R23 Model


R-2-3: A Simple Global CGE Analysis Focusing on Macroeconomic Links

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1. Introduction

The R-2-3 model (r region country, two sector, and three goods) is a simple and static global computable general equilibrium (CGE) model. It is a generalisation of a user-friendly fiscal tool developed at the World Bank for single country analysis - the 1-2-3 model (1 country, with 2 producing sectors, and 3 goods). In the R-2-3 model, many single country 1-2-3 models are linked via trade flows and other macroeconomic variables such as foreign aid, remittances, and other transfers. The R-2-3 model can be used as a as one region model with one trading partner, a one region model with multiple trading partners or as an r region model with multiple trading partners.

The database that underpins the model is a global, macro Social Accounting Matrix (SAM), which consists of a series of intra-regional transactions similar to the standard 1-2-3 model but with additional tax/subsidy instruments – import duties, export taxes, other commodity taxes, production taxes, factor use taxes, direct taxes – and total imports and exports by source and destination regions valued \(fob\), trade margins (differences between \(fob\) and \(cif\) valuations), other current account transactions, e.g., migrant remittances, aid transfers, etc. This database combines and augments in a unique way the GTAP database used in global models and the national and fiscal accounts used in the 1-2-3 model.

The GTAP database targets a limited number of macro totals during its construction, namely GDP and the trade data. Other macro totals are collected and used in the reconciliation process (C, I, G), however because these data may be inconsistent with the trade data, they cannot also be targeted. Moreover, certain tax instruments and current account transactions such as remittances and aid flows are missing. This is likely to mean that certain important macro totals may not be consistent with the expectations of country specialists or policy makers.

Unfortunately, the detailed SAMs (or IO tables) in GTAP are not easily updated and available for all countries. In Sub-Saharan Africa, only 18 countries are available (out of more than 45 individual countries). On the other hand, the national accounts that are used in the 1-2-3 model are available for almost for all countries, but lack trade flows to link the interactions of countries and regions and for the global transmission of shocks. The GTAP database will therefore provide trade data – imports and exports by source and destination region and trade margins, trade tax data, absorption data, value added data (labour, capital, land and natural resources) and intermediate input data. These data are augmented with details for the current accounts and inter institutional transactions. In combining the two approaches, there are therefore significant gains to be made, including the use of the World Bank’s World Development Indicators, to improve the macro totals of future releases of GTAP database. We develop a globally consistent set of macro SAMs for as many as 244 regions/countries and for
a more recent year, thus contributing to enhancement of the GTAP database and the quality of its macro information.

While simple, the R-2-3 model allows for a forecast of welfare measures and poverty outcomes consistent with a set of macroeconomic policies in the context of a simple global general equilibrium model. For a given set of macroeconomic policies, R-2-3 generates a set of wages and employment, relative prices, nominal and real exchange rates and internal and external balances that are mutually consistent in the presence of global interdependencies. The projected changes can then be inputted into household models to estimate the policy impacts on each household in the samples in terms of welfare and income distribution.

The analytical potential of the R-2-3 model is demonstrated by a series of simulations based on changes in aid flows from the developed market economies (DMEs) to developing African economies. The aggregation of the database separately identifies the several aggregates of OECD countries – the principal sources of aid – and other countries that contribute to aid, the least developed African economies and other regions. The scenarios analysed include increasing aid flows to 0.7% of the GDP of each OECD region to existing recipient regions in the ratios currently realised and a redistribution of aid flows in ratios that in inverse proportion to the wealth of the developing African economies. The analyses are conducted using a variety of macroeconomic closure rules that include equivalent reductions in tax instruments in the developing African economies, increases in social expenditures by the governments and reductions in government deficits that facilitate increases in investment.

2. Data

The data used in the model were derived from the GTAP database (see Hertel, 1997) using a three dimensional Social Accounting Matrix (SAM) method for organising the data. Details of the method used to generate a SAM representation are reported in McDonald and Thierfelder (2004a) while a variety of reduced form representations of the SAM and methods for augmenting the GTAP database are reported in McDonald and Thierfelder (2004b) and McDonald and Sonmez (2004) respectively. Detailed descriptions of the data are provided elsewhere so the discussion here is limited to the general principles.

2.1 Global Social Accounting Matrix

The Global SAM can be conceived of as a series of single region SAMs that are linked through the trade accounts; it is particularly valid in the context of the GTAP database to note that the ONLY way in which the regions are linked directly in the database is through commodity trade transactions although there are some indirect links through the demand and supply of trade and transport services. Specifically the value of exports, valued free on board
(fob) from source \( x \) to destination \( y \) must be exactly equal to the value of imports valued fob to destination \( y \) from source \( x \), and since this holds for all commodity trade transactions the sum of the differences in the values of imports and exports by each region must equal zero. However the resultant trade balances do not fully accord with national accounting conventions because other inter-regional transactions are not recorded in the database (see McDonald and Sonmez, 2004). A description of the transactions recorded in a representative SAM for a typical region in the database is provided in Table 1.

