Foreign Currency Debt and Exchange Rate Regimes in the Prospective Monetary Union of the ECOWAS Countries

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Abstract: Corporates in developing countries often issue foreign currency denominated debt. Liability dollarization carries additional risks since large devaluation of the real exchange rate can suddenly raise default probabilities. We use a small open economy Dynamic Stochastic General Equilibrium model with the “balance sheet channel” similar to Bernanke et al (1999) explicitly modeled to study the implications of liability dollarization for the conduct of monetary policy. Bayesian estimation methods are employed and the model is estimated for the Economic Community of the West African States (ECOWAS). We find that a floating regime offers greater stability than a hard peg. Results are robust to different model parameters, except for the degree of openness, highlighting the role of demand-switching effects.

Keywords: Corporate debt default risk; Exchange rate policy; DSGE models; Bayesian estimation; ECOWAS

JEL Classification: E3, E4, F3

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1. Introduction

Developing economies like those in the ECOWAS often finance the accumulation of physical capital by issuing foreign currency denominated debt, a phenomenon referred to as “original sin” by Eichengreen and Hausmann (1999). For the first time, many of them are able to borrow in international financial markets, selling so-called Eurobonds, which are usually denominated in dollars or euros. The World Bank Debtor Reporting System (DRS) confirms that in 2010 around 30 percent of developing countries’ external net debt inflows are denominated in developed countries currencies such as the U.S dollar. More recently, countries such as Angola, Cote d’Ivoire, Gabon, Ghana, Namibia, Nigeria, Rwanda, Senegal, Seychelles, and Zambia have tapped into international debt markets (figure 1 and 2). In total, more than 20 percent of the 48 countries in sub-Saharan Africa have sold Eurobonds. Moreover, a few corporate entities in sub-Saharan Africa (SSA) have also successfully issued Eurobonds. For example, Guarantee Trust Bank in Nigeria raised 500 million dollar in a five-year bond offering in 2011, and Ghana Telecom issued 300 million dollar in five-year bonds in 2007 (Masetti, 2013).

The factors propelling the reversal include the dire infrastructure gaps, and large borrowing needs which often exceed aid flows and domestic saving. In addition, local currency bond markets in SSA in general are at a nascent stage of development, which otherwise could offer alternative sources of finance. First, market capitalization of government securities and corporate bonds are typically much lower than those of other developing, emerging and developed economies (figure 3). Second, there is more apparent disparity for corporate bond in bond. Indeed, the average corporate capitalization of corporate bond was 1.8 percent of GDP in 2010 for sub-Saharan Africa, whereas the figure was generally much larger for other developing and emerging economies. Also evident is that the local currency bond market in the region is denominated by government securities, with a share of 89.2 percent of the total market capitalization, compared to the share of corporate bond which stands at just 10.8 percent in 2010 (Sy, 2013).

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2 Kenya, Tanzania and Uganda are expected to join the crowd in the near future.
3 See Foster and Briceño-Garmendia (2010).
4 In recent year, there is a reversal. The government securities market capitalization has tended to fall, from 18.7 percent of GDP in 2006 to 14.1 percent in 2009. When taken together, the share of corporate bond in total bonds has increased rapidly from 5.1 percent in 2006 to 10.8 percent in 2010 (Mu, 2013).
Figure 1: Yields on Eurobonds issued by sub-Saharan African countries
Source: Bloomberg L.P.

Figure 2: Bond selling by sub-Saharan African countries in international capital markets
Source: Sy (2013)

Figure 3: Local Bond Market Comparisons, 2010 (percent GDP)
Source: World economic and financial surveys
*Excluding South Africa
When credit-constrained corporates’ assets are denominated in domestic currency while liabilities are denominated in foreign currency, an exchange rate depreciation is likely to wreak havoc on their net worth by raising the debt burdens and making it more expensive to repay.\(^5\) As a result, firms’ costs of capital increase, leading to a contraction in equilibrium investment. In addition, whatsoever access firms in these countries have on world capital markets, their borrowing tends to be collateralized and subject to nontrivial finance premiums above the international lending rate. The effect of foreign currency denominate debt along with the (sizeable) risk premium give rise to what is referred to as the “financial accelerator mechanism” pioneered by Bernanke et al (1999).\(^6\)

Even though a depreciation can potentially boost export volumes by promoting competitiveness, it can also trigger a potentially severe recession due to balance sheet effects.

The paper aims to study how the balance sheet channel can shape the choice of appropriate exchange rate policy. In the model, credit-constrained firms are exposed to foreign currency denominated debt and their borrowing constraints depend on the state of their balance sheets. An unexpected nominal exchange rate depreciation is likely to increase their default risk, and potentially offset the standard expansionary effects of a depreciation. The model is calibrated and estimated for Nigeria, Ghana and the West African Economic Monetary Union (WAEMU).\(^7,8\)

Specifically, we compare the performance of two exchange rate regimes: Pure fixed and floating regimes.\(^9\)

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\(^5\)The 1997 Asian crisis is a good example. The crisis was in part due to the maintenance by these countries of soft peg for too long, which encouraged external borrowing and led to excessive exposure to foreign exchange risk in both the financial and corporate sectors. Monetary authorities have been reluctant to allow their currency to float freely—the “Fear of Floating” argument advanced by Calvo and Reinhart (2001).

\(^6\) We use interchangeably the “financial accelerator mechanism” the “balance sheet channel” to express the same phenomenon.

\(^7\) The ECOWAS is comprised of Benin, Burkina Faso, Cape-Verde, Cote-d’Ivoire, The Gambia, Ghana, Guinea, Guinea-Bissau, Liberia, Mali, Niger, Nigeria and Sierra Leone, Senegal and Togo. The WAEMU is a subset of the ECOWAS and encompasses 8 countries, namely Benin, Burkina Faso, Cote-d’Ivoire, Guinea, Mali, Niger, Senegal and Togo. Five non-WAEMU countries including The Gambia, Ghana, Guinea, Nigeria and Sierra Leone have created the West African Monetary Zone (WAMZ) in 2003, which later is meant to merge with the WAEMU and form the currency union of the ECOWAS.

\(^8\) Sy (2013) shows that after the middle income African countries with a relatively developed corporate external bond market capitalization, the WAEMU zone display a significant one, yet at infancy level. Diouf and Boutin-Dufresne (2012) and Sy (2007) provide a detailed description of the local bond market in the WAEMU region.

\(^9\) These two regimes are in line with the surge in the corner solutions in recent year around the world in the wake of the Asian crisis. See Frankel, Schmukler, and Servén (2001) and Fischer (2001), Obstfeld and
In the wake of the financial crisis of the 1990s, a number of studies have considered issues relating the balance sheet channel with the conduct of monetary policy (Krugman (1998); Aghion, Bacchetta, Banerjee (200, 2001)). Recent studies carried out by Cook (2004), Eleckdag and Tchakarov (2007), Cespedes et al. (2004), Devereux et al. (2006) and Gertler et al. (2007) have analyzed the credit channel in developed and emerging economies with well-functioning financial markets and the role of exchange rate policy. The results of these studies allow us to classify them into two groups: Cook (2004), Eleckdag and Tchakarov (2007) found a greater role for the fixed exchange rate in macroeconomic stabilization of the emerging economies, while Cespedes et al. (2004), Devereux et al. (2006) and Gertler et al. (2007) emphasize the primacy of the flexible exchange rate regime on the fixed exchange rate, which is consistent with the recommendation of the standard Mundell-Fleming framework.

From a theoretical point of view, these studies are subject to criticisms because they assume a complete exchange rate pass through, perfect mobility of capital and flexible domestic import prices. The assumption of complete pass-through is in stark contrast with the well-established empirical evidence that deviations from the law of one price for traded goods prices are large and pervasive. Empirical evidence by Akofio-Sowah (2009) on Sub-Saharan Africa (SSA) and on Latin American countries points to incomplete pass-through as a result of low inflationary environments. Within the SSA region, the Common Market for Eastern and Southern Africa (COMESA) countries have the highest inflation and therefore the exchange rate pass-through in those countries is 25 to 50 percent higher than that in the WAMZ, the WAEMU and the Central African Economic and Monetary Community (CEMAC). In the same vein, Diop et Fall (2011) also assume incomplete pass-through in a dynamic stochastic general equilibrium model to study which exchange rate regime would be relevant for the ECOWAS members. They find that a fixed regime is likely to foster more stability without undermining growth performances of those countries. With respect to this assumption, our framework is similar to theirs, but different in that it focus on the balance sheet channel of monetary transmission.

