Equation E_phia # Equation (4.5) #
(All,c,COM)(All,s,CNT)
phia(c,s) = [1/sum(dd,CNT, C_QT(c,s,dd)*C_N(c,s,dd))]
            *sum(d,CNT, C_QT(c,s,d)*C_N(c,s,d)*[q_tsd(c,s,d) + n_sd(c,s,d)])
              - l(c,s) +slack_phia(c,s);

Equation E_ta # Equation (4.6) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
C_TA(c,s,d)*ta(c,s,d) = 100/[C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d) - C_REV(c,s,d)]*d_rev(c,s,d)
-C_REV(c,s,d)/{[C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d) - C_REV(c,s,d)]^2}*
[C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d)*(p_tsd(c,s,d)+q_tsd(c,s,d) + n_sd(c,s,d))-100*d_rev(c,s,d)]
   +100*d_slack_ta(c,s,d);

! E_deltaa should be turned off (by making deltaa exogenous and sl_deltaa
endogenous) if SIGMAA is not equal to SIGMA !
Equation E_deltaa # Equation (4.7) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
deltaa(c,s,d) =(1/SIGMA)* (1/{C_PHIA(c,s)
     *[C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d) -
C_REV(c,s,d)]/C_W(s)} )*{[C_PHIA(c,s)*C_PT(c,s,d)*C_QT(c,s,d)*C_N(c,s,d)/C_W(s)]*
           [phia(c,s)+p_tsd(c,s,d)+q_tsd(c,s,d) + n_sd(c,s,d)- w(s)]
 -[C_PHIA(c,s)* C_REV(c,s,d)/C_W(s)]* (phia(c,s) -w(s))
-100*[C_PHIA(c,s)/C_W(s)]* (d_rev(c,s,d))}
+(1- 1/SIGMA)*q_d(c,d)

+[ w(s)+ta(c,s,d)-phia(c,s)]
-[1/Sum(r,CNT,C_PT(c,r,d)*C_QT(c,r,d)*C_N(c,r,d))]*Sum(k,CNT,C_PT(c,k,d)*C_QT(c,k,d)*C_N(c,k,d)
*[p_tsd(c,k,d)+q_tsd(c,k,d) + n_sd(c,k,d)])   +  sl_deltaa(c,s,d);
```

```
! Transfers  aggregate employment and expenditure shares from Melitz to Armington !
Equation E_f_ltota
(All,d,CNT) ltota(d) = ltot(d) +  f_ltota(d);

Equation E_f_mua
(All,c,COM)(All,d,CNT) mua(c,d) = mu(c,d) +  f_mua(c,d);

! Armington model from Table 3 in percentage change form !

Equation E_pa # Equation (T3.1) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
  pa(c,s,d) = wa(s) + ta(c,s,d) - phia(c,s) ;

Equation E_pca # Equation (T3.2) #
(All,c,COM)(All,d,CNT)
 pca(c,d) = (1/(1-SIGMAA))
     *{sum(s,CNT, C_SHA(c,s,d)*[SIGMAA*deltaa(c,s,d) + (1-SIGMAA)*(pa(c,s,d))])};

Equation E_qa # Equation (T3.3) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
  qa(c,s,d) = q_da(c,d) + SIGMAA*[deltaa(c,s,d) + pca(c,d) - pa(c,s,d)];

Equation E_wa # Equation (T3.4) #
(All,s,CNT)
  C_LTOTA(s)*ltota(s) = sum(c,COM, sum(d,CNT,
     [C_QA(c,s,d)/C_PHIA(c,s)]*[qa(c,s,d) - phia(c,s)] ));

Equation E_d_reva # Equation (T3.5) #
(All,c,COM)(All,s,CNT)(All,d,CNT)
100*d_reva(c,s,d) = C_TA(c,s,d)*C_QA(c,s,d)*C_WA(s)*(1/C_PHIA(c,s))
                      *[ ta(c,s,d)+qa(c,s,d) + wa(s) - phia(c,s) ]
                    - C_QA(c,s,d)*C_WA(s)/C_PHIA(c,s)
                      *[ qa(c,s,d) + wa(s) - phia(c,s)];
```

```
Equation E_gdpa # Equation (T3.6) #
(All,d,CNT)
 C_GDPA(d)*gdpa(d) = [C_WA(d)*C_LTOTA(d)]*[wa(d) + ltota(d)]
                    + 100*sum(c,COM, sum(s,CNT, d_reva(c,s,d) ));

Equation E_q_da # Equation (T3.7) #
(All,c,COM)(All,d,CNT)
 pca(c,d) + q_da(c,d) = mua(c,d) + gdpa(d) ;

! Other useful equations for the Armington model !