A SAM is a transactions matrix; hence each cell in a SAM simply records the values of the transactions between the two agents identified by the row and column accounts. The selling agents are identified by the rows, i.e., the row entries record the incomes received by the identified agent, while the purchasing agents are identified by the columns, i.e., the column entries record the expenditures made by agents. As such a SAM is a relatively compact form of double entry bookkeeping that is complete and consistent and can be used to present the National Accounts of a country in a single two-dimensional matrix (see UN, 1993, for a detailed explanation of the relationship between conventional and SAM presentations of National Accounts). A SAM is complete in the sense that the SAM should record ALL the transactions within the production boundary of the National Accounts, and consistent in the sense that income transactions by each and every agent are exactly matched by expenditure transactions of other agents. A fundamental consequence of these conditions is that the row and column totals of the SAM for each region must be identical, and hence the SAM provides a complete characterisation of current account transactions of an economy as a circular (flow) system. In the context of a global SAM the complete and consistent conditions need extending to encompass transactions between regions; this simply requires that each and every import transaction by a region must have an identical counterpart export transaction by another region. This is enough to ensure that the resultant global SAM provides a characterisation of current account transactions of the global economy as a circular (flow) system.
Table 1  
Social Accounting Matrix for a Region in the Global Social Accounting Matrix

<table>
<thead>
<tr>
<th></th>
<th>Commodities</th>
<th>Activities</th>
<th>Factors</th>
<th>Households</th>
<th>Government</th>
<th>Capital</th>
<th>Rest of World</th>
<th>GLOBE</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodities</td>
<td>0</td>
<td>Combined</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Activities</td>
<td>Domestic Supply Matrix</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Factors</td>
<td>0</td>
<td>Expenditure on Primary Inputs</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Households</td>
<td>0</td>
<td>Distribution of Factor Incomes</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Government</td>
<td>Taxes on Commodities</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Capital</td>
<td>0</td>
<td>Depreciation Allowances</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rest of World</td>
<td>Imports of Commodities (fob)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GLOBE</td>
<td>Imports of Trade and Transport Margins</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>Total Supply of Commodities</td>
<td>Total Expenditure on Inputs by Activities</td>
<td>Total Factor Expenditure</td>
<td>Total Household Expenditure</td>
<td>Total Government Expenditure</td>
<td>Total Investment</td>
<td>Total Expenditure on Exports</td>
<td>Total Expenditure on Margin Exports</td>
<td></td>
</tr>
</tbody>
</table>
Given these definitions of a SAM the transactions recorded in a SAM are easily interpreted. In Table 1 the row entries for the commodity accounts are the values of commodity sales to the agents identified in the columns, i.e., intermediate inputs are purchased by activities (industries etc.), final consumption is provided by households, the government and investment demand and export demand is provided by the all the other regions in the global SAM and the export of margin services. The commodity column entries deal with the supply side, i.e., they identify the accounts from which commodities are purchased so to satisfy demand. Specifically commodities can be purchased from either domestic activities – the domestic supply matrix valued inclusive of domestic trade and transport margins – or they can be imported – valued exclusive of international trade and transport margins. In addition to payments to the producing agents – domestic or foreign – the commodity accounts need to make expenditures with respect to the trade and transport services needed to import the commodities and any commodity specific taxes.

The GTAP database provides complete coverage of bilateral transactions in commodities – these are valued free on board (fob) - but only provides partial coverage of transactions in trade and transport margins. Specifically the imports of trade and transport margins by each region are directly associated with the imports of specific commodities, hence for each commodity import valued fob the source and destination regions are identified and the value of each trade and transport margin service used is identified. The sum of the values of trade and transport services and the fob value of the commodity imports represent the carriage insurance and freight (cif) paid value of each imported commodity. But the source regions of the trade and transport services are NOT identified, and similarly the values of exports of trade and transport services by a region do NOT identify the destination regions. To overcome this lack of information an artificial region called Globe is included in the database. This region collects together all the exports of trade and transport services by other regions as its imports and then exports these to other regions to satisfy their demand for the use of trade and transport services associated with commodity imports. By construction the value of imports by Globe for each and every trade and transport margin service must exactly equal the value of exports for the corresponding trade and transport service. However this does not mean that the trade balance between Globe and each and every region must exactly balance, rather it requires that the sum of Globe’s trade balances with other regions is exactly equal to zero.

