ESTIMATING PARAMETERS AND STRUCTURAL CHANGE IN CGE MODELS USING A BAYESIAN CROSS-ENTROPY ESTIMATION APPROACH

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ABSTRACT

This paper uses a three-step Bayesian cross-entropy estimation approach in an environment of noisy and scarce data to estimate behavioral parameters for a computable general equilibrium (CGE) model and to measure how labor augmenting productivity and other structural parameters in the model may have shifted over time to contribute to the generation of historically observed changes in the economic arrangement. In this approach, the parameters in a CGE model are treated as fixed but unobserved, which we represent as prior mean values with prior error mass functions. Estimation of the parameters involves using an information-theoretic Bayesian approach to exploit additional information in the form of new data from a series of Social Accounting Matrices (SAMs), which we assume were measured with error. The estimation procedure is "efficient" in the sense that it uses all available information and makes no assumptions about unavailable information. As illustration, we apply the methodology to estimate the parameters of a CGE model using alternative data sets for South Korea and for Sub-Saharan Africa.

JEL codes: C68, C61, C11, C13, E16, J24

Keywords: Computable general equilibrium (CGE) models, Bayesian estimation, cross-entropy estimation, Kullback-Leibler objective function, total factor productivity, labor augmenting productivity.

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INTRODUCTION

CGE models are often criticized for weak empirical estimation of the parameters and for preserving static economic structures in long-term simulations. The problem is associated with the lack of reliable time-series data, most severe in developing countries, to support standard econometric estimation of parameters and shifts in structure. In particular, we have lacked a times series of social accounting matrices (SAMs) and their associated prices and quantities that provide the information base for these models. There are examples of econometric estimation of parts of the CGE model where some time series data are available—see Jorgenson and Yun (2013) and other work by Jorgenson and various coauthors (Jorgenson, D. W. 2011, Jorgenson, D. W. and M. P. Timmer 2011, Jorgenson, D. W., et al. 2012).

This paper uses an information-theoretic cross-entropy estimation approach in an environment of scarce data measured with error to estimate behavioral parameters, labor augmenting productivity, and other parameters in the model that shift over time to generate historically observed changes in economic structure. With noisy and limited observations common to developing countries, the unseen parameters of a CGE model cannot generally be observed and measured directly. In systems and information theory, this is described as a *stochastic inverse problem:* how to use available "information" to recover these unobserved and uncontrolled components. Robust solution methods consistent with the underlying ill-posed noisy information recovery problem have been developed under Bayesian inference and decision theory (see, for example, Golan et al. 1996, and Judge and Mittelhammer 2012).[2] In CGE model estimation, Arndt et al. (2002) used the cross-entropy method from information theory to estimate the trade substitution elasticities in a CGE model of Mozambique. Robinson et al. (2001) also utilized the cross-entropy method in data updating and estimation when available information was insufficient to construct a balanced new SAM.

In this paper, we extend and refine the cross-entropy estimation method to account for different levels of noise and amount of information in a sequential and particular way. We start with a collection of SAMs with identical accounts for the Republic of Korea (South Korea) for various years. Even though the availability of such data to support is uncommon, it is still not enough to support standard econometric methods. Each new data set for later years provides new information to improve the estimation of parameters of a CGE model, updating priors that initially were based on scattered data and theoretical properties of various functions in the model. The statistical results recovered from this noisy and limited informational environment depend on how the CGE model is formulated; the more data, the more reliable the estimates of structural parameters. Furthermore, are the parameters stable? That is, even within a given set of specifications for a CGE model, the transformation of economic structure over time may be captured in alternative ways by the evolution of different parameters in the model and the entropy method described here provides a statistical approach for making model selection in this context. When only one SAM is available, a priori parameters such as trade elasticities are employed so that structure of economy such as trade shares are replicated and as depicted by the CGE model. With more SAMs, one can improve not only the posterior

[2] Not always initially accepted, the Bayesian approach is now applied widely in decision and information theory, operations research, and more recently, in finance and macroeconomics, such as the recent methodology regarding dynamic stochastic general equilibrium (DSGE) modeling. A popular history of its development, the debates about its validity and uses, its triumphant applications in the great wars of the last century, and its recent uses is available in McGrayne, S. B. (2011).

estimates of the trade elasticities, but also check if the trade structure is changing. This may be reflected by rising elasticities (the economy is becoming more flexible and responsive to trade prices) or the trade shares are shifting due to greater competitiveness and global integration, i.e., there is a trend in the trade delta shares of common functions such as the Armington and CET functions. Model selection even within the same general specifications of the CGE model means choosing among alternative formulations about how the economic structure is changing or whether it is stable. The estimation procedure is "efficient" in the sense that it uses all available information and makes no assumptions about unavailable information. Importantly, estimated parameters maintain consistency with the microeconomic foundations of the general equilibrium theory embodied in CGE models. [3]

We make use of the collection of available comparable SAMs for different years to estimate parameters and to account for equilibrium dynamics of the country's economic flows as postulated in the CGE model. Since SAMs are expressed in nominal values and information about relative prices may be limited, we also implement a data step by using the cross-entropy method to estimate real SAMs of the same base year and that are consistent with available information on changes in prices before parameter estimation. We use scattered information on relative prices and factor accumulation quantities, where available in order to estimate real SAMs. With each new real SAM, a parameter estimation step checks if the posterior estimates of the parameters converge to new values and whether they are stable (e.g. no trends). We consider various discrete prior probability distributions, specifying additive or multiplicative errors on behavioral and structural parameters and SAM targets to examine convergence and stability results. Since developing countries are undergoing fundamental economic transformations as they grow, the next step in our method for model selection calculates for evolution of parameters, including the elasticities, factor specific technical change, or shifts in value added and trade shares that may be important in the changing economic structure. As more estimates are made for many countries, any regularity in the estimates may further inform the pattern of development and structural change, following the work of Chenery and various coauthors (Chenery, H. B., et al. 1975, Chenery, H. B. and H. Elkington 1979, Chenery, H. B., et al. 1986).

