Trade Under the Threat of Tariff Hikes in General Equilibrium

Abstract

In this paper, I develop a multi-country, multi-industry dynamic stochastic general equilibrium model of trade with firms that are heterogeneous in productivity, and which must incur a one time sunk cost to begin producing as well as a one time sunk cost to begin exporting to a new market. I model tariffs as stochastic, with the bilateral tariff each period and for each country pair drawn from one of two possible values and where the draws follow a simple Markov process. I find that in this setting, fewer firms choose to enter into the export market in equilibrium when there exists the threat of a tariff hike relative to a deterministic setting where tariffs are fixed at their current value. Based on this model, I obtain numerical results for the impacts of an (exogenous) threat of reverting to a permanent non-cooperative tariff level. In a symmetric two-country setting, the effects are a 4.55% reduction in trade and a 0.02% reduction in welfare. I am further able to derive the effect of a tariff threat on third-countries and outside sectors not directly targeted, and find these effects to be small.

1 Introduction

In the policy world, trade agreements are often evaluated using Computable General Equilibrium (CGE) models to simulate the effects of a particular trade policy shock. Generally, this is done by using estimates of price elasticities to simulate the effect of reducing applied tariffs as prescribed by the agreement in a general equilibrium framework. Recently, however, a strand of literature has been developed that examines a potential secondary effect of trade agreements; namely the reduction in policy uncertainty provided by such agreements. Due to the static nature of most CGE models, this effect is generally not taken into account when modeling the impact of trade agreements, despite the fact that recent empirical evidence has shown that trade policy uncertainty can negatively impact a firm’s entry decision into a given export market. Indeed, certain trade agreements may have no impact on applied tariff rates, while still reducing uncertainty over future applied tariffs for exporters. For example, multilateral negotiations in the WTO often include agreements on “bound” tariff rates which have no effect on applied tariff rates, but may well induce firms to enter an export market where they have increased certainty that they will not be subjected to tariff
spikes in the future. In order to evaluate the likely impact of a given trade policy agreement that takes this change in tariff expectations into account, it is necessary to build a dynamic model that is able to capture both the direct effect of applied tariff reduction and the effect of changes in tariff expectations in a general equilibrium setting. In this chapter, I extend theoretical analysis of tariff uncertainty in general equilibrium to a setting with endogenous entry not only into exporting but also into production with multiple countries and sectors. Based on this model, I obtain numerical results for the impacts of an (exogenous) threat of reverting to a permanent non-cooperative tariff level. In a symmetric two-country setting, the effects are a 4.55% reduction in trade and a 0.02% reduction in welfare. I am further able to derive the effect of a tariff threat on third-countries and outside sectors not directly targeted, and find these effects to be small.

2 Related Literature

This chapter is related to many studies modeling trade in a general equilibrium framework, though as mentioned above, typical CGE models do not include a stochastic component. It is also the case that CGE models tend to assume homogeneous firms, and therefore are not able to generate extensive margin effects of trade liberalization that have been identified as important by several empirical studies using a Melitz-type model. One exception to this is Zhai (2008) which introduces a Melitz framework with heterogeneous firms into a traditional CGE model and runs various simulations to demonstrate the additional welfare gains that may be obtained following a trade liberalization when allowing for effects through the extensive margin. Similarly, Balistreri and Rutherford (2012) develop a richer CGE model including firms heterogeneous in productivity, two factors of production and multiple industries, each of which may be modeled as either Armington, Krugman or Melitz. Arkolakis et al. (2009) investigate the theoretical implications for welfare effects of trade liberalization under various assumptions in such micro-founded models and provide conditions under which welfare implications depend on two sufficient statistics. Unlike the model presented here, however, these papers all present the results for a steady state equilibrium in a deterministic framework.

In regards to methodology, this chapter is more closely related to the strand of macro literature which simulations the effect of shocks in a DSGE framework. Alessandria and Choi (2012) attempt to infer the change in iceberg trade costs over time using US establishment data and their model’s prediction that total amount exported relative to total sales among exporters is solely determined by iceberg costs. They create model with a heterogeneous firms, sunk costs of exporting, and uncertainty over both idiosyncratic productivity and fixed export costs, each of which is assumed to follow a Markov process. Another paper which develops a heterogeneous firms model in general equilibrium with uncertainty over idiosyncratic productivity is Impullitti et al. (2013), in which the authors allow for continuous time. Because the stochastic variables
these models are idiosyncratic, one is able to define steady state aggregate variables which are not stochastic, which is not the case in my model, where I allow stochastic tariffs, which affect aggregate variables.

Another paper closely related to this chapter is Limão and Maggi (2013), which also explores the role of trade policy uncertainty in a general equilibrium framework. Unlike this chapter, however, their focus is primarily on the uncertainty-managing incentives to form trade agreements in an environment with underlying political economy (or economic) shocks. That is, in their model, they allow trade policy to be endogenous and explore the conditions under which an agreement maximizing joint welfare between contracting parties will reduce or increase trade policy uncertainty. In my case, I take trade policy (with a stochastic component) as exogenous and ask how the presence of the stochastic component affects endogenous variables of interest (such as utility, trade flows, etc.).

The underlying hypothesis behind constructing a model with tariff uncertainty is that this has an impact on exporter behavior. Recent empirical work provides several examples of studies that support this hypothesis while recent theoretical work provides mechanisms through which this may occur. Evenett et al. (2004) provide some evidence of increased exports after WTO accession to developed countries in products with lower gaps between preferential and Most Favored Nation (MFN) tariff rates, though this evidence is somewhat mixed. Sala et al. (2009) develop a purely theoretical model, using heterogeneous firms and an option value approach, and show that a reduction in bound rates can move forward export time and that we see a larger effect for “high risk” destination markets. Handley and Limão (2012) find increased entry into the export market for Portuguese exporters as a result of the reduction in uncertainty from EC accession, and Handley (forthcoming, JIE) finds evidence that product level uncertainty negatively impacts the level and responsiveness of exports to applied tariff reductions for exports to Australia. Handley and Limão (2013) is perhaps most closely related to the model presented in this chapter as they examine the impact of uncertainty faced by Chinese exporters to the United States in a general equilibrium framework prior to the granting of Permanent Normal Trade Relations to China in 2001. They also provide estimates of the welfare impact of trade policy uncertainty in this context. Pierce and Schott (2014) provide further evidence that the reduction in tariff uncertainty for Chinese exporters to the United States following the granting of Permanent Normal Trade Relations to China in 2001 contributed to an increase in Chinese exports as well as a reduction in US manufacturing employment in sectors with larger reductions in tariff uncertainty.

The model I present here offers several extensions beyond what has been done in other general equilibrium frameworks. First, I allow for free entry of firms rather than assuming a fixed number of firms as is done in Handley and Limão (2013). This results in a model whose structure aligns more closely with standard CGE models used to evaluate trade agreements and allows us to understand the effects of tariff threats not only on the mass of exporters, but the number of producing firms in general equilibrium. Additionally, I
am able to include arbitrarily many countries and differentiated goods industries in my model, rendering it a practical framework in which to study trade agreements which may affect trade in multiple countries and/or industries. This also speaks to the added value of considering the effect of tariff threats in a general equilibrium as opposed to a partial equilibrium framework; while a given trade agreement may directly impact the tariffs and tariff uncertainty for a bilateral pair or regional group, in general equilibrium these tariffs/potential tariffs will affect the economies of all regions. My model allows one to quantify such effects not just for countries party to a particular agreement, but also effects on their trading partners. For example, while previous partial equilibrium or two-country general equilibrium models allow one to examine the effects of tariff uncertainty faced by China prior to joining the WTO, my model allows one to examine the effects of this tariff uncertainty on third country trading partners, such as Mexico.

3 Model

I model a world with \( C \geq 2 \) countries, \( K \) differentiated goods industries and one industry which produces a homogenous good. In a multi-period framework with uncertain tariffs, in each period, producers of differentiated good \( k \) in country \( r \) decide whether or not to incur the one time sunk cost \( f_{k,r,s}^{exp} \) required to begin exporting to market \( s \). In this set-up, there are \( K \) industries that produces differentiated goods (in all countries), and there is a continuum of differentiated goods within each industry. Consumers (in each country) have a constant elasticity of substitution between varieties, \( \sigma \). Countries are indexed by \( r \) or \( s \), sector by \( k \), and varieties by \( \omega \).

3.1 Consumer Demand

It is assumed that in a given period, consumers in each region \( r \) derive utility

\[
U_r = q_0 r^\mu_0 \prod_k Q^\omega_{k,r}
\]

where \( \mu_0 = 1 - \sum_k \mu_k \) based on a Cobb-Douglas utility function aggregating a homogenous numeraire good, \( q_0 \) and CES aggregates of differentiated goods defined by the sub-utility function of imperfectly substitutable varieties:

\[
Q_{k,r} = \left[ \sum_s \lambda_{k,s,r} \int_{\Omega_{k,s,r}} q_{k,s,r}(\omega) \frac{1}{\sigma} d\omega \right]^\frac{\sigma}{\sigma-1}
\]

where \( \Omega_{k,s,r} \) is the set of industry \( k \) varieties produced in market \( s \) available for purchase in market \( r \) and \( q_{s,r}(\omega) \) is the quantity consumed in \( r \) of variety \( \omega \) produced in \( s \). Parameters \( \lambda_{k,r,s} \) are preference share parameters to allow for non-symmetric preferences across domestic vs. foreign varieties.\(^1\)

\(^1\)These parameters are included so that when applying the model to actual data, these can be calibrated to account for differing trade flows between partners which could not be accounted for in the price index \( P_{k,r} \) in a model with more than two countries, as this would be fixed across bilateral pairs \((r,s)\). These preference parameters can alternatively be thought of as
Maximizing the utility as defined in (1) subject to a budget constraint yields the Dixit-Stiglitz price index of differentiated goods sector $k$ in region $r$ (that is, the marginal price of $Q_{k,r}$):\[ P_{k,r} = \left[ \sum_s \lambda_{k,s,r} \int_{\Omega_{k,s,r,t}} \left( \tau_{k,s,r} p_{k,s,r}(\omega) \right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \] (2)

Where $p_{k,s,r}(\omega)$ is the price received by the industry $k$, region $s$ firm producing variety $\omega$ and selling in market $r$. It is assumed that the consumer will pay this firm price times a factor $\tau_{k,s,r}$ for imported goods where $\tau$ represents 1 plus the ad valorem tariff (and it is assumed that $\tau_{k,r,r} = 1 \forall k$). Tariff revenue is paid to the government, which then redistributes all tariff revenue as a lump sum in each period to the population.

The compensated demand function for each domestic and foreign variety in each industry resulting from the maximizing utility is given by:
\[ q_{k,r,s}(\omega) = \lambda_{k,r,s} Q_{k,s} \left( \frac{P_{k,s}}{\tau_{r,s} p_{k,r,s}(\omega)} \right)^{\sigma} \] (3)

Because of the Cobb-Douglas utility assumption, expenditures $E_{k,r}$ on differentiated goods sector $k$ will be equal to a constant share of GDP (total expenditures in the economy):
\[ E_{k,r} = \mu_k GDP_r \]
and aggregate quantity $Q_{k,r}$ can be thought of as the nominal expenditures in differentiated good sector $k$ deflated by the price index:
\[ Q_{k,r} = \frac{E_{k,r}}{P_{k,r}} \] (4)

3.2 Supply

Turning to supply, I assume that the homogenous good is produced in each country with constant marginal product of labor, and is freely traded on world markets. I assume that each variety in a differentiated goods sector is produced using only one factor of production (labor) which has factor price $w_r$. Throughout the model, I will impose the assumption that the labor force in each country is sufficiently large that the homogenous good is produced in every country in equilibrium. This then fixes the wage in each country at $w_r = w_s$, regardless of the state of the economy, and taking the homogenous good as the numeraire, we have that $w_r = w_s = 1$.\footnote{I assume that consumers are credit constrained and also do not allow for inter-temporal substitution by saving; they solve a static optimization problem in each period.}

Prospective firms in an industry $k$, region $r$ face a sunk entry cost $f^{\text{sunk}}_{k,r}$ which much be paid to receive a productivity draw and begin production. Prospective firms observe the current period tariff before making sector-bilateral pair specific iceberg trade costs such that for each amount $q_{k,s,r}(\omega)$ of variety $\omega$ purchased by some consumer in region $r$, only $\lambda_{k,s,r} q_{k,s,r}(\omega)$ of the product is actually consumed (enters into the utility function).\footnote{In the notation below, I retain the country specific subscript on $w$.}
their entry decision, and after paying the sunk cost, receive a random productivity draw $\varphi$ from distribution $G(\varphi)$ which determines their marginal cost of production, $w_r/\varphi$. After learning their (time invariant) productivity, the firm begins producing and sells in the domestic market. At the end of each period, the firm faces a $\delta$ probability of being hit by an exogenous death shock in which case it exits. If it survives until the next period, it can then choose to enter into export market $s$ by paying the sunk cost of exporting, $f_{k,r,s}^{exp}$. I assume no per-period fixed costs to sell in either the domestic or the export market (once the sunk cost has been paid) so that once firms have entered into production, they will always choose to sell in the domestic market, and once they have paid the sunk cost of exporting, they will continue to export for as long as they survive. Firms are monopolistically competitive, and so in any market where they choose to sell, they will charge the standard markup over marginal cost:

$$p_{k,r,s}(\varphi) = \frac{w_r}{\varphi} \left( \frac{\sigma}{\sigma - 1} \right)$$

where $\sigma$ is the constant elasticity of demand (equal to the elasticity of substitution between varieties).

Gross profits for a firm with productivity $\varphi$ in sector $k$ are given by:

$$\pi_{k,r}(\varphi) = \sum_s \pi_{k,r,s}(\varphi) = \sum_s \frac{p_{k,r,s}(\varphi)q_{k,r,s}(\varphi)}{\sigma}$$

I define aggregate economic conditions faced by a sector $k$, country $r$ firm selling in market $s$ as

$$A_{k,r,s} = \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right) w_r \lambda_{r,s} Q_{k,s} \left( \frac{P_{k,s}}{\tau_{k,r,s}} \right)^{\sigma}$$

so that the gross profits realized by a firm of productivity $\varphi$ in sector $k$ region $r$ are $\pi_{\varphi,k,r,r} = \varphi^{\sigma - 1} A_{k,r,r}$ from domestic sales and $\pi_{\varphi,k,r,s} = \varphi^{\sigma - 1} A_{k,r,s}$ from exports to country $s \neq r$, for firms sufficiently productive to export.

### 3.3 Tariff Process

I assume a simple tariff shock process that allows me to maintain a tractable model under the assumption of free entry. Bilateral tariffs applied to sector $k$ exports from country $s$ into country $r$ can take on only two values: $L_{k,s,r}$ or $H_{k,s,r}$, with $L_{k,s,r} \leq H_{k,s,r}$. In the initial period, tariffs are (and have always been) at $L_{k,s,r}$ in each country $r$. In each subsequent period for which tariffs have always been at $L$, there is a probability $\gamma$ that tariffs will go up to $H$. Once this happens, tariffs remain at level $H$ forever after.

The timing is as follows: at the beginning of each period, there are $N_{k,r}$ firms in sector $k$, region $r$ who have already produced in the previous period and survived the death shock. The current period tariff
is observed. Then, potential firms decide whether or not to incur the sunk entry cost and \( NE_{k,r} \) new firms enter and can sell only in the domestic market for this period (they can begin exporting in the next period if they so choose). Then, the death shock arrives, killing a share \( \delta \) of the \( N_{k,r} + NE_{k,r} \) firms so that the state transition equation is: 

\[
N'_{k,r} = (1 - \delta)(N_{k,r} + NE_{k,r}).
\]

I assume that prior to the arrival of the high tariff shock, the economy is in a steady state equilibrium (that is, all endogenous variables are the same in any state with low tariffs), and so the number of firms in this state will be determined by the relationship 

\[
\frac{\delta}{1-\delta} N_{k,r} = NE_{k,r}.
\]

In this set-up, in each period the economy will be in one of the following discrete states \( \zeta \): \( L \) in the case that the high tariff shock has not arrived (so that tariffs are at level \( L_{k,s,r} \) in country \( r \) for sector \( k \) goods imported from \( s \)), or \( H^{(n)} \), \( n \in \{0, 1, \ldots \} \), where \( H^{(0)} \) is the state in which the high tariff shock arrives, \( H^{(1)} \) is the state the following period, etc. The state describes not only the current tariff level(s), but also the distribution and mass of firms who are present in the domestic and export markets. Because of the sunk cost to export, once a firm chooses to begin exporting, he will continue to do so even after a shock arrives that causes his profitability to fall below the profitablility of the cutoff firm who is just indifferent between entering the export market or not. Thus, after the arrival of the high tariff shock, in state \( H^{(0)} \), there may be exporters with productivity levels such that they would not choose to enter the export market under current conditions had they not already paid the sunk cost to export, but will continue to export in this and all future periods (until hit by the exogenous death shock); I refer to these as “legacy firms”. I let \( H^{det} \) refer to the steady state of the deterministic model with tariffs at level \( H \) in all periods.

### 3.4 Preliminary Definitions

As will be shown in the following sections, in equilibrium in each state \( \zeta \), there will exist a cutoff productivity \( \varphi^c_{k,r,s}(\zeta) \) such that all sector \( k \), country \( r \) firms with \( \varphi < \varphi^c_{k,r,s}(\zeta) \) will not choose to enter export market \( s \), while firms with \( \varphi \geq \varphi^c_{k,r,s}(\zeta) \) will want to enter into this export market. That is, I let \( \varphi^c_{k,r,s}(L) \) be the cutoff productivity for a sector \( k \), country \( r \) firm to want to enter into export market \( s \) when the state is \( L \), and similarly I let \( \varphi^c_{k,r,s}(H^{(n)}) \) be the cutoff productivity for entering into this export market when the state is \( H^{(n)} \).\(^4\) Once the high tariff shock has arrived, legacy firms may remain, though a share \( \delta \) of them will be killed in each subsequent period, and not replaced in the export market. Thus, once the high tariff shock arrives, in each subsequent period, conditions will change, and the cutoff productivity for exporting may depend on how many periods have passed since the high tariff shock was first realized. In order for the model to be tractable, I make the explicit assumption that any change in export cutoff for a given

\(^4\)Throughout the paper I maintain the assumption that the sunk cost to begin exporting in any market, \( f^{exp} \), is sufficiently large that the lowest productivity firm will never choose to export.
The bilateral flow is either non-decreasing or non-increasing in \( n \), that is, I assume that for any sector \( k \) and any pair of countries \( r \) and \( s \), it is either the case that \( \varphi_{k,r,s}^c(H^{(n+1)}) \leq \varphi_{k,r,s}^c(H^{(n)}) \) \( \forall n = 0,1,2,\ldots \) or that \( \varphi_{k,r,s}^c(H^{(n+1)}) \geq \varphi_{k,r,s}^c(H^{(n)}) \) \( \forall n = 0,1,2,\ldots \). Under the further assumption that the economy eventually reaches its long-term steady state equilibrium, it turns out to be the case that cutoff productivities in each high tariff state \( H^{(n)} \) do not depend on \( n \), that is

\[
\varphi_{k,r,s}^c(H^{(n+1)}) = \varphi_{k,r,s}^c(H^{(n)}) = \varphi_{k,r,s}^c(H^{det}) = \varphi_{k,r,s}^c(H) \tag{7}
\]

where \( \varphi_{k,r,s}^c(H^{det}) \) is the cutoff productivity to enter export market \( s \) in a deterministic model where tariffs are at level \( H \) in every period and the economy is in its steady state (see Appendix A for proof of this result). Also shown in this appendix, as a corollary to this result, it will be the case that aggregate economic conditions, \( A_{k,r,s}(H^{(n)}) \) do not vary with \( n \), and so I define \( A_{k,r,s}(H) \equiv A_{k,r,s}(H^{(n)}) = A_{k,r,s}(H^{(n+1)}) = A_{k,r,s}(H^{det}) \) \( \forall n \).