An important feature of the construction of a SAM can be deduced from the nature of the entries in the commodity account columns. By definition the column and row totals must equate and these transaction totals can be expressed as an implicit price times a quantity, and the quantity of a commodity supplied must be identical to the quantity of a commodity demanded. The column entries represent the expenditures incurred in order to supply a commodity to the economy and hence the implicit price must be exactly equal to the average cost incurred to
supply a commodity. Moreover since the row and column totals equate and the quantity represented by each corresponding entry must be same for the row and column total the implicit price for the row total must be identical to average cost incurred to supply the commodity. Hence the column entries identify the components that enter into the formation of the implicit prices in the rows, and therefore identify the price formation process for each price in the system. Typically a SAM is defined such that the commodities in the rows are homogenous and that all agents purchase a commodity at the same price.

Total income to the activity accounts is identified by the row entries. In the simple representation of production in the GTAP database each activity makes a single commodity and each commodity is made by a single activity, which means that the domestic supply matrix is a diagonal (square) matrix. The expenditures on inputs used in production are recorded in the activity columns. Activities use intermediate inputs, which in this version of the database are record as composites of domestically produced and imported commodities, primary inputs and pay taxes on production and factor use. For each region the sum of the payments to primary inputs and on production and factor use taxes by activity is equal to the activity’s contribution to the value added definition of GDP while the sum over activities equals the region’s value added measure of GDP.

The remaining accounts relate to the institutions in the SAM. All factor incomes are distributed to the single private household after making allowance for depreciation of physical capital and the payment of direct (income) taxes on factor incomes. Incomes from factor sales are also the sole source of income to the household account. Three categories of expenditures by the household account are recorded; direct (income) taxes, savings and consumption. The government receives incomes from commodity taxes, production taxes and direct taxes on factor and household incomes, and uses that income to pay for consumption and for savings. In the basic form of the database government savings are set to zero for all regions; this stems from the reduced form representation of intra institutional transactions provided by the GTAP database (see McDonald and Thierfelder, 2004b).1 There are therefore five sources of savings in each region: depreciation, household/private savings, government savings, balances on trade in margin services and balances on trade in commodities, but only a single expenditure activity – investment (commodity) demand.

As should be apparent from the description of the SAM for a representative region the database is strong on inter regional transactions but relatively parsimonious on intra regional transactions.

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1 McDonald and Sonmez (2004) demonstrate that it is straightforward to overcome this limitation of the database. The model described in this paper operates whether the government savings are zero or non-zero.
2.2 Other GTAP Data

In addition to the transactions data the GTAP database contains other data that can be used with this model, and/or variants of the model. The most obviously useful data are the import and primary factor elasticity data used in the GTAP model; the programme used to derive an aggregation of the SAM also contains a routine for aggregating these elasticities for use in this model. However, the GTAP elasticities are only a subset of the elasticities used in this model and it is therefore necessary to provide other elasticities even when using the GTAP elasticity data.

Other data of interest to modellers include estimates of energy usage and emissions and land use (carbon sinks). None of these data are used in this variant of the model.

2.3 Database Dimensions

The dimensions of the SAM are determined by the numbers of accounts within each aggregate group identified in Table 1, while the actual numbers of accounts in each group of accounts are defined for version 5.4 and 6.0 of the GTAP database in Table 2. Given the large number of accounts in the SAMs for each region and the relatively large number of regions the total number of cells in the global SAM is very large, although only slightly over 10 percent of the cells actually contain non zero entries; nevertheless this still means that the GTAP database contains some 4 million transaction values, which implies that there are some 8 million possible prices and quantities that can be deduced from the database. Even allowing for the implications of adopting the law of one price for transactions in the rows of each region’s SAM and for other ways of reducing the numbers of independent prices and quantities that need to be estimated in a modelling environment, it is clear that the use of the GTAP database without aggregation is likely to generate extremely large models (in terms of the number of equations/variables). Consequently, except in exceptional circumstances all CGE models that use the GTAP data operate with aggregations of the database.

Table 2: Dimensions of the Global Social Accounting Matrix

<table>
<thead>
<tr>
<th>Account Groups</th>
<th>Sets</th>
<th>Total Number of Accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>GTAP 5.4</td>
</tr>
<tr>
<td>Commodities</td>
<td>C</td>
<td>57</td>
</tr>
<tr>
<td>Activities</td>
<td>A</td>
<td>57</td>
</tr>
<tr>
<td>Factors</td>
<td>F</td>
<td>5</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Category</th>
<th>Formula</th>
<th>234</th>
<th>261</th>
<th>336</th>
<th>387</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes</td>
<td>((2<em>r)+(1</em>f) + 3)</td>
<td>164</td>
<td>182</td>
<td>232</td>
<td>395</td>
</tr>
<tr>
<td>Other Domestic Institutions</td>
<td>(3)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Margins</td>
<td>(3*r)</td>
<td>234</td>
<td>261</td>
<td>336</td>
<td>387</td>
</tr>
<tr>
<td>Trade</td>
<td>(R)</td>
<td>78</td>
<td>87</td>
<td>112</td>
<td>129</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>598</td>
<td>652</td>
<td>802</td>
<td>1,033</td>
</tr>
<tr>
<td>Total Number of Cells in the Global SAM</td>
<td></td>
<td>27,893,112</td>
<td>36,984,048</td>
<td>72,038,848</td>
<td>137,654,481</td>
</tr>
</tbody>
</table>
2.1 GTAP macro data