In the case of South Korea, the CGE-SAM-cross-entropy (CGE-SAM-CE) method is applied to the period 1990 to 2011 to estimate economic a set of behavioral parameters: the elasticities of substitution between traded goods and domestic goods and between factors in production functions. We also explore for possible changes in the relative levels of sectoral value-added and its factor components through the growth of total factor productivity (TFP) and labor-augmenting productivity (LAP) in the value added function of each sector, as well as the evolution in economic and trade structure as reflected in trends of trade elasticities and trade shares over time.

As the availability of datasets for global CGE modeling is improving thanks to the Global Trade Analysis Project (GTAP) at Purdue University and other activities, regional units that are aggregation of multiple countries and their corresponding SAMs are becoming popular in CGE modeling, but the availability of regional price data still lags. We therefore also look at a regional application to Sub-Saharan

[3] See Arnold Zellner's definition of an efficient "information processing rule" in Zellner, A. (1988).

Africa (SSA), where data noise and constraints can be severe in order to demonstrate how the approach may be applied.

As a last point, the framework permits both the recovery of behavioral (unobserved) parameters in the estimation model and a switch to policy simulations in the estimated CGE application mode once the parameters are estimated.

The rest of the paper is organized as follows. Section II presents the methodology where we describe the three-step, cross-entropy approach employed in the paper for estimating the structural parameters and shifts in the economic arrangement. Using the CGE-SAM-CE method, Section III discusses the estimates and results as well as the error measures corresponding to the estimates. In the last section, we conclude and suggest areas for future research.

METHODOLOGY

Estimation of parameters of a CGE model is often a complex task because of incomplete, infrequent, inaccessible, or uncertain data, including dated or poorly constructed SAMs and auxiliary information.[4] To simplify the estimation task, some practical procedures are commonly applied: subjective and expert judgments about the parameters and other assumptions; borrowing estimates from other studies; and sensitivity testing to refine their values and the economic reasonableness of simulation results. With somewhat more information, "back-casting" or "double-calibration" procedures have been employed to improve the validation of models; see e.g., Dawkins et al. (2001) and Okushima and Tamura (2009). Although Bayesian in spirit, these procedures do not assess the inherent errors or noise associated with weak data or with the estimates recovered in the process. To deal with noisy and limited data more directly, the method of cross-entropy (CE) estimation from information theory and Bayesian econometrics has recently been deployed to update and rebalance a SAM or to separately estimate related parameters in a CGE model (Robinson et al. 2001, and Arndt et al. 2002).[5] It can clearly be beneficial to integrate the two tasks of SAM updating and CGE parameter estimation. Prior guesses of parameters are refined with new information so that they maximize the probability of matching the historical record of vital aggregates in a SAM while the entropy method minimizes a statistically measured pseudo-distance of the calibration to the unseen parameters.

In this paper, we employ the cross-entropy method to integrate three steps critical to calibrating and validating CGE models: the SAM data preparation, the CGE parameter estimation, and the statistical analysis. Accordingly, we define the three-step procedure as follows: i) a data step that adjusts the historical nominal SAMs to real terms with a common base year taking into account that the SAMs are measured with errors and relative price indices are scant; ii) a parameter estimation step that calculates (filters) parameters

[4] This is often the case for developing countries, but those working on developed countries may face similar problems, in part because they try to carry out more complicated and data-demanding analysis.
[5] There are links between cross-entropy and empirical-likelihood estimation procedures. See Golan, A., et al. (1996) and Judge, G. G. and R. C. Mittelhammer (2012).

and structural change simultaneously within the specifications of the CGE model; and iii) an integrated statistical analysis to check for model precision and prediction and to test alternative ways of modeling historical structural change with the evolution of different parameters in the CGE model. In order to provide a practical approach and to ensure the consistency with economic theory in the behavioral equations, we follow the customary calibration process of CGE modeling. Finally, we develop a reference GAMS programming code that integrates the three steps of the procedure to facilitate use and further development of the technique by other analysts.[6] The code also matches CGE variables with the SAM cells, an approach that turn greatly simplifies the structure of the code. In what follows, we describe in more detail the different steps of the methodology and how it is applied to estimate and calibrate a CGE model.

The data step

As the basic source of information for a CGE model, a SAM provides the starting point for the calibration process. Although a time series of country SAMs is still not common, it is becoming less so thanks to the efforts of national statistical agencies. At the global level, country SAMs are compiled and reconciled in globally consistent data sets as part of the research activities of GTAP and others. As more SAMs are estimated, each additional SAM provides new information to combine with previous SAMs to test and adjust prior assumptions about the parameters and components in the CGE model.