In the limit as \( n \to \infty \), the mass of legacy firms approaches zero and all endogenous variables \( X \) approach their deterministic steady state level: \( X(H^{(n+1)}) \to X(H^{det}) \).

Under the above assumptions, each \( k-r-s \) productivity cutoff to export in state \( L \) versus that in state \( H^{(n)} \) must be ranked in one of two ways: either

\[
\text{RANKING A} : \varphi_{k,r,s}^c(L) \leq \varphi_{k,r,s}^c(H) \tag{8}
\]

or

\[
\text{RANKING B} : \varphi_{k,r,s}^c(L) > \varphi_{k,r,s}^c(H)
\]

In a perfectly symmetric setup where the high tariff threat level is symmetric across all countries and there is only one differentiated goods industry, we will be in the first case for each country, that is, the arrival of a high tariff shock resulting in a high tariff that is the same in all countries will result in higher productivity cutoffs in all countries. As we will see below, however, in asymmetric settings (for example when the tariff shock affects only one bilateral tariff), it may be the case that the arrival of the tariff shock \( H \) actually lowers the productivity cutoff to export in one of the regions. I let \( 1_{\text{RankB}_{k,r,s}} \) be a binary indicator equal to 1 if \( \varphi_{k,r,s}^c(L) > \varphi_{k,r,s}^c(H) \) and zero otherwise.

Letting \( F \) be the distribution from which productivity is drawn (which I will later assume to be Pareto) and assuming a lower bound of \( b \) for the support of this distribution, I define the “average” domestic productivity of all producing firms in country \( r \) as \( \bar{\varphi}_{k,r,r} = \left[ \int_b^\infty \varphi^{\sigma-1} dF(\varphi) \right]^{\frac{1}{\sigma-1}} \) which does not depend on the state since the distribution of firms remains constant over time with exit occurring only via the exogenous
death shock. I also define the (state contingent) average export productivity for sector $k$ exporters\footnote{Here, “exporters” means the set of firms above the productivity cutoff for exporting; it does not include legacy firms.} from $r$ to $s$ as:

$$
\tilde{\varphi}_{k,r,s}(\zeta) = \left[ \int_{\varphi_{k,r,s}(\zeta)}^{\infty} \varphi \sigma^{-1} dF(\varphi) \right]^{\frac{1}{\sigma-1}}
$$

Note that under the assumption that $\varphi$ follows a Pareto distribution with shape parameter $\alpha$, it will be the case the average export productivity can be written as a constant multiple of the cutoff productivity in each state:

$$(\tilde{\varphi}_{k,r,s}(\zeta))^{\sigma-1} = \frac{\alpha}{\alpha + 1 - \sigma} \left( (\varphi_{k,r,s}(\zeta))^{\sigma-1} \right) \text{ for } \varphi \sim \text{Pareto}$$

### 3.5 Value Functions and Entry

For a firm of productivity level $\varphi$ in sector $k$, region $r$, I let $V_{\varphi,k,r,s}(\zeta)$ be the present discounted value of selling in market $s$ in state $\zeta$, taking into account current and expected future profit flows from exporting from $r$ to $s$ (or selling domestically for $r = s$).

The value of being a sector $k$ region $r$ producer selling in region $s$ in each state for a productivity $\varphi$ firm is then:

$$V_{\varphi,k,r,s}(L) = \varphi^{\sigma-1} A_{k,r,s}(L) + (1 - \delta) \left[ (1 - \gamma) V_{\varphi,k,r,s}(L) + \gamma V_{\varphi,k,r,s}(H(0)) \right]$$

$$\Rightarrow V_{\varphi,r,s}(L) = \frac{\varphi^{\sigma-1} A_{k,r,s}(L) + (1 - \delta) \gamma V_{\varphi,k,r,s}(H(0))}{\delta + \gamma - \delta \gamma}$$

and

$$V_{\varphi,k,r,s}(H(n)) = \varphi^{\sigma-1} A_{k,r,s}(H(n)) + (1 - \delta) V_{\varphi,k,r,s}(H(n+1))$$

$$\Rightarrow V_{\varphi,k,r,s}(H(n)) = \sum_{i=n}^{\infty} (1 - \delta)^{i-n} \varphi^{\sigma-1} A_{k,r,s}(H(i)) = \varphi^{\sigma-1} \frac{A_{k,r,s}(H)}{\delta}$$

while the value of waiting to enter a given export market is:

$$V_{\varphi,k,r,s}^{\text{wait}}(L) = (1 - \delta) \left[ (1 - \gamma) \max \left\{ \left( V_{\varphi,k,r,s}(L) - w_r f_{r,s}^{\text{exp}} \right), V_{\varphi,k,r,s}^{\text{wait}}(L) \right\} \right] + \gamma \max \left[ \left( V_{\varphi,k,r,s}(H(0)) - w_r f_{r,s}^{\text{exp}} \right), V_{\varphi,k,r,s}^{\text{wait}}(H(0)) \right]$$

and

$$V_{\varphi,k,r,s}^{\text{wait}}(H(n)) = (1 - \delta) \max \left[ \left( V_{\varphi,k,r,s}(H(n+1)) - w_r f_{r,s}^{\text{exp}} \right), V_{\varphi,k,r,s}^{\text{wait}}(H(n+1)) \right]$$

For a given state $\zeta$, the value of entering into production for a firm who draws productivity $\varphi$ will be:
by two countries denotes the value of waiting to begin exporting to region $s$ and so the ex-ante (prior to drawing a productivity) value of entry for a prospective firm in sector $k$ region $r$ will be:

$$V_{k,r}^{entry}(\zeta) = \int \varphi V_{\varphi,k,r}^{entry}(\zeta) dF(\varphi)$$ (15)

I assume that the “death shock” applies equally to current and potential firms so that the same discount factor is used by all when discounting future periods. Then, the value of waiting to enter into production in a given state will be given by

$$V_{k,r}^{wait to enter}(\zeta) = 0 + (1 - \delta) \max \left[ \left( V_{k,r}^{entry}(\zeta') - w_r f_{k,r}^{sunk} \right), V_{k,r}^{wait to enter}(\zeta') \right]$$ (16)

where the final equality follows from the fact that in equilibrium, free entry implies that $V_{k,r}^{entry}(\zeta) - w_r f_{k,r}^{sunk} \leq V_{k,r}^{wait to enter}(\zeta') \forall \zeta$ (with strict inequality only in the case when the constraint $NE_{k,r}(\zeta) \geq 0$ is binding).$^6$ I make the assumption that no tariff shock is sufficiently large to cause the constraint $NE_{k,r}(\zeta) \geq 0$ to bind, so that in equilibrium, $V_{k,r}^{entry}(\zeta) - w_r f_{k,r}^{sunk} = V_{k,r}^{wait to enter}(\zeta)$. Writing out the expression for $V_{k,r}^{entry}(\zeta)$ and simplifying (see Appendix B for details) we obtain the free entry condition determining the number of new entrants into sector $k$, region $r$ in state $L$ as:

$$V_{k,r}^{wait to enter}(\zeta) = 0 + (1 - \delta) \max \left[ \left( V_{k,r}^{entry}(\zeta') - w_r f_{k,r}^{sunk} \right), V_{k,r}^{wait to enter}(\zeta') \right]$$ (16)

where the final equality follows from the fact that in equilibrium, free entry implies that $V_{k,r}^{entry}(\zeta) - w_r f_{k,r}^{sunk} \leq V_{k,r}^{wait to enter}(\zeta') \forall \zeta$ (with strict inequality only in the case when the constraint $NE_{k,r}(\zeta) \geq 0$ is binding).$^6$ I make the assumption that no tariff shock is sufficiently large to cause the constraint $NE_{k,r}(\zeta) \geq 0$ to bind, so that in equilibrium, $V_{k,r}^{entry}(\zeta) - w_r f_{k,r}^{sunk} = V_{k,r}^{wait to enter}(\zeta)$. Writing out the expression for $V_{k,r}^{entry}(\zeta)$ and simplifying (see Appendix B for details) we obtain the free entry condition determining the number of new entrants into sector $k$, region $r$ in state $L$ as:

$$
\bar{\phi}_{k,r,s}^{\sigma-1} \left[ \frac{M_{k,r,s}(H(0))}{N_{k,r}(L) + NE_{k,r}(L)} \right] \left( \frac{\tilde{\phi}_{k,r,s}^{-1}(L)}{\tilde{\phi}_{k,r,s}^{-1}(H)} \frac{A_{k,r,s}(H)}{\delta} - w_r f_{k,r,s}^{exp} \right) \\
+ (1 - \delta) \sum_{s \neq r} \left[ (1 - \gamma) \left( \frac{M_{k,r,s}(L)}{N_{k,r}(L)} \right) \left( \frac{\tilde{\phi}_{k,r,s}^{-1}(L)}{\gamma} \left( A_{k,r,s}(L) + \gamma(1 - \delta) \frac{A_{k,r,s}(H)}{\delta} \right) - (\delta + \gamma - \delta) w_r f_{k,r,s}^{exp} \right) \right] + \\
+ (1 - \delta) \sum_{s \neq r} (1 - \gamma) (1 - \delta) \left( F(\varphi_{k,r,s}^{c}(L)) - F(\varphi_{k,r,s}^{c}(H)) \right) \left( \frac{\phi_{k,r,s}^{L-H+1}}{\varphi_{k,r,s}^{L-H}} \frac{A_{k,r,s}(H)}{\delta} - w_r (H^0) f_{k,r,s}^{exp} \right) \right) * 1_{\text{RankB}_{k,r,s}}
= \delta w_r f_{k,r}^{sunk}
$$

$^6$Note that $V_{k,r}^{wait to enter}$ indexed by one country denotes the value of waiting to enter into production while $V_{k,r,s}^{wait to enter}$ indexed by two countries denotes the value of waiting to begin exporting to region $s$. These are two distinct values.
where

\[
\tilde{\varphi}_{k,r,s}^{LH} = \left\{ \begin{array}{ll}
\varphi_{k,r,s}^{c}(L) & \text{if } \varphi_{k,r,s}^{c}(L) = \varphi_{k,r,s}^{c}(H) \\
\frac{\varphi_{k,r,s}^{c}(H) - \varphi_{k,r,s}^{c}(L)}{F(\varphi_{k,r,s}^{c}(H)) - F(\varphi_{k,r,s}^{c}(L))} & \text{otherwise}
\end{array} \right.
\]

\( \varphi_{k,r,s}^{c}(L) \neq \varphi_{k,r,s}^{c}(H) \)

is the average productivity level between high and low state cutoffs.

In state \( H^{(n)} \), the number of new entrants is determined by:

\[
\varphi_{k,r,s}^{\sigma-1}(H^{(n)}) A_{k,r,s}(H^{(n)}) + \sum_{s \neq r} \left( \frac{M_{k,r,s}(H^{(n+1)})}{N_{k,r}(H^{(n)}) + NE_{k,r}(H^{(n)})} \right) \left( \varphi_{k,r,s}^{\sigma-1}(H^{(n+1)}) A_{k,r,s}(H^{(n+1)}) - \delta w_{r} f_{k,r,s}^{exp} \right) = \delta w_{r} f_{k,r,s}^{sunk}
\]

The complete derivations of both conditions are detailed in Appendix B.

### 3.6 Export Productivity Cutoffs

In a given state \( \zeta \), sector \( k \)-region \( r \) firms who have entered into production in some previous period will choose to incur the sunk cost \( f_{k,r,s}^{exp} \) and begin exporting to \( s \) if the value of exporting exceeds the value of waiting to begin exporting to \( s \) by at least the amount of this sunk cost. That is, in state \( \zeta \) a country \( r \) firm with productivity \( \varphi \) will enter export market \( s \) if

\[
V_{\varphi,k,r,s}(\zeta) - V_{\varphi,k,r,s}^{wait}(\zeta) \geq w_{r} f_{k,r,s}^{exp}
\]

As shown in Appendix C, for a sector \( k \) region \( r \) with export cutoffs to \( s \) ranked according to Ranking A (see 8):

\[
(\varphi_{k,r,s}^{c}(L))^{\sigma-1} = \frac{(\delta + \gamma - \delta \gamma) w_{r} f_{k,r,s}^{exp}}{[A_{k,r,s}(L) + \gamma \left( \sum_{n=0}^{\infty} (1 - \delta)^{n+1} A_{k,r,s}(H^{(n)}) \right)]^{\frac{1}{\sigma}}} \varphi_{k,r,s}^{c}(L)
\]

(17)

Note that one can show that this cutoff is increasing in \( \gamma \) (see Appendix D) under certain conditions.

That is, for a country with \( \varphi_{k,r,s}^{c}(L) \leq \varphi_{k,r,s}^{c}(H^{det}) \), it is also the case that the export cutoff productivity in state \( L \) is increasing in \( \gamma \), the probability that the high tariff shock will arrive in any given period.

For a sector \( k \) region \( r \) with export cutoffs to \( s \) ranked according to Ranking B, as shown in Appendix C we have

\[
(\varphi_{k,r,s}^{c}(L))^{\sigma-1} = \frac{(\delta + \gamma - \delta \gamma) w_{r} f_{k,r,s}^{exp} - (1 - \delta) \gamma w_{r}(H^{(0)}) f_{k,r,s}^{exp}}{A_{k,r,s}(L)} \varphi_{k,r,s}^{c}(L)
\]

(18)

The cutoff productivity in high tariff states, \( \varphi_{k,r,s}^{c}(H^{(n)}) \), is derived in appendix A and is given by

\[
(\varphi_{k,r,s}^{c}(H^{(n)}))^{\sigma-1} = \frac{\delta w_{r} f_{k,r,s}^{exp}}{A_{k,r,s}(H^{(n)})}
\]

(19)
3.7 Number of Exporters

Here, I let $M_{k,r,s}(\zeta)$ denote the mass of sector $k$ firms in country $r$ at state $\zeta$ who lie above the export cutoff $\varphi_{k,r,s}^c(\zeta)$ in that state and export to $s$. Because new entrants this period cannot export, this will be given by the share of the $N_{k,r}(\zeta)$ firms who lie above this cutoff, that is:

$$M_{k,r,s}(\zeta) = N_{k,r}(\zeta)(1 - F(\varphi_{k,r,s}^c(\zeta))) \text{ for } \zeta \in \{L, H^{(0)}, H^{(1)}, \ldots\}$$

In states where legacy firms are present, the total number of sector $k$ region $r$ exporters to $s$ will be given by $M_{k,r,s}(\zeta) + \text{Leg}_{k,r,s}(\zeta)$ where the mass of legacy exporters satisfies:

$$\begin{align*}
\text{Leg}_{k,r,s}(L) & = 0 \\
\text{Leg}_{k,r,s}(H^{(n)}) & = \begin{cases} 
N_r(H^{(0)})(1 - \delta)^n (F(\varphi_{r,s}^c(H^{(n)})) - F(\varphi_{r,s}^c(L))) & , 1_{\text{Rank}B_{k,r,s}} = 0 \\
0 & , 1_{\text{Rank}B_{k,r,s}} = 1
\end{cases}
\end{align*}$$

because there will be no legacy exporters in states where the tariff remains at the low level, and in state $H^{(n)}$ only firms who were present when the high tariff shock first arrives (that is, those who form part of the mass $N_{k,r}(H^{(0)}) = N_{k,r}(L)$) and who have not been killed by the exogenous shock, and who lie above cutoff $\varphi_{k,r,s}^c(L)$ and below $\varphi_{k,r,s}^c(H^{(n)})$ will be legacy firms.

3.8 Number of New Exporters

I denote the mass of new exporters to country $s$ from country $r$ in sector $k$ and state $\zeta$ by $NX_{k,r,s}(\zeta)$. In state $L$ (for which I assume we are in a steady-state equilibrium), this will just be the mass of firms required to replace exporters killed off by the exogenous death shock:

$$NX_{k,r,s}(L) = \delta M_{k,r,s}(L)$$

For $1_{\text{Rank}B_{k,r,s}} = 0$ (see (8)), new exporters in states $H^{(n)}$ will include the share of new entrants in the previous period who have survived and are above today’s cutoff. This gives:

$$\begin{align*}
NX_{k,r,s}(H^{(0)}) & = (1 - \delta)NE_{k,r}(L) \left(1 - F(\varphi_{k,r,s}^c(H^{(0)}))\right), 1_{\text{Rank}B_{k,r,s}} = 0 \\
NX_{k,r,s}(H^{(1)}) & = (1 - \delta)NE_{k,r}(H^{(0)}) \left(1 - F(\varphi_{k,r,s}^c(H^{(1)}))\right) \\
NX_{k,r,s}(H^{(n)}) & = (1 - \delta)NE_{k,r}(H^{(n-1)}) \left(1 - F(\varphi_{k,r,s}^c(H^{(n)}))\right), n = 2, 3, \ldots
\end{align*}$$
The equations determining the number of new \( k - s \) exporters in regions \( r \) with cutoffs ranked according to ranking \( B \) will be the same in each state with the exception of state \( H(0) \) for which we have:

\[
NX_{k,r,s}(H(0)) = (1 - \delta)NE_{k,r}(L) \left( 1 - F \left( \varphi_{k,r,s}^c(H(0)) \right) \right) \\
+ N_{k,r}(H(0)) \left( F(\varphi_{k,r,s}^c(L)) - F(\varphi_{k,r,s}^c(H(0))) \right), \quad \text{Rank}_{B_{k,r,s}} = 1
\]

because in this case, the cutoff export productivity in state \( H(0) \) is lower than that in state \( L \), so firms who have already entered into production when the high tariff shock arrives and who have productivity \( \varphi_{k,r,s}^c(H(0)) \leq \varphi \leq \varphi_{k,r,s}^c(L) \) will also enter the export market in state \( H(0) \).