Rogoff (1995), for interesting discussion on the corner solutions hypothesis. Since the intermediate case, soft peg, varies within a band with the pure fixed and floating exchange rate regime being its lower and upper bound respectively, we do not explicitly implement the experiment under this regime; rather we provide a perception of its effects drawing on the two mentioned regimes.
The main findings of this paper can be summarized as follows. We find that a floating regime offers greater stability of corporates’ balance sheet in the presence of risk premium shocks and leads to greater business cycle stability than pure fixed regime. These findings are consistent with the conventional framework of Mundell-Fleming which highlights that free float acts as a shock absorber. It follows that the standard policy recommendation holds: small open economies with greater exposure to being adversely affected by external disturbances should implement a float. The results are robust to important model parameters, yet affected by the degree of openness of the economy, which reveals the role of demand switching effects.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 explains the calibration and econometric strategies used to estimate the parameters of the model, the estimation results and impulse response functions of the shocks. Finally, section 4 conducts robustness checks and section 5 concludes.

2. The Model

The core framework in this paper is a typical new Keynesian small open economy DSGE model with nominal rigidities, which is key for investigation of monetary policy. It builds on Sangaré (2013), focusing on three aspects of the model: first it accounts for incomplete pass-through (Monacelli, 2005); second, it includes the financial accelerator mechanism à la Bernanke et al. (1999) by linking domestic firms’ borrowing conditions—the cost of capital induced by the risk premium—to the state of their balance sheets; and third, the model assumes imperfect capital mobility. The phenomenon of the original sin is captured in the framework by assuming that an important part of the debt in the economy is denominated in foreign currency. Through the borrowers’ balance sheets, the financial accelerator mechanism works to amplify and ensure the persistence of shocks to the economy. We then extend on these model features to include habit formation in consumption utility (Justiniano and Preston, 2004) to allow for a smoothed consumption path and to avoid unrealistically drastic adjustments. Furthermore, the extended model exhibits two types of firms encompassing firms adopting forward-looking behavior on the one hand, and firms endowed with backward-looking behavior in price setting on the other. Previous studies consider only the presence of forward-looking firms thus overlooking the well-established evidence on the rule of thumb price-setting behavior of some firms (see Fuhrer and Moore (1995) and Rudd and Whelan (2005) among others). In addition, while most studies focuses
on emerging Asian countries, this paper is interested in linking financial market frictions with the choice of monetary policy regime in the prospective currency union of the founding members of the ECOWAS or a subset of countries of this.

The framework contains the salient features of the standard new Keynesian small economy DSGE model with respect to the optimizing behavior of the microeconomic units, entrepreneurs, capital producers and household, government, the monetary authority and a foreign sector. Households supply labor to entrepreneurs and consume tradable goods that are produced both domestically (H) and abroad (M). Credit-constrained firms borrow both in foreign currency and in domestic currency (see chart 1). Their demand for capital depends on their net worth via payment of a risk premium. This is the key aspect of the financial accelerator channel.

Chart 1: Flow Chart of the Economy

A continuum of monopolistically competitive firms (retailers) operating through domestic and foreign market set their prices in the local market on a staggered basis à la Calvo (1983). This
helps to explain inflation inertia and output persistence. Capital accumulation is subject to adjustment costs. The deviation from the law of one price is introduced in the model to account for the assumption of incomplete pass-through.

2.1. Households

The domestic small open economy is populated by a continuum of infinitely-lived maximizing households. The intertemporal utility function of the households depends positively on consumption \( C_t \) relative to an external habit formation \( hC_{t-1} \) and negatively on labor supply \( L_t \):

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{(L_t)^{1+\eta}}{1+\eta} \right)
\]

(1)

Where \( 0 < \beta < 1 \) is the discount factor; \( \sigma > 0 \) is the coefficient of relative risk aversion (or inverse of the intertemporal elasticity of consumption, and \( \eta > 0 \) is the inverse elasticity of labor supply; \( C_t \) is a CES function defined over domestic goods and imported goods:

\[
C_t = \left[ (1 - a)^{\frac{1}{\theta}}(C_{H,t})^{\frac{\theta-1}{\theta}} + a^{\frac{1}{\theta}}(C_{M,t})^{\frac{\theta-1}{\theta}} \right]^\frac{\theta}{\theta-1}
\]

(2)

Where \( C_{H,t} \) and \( C_{M,t} \) stand for the usual CES aggregators of the quantities of domestic and foreign goods respectively, and \( \theta > 0 \) is the elasticity of substitution between both types of goods; \( 0 < a < 1 \) is the share of foreign-produced goods in the consumption bundle reflecting the degree of openness of the domestic economy.\(^{10}\)

\[
C_{H,t} = \left( \int_0^1 C_{H,t}(j) \frac{x^{\chi-1}}{\chi} \; dj \right)^{\frac{\chi}{\chi-1}} \quad \text{and} \quad C_{M,t} = \left( \int_0^1 C_{M,t}(j) \frac{x^{\chi-1}}{\chi} \; dj \right)^{\frac{\chi}{\chi-1}},
\]

\( \chi > 1 \) is the elasticity of substitution between the different varieties of goods and \( C_{H,t}(j) \) stands for the consumption of the variety \( j \) of the domestic and foreign good. The consumer price index associated with equation (2) is defined as:

\(^{10}\) We pay attention to this parameter later in this paper for robustness check purpose.
\[ P_t = \left[ (1 - a)(P_{H,t})^{\theta-1} + a(P_{M,t})^{\theta-1} \right]^{\frac{1}{\theta-1}} \]  

(3)

In the same vein, the corresponding aggregate prices over the varieties \( j \) of domestic and foreign goods are given by:

\[ P_{H,t} = \left( \int_0^1 P_{H,t}(j) x^{-1} d_j \right)^{\frac{1}{x-1}} \quad \text{and} \quad P_{M,t} = \left( \int_0^1 P_{M,t}(j) x^{-1} d_j \right)^{\frac{1}{x-1}}. \]

Optimal allocation of expenditures between domestic and foreign goods can be written as

\[
\min_{C_{H,t}, C_{M,t}} P_{H,t} C_{H,t} + P_{M,t} C_{M,t} = P_t C_t
\]

s.t \quad C_t = \left[ (1 - a)\frac{1}{\theta} \left( \frac{P_{H,t}}{P_t} \right)^{\theta-1} C_t + a\frac{1}{\theta} \left( \frac{P_{M,t}}{P_t} \right)^{\theta-1} C_t \right]^{\frac{1}{\theta}}

(4)

This expenditure minimization on domestic and foreign goods yields the demand functions for domestically produced and imported goods as in the following:

\[ C_{H,t} = (1 - a) \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C_t; \quad C_{M,t} = a \left( \frac{P_{M,t}}{P_t} \right)^{-\theta} C_t \]

The household budget constraint is given by:

\[ P_t C_t + R_{t-1} B_{t-1} + R_{t-1}^{W} \Psi_{D,t-1} S_t D_{H,t-1} + \tau_t = W_t L_t + B_t + S_t D_{H,t} + \Lambda_t + T_t \]  

(5)

Following Devereux et al. (2006), we assume that households purchase public bond in local currency \( B_t \) at a nominal interest rate \( r_t = R_t - 1 \), and that part of their debt is denominated in foreign currency, \( D_{H,t} \). The nominal interest rate associated to the latter debt is \( r_t^w = R_t^w - 1 \), while \( \Psi_{D,t} \) stands for the country borrowing premium (detailed description of that follows later in this section). We introduce this country borrowing premium to account for the assumption of imperfect international capital mobility and partly for technical reasons on the stationarity of the total net foreign indebtedness (Schmitt-Grohe and Uribe 2003). Following Sangaré (2013), the country borrowing premium is a modified version of Adolfson et al. (2008) as follows:

\[ \Psi_{D,t}(d_t, ER, Z_t) = r_t - r_t^w = Z_t \exp \left( \psi_D \left( \frac{S_t D_t}{Y_{Pt}} \right) \right) E_t \left( \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \right), \]
Where \( d_t = \frac{S_t D_t}{Y P_t} \) is the total debt to GDP ratio in period \( t \), \( ER_t = \frac{S_{t+1}}{S_t} \) is changes in the exchange rate and \( R^w \) is the risk-free world interest rate. \( \Psi_{D,t} \) is an increasing function of the total net foreign indebtedness \((\Psi_{D,t})_d > 0 \) and \( \Psi_{D,t}(0,0) = 1 \); \( D_t \) is total debt of the country and comprises \( D_{H,t} \) (the households foreign debt) and \( D_{E,t} \) (the entrepreneurs foreign debt) \((D_t = D_{H,t} + D_{E,t})\). We elaborate on \( D_{E,t} \) in the next sections; \( \psi_D \) is the elasticity of the country’s borrowing premium with respect to the debt and \( Z_t \) stands for a random shock:

\[
Z_t \sim AR(1), \quad \log(Z_t) = \zeta z_{t-1} + \varepsilon_{z,t}, \text{with } \varepsilon_{z,t} \sim i.d(\sigma^2).
\]

Besides the financial borrowing, the flow of the households’ income is composed of nominal wages \( W_t \) from labor services and profits \( \Lambda_t \) of monopolistically competitive firms they own. They also receive transfers \( T_t \) from government, which represents the lump sum tax payment \( \tau_t \). \( S_t \) stands for the nominal effective exchange rate and \( R_{t-1} B_{t-1} + R^w_{t-1} \Psi_{D,t-1} S_{t-1} D_{H,t-1} \) is the total gross refund on the borrowings contracted by the households at \( t - 1 \).