2.2 Augmented GTAP database

3. Model Structure

3.1 A Basic 123 Model

The basic 123 model has **one** country with **two** production sectors and **three** commodities. A key objective of the model is to demonstrate the incorporation of the Armington trade insight into a CGE model in the simplest possible context, and demonstrate the consistency of the model with (neoclassical) trade theory. In its simplest form the model contains no factor markets, government or investment, and can be expressed in 14 equations.

The basic mechanics of the model can be illustrated in a four-quadrant trade diagram (Figure 1), where all the dimensions are positive. The bottom right-hand quadrant of this diagram illustrates how, in this model, domestic production, represented by the production possibility frontier \(ppf\), can either be consumed domestically, commodity \(QDS\), or exported, commodity \(QE\). The commodities \(QDS\) and \(QE\) are presumed to be imperfect substitutes; hence the \(ppf\) is concave to the origin. Given the relative prices of \(QDS\) and \(QE\), i.e., \(P_e/P_{ds}\), represented by the budget constraint, the producer decides the optimal (relative) quantities of \(QDS\) and \(QE\) to produce so as to maximise ‘profit’, i.e., at the point \(P\). If the exchange rate is the numéraire, and it is assumed (for the moment) that the balance of trade is zero, and the export, \(P_e\), and import, \(P_m\), prices are equal, then the relationship between exports and imports is one to one, and the balance of trade condition, in quadrant I, is represented by a 45° line through the origin. Similarly, the transformation of domestic commodity production, \(QDS\), into the domestic commodity for consumption, \(QDD\), is a one to one mapping, and hence the domestic market is characterised by a 45° line through the origin in quadrant III.
The information in quadrants III, IV, and I can now be used to map out the consumption possibility frontier (cpf) in the remaining quadrant, II. Well-being/utility is maximised where the line representing the relative prices of the domestic commodity and the imported commodity is tangential to the cpf, C, which is also where the indifference curve is at a tangent to the cpf. In this case where world prices are equal and trade is balanced, it is clear that the cpf is a mirror image of the ppf; this is a special case chosen for simplicity of exposition.

The data requirements can be presented as a Social Accounting Matrix (SAM), see Table 2. Note how in this SAM all income is spent on consumption in the current period and how there is balanced commodity trade. The former assumption cannot be relaxed because of the absence of a capital account, but the latter assumption can be relaxed by allowing the exogenously determined current account of the balance of payments to be in surplus or deficit via a transfer to the household from the rest of the world. It is relatively simple to extend the 123 model to include factor, government and saving-investment accounts (see Deverajan et al., 1997; and McDonald, 2006).

Table 2  
Social Accounting Matrix for a Basic 1*2*3 Model

<table>
<thead>
<tr>
<th></th>
<th>Commodity</th>
<th>Activity</th>
<th>Household</th>
<th>Rest of World</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Activity</td>
<td>80</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

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Armington 'Insight'

Standard neoclassical models presume that all commodities are tradable and all tradable commodities are perfect substitutes. This is scarcely plausible, since such models have a propensity to yield extreme specialisation and wild swings in relative prices when world prices or trade policies change. Moreover it does not accord with observed patterns of trade, even when the data are recorded with very high degrees of disaggregation (several thousand categories of commodities). The Salter (1959) and Swan (1960) models recognised this by distinguishing between ‘tradables’ and ‘nontradables’, which resolved the problem at least at a theoretical level, but had little impact on empirical work. In the Salter-Swan model the link between the domestic and world prices depends solely upon whether a commodity/sector is traded. If a commodity is traded the domestic price equals the world price (cif or fob). If a commodity is non-traded the domestic price is determined by the interaction of demand and supply on the domestic market. Thus tradables and nontradables are treated asymmetrically.

The Armington insight extends the Salter-Swan approach by assuming imperfect substitutability and hence treating all sectors symmetrically. The link between domestic and world prices then depends upon the trade shares and the elasticities of substitution. For any given elasticity the domestic price will be closer to the world price the greater the trade share, and similarly for any given trade share the domestic price will be closer to the world price the greater the elasticity (see de Melo and Robinson, 1985).