In this data step, we exploit the fact that SAM provide a transaction (flow) matrix approach to national accounting (see Stone 1962, and Pyatt and Round 1985). We also draw on Robinson et al. (2001) who, using limited up-to-date information (such as macro aggregates), employ the cross-entropy estimation method to update an existing SAM. Here, we apply the methodology for the purpose of pooling and adjusting a set of historical SAMs when auxiliary data to directly do so are incomplete, essentially making use of the SAM's necessary consistency with macroeconomic aggregates in the national accounts.

More precisely, the first step is to adjust the nominal SAMs associated with different years by properly deflating them with price indices so that they are measured in the same base year prices and are hence comparable. The ideal way to do this is to separate the nominal magnitudes into their respective prices and quantities; and each cell in the SAM should be adjusted by the correct relative price index so that all individual cells are expressed in equivalent real terms. Where that is possible, it should clearly be employed. For many developing countries, however, the informational requirements will be daunting relative to the state of their statistical capacities (e.g. countries in Sub-Saharan Africa or newly formed countries after civil conflicts). Noise in the data, including the quality of price indices if available and the values of the specific SAM cells, will likely be very high.

In the absence of full information about prices and quantities, each SAM still brings new information about the economic structure of the country, albeit in nominal values. It adds new observational data and the

[6] GAMS refers to the General Algebraic Modeling System; for more on GAMS, see www.gams.com. The GAMS code in this paper is available from the authors in the future when documentation is completed.

new evidence can be used to update the old priors about the parameters. Mindful of the data noise and limitations however, we implement the following practical steps:

a. First, deflate all cells in the historical SAMs by a single overall GDP deflator so that they are broadly expressed in the same base year prices as a first step. For models with regional aggregation of countries, this may be the only means possible if deriving weighted aggregation of country specific prices is not feasible.

b. We assign greater weight to known data in a limited informational environment. Accordingly, as a second step, we apply the cross-entropy estimation method to target known macroeconomic aggregates in constant prices from the national accounts, which are generally available for most countries; and we rebalance the SAM to preserve, given priors on measurement error, these critical economic aggregates. This approach in effect extends the SAM cross-entropy estimation method to a constant price application.

Commented [DG1]: Both receipts and outlays are adjusted to the same prices.

c. Alternatively, one could use known relative price indices (relative to the GDP deflator) to target the macroeconomic aggregates. The latter could conceivably be expanded information-wise if auxiliary data such as world prices of exports and imports for various commodities are also available. As mentioned, the more cells in the SAMs that can be deflated to the right base year prices, the better the economic representation that is reflected in the SAM. In any case, to ensure that the SAM adheres to known national account aggregates in constant prices, it should still be estimated and rebalanced appropriately.

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d. In the extreme case, where there are no price and real national accounts data to work with (e.g. in post-conflict or other very data-poor economies), the evolution of the economic structure reflected in the nominal SAMs still adds information to test various prior assumptions about the unobserved parameters. It is still better than using less information from a single SAM.

The method used here follows the earlier approach in Robinson et al. (2001) in that standard additive errors and discrete probability distributions are defined at three levels of information – (1) each cell value in the SAM; (2) the row and column totals (total expenditures and total incomes) in the SAM (SAM balance condition); and (3) the macroeconomic aggregates in the SAM. The formal mathematical statement generally follows the earlier work and is not repeated here. In adjusting the historical SAMs to constant prices under limited information, we assign the smallest standard errors to the macro aggregates since they are known from the national accounts. The end result of the data step is a series of real SAMs that are consistent with the aggregates of national income accounts in constant prices.

The parameter estimation step

In this section, we describe the cross-entropy method that is used to estimate the unseen parameters and other components of the economic framework, subject to the outputs (i.e., the real SAMs) in the data step.

Prior and posterior probability distributions of parameter error. Our basic approach explicitly recognizes that the value of each parameter θ_i is not observed, but can be represented by its prior θ_i^0 and an error term ε_{θ_i}, in additive or multiplicative way, i.e., $\theta_i = \theta_i^0 + \varepsilon_{\theta_i}$ or $\ln(\theta_i) = \ln(\theta_i^0) + \varepsilon_{\theta_i}$, respectively. Moreover, the error term has an assigned prior error probability mass distribution, such that $Prob\left(\varepsilon_{\theta_i} \in E_{j,\varepsilon_{\theta_i}}\right) = \pi\left(\varepsilon_{j,\theta_i}\right) \in [0,1]$, $\varepsilon_{\theta_i} \in \left\{E_{1,\varepsilon_{\theta_i}}, E_{2,\varepsilon_{\theta_i}}, \dots, E_{n,\varepsilon_{\theta_i}}\right\}$, $Prob\left(E_{j,\varepsilon_{\theta_i}} \cap E_{k\neq j,\varepsilon_{\theta_i}}\right)=0$ and $\sum_j \pi\left(\varepsilon_{j,\theta_i}\right)=1$. Then, our information-theoretic cross-entropy estimation centers on the posterior parameter values, $\mathbf{\Theta} = f(\boldsymbol{F}(\cdot) = \mathbf{0}, \mathbf{Y}, \mathbf{V}, \ \mathbf{\Pi}(\boldsymbol{\varepsilon}|\cdot))$, conditioned on the specifications of the CGE model $\boldsymbol{F}(\dots) = \mathbf{0}$, the SAM targets, $\mathbf{Y}$, the SAM data, $\boldsymbol{V}$, and the posterior error distribution $\mathbf{\Pi}(\varepsilon_{\theta_1}, \dots, \varepsilon_{\theta_I}|\boldsymbol{Y}, \boldsymbol{V}, \boldsymbol{F}(\cdot) = \mathbf{0})$, which is a conditional joint distribution of all the error parameters, $\boldsymbol{\varepsilon} = (\varepsilon_{\theta_1}, \dots, \varepsilon_{\theta_I})$.