### 3.9 Price Index

The price index in country \( r \), sector \( k \) in state \( \zeta \) will be affected by the firm level prices of domestic firms and foreign firms exporting into \( r \), which themselves are made up of firms above the current state export cutoff and legacy exporters.

\[
P_{1-\sigma}^{\text{cat}}(\zeta) = \lambda_{k,r,r}(N_{k,r}(\zeta) + NE_{k,r}(\zeta)) \left( \hat{p}_{k,r,r}(\zeta) \right)^{1-\sigma} + \\
+ \sum_{s \neq r} \lambda_{k,s,r} \left[ (M_{k,s,r}(\zeta)) \left( \hat{p}_{k,s,r}(\zeta) \tau_{k,s,r}(\zeta) \right)^{1-\sigma} + Leg_{k,s,r}(\zeta) \left( \hat{p}_{k,s,r}^{\text{leg}}(\zeta) \tau_{k,s,r}(\zeta) \right)^{1-\sigma} \right]
\]

where \( \hat{p}_{k,s,r}(\zeta) \) is the firm level price charged by the firm of average productivity \( \bar{\varphi}_{k,s,r} \), and \( \hat{p}_{k,s,r}^{\text{leg}}(H(n)) \) is the price charged by the country \( s \) firm with average \( r \) legacy productivity.

\[
\hat{p}_{k,s,r}^{\text{leg}}(H(n)) = \left[ \frac{\int_{\varphi_{k,s,r}^c(H(n))}^{\varphi_{k,s,r}^c(H(n))} \varphi^{-1} dF(\varphi)}{F(\varphi_{k,s,r}^c(H(n))) - F(\varphi_{k,s,r}^c(L))} \right]^{\frac{1}{\sigma - 1}}
\]

with each price depending on productivity according to \( \hat{p}_{\varphi,k,r,s}^{\text{cat}}(\zeta) = \frac{w_{\text{sr}}}{\varphi_{k,r,s}} \left( \frac{\sigma}{\sigma - 1} \right) \), \( \text{cat} \in \{\cdot, \text{leg} \} \).

### 3.10 Labor Market Clearing Condition

I assume that the population in each country is sufficiently large for the numeraire good to always be produced in all countries in equilibrium, which implies that wages in all countries and all periods will be pinned down at \( w_r \) (which I take to be equal to 1 in each country), independent of the state \( \zeta \).

### 3.11 Income-Expenditure Closure

Finally, in each region, total income must equal total expenditure:
\[ \text{GDP}_r(\zeta) = w_r \bar{L}_r + \sum_k \sum_{s \neq r} (\tau_{k,s,r} - 1) \left[ M_{k,s,r}(\zeta) \hat{p}_{k,s,r}(\zeta) \hat{q}_{k,s,r}(\zeta) + Leg_{k,s,r}(\zeta) \hat{p}_{z_{k,s,r}}(\zeta) \hat{q}_{z_{k,s,r}}(\zeta) \right] \\
+ \sum_k \left( N_{k,r}(\zeta) + NE_{k,r}(\zeta) \right) \frac{\hat{p}_{k,r,r}(\zeta) \hat{q}_{k,r,r}(\zeta)}{\sigma} \\
+ \sum_k \sum_{s \neq r} M_{k,r,s}(\zeta) \frac{\hat{p}_{k,r,s}(\zeta) \hat{q}_{k,r,s}(\zeta) + Leg_{k,r,s}(\zeta) \hat{p}_{z_{k,r,s}}(\zeta) \hat{q}_{z_{k,r,s}}(\zeta)}{\sigma} \\
- \sum_k w_r f_{\text{sunk}}^{k,r} NE_r(\zeta) - \sum_k \sum_{s \neq r} w_r f_{\text{exp}}^{k,s} N X_{k,r,s}(\zeta) \]

Here, income is equal to the sum of labor income, tariff revenue and firm level profits minus the sunk costs incurred by firms to begin production or exporting.

4 Computation

Here, I summarize the set of equations and complementarity conditions determining the equilibrium values of all endogenous variables under the tariff process and set-up described above. Recall that some of these derivations relied on the assumption that no tariff shock is large enough to cause the condition \( NE_{k,r} \geq 0 \) to bind and that fixed costs of exporting are sufficiently large that the productivity cutoff to export in any market is strictly greater than the minimum productivity draw, \( b \), so I must verify that this is the case in equilibrium. Appendix E lists the set of complementarity conditions that define equilibrium under the assumptions of Pareto distributed productivity with parameters \( a \) and \( b \), and sufficiently large labor forces in each country such that the wage \( w_r \) is not state-dependent.

4.1 Solution Method

The equations and complementarity conditions in the table in Appendix E yield a square system of \( C \ast (n + 3) \ast [3 + 10KC + 2(C - 1)K] \) endogenous variables and the same number of equations determining these where \( C \) is the number of regions in the model, \( K \) is the number of sectors, and \( n = 1, 2, \ldots \) is the number of periods that have passed since the initial arrival of the high tariff shock. In order to reduce the system to a finite number of equations and unknowns, I assume that after period \( n^* \) the economy has returned to its steady state “Hdet”, the equilibrium of the deterministic model with tariffs at level \( H \) in every period. I take \( n^* = 100 \), and then check that increasing this value has no effect on my equilibrium values.

This then yields a square mixed complementarity problem which is straightforward to program and solve using GAMS software. I use the PATH solver, a generalization of Newton’s method to solve the system (Ferris and Munson (2000)).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ</td>
<td>Exogenous probability of death each period</td>
<td>0.1</td>
</tr>
<tr>
<td>a</td>
<td>Pareto shape parameter</td>
<td>4.6</td>
</tr>
<tr>
<td>b</td>
<td>Minimum Productivity Level</td>
<td>0.5</td>
</tr>
<tr>
<td>σ</td>
<td>Elasticity of Substitution between Varieties</td>
<td>3.8</td>
</tr>
<tr>
<td>μ0</td>
<td>Share of Total Expenditures on Homogenous Good Sector</td>
<td>0.3</td>
</tr>
<tr>
<td>γ</td>
<td>Probability of a High Tariff Shock in each period</td>
<td>0.03</td>
</tr>
<tr>
<td>λ_{r,r} = R1, R2</td>
<td>Preference parameter for domestic goods</td>
<td>0.046</td>
</tr>
<tr>
<td>λ_{r,s}, r \neq s</td>
<td>Preference parameter for imported goods</td>
<td>0.032</td>
</tr>
<tr>
<td>f^{\text{sunk}}_{r} = R1, R2</td>
<td>fixed cost to enter into production (in units of Labor)</td>
<td>2.183</td>
</tr>
<tr>
<td>f^{\text{exp}}_{r,s}, r \neq s</td>
<td>fixed cost to enter into exporting (in units of Labor)</td>
<td>2.059</td>
</tr>
<tr>
<td>L_{r}, r = R1, R2</td>
<td>Labor Supply</td>
<td>143</td>
</tr>
</tbody>
</table>

### Table 1: Assumed Parameter Values

#### 4.2 Parametrization

For all results presented in this section, I assume the baseline parameter values reported in Table 1, except where otherwise noted. The parameter values in rows 7-11 are based on simulated data. The values reported here are those calibrated using the two-country, one differentiated good industry deterministic model with zero tariffs and assumed values in the differentiated sector of 80 for domestic sales, 20 for imports and 100 domestic firms in the differentiated goods sector (prior to entry) with 10 of these exporting. The value chosen for $1 - \mu0$, the share of total expenditures which go towards the differentiated goods sector, is admittedly somewhat arbitrary, though may be rationalized by assuming that the economy is comprised of agriculture, which is characterized by constant returns to scale, and manufacturing, as in Krugman (1991). Using this framework, I choose a value of 70% for expenditures on the non-agricultural sector, as this is close to the share of spending in the United States. Section 7 provides a detailed analysis of the sensitivity of the main results presented below to the assumed parameter values for $\delta, a, \sigma, \mu0$, and $\gamma$, showing that in general, these results are not sensitive to the assumed parameter value of $\mu0$, somewhat sensitive to the assumed values of $a$ and $\sigma$, and quite sensitive to the assumed values for $\delta$ and $\gamma$.

### 5 Results

In this section, I present the equilibrium values for endogenous variables of interest for various scenarios where tariffs are at level $L$ but there exists the threat of a tariff hike to level $H$, and compare these values to the equilibrium values which would be obtained in a deterministic version of the model with tariffs at level $L$ in all periods. Thus, I am able to isolate the effect of the mere threat of a tariff hike by comparing equilibrium values from two scenarios, both of which have the same current applied tariffs, but where in one...
case, applied tariffs in future periods may rise. In cases where understanding the dynamics of endogenous variables following the realization of a tariff threat are useful in understanding the results of these static comparisons, I also present these dynamics.

5.1 Symmetric Two-Country, One Differentiated Industry Model

Here, I present the equilibrium of the model where it is assumed that the two countries are perfectly symmetric (including labor endowments, preference parameters $\lambda$, and high and low tariff levels). I assume that the low tariff in each region is 0%, while the high tariff (which may arrive in any subsequent period with probability $\gamma$) is assumed to be $(\tau^H - 1) \times 100\%$, that is, in any period where the high tariff shock has not yet arrived, with probability $\gamma$ it will arrive and set the tariffs in both countries to 19% in the case where $\tau^H = 1.19$. This particular level is chosen as the high tariff level in this example because it corresponds to the optimal tariff which will be set by each country in a non-cooperative equilibrium. As such, in interpreting the results below in Table 2, one can view the difference between the equilibrium where tariffs are locked in at the low tariff level and that where there exists a threat of each country imposing the optimal tariff in a non-cooperative equilibrium (that is, between columns (1) and (2) or (1) and (3)) as representing the value of a trade agreement in a scenario where without the agreement, there exists a positive probability that the countries will enter into a tariff war, ending with each country imposing the optimal tariff in the non-cooperative equilibrium.

The final two columns of Table 2 present the analogous results for a higher tariff threat level, in order to provide an idea of how a change in the magnitude of the tariff threat level will affect the equilibrium.

The first column of Table 2 presents the equilibrium values of endogenous variables in a deterministic version of the model where tariffs are at their initial $(L)$ level in every period. Columns two and three present these equilibrium values in the low tariff state where there is now a positive probability in each period of a high tariff shock. We can see by comparing columns one and two that the threat of a tariff hike actually causes the equilibrium mass of producing firms to increase relative to the deterministic setting. This increase in the mass of producing firms induced by uncertainty over future tariffs is however more than offset by an increase in cutoff productivity to export, yielding a smaller mass of exporters in this symmetric model with a positive threat of tariff hikes in the future. The mass of exporters in each region under the presence of a tariff threat is 13.49% lower than in the case without a tariff hike threat. Further, total utility $(U)$ in each

---

8In a perfectly symmetric deterministic set-up where firms are modeled as heterogeneous in productivity and face fixed costs to sell in any given market, it turns out there is an analytic expression which implicitly defines the optimal tariff of each country in the non-cooperative equilibrium [see Felbermayr et al. (2013)]. As the ratio of fixed export to fixed domestic costs approaches infinity, this expression implies an explicit expression which this optimal tariff approaches. Because I have no period fixed costs to sell in the domestic market in my set-up, I am able to use this to obtain an explicit expression for the optimal tariff in the symmetric country set-up which depends only on the elasticity of substitution and productivity dispersion parameters. For my assumed parameters, this yields $\tau^{optimal} = 1 + \frac{\sigma}{\sigma - (\sigma - 1)} = 1.190736$. 

Table 2: Symmetric Two Country Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equilibrium in Deterministic Model with $\tau_{r,s} = 1$</th>
<th>Equilibrium in state $\tau = L = 1$</th>
<th>Equilibrium in state $\tau = L = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>100.000</td>
<td>100.104</td>
<td>100.285</td>
</tr>
<tr>
<td>$M$</td>
<td>10,000</td>
<td>8.651</td>
<td>7.048</td>
</tr>
<tr>
<td>$N$</td>
<td>100,000</td>
<td>100.645</td>
<td>101.139</td>
</tr>
<tr>
<td>$NE$</td>
<td>11.111</td>
<td>11.183</td>
<td>11.238</td>
</tr>
<tr>
<td>$NX$</td>
<td>1.000</td>
<td>0.865</td>
<td>0.705</td>
</tr>
<tr>
<td>$P$</td>
<td>1.000</td>
<td>1.002</td>
<td>1.006</td>
</tr>
<tr>
<td>$Q$</td>
<td>100,000</td>
<td>99.921</td>
<td>99.735</td>
</tr>
<tr>
<td>$U$</td>
<td>77.554</td>
<td>77.536</td>
<td>77.477</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.825</td>
<td>0.852</td>
<td>0.892</td>
</tr>
</tbody>
</table>

region is lower by 0.02% when there exists a positive probability of tariff hikes relative to the deterministic model. The value of exports from each region under the tariff threat is 4.55% lower than without the threat.\(^9\)

Comparing columns two and three (or four and five), we see that the direction change of endogenous variables from a scenario with no threat to one with a positive threat is the same as from the scenario with a positive threat, to one with an even higher probability of tariff hikes.

The final two columns present more comparative statics, now for a higher magnitude tariff shock, $\tau^H = 2$, that is, an ad-valorem tariff of 100% which may arrive in any subsequent period with probability $\gamma$. As expected, the threat of a higher tariff decreases the mass of exporters and increases the export productivity cutoff, as seen by comparing columns two and four (or three and five). I note that the cumulative effect on number of exporters of both raising the tariff hike probability from 3% to 10% and raising the tariff shock magnitude from 19% to 100% is greater than the sum of each individual effect.

The positive correlation between probability (or magnitude) of a tariff hike and number of firms $N$ (or entry, $NE$) can be understood intuitively by noting that in a symmetric set-up, higher tariff levels in both regions imply less domestic competition from imports and thus higher profitability from domestic sales; this implies a direct channel through which firm entry will increase. As will be seen later in an asymmetric set-up (See section 5.2), in a model with asymmetric high tariff levels, the equilibrium mass of producing firms in the country threatening a tariff hike is higher than that of the other country, as these firms face less import competition in the event that the tariff threat is realized. The fact that $N$ increases in the presence of a symmetric tariff threat is also partially driven by the increase in aggregate expenditures, which themselves

\(^9\)Note that while this effect may seem somewhat large for only a 3% chance of tariff hike to 19%, a back of the envelope calculation shows that the expected reduction in log trade value in a deterministic model where the tariff is raised by 0.57% (the expected tariff in the next period, =0.03*0.19) is $\sigma_{\text{expected}}^2 = 0.0057 = 0.03558$, that is, a 3.6% reduction in trade, which accounts for most of this 4.55% reduction. That is, this reduction in trade results not from simply reducing uncertainty over future applied tariff rates, but largely from the change in expected tariff value, which in the long run, is given by the tariff threat level of 19%.
Table 3: Applied Tariff Equivalents of Tariff Threat

<table>
<thead>
<tr>
<th>Equivalence Variable</th>
<th>Tariff threat: 19%</th>
<th>Tariff threat: 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 0.03$</td>
<td>$\gamma = 0.1$</td>
</tr>
<tr>
<td>$U$: Utility</td>
<td>1.30% (2)</td>
<td>3.96% (2)</td>
</tr>
<tr>
<td>$M$: Mass of Exporters</td>
<td>3.09% (5)</td>
<td>7.42% (4)</td>
</tr>
<tr>
<td>$\varphi^c$: Export productivity cutoff</td>
<td>3.03% (5)</td>
<td>7.22% (4)</td>
</tr>
<tr>
<td>Decline in Differentiated Good Trade (share of Level in case with no threat)</td>
<td>4.55%</td>
<td>10.57%</td>
</tr>
</tbody>
</table>

*Value in parentheses indicates number of periods in the future before expected applied tariff will be greater than this applied tariff equivalent.

are a result of higher GDP. GDP increases in the presence of a tariff threat though higher domestic profits and decreased fixed cost payments of new exporters which together outweigh the decrease in export profits and increase in aggregate entry costs.\textsuperscript{10}

As another way to measure the importance of the tariff threat on the economy in equilibrium, I compute the ad-valorem tariff(s) that would yield the same equilibrium values in column 3 for either Utility ($U$), Mass of Exporters ($M$), or Export Productivity Cutoff ($\varphi^c$) in a deterministic model. In this symmetric model, the applied tariff in each country which would yield the same outcome utility in each country as the threat of a 19% tariff (with 0.03 probability) in a future period is 1.3%. The applied tariff which would yield the same mass of exporters is 3.09% and that which would yield the same cutoff productivity in each country is 3.03%. The value of bilateral trade in the differentiated good sector predicted by the model decreases from 20 units to 19.1 units in the case where there exists a 3% threat of a 19% tariff hike. Thus, the model indicates that the effect of the threat of a tariff hike to 19% (with a 3% chance of the shock arriving each period) in both countries is roughly equivalent to an applied tariff of between 1.3 and 3.1 percent in each country, and causes bilateral trade to be 4.55% lower than would be expected without the presence of the tariff threat. Table 3 presents the applied tariff equivalents as well as the share of lost trade computed for the different tariff threat scenarios discussed above in this symmetric model.

5.2 Asymmetric Case: Single Policy Active Country

Here, I present the equilibrium values of endogenous variables for the general equilibrium model with symmetric country size and parameters, but now assume that the threat of a tariff hike exists in only one of the two regions. Specifically, I assume that in the initial state tariffs in both regions are 0, and that in each subsequent period there is a probability $\gamma = 0.03$ that the tariff in region 1 will jump to 25.3%, remaining there forever after, while the region 2 tariff remains 0 in all states. This tariff threat level for region 1 is

\textsuperscript{10}This channel is not obvious in the one-sector example shown here, however, later in Section 5.3.2 when an outside sector is present for which domestic and export profitability do not change with the presence of a tariff threat on the other sector, we still see an increase in the equilibrium number of firms in both sectors.
Table 4: Asymmetric Case: One Policy Active Country

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equilibrium in Deterministic Model $\tau_{R1,R2} = \tau_{R2,R1} = 1$</th>
<th>Equilibrium in state $\tau_{R1,R2}^H = \tau_{R2,R1}^H = 1$ Under R1 Threat ($\tau_{R2,R1}^H = 1.253$, $\gamma = 0.03$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Region</td>
<td>Region</td>
</tr>
<tr>
<td>$E_r$</td>
<td>$r = R1, s = R2$</td>
<td>$r = R1, s = R1$</td>
</tr>
<tr>
<td></td>
<td>100.000</td>
<td>99.970</td>
</tr>
<tr>
<td></td>
<td>100.000</td>
<td>100.139</td>
</tr>
<tr>
<td>$GDP_r$</td>
<td>142.857</td>
<td>142.814</td>
</tr>
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chosen because this is the tariff that maximizes R1 utility in the case where R2 is constrained to leave its tariff at zero.\(^{11}\) In terms of real world examples, one can think of a case where perhaps R2 is committed to leave its tariff rates at zero in the event of a tariff hike by R1 because of some international agreement, such as MFN rates in the WTO, while R1 may have the ability to raise its rates because its initially low tariff rates were only granted as part of a unilateral preference scheme, such as the Generalized System of Preferences (GSP) schemes offered by several developed nations towards developing nations.\(^{12}\)

Here we see that the effect of tariff uncertainty inducing entry into the domestic market appears to have been a consequence of the symmetric set-up previously assumed. When I allow a positive threat of a tariff hike in only one region (R1, in this case), this induces higher entry into the domestic market in R1, as one might expect given that a higher tariff in R1 reduces import competition for domestic firms. In R2, where exporters face the possibility of a high tariff level in R1 in the future, uncertainty leads to a lower equilibrium mass of firms. Similarly, uncertainty over the tariffs that will be applied on region 1 imports leads to a lower equilibrium mass of exporters and higher productivity cutoff in R2, and the opposite in R1. This is due to the expectations that the region 1 import tariff may rise, leading to improved export conditions for region 1 exporters driven by the increase in the price index of the differentiated goods sector in region 2, which itself will be driven by the decrease in the number of firms producing in region 2.