This ensures there are links between domestic and world prices, as in the standard neoclassical model, but these are less strong than in the standard model. Also the model can accommodate two-way trade within the same sector, intra-industry trade; a feature of trade flows that is commonly observed. The approach is therefore consistent with a view of trade, imports and exports, as taking place in differentiated products and is consistent with the Salter-Swan model in that it gives rise to normally shaped offer curves. The exchange rate is also appropriately defined: if the domestic commodity is the numéraire, i.e., $P^d$ is set equal to one, then the exchange rate, $R$, is the real exchange rate of neoclassical theory (the relative price of tradables ($QM$ & $QE$) to nontradables ($QDD$ & $QDS$). If $R$ is set equal to one then $P^d$ defines the real exchange rate, while for other choices of numéraire $R$ is a monotonic transformation of the real exchange rate.
Typically, despite their limitations, it is common practice to use CES/CET functions. (CES(substitution) is used for consumption, CET(transformation) for production). Specifically we can write a general (primal) form, $QX$, of the CES/CET specification as

$$QX = \bar{A}\left[\alpha.QDS^\rho + (1 - \alpha).QE^\rho\right]^\frac{1}{\rho}$$

(1)

and the price dual, $P$, as

$$P = \bar{A}^{-1}\left[\alpha^\frac{1}{(1-\rho)} .PDS^\rho + (1 - \alpha)^\frac{1}{(1-\rho)} .PE^\rho\right]^\frac{\rho-1}{\rho}$$

(2)

where the substitution/transformation elasticity is given by

$$\sigma \text{ or } \Omega = \frac{1}{(1-\rho)} \quad -\infty < \rho < +1$$

(3)

The export supply and import demand functions are then given by

$$\frac{QE}{QDS} = \left[\frac{(1-\delta).PE}{\delta.PDS}\right]^\Omega$$

(4)

$$\frac{QM}{QD} = \left[\frac{\beta.PD}{(1-\beta).PM}\right]^\sigma$$

(5)

where $\beta$ and $\delta$ are the respective share parameters and $\sigma$ and $\Omega$ are the elasticities.

Exploiting Euler’s theorem for linearly homogeneous functions we can replace the dual price equations for export transformation and import aggregation by expenditure identities, i.e.,

$$PQ = \frac{PM.QM + PD.QD}{QD}$$

(6)

$$PX = \frac{PE.QE + PD.QD}{QX}$$

(7)

This approach to the modelling of trade is the approached adopted for the R23 model. The key differences are that world prices are endogenous and each region imports from and exports to multiple other regions.

### 3.2 Behavioural Relationships in the R23 Model

The within regional behavioural relationships are fairly standard in the R23 but given the degree of aggregation there is relatively little room to make them more elaborate. The focus here is therefore on international trade relationships. The activities are assumed to maximise profits using technology characterised by Constant Elasticity of Substitution (CES) and/or Leontief
production functions between aggregate primary inputs and aggregate intermediate inputs, with CES production functions over primary inputs and Leontief technology across intermediate inputs. The household maximises utility subject to the price of the consumption good after meeting its direct tax obligations and saving.

The Armington assumption is used for trade. Domestic output is distributed between the domestic market and exports according to a two-stage Constant Elasticity of Transformation (CET) function. In the first stage the domestic producer allocates output to the domestic or export market according to the relative prices for the commodity on the domestic market and the composite export commodity, where the composite export commodity is a CET aggregate of the exports to different regions – the distribution of the exports between regions being determined by the relative export prices to those regions. Consequently domestic producers are responsive to prices in the different markets – the domestic market and all other regions in the model – and adjust their volumes of sales according relative prices. The elasticities of transformation are commodity and region specific.

Domestic demand is satisfied by a composite commodity that is formed from domestic production sold domestically and composite imports. This process is modelled by a two-stage CES function. At the bottom stage the composite import commodity is a CES aggregate of imports from different regions with the quantities imported from different regions being responsive to relative prices. The top stage defines a composite consumption commodity as a CES aggregate of a domestic commodity and a composite import commodity with the mix being determined by the relative prices. The elasticities of substitution are commodity and region specific. Hence the optimal ratios of imports to domestic commodities and exports to domestic commodities are determined by first order conditions based on relative prices. The price and quantity systems are described in greater detail below.

Most commodity and activity taxes are expressed as *ad valorem* tax rates, while income taxes are defined as fixed proportions of household incomes. Import duties and export taxes apply to imports and exports, while sales taxes are applied to all domestic absorption, i.e., imports are subject to sequential import duties and sales taxes, and VAT is applied to household demand. Production taxes are levied on the value of output, while activities also pay taxes on the use of specific factors. Factor income taxes are charged on factor incomes after allowance for depreciation after which the residual income is distributed to households. Income taxes are taken out of household income and then the households are assumed to save a proportion of disposable income. This proportion is either fixed or variable according to the closure rule chosen for the capital account.