Equation E_f_wa #  Equation for equalizing Armington & Melitz numeraires#
(All,d,CNT) wa(d) = w(d) +  f_wa(d);

Equation E_mua2 # Allows movements in total cons, to GDP ratio in d: useful for Walras' law #
(All,c,COM)(All,d,CNT)
mua(c,d) = f_muan(c,d) +ff_mua(d);

! *************************************************************************** !
! Definitions of GDP and and other macro variables in the Armington Model      !
! *************************************************************************** !
Coefficient
        (All,r,CNT)
C_GDPEXPA(r)  # GDP expenditure, Armington #;
        (All,c,COM)(All,s,CNT)
C_LA(c,s) # Employment in industry c country s #;

Formula
(Initial)(All,d,CNT) C_GDPEXPA(d) = SUM{c,COM, C_PCA(c,d)*C_QDA(c,d)}
    + SUM{c,COM, SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt))}
      - SUM{c,COM, SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d))} ;

(Initial)(All,c,COM)(All,s,CNT) C_LA(c,s) = Sum(d,CNT, C_QA(c,s,d))/C_PHIA(c,s);
```

```
WRITE (postsim) C_GDPEXPA to terminal ;
(postsim) C_GDPA to terminal ;

Variable
        (All,r,CNT)
gdprealexpa(r) # GDP real expenditure, Armington #;
        (All,r,CNT)
gdprealinca(r) # GDP real income, Armington # ;
        (All,d,CNT)
gdpexpa(d) # Nominal GDP expenditure side, Armington #;
        (All,c,COM)(All,s,CNT)
la(c,s) # Employment in industry c in country s #;
        (all,d,CNT)
exports(d) # quantity of exports #;
        (all,d,CNT)
pexports(d) # price of exports fob #;
        (all,d,CNT)
imports(d) # quantity of imports #;
        (all,d,CNT)
pimports(d) # price of imports cif #;
ave_wagea # average worldwide wage,Armington #;

Update
(All,d,CNT) C_GDPEXPA(d) = gdpexpa(d) ;
(All,c,COM)(All,s,CNT) C_LA(c,s) = la(c,s);

Equation E_gdpexpa # GDP nominal expenditure, Armington #
(All,d,CNT) C_GDPEXPA(d)*gdpexpa(d) = SUM{c,COM, C_PCA(c,d)*C_QDA(c,d)*[pca(c,d)+q_da(c,d)]}
    + SUM{c,COM, SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt)*[pa(c,d,tt) -ta(c,d,tt) +qa(c,d,tt)])}
      - SUM{c,COM, SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*[pa(c,s,d) -ta(c,s,d) +qa(c,s,d)])} ;

Equation E_gdprealexp   # GDP real expenditure, Armington #
(All,d,CNT) C_GDPEXPA(d)*gdprealexpa(d) =SUM{c,COM, C_PCA(c,d)*C_QDA(c,d)*q_da(c,d)}
```

```
 + SUM{c,COM,
   SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt)*qa(c,d,tt) )}

   - SUM{c,COM,
        SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*qa(c,s,d))} ;

Equation E_gdprealinc # GDP real income, Armington #
(All,d,CNT) C_GDPA(d)*gdprealinca(d) =
C_LTOT(d)*C_WA(d)*[ltota(d)]
   + SUM(c,COM, SUM(s,CNT,
     [C_TA(c,s,d)-1]*[C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*qa(c,s,d) ))

   + SIGMAA/(SIGMAA-1)*SUM(c,COM, SUM(s,CNT,
                C_PA(c,s,d)*C_QA(c,s,d)*deltaa(c,s,d) ))

   + SUM(c,COM, {SUM(tt,CNT,
                [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt) )}*phia(c,d)) ;

Equation E_la  # Employment by industry and country #
(All,c,COM)(All,s,CNT) C_LA(c,s)*la(c,s)
  = Sum(d,CNT, (C_QA(c,s,d)/C_PHIA(c,s))*(qa(c,s,d) - phia(c,s)));