For region 1, the presence of the tariff threat implies an increase in utility by 0.09%, an increase in the mass of exporters of 5.34% and an increase in the value of differentiated goods exports by 5.35%. For region 2, this tariff threat induces a 0.13% decrease in utility level, a 22.2% decrease in the mass of exporters, and a 11.1% decrease in the value of differentiated goods exports.

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\(^{11}\)This optimal tariff is computed numerically.

\(^{12}\)Of course, to properly model such a scenario would require including more than two countries, and likely more than one industry. Such an example is presented in Chapter ?? of this dissertation for the case of Chile-USA trade in the context of a four region, multi-sector model.
As in the previous section, I compute the applied tariff level that would need to be imposed by region R1 on imports in a deterministic model in order to yield the same equilibrium value of utility (U), mass of exporters (M), and export productivity cutoff (ϕc) for region 2. These applied tariff equivalents in this case are 0.52% for equivalent utility effect, 3.01% for equivalent effect on mass of exporters 3.49% for equivalent export productivity cutoff. In terms of trade flows, the level of differentiated good exports from R2 to R1 will be 11.1% lower with the threat of the tariff hike than without, while differentiated good exports from R1 to R2 will be 5.3% higher with the threat.

An Optimal γ

In this example, it is clearly to region R1’s benefit to maintain a threat of raising tariffs. Of course, if R1 has control over this parameter γ (by sending signals about the likelihood of a tariff hike) it is obvious that if the goal is to maximize welfare in the current “L” state, setting γ = 1 will produce the highest utility for region 1. If, however, the policy maker in region 1 is able to credibly commit to some other γ and cares not just about current period and long term utility levels but rather the present discounted value (PDV) of expected utility for region 1, it turns out that γ = 1 is not optimal. Under the assumption that a high shock will arrive with probability γ in any subsequent period, the present discounted value of expected utility for a region is given by:

\[
PDEV(r) = \frac{δ}{(1 - (1 - δ)(1 - γ))} \left[ U_r(L) + γ \sum_{n=0}^{∞} (1 - δ)^{n+1} U_r(H^{(n)}) \right]
\]

In order to understand how the utility levels \( U_r(H^{(n)}) \) evolve in each region following the arrival of a high tariff shock, I present here figure 1. We see that although the imposition of a high tariff by R1 will leave it with higher utility in the long run, transitioning from the low tariff state requires going through several periods of low utility levels. The main mechanism driving this lower utility in the short run is due to the presence of legacy firms exporting from R2 to R1, whose mass gradually decreases over time as they are randomly hit by the exogenous death shock. The presence of these legacy firms raises the price index in R1 because consumers in this region must now pay the price of the differentiated goods as well as the tariff markup resulting from the R1 imposed tariff. Figure 2 presents the dynamics of these two endogenous variables following a high tariff shock.

We now see from these figures and equation (20) that there exists a tradeoff for region 1 with respect to γ: a higher level of γ raises current period utility \( U(L) \) and utility levels further than about 6 periods into the future, however it also increases the weight given to utility in periods just after the imposition of the high tariff, which are lower. I numerically compute the optimal level of γ from the perspective of R1 where this is the level that maximizes PDV of R1 utility, under the assumption that the high tariff shock
Figure 1: Utility Dynamics

Dynamics for U Following High Tariff Shock

Figure 2: Price and Legacy Firm Dynamics

Dynamics for P Following High Tariff Shock

Dynamics for Leg Following High Tariff Shock
does indeed have a probability $\gamma$ of arriving in any future period. This yields a value of $\gamma = .69$. In terms of real world applications, however, it is difficult to imagine a situation where a region can commit to its probability of invoking a tariff in the future. More reasonable, perhaps, is a situation where a policy maker may intend to leave tariffs at their low level (L), and therefore optimizes only utility in this low tariff state, $U(L)$, by sending signals about raising tariffs in the future. In the example presented in this section, it is to this policy maker's advantage to generate signals as close to $\gamma = 1$ as possible, though in practice, he may experience a lack of credibility if the tariff is never raised.

5.3 Broader Tariff Threat Effects

In this section, I present the results of versions of the model that include countries or sectors not directly targeted by the tariff threat in order to assess the general equilibrium effects that such a threat has on outside countries and sectors. This is an important topic, as the effects of Preferential Trade Agreements on non-member countries as well as the effects of such agreements on incentives for multilateral trade liberalization have become a topic of debate over the past two decades (see, for example, Baldwin (1997) and Bhagwati (1998)). By including the uncertainty reducing effects of trade agreements in a general equilibrium model with multiple regions and sectors, I allow for another mechanism, apart from the direct effect of applied tariffs, through which the trade, welfare, and incentive to enter into multilateral agreements of non-member countries to a given agreement may be affected by the presence of the agreement.

5.3.1 Third Country Effects

Here I present the equilibrium values of endogenous variables for a model similar to that presented in section 5.1 where two symmetric countries threaten to raise tariffs against each other, but I now add in a third symmetric country (R3) whose tariffs remain at level L in all periods, as do the tariffs faced by its exporters to regions R1 and R2. That is, initially all tariffs are zero (state L) and when the high tariff shock arrives (which happens with probability $\gamma$ in each subsequent period), it causes only the tariffs governing region 2 exports to region 1 (R2-R1) and region 1 exports to region 2 (R1-R2) to rise to 17.5%, the non-cooperative equilibrium tariff level for R1 and R2 when all R3 imports are constrained to have zero tariffs. One may think of this as a situation where initially two of the three pairs of countries in this 3-country model, (R1,R3) and (R2,R3) have an explicit agreement (such as an FTA) constraining tariffs to remain at 0% while the zero tariffs imposed by regions R1 and R2 are not locked in by any explicit contract. In a partial equilibrium model where the effects of a tariff shock do not affect aggregate expenditure and price indexes, we would see no effect of this tariff uncertainty on the equilibrium for the third country, R3. In this general equilibrium
framework, however, region 3 will be affected by the imposition of a tariff on R1-R2 bilateral trade flows, and so its state $L$ equilibrium is also affected by the mere threat of such a tariff hike. I now focus on the effect of this uncertainty over future tariffs between R1 and R2 not on the outcomes for these countries, as these will be similar to those presented in section 5.1, but for the outside region, R3.

Table 5 presents the equilibrium values of endogenous variables for region R3 in both the deterministic model with tariffs at zero in all states and the $L$ state of the model with a positive threat of a tariff hike on R1-R2 and R2-R1 trade flows in some future period. We see that although the threatened tariff hike will have no direct impact on region 3, it does affect this region in general equilibrium. Specifically, compared with the case of no tariff threat, we see a lower productivity cutoff and higher mass of exporters to regions R1 and R2 (where, in the event of the imposition high tariffs, the price index of the differentiated goods sector will rise, making it more attractive as an export market). Overall, R3 benefits from this tariff threat that directly affects only the other two countries, and realizes a utility level slightly higher than it would in the case where zero tariffs are locked in between all pairs.

### 5.3.2 Cross-Industry Effects

Until now, I have only presented the results of models where there is one differentiated goods sector. I now expand this to include two such sectors, labeled $K1$ and $K2$, in a two-country model, and present the equilibrium values of endogenous variables for this model when the event of a high tariff shock results in the imposition of a 19.25% tariff on all trade flows for sector $K1$ goods, where this tariff level is chosen because it is the tariff that will be optimally chosen by both regions in sector K1 in the non-cooperative equilibrium.
when sector K2 tariffs are constrained to remain at zero. In the results presented here, I explore whether the addition of another sector (of the same size as K1) which is not subject to a tariff hike qualitatively changes the effect of this tariff threat on equilibrium values in sector K1, as well as how the indirect general equilibrium effects of a tariff threat in sector K1 affect sector K2. As with the previous section, where there was an additional region not affected directly by the tariff shock, here, in the absence of general equilibrium effects, there would be no effect of this tariff threat on sector K2. We will see that this is not the case in general equilibrium, and will explore the channels through which this outside sector is affected.

I do not present here the equilibrium values of endogenous variables for sector K1, as they are qualitatively the same as those found in the 2-country 1-sector model presented in section 5.1, namely that higher export productivity cutoffs in both regions for sector K1 and a smaller mass of exporters in this sector.

Table 6 shows these equilibrium values for sector K2. In this sector, not directly affected by the potential tariff hike, we see that export productivity cutoffs in each country remain unaffected by the threat of a tariff hike in sector 1 (see Appendix F for a discussion of the hypothesis that this will hold under the assumption of Pareto distributed productivity for any sector not directly impacted by tariff changes). Thus, any effect on the number of sector K2 exporters occurs as a result of a change in the number of firms producing domestically in this sector. We see that the equilibrium mass of sector K2 firms producing in each region is higher when there exists the threat of a symmetric tariff hike on sector K1. This can be understood as a sort of spillover effect resulting from the slightly higher GDP, and therefore sector level expenditures resulting from the tariff threat in sector K1. As mentioned in the symmetric two-country one-sector model (see section 5.1), this higher GDP in the presence of a tariff threat is the result of higher aggregate domestic revenue in

<table>
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both sectors which outweighs the decreased export revenues of sector \( K1 \) and the increase in aggregate sunk costs which must be paid in both sectors. I note that although the number of decimals presented in table 6 do not show a difference in the price index for sector \( K2 \), it is the case that this price is slightly lower in the presence of the sector \( K1 \) tariff threat (due to the increased number of domestic firms and imports in \( K2 \)) which balances the slight increase in \( Q \) in this sector such that aggregate economic conditions remain the same in \( K2 \) with or without the presence of a tariff threat in \( K1 \).

6 Discussion

In this section I discuss the added value obtained by using a general equilibrium rather than a partial equilibrium framework to model the effects of tariff threats as well as the reasons for and implications of various assumptions, and the economic forces that act to maintain constant cutoff productivities in all states after the arrival of the tariff shock, even as legacy firms decay away.

6.1 Added Value of General Equilibrium Analysis

As already noted in the results sections above, one immediate advantage to using a general equilibrium framework is that it allows us to analyze the effects of tariff threats and shocks on countries not directly impacted by the tariffs, but whose trade may be affected by the policies. Similarly, in a multi-sector setting, one can obtain results for the effects of tariff policies on sectors not directly targeted.

Beyond this, however, the general equilibrium model captures additional feedback effects that would be absent in a partial equilibrium analysis. Turning back to the results of the two-country, one-industry model where there exists the threat of a tariff hike to 25.3\% on \( R1 \) imports from \( R2 \) (see Section 5.2), one can ask how the general equilibrium results obtained compare with those from a partial equilibrium framework which does not take into account general equilibrium effects on aggregate region 1 variables. Specifically, I note that the equation governing the productivity cutoff for region 2 exports to region 1 in states prior to the arrival of the high tariff shock is given by\(^{13}\):

\[
(p_{R2,R1}(\zeta))^{-1} = \frac{(\delta + \gamma - \delta \gamma) w_{R2,R1} f_{R2,R1}^{exp} f_{R2,R1}^{exp} A_{R2,R1}(L) + \gamma(1-\delta) A_{R2,R1}(H)}{A_{R2,R1}(L) + \gamma(1-\delta) A_{R2,R1}(H)}
\]

Recall that \( A_{R2,R1} \) summarizes aggregate economic conditions for a region \( R2 \) exporter to region \( R1 \) and is defined as \( A_{R2,R1}(\zeta) = \frac{1}{\delta} \left[ \left( \frac{\sigma}{\gamma-1} \right) w_{R2} \right]^{1-\sigma} \lambda_{R2,R1} Q_{R1}(\zeta) \left( \frac{P_{R1}(\zeta)}{\gamma(1-\delta)(\zeta)} \right)^{\sigma} \), that is, a constant multiple of \( Q_{R1} \left( \frac{P_{R1}}{\gamma(1-\delta)} \right)^{\sigma} \) in a given state. In a partial equilibrium analysis, one may assume that \( Q_{R1} \) and \( P_{R1} \) are

\(^{13}\)I have taken advantage of the fact that \( A_{r,s} \) remains constant in all \( H^{(n)} \) periods to simplify the expression given in (17). See Appendix A for proof of this fact.
unaffected by the arrival of a high tariff shock in region 1, and a tariff threat will still result in a lower export productivity cutoff in R2 as the forward looking exporter takes into account future economic conditions $A_{R2,R1}(H)$ which include the direct effect of the tariff, $\tau_{R2,R1}$. In general equilibrium, however, any additional feedback effects on $Q_{R1}$ and $P_{R1}$ are also taken into account when the exporter makes his decision. Using the equilibrium $Q$ and $P$ values for $R1$ from the initial $L$ state (one could think of these as observed quantity and price indexes in some initial period) we obtain that $Q_{R1}(L)\left(\frac{P_{R1}(L)}{\tau_{R2,R1}(H)}\right)^{\sigma} = 42.2$ while using the general equilibrium $Q$ and $P$ values, we obtain $Q_{R1}(H_{det})\left(\frac{P_{R1}(H_{det})}{\tau_{R2,R1}(H)}\right)^{\sigma} = 41.1$. That is, if we ignore the additional general equilibrium effects that cause $Q_{R1}$ to rise and $P_{R1}$ to fall once region 1 imposes a tariff on imports, we obtain a higher value for $A_{R2,R1}(H)$ and thus a lower cutoff productivity $\varphi_{R2,R1}(L)$ for region 2 exporters than if we take these feedback effects into account. In other words, in this case, using a partial equilibrium model to evaluate the effects of a tariff threat would underestimate the impact on the export productivity cutoff in the region directly impacted by the tariff. What are the mechanisms driving these feedback effects?

After the high tariff shock arrives, in region 1 we have that $Q$ rises while $P$ falls (in the long term; in the short term the opposite is true, however $QP^\sigma$ remains constant once the shock arrives). The lower price index results from more domestic firms entering over time as import competition is decreased and (for sufficiently large $\sigma$) this effect is enough to outweigh the rise in demand, $Q$ that results, yielding a lower value of $QP^\sigma$ in state $H$ relative to state $L$.\textsuperscript{14}

In a similar vein, I note that without allowing the number of producing firms to be determined endogenously via the free-entry condition (as might be the case in certain partial equilibrium analyses), the post-shock mass of exporters from region 2 predicted by the model would be higher than is the case here, since in general equilibrium, fewer firms produce domestically and so for a given fraction of producers who export, this mass will also be reduced. That is, in a partial equilibrium analysis that takes the number of domestic firms as given, one would underestimate the effect of an applied tariff or a tariff threat on the mass of exporting firms.

### 6.2 No Per-Period Fixed Costs

The assumption of no per-period fixed costs to sell either domestically or in a given export market, though not very realistic, is made in order to maintain tractability of the model in the presence of uncertain future tariff levels. Positive per-period fixed costs to sell domestically and/or in a given export market would generate an endogenous productivity cutoff below which firms would exit the given market, and forward looking potential firms would take this into account when making their entry decision. This would greatly complicate the

\textsuperscript{14}In the short term, the mechanisms driving the higher $P$ are that import prices are now higher, as they take into account the tariff, and many $R2$ legacy firms remain, so that there are still a substantial mass of firms exporting from region 2 and driving up the price index, which also decreases demand, $Q$, in region 1.
model, as in this case, expressions (10) and (11) would need to take into account the probability of falling below an endogenous exit cutoff when computing the present discounted value of selling in a given market. It is not clear that this added feature would allow additional insight about the mechanisms causing reduced entry in the presence of a threatened tariff hike with sunk entry costs, which is my main focus. For this reason, I do not include fixed per-period costs, but may wish to explore this in the future.

6.3 One-Period Delay to Begin Exporting

The primary rationale for not allowing firms to begin exporting until they have produced domestically for at least one period is so that firms can learn their productivity (which, in practice, would require one to actually produce) before deciding whether or not to enter a given export market. If the timing of the model were changed so that firms could export immediately (but still know their productivity level when making the export decision), this would not affect the results in any significant way; it would simply change the mass of exporters to be the share of the $N + NE$ firms above the export cutoff (rather than the share of $N$), and would cause the consideration of potential export profits to appear in the entry decision equation without the time discount $(1 - \delta)$ coefficient. Under this alternate timing scenario, the proof that

$$\varphi_{r,s}^{c}(H^{(n+1)}) = \varphi_{r,s}^{c}(H^{(n)}) = \varphi_{r,s}^{c}(H^{det}) \forall n = 0, 1, 2, \ldots, \forall r, s \text{ will be slightly more complicated, as in this case the period } H^{(n)} \text{ free entry equation involving } \varphi_{r,s}^{c}(H^{(n)}) \text{ will involve not only variables in future states but also state } H^{(n)} \text{, however, the result will continue to hold.}

6.4 Economic Intuition of Constant Productivity Cutoffs in states $H^{(n)}$

As shown in Appendix A, under the assumption that the economy does eventually reach its steady state equilibrium, we have that the productivity cutoffs to enter a given export market remain constant once the high tariff shock arrives, despite the changes in other endogenous variables. What are the economic forces at work to maintain constant export cutoffs? First, the cutoffs are constant multiples of the aggregate economic conditions, $A_{k,r,s}(H^{(n)})$, which are themselves constant multiples of the term $Q_{k,s}(H^{(n)}) \left( \frac{P_{k,s}(H^{(n)})}{\tau_{k,r,s}(H^{(n)})} \right)^{\sigma}$. In this model, the tariff process is such that $\tau_{k,r,s}(H^{(n)})$ does not change with $n$ (once the high tariff shock arrives, it remains at that level forever), and so the question is how $Q_{k,s}(H^{(n)})P_{k,s}(H^{(n)})^{\sigma}$ remains constant.

Looking at the two-country, one-industry results presented in section 5.2 as an example, we see that after the arrival of the high tariff shock imposed on region 1 imports, the price index in region 2 gradually rises as entry into production in that region becomes less valuable with the imposition of the tariff, and so the number of firms decreases as firms die and are not completely replaced by new entrants (see figure 3). This fall in the price index increases demand $Q$ so that the aggregate quantity $QP^{\sigma}$ remains constant. That this
Figure 3: Dynamics for Selected Endogenous Variables

Dynamics for Q Following High Tariff Shock

Dynamics for P Following High Tariff Shock

Dynamics for Leg Following High Tariff Shock
remains constant across periods where the tariff is not changing is a direct result of the free-entry condition and the assumption of no fixed costs to produce domestically. As conditions change over time following the arrival of the high tariff shock, the number of new entrants into production adjusts until the value of entry into production (minus the value of waiting to enter) is exactly equal to the sunk cost of entry; because exit is entirely exogenous, this value is directly linked to the aggregate economic conditions in each country, and so the number of new entrants adjusts until $QP^e$ is at its constant (over $n$ for all states $H^{(n)}$) level.

7 Robustness to Alternative Parametrization

In order to assess the sensitivity of the results presented above to the parameter values assumed, I allow several parameters to vary within a certain range, and see how this changes the magnitude of the impact of a tariff hike threat on utility, mass of exporters, and total trade value. Recall that for my assumed baseline parameters of $a = 4.6, \sigma = 3.8, \mu_0 = 0.3, \delta = 0.1, \gamma = 0.03$, the model indicated that for the 2-country, 1-industry model presented in section 5.1, a reversion to the non-cooperative equilibrium that was feared to occur with probability $\gamma$ in any future period had the effect of reducing each country’s utility by 0.02%, the mass of exporters by 13.49%, and the value of exports by 4.55%. Below, I allow the parameters to vary between the following values: $3.6 \leq a \leq 5.6, 2.8 \leq \sigma \leq 4.8, 0.1 \leq \mu_0 \leq 0.4, 0.05 \leq \delta \leq 0.5, 0.01 \leq \gamma \leq 0.2$ and look at the range of percent changes in utility, mass of exporters, and total trade value implied by this range of parameters for this symmetric 2-country, 1-industry model. In the graphs presented in this section, I show how variables of interest change along with a given parameter for every other possible combination of parameter values in this set, highlighting the case where all other parameters are at their baseline value. In general, I find that the results presented above are not sensitive to the assumed parameter value of $\mu_0$, somewhat sensitive to the assumed values of $a$ and $\sigma$, and quite sensitive to the assumed values for $\delta$ and $\gamma$.