Government expenditure consists of commodity (final) demand, which is assumed to be fixed in real/volume terms. Hence government saving, or the internal balance, is defined as a residual. However, the closure rules for the government account allow for various permutations.
In the base case it is assumed that the tax rates and volume of government demand are fixed and government savings are calculated as a residual. However, the tax rates can all be adjusted using various forms of scaling factors; hence for instance the value of government savings can be fixed and one of the tax scalars can be made variable thereby producing an estimate of the constrained optimal tax rate. If the analyst wishes to change the relative tax rates across commodities (for import duties, export taxes and sales taxes) or across activities (for production taxes) then the respective tax rate parameters can be altered or solved for endogenously. Equally the volume of government consumption can be changed by adjusting the closure rule. The patterns of government expenditure are altered by changing the parameters that controls the pattern of government expenditure ($qgdconst$).

Total savings come from the households, the internal balance on the government account and the external balance on the trade account. The external balance is defined as the difference between the value of total exports and total imports, converted into domestic currency units using the exchange rate. In the base model it is assumed that the exchange rates are flexible and hence that the external balances are fixed. Alternatively the exchange rates can be fixed and the external balances can be allowed to vary. Expenditures by the capital account consist solely of commodity demand for investment. In the base solution it is assumed that the shares of investment in total domestic final demand are fixed and that household savings rates adjust so that total expenditures on investment are equal to total savings, i.e., the closure rule presumes that savings are determined by the level of investment expenditures. The volumes of investment can be changed exogenously or solved for endogenously.
### Table 3  Behavioural Relationships for the R23 Model

<table>
<thead>
<tr>
<th></th>
<th>Commodities</th>
<th>Activities</th>
<th>Factors</th>
<th>Households</th>
<th>Government</th>
<th>Capital</th>
<th>Margins</th>
<th>Rest of World</th>
<th>Prices</th>
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<td>Two-Stage CET Functions</td>
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<td>Shares of household income</td>
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<td>Rest of World</td>
<td>Two-Stage CES Functions</td>
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<td>Prices</td>
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<td>Domestic and World Prices for Imports</td>
<td>Value Added Prices</td>
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### Table 4: Transactions Relationships for the R23 Model

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<th>Factors</th>
<th>Households</th>
</tr>
</thead>
<tbody>
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<td>Commodities</td>
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<td>0</td>
<td>(PQD*(1+TV)*QCD)</td>
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<tr>
<td>Activities</td>
<td>(PDS*QDS)</td>
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<td>0</td>
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<tr>
<td>Factors</td>
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<td>(WF&lt;sub&gt;f&lt;/sub&gt;*FD&lt;sub&gt;f&lt;/sub&gt;)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Households</td>
<td>0</td>
<td>∑&lt;sub&gt;f&lt;/sub&gt; (hva$h&lt;sub&gt;f&lt;/sub&gt;)</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

**Government**

- (TM<sub>W</sub>*PWM<sub>W</sub>)
- (*QMR<sub>W</sub>*ER)
- (TE<sub>W</sub>*PWE<sub>W</sub>)
- (*QER<sub>W</sub>*ER)
- (TS*PQS*QQ)
- (TV*PQD*QCD)

- (TX*PX*QX) (TYF<sub>f</sub>) *(deprec<sub>f</sub>*YF<sub>f</sub>) (TYH*YH)

**Capital**

- 0
- 0
- (deprec<sub>f</sub>*YF<sub>f</sub>) (YH*(1-TYH))*(SHH)

**Margins**

- (PT*QT<sub>w</sub>)
- 0
- 0

**Rest of World**

- (PWMFOB<sub>W</sub>)
- 0
- 0

**Total**

- (PQD*QQ)
- (PX*QX)
- YF<sub>f</sub>
- YH
Table 4 (cont) Transactions Relationships for the R23 Model

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Government</th>
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<th>Margins</th>
<th>RoW</th>
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<td>(PQD*QGD)</td>
<td>(PQD*QINVD)</td>
<td>(PWE\text{<em>}QER\text{</em>}ER)</td>
<td>(PWE\text{<em>}QER\text{</em>}ER)</td>
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<tr>
<td>Activities</td>
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<tr>
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<tr>
<td>Households</td>
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<td>Government</td>
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<tr>
<td>Capital</td>
<td>(YG - EG)</td>
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<td>(KAPREG*ER)</td>
<td>(KAPREG*ER)</td>
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<td>Margins</td>
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<tr>
<td>Rest of World</td>
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<tr>
<td>Total</td>
<td>YG</td>
<td>INVEST</td>
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3.2 Price and Quantity Systems for a Representative Region