The representative household chooses the set \( \{C_t, L_t, B_t, D_{H,t}\}_0^\infty \) that maximizes its intertemporal utility (1) subject to its budget constraint (5). The first order conditions of the maximization problem are given by:

\[
\frac{(L_t)^n}{Q_t} = \frac{W_t}{P_t} \tag{6}
\]

\[
1 = \beta R_t E_t \left( \frac{Q_{t+1} P_t}{Q_t P_{t+1}} \right) \tag{7}
\]

\[
Q_t = \beta R^w_t \Psi_{D,t}(d_t, Z_t) E_t \left( \frac{Q_{t+1} P_t S_{t+1}}{P_{t+1} S_t} \right) \tag{8}
\]

Where \( Q_t = (C_t - h C_{t-1})^{-\sigma} \)

The first order conditions of the consumer’s problem are standard and can be written in a log-linearized form as:
\[ w_t - p_t = \eta L_t + \frac{\sigma}{1 - h} (C_t - hC_{t-1}) \] (9)

\[ C_t = \frac{h}{1 + h} C_{t-1} + \frac{1}{1 + h} E_t C_{t+1} - \frac{1 - h}{\sigma(1 + h)} (r_t - E_t \pi_{t+1}) \] (10)

Where \( \pi_{t+1} \) is the next period’s overall inflation in the economy defined as \( P_{t+1} - P_t \). Condition (9) and (10) can be viewed as the marginal rate of substitution between consumption and labor while (8) is the famous Euler equation of consumption. Combining equations (7) and (8) yields the usual condition of the Uncovered Interest Parity (UIP) adjusted for the risk premium.

2.2. The Real Exchange Rate, the Terms of Trade, and Incomplete Pass-Through

One of the recent developments in open economy New Keynesian DSGE is the modeling of the deviation of prices from the Law of one price referred to as the law of one price gap (Monacelli, 2005). The claim is that monopolistically competitive firms exert some power on price of goods they import and distribute thus creating a distortion between the domestic and foreign prices of these imported goods when expressed in the same currency. It is this distortion that is referred to as the law of one price gap. It is assumed that the Law of one price does not hold for import goods in this study.

In this section, we are concerned with the link between inflation, the real exchange rate (RER) and terms of trade (TOT). We define three types of inflation in the economy: the domestic inflation \( \pi_{H,t} \) which stems from price setting rules of domestic goods by firms, the imported inflation \( \pi_{M,t} \) resulting from price setting rules of imports by firms, and finally the consumer price-based inflation \( \hat{\pi}_t \). Taking the log-linearized form of equation (3) and then taking the first-difference yields equation (11) which is a weighted average of the two types of inflation we just mentioned.

\[ \hat{\pi}_t = (1 - a) \hat{\pi}_{H,t} + a \hat{\pi}_{M,t} \] (11)

The terms of trade is defined as follows:

\[ TOT_t = \frac{P_{M,t}}{P_{H,t}} \] (12)
Log-linearizing (12) around the steady-state yields the following:

\[ \hat{t}_{t} = \hat{p}_{M,t} - \hat{p}_{H,t} \]

Taking the first-difference yields \( \Delta \hat{t}_{t} = \hat{p}_{M,t} - \hat{p}_{H,t} \). We then substitute this in (11) to get

\[ \hat{\pi}_{t} = \hat{\pi}_{H,t} + a \Delta \hat{t}_{t} \tag{13} \]

From equation (13), it is possible to say that the difference between the total and domestic inflation rates is proportional to the terms of trade and that proportionality increases with the degree of openness of the domestic economy.

Furthermore, we define the real exchange rate \( RER_t \) through the following relationship:

\[ RER_t = \frac{S_t p_t^w}{p_t} \tag{14} \]

Under the hypothesis of complete pass-through the price of import in domestic currency is given by \( p_{M,t} = S_t p_t^w \), which means that any idiosyncratic change in exchange rate completely spills over into domestic prices. In contrast, under incomplete pass-through—as it is the case in this study—the law of one price does not hold and therefore \( p_{M,t} \neq S_t p_t^w \).

The Law of one price gap is therefore given by the ratio of the foreign price index in terms of domestic currency to the domestic currency price of imports.

\[ LOPG_t = \frac{S_t p_t^w}{p_{M,t}} \tag{15} \]

Note that the law of one price holds only if \( LOPG_t = 1 \). Otherwise, the Law of one price does not hold. It is worth mentioning that through this study, the law of one price holds for exports. This is a realistic assumption since it assumes that the economies we are concerned with in this study are price takers in international markets for their exports. In contrast, importing firms are monopolistically competitive and have a small degree of pricing power in the domestic market, a novelty of Monacelli (2005)’s model (see section 2.3 for more details on that). This means that when retail firms sell imported goods to domestic consumers, they charge a mark-up over their
costs, creating a wedge between the world market price of foreign goods and domestic currency price of these goods.

Ultimately, the link between the Law of one price gap, the terms of trade and the real exchange rate is obtained by combining the log-linearized versions of (12), (13), (14) and (15) as follows:

\[
\hat{r}e_{t} = \log g_{t} + (1 - a)\hat{o}_{t, t} \tag{16}
\]

Equation (16) deserves some explanations. It shows that the deviation from aggregate PPP is driven by two factors. The first one is due to the heterogeneity of consumption basket between domestic goods and imported goods, an effect captured by the term \((1 - a)\hat{o}_{t, t}\), as long as \(a < 1\). For \(a \to 1\), in fact, the two aggregate consumption baskets coincide and relative price variations are not required in equilibrium. The second source of deviation from PPP is due to the deviation from the law of one price, captured by movements in \(\log g_{t}\).

### 2.3. Firms

There are four types of entrepreneurs in the economy: wholesale entrepreneurs, capital producers, domestic goods retailers operating both on domestic and international markets, and imported goods retailers.

#### 2.3.1 Wholesalers and the Financial Accelerator

There is a continuum of perfectly competitive wholesale firms \(j \in [0,1]\) producing wholesale goods with a Cobb-Douglas-type technology of production:

\[
Y_{t} = A_{t}K_{t}(j)^{\alpha}L_{t}(j)^{1-\alpha} \tag{17}
\]

Where \(A_{t}\) is a technology shock following an AR(1) process:

\[
\log(A_{t}) = \zeta_{A} \log A_{t-1} + \epsilon_{A,t}, \text{ where } 0 < \zeta_{A} < 1 \text{ is a persistence parameter and } \epsilon_{A,t} \sim i.i.d(o, \sigma_{t}^{2})
\]

\(K_{t}\) is the capital factor and \(L_{t}\) is the labor factor supplied by households; \(0 < \alpha < 1\) is the share of the capital factor in the production function.

Following Bernanke et al. (1999), we assume that firms are credit-constrained and never accumulate enough funds to fully self-finance their capital acquisitions. This assumption is taken
into account by assuming that firms have a finite expected horizon. Each survives until the next period with a probability \( \nu \). Accordingly, the expected horizon is given by \( 1/(1 - \nu) \). We assume that firms borrow only in foreign market and that their borrowing is denominated in foreign currency (Eleckdag and Tchakarov, 2007). Borrowings from foreign lenders are subject to payment of a risk premium denoted by \( \Phi \). If \( Q_t \) and \( N_t \) represent the price of capital and entrepreneur’s net worth respectively, then the entrepreneurs’ net worth is expressed in each period \( t \), by the following budget constraint:

\[
Q_t N_{t+1} = Q_t K_{t+1} - S_t D_{E,t+1}
\]  

(18)

Where \( S_t \) is the exchange rate and \( D_{E,t+1} \) is the entrepreneur’s foreign debt in period \( t + 1 \). Equation (18) tells us that the entrepreneur’s net worth is the difference between it asset and liability. Any unanticipated depreciation of the exchange rate raises the cost of capital and worsens the entrepreneur’s net worth. The framework assumes that entrepreneurs are risk neutral and choose the level of capital \( K_{t+1} \) and the associated borrowing \( D_{E,t+1} \) which maximize their profits. When the optimality conditions satisfying the financial contract between the borrower and the foreign lender are reached, \(^{11}\) then the expected return on capital \((E_t R_{K,t+1})\) is equal to the marginal cost of the external fund, that is, the gross interest rate of the rest of the world \( R_t^w \) adjusted for changes in the exchange rate \( \frac{S_{t+1}}{S_t} \) plus country-specific risk-premium \( \Psi_{D,t} \) and external finance premium \( \Phi \).

\[
E_t R_{K,t+1} = \Phi \left\{ R_t^w \left( \Psi_{D,t} \right) E_t \left( \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \right) \right\}
\]  

(19)

The external finance premium \( \Phi \) depends on the entrepreneur’s net worth \( \frac{P_t N_{t+1}}{Q_t K_{t+1}} \). In general, it varies inversely with the entrepreneur’s net worth. Therefore, the greater the share of capital the entrepreneur can either self-finance or finance with collateralized debt, the smaller the expected bankruptcy costs and, the smaller the external finance premium: \( \Phi = \left( \frac{N_{t+1}}{Q_t K_{t+1}} \right)^{-\gamma} \) where \( \gamma \) is the

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\(^{11}\) Interested readers should refer to Bernanke et al (1999) for more details on the optimization problems arising from the financial contracts between the two parties.
elasticity of the external finance premium with respect to entrepreneurs’ net worth capital ratio, and \( q_t \) is the price of capital in real terms \( (q_t = \frac{Q_t}{P_t}) \) and \((\Phi)'<0; \Phi(1)=1\).