7.1 Productivity Dispersion Parameter

The Pareto parameter $a$ governs the shape of the Pareto distribution from which firm level productivity is drawn. As $a$ increases, the dispersion of firm productivity decreases such that as $a \to \infty$, we reach the degenerate distribution where all firms share the same productivity level, while as $a$ decreases, the distribution becomes closer to a uniform distribution. Figure 4 shows the relative change in utility, exporter mass, and trade value obtained in the symmetric model by going from a model with no tariff threat to one with a tariff hike threat, for various values of $a$, holding all other parameters constant. The black line shows these values for the set of parameter values where all other parameters are at their baseline value (the

\footnote{Similar graphs for other scenarios with results presented above are available upon request.}
values assumed for results presented above) while the red marker on this line represents the point where all parameters (including $a$) are at these baseline values. We see that the higher the value of $a$, that is, the more compressed the productivity distribution such that there is less dispersion in productivity, the larger the effect of a tariff threat on all three measures. For the baseline values assumed for all other parameters, allowing the Pareto shape parameter to vary between 3.6 and 5.6, we obtain relative losses in utility of between 0.02%-0.03%, in exporter mass of 11.1%-15.8%, and in trade value of 2%-7.1%.

### 7.2 Elasticity of Substitution

The parameter $\sigma$, which governs substitutability between varieties of goods in consumers’ utility function, is varied between 2.8 and 4.8. As this assumed parameter value increases, that is, goods become more substitutable, we see that there are smaller effects of the tariff threat on utility, exporter mass and total export value. Allowing all other parameters to remain fixed at their baseline values, we see that as $\sigma$ varies between 2.8 and 4.8, the decrease in utility goes from 0.05% to 0.01%, the decrease in exporter mass from 15.3% to 12.6%, and the decrease in export value from 8.5% to 1.8%.
7.3 Share of Expenditures towards Homogenous Good Sector

Here, I allow the parameter governing the share of expenditures going towards the homogenous good, $\mu_0$, to vary between 0.1 and 0.4. In general, this parameter has very little effect on the overall effect of a tariff threat for utility, and especially for exporter mass and trade value. For baseline values of other parameters, as $\mu_0$ increases from 0.1 to 0.4, the utility, exporter mass, and trade value decrease resulting from a tariff hike threat increases only very slightly in magnitude, with all exhibiting the same percent change (to one decimal place).

7.4 Discount Factor

The discount factor $1 - \delta$ governs the rate at which agents in the model discount future periods. For high values of $\delta$, forward looking agents (such as profit maximizing firms) place less weight on expected values in future periods, and so it makes sense that as $\delta$ increases, we see the magnitude of the effect of a tariff threat on utility, exporter mass, and export value decrease. That is, in a limiting case where $\delta \to 1$, firms making export decisions based on beliefs about the future will act just like firms in a deterministic model with fixed tariffs because they are concerned only with their profits in the current period. As I allow $\delta$ to vary between 0.05 and 0.5 (holding all other parameters constant at baseline values), I obtain a negative effect of tariff threat presence in the model of between 0.054% and 0.007% for utility, between 22.9% and 1.68% for exporter mass, and between 8.2% and 0.4% for trade value.

7.5 Probability of Realization of Tariff Threat

Finally, I experiment with various assumed values for $\gamma$, the parameter governing the probability with which agents believe a the tariff threat may be realized in any subsequent period. As would be expected, larger values of $\gamma$, meaning a more likely high tariff imposition, lead to a larger magnitude effect of a tariff hike
threat on utility, exporter mass, and export value. Holding other parameters constant at assumed baseline values, varying $\gamma$ between 0.01 and 0.2 yields an effect of the tariff threat that reduces utility by between 0.006% and 0.193%, reduces exporter mass by between 5.3% and 39.7% and reduces export value by between 1.7% and 14.7%.

8 Conclusion

I have presented a simple multi-country, multi-industry general equilibrium model of trade with heterogeneous firms in which exporters face uncertainty over future tariff levels and must incur a sunk cost to begin exporting. In this set-up, assuming a simple Markov process for tariffs, I have shown that in a steady state equilibrium where tariffs have been at their current level for all time, the equilibrium number of firms who choose to export is lower when there exists a threat of a tariff hike on the tariff that they will face in future periods relative to the case where tariffs are deterministic and remain at current levels in all periods. For the parameter values assumed, the model finds that for a global economy with one differentiated goods industry and two identical countries, compared with the equilibrium where tariffs are set at zero and expected
to remain there forever, in an equilibrium where there exists a 3% chance that in any future period the
countries will revert to the non-cooperative equilibrium (of each imposing a 19% tariff on the differentiated
good sector), utility levels for each region will be lowered by 0.02%, the mass of exporters will be lowered by
13.49%, and the value of exports will be lowered by 4.55%. This corresponds to an applied tariff equivalent to
generate the same utility loss of 1.3%, an applied tariff equivalent of 3.09% with respect to mass of exporters,
and of 3.03% with respect to export productivity cutoff level.

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A Proof: Constant Export Productivity Cutoffs over $H^{(n)}$ states

In this appendix, I drop sector subscript $k$ for convenience, with the understanding that the proof is valid for each sector $k$. I assume that for each $r \neq s$ either

$$
\varphi_{r,s}^c(H^{(n+1)}) \leq \varphi_{r,s}^c(H^{(n)}) \quad \forall n = 0, 1, 2, \ldots \tag{21}
$$

or

$$
\varphi_{r,s}^c(H^{(n+1)}) \geq \varphi_{r,s}^c(H^{(n)}) \quad \forall n = 0, 1, 2, \ldots \tag{22}
$$

and further assume that productivity follows some continuous distribution, that no shock is large enough to cause the constraint $NE_r(\zeta) \geq 0$ to bind, and that the economy eventually reaches its long term steady state equilibrium, $H^{det}$. I show that under these assumptions, it must be the case that in equilibrium, $\varphi_{r,s}^c(H^{(n+1)}) = \varphi_{r,s}^c(H^{(n)}) = \varphi_{r,s}^c(H^{det}) \forall n = 0, 1, 2, \ldots, \forall r, s$ (see section 3.4 for notation definitions). This proof relies on the equilibrium equations determining cutoff productivities $\varphi_{r,s}^c(H^{(n)})$ as well as the free entry conditions that determine the mass of new entrants into production, $NE_r(H^{(n)})$, in each high-tariff period, and so the two subsections below derive these equations under each possible ordering of the high tariff cutoffs given in (22) and (21) and the final subsection uses these equations to prove the result.

Export Productivity Cutoffs in High Tariff States

For a firm of productivity level $\varphi$ in region $r$, I let $V_{\varphi,r,s}(\zeta)$ be the present discounted value of selling in market $s$ in state $\zeta$, taking into account current and expected future profit flows from exporting from $r$ to
s (or selling domestically for \( r = s \)). In a given state \( \zeta \), region \( r \) firms who have entered into production in some previous period will choose to incur the sunk cost to export to region \( s \), \( f_{r,s}^{exp} \), and begin exporting to \( s \) if the value of exporting exceeds the value of waiting to begin exporting to \( s \) by at least the amount of this sunk cost. That is, in state \( \zeta \) a country \( r \) firm with productivity \( \varphi \) will enter export market \( s \) if
\[
V_{\varphi,r,s}(\zeta) - V_{\varphi,r,s}^{wait}(\zeta) \geq w_{r} f_{r,s}^{exp}
\]

The value of being a region \( r \) producer selling in region \( s \) in each state for a productivity \( \varphi \) firm in state \( H^{(n)} \) is then:
\[
V_{\varphi,r,s}(H^{(n)}) = \varphi^{\sigma-1} A_{r,s}(H^{(n)}) + (1 - \delta) V_{\varphi,r,s}(H^{(n+1)})
\]
\[
\Rightarrow V_{\varphi,r,s}(H^{(n)}) = \varphi^{\sigma-1} \sum_{i=n}^{\infty} (1 - \delta)^{i-n} A_{r,s}(H^{(i)})
\]
expression (23) gives the present discounted value of exporting from \( r \) to \( s \) for a productivity \( \varphi \) firm regardless of the ordering of the cutoffs over time, as once a firm pays the fixed cost and enters a given export market he will continue to sell in that market until hit by the exogenous death shock. The value of waiting to begin exporting in state \( H^{(n)} \), \( V_{\varphi,r,s}^{wait}(H^{(n)}) \), however can be simplified further and will depend on the ordering of the cutoffs \( \varphi_{r,s}^{c}(H^{(n)}) \) over time:

Cutoffs Ranked According to (21) In the case where \( \varphi_{r,s}^{c}(H^{(n+1)}) \leq \varphi_{r,s}^{c}(H^{(n)}) \), for firms below the cutoff productivity in state \( H^{(n)} \), there may be a positive value of waiting to enter in a later period when the cutoff is lower. For the marginal firm who is just indifferent between entering and waiting in state \( H^{(n)} \) in this case, we can be sure he will choose to enter in state \( H^{(n+1)} \) and so the value of waiting for \( \varphi = \varphi_{r,s}^{c}(H^{(n)}) \) in this case is
\[
V_{\varphi,r,s}^{wait}(H^{(n)}) = (1 - \delta) \left[ V_{\varphi,r,s}^{c}(H^{(n+1)}) - w_{r} f_{r,s}^{exp} \right]
\]
\[
\Rightarrow V_{\varphi,r,s}^{wait}(H^{(n)}) = \varphi_{r,s}^{c}(H^{(n)})^{\sigma-1} \sum_{i=n+1}^{\infty} (1 - \delta)^{i-n} A_{r,s}(H^{(i)}) - (1 - \delta) w_{r} f_{r,s}^{exp}
\]

For a firm with productivity \( \varphi = \varphi_{r,s}^{c}(H^{(n)}) \), we have
\[
\left( V_{\varphi,r,s}(H^{(n)}) - V_{\varphi,r,s}^{wait}(H^{(n)}) \right) = \left( \varphi_{r,s}^{c}(H^{(n)}) \right)^{\sigma-1} A_{r,s}(H^{(n)}) + (1 - \delta) \left( w_{r} f_{r,s}^{exp} \right)
\]
And so for this marginal firm who is just indifferent between selling only in the domestic market and entering
the export market in state \( H^{(n)} \), it must be that

\[
w_r f_{r,s}^{\text{exp}} = (\varphi_{r,s}^c(H^{(n)}))^{\sigma-1} A_{r,s}(H^{(n)}) + (1 - \delta) (w_r f_{r,s}^{\text{exp}})
\]

\[
\Rightarrow (\varphi_{r,s}^c(H^{(n)}))^{\sigma-1} = \frac{\delta w_r f_{r,s}^{\text{exp}}}{A_{r,s}(H^{(n)})}, r \neq s
\]  

(24)

Cutoffs Ranked According to (22) If \( \varphi_{r,s}^c(H^{(n+1)}) \geq \varphi_{r,s}^c(H^{(n)}) \), that is, if the cutoff productivity to export from \( r \) to \( s \) is increasing over time after the arrival of the high tariff shock, it will be the case that for

\( \varphi \leq \varphi_{r,s}^c(H^{(n)}) \), \( V_{r,s}^{\text{wait}}(H^{(n)}) = 0 \) since for firms below the \( H^{(n)} \) state productivity cutoff, export conditions from \( r \) to \( s \) will only continue to worsen over time and they will never enter that export market.

In this case, for a firm with productivity \( \varphi = \varphi_{r,s}^c(H^{(n)}) \), we have

\[
(V_{\varphi,r,s}(H^{(n)}) - V_{\varphi,r,s}^{\text{wait}}(H^{(n)})) = \varphi_{r,s}^c(H^{(n)})^{\sigma-1} \sum_{i=n}^{\infty} (1 - \delta)^{i-n} A_{r,s}(H^{(i)})
\]

And so for this marginal firm who is just indifferent between selling only in the domestic market and entering the export market in state \( H^{(n)} \), it must be that

\[
w_r f_{r,s}^{\text{exp}} = \varphi_{r,s}^c(H^{(n)})^{\sigma-1} \sum_{i=n}^{\infty} (1 - \delta)^{i-n} A_{r,s}(H^{(i)})
\]

\[
\Rightarrow (\varphi_{r,s}^c(H^{(n)}))^{\sigma-1} = \frac{w_r f_{r,s}^{\text{exp}}}{\sum_{i=n}^{\infty} (1 - \delta)^{i-n} A_{r,s}(H^{(i)})}, r \neq s
\]  

(25)

Free Entry Condition in High Tariff States

Under the assumption of free entry, a prospective firm will chose to pay the sunk cost, \( f_{r}^{\text{sunk}} \), and enter into production in state \( \zeta \) if and only if \( V_{r}^{\text{entry}}(\zeta) - w_r f_{r}^{\text{sunk}} \geq V_{r}^{\text{wait to enter}}(\zeta) \) where \( V_{r}^{\text{entry}}(\zeta) \) is the value of entering into production in state \( \zeta \) and \( V_{r}^{\text{wait to enter}}(\zeta) \) is the value of not entering, but retaining the option of entering in some future period.\(^{17}\)

For a given state \( \zeta \), the value of entering into production for a firm who draws productivity \( \varphi \) will be:

\[
V_{\varphi,r}^{\text{entry}}(\zeta) = V_{\varphi,r,r}(\zeta)
\]

\[
+(1 - \delta) E_{\zeta'|\zeta} \left[ \sum_{s \neq r} 1(\varphi \geq \varphi_{r,s}^c(\zeta')) (V_{\varphi,r,s}(\zeta') - w_r f_{r,s}^{\text{exp}}) + 1(\varphi < \varphi_{r,s}^c(\zeta')) V_{\varphi,r,s}^{\text{wait}}(\zeta') \right]
\]

Thus, the expected (over \( \varphi \)) value of entry in state \( H^{(n)} \) is given by:

\(^{16}\) I assume throughout the paper that fixed exporting costs are sufficiently high that this cutoff never falls below the minimum possible productivity draw, \( b \).

\(^{17}\) Note that \( V_{r,s}^{\text{wait to enter}} \) indexed by one country denotes the value of waiting to enter into production while \( V_{r,s}^{\text{wait}} \) indexed by two countries denotes the value of waiting to begin exporting to region \( s \). These are two distinct values.
\[ V_{r, entry}(H^{(n)}) = \int_{b}^{\infty} V_{\varphi, r, r}(H^{(n)})dF(\varphi) + (1 - \delta) \left[ \sum_{s \neq r} \int_{b}^{\varphi_c^r, r}(H^{(n+1)}) \left( V_{\varphi, r, s}(H^{(n+1)}) - w_r f_{r, s}^{\text{exp}} \right) dF(\varphi) \right. \\
+ \left. \int_{b}^{\varphi_c^r, r}(H^{(n+1)}) V_{\varphi, r, s}(H^{(n+1)})dF(\varphi) \right] \]

I assume that the “death shock” applies equally to current and potential firms so that the same discount factor is used by all when discounting future periods. Then, the value of waiting to enter into production in a given state will be given by

\[ V_{r, wait to enter}(H^{(n)}) = 0 + (1 - \delta) \max \left[ \left( V_{r, entry}(H^{(n+1)}) - w_r f_{r, s}^{\text{sunk}} \right), V_{r, wait to enter}((H^{(n+1)}) \right] = (1 - \delta) \left[ V_{r, wait to enter}(H^{(n+1)}) \right] \]

where the final equality follows from the fact that in equilibrium, free entry implies that \( V_{r, entry}(\zeta) - w_r f_{r, s}^{\text{sunk}} \leq V_{r, wait to enter}(\zeta) \forall \zeta \) (with strict inequality only in the case when the constraint \( NE_r(\zeta) \geq 0 \) is binding). I make the explicit assumption that the condition \( NE_r(\zeta) \geq 0 \) is never binding, so that in any state, in equilibrium we have that \( V_{r, entry}(\zeta) - w_r f_{r, s}^{\text{sunk}} = V_{r, wait to enter}(\zeta) \).