Equation E_exports # quantity of exports #
(All,d,CNT) exports(d) =
   (1/SUM{c,COM, SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt))})
   *SUM{c,COM, SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt)*qa(c,d,tt) )} ;

Equation E_pexports # price of exports fob #
(All,d,CNT) pexports(d) =
   (1/SUM{c,COM, SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt))})
   *SUM{c,COM, SUM(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt)*[w(d)-phia(c,d)] )} ;
```

```
Equation E_imports # quantity of imports #
(all,d,CNT) imports(d) =
      (1/SUM{c,COM, SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d))})
      *SUM{c,COM, SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*qa(c,s,d))} ;

Equation E_pimports # price of imports cif #
(all,d,CNT) pimports(d) =
      (1/SUM{c,COM, SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d))})
      *SUM{c,COM, SUM(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*[w(s)-phia(c,s)])} ;

Equation E_ave_wagea # average world-wide wage rate, Armington #
 Sum(tt,CNT, C_LTOTA(tt))*ave_wagea = sum(s,CNT, C_LTOTA(s)*wa(s));

! ************************************************************************* !
!             Welfare decomposition                                         !
! ************************************************************************* !
Coefficient
        (All,d,CNT)
WELFAREINDEX(d) # Welfare index #;

Formula
(initial) (All,d,CNT) WELFAREINDEX(d) = 1.0 ;

Variable
        (All,d,CNT)
welfarea(d)  #  Welfare, calculated in Armington model #;
wld_welfarea #  World welfare, calculated in Armington model #;
        (All,d,CNT)
tot(d) #  Gain from terms of trade movement expressed as percent of GDP #;
        (All,d,CNT)
slackw(d) # Will be zero if balance of trade is held on zero # ;
        (change)(All,d,CNT)
cont_toft(d) # Welfare contribution, terms of trade #;
```

```
        (change)(All,d,CNT)
cont_prim(d) # Welfare contribution, primary factors #;
        (change)(All,d,CNT)
cont_tcf(d) # Welfare contribution, tax-carrying flows #;
        (change)(All,d,CNT)
cont_techmix(d) # Welfare contribution, variety #;
        (change)(All,d,CNT)
cont_techprod(d) # Welfare contribution, production technology #;
        (change)(All,d,CNT)
cont_total(d) # Total of welfare contributions #;

Update (All,d,CNT) WELFAREINDEX(d) = welfare(d) ;

Equation E_welfarea  #  Welfare, calculated in Armington as real consumption #
(All,d,CNT)
welfarea(d) = (1/sum{cc,COM, C_PCA(cc,d)*C_QDA(cc,d)})
                                  *sum{c,COM, C_PCA(c,d)*C_QDA(c,d)*q_da(c,d)};

Equation E_wld_welfarea  #  World welfare, calculated in Melitz as real consumption #
wld_welfarea = (1/sum(d,CNT, sum(cc,COM, C_PCA(cc,d)*C_QDA(cc,d))))
                   *sum(dd,CNT, sum{c,COM, C_PCA(c,dd)*C_QDA(c,dd)*q_da(c,dd)});

Equation E_tot #  Gain from terms of trade movement expressed as percent of GDP #
(all,d,CNT) tot(d) = (1/C_GDPEXPA(d))*{
sum{c,COM,
 sum(tt,CNT:tt NE d, [C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt)*(wa(d)-phia(c,d)))}

   - sum{c,COM,
      sum(s,CNT:s NE d, [C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*(wa(s)-phia(c,s)))}
                                      } ;

Equation E_slackw  # Disaggregation of welfare #
(All,d,CNT) welfarea(d) = (1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})*C_GDPEXPA(d)*tot(d)
    +  (1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})
 *{  sum{c,COM, C_W(d)*C_LA(c,d)*la(c,d)}
```

```
    + sum(c,COM, sum(s,CNT,[C_TA(c,s,d)-1]*[C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*qa(c,s,d) ))
     + SIGMAA/(SIGMAA-1)*sum(c,COM, sum(s,CNT, C_PA(c,s,d)*C_QA(c,s,d)*deltaa(c,s,d) ))
     + sum(c,COM, {sum(tt,CNT,[C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt) )}*phia(c,d))
                                              } + slackw(d) ;