Mathematical statement of the problem. Two concepts, precision and prediction, play important roles in the interpretation of the method. *Precision* refers to the behavioral and structural parameters and to the difference between the posterior and prior values of these parameters. *Prediction* refers to the sample data and the difference between the estimated values of the targeted (selected) SAM cells and their prior values.

We setup the CGE-SAM-CE method as a minimization problem of an objective function (1) subject to equations (2) through (13) as its constraints (appendix 1 lists various notations and their definitions). More specifically, the objective function minimizes the sum of the Kullback-Leibler divergence of the prior and estimated probabilities, for all the discrete error distributions that characterize unobserved parameters and SAM targets.

Commented [DG3]: A little work and investment for the reader is okay…A journal paper will want it short. For the working paper, I will leave it to Fabian if he wants to put it here.

$$(1) \qquad \min_{\{\boldsymbol{w},\boldsymbol{X_t}\}} \left\{ \underbrace{\alpha_1 \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{k_m=1}^{K_m} w_{B,t,k_m} \ln\left(\frac{w_{B,t,k_m}}{\bar{w}_{B,t,k_m}}\right)}_{precision} + \underbrace{\alpha_2 \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{k_s=1}^{K_s} w_{Z,t,k_s} \ln\left(\frac{w_{Z,t,k_s}}{\bar{w}_{Z,t,k_s}}\right)}_{precision} + \underbrace{\alpha_3 \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{k_n=1}^{K_n} w_{Y,t,k_n} \ln\left(\frac{w_{Y,t,k_n}}{\bar{w}_{Y,t,k_n}}\right)}_{prediction} \right\}$$

subject to [7]

The CGE block:

$$(2) \qquad \boldsymbol{F}(\boldsymbol{X_t}, \boldsymbol{Z}_t^o, \boldsymbol{Z}_t^u, \boldsymbol{B_t}, \boldsymbol{\delta_t}) = \mathbf{0}, \ \forall \ t \in T.$$

[7] The vector of posterior error probabilities ($\boldsymbol{w}$) is partitioned in three components: the behavioral parameter, the SAM target and the non-behavioral parameters, i.e., $\boldsymbol{w} = \{\boldsymbol{w_B}, \boldsymbol{w_Y}, \boldsymbol{w_Z}\}$.

The calibration block:

(3) $\boldsymbol{\delta_t} = \Phi(\boldsymbol{Z_t}, \boldsymbol{B_t}),\ \forall\, t \in T.$

The behavioral parameter (precision) block:

(4) $\boldsymbol{B}_t = \boldsymbol{B}_t^0 \exp(\boldsymbol{e}_{B,t}),\ \forall t \in T.$

(5) $e_{B,t,m} = \sum_{k_m=1}^{K_m} w_{B,t,k_m} v_{B,t,k_m},\forall\, t \in T,\ \forall\, m \in M.$

(6) $\sum_{k_m=1}^{K_m} w_{B,t,k_m} = 1,\forall\, t \in T,\ \forall\, m \in M.$

The unobserved non-behavioral parameter (precision) block:

(7) $\boldsymbol{Z}_t^u = \boldsymbol{Z}_t^{u,0} \exp(\boldsymbol{e}_{Z,t}),\ \forall t \in T.$

(8) $e_{Z,t,s} = \sum_{k_s=1}^{K_s} w_{Z,t,k_s} v_{Z,t,k_s},\ \forall\, t \in T,\ \forall\, s \in S.$

(9) $\sum_{k_s=1}^{K_s} w_{Z,t,k_s} = 1,\ \forall\, t \in T,\ \forall\, s \in S.$

The SAMs target (prediction) block:

(10) $\boldsymbol{Y}_t = \boldsymbol{Y}_t^0 \exp(\boldsymbol{e}_{Y,t}),\ \forall\, t \in T.$

(11) $\boldsymbol{V}_t = \boldsymbol{G}(\boldsymbol{X}_t, \boldsymbol{Y}_t, \boldsymbol{Z}_t^o, \boldsymbol{Z}_t^u, \boldsymbol{B}, \boldsymbol{\delta}),\ \forall\, t \in T.$

(12) $e_{Y,t,n} = \sum_{k_n=1}^{K_n} w_{Y,t,k_n} v_{Y,t,k_n},\forall t \in T,\ \forall n \in N.$

(13) $\sum_{k_n=1}^{K_n} w_{Y,t,k_n} = 1,\forall t \in T,\ \forall n \in N.$

A brief description of each block is provided next while the definition of various symbols is found in the appendix 1 and appendix 2.