Letting \( DEC_{r, s} \) be a binary indicator equal to 1 when the export cutoffs from \( r \) to \( s \) are ranked according to (21) (that is, decreasing over time), and zero otherwise, I can write the value of entry into production as:

\[ V_{r, entry}(H^{(n)}) = \int_{b}^{\infty} V_{\varphi, r, r}(H^{(n)})dF(\varphi) + (1 - \delta) \left[ \sum_{s \neq r} \int_{b}^{\varphi_c^r, r}(H^{(n+1)}) \left( V_{\varphi, r, s}(H^{(n+1)}) - w_r f_{r, s}^{\text{exp}} \right) dF(\varphi) \right. \\
+ \left. \int_{b}^{\varphi_c^r, r}(H^{(n+1)}) V_{\varphi, r, s}(H^{(n+1)})dF(\varphi) \right] \]

\[ = \int_{b}^{\infty} V_{\varphi, r, r}(H^{(n)})dF(\varphi) + (1 - \delta) \left[ \sum_{s \neq r} \int_{b}^{\varphi_c^r, r}(H^{(n+1)}) \left( V_{\varphi, r, s}(H^{(n+1)}) - w_r f_{r, s}^{\text{exp}} \right) dF(\varphi) \right. \\
+ \left. DEC_{r, s} \times \sum_{i=n+1}^{\infty} (1 - \delta)^{i-n} \int_{b}^{\varphi_c^r, r}(H^{(i+1)}) \left( V_{\varphi, r, s}(H^{(i+1)}) - w_r f_{r, s}^{\text{exp}} \right) dF(\varphi) \right] \]

since under the assumption that \( \varphi_c^r, s(H^{(n+1)}) \leq \varphi_c^r, r(H^n) \forall n \), we have that the value of waiting to export will be given by

\[ V_{\varphi, r, s}(H^{(n+1)}) = (1 - \delta)^i - n \left( V_{\varphi, r, s}(H^{(i+1)}) - w_r f_{r, s}^{\text{exp}} \right) \text{ for } \varphi_{r, s}(H^{(i+1)}) \]

\[ \leq \varphi \]

\[ \leq \varphi_c^r, s(H^n) \]

while in the case where \( \varphi_c^r, s(H^{(n+1)}) > \varphi_c^r, r(H^n) \) (that is, \( DEC_{r, s} = 0 \)), we have that

\[ \int_{b}^{\varphi_c^r, r}(H^{(n+1)}) V_{\varphi, r, s}(H^{(n+1)})dF(\varphi) = 0. \]

Then, in state \( H^{(n)} \), we have that \( V_{r, entry}(H^{(n)}) - V_{r, wait to enter}(H^{(n)}) = \)
\[= \int_b^\infty V_{\varphi,r,r}(H^{(n)}) dF(\varphi) + (1 - \delta) \sum_{s \neq r} \int_b^\infty V_{\varphi,r,s}(H^{(n+1)}) - w_r f_{r,s}^{\text{exp}} dF(\varphi) + \int_b^\infty \varphi_{r,s}^{\text{wait}}(H^{(n+1)}) dF(\varphi) \]

\[- V_{r,\text{wait to enter}}(H^{(n)}) \]

\[= \int_b^\infty V_{\varphi,r,r}(H^{(n)}) dF(\varphi) + (1 - \delta) \left[ \sum_{s \neq r} \int_b^\infty V_{\varphi,r,s}(H^{(n+1)}) - w_r f_{r,s}^{\text{exp}} dF(\varphi) \right. \]

\[+ (1 - \delta) \left( \int_b^\infty \varphi_{r,s}^{\text{wait}}(H^{(n+1)}) dF(\varphi) + \right. \]

\[+ \ \text{DEC}_{r,s} \left( \int_b^\infty V_{\varphi,r,s}(H^{(n+1)}) - w_r f_{r,s}^{\text{exp}} dF(\varphi) \right) \]

\[- (1 - \delta) V_{r,\text{wait to enter}}(H^{(n+1)}) \]

\[= \varphi_{r,r}^{-1}(H^{(n)}) A_{r,r}(H^{(n)}) + (1 - \delta) \left[ \int_b^\infty V_{\varphi,r,r}(H^{(n+1)}) dF(\varphi) + \sum_{s \neq r} (1 - \delta) \left( \int_b^\infty \varphi_{r,s}^{\text{wait}}(H^{(n+1)}) dF(\varphi) + \text{DEC}_{r,s} \int_b^\infty \varphi_{r,s}^{\text{wait}}(H^{(n+1)}) dF(\varphi) \right) \right. \]

\[+ (1 - \delta) \sum_{s \neq r} \left( 1 - F(\varphi_{r,s}^{\text{wait}}(H^{(n+1)}))) \right) \left( \varphi_{r,s}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{\text{exp}} \right) \]

\[+ (1 - \delta) \sum_{s \neq r} \left( 1 - F(\varphi_{r,s}^{\text{wait}}(H^{(n+1)}))) \right) \left( \varphi_{r,s}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{\text{exp}} \right) \]

\[= \varphi_{r,r}^{-1}(H^{(n)}) A_{r,r}(H^{(n)}) + (1 - \delta) \left[ \int_b^\infty V_{\varphi,r,r}(H^{(n+1)}) dF(\varphi) + \right. \]

\[+ \text{DEC}_{r,s} \left( \int_b^\infty V_{\varphi,r,s}(H^{(n+1)}) - w_r f_{r,s}^{\text{exp}} dF(\varphi) \right) \]

\[+ (1 - \delta) \sum_{s \neq r} \left( 1 - F(\varphi_{r,s}^{\text{wait}}(H^{(n+1)}))) \right) \left( \varphi_{r,s}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{\text{exp}} \right) \]

\[= \varphi_{r,r}^{-1}(H^{(n)}) A_{r,r}(H^{(n)}) + (1 - \delta) \left[ \int_b^\infty V_{\varphi,r,r}(H^{(n+1)}) dF(\varphi) + \right. \]

\[+ \text{DEC}_{r,s} \left( \int_b^\infty V_{\varphi,r,s}(H^{(n+1)}) - w_r f_{r,s}^{\text{exp}} dF(\varphi) \right) \]

\[+ (1 - \delta) \sum_{s \neq r} \left( 1 - F(\varphi_{r,s}^{\text{wait}}(H^{(n+1)}))) \right) \left( \varphi_{r,s}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{\text{exp}} \right) \]
\[= \varphi_{r,r}^{-1}(H^{(n)})A_{r,r}(H^{(n)}) + (1 - \delta)w_r f_{r,s}^{sunk}
\]
\[-(1 - \delta)^2 \sum_{s \neq r} DEC_{r,s} \left(1 - F(\varphi_{r,s}(H^{(n+1)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+1)}) \sum_{i=n+2}^{\infty} (1 - \delta)^{i-(n+2)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)
\]
\[-(1 - \delta)^2 \sum_{s \neq r} (1 - DEC_{r,s}) \left(1 - F(\varphi_{r,s}^c(H^{(n+2)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+2)}) \sum_{i=n+2}^{\infty} (1 - \delta)^{i-(n+2)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)
\]
\[+ (1 - \delta) \sum_{s \neq r} \left(1 - F(\varphi_{r,s}^c(H^{(n+1)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)
\]
\[= \varphi_{r,r}^{-1}(H^{(n)})A_{r,r}(H^{(n)}) + (1 - \delta)w_r f_{r,s}^{sunk}
\]
\[+ (1 - \delta) \sum_{s \neq r} DEC_{r,s} \left(1 - F(\varphi_{r,s}^c(H^{(n+1)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)
\]
\[+ (1 - \delta) \sum_{s \neq r} (1 - DEC_{r,s}) \left(1 - F(\varphi_{r,s}^c(H^{(n+2)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+2)}) \sum_{i=n+2}^{\infty} (1 - \delta)^{i-(n+2)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)
\]
\[-(1 - \delta) \left(1 - F(\varphi_{r,s}^c(H^{(n+2)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+2)}) \sum_{i=n+2}^{\infty} (1 - \delta)^{i-(n+2)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)
\]
Setting this expression equal to the sunk cost of entry we have that in equilibrium:

\[\varphi_{r,r}^{-1}(H^{(n)})A_{r,r}(H^{(n)}) + (1 - \delta) \sum_{s \neq r} \left(- DEC_{r,s} \left(1 - F(\varphi_{r,s}^c(H^{(n+1)}))\right) \left(\varphi_{r,r}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1 - \delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp}\right)\right)
\]
\[= \delta w_r f_{r}^{sunk}
\]
\[(26)
\]

**Proof that** \(\varphi_{r,s}^c(H^{(n+1)}) = \varphi_{r,s}^c(H^{(n)}) = \varphi_{r,s}^c(H^{det})\) \(\forall n = 0, 1, 2, \ldots\)

Now, by definition, \(A_{r,r}(H^{(n)}) = A_{s,r}(H^{(n)}) = \frac{\lambda_{s,r}}{\lambda_{s,r}} \varphi_{r,r}^c(H^{(n)}) \left(\frac{w_s}{w_r}\right)^{1-\sigma}\) for any \(s\). Given this, and the fact that tariffs do not change as \(n\) increases, (so that \(\tau_{s,r} \equiv \tau_{s,r}(H^{(n)}) \forall n\)) I can write this as

\[K_{q,r}^1 \left(\varphi_{q,r}^c(H^{(n)})\right)^{1-\sigma} - (1 - DEC_{q,r})(1 - \delta) \left(\varphi_{q,r}^c(H^{(n+1)})\right)^{1-\sigma} + \Theta_{r}^n
\]
\[= \delta w_r f_{r}^{sunk}, \forall n, q
\]
\[\neq r
\]

where \(K_{q,r}^1 = \frac{a}{a+1-\sigma} (b^{\sigma-1}) \frac{\lambda_{q,r}}{\lambda_{q,r}} \varphi_{q,r}^c(H^{(n)}) w_r^{1-\sigma} w_q^{1-\sigma} f_{q,r}^{exp}\) is constant across \(n\) and the function \(\Theta_{r}^n\) is defined by:
\[
\Theta^n_r \left( \{ \varphi_{r,s}(H^{(n+j)}) \}_{n=1}^{\infty} \right)_{s \neq r,j=1,2} \oplus \{ \tilde{\varphi}_{r,s}(H^{(n+j)}) \}_{s \neq r,j=1,2} \oplus \{ A_{r,s}(H^{(n)}) \}_{s \neq r,i \geq n+1} \\
= (1-\delta) \sum_{s \neq r} DEC_{r,s} \left[ (1-F(\varphi_{r,s}(H^{(n+1)}))) \left( \tilde{\varphi}_{r,s}^{-1}(H^{(n+1)}) A_{r,s}(H^{(n+1)}) - \delta w_r f_{r,s}^{exp} \right) + (1-\delta) \sum_{s \neq r} \right] \\
- DEC_{r,s} \left[ (1-F(\varphi_{r,s}(H^{(n+1)}))) \left( \tilde{\varphi}_{r,s}^{-1}(H^{(n+1)}) \sum_{i=n+1}^{\infty} (1-\delta)^{i-(n+1)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp} \right) \\
- (1-\delta) \left( 1-F(\varphi_{r,s}(H^{(n+2)})) \right) \left( \tilde{\varphi}_{r,s}^{-1}(H^{(n+2)}) \sum_{i=n+1}^{\infty} (1-\delta)^{i-(n+2)} A_{r,s}(H^{(i)}) - w_r f_{r,s}^{exp} \right) \right]
\]

It follows that \( \Theta^n_r \) is a continuous function of export cutoff productivities (assuming that the distribution function of productivities is continuous), average export productivities, and aggregate economic conditions in periods \( n \to \infty \), and in implementing the model numerically, I make the further assumption that after \( N \) periods, all endogenous variables have returned to their (high tariff) steady state value, denoted by state \( H^{det} \). Given this, and the fact that for any \( n \), given all endogenous variable values for periods \( H^{(n+1)} \) and later, we can uniquely solve for \( \varphi_{q,r}(H^{(N-1)}) \), by plugging in these the steady state equilibrium \( H^{det} \) values for each of these endogenous variables in states \( N, N+1, \ldots \). Taking limits as \( n \to \infty \) of (27), it is clear that \( \varphi_{q,r}(H^{(N-1)}) = \varphi_{q,r}(H^{det}), \forall q,r,q \neq r \). This then implies that \( \tilde{\varphi}_{q,r}(H^{(N-1)}) = \tilde{\varphi}_{q,r}(H^{det}), \forall q,r,q \neq r \) (from the definition of \( \tilde{\varphi}_{q,r} \), see (9)) and \( A_{q,r}(H^{(N-1)}) = A_{q,r}(H^{det}) \), as implied by (24) and (25). Back substituting again to solve for \( \varphi_{q,r}(H^{(N-2)}) \) then yields \( \varphi_{q,r}(H^{(N-2)}) = \varphi_{q,r}(H^{det}), \forall q,r,q \neq r \), and so on, so that for all \( n = 1,2,\ldots \), we have that \( \varphi_{q,r}(H^{(n)}) = \varphi_{q,r}(H^{det}), \forall q,r,q \neq r \) and \( A_{q,r}(H^{(n)}) = A_{q,r}(H^{det}), \forall q,r,q \neq r \).

In understanding the assumptions driving this result, I first note that the functional form assumed for productivity distribution plays no role in this proof, other than the requirement that the cumulative distribution function be continuous. The result follows from the fact that under free entry, the aggregate economic conditions in the domestic market adjust (via the number of firms who enter) to compensate for any changes in economic conditions or cutoffs in export markets so that the value of entering into production for the marginal firm is the same, regardless of state. Further, the aggregate economic conditions for export, \( A_{r,s} \), can be pinned down from just knowing the corresponding export productivity cutoff. Then, since \( A_{r,s} \) completely determines \( \varphi_{r,s} \) and vice versa, and since each is a function of next period (and later) values, it follows that for both to eventually equal some final limiting value, they must equal this in every high tariff period.

**B Derivation of Free Entry Condition**

In this appendix I derive the free entry condition for each sector-country based on the equilibrium condition, \( V_{k,r}^{entry}(\zeta) - w_r f_{k,r}^{sink} = V_{k,r}^{waittoenter}(\zeta) \). In the derivations below, I drop the sector subscript \( k \) for ease of
notation; it is understood that all values and endogenous variables are also sector specific.

The expected (over $\varphi$) value of entry in a given state is given by:

$$V_{r,\text{entry}}(L) = \int_b^\infty V_{\varphi,r,r}(L)dF(\varphi)$$

$$+ (1 - \delta) \sum_{s \neq r} \left[ (1 - \gamma)(\int_{\varphi_{r,s}(L)}^\infty (V_{\varphi,r,s}(L) - w_r f_{r,s}^{\exp})dF(\varphi) + \int_b^{\varphi_{r,s}(L)} V_{\varphi,r,s}(L)dF(\varphi) \right]$$

$$+ \gamma \left( \int_{\varphi_{r,s}(H)}^\infty (V_{\varphi,r,s}(H(0)) - w_r f_{r,s}^{\exp})dF(\varphi) + \int_b^{\varphi_{r,s}(H)} V_{\varphi,r,s}(H(0))dF(\varphi) \right)$$

$$= \frac{\varphi_{r}^{-1}A_{r,r}(L)}{\delta + \gamma - \delta \gamma} + \frac{(1 - \delta)\gamma}{\delta + \gamma - \delta \gamma} \left[ V_{r,\text{entry}}(H(0)) - (1 - \delta) \left[ \sum_{s \neq r} \int_{\varphi_{r,s}(H)}^\infty (V_{\varphi,r,s}(H(1)) - w_r f_{r,s}^{\exp})dF(\varphi) \right] \right]$$

$$+ (1 - \delta) \sum_{s \neq r} \left[ (1 - \gamma)(\int_{\varphi_{r,s}(L)}^\infty (V_{\varphi,r,s}(L) - w_r f_{r,s}^{\exp})dF(\varphi) + \int_b^{\varphi_{r,s}(L)} V_{\varphi,r,s}(L)dF(\varphi) \right]$$

$$+ \gamma \left( \int_{\varphi_{r,s}(H)}^\infty (V_{\varphi,r,s}(H(0)) - w_r f_{r,s}^{\exp})dF(\varphi) + \int_b^{\varphi_{r,s}(H)} V_{\varphi,r,s}(H(0))dF(\varphi) \right)$$

since

$$V_{r,\text{entry}}(H(n)) = \int_b^\infty V_{\varphi,r,r}(H(n))dF(\varphi) + (1 - \delta) \left[ \sum_{s \neq r} \int_{\varphi_{r,s}(H(n+1))}^\infty (V_{\varphi,r,s}(H(n+1)) - w_r f_{r,s}^{\exp})dF(\varphi) \right]$$

$$+ \int_b^{\varphi_{r,s}(H(n+1))} V_{\varphi,r,s}(H(n+1))dF(\varphi)$$

$$= \int_b^\infty V_{\varphi,r,r}(H(n))dF(\varphi) + (1 - \delta) \left[ \sum_{s \neq r} \int_{\varphi_{r,s}(H(n+1))}^\infty (V_{\varphi,r,s}(H(n+1)) - w_r f_{r,s}^{\exp})dF(\varphi) \right]$$

which follows from the fact that $\varphi_{r,s}(H(n+1)) = \varphi_{r,s}(H) \forall n$, and so we have that the value of waiting to export for $\varphi < \varphi_{r,s}(H(n+1))$ is zero.

I assume that the “death shock” applies equally to current and potential firms so that the same discount factor is used by all when discounting future periods. Then, the value of waiting to enter into production in a given state will be given by

$$V_{r,\text{wait to enter}}(\zeta) = 0 + (1 - \delta)E_{\zeta'|\zeta} \max \left[ \left( V_{r,\text{entry}}(\zeta') - w_r f_{r}^{\text{shock}} \right), V_{r,\text{wait to enter}}(\zeta') \right]$$

$$= (1 - \delta)E_{\zeta'|\zeta} \left[ V_{r,\text{wait to enter}}(\zeta') \right]$$
where the final equality follows from the fact that in equilibrium, free entry implies that $V^{\text{entry}}(\zeta) - w_r^{f_{r,\text{sunk}}} \leq V^{\text{wait}}(\zeta) \forall \zeta$ (with strict inequality only in the case when the constraint $NE_r(\zeta) \geq 0$ is binding). So,

$$V^{\text{wait to enter}}(L) = (1 - \delta) \left( (1 - \gamma) V^{\text{wait to enter}}(L) + \gamma V^{\text{wait}}(H(0)) \right)$$

$$\Rightarrow V^{\text{wait to enter}}(L) = \frac{\gamma(1 - \delta)}{(\delta + \gamma - \delta \gamma)} V^{\text{wait to enter}}(H(0))$$

and

$$V^{\text{wait to enter}}(H^{(n)}) = (1 - \delta) V^{\text{wait to enter}}(H^{(n+1)}), \quad n = 0, 1, 2, \ldots$$

Then, I can write the difference between the value of entering into production and waiting to enter in state $L$ as:

$$V^{\text{entry}}(L) - V^{\text{wait to enter}}(L) = \frac{\phi^{\sigma - 1}(L) A_{r,s}(L)}{\delta + \gamma - \delta \gamma} + \frac{(1 - \delta) \gamma}{(\delta + \gamma - \delta \gamma)} \left( V^{\text{entry}}(H(0)) - V^{\text{wait to enter}}(H(0)) \right)$$

$$- (1 - \delta) \sum_{s \neq r} \int_{\phi^{\sigma},r,s}^{\infty} \left( V_{\phi^{\sigma},r,s}(H(1)) - w_r^{f_{r,\text{sunk}}} \right) dF(\varphi)$$

$$+ (1 - \delta) \sum_{s \neq r} \left[ (1 - \gamma) \left( \int_{\phi^{\sigma},r,s}^{\infty} (V_{\phi^{\sigma},r,s}(L) - w_r^{f_{r,\text{sunk}}} \right) dF(\varphi)$$

$$+ \int_{\phi^{\sigma},r,s}^{\phi^{\sigma},r,s}(L) V^{\text{wait to enter}}(L) dF(\varphi)$$

$$+ \gamma \left( \int_{\phi^{\sigma},r,s}^{\infty} (V_{\phi^{\sigma},r,s}(H(0)) - w_r^{f_{r,\text{sunk}}} \right) dF(\varphi)$$

$$+ \int_{\phi^{\sigma},r,s}^{\phi^{\sigma},r,s}(H) V^{\text{wait to enter}}(H(0)) dF(\varphi)$$

$$- \frac{\gamma(1 - \delta)}{(\delta + \gamma - \delta \gamma)} V^{\text{wait to enter}}(H(0))$$

$$= \frac{\phi^{\sigma - 1}(L) A_{r,s}(L)}{\delta + \gamma - \delta \gamma} + \frac{(1 - \delta) \gamma}{(\delta + \gamma - \delta \gamma)} \left( V^{\text{entry}}(H(0)) - V^{\text{wait to enter}}(H(0)) \right)$$

$$- (1 - \delta) \sum_{s \neq r} \left( 1 - F(\phi^{\sigma},r,s)(H) \right) \left( \frac{\phi^{\sigma - 1}(L) A_{r,s}(H)}{\delta} - w_r^{f_{r,\text{sunk}}} \right)$$

$$+ (1 - \delta) \sum_{s \neq r} \left( 1 - \gamma \right) \left( \frac{\phi^{\sigma - 1}(L)}{\delta + \gamma - \delta \gamma} \right) \left( A_{r,s}(L) + \gamma(1 - \delta) \frac{A_{r,s}(H)}{\delta} \right)$$

$$- w_r^{f_{r,\text{sunk}}}$$

$$+ (1 - \delta) \sum_{s \neq r} \left( 1 - \gamma \right) \frac{\gamma(1 - \delta)}{\delta + \gamma - \delta \gamma} \left( F(\phi^{\sigma},r,s)(L) - F(\phi^{\sigma},r,s)(H) \right) \left( \phi^{\sigma - 1}(L) A_{r,s}(H) - w_r^{f_{r,\text{sunk}}} \right)$$