Equation E_cont_toft # Welfare contribution, terms of trade #
(All,d,CNT) cont_toft(d) = WELFAREINDEX(d)*(1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})
                                                        *C_GDPEXPA(d)*tot(d) ;

Equation E_cont_prim  # Welfare contribution, primary factors #
(All,d,CNT) cont_prim(d)
= WELFAREINDEX(d)*(1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})
                                        *{ sum{c,COM, C_W(d)*C_LA(c,d)*la(c,d)}};

Equation E_cont_tcf  # Welfare contribution, tax-carrying flows #
(All,d,CNT) cont_tcf(d)
= WELFAREINDEX(d)*(1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})*sum(c,COM, sum(s,CNT,[C_TA(c,s,d)-1]
                           *[C_PA(c,s,d)/C_TA(c,s,d)]*C_QA(c,s,d)*qa(c,s,d) ));

Equation E_cont_techmix # Welfare contribution, variety #
(All,d,CNT) cont_techmix(d)
= WELFAREINDEX(d)*(1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})*SIGMAA/(SIGMAA-1)
                *sum(c,COM, sum(s,CNT, C_PA(c,s,d)*C_QA(c,s,d)*deltaa(c,s,d) ));

Equation E_cont_techprod  # Welfare contribution, production technology #
(All,d,CNT) cont_techprod(d)
= WELFAREINDEX(d)*(1/sum{cc,COM, C_PCA(cc,d)*C_QD(cc,d)})
*sum(c,COM, {sum(tt,CNT,[C_PA(c,d,tt)/C_TA(c,d,tt)]*C_QA(c,d,tt) )}*phia(c,d));

Equation E_cont_total # Total of welfare contributions #
(All,d,CNT) cont_total(d)
= cont_toft(d) + cont_prim(d) + cont_tcf(d) + cont_techmix(d) + cont_techprod(d) ;

!  CONDENSATION suitable for standard linked Melitz-Armington simulations
We omit high dimension unshocked exogenous variables and substitute out high
```

```
dimension endogenous variables. Movements in these endogenous variables are
recovered via backsolving.  Omissions and backsolve substitutions are not
necessary with small models.  !

Omit f_sd; !  Normally exogenous and unshocked, hence can usually be omitted!
Omit delta ; ! Normally exogenous and unshocked, hence can usually be omitted!
Omit sl_deltaa; ! Can't be omitted if Armington and Melitz are disconnected
because it must be endogenized to turn off E_deltaa   !
Omit d_slack_ta ; ! If the Armington auxiliary model is disconnected from
Melitz then this variable is endogenous (swapped with ta) and can't be omitted!

backsolve phi_min using E_phi_min ;
backsolve q_min using E_q_min ;
backsolve qa using E_qa ;
backsolve pa using E_pa ;
backsolve d_reva using E_d_reva ;
backsolve d_rev using E_rev ;
backsolve deltaa using E_deltaa ;
backsolve ta using E_ta ;  !  Not applicable if ta exogenous !
backsolve n_sd using E_n_sd ;
backsolve q_tsd using E_q_tsd ;
backsolve phi_tsd using E_phi_tsd ;
backsolve pi_tsd using E_pi_tsd ;
backsolve p_tsd using E_p_tsd ;
backsolve q_sd using E_q_sd ;
```

A6.2. Closure file for running Melitz and Armington in linked mode

```
! Closure for linked Melitz-Armington simulation
Exogenous
ltot
!delta    ***** Omitted
t
!f_sd  ****** Omitted
h
f_mu
ff_mu("CNT1")    ! Allows for Walras' law
ave_wage         ! Melitz numeraire
! End of Melitz closure

slack_phia
!d_slack_ta  ****** Omitted
!sl_deltaa   ****** Omitted

f_muan
ff_mua("CNT1")  ! Allows for Walras' law in Armington model
f_wa("CNT1")    ! Equalizes Armington and Melitz numeraires
f_ltota; !  Transfers Melitz setting for aggregate employment to Armington

Rest endogenous;
!These swaps are activated if we want to disconnect Melitz and Armington
!swap slack_phia = phia;
!swap sl_deltaa = deltaa;
!swap d_slack_ta = ta;
```