The first set of constraints belongs to the standard CGE block of equations as presented in (2). This is the representation of the general equilibrium model, a square system of non-linear equations that satisfy the property of homogeneity of degree zero in prices. This $\boldsymbol{F}(\cdot)$ vector function depends on the CGE endogenous variables $\boldsymbol{X_t}$, the observed exogenous variables $\boldsymbol{Z_t^o}$, the unobserved exogenous variables $\boldsymbol{Z_t^u}$, the behavioral parameters $\boldsymbol{B_t}$, and the calibration parameters $\boldsymbol{\delta_t}$.

The next constraints belong to the calibration block (3). This set of equations depends on the vector of exogenous variables $\boldsymbol{Z_t}$ and on the behavioral parameters $\boldsymbol{B_t}$. Note that $t_0 \subset t$, which means that this base period can be selected from any element of the sequence corresponding to SAM's periods.

The behavioral parameter block is given by equation (4) to (6). The entropy setup requires modeling the unobserved (behavioral) parameters $\boldsymbol{B}$ by using prior information $\boldsymbol{B^0}$ and an error term $\boldsymbol{e_B}$. In equation (4), the error term is entered in multiplicative form, such that $\boldsymbol{e_B}$ measures some amount of error between the prior $\boldsymbol{B^0}$ and posterior $\boldsymbol{B}$ information in logarithmic units. However, this can also be written in additive manner, i.e., $\boldsymbol{B} = \boldsymbol{B^o} + \boldsymbol{e_B}$, then $\boldsymbol{e_B}$ will be in the same units as the behavioral parameters. Equation (5) models the error terms $e_{B,t,m}$ as an expected mean using error support values v_{B,k_m} and their respective endogenously estimated probability weights w_{B,t,k_m}. For coherence, this behavioral parameter block requires that these discrete error probability weights sum to one (6). The prior specified for the probability weights w_{B,t,k_m} in the estimation reflect the degree of information available about the distributions—how "informative" are the priors.

Additionally, in this minimization problem the restrictions presented in equations (7) through (9), pertains to the unobserved exogenous (non-behavioral) parameters $\boldsymbol{Z}_t^u$. The vector of parameters $\boldsymbol{Z}_t^u$ depends on a vector of prior information $\boldsymbol{Z}_t^{u,0}$ and their error terms $\boldsymbol{e}_{Z,t}$, which are incorporated in multiplicative way as presented in (7). The unobserved exogenous parameters $\boldsymbol{Z}_t^u$ may contain parameters such as technical and productivity coefficients and rates. As in the two previous blocks, equation (8) also restricts the error terms with the expected value formula, considering specific support values and particular posterior weights, v_{Z,t,k_s} and w_{Z,t,k_s}, respectively. The corresponding discrete error probabilities are required to sum to one, equation (9).

Finally, the last set of constraints defines the SAMs target block through equations (10) to (13). The SAM targets are selected cells of the SAMs that serve to adjust the system for an easier convergence and a closer representation of the historical data. One of the main differences between Arndt, C., et al. (2002) and our method is that our targets are specific SAM cells (flows in constant currency units), while the model of 2002 was targeting a subset of the endogenous CGE variables ($\boldsymbol{X}_t$). Note that this method permits targeting specific cells of the SAM at particular periods, for instance, household savings could be a SAM target variable for period t_1 but not for time t_T, this is useful predominantly when there is information that indicates that some years were noisier than others. The first equation in this block, (10), models the SAM targets $\boldsymbol{Y}_t$ as function of their prior values $\boldsymbol{Y}_t^0$and their exponential error terms $e^{e_{Y,t}}$. Additionally, equation (11) is the representation of the endogenous recovery of the complete SAMs $\boldsymbol{V}_t$. That is, the full posterior SAMs are recovered as functions of the endogenous CGE variables $\boldsymbol{X}_t$, the SAM targets $\boldsymbol{Y}_t$, the observed and unobserved exogenous variables, $\boldsymbol{Z}_t^o$ and $\boldsymbol{Z}_t^u$, respectively, the behavioral parameters $\boldsymbol{B}_t$, and the calibration constraints $\boldsymbol{\delta}_t$. Equation (12) also imposes the expected mean value restriction on the error terms of the SAM targets using error supports v_{Y,t,k_n} and endogenously estimated posterior weights w_{Y,t,k_n}. The last set of constraints of this SAM target block is presented in equation (13) which again requires that the discrete error probability weights sum to one.

Considering that the selection of error types and prior distributions for the SAM targets and unobserved parameters are fixed, then the number of possible combinations of SAM targets depends on the number of non-empty cells in each SAM and the number of available SAM periods. Thus, the maximum number of combinations is $2^{\left(N_{t_1}^{SAM}+N_{t_2}^{SAM}+\cdots+N_{t_T}^{SAM}\right)}$, where $N_{t_1}^{SAM}$ represents the total number of non-empty cells in the SAM of period t_1, and the subindex T stands for the last period of the available SAMs. Furthermore, the possibility to update new SAM observations plus the error term prior distributions with old optimal posteriors will make this procedure a Bayesian estimation process of successive prior-to-posterior-to prior, etc. steps.