Equation (19) provides the basis for the financial accelerator since it links movements in the borrower financial position to the marginal cost of funds and, hence, to the demand for capital.

Now we link the return to the entrepreneur’s capital, \( R_{K,t} \), with the marginal productivity of capital \( mpc_t \). The gross return on investment per unit of capital is measured as the sum of the marginal productivity of capital arising from the production process plus non depreciated value of capital:

\[
R_{K,t}q_{t-1} = mpc_t + (1 - \delta)q_t
\]  

(20)

Where \( \delta \) is the rate of depreciation of capital.

Finally, the relation describing the evolution of entrepreneurial net worth \( N_{t+1} \) is worth mentioning. It can be expressed as a function of the value of entrepreneurial firms’ capital, net of borrowing costs carried over from the previous period, \( \nu \left[ R_{K,t}q_{t-1}K_t - R_{t}^w\Psi_{D,t}\left(\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}\right)\left(\frac{N_t}{q_{t-1}K_t}\right)^\gamma(q_{t-1}K_t - N_t)\right] \), plus the net worth left by firms who did not survive, \((1-\nu)\Omega_t\):

\[
N_{t+1} = \nu \left[ R_{K,t}q_{t-1}K_t - R_{t}^w\Psi_{D,t}\left(\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}\right)\left(\frac{N_t}{q_{t-1}K_t}\right)^\gamma(q_{t-1}K_t - N_t)\right] + (1 - \nu)\Omega_t
\]  

(21)

Where \( \nu \) is the proportion of firms who survive in the economy and \( \Omega_t \) is the net worth of firms who do not survive and leave the economy each time.

Equation (21) clearly shows that the evolution of entrepreneurs’ net worth is driven by the return on investment \( R_{K,t} \) and the world interest rate on borrowings, supplemented with country-specific risk premium \( (R_t^w\Psi_{D,t}) \). As the interest rate increases, the entrepreneur is not inclined to borrow in the foreign market, everything else being equal, and this reduces the availability of resource in the next period. The last source of fluctuations in the firms’ net worth is the variation of the exchange rate whose depreciation reduces the net worth.

2.3.2 Capital Producers
The activity pertaining to the role of capital producers in the economy consists of repairing depreciated capital goods and building new ones, all this being carried over in a competitive way. The production of new capital is subject to adjustment costs while the repair of old capital goods is not as in Eisner and Strotz (1963), Lucas (1967) and Gertler et al. (2006). It is also assumed that there is not possibility of substitution between old capital and new capital. The claim is that for the old capital to be productive, it should be repaired.

Both activities—old capital maintenance and production of new capital—use as input a composite investment good that is composed of domestic and foreign final goods:

\[ I_t = \left[ (1 - a)^\frac{1}{\sigma} (I_{H,t})^\frac{\sigma - 1}{\sigma} + (a)^\frac{1}{\sigma} (I_{M,t})^\frac{\sigma - 1}{\sigma} \right] ^{\frac{1}{\sigma - 1}} \]

(22)

The associated investment price index is denoted by \( P_t \). The number of units of investment goods required to replace the depreciated capital is \( \delta K_t \) whose costs are bore by the entrepreneurs who own the capital stock. Therefore the amount of the investment good used for the construction of new capital goods is given by \( I_t - \delta K_t \).

The adjustment costs associated with the production of new capital is given by the following quadratic form: \( \frac{\psi_I}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \). Similar to other attempts applying DSGE models to LDCs,\(^{12}\) this reflects absorptive capacity constraints, the fact that skilled administrators are in scarce supply in LDCs and, therefore, any sudden investment scaling up can create additional costs. The parameters \( \psi_I \) determine the severity of these absorptive capacity constraints. Accordingly, the law of motion of capital in the economy is given by:

\[ K_{t+1} = \left[ \frac{l_t}{K_t} \psi_I \left( \frac{l_t}{K_t} - \delta \right)^2 \right] K_t + (1 - \delta) K_t \]

(23)

Individual capital producers choose inputs \( I_t \) and \( K_t \) to maximize expected profits from the construction of new investment goods. If \( q_t \) denotes the price of capital, then capital producers

---

solve the following programme, \(\max_t q_t l_t - l_t - \frac{\psi_l}{2} \left( \frac{l_t - \delta K_t}{K_t} \right)^2 K_t\), which yields the following optimality conditions:

\[
q_t - \psi_l \left( \frac{l_t - \delta K_t}{K_t} \right) = 1
\]  

(24)

Equation (24) is the famous Tobin Q, which stems from a distortion induced by the cost of capital on price of capital. Therefore, the price of capital is variable by virtue of the adjustment cost. In the absence of this investment cost (\(\psi_l = 0\)), then \(q_t\) is identically the unity. The more the adjustment cost increases, the less the producers of capital are inclined to produce new capital. Accordingly, the price of capital increases which in turn increases affects negatively the entrepreneurial balance sheet in (21).

2.3.3 Price Setting

2.3.3.1 Price setting by domestic retailers

One important feature of the model is the accommodation of the assumption of Calvo (1983) type staggered-price setting.\(^{13}\) We assume there is a continuum of domestic firms buying wholesale goods from producers in a competitive way and then repackage them as final goods without any cost. It is assumed that retailers of the final goods are monopolistically competitive on domestic market whereas they are perfectly competitive on the international market. Put another way, the law of one price holds when they export, which is not the case for imports.\(^{14}\) Therefore, they sell in the foreign markets at the domestic price adjusted with the exchange rate as follows: \(P_{X,t} = \frac{P_{H,t}}{S_t}\).

In the domestic market the retailers set prices in the Calvo (1983) type price rigidity as follows: at a given point in time, a constant fraction \((1 - \phi^H)\) of randomly selected domestic retailers set prices optimally, while the other fraction \(\phi^H \in [0,1]\) keeps its price unchanged. Accordingly, the expected time the price of domestic goods remains unchanged is \(1/(1 - \phi^H)\). Furthermore, we assume that those firms who can reset their prices are of two types in the economy: “forward-

\[^{13}\] There are many reasons for the firm to charge a price level different from the optimal price level in the short run: menu cost, staggered prices, coordination failure, etc. (Snowdon and Vane, 2005).

\[^{14}\] This is realistic since it assumes that the country has no market power when selling in the international market.
looking firms” and “backward-looking firms”. Forward-looking firms are those firms that reset their prices optimally by exploiting all available information at the time of making decision. Backward-looking firms unlike set their prices on a rule of thumb basis. They assume the information they use is “sticky” so they collect and process it with delay at the time they set their prices optimally. Basically, they use their knowledge of the historical development of price levels (which is referred to as backward-looking).

Following Galí and Gertler (1999) and more recently Smeets and Wouters (2002), backward-looking firms are assumed to reset their prices, \( P_{H,t}^f(f) \) by indexing it to the last period inflation. Therefore the parameter \( \phi^H \) becomes a natural index of price stickiness. The index of domestic prices is therefore defined as:

\[
p_{H,t}^f(f) = P_{H,t-1}(f) \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\phi^H}
\]  
(25)

The aggregate domestic price is given by:

\[
P_{H,t} = \left\{ (1 - \phi^H) \bar{P}_{H,t}^{1-\rho} + \phi^H \left[ P_{H,t-1} \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\phi^H(1-\rho)} \right] \right\}^{\frac{1}{1-\rho}}
\]  
(26)

Where \( \bar{P}_{H,t}(f) \) is the new price each domestic firm \( f \) sets in order to maximize the present market value of its stream of expected future profits.

Log-linearizing (26) around the steady-state and taking the first difference yields the following relation for domestic inflation:

\[
\pi_{H,t} = (1 - \phi^H)(\bar{P}_{H,t} - P_{H,t-1}) + (\phi^H)^2 \pi_{H,t-1}
\]  
(27)

Firms re-optimize their prices and maximize their profits after setting the new price \( \bar{P}_{H,t}(f) \) at time \( t \) as:

\[15\] This is a crude assumption since it assumes that the degree of price stickiness is the same as the fraction of past inflation indexation. However, it validates a basic rationale of Philips curve. “In the long-run Philips curve is vertical”.  