3.2.1 Price System

The price system is built up using the principle that the components of the ‘price definitions’ for each region are the entries in the columns of the SAM. Hence there are a series of explicit accounting identities that define the relationships between the prices and thereby determine the processes used to calibrate the tax rates for the base solution. However, the model is set up using a series of linear homogeneous relationships and hence is only defined in terms of relative prices. Consequently as part of the calibration process it is necessary set some of the prices equal to one (or any other number that suits the modeller) – this model adopts the convention that prices are normalised at the level of the CES and CET aggregator functions \( PQS \), the supply price of the domestic composite consumption commodity and \( PXC \), the producer price of the composite domestic output. The price system for a typical region in a 4-region global model is illustrated by Figure 1 – note that this representation abstracts from the Globe region.
The relationships between the various prices in the model are illustrated in Figure 2. The domestic consumer prices \((PQD)\) are determined by the domestic prices of the domestically supplied commodities \((PD)\) and the domestic prices of the composite imports \((PM)\), and by the sales taxes \((TS)\) that are levied on all domestic demand. The prices of the composite imports are determined as aggregates of the domestic prices paid for imports from all those regions that supply imports to this economy \((PMR)\) under the maintained assumption that imports are differentiated by their source region. The region specific import prices are expressed in terms of the domestic currency units after paying for trade and transport services and any import duties. Thus a destination region is assumed to purchase a commodity in a source economy where the price is defined in “world dollars” at the basket exchange rate and is valued free on board \((fob)\), i.e., \(PWMFOB\). The carriage insurance and freight \((cif)\) price \((PWM)\) is then defined as the \(fob\) price.
plus trade and transport margin services \((\text{margcor})\) times the unit price of margin services \((PT)\). The \(cif\) prices are related to the domestic price of imports by the addition of any import duties \((TM)\) and then converted into domestic currency units using the nominal exchange rate \((ER)\).

The prices for output \((PX)\) are determined by the domestic prices \((PD)\) and the composite export prices \((PE)\). The composite export prices are a CET aggregates of the export prices received by the source economy for exports to specific destinations \((PER)\). The prices of the composite exports are determined as aggregates of the domestic prices paid for exports by all those regions that demand exports from this economy under the maintained assumption that exports are differentiated both by their destination region. The prices paid by the destination regions \((PWE)\) are net of export taxes \((TE)\) and are expressed in the currency units of the model’s reference region by use of the nominal exchange. Note how the export prices by region of destination \((PER)\), and the intermediate aggregates, are all normalised on 1, but the seeming counterpart of normalising import prices by source region \((PMR)\) are not normalised on 1. The link between the regions is therefore embedded in the identification of the quantities exchanged rather than the normalised prices and is a natural consequence of the normalisation process. The CET function can be switched off so that the domestic and export commodities are assumed to be perfect substitutes; this is the assumption in the GTAP model and is an option in this model.

The price system also contains a series of equilibrium identities. Namely the \(fob\) export price \((PWE)\) for region \(x\) on its exports to region \(y\) must be identical to the \(fob\) import price \((PWMFOB)\) paid by region \(y\) on its imports from region \(x\). These equilibrium identities are indicated by double headed arrows.

### 3.2.2 Quantity System

The quantity system for a representative region is somewhat simpler. The composite consumption commodity \((QQ)\) is a mix of the domestically produced commodity \((QD)\) and the composite import commodity \((QM)\), where the domestic and imported commodities are imperfect substitutes. The equilibrium conditions require that the quantities imported from different regions \((QMR)\) are identical to the quantities exported by other regions to the representative region \((QER)\).
The composite consumption commodity is then allocated between domestic intermediate demands ($Q_{INT}$), private consumption demand ($Q_{CD}$), government demand ($Q_{GD}$) and investment demand ($Q_{INVD}$).

On the output side, domestic output ($Q_{X}$) is allocated between the domestic market ($Q_{D}$) and composite export commodities ($Q_{E}$) under the maintained assumption of imperfect transformation. Exports are allocated between the different destination regions ($Q_{ER}$) under the maintain assumption of imperfect transformation.

3.2.3 Production System

The production system is set up as a two-level nest of CES production functions. At the top level aggregate intermediate inputs ($Q_{INT}$) are combined with aggregate primary inputs ($Q_{VA}$) to produce the output of an activity ($Q_{X}$). This top level production function can take either CES or Leontief form, with CES being the default and the elasticities being activity and region specific.²

² The model allows the user to specify the share of intermediate input cost in total cost below which the Leontief alternative is automatically selected. The user also has the option to make activity and region specific decisions about the selection of CES or Leontief forms.
Aggregate intermediate inputs are defined as a fixed proportion \((ioqint)\) of the output. The value added production function is a standard CES function over capital, land and two types of labour, with the elasticities being region specific. The notation for all primary inputs, natural and aggregates, is the same: quantity demand is \(FD\), quantity supplied is \(FS\) and the factor prices is \(WF\). The operation of this aggregator function can, of course, be influenced by choices over the closure rules for the factor accounts.