An alternative way to express the objective function is using expected mean values for the error probabilities as in (14), with expected values defined as $\mathbb{E}_{w_{B,t}}\left[\ln\left(w_{B,t,k_m}\right)\right]=\sum_{k_m=1}^{K_m} w_{B,t,k_m} \ln\left(w_{B,t,k_m}\right)$ and $\mathbb{E}_{w_{B,t}}\left[\ln\left(\bar{w}_{B,t,k_m}\right)\right] = \sum_{k_m=1}^{K_m} w_{B,t,k_m} \ln\left(\bar{w}_{B,t,k_m}\right)$. In this formulation, $\boldsymbol{w}$ and $\bar{\boldsymbol{w}}$ represent the matrices with the posterior and prior error probability mass function values of the correspondent error elements. Each of the expected mean value sub-indexes expresses their own and specific probability spaces. Under the afore-mentioned definitions, we can explain the value of the objective function as a noise measure of the observed data, or more precisely, as a pseudo-distance[8] of expected data noise. In this case, the bigger the value of the objective function, the noisier the data, and the more informative the parameters. Our optimization problem therefore measures and minimizes the expected pseudo-measure of error probabilities or data noise.

(14) $\min_{\{\boldsymbol{w},\boldsymbol{X}_t\}} \{\alpha_1 \sum_{t=1}^{T} \sum_{m=1}^{M} \left(\mathbb{E}_{w_{B,t}}\left[\ln\left(w_{B,t,k_m}\right)\right] - \mathbb{E}_{w_{B,t}}\left[\ln\left(\bar{w}_{B,t,k_m}\right)\right]\right)$

$$\min_{\{\boldsymbol{w},\boldsymbol{X}_t\}} \left\{\alpha_1 \sum_{t=1}^{T} \sum_{m=1}^{M} \left(\mathbb{E}_{w_{B,t}}\left[\ln\left(w_{B,t,k_m}\right)\right] - \mathbb{E}_{w_{B,t}}\left[\ln\left(\bar{w}_{B,t,k_m}\right)\right]\right)\right.$$

$+\alpha_2 \sum_{t=1}^{T} \sum_{s=1}^{S} \left(\mathbb{E}_{w_{Z,t}}\left[\ln\left(w_{Z,t,k_s}\right)\right] - \mathbb{E}_{w_{Z,t}}\left[\ln\left(\bar{w}_{Z,t,k_s}\right)\right]\right)$

$+\alpha_3 \sum_{t=1}^{T} \sum_{n=1}^{N} \left(\mathbb{E}_{w_{Y,t}}\left[\ln\left(w_{Y,t,k_n}\right)\right] - \mathbb{E}_{w_{Y,t}}\left[\ln\left(\bar{w}_{Y,t,k_n}\right)\right]\right)\}$.

As introduced above, equation (1) includes the precision and prediction parts; the precision part is represented by every term that is multiplied by α_1 and α_2, while α_3 corresponds to the section related with the "prediction" part embedded on the SAM target errors. Thus, at the optimal levels, the weights α_1, α_2 and α_3 partially define the noise level of this cross-entropy objective function, and the more weight on any of the above alpha parameters, the bigger expected marginal values of the correspondent error probabilities.

Error specification. The typical specification of the error is either additive or multiplicative: $X = \bar{X} + e$ or $X = \bar{X}\exp(e)$, where $\bar{X}$ is the prior mean and e is the error term with mean zero. The error is written as the discrete probability weighted sum of an error support set.

(15) $e = \sum_k w_k v_k$; $0 \leq w_k \leq 1$ and $\sum_k w_k = 1$.

The v parameters are fixed and have the units of the item (X) being estimated. They define the domain of possible values that X can take and so contain information for estimation. The w's are discrete probabilities, defining the probability mass function for the distribution of the error. This specification converts the problem of estimating errors in "natural" units into a Bayesian problem of estimating a set of probabilities. Instead of directly estimating the mean and variance of a random variable, we are now estimating a discrete probability distribution. The specified support set provides the link.

[8] See Judge, G. G. and R. C. Mittelhammer (2012, p. 107) for a more detailed discussion.

The number of elements in the support set, k, defines the number of probability weights (w) that need to be estimated and will determine the parameters of the distribution of the errors that can be "recovered" from the data. To estimate the errors, one starts with the error support set (v) and a prior on the probability weights $(\overline{w})$. This prior can be "uninformative" or "informative", depending on the choice of the prior probability weights – (see appendix 2 for more details).

The statistical and data validation step

Bayesian inference offers a way to derive posterior estimation of the unseen parameters and their likelihood given the model, limited data, and prior judgment about their initial values and distribution. Hence, the Bayesian inference is not like traditional statistics, which relies on repeated observations to form a large sampling distribution(s). In the context of scant and noisy data circumstances, the Bayesian procedure raises the question regarding what the economy would look like, what values are plausible, what are not – that is, if the parameters (Θ) of the CGE model were to take a set of values versus another – and what plausible values and likelihood may be associated with each parameter. The prior values and likelihood or probability distributions can be postulated from theory, taken from other studies, or derived from subjective or expert judgment. In CGE modeling, for example, historical applications or backcasting exercises may suggest a plausible prior, say 2.0, for the Armington elasticity, the substitution possibility between domestic good and imports. The cross-entropy method checks these postulations with the observations from the historical SAMs in order to derive new estimates of the parameters and their likelihoods. Summarizing, on the basis of what we know – the model, the data, and the initial guesses and likelihoods – the method refines the calculation with posterior error distribution estimates, thus providing an inferred probability distribution of the unobserved parameters. These inferred probability distributions of the unseen parameters can be used, in principle, to assess other type of economic analysis that involve the use of more detailed parameter-statistics than the merely use of their (posterior) mean.