18
\[
\max_{\bar{P}_{t+k}} \sum_{k=0}^{\infty} \left( \phi^H \right)^k E_t \left[ Q_{t+k}Y_{t+k}(f)(\bar{P}_{t+k} - MC_{t+k}P_{t+k}) \right]
\]

Subject to the following demand function:

\[
Y_{t+k} \leq \left( C_{t+k} + \bar{c}_{t+1} \right) \left[ \frac{\bar{P}_{t+k}}{P_{t+k}} \right]^{-\epsilon}
\]

Where \( MC_{t+k} \) is the real marginal cost and \( Q_{t+k} = \frac{1}{R_{t+1}} \) is a discount factor. The first order condition of the above programme is given by:\footnote{See Chuantantikamon (2008) and Haider and Khan (2008) for more detail.}

\[
\sum_{k=0}^{\infty} \left( \phi^H \right)^k E_t \left[ Q_{t+k}Y_{t+k}(f)(\bar{P}_{t+k} - \frac{\epsilon}{\epsilon - 1} NMC_{t+k}) \right] = 0 \tag{28}
\]

Where \( NMC_{t+k} = MC_{t+k}P_{t+k} \) and \( \frac{\epsilon}{\epsilon - 1} \) is considered as the marginal cost when all prices are flexible (see Gali, 2008).

Replacing \( Q_{t,t+1} \) by its expression in equation (7) and log-linearizing around zero-inflation steady-state, we obtain:

\[
\bar{P}_{t+1} = \bar{P}_{t+1} + \hat{\pi}_{t+1} + \sum_{k=1}^{\infty} \left( \beta \phi^H \right)^k \left\{ E_t \left[ \hat{\pi}_{t+k} \right] + (1 - \beta \phi^H) E_t \left[ \bar{c}_{t+k} \right] \right\} \tag{29}
\]

Solving equation (29) recursively and rearranging it yields the following New Keynesian Phillips Curve (NKPC):

\[
\bar{P}_{t+1} - \bar{P}_{t+1} = \beta \phi^H E_t \left[ \hat{\pi}_{t+1} \right] + \hat{\pi}_{t+1} + (1 - \beta \phi^H) \bar{c}_t \tag{30}
\]

Furthermore, replacing equation (27) in (30), we obtain ultimately the following hybrid Phillips curve:

\[
\hat{\pi}_{t+1} = (1 - \beta \phi^H) E_t \left[ \hat{\pi}_{t+1} \right] + \phi^H \hat{\pi}_{t+1} + \frac{(1-\beta \phi^H)(1-\phi^H)}{\phi^H} \bar{c}_t \tag{31}
\]
Where \( \hat{m}c_t \) denotes log-deviation of marginal cost from its steady state value. The NKPC equation (31) implies that home country’s inflation dynamics derives from both forward-looking \( E_t[\hat{\pi}_{H,t+1}] \) and backward-looking \( \hat{\pi}_{H,t-1} \) components. The above NKPC representation is also known as a hybrid version of NKPC with forward-looking and backward-looking behavior. Furthermore, the equation shows that real marginal cost is also a main determinant of domestic inflation.

2.3.3.2 Price setting by import goods retailers

As with the domestic firms, we assume that the importing firms operate in a monopolistically competitive market. What is distinguishing here is that the law of one price gap plays an import role in determining the inflation dynamics of imported goods. Since the law of one price fails to hold, then the price index of imports in domestic currency is no longer equal to the nominal exchange rate times the foreign price index \( (P_{M,t} \neq S_tP^W_t) \). Like previously shown as regard the domestic price setting of the domestic goods, domestic price of imported goods follows a Calvo-type price staggering. It implies that at a given point in time a fraction \( (1 - \phi^M) \) of firms adjust their prices while the remaining \( \phi^M \) cannot. Furthermore, we assume that among those firms who reset their prices some are “forward-looking” while the others are “backward-looking” firms.

The process underpinning the domestic price setting of importing firms is similar to the one defined in the case of domestic goods in equation (29). Therefore the monopolistically competitive importer’s optimal price behavior could be defined as:

\[
\hat{P}_{M,t} = \hat{P}_{M,t-1} + \pi^M_{M,t} + \sum_{k=1}^{\infty} (\beta^M)^k \{ E_t[\hat{\pi}_{M,t+k}] + (1 - \beta^M) E_t[\hat{m}c_{t+k}] \} 
\]

(32)

Where \( \hat{m}c_t = \frac{S_tP^W_t}{P_{M,t}} = LOPG_t \) is the real marginal cost of imported goods. Solving recursively (32) and rearranging, we obtain the following new Keynesian Phillips curve relation which relates the rate of inflation in the average domestic currency price of imports to three factors: The lagged inflation rates, the expected future inflation rates and the law of one price gap.

\[
\hat{\pi}^M_{M,t} = (1 - \beta^M) E_t[\hat{\pi}_{M,t+1}] + \phi^M \hat{\pi}_{M,t-1} + \frac{(1-\beta^M)(1-\phi^M)}{\phi^M} LOPG_t
\]

(33)
Finally, log-linearizing the consumer price index given by equation (3) around the steady-state and then taking the first difference we obtain the following log-linear form of overall inflation which is an average of domestic and imported inflations.

\[ \hat{\pi}_t = (1 - a)\hat{\pi}_{H,t} + a\hat{\pi}_{M,t} \]  

(34)

**2.4. Policy, the External Sector and Shocks**

The monetary authority a general inflation targeting interest rate rule to stabilize three targets as in Kollmann (2002): inflation, current output gap and nominal exchange rate depreciation.

\[ \hat{r}_t = \beta_0\hat{r}_{t-1} + (1 - \beta_0)[\beta_1\hat{\pi}_t + \beta_2\hat{y}_t + \beta_3\Delta\hat{s}_t] \]

(35)

where \( \beta_1, \beta_2, \) and \( \beta_3 \) are weight put by monetary authorities on inflation, GDP and variations of exchange rate respectively. When \( \beta_1 \to \infty \), the central bank is strictly targeting inflation and when \( \beta_3 \to \infty \), the central bank is implementing a fixed exchange rate regime. In the intermediate case, the central bank is implementing a managed floating regime.

The external sector is modeled in a symmetric way relative to the domestic economy since exports from the domestic country are defined as imports of the rest of the world from that country. Therefore, similar to the optimal domestic demand for imported goods in equation (4), the optimal demand of domestically produced goods by the rest of the world is given by:

\[ C_{H,t}^\omega = a\left(\frac{P_{H,t}^\omega}{P_t}\right)^{-\theta} Y_t^\omega \]  

(36)

Where \( Y_t^\omega = C_t^\omega \) is the total demand of the rest of the world. Since the law of one price holds for exports, the price of the domestic goods in the foreign market is \( P_{H,t}^\omega = \frac{P_{H,t}^\omega}{S_t} \).

Then, rearranging equation (36) yields the following expression of foreign demand as a function of the real exchange rate:

\[ C_{H,t}^\omega = a\left(\frac{P_{H,t}^\omega}{P_t}\right)^{-\theta} \left(\frac{P_t}{S_t P_t^\omega}\right)^{-\theta} Y_t^\omega = a\left(\frac{P_{H,t}^\omega}{P_t}\right)^{-\theta} \left(\frac{1}{RER_t}\right)^{-\theta} Y_t^\omega \]  

(37)

Finally the external interest rate, output and inflation follow a dynamic stochastic process given by
\[ \hat{r}_t^{\omega} = \zeta \hat{r}_{t-1}^{\omega} + \epsilon_{r_t,t} \quad (38) \]
\[ \hat{y}_t^{\omega} = \zeta \hat{y}_{t-1}^{\omega} + \epsilon_{y_t,t} \quad (39) \]
\[ \hat{\pi}_t^{\omega} = \zeta \hat{\pi}_{t-1}^{\omega} + \epsilon_{\pi_t,t} \quad (40) \]

where \( \hat{r}_t^{\omega} \), \( \hat{y}_t^{\omega} \) and \( \hat{\pi}_t^{\omega} \) represent the log-deviation of foreign interest rate, foreign GDP and foreign inflation respectively from their steady-state and \( \epsilon_{i,t} \) is an i.i.d normal error term with zero-mean and standard deviation of \( \sigma_i \), where \( i = r\omega, y\omega \) and \( \pi\omega \).

Besides the shocks pertaining to the external sector, the economy is subject to two other orthogonal AR (1) stochastic shocks:\(^{17}\) (i) a country risk premium shock (\( \hat{z}_t \)): \( \hat{z}_t = \zeta A \hat{z}_{t-1} + \epsilon_{z,t} \) and (ii) a domestic productivity shock (\( \hat{A}_t^{\nu} \)): \( \hat{A}_t = \zeta A \hat{A}_{t-1} + \epsilon_{A,t} \)

### 2.5. Equilibrium

The system of the model consists of the optimality conditions (see equation 1 till 34), the government budget constraint, the monetary policy regimes, market clearing conditions, the balance of payment, and processes of the exogenous shocks.