**Figure 4 Production Quantity System for a Typical Region**

\[
QX \quad \sigma_c \\
| \quad | \\
QINT = ioqint*QX \\
| \quad | \\
QVA \quad \sigma_{va} \\
| \quad | \\
FD_{i1} \quad FD_{i2} \quad FD_k \quad FD_n
\]

In the price system for production the value added prices \((PVA)\) are determined by the activity prices \((PX)\), the production tax rates \((TX)\), the input-output coefficient \((ioqint)\) and the commodity price \((PQD)\). The price of value added is determined by the factor prices \((WF)\) and any factor use taxes \((tf)\).
3.3 The Globe Region

An important feature of the model is the use of the concept of a region known as Globe. While the GTAP database contains complete bilateral information relating to the trade in commodities, i.e., in all cases transactions are identified according to their region of origin and their region of destination, this is not the case for trade in margins services associated with the transportation of commodities. Rather the GTAP database identifies the demand, in value terms, for margin services associated with imports by all regions from all other regions but does not identify the region that supplies the margin services associated with any specific transaction. Consequently the data for the demand side for margin services is relatively detailed but the supply side is not. Indeed the only supply side information is the total value of exports of margin services by each region. The Globe construct allows the model to get around this shortage of information, while simultaneously providing a general method for dealing with any other transactions data where full bilateral information is missing.
The price system for the Globe region is illustrated in Figure 6. On the import side Globe operates like all other regions. The commodities used in trade and transport services are assumed to be differentiated by source region and the proportion of imports accounted for by the source region.
region. Thus a two-level CES aggregation nest is used. It is assumed that imports of trade and transport services can potentially incur trade and transport margins \((\text{margcor})\) and face tariffs \((\text{TM})\); in fact the database does not include any transport margins or tariff data for margin services in relation to the destination region, although they can, and do, incur export taxes levied by the exporting region.

The export side is slightly different. In effect the Globe region is operating as a method for pooling differentiated commodities used in trade and transport services and the only differences in the use of trade and transport services associated with any specific import are the quantities of each type of trade service used and the mix of types of trade services. Underlying this is the implicit assumption that each type of trade service used is homogenous, and should be sold therefore at the same price. Hence the export price system for Globe needs to be arranged so that Globe exports at a single price, i.e., there should be an infinite elasticity of substitution between each type of trade service exported irrespective of its destination region. Therefore the average export price \((PE)\) should equal the price paid by each destination region \((PER)\), which should equal the export price in world currency units \((PWE)\) and will be common across all destinations \((PT)\).

The linked quantity system contains the same asymmetry in the treatment of imports and exports by Globe (see Figure 7). The imports of trade and transport commodities are assumed to be differentiated by region and the proportion of imports accounted for by the source region, hence the elasticity of substitution is greater than or equal to zero but less than infinity, while the exports of trade and transport commodities are assumed to be homogenous and hence the elasticities of transformation are infinite.

One consequence of using a Globe region for trade and transport services is that Globe runs trade balances with all other regions. These trade balances relate to the differences in the values of trade and transport commodities imported from Globe and the value of trade and transport commodities exported to Globe; however the sum of Globe’s trade balances with other regions must be zero since Globe is an artificial construct rather than a real region. But the demand for trade and transport services by any region is determined by technology, i.e., the coefficients \(\text{margcor}\), and the volume of imports demanded by the destination region. This means that the prices of trade and transport commodities only have an indirect effect upon their demand – the only place these prices enter into the import decision as a variable is as a partial determinant of the difference between the \textit{fob} and \textit{cif} valuations of other imported commodities. Consequently the primary market clearing mechanism for the Globe region comes through the quantity of trade and transport commodities it chooses to import.
The Globe concept has other potential uses in the model. All transactions between regions for which there is an absence of full bilateral information can be routed through the Globe region. While this is not a ‘first best’ solution, it does provide a ‘second best’ method by which augmented versions of the GTAP database can be used to enrich the analyses of international trade in a global model prior to availability of full bilateral transactions data (see McDonald and Sonmez (2006) for application).
4. Simulations: Increased Aid Flows to Developing African Economies

4.1 Aid Flows and Lower Taxes in Recipient Countries

4.2 Aid Flows and Increased Government Spending

4.3 Aid flows and Increased Private Investment

5. Results and Analysis

6. Concluding Comments

References