Measuring uncertainty and uncertainty gains about the recovered parameters and error terms involve the usage of cross-entropy derived statistics. First, note that each of the elements of $\boldsymbol{B}$, $\boldsymbol{Z}_t^u$, and SAM targeted cells $\boldsymbol{Y}_t$ can be generalized and expressed in the alternative forms: $\theta = \theta^0 + \varepsilon$ or $\ln\theta = \ln\theta^0 + \ln\varepsilon$, where ε is the stochastic error part with probability weights $\boldsymbol{w}$, prior weights $\overline{\boldsymbol{w}}$ and support $\boldsymbol{v}$. After minimizing our CGE-SAM-CE objective function, the recovered error terms $\hat{\varepsilon}$ measure how good the CGE system and cross-entropy priors $\boldsymbol{B^o}$, $\boldsymbol{Z}_t^{u,o}$, and $\boldsymbol{Y}_t^o$ are capturing the (information) noise. On this regard, we are interested to test the behavior of the estimated error terms, $\hat{\varepsilon}$.

For individual and subgroup parameter estimates, we compute model pseudo -R^2 statistics – henceforth cross-entropy $\widehat{\mathrm{R}}^2$ – as presented in equation (16), where $\hat{S}(\boldsymbol{w}) = (-\sum_{k=1}^{K} w_k \ln w_k)/\ln K$ stands for the normalized entropy of the error terms. Since $\hat{S}(\boldsymbol{w}) \in [0,1]$, a value $\hat{S}(\boldsymbol{w}) = 0$ implies no uncertainty while $\hat{S}(\boldsymbol{w}) = 1$ redirects total uncertainty in the sense that $\boldsymbol{w}$ is uniformly distributed. Thus, in our CGE-SAM-CE error arrangement as pointed out in appendix 2, $\widehat{\mathrm{R}}^2 = 0$ means that the CGE-SAM-

CE system has total uncertainty about the posterior behavior of θ, while $\hat{R}^2 = 1$ reveals that the model arrangement has total certainty on the posterior and prior error behavior of θ[9].

$$(16) \qquad \hat{R}^2 = 1 - \hat{S}(\boldsymbol{w}).$$

Next, for model selection and data validation purposes we incorporate some goodness of fit statistics. In our CGE-SAM-CE model we rely on assumptions like parameters are evolving over time and structural transformation is present, hence, we are able to test model specifications using certain statistics in order to select the one that best validate the CGE-SAM-CE results on the prediction side of the estimation. First, we calculate an adjusted-R^2 [10] to compare the prior and the posterior SAM cells[11]. The $\bar{R}_t^2$ statistics in (17) includes all the non-empty SAM cells inside the SAM, N_t^{SAM}, while n_t^{SAM} denotes the number of targeted SAM cells[12]. Finally, we assess goodness of fit through an information-theoretic measure which imposes a penalty for adding/modifying SAM targets, n_t^{SAM}, the Akaike information criterion[13] (AIC) as defined in equation (18).

$$(17) \qquad \bar{R}_t^2 = 1 - \frac{\left[\sum_j \left(SAM_{posterior,j} - SAM_{prior,j}\right)^2 / \left(N_t^{SAM} - n_t^{SAM}\right)\right]}{\left[\sum_j \left(SAM_{prior,j} - \overline{SAM}_{prior}\right)^2 / \left(N_t^{SAM} - 1\right)\right]}.$$

$$(18) \qquad AIC = \exp\left(2\frac{n_t^{SAM}}{N_t^{SAM}}\right)\frac{1}{N_t^{SAM}}\sum_j \left(SAM_{posterior,j} - SAM_{prior,j}\right)^2.$$

Given that the AIC statistics measure a weighted squared difference between the data inside the SAM and the recovered (estimated) SAM, then the model estimation with the lowest AIC value would be preferred. As well, note that the addition of more targets inside the SAM is more penalized by the AIC than by the $\bar{R}_t^2$.

In contrast, from the prediction section of the objective function in (1) we derive the statistic $D(\cdot)_Y$ as presented in equation (19). This Kullback-Leibler statistic measures data and model generated noise, such that the smaller the number, the less noise produced by our model assumptions given the information

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[9] A third type of parameter statistic is $\tilde{S}(\boldsymbol{w}) = \left(-\sum_{k=1}^{K} w_k \ln w_k\right) / \left(-\sum_{k=1}^{K} \bar{w}_k \ln \bar{w}_k\right)$. In this case, $\tilde{S}(\boldsymbol{w}) \in [0, \infty^+)$, thus, a value close to 1 means that the system has a proper prior selection of the parameter error distribution while something different than one reflects the noise inconsistencies between our initial prior and the posterior error values, i.e., we over/under predict the parameter error behavior.

[10] Alternatively, we might estimate a percentage change deviation from the posterior SAM cells to their prior values: $pc_j = \left(\frac{abs[SAM_{posterior,j} - SAM_{prior,j}]}{Min\{abs[SAM_{posterior,j}], abs[SAM_{prior,j}]\}}\right) * 100$, such that $pc = \sum_j pc_j$.

[11] $SAM_{posterior,j}$ and $SAM_{prior,j}$ denote and consider all the non-empty SAM cells with the exception of the total expenditures and incomes.