For simplicity, it is assumed that the sole role of the government in the economy consists of receiving lump-sum tax (\( \tau_t \)) from households and then transferring it (\( T_t \)) to the same households.\(^{18}\) Therefore, the government budget constraint is simply given by:

\[ T_t = \tau_t \]

The equilibrium conditions in each market are as follows:

The financial market: \( B_t = 0 \)

The labor market: \( L_t = \int_0^1 L_t (j) \, dj \)

\(^{17}\) For the sake of economy of space, we report only the results from country risk premium shock. The other results are available upon request.

\(^{18}\) See Medina and Soto, 2007; Fernández-Villaverde and Ohanian, 2009 for a model with a fully-fledged fiscal sector.
The domestic goods market: \( Y_t = C_{H,t} + I_{H,t} + X_t \), where \( X_t \) stands for total exports of the country.

Using demand functions defined in (4) and (37), the aggregate demand is given by:

\[
Y_t = (1 - a) \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left( C_t + I_t \right) + \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left[ a \left( \frac{1}{RER_t} \right)^{-\theta} Y^\omega_t \right]
\]  

(41)

Net foreign asset position (balance of payments) of the country is given by:

\[
S_t D_t = S_t R^\omega_{t-1} D_{t-1} \Psi_{D,t-1} + X_t - M_t
\]

(42),

where \( M_t \) stands for total imports of the country.

The dynamic of the net foreign position of the country therefore depends on the current account balance as well as the interest payments on the previous period debt. We express equation (42) relative to GDP as follows:

\[
d_t = R^\omega_{t-1} d_{t-1} \Psi_{D,t-1} \frac{1}{\pi_t} + \frac{P_{H,t}}{P_t} Y^\omega_t - \frac{C_t}{Y} - \frac{I_t}{Y}
\]

(43)

3. Calibration and Estimation Strategies

The empirical literature offers numerous strategies for the determination of the parameters of New Keynesian DSGE models ranging from pure calibration to econometric estimation or a mix of both. This study builds on the latter strategy. The estimation strategy uses actual data for two founding members of the WAMZ (Ghana and Nigeria) and the WAEMU. The model is solved numerically using DYNARE\(^{19}\) toolbox.

3.1. Calibration

The complete list of the calibrated parameters, their values and their sources are in table 1.
Table 1: Baseline parameters calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>inverse of intertemporal elasticity substitution for consumption</td>
<td>$1/0.34$</td>
<td>Ogaki et al. (1996)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>inverse of Frisch elasticity of labour supply</td>
<td>10.0</td>
<td>Berg et al. (2012)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>agents’ discount factor</td>
<td>0.91</td>
<td>Berg et al. (2012)</td>
</tr>
<tr>
<td>$\psi_D$</td>
<td>elasticity of country risk premium on FX borrowing</td>
<td>0.0007</td>
<td>Schmitt-Grohe and Uribe (2003)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>production parameter for private capital</td>
<td>0.40</td>
<td>Araujo et al. (2013)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>probability of firms surviving in the economy</td>
<td>0.9728</td>
<td>Bernanke et al (1999)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>elasticity of firms’ risk premium on FX borrowing</td>
<td>1.00</td>
<td>Elekdag and Tchakarov (2007)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate for private capital</td>
<td>0.1</td>
<td>Berg et al. (2012)</td>
</tr>
<tr>
<td>$\psi_I$</td>
<td>capital adjustment cost</td>
<td>0.25</td>
<td>Araujo et al. (2013)</td>
</tr>
<tr>
<td>$K/N$</td>
<td>capital/firms’ net worth in initial state</td>
<td>3.00</td>
<td>Devereux et al. (2006)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>elasticity of substitution between different varieties of goods</td>
<td>0.44</td>
<td>Berg et al. (2012)</td>
</tr>
</tbody>
</table>

### 3.2. Estimation Strategies

The structural parameters characterizing the economies in the theoretical model described above are estimated. These are basically the parameters related to the monetary policy rule; elasticities of substitution between domestic and foreign goods, parameters related to the Calvo-type price rigidity, parameters pertaining to the persistence of stochastic shocks, and standard errors related to the shocks.

Many estimation methods of the DSGE models have been put forward in the literature. We distinguish among them the maximum likelihood method, the generalized moment method and Bayesian method. In this study we use Bayesian estimation techniques for the model estimation.\textsuperscript{20} We follow the same presentation form of Beidas-Strom and Poghosyan (2011) for our model estimation. The complete log-linearized version of the model previously described is presented in the appendix and can be written in the form of linear system with rational expectation as follows:

$$
\Omega_0(\theta)z_t = \Omega_1(\theta)z_{t-1} + \Omega_2(\theta)\epsilon_t + \Omega_3(\theta)\xi_t
$$

\textsuperscript{20} See Ruge-Murcia (2007) for a comparative study on these methods.
where

\[ z_t = \left\{ \hat{y}_t, \hat{c}_t, \hat{q}_t, \hat{l}_t, \hat{w}_t, \hat{mp}_t, \hat{G}_t, \hat{H}_t, \hat{M}_t, \hat{r}_t, \hat{r}_t, \hat{D}_t, \hat{D}_t, \hat{K}_t, \hat{K}_t, \hat{d}_t \right\} \]

is a vector containing the model’s endogenous variables expressed as log-deviations from their steady-state values, and

\[ \varepsilon_t = \left\{ \varepsilon_{\hat{A}t}, \varepsilon_{\hat{z}t}, \varepsilon_{\hat{y}_t}, \varepsilon_{\hat{w}_t}, \varepsilon_{\hat{M}_t}, \varepsilon_{\hat{K}_t} \right\} \]

is a vector of innovations to stochastic shocks and coefficients matrices, \( \Omega_t \), are non-linear functions of the structural parameters contained in \( \theta \). The solution to the system can be written as follows:

\[ z_t = \Omega_z(\theta)z_{t-1} + \Omega_\varepsilon(\theta)\varepsilon_t \tag{44} \]

Relations (43) and (44) stems from measurement equations linking observable variables used in the estimation with endogenous and exogenous variables. We can express them through a single equation as follows:

\[ y_t^T = Hz_t \tag{45} \]

Where \( y_t^T = \{ \hat{y}_t, \hat{c}_t, \hat{q}_t, \hat{L}_t, \hat{R}_t \} \) is a vector of observable variables used in the estimation and \( H \) is a deterministic matrix. Equations (43), (44) and (45) form the state-space representation of the model, the likelihood of which can be evaluated using Kalman filter, provided the white innovations are normally distributed. In practice, the Bayesian approach first place a prior distribution with density \( P(\theta) \) on structural parameters \( \theta \). It then uses the data, \( y_t^T \), to update the prior distribution through the likelihood function, \( L(\theta|y_t^T) \). From this updating process we obtain the posterior distribution of \( \theta \) according to Bayes’ theorem:

\[ P(\theta|y_t^T) = \frac{L(\theta|y_t^T)P(\theta)}{\int L(\theta|y_t^T)P(\theta)d\theta} \tag{46} \]

Posterior distributions are generated using Markov-Chain-Monte-Carlo (MCMC) simulation methodology which is briefly discussed in Lubik and Schorfheide (2005); Gelman et al. (2006). Finally, the simulation techniques use the random walk Metropolis-Hastings (MH) algorithm.
The parameter vector to be estimated in this study is:

\[ \vartheta = \{ h, \theta, \phi^H, \phi^M, \zeta_A, \zeta_r, \zeta_y, \zeta_\pi, \sigma_\varepsilon, \sigma_{\varepsilon_A}, \sigma_{\varepsilon_r}, \sigma_{\varepsilon_y}, \sigma_{\varepsilon_\pi}, \beta_0, \beta_1, \beta_2, \rho_\nu, \rho_g, \sigma_{\varepsilon_g} \} \]

### 3.2.1. Data

For the five countries of the WAMZ plus the WAEMU sub-region under study, we rely on data drawn from the *World Economic Outlook (2010)* and the IMF’s *International Financial Statistics (2012)*. The data are of annual frequency spanning from 1980 to 2010 and the selected observable variables include real GDP; consumption; overall domestic inflation; real exchange rate; and nominal interest rate. Since the model variables are expressed in terms of log-deviations from their steady-state values, we pre-process them. Basically, this consists of seasonally adjusting the variables using filtering techniques. The most commonly used approach is the Hodrick-Prescott (HP) filter we build on in this paper. In the case of real GDP, we detrend the series in order to work with stationary series. Consumer price inflation is used as a measure of the overall domestic inflation as well as to construct real exchange rate.

### 3.2.2. Prior distribution

Priors’ distributions (mean and standard deviation) are gleaned from personal belief about parameter value and economic theory (Schorfheide, 2000). In practice, priors are chosen on the base of theoretical restrictions on the parameter values (non-negativity or confidence interval) given in the existing literature. *Beta* distribution is chosen for parameters with values constrained in interval [0, 1]. *Gamma* and *normal* distributions pertain to parameters values that are non-negative while *inverse gamma* distribution is used for the distribution of standard deviation of shocks.