$$\ast \text{Rank}_{r,s}$$

$$+ (1 - \delta) \sum_{s \neq r} \gamma \left( 1 - F(\phi^{\sigma},r,s)(H) \right) \left( \phi^{\sigma - 1}(L) A_{r,s}(H) - w_r^{f_{r,\text{sunk}}} \right)$$

\[^{18}\text{Note that } V^{\text{wait to enter}} \text{ indexed by one country denotes the value of waiting to enter into production while } V^{\text{wait}} \text{ indexed by two countries denotes the value of waiting to begin exporting to region } s. \text{ These are two distinct values.}\]
where

\[
\begin{aligned}
\phi_{r,s}^{LH} &= \begin{cases} 
\varphi_{r,s}^e(L) & \text{if } \varphi_{r,s}^e(L) = \varphi_{r,s}^e(H) \\
\frac{f'_{\varphi_{r,s}^e(H)}(\varphi^e - 1)d\varphi}{F(\varphi_{r,s}^e(H)) - F(\varphi_{r,s}^e(L))} & \text{if } \varphi_{r,s}^e(L) \neq \varphi_{r,s}^e(H)
\end{cases}
\end{aligned}
\]

is the average productivity level between high and low state cutoffs and \(1_{\text{Rank}B_{r,s}}\) is a binary indicator equal to one when \(\varphi_{r,s}^e(L) > \varphi_{r,s}^e(H)\). Then, in equilibrium, the mass of new entrants in state \(L\) is determined by:

\[
\begin{aligned}
\frac{\varphi_{r,s}^{L-1}(L)A_{r,s}(L) + \gamma(1 - \delta)}{1 - \delta} &\left(1 - \frac{M_{r,s}(H^{(1)})}{N_r(H^{(1)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H)}{A_{r,s}(H)} - \frac{A_{r,s}(H)}{\delta} - w_r f_r \exp\right) \\
&+ \left(1 - \gamma\right) \left(\frac{M_{r,s}(L)}{N_r(L)}\right) \left(\frac{\varphi_{r,s}^{L-1}(L)}{A_{r,s}(L) + \gamma(1 - \delta) A_{r,s}(H)} - \delta + \gamma - \delta \gamma w_r f_r \exp\right) \\
&+ \left(1 - \delta\right) \left(1 - \gamma\right) \left(1 - \delta\right) \left(F(\varphi_{r,s}^e(L)) - F(\varphi_{r,s}^e(H))\right) \left(\frac{\varphi_{r,s}^{L-1}(H)}{A_{r,s}(H)} - \frac{A_{r,s}(H)}{\delta} - w_r f_r \exp\right) \\
&\times (1_{\text{Rank}B_{r,s}} + \sum_{s \neq r} \gamma(1 - \delta) \left(\frac{M_{r,s}(H^{(0)})}{N_r(L) + NE_r(H^{(n)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H)}{A_{r,s}(H)} - \frac{A_{r,s}(H)}{\delta} - w_r f_r \exp\right) = \delta w_r f_r \text{ sunk}
\end{aligned}
\]

And in state \(H^{(n)}\),

\[
\begin{aligned}
V_{\text{entry}}(H^{(n)}) - V_{\text{wait to enter}}(H^{(n)}) &= \int_b^{\infty} V_{\varphi, r, r}(H^{(n)})d\varphi \\
&+ \left(1 - \delta\right) \sum_{s \neq r} \int_b^{\infty} V_{\varphi, r, s}(H^{(n+1)}) d\varphi \\
&- V_{\text{wait}}(H^{(n+1)}) \\
&= \frac{\varphi_{r,s}^{L-1}(H)A_{r,s}(H)}{1 - \delta} \int_b^{\infty} V_{\varphi, r, r}(H^{(n+1)})d\varphi \\
&+ \sum_{s \neq r} \left(\frac{M_{r,s}(H^{(n+1)})}{N_r(H^{(n)}) + NE_r(H^{(n)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H)}{A_{r,s}(H)} - \frac{A_{r,s}(H)}{\delta} - w_r f_r \exp\right) \\
&- (1 - \delta) V_{\text{wait}}(H^{(n+1)}) \\
&= \frac{\varphi_{r,s}^{L-1}(H)A_{r,s}(H)}{1 - \delta} \left(V_{\text{entry}}(H^{(n+1)}) - \delta \sum_{s \neq r} \left(\frac{M_{r,s}(H^{(n+2)})}{N_r(H^{(n+2)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H^{(n+2)})}{A_{r,s}(H^{(n+2)})} - \frac{A_{r,s}(H^{(n+2)})}{\delta} - w_r f_r \exp\right) \\
&+ \sum_{s \neq r} \left(\frac{M_{r,s}(H^{(n+1)})}{N_r(H^{(n)}) + NE_r(H^{(n)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H^{(n)})}{A_{r,s}(H^{(n)})} - \frac{A_{r,s}(H^{(n)})}{\delta} - w_r f_r \exp\right) \\
&- (1 - \delta) V_{\text{wait}}(H^{(n+1)}) \\
&= \frac{\varphi_{r,s}^{L-1}(H)A_{r,s}(H)}{1 - \delta} \left(V_{\text{entry}}(H^{(n+1)}) - \delta \sum_{s \neq r} \left(\frac{M_{r,s}(H^{(n+2)})}{N_r(H^{(n+2)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H^{(n+2)})}{A_{r,s}(H^{(n+2)})} - \frac{A_{r,s}(H^{(n+2)})}{\delta} - w_r f_r \exp\right) \\
&+ \sum_{s \neq r} \left(\frac{M_{r,s}(H^{(n+1)})}{N_r(H^{(n)}) + NE_r(H^{(n)})}\right) \left(\frac{\varphi_{r,s}^{L-1}(H^{(n)})}{A_{r,s}(H^{(n)})} - \frac{A_{r,s}(H^{(n)})}{\delta} - w_r f_r \exp\right)
\end{aligned}
\]

\[
43
\]
\[\varphi_{r,r}^{-1}(H)A_{r,r}(H) + (1 - \delta)w_{r} f_{r}^{\text{ sunk}} + \sum_{s \neq r} \left( \frac{M_{r,s}(H^{(n+1)})}{N_{r}(H^{(n)}) + NE_{r}(H^{(n)})} \right) \left( \varphi_{r,s}^{-1}(H)A_{r,s}(H) - \delta w_{r} f_{r,s}^{\text{ exp}} \right) \]

So that the number of new entrants in state \(H^{(n)}\) is determined by:

\[\varphi_{r,r}^{-1}(H)A_{r,r}(H) + \sum_{s \neq r} \left( \frac{M_{r,s}(H^{(n+1)})}{N_{r}(H^{(n)}) + NE_{r}(H^{(n)})} \right) \left( \varphi_{r,s}^{-1}(H)A_{r,s}(H) - \delta w_{r} f_{r,s}^{\text{ exp}} \right) = \delta w_{r} f_{r}^{\text{ sunk}}\]

## C Derivation of Cutoff Export Productivity in Low Tariff State

In state \(\zeta\) a country \(r\)-sector \(k\) firm with productivity \(\varphi\) will enter export market \(s\) if

\[V_{\varphi,k,r,s}(\zeta) - V_{\varphi,k,r,s}^{\text{ wait}}(\zeta) \geq w_{r} f_{k,r,s}^{\text{ exp}}\]

The expressions for these values as a function of the state and productivity level are as follows:

For \(\varphi \leq \varphi_{k,r,s}^{\text{ c}}(L)\) and \(r \neq s\):

\[V_{\varphi,k,r,s}(L) = \begin{cases} 0 & \text{if } 1_{\text{RankB}_{k,r,s}} = 0 \text{ or } 1_{\text{RankB}_{k,r,s}} = 1 \text{ and } \varphi < \varphi_{k,r,s}^{\text{ c}}(H^{(0)}) \\ \frac{(1-\delta)\gamma}{\delta + \gamma - \delta \gamma} \left( V_{\varphi,k,r,s}(H^{(0)}) - w_{r} f_{k,r,s}^{\text{ exp}} \right) & \text{if } 1_{\text{RankB}_{k,r,s}} = 1 \text{ and } \varphi \geq \varphi_{k,r,s}^{\text{ c}}(H^{(0)}) \end{cases} \]

\[V_{\varphi,k,r,s}(L) = \frac{\pi_{\varphi,k,r,s}(L) + (1 - \delta)\gamma V_{\varphi,k,r,s}(H^{(0)})}{\delta + \gamma - \delta \gamma} \]

Which implies that for \(\varphi \leq \varphi_{k,r,s}^{\text{ c}}(L)\) (and \(\varphi > \varphi_{k,r,s}^{\text{ c}}(H^{(0)})\) in the case where cutoffs are ranked according to B),

\[(\delta + \gamma - \delta \gamma) \left( V_{\varphi,k,r,s}(L) - V_{\varphi,k,r,s}^{\text{ wait}}(L) \right) = \begin{cases} \pi_{\varphi,k,r,s}(L) + (1 - \delta)\gamma \sum_{i=0}^{\infty} (1 - \delta)^i \varphi_{k,r,s}^{\text{ c}}(H^{(i)}) & \text{if } 1_{\text{RankB}_{k,r,s}} = 0 \\ \pi_{\varphi,k,r,s}(L) + (1 - \delta)\gamma w_{r} f_{k,r,s}^{\text{ exp}} & \text{if } 1_{\text{RankB}_{k,r,s}} = 1 \end{cases} \]

And, since for the marginal firm who is just indifferent between selling in the domestic market and entering the export market in state \(L\), \(V_{\varphi,k,r,s}(L) - V_{\varphi,k,r,s}^{\text{ wait}}(L) = w_{r} f_{k,r,s}^{\text{ exp}}\), we have:

For a sector \(k\) region \(r\) with export cutoffs to \(s\) ranked according to Ranking A (see 8), then:

\[\varphi_{k,r,s}(L) = \frac{(\delta + \gamma - \delta \gamma) w_{r} f_{k,r,s}^{\text{ exp}}}{\pi_{\varphi,k,r,s}(L) + (1 - \delta)\gamma \sum_{i=0}^{\infty} (1 - \delta)^i \varphi_{k,r,s}^{\text{ c}}(H^{(i)})} \]

\[\Rightarrow \varphi_{k,r,s}(L) = \left( \varphi_{k,r,s}^{\text{ c}}(L) \right)^{\sigma - 1} \left[ A_{k,r,s}(L) + (1 - \delta)\gamma \left( \sum_{n=0}^{\infty} (1 - \delta)^n A_{k,r,s}(H^{(n)}) \right) \right] \]

\[\Rightarrow \left( \varphi_{k,r,s}^{\text{ c}}(L) \right)^{\sigma - 1} = \frac{(\delta + \gamma - \delta \gamma) w_{r} f_{k,r,s}^{\text{ exp}}}{\left[ A_{k,r,s}(L) + \gamma \left( \sum_{n=0}^{\infty} (1 - \delta)^{n+1} A_{k,r,s}(H^{(n)}) \right) \right]} \quad (28)\]
For a region \( r \) with sector \( k \) cutoffs for exporting to market \( s \) ranked according to Ranking B, (see (8)), we have:

\[
(\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} = \pi_{\varphi_r,s}(L) + (1 - \delta)\gamma w_r(H^{(0)})f_{k,r,s}^{exp}
\]

\[
\Rightarrow (\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} = \left(\varphi_{k,r,s}^c(L)\right)^{\sigma-1} A_{k,r,s}(L) + (1 - \delta)\gamma w_r(H^{(0)})f_{k,r,s}^{exp}
\]

\[
\Rightarrow \left(\varphi_{k,r,s}^c(L)\right)^{\sigma-1} = \frac{(\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} - (1 - \delta)\gamma w_r(H^{(0)})f_{k,r,s}^{exp}}{A_{k,r,s}(L)} \quad (29)
\]

### D Proof: Cutoffs Increasing in Probability of Tariff Hike

As shown in section 3.6, the cutoff productivity in sector \( k \), country \( r \) to choose to enter export \( s \) market in state \( H^{(n)} \) satisfies \( \left(\varphi_{k,r,s}^c(H^{(n)})\right)^{\sigma-1} = \frac{\delta w_r f_{k,r,s}^{exp}}{A_{k,r,s}(H^{(n)})} \) and for a sector \( k \) and country \( r \) with \( s \) cutoffs ranked according to Ranking A, that is, for which \( \varphi_{k,r,s}^c \leq \varphi_{k,r,s}^c(H^{det}) = \varphi_{k,r,s}^c(H^{(n)}) \forall n = 0,1, \ldots \), we have that \( \left(\varphi_{k,r,s}^c(L)\right)^{\sigma-1} = \frac{(\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp}}{A_{k,r,s}(L) + \gamma \frac{1-\delta}{\delta} A_{k,r,s}(H^{det})} \), and so

\[
(\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} \leq \frac{\delta w_r f_{k,r,s}^{exp}}{A_{k,r,s}(H^{det})}
\]

\[
\Rightarrow (\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} A_{k,r,s}(H^{det}) \leq \delta w_r f_{k,r,s}^{exp} \left(A_{k,r,s}(L) + \gamma \frac{1-\delta}{\delta} A_{k,r,s}(H^{det})\right)
\]

\[
\Rightarrow A_{k,r,s}(H^{det}) \leq A_{k,r,s}(L)
\]

Further, for a country with cutoffs ranked according to Ranking A, we have that

\[
\frac{\partial}{\partial \gamma} \left(\varphi_{k,r,s}^c(L)\right)^{\sigma-1} = \frac{\partial}{\partial \gamma} \left[\left(\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} \right] \right.
\]

\[
= \frac{(A_{k,r,s}(L) + \gamma \frac{1-\delta}{\delta} A_{k,r,s}(H^{det})) (1 - \delta) w_r f_{k,r,s}^{exp} - (\delta + \gamma - \delta \gamma) w_r f_{k,r,s}^{exp} 1 - \frac{1-\delta}{\delta} A_{k,r,s}(H^{det})}{(A_{k,r,s}(L) + \gamma \frac{1-\delta}{\delta} A_{k,r,s}(H^{det}))^2} \geq 0
\]

So that for a country with \( \varphi_{k,r,s}^c \leq \varphi_{k,r,s}^c(H^{det}) \), it is also the case that the export cutoff productivity is increasing in \( \gamma \), the probability that the high tariff shock will arrive in any given period.

### E Summary of Equilibrium Conditions
<table>
<thead>
<tr>
<th>Variable</th>
<th>Associated Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_r(c) )</td>
<td>( U_r(c) = q_0 r(c)^{1 - \sum_k \mu_k} \prod_k Q_k^{\mu_k} Q_{k, r}(c) )</td>
</tr>
<tr>
<td>( Q_{k, r}(c) )</td>
<td>( E_{k, r}(c) = P_{k, r}(c) Q_{k, r}(c) )</td>
</tr>
<tr>
<td>( q_0 r(c) )</td>
<td>( q_0 r(c) = (1 - \sum_k \mu_k) G D_P r(c) )</td>
</tr>
<tr>
<td>( E_{k, r}(c) )</td>
<td>( E_{k, r}(c) )</td>
</tr>
<tr>
<td>( P_{k, r}(c) )</td>
<td>( P_{k, r}(c) = \lambda_{k, r} \left( N_{k, r}(c) + NE_{k, r}(c) \right) \left( \hat{p}<em>{k, r}(c) \right)^{1 - \sigma} + \sum</em>{x \neq r} \lambda_{k, x, r} \left( M_{k, x, r}(c) \right) \left( \hat{p}<em>{k, x, r}(c) \right)^{1 - \sigma} + \sum</em>{x \neq r} \lambda_{k, x, r} \left( \hat{p}_{k, x, r}(c) \right)^{1 - \sigma} ) )</td>
</tr>
<tr>
<td>( A_{k, r, s}(c) )</td>
<td>( A_{k, r, s}(c) = \frac{1}{\left( \frac{\sigma_{k, r, s}}{\sum_{n=0}^{\infty}} \right)} \left( \frac{\sigma_{k, r, s}}{\sum_{n=0}^{\infty}} \right) )</td>
</tr>
<tr>
<td>( x_{k, r, s}(c) )</td>
<td>( x_{k, r, s}(c) )</td>
</tr>
<tr>
<td>( D_{k, r, s}(c) )</td>
<td>( D_{k, r, s}(c) )</td>
</tr>
<tr>
<td>( 1 \text{Rank}_{k, r, s}(c) )</td>
<td>( 1 \text{Rank}_{k, r, s}(c) )</td>
</tr>
<tr>
<td>( q_{k, r, s}(c) )</td>
<td>( q_{k, r, s}(c) )</td>
</tr>
<tr>
<td>( M_{k, r, s}(c) )</td>
<td>( M_{k, r, s}(c) )</td>
</tr>
<tr>
<td>( L_{k, r, s}(c) )</td>
<td>( L_{k, r, s}(c) )</td>
</tr>
</tbody>
</table>

\[ P_{k, r}(c) = \lambda_{k, r} \left( N_{k, r}(c) + NE_{k, r}(c) \right) \left( \hat{p}_{k, r}(c) \right)^{1 - \sigma} + \sum_{x \neq r} \lambda_{k, x, r} \left( M_{k, x, r}(c) \right) \left( \hat{p}_{k, x, r}(c) \right)^{1 - \sigma} + \sum_{x \neq r} \lambda_{k, x, r} \left( \hat{p}_{k, x, r}(c) \right)^{1 - \sigma} \]

\[ A_{k, r, s}(c) = \frac{1}{\left( \frac{\sigma_{k, r, s}}{\sum_{n=0}^{\infty}} \right)} \left( \frac{\sigma_{k, r, s}}{\sum_{n=0}^{\infty}} \right) \]

\[ x_{k, r, s}(c) \]

\[ D_{k, r, s}(c) \]

\[ 1 \text{Rank}_{k, r, s}(c) \]

\[ q_{k, r, s}(c) \]

\[ M_{k, r, s}(c) \]

\[ L_{k, r, s}(c) \]

\[ N_{k, r}(c) \]

\[ N_{k, r}(c) \]

\[ N_{k, r}(c) \]

\[ N_{k, r}(c) \]

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<tr>
<td>$NE_{k,r}(c)$</td>
<td>$\psi_{k,r,c}^{\sigma -1}(L)A_{k,r,c}(L)+\sum_{x \not \in r}(\gamma (\delta +1-\delta)) \left(\frac{M_{k,r,c}(H^{(n)})}{NE_{k,r,c}(H^{(n)})}\right)\left(\psi_{k,r,c}^{\sigma -1}(H)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}-wr_{k,r,c}\right)$</td>
</tr>
<tr>
<td></td>
<td>$+(1-\delta)\sum_{x \not \in r}(1-\gamma)\frac{M_{k,r,c}(L)}{NE_{k,r,c}(L)}\left(\psi_{k,r,c}^{\sigma -1}(L)\left(A_{k,r,c}(L)+\gamma (1-\delta)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}\right)-(\delta+\gamma-\delta)wr_{k,r,c}\right)\right)+$</td>
</tr>
<tr>
<td></td>
<td>$+(1-\delta)\sum_{x \not \in r}(1-\gamma)(1-\delta)\left(F(\psi_{k,r,c}(L))-F(\psi_{k,r,c}(H))\right)\left(\psi_{k,r,c}^{\sigma -1}(H)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}-wr_{k,r,c}\right)\left(1-Rank_{E_{k,r,c}}\right)$</td>
</tr>
</tbody>
</table>

\[
\phi_{k,r,c}^{\sigma -1}(H^{(n)}) = \frac{M_{k,r,c}(H^{(n)})}{NE_{k,r,c}(H^{(n)})}\left(\phi_{k,r,c}^{\sigma -1}(H)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}-wr_{k,r,c}\right) + \sum_{x \not \in r}(1-\gamma)(1-\delta)\left(F(\psi_{k,r,c}(L))-F(\psi_{k,r,c}(H))\right)\left(\phi_{k,r,c}^{\sigma -1}(H)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}-wr_{k,r,c}\right)\left(1-Rank_{E_{k,r,c}}\right)
\]

\[
\phi_{k,r,c}^{\sigma -1}(H^{(n+1)}) = \frac{M_{k,r,c}(H^{(n+1)})}{NE_{k,r,c}(H^{(n+1)})}\left(\phi_{k,r,c}^{\sigma -1}(H^{(n+1)})\frac{A_{k,r,c}(H^{(n+1)})}{\phi_{\sigma}}-\delta wr_{k,r,c}\right)
\]

\[
 \phi_{k,r,c}^{\sigma -1}(H^{(n)}) = \frac{M_{k,r,c}(H^{(n)})}{NE_{k,r,c}(H^{(n)})}\left(\phi_{k,r,c}^{\sigma -1}(H)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}-wr_{k,r,c}\right) + \sum_{x \not \in r}(1-\gamma)(1-\delta)\left(F(\psi_{k,r,c}(L))-F(\psi_{k,r,c}(H))\right)\left(\phi_{k,r,c}^{\sigma -1}(H)\frac{A_{k,r,c}(H)}{\phi_{\sigma}}-wr_{k,r,c}\right)\left(1-Rank_{E_{k,r,c}}\right)
\]

F Conjecture: Constant Cutoffs across $L$ and $H$ states in sectors with constant tariffs

In this section, I hypothesize that under the assumption of Pareto distributed productivity, for any sector $k$ with $\tau_{k,r,s} \equiv \tau_{k,r,s}(L) = \tau_{k,r,s}(H) \forall r \neq s$, that is, any sector for which no bilateral tariffs change with the arrival of the high tariff shock, it will be the case that $\psi_{k,r,s}(L) = \psi_{k,r,s}(H)$, that is, that productivity cutoffs to export in this sector do not change, despite changes in the aggregate economy that may be caused by tariff shocks in other sectors. To simplify notation, for the remainder of this section I drop the $k$ industry subscript; it is assumed that all industry specific variables in this section are for an industry $k$ which is not directly affected by the tariff shock.