In line with the empirical onslaught pertaining to DSGE models with application in Sub-Saharan African economies (Peiris and Saxegaard, 2010, Dagher et al. 2010; Berg et al. 2010; Senbeta 2011; Diop et Fall. 2011; Berg et al. 2012; Araujo et al. 2013), we draw the prior distribution for each parameter contained in \( \vartheta \), its mean and standard deviation. For the degree of habit persistence in consumption, \( h \), we assume a truncated normal distribution with mean 0.70 and standard deviation equal to 0.15. Similar to Diop et Fall. (2011), the parameters measuring the degree of Calvo price stickiness (\( \phi^H \)) and (\( \phi^M \)) are assumed to have the same mean 0.50 and standard
deviation 0.15. As regards the priors in the coefficients of the monetary policy, we place a relatively high mean on inflation coefficient ($\beta_1$) with mean 1.50 and standard deviation 0.25 and identically low coefficient mean value equal to 0.70 and standard deviation 0.10 for output growth coefficient ($\beta_2$) and exchange rate coefficient ($\beta_3$). The interest rate smoothing coefficient ($\beta_0$) is assumed to follow a gamma distribution with mean set to 0.75 and standard deviation 0.15. The elasticity of substitution between foreign and domestic goods, $\theta$, follows an inverse gamma distribution with mean 1.50 and standard deviation 0.75. Finally the AR (1) parameters (persistence coefficients) of the stochastic shocks $\zeta_A$, $\zeta_{y,0}$, $\zeta_{\pi,0}$ have gamma distribution with the same mean set at 0.50 and standard deviation at 0.20. The mean of the world interest rate smoothing parameter, $\zeta_{r,0}$, and country risk premium $\zeta_e$ are identically set to 0.46 as is the value of the standard deviation set at 0.15, as in Devereux et al. (2006).

### 3.3. Estimation Results

In this section, we outline the estimation results of the model. First, we lay out the parameters estimates and then we discuss the impulse response functions from a risk premium shock on the dynamic of some key macroeconomic variables.

#### 3.3.1. Parameter Estimates

The parameters estimates fall within plausible ranges (figure A1-3). The parameter of habit formation in consumption, $h$, is lower than its prior mean of 0.7 for all countries, meaning that the degree of consumption persistence in these countries is quite low as compared with developed economies (see for instance Lubik and Schorfheide (2005)). The parameter estimates of the elasticity of substitution between home and foreign goods in the consumption basket of domestic households, $\theta$, is higher than its prior mean value of 1.5 for all countries. It should be noted that a high value of this parameter points to a high degree of adjustment of consumers from these countries in response to changes in relative prices of domestic goods and imported goods.

The posterior estimates of Calvo price stickiness provides insights about the frequencies of price changes, through the probability of firms who do not reset their prices in a given year. The estimated posterior mean value of the Calvo probability is lower than the prior mean of 0.5 for home goods prices ($\phi^H$) and for foreign goods prices ($\phi^M$) for Nigeria and the WAEMU and
higher for Ghana. Lower values of \( (\phi^H) \) and \( (\phi^M) \) show that domestic goods prices and foreign goods prices respectively are re-optimized frequently in a given year. The more the firms reset their prices in a given year, the more inflation is subdued and inversely when price setting is staggered. Therefore, the lower posterior mean \(<0.5\) of the probability of not resetting prices in Nigeria and the WAEMU brings into the fore that inflation is subdued in this set of countries while the reverse holds for Ghana\(^{21}\). Since the expected time a price is reset is \( 1/(1 - \phi^i) \), with \( i = H, M \), then the average duration retailers of both home goods and foreign goods reset their prices is less than 2 years in Nigeria and the WAEMU while prices are sticky over more than 2 years for Ghana. These results are in line with findings in Diop et Fall (2011).

The posterior estimates of the policy rule coefficients, \( \beta_0, \beta_1, \beta_2 \) and \( \beta_3 \) provide plausible reaction function of the future central bank of the currency union to inflation, output gap, and exchange rate depreciation. First, the degree of interest rate persistence \( (\beta_0) \) falls below the prior mean of 0.75. In particular, its mean value for Nigeria, Ghana is quite large, 0.44 and 0.57 respectively, and close to Diop et Fall (2011)’s estimates. Second, the response of the interest rate to inflation’s deviation from its target \( (\beta_1) \) is estimated to be higher than the prior mean value of 1.5 for all the countries.\(^22\) Likewise, the output gap coefficient \( (\beta_2) \) is above its prior mean of 0.70 for all the countries. This finding shows that central banks in these countries overreact to inflation and output. The rational for the central banks overreaction to inflation and growth is to cope with demand side shocks in these countries. Third, the estimated coefficient of the exchange rate depreciation \( (\beta_3) \) is also above its prior mean of 0.70 for all countries. Again, these estimates are in line with Diop et Fall (2011).

### 3.1.2. Dynamics of the Model: Impulse Response Analysis

Figure B1-3 displays the response of important endogenous variables to a 1 standard deviation negative shock to the external risk premium \( (\epsilon_{z,t}) \).\(^{23}\) In other words, the shock is similar to a reduction in the risk premium on the external debt which in turn reduces the default probability.

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\(^{21}\)Ghana adopted inflation-targeting policy since 2007 which is deemed to keep inflation within a target band. Indeed, the country experienced many double digit-inflations, with inflation reaching 20-percent levels in 2004 to 10.7 percent by end-2010, above the mid-point target, according to IMF staff report.\(^{22}\)Diop et Fall (2011)’s estimates of these parameters points to similar finding for Ghana, as can be expected given the prevailing strict interest rate rule policy in these countries.\(^{23}\)Results from the other shocks mentioned above are not reported here but are available upon request.
The shock is carried out under both regimes: pure fixed (solid line) and floating exchange rate regime (dotted red line).

There is a clear perception of the response of inflation, output, investment, real interest rate, real exchange rate and terms of trade and entrepreneurs’ net worth to the shock. Since corporates’ financial position—measured in terms of factors such as net worth—is procyclical (Bernanke et al. 1999), and that it depends inversely on the external finance premium, then accordingly the latter tends be countercyclical. Hence, a sound financial condition has a declining effect on the external finance premium which in turn leads to an increase in the capital accumulation. The balance sheet channel therefore creates a situation through which otherwise short-lived economic shocks may have long-lasting effects. Formally, this is equivalent to a fall in foreign interest rate, manifested in the uncovered interest rates parity relations in equations 7 and 8; everything else being equal, the real exchange rate decreases as a result (nominal and real appreciation). Because liabilities are “dollarized”, the real appreciation has expansionary effects through firms’ balance sheet. This phenomenon linking the entrepreneurs’ balance sheet with capital accumulation, and hence output illustrates how the traditional contractionary effect of an appreciation of the real exchange rate could sometimes be misleading.

For all individual countries, the impulse responses of the variables—especially firms’ financial position, investment and output—highlight the superiority of flexible exchange rate over fixed rates in amplifying positive effects or cushioning adverse effects. For example, a 1 standard deviation negative shock on the risk premium on Ghana creates 2.3 percent and 0 percent increase

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24 This is so because when borrowers have little wealth to contribute to capital financing the potential divergence of interests between the borrower and the suppliers of external funds is greater, implying increased agency costs; in equilibrium, lenders must be compensated for higher agency costs by a larger premium (Bernanke et al. 1999).

25 Adolfson et al. (2008) noted that the uncovered interest rate parity (UIP) condition is a key equation in open economy DSGE models. It shows the difference between domestic and foreign nominal interest rates equals the expected future change in the nominal exchange rate. The UIP condition is a key equation in open economy models not only for the exchange rate but also for many macroeconomic variables, since there is a lot of internal propagation of exchange rate movements working through fluctuating relative prices. There is, however, strong empirical evidence against the standard UIP condition, see for instance, e.g., Eichenbaum and Evans (1995); Faust and Rogers (2003). Moreover, a DSGE model with a standard UIP condition cannot account for the so-called ‘forward premium puzzle’ recorded in the data, i.e. that a currency whose interest rate is high tends to appreciate, which implies that the risk premium must be negatively correlated with the expected exchange rate depreciation see, e.g., Fama (1984); Froot and Frankel (1989).
above the steady state in the net worth, and 6 percent and 0.2 percent increases above the steady state in investment under floating exchange rate and fixed exchange rate policy respectively. A mild decrease in output emerges yet the superiority of the flexible regime over the fixed regime still prevails (0.18 percent decrease vs. 0.27 percent decrease). With regard to Nigeria and the WAEMU, the results also reveal the primacy of floating regime over pure fixed regime. The findings are consistent with the conventional wisdom that flexible exchange rates are better absorber of real foreign shocks than are fixed exchange rates (see Flood and Marion (1982) and Aizenman and Frenkel (1985), among many others).

In addition, the direction of the response to shock to the external risk premium depends on the size of the debt risk premium parameter($\psi_D$). Nonetheless, the overall message holds with regard to this parameter values.