Now using fact that cutoffs $\psi_{r,s}(H) \equiv \psi_{r,s}(H^{(n)})$ as well as aggregate economic conditions $A_{r,s}(H) \equiv A_{r,s}(H^{(n)})$ are constant across $n$ and also assuming homogenous good produced in all countries (so that wages
do not depend on the state \( \zeta \in \{ L, H^{(n)} \} \) and Pareto distributed productivity, the free entry condition in state \( L \) implies that in equilibrium, for each \( r \):

\[
\tilde{\varphi}_{r,r}^{-1}(L) A_{r,r}(L) + \gamma (1 - \delta) \left( w_r f_{r,s}^{\text{sunk}} - \sum_{s \neq r} \left( \frac{b}{\varphi_{r,s}^c(H)} \right)^a \left( \frac{a}{a + 1 - \sigma} \varphi_{r,s}^c(H)^{\sigma - 1} \left( \frac{1 - \delta}{\delta} \right) A_{r,s}(H) - (1 - \delta) w_r f_{r,s}^{\text{exp}} \right) \right) + (1 - \delta) \sum_{s \neq r} \left( 1 - \gamma \right) \left( \left( \frac{b}{\varphi_{r,s}^c(L)} \right)^a \left( \frac{a}{a + 1 - \sigma} \varphi_{r,s}^c(L)^{\sigma - 1} \right) \left[ A_{r,s}(L) + \gamma \left( 1 - \delta \right) \frac{1 - \delta}{\delta} A_{r,s}(H) - (\delta + \gamma - \delta \gamma) w_r f_{r,s}^{\text{exp}} \right) \right) \right) + (1 - \delta) \sum_{s \neq r} (1 - \gamma) (1 - \delta) \left( a b^a \left( \varphi_{r,s}(H)^{\sigma - a - 1} - \varphi_{r,s}(L)^{\sigma - a - 1} \right) \frac{A_{r,s}(H)}{\delta} \right) - \left( \left( \frac{b}{\varphi_{r,s}^c(H)} \right)^a - \left( \frac{b}{\varphi_{r,s}^c(L)} \right)^a \right) w_r f_{r,s}^{\text{exp}} \right) * 1_{\text{RankB}_{r,s}} + (\delta + \gamma - \delta \gamma) \sum_{s \neq r} (1 - \gamma) (1 - \delta) \left( \left( \varphi_{r,s}^c(L)^{\sigma - a - 1} - \varphi_{r,s}^c(H)^{\sigma - a - 1} \right) \frac{A_{r,s}(H)}{\delta} \right) \right)

Now, using the condition governing export productivity cutoffs, namely

\[
\varphi_{r,s}^c(L)^{\sigma - 1} = \frac{\delta w_r f_{r,s}^{\text{exp}} + (1 - \delta) w_r f_{r,s}^{\text{exp}} * (1 - 1_{\text{RankB}_{r,s}})}{A_{r,s}(L) + \gamma (1 - \delta) w_r f_{r,s}^{\text{exp}} * (1 - 1_{\text{RankB}_{r,s}})}
\]

and

\[
\varphi_{k,r,s}^c(H)^{\sigma - 1} = \frac{\delta w_r f_{r,s}^{\text{exp}}}{A_{k,r,s}(H)}
\]

this can be written as:

\[
\tilde{\varphi}_{r,r}^{-1}(L) A_{r,r}(L) + \gamma (1 - \delta)^2 \left( - \sum_{s \neq r} \left( \frac{b}{\varphi_{r,s}^c(H)} \right)^a \left( \frac{a - 1}{a + 1 - \sigma} w_r f_{r,s}^{\text{exp}} \right) \right) + (1 - \delta) \sum_{s \neq r} \left( 1 - \gamma \right) \left( \left( \frac{b}{\varphi_{r,s}^c(L)} \right)^a \left( \frac{a}{a + 1 - \sigma} \varphi_{r,s}^c(L)^{\sigma - 1} \right) \left[ A_{r,s}(L) + \gamma \left( 1 - \delta \right) \frac{1 - \delta}{\delta} A_{r,s}(H) - (\delta + \gamma - \delta \gamma) w_r f_{r,s}^{\text{exp}} \right) \right) \right) + (1 - \delta) \sum_{s \neq r} (1 - \gamma) (1 - \delta) \left( \left( \varphi_{r,s}(H)^{\sigma - a - 1} - \varphi_{r,s}(L)^{\sigma - a - 1} \right) \frac{A_{r,s}(H)}{\delta} \right) - \left( \left( \frac{b}{\varphi_{r,s}^c(H)} \right)^a - \left( \frac{b}{\varphi_{r,s}^c(L)} \right)^a \right) w_r f_{r,s}^{\text{exp}} \right) * 1_{\text{RankB}_{r,s}} + (\delta + \gamma - \delta \gamma) \sum_{s \neq r} (1 - \gamma) (1 - \delta) \left( \left( \varphi_{r,s}^c(L)^{\sigma - a - 1} - \varphi_{r,s}^c(H)^{\sigma - a - 1} \right) \frac{A_{r,s}(H)}{\delta} \right) \right)

* 1_{\text{RankB}_{r,s}} + (\delta + \gamma (1 - \delta)) \sum_{s \neq r} (1 - \delta) \left( \left( \frac{b}{\varphi_{r,s}^c(H)} \right)^a \left( \frac{\sigma - 1}{a + 1 - \sigma} w_r f_{r,s}^{\text{exp}} \right) \right) = \delta w_r f_{r,s}^{\text{sunk}}
$$\Rightarrow \varphi_{r,r}^{-1}(L)A_{r,r}(L) + [(\delta + \gamma - \delta \gamma)(1 - \delta) - \gamma(1 - \delta)^2] \sum_{s \neq r} \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} \left( \frac{\sigma - 1}{a + 1 - \sigma} w_{r,f,x}^{exp} \right)$$

$$+ (1 - \delta) \sum_{s \neq r} (1 - \gamma) \left( \left( \frac{b}{\varphi_{r,s}^{c}(L)} \right)^{a} \left( \frac{\sigma - 1}{a + 1 - \sigma} w_{r,s}^{exp} \right) \right) \ast (1 - 1_{RankB_{r,s}})$$

$$+ (1 - \delta) \sum_{s \neq r} (1 - \gamma) \left( \left( \frac{b}{\varphi_{r,s}^{c}(L)} \right)^{a} \left( \frac{a \sigma - 1}{a + 1 - \sigma} + a \gamma (1 - \delta) \left( \frac{\varphi_{r,s}^{c}(L)}{\varphi_{r,s}^{c}(H)} \right)^{\sigma - 1} - (\delta + \gamma - \delta \gamma) \right) \right) w_{r,s}^{exp}$$

$$\ast 1_{RankB_{r,s}} + (1 - \delta) \sum_{s \neq r} (1 - \gamma)(1 - \delta) \left( \left( \frac{1}{\varphi_{r,s}^{c}(H)} \right)^{a} - \left( \frac{\varphi_{r,s}^{c}(L)}{\varphi_{r,s}^{c}(H)} \right)^{\sigma - 1} \left( \frac{1}{\varphi_{r,s}^{c}(L)} \right)^{a} \right)$$

$$- \left( \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} - \left( \frac{b}{\varphi_{r,s}^{c}(L)} \right)^{a} \right) w_{r,s}^{exp} * 1_{RankB_{r,s}}$$

$$= \delta w_{r,s}^{f_{r,x}^{exp}}$$

(30)

The free entry condition in any \( H \) state yields

$$\Rightarrow \varphi_{r,r}^{-1}(H)A_{r,r}(H) + (1 - \delta) \sum_{s \neq r} \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} \left( \frac{\sigma - 1}{a + 1 - \sigma} \right) \varphi_{r,s}^{c}(H) A_{r,s}(H) - \delta w_{r,s}^{f_{r,x}^{exp}} = \delta w_{r,s}^{f_{r,x}^{exp}}$$

$$\Rightarrow \varphi_{r,r}^{-1}(H)A_{r,r}(H) + (1 - \delta) \sum_{s \neq r} \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} \left( \frac{a}{a + 1 - \sigma} \right) \varphi_{r,s}^{c}(H) A_{r,s}(H) - \delta w_{r,s}^{f_{r,x}^{exp}} = \delta w_{r,s}^{f_{r,x}^{exp}}$$

$$\Rightarrow \varphi_{r,r}^{-1}(H)A_{r,r}(H) + (1 - \delta) \sum_{s \neq r} \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} \left( \frac{\sigma - 1}{a + 1 - \sigma} \right) \delta w_{r,s}^{f_{r,x}^{exp}}$$

(31)

Subtracting the free entry condition (30) in state \( L \) from (31) in state \( H \), we have that for each \( r \):

$$\varphi_{r,r}^{-1} [A_{r,v}(L) - A_{r,r}(H)] + (1 - \delta) \sum_{s \neq r} \left[ \Gamma_{r,s} * (1 - 1_{RankB_{r,s}}) + A_{r,s} * 1_{RankB_{r,s}} \right] = 0$$

where

$$\Gamma_{r,s} = (1 - \gamma) \left( \left[ \left( \frac{b}{\varphi_{r,s}^{c}(L)} \right)^{a} - \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} \right] \left[ \frac{(\sigma - 1) (\delta + \gamma - \delta \gamma)}{a + 1 - \sigma} \right] \right)$$

and

$$A_{r,s} = -(1 - \gamma) \left( \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} \right) \left[ \frac{(\sigma - 1) (\delta + \gamma - \delta \gamma)}{a + 1 - \sigma} \right] w_{r,s}^{f_{r,x}^{exp}}$$

$$+ \left( \frac{a \delta - 1}{a + 1 - \sigma} + a (1 - \delta) \left( \frac{\varphi_{r,s}^{c}(L)}{\varphi_{r,s}^{c}(H)} \right)^{\sigma - 1} - (\delta + \gamma - \delta \gamma) \right) w_{r,s}^{f_{r,x}^{exp}}$$

$$+ (1 - \delta) \gamma (1 - \delta) \left( \left( \frac{1}{\varphi_{r,s}^{c}(H)} \right)^{a} - \left( \frac{\varphi_{r,s}^{c}(L)}{\varphi_{r,s}^{c}(H)} \right)^{\sigma - 1} \right) \left( 1 \right) - \left( \left( \frac{b}{\varphi_{r,s}^{c}(H)} \right)^{a} - \left( \frac{b}{\varphi_{r,s}^{c}(L)} \right)^{a} \right) \right) w_{r,s}^{f_{r,x}^{exp}}$$

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Then, using the expressions for cutoff productivities and the fact that $A_{r,r}(\zeta) = A_{q,r}(\zeta) \frac{\lambda_{q,r}}{\lambda_{r,r}} \Gamma_{q,r}(\zeta) \left( \frac{w_r}{w_q} \right)^{1-\sigma}$, for $q \neq r$, $1_{Rank_{B_{q,r}}} = 0$, this gives

$$
\frac{a}{a+1-\sigma} \left( b^{\sigma-1} \frac{\lambda_{r,r}}{\lambda_{q,r}} \frac{r_{q,r}}{r_{q,r}} \right) \left( \frac{w_r}{w_q} \right)^{1-\sigma} \left[ A_{q,r}(L) + \frac{\gamma(1-\delta)}{\delta} A_{q,r}(H) - \frac{\delta + \gamma - \delta \Gamma}{\delta} A_{q,r}(H) \right] \\
+ (1-\delta) \sum_{s \neq r} \left[ \Gamma_{r,s} * (1 - 1_{Rank_{B_{r,s}}}) + \Lambda_{r,s} * 1_{Rank_{B_{r,s}}} \right] = 0
$$

$$
\Rightarrow K_{q,r} (\delta + \gamma - \delta \Gamma) \left[ \varphi_{q,r}^c(L)^{1-\sigma} - \varphi_{q,r}^c(H)^{1-\sigma} \right] + (1-\delta) \sum_{s \neq r} \left[ \Gamma_{r,s} * (1 - 1_{Rank_{B_{r,s}}}) + \Lambda_{r,s} * 1_{Rank_{B_{r,s}}} \right] = 0
$$

while for $q \neq r$, $1_{Rank_{B_{q,r}}} = 1$, this gives

$$
\frac{a}{a+1-\sigma} \left( b^{\sigma-1} \frac{\lambda_{r,r}}{\lambda_{q,r}} \frac{r_{q,r}}{r_{q,r}} \right) \left( \frac{w_r}{w_q} \right)^{1-\sigma} \left[ \delta w_q f_{q,r}^{exp} - \delta w_q f_{q,r}^{exp} \right] \\
+ (1-\delta) \sum_{s \neq r} \left[ \Gamma_{r,s} * (1 - 1_{Rank_{B_{r,s}}}) + \Lambda_{r,s} * 1_{Rank_{B_{r,s}}} \right] = 0
$$

$$
\Rightarrow K_{q,r} [\varphi_{q,r}^c(L)^{1-\sigma} - \varphi_{q,r}^c(H)^{1-\sigma}] + (1-\delta) \sum_{s \neq r} \left[ \Gamma_{r,s} * (1 - 1_{Rank_{B_{r,s}}}) + \Lambda_{r,s} * 1_{Rank_{B_{r,s}}} \right] = 0
$$

So, we have that for any $q \neq r$,

$$
K_{q,r} \left[ \varphi_{q,r}^c(L)^{1-\sigma} - \varphi_{q,r}^c(H)^{1-\sigma} \right] \left[ 1_{Rank_{B_{q,r}}} + (\delta + \gamma - \delta \Gamma) * (1 - 1_{Rank_{B_{q,r}}}) \right] \\
+ (1-\delta) \sum_{s \neq r} \left[ \Gamma_{r,s} * (1 - 1_{Rank_{B_{r,s}}}) + \Lambda_{r,s} * 1_{Rank_{B_{r,s}}} \right] = 0 \tag{32}
$$

where $K_{q,r} = \frac{a}{a+1-\sigma} \left( b^{\sigma-1} \frac{\lambda_{r,r}}{\lambda_{q,r}} \frac{r_{q,r}}{r_{q,r}} \right) \left( \frac{w_r}{w_q} \right)^{1-\sigma} w_q f_{q,r}^{exp}$ as before, and now $\tau_{q,r}$ does not depend on the state since we are considering an industry $k$ which is not directly affected by the tariff shock.

Now, the free entry condition in state $H$, (31), can be rewritten entirely in terms of cutoff productivities in state $H$. For $q \neq r$,

$$
K_{q,r} \varphi_{q,r}^c(H)^{1-\sigma} + (1-\delta) \sum_{s \neq r} \left( \frac{b}{\varphi_{q,r}^c(H)} \right)^a \left( \frac{\sigma - 1}{a+1-\sigma} \delta w_r f_{r,s}^{exp} \right) = \delta w_r f_{r,s}^{sunk} \tag{33}
$$

Equations (32) and (33) together yield $2 * C(C-1)$ equations (where $C$ is the number of countries) in $2 * C(C-1)$ unknowns, $\varphi_{q,r}^c(L)$ and $\varphi_{q,r}^c(H)$ for each $q \neq r$. Although I cannot solve for these variables explicitly in order to verify that there is a unique solution to the system of equations, I can verify that there is a solution which satisfies $\varphi_{q,r}^c(L) = \varphi_{q,r}^c(H)$ $\forall q \neq r$. If $\varphi_{q,r}^c(H) = \varphi_{q,r}^c(L)$, then by definition $1_{Rank_{B_{r,s}}} = 0$, and it is clear that $\Gamma_{r,s} = 0$. Then, for $\varphi_{q,r}^c(H) = \varphi_{q,r}^c(L) \forall q \neq r$, we have that equation (32) is satisfied.

\textsuperscript{19}It is also true that in this case $\Lambda_{r,s} = 0$, as one would expect given that $1_{Rank_{B_{r,s}}}$ changes values from 0 to 1 exactly at when $\varphi_{r,s}^c(H) = \varphi_{r,s}^c(L)$; that is, all of the above equations continue to be valid if we define $1_{Rank_{B_{r,s}}} = \begin{cases} 0 & \text{for } \varphi_{r,s}^c(L) < \varphi_{r,s}^c(H) \\ 1 & \text{for } \varphi_{r,s}^c(L) \geq \varphi_{r,s}^c(H) \end{cases}$ rather than $1_{Rank_{B_{r,s}}} = \begin{cases} 0 & \text{for } \varphi_{r,s}^c(L) \leq \varphi_{r,s}^c(H) \\ 1 & \text{for } \varphi_{r,s}^c(L) > \varphi_{r,s}^c(H) \end{cases}$ as is done here.