4. Robustness Checks

Now, we check the sensitivity of the results to important model parameters. In particular, we examine cases where the openness of the economy, the intertemporal elasticity of substitution vary under both regimes. We find that the degree of openness makes the balance sheet effect more potent in flexible regime than in fixed regime (figure 4). This comes as a result of the demand-switching effect that arises with the depreciation of the exchange rate. Furthermore, we find that the results are not sensitive to the intertemporal elasticity of substitution, which supports the overall message of the results.
5. Conclusion

Using a model of a small open economy Dynamic Stochastic General Equilibrium (DSGE) which features real and nominal rigidity, habit formation in consumer’s utility function, backward-looking and forward-looking firms, operating costs in firm’s capital utilization and imperfect capital mobility, inefficiencies in private investment and absorptive capacity constraints, this paper evaluates the performances of two exchange rate regimes for the five founding member of the WAMZ under a foreign shock namely a country risk premium shock. The model embeds the financial accelerator mechanism through which the terms of access to credit in international credit market and hence of demand for capital are linked with the state of borrower balance sheets. It also incorporates the phenomena of incomplete pass-through and foreign currency debt mechanism. Some parameters of the model have been calibrated while the remaining parameters have been estimated using the Bayesian simulation approach, which combines prior information drawn from the literature and from historical data covering the period 1980 to 2010. The estimates of the key structural parameters of the model fall within plausible ranges. To try to pin down how the economy responds to foreign shock and how the choice of an exchange rate regime influences that response for the ECOWAS countries, we simulate the model by modifying different policy parameters and compare the results under two policy rules: fixed exchange rate and floating exchange rate.
The main results can be summarized as follows. First, a negative shock to country risk premium is equivalent to a falling borrowing cost in the international capital market. The real exchange rate decreases through the uncovered interest rate parity conditions. Since the entrepreneurial liabilities are foreign currency denominated, the real exchange rate appreciation tends to improve the entrepreneurial net worth as well as the accumulation of capital. Second, there is an offsetting effect since real appreciation makes export goods more expensive relative to import goods with a detrimental effect on the net foreign position. Third, the model suggests the superiority of the insulating role of a flexible exchange rate regime over that of a peg. Indeed, we find that the offsetting effect seems to dominate since there is a drop in output in all the countries. Nonetheless, the contraction in economic activity remains smaller under flexible exchange rate than under fixed rates. Therefore, the conventional policy advice prevails: small open economies with greater exposure to external disturbances should implement a float. The latter acts as a shock absorber while containing adverse external shocks. Finally, we find that the effect of the balance sheet channel is more pronounced with regard to the degree of openness of the economy, yet robust to the intertemporal elasticity of substitution.

Our results suggest that the choice of exchange rate regime by the monetary authorities of the future common currency of the founding member of the ECOWAS should wisely account for the risk premium associated with liabilities dollarization their corporates and governments face. This seems to be a compelling message, with local bond market of these countries at infancy level and more dependence on external currency debt. While it could be optimal for such countries to pursue policies implying a flexible exchange rate, we concede that it requires strong institutional capacity.

References


Devereux M.B., Engel C. (2001), ”Monetary Policy in the Open Economy Revisited: Exchange Rate Flexibility and Price Setting Behavior”, Mimeo.


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Appendix

A1. Log-linearized Version of the Model

(a) Demand

\[ \hat{y}_t = (1 - \alpha) \left( \frac{c}{y} \hat{c}_t + \frac{i}{y} \hat{i}_t + \frac{g}{y} \hat{g}_t \right) + a \hat{y}_t^{\omega} + \theta a \left( \frac{2 - \alpha}{1 - \alpha} \right) r \hat{r}_t - \frac{\theta a}{1 - \alpha} \text{log} \hat{g}_t \]

\[ C_t = \frac{h}{1 + h} C_{t-1} + \frac{1}{1 + h} E_t C_{t+1} - \frac{1 - h}{\sigma(1 + h)} (r_t - E_t \pi_{t+1}) \]

\[ E_t (\hat{r}_{K,t+1}) = \hat{r}_t - E_t \hat{\hat{r}}_{t+1} - \gamma (\hat{n}_t + \hat{q}_t - \hat{k}_{t+1}) \]

\[ \hat{r}_{K,t+1} = \left( 1 - \left( \frac{1 - \delta}{r_K} \right) \right) \text{mpc}_t + \left( \frac{1 - \delta}{r_K} \right) \hat{q}_t - \hat{q}_{t-1} \]

\[ \hat{q}_t = \psi_t (\hat{i}_t - \hat{k}_t) \]

(b) Supply

\[ \hat{y}_t = \hat{A}_t^{\gamma} + a \hat{k}_t + (1 - \alpha) \hat{l}_t \]

\[ w_t = \eta L_t + \frac{\sigma}{1 - h} (C_t - h C_{t-1}) \]

\[ \hat{w}_t = \hat{y}_t + \hat{m} c_t - \hat{l}_t - \frac{a}{1 - \alpha} (r \hat{r}_t - \text{log} \hat{g}_t) \]

\[ \text{mpc}_t = \hat{y}_t + \hat{m} c_t - \hat{k}_t - \frac{a}{1 - \alpha} (r \hat{r}_t - \text{log} \hat{g}_t) \]

\[ \hat{\pi}_t = (1 - \alpha) \hat{\pi}_{H,t} + a \hat{\pi}_{M,t} \]

\[ \hat{\pi}_{H,t} = (1 - \beta \phi^H) E_t [\hat{\pi}_{H,t+1}] + \phi^H \hat{\pi}_{H,t-1} + \frac{(1 - \beta \phi^H)(1 - \phi^H)}{\phi^H} \hat{m} c_t \]
\[ \hat{\pi}_{M,t} = (1 - \beta \phi^M)E_t[\hat{\pi}_{M,t+1}] + \phi^M \hat{\pi}_{M,t-1} + \left(1 - \beta \phi^M \right) \left(1 - \frac{1}{\phi^M} \right) \text{log} g_t \]

\[ \hat{r}_t = \text{log} g_t + (1 - a) \hat{o}t_r \]

\[ \Delta \text{log} g_t = \Delta \hat{S}_t + \hat{r}_t - \hat{\pi}_{M,t} \]

\[ \Delta \hat{o}t_r = \hat{\pi}_{M,t} - \hat{\pi}_{H,t} \]

(c) Evolution of State Variables

\[ \hat{k}_{t+1} = \delta i_t + (1 - \hat{\delta}) \hat{k}_t \]

\[ \hat{n}_{t+1} = v r_K \left[ \left( \frac{k}{n} \right) \hat{r}_{K,t} + \left( 1 - \frac{k}{n} \right) (\hat{r}_{t-1} - E_t \hat{\pi}_t) + \gamma \left( 1 - \frac{k}{n} \right) (\hat{q}_{t-1} + \hat{k}_t) + \left( 1 + \gamma \left( \frac{k}{n} - 1 \right) \right) \hat{n}_t \right] \]

\[ \hat{a}_t = \frac{1}{\hat{b}} \hat{a}_{t-1} + \hat{y}_t - \frac{c}{y} \hat{\epsilon}_t - \frac{i}{y} \hat{i}_t - \frac{g}{y} \hat{g}_t - \left( \frac{a}{1 - \hat{a}} \right) (\hat{r}_{t} - \text{log} g_t) \]

\[ \hat{r}_t - E_t \hat{r}_{t+1} = \hat{r}_{t} + E_t \hat{\pi}_{t+1} + \psi_D \hat{a}_t + \hat{z}_t + E_t \hat{r}_{t} + \hat{r}_{t+1} - \hat{r}_t \]

\[ \Delta \hat{S}_t = \Delta \hat{r}_t - \hat{\pi}_{t+1} \]

(d) Monetary Policy Rule

\[ \hat{r}_t = \beta_0 \hat{r}_{t-1} + (1 - \beta_0) \left( \beta_1 \hat{\pi}_t + \beta_2 \hat{y}_t + \beta_3 \Delta \hat{S}_t \right) + \epsilon_r \]

(e) Foreign Variables

\[ \hat{r}_t = \zeta_{r\omega} \hat{r}_{t-1} + \epsilon_{r\omega,t} \]

\[ \hat{y}_t = \zeta_{y\omega} \hat{y}_{t-1} + \epsilon_{y\omega,t} \]

\[ \hat{n}_t = \zeta_{\pi\omega} \hat{n}_{t-1} + \epsilon_{\pi\omega,t} \]

(f) AR(1) Process of Stochastic Shocks

\[ \hat{A}_t = \zeta_A \hat{A}_{t-1} + \epsilon_{A,t} \]

\[ \hat{z}_t = \zeta_z \hat{z}_{t-1} + \epsilon_{z,t} \]
\[ \hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \]

A2. Estimated Parameters and Impulses Response Functions

Figure A1: Ghana: Estimated parameters
Figure B1: Ghana: Impulse response to a country risk premium shock—$\varepsilon_{z,t}$
Figure A2: WAEMU: Estimated parameters
Figure B2: WAEMU: Impulse response to a country risk premium shock—$\epsilon_{z,t}$
Figure A3: Nigeria: Estimated parameters
Figure B3: Nigeria: Impulse response to a country risk premium shock—$\varepsilon_{z,t}$