[12] N_t^{SAM} and n_t^{SAM} take into account all the non-empty SAM cells but the total expenditures and incomes.

[13] Additionally, the Bayesian information criterion might be estimated as:
$\mathrm{BIC} = \left(\mathrm{N_t^{SAM}}\right)^{\frac{\mathrm{n_t^{SAM}}}{\mathrm{N_t^{SAM}}}} \frac{1}{\mathrm{N_t^{SAM}}} \sum_j \left(\mathrm{SAM_{posterior,j}} - \mathrm{SAM_{prior,j}}\right)^2$. Note that the AIC statistic penalizes in less severe form the addition of targets than the BIC.

embedded in the data, and then the best model-data fit. Note that AIC, BIC and $D(\cdot)_Y$ do not always point out in the same direction, for instance, if we have two different and hypothetical models A and B, we might have a case with the lowest AIC and $D(\cdot)_Y$ in model A, however, we might also have results that generate the lowest AIC in model A and the lowest $D(\cdot)_Y$ statistic in model B.

(19) $$\mathbf{D}(\mathbf{w}||\overline{\mathbf{w}})_Y = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{k_n=1}^{K_n}\mathbf{D}(\mathbf{w}||\overline{\mathbf{w}})_{Y,t,n,k_n} = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{k_n=1}^{K_n} w_{Y,t,k_n}\ln\left(\frac{w_{Y,t,k_n}}{\overline{w}_{Y,t,k_n}}\right).$$

ESTIMATION AND RESULTS

The cross-entropy estimation method is technically compatible with any CGE model. In the implementation, we choose a widely used standard, the IFPRI model (Lofgren, H., et al. 2002) as recently implemented by Cicowiez, M. and H. Lofgren (2006). This is a static CGE model for a single open economy in the tradition of Dervis, K., et al. (1982) and De Melo, J. and S. Robinson (1989). Over the years, CGE models of this type have been applied to a wide range of analysis of economic policy and external shocks in developing countries; hence, it is a good starting point for illustrations; see a recent survey of CGE models with policy applications to developing countries by Devarajan, S. and S. Robinson (2013). Since the specifications of the standard IFPRI CGE model is well documented, we only briefly sketch its outline to emphasize the key unobserved parameters of behavioral relationships for estimation. figure 1 of appendix 4 depicts the general structure of the production and consumption sides in the model, respectively. For illustrations of the methodology, we select South Korea as a country case and Sub-Saharan Africa as a regional application.

SAM preparation

Although there is good and extensive data available in the case of South Korea, we do not include all of them in order to demonstrate parameter estimation in a more limited data circumstances that are likely to prevail in other developing countries. Accordingly, we constructed five specific SAMs of South Korea for the following years, 1990, 1995, 2000, 2005, and 2011. We aggregate the SAMs to cover 6 sectors – agriculture, mining, manufacturing, utilities, construction, and services.

As described in the data step, we generate SAM cells error distributions. We assign to the SAM total expenditure and total income cells an error support that covers three standard deviations of the correspondent prior SAM cell value. For the macro aggregates, we generate error distributions with smaller standard deviations that account for only 0.01 the macro aggregate prior mean value, i.e., the symmetric macro aggregate error supports cover a range $\pm$0.03 the macro aggregate prior mean values. Table 1 illustrates how the South Korean macro results deviate (in percent) from the numbers of the national accounts.

Table 2 shows a similar application at a regional level, the case of Sub-Saharan Africa, using the regional SAMs for 2004, 2007, and 2011 from the GTAP database. The GTAP SAMs are already expressed in U.S. dollar, which is convenient for the data step since regional income accounts are available in the World Development Indicators at the World Bank in both current and constant price series. This means relevant price indices are derivable for regional GDP and for several of its components. The data step in this case consists of deflating the SAM values by first using the regional U.S. GDP deflator and then targeting macroeconomic aggregates that are available.[14] Regional accounts were available for the GDP components of government consumption, investment, exports, imports, as well as aggregated value added and disaggregated value added figures for agriculture, manufacturing, and services. The GTAP SAM for SSA in 2011 is a preliminary version and the deviations from the regional accounts targets are higher than those in the other two years.[15] Even so, they still provide valuable observations to use, albeit more noisy, in the Bayesian entropy approach.

Model definitions

Our Bayesian method estimates all the parameters in the CGE model simultaneously relative to their initial or prior values, subject to the specifications of the CGE model and the information from all available SAMs. In table 3 we state four model specifications or cases that define the behavior of the CES-CET functions in the standard CGE (figure 1) and illustrate technical aspects of the procedure. Given convergence issues and to make comparable the results among the country/regional applications, models 1, 2 and 3 are applied for South Korea, while models 1, 2 and 4 are designed for SSA. Whereas the model specifications are a debate in theoretical econometrics, DSGE, and CGE literature, in our specifications we merely look for flexible features of the method and their link to entropy (information) gains. In next subsections we explicitly state the behavior of the CES functions.

Elasticities

Elasticities are critical unobserved behavioral parameters in the standard CGE model, which are usually defined by using the CES functions (Arrow, K. J., et al. 1961). For example, the trade elasticities express the substitution possibilities between domestic and foreign goods in the CET (constant elasticity of transformation) and CES (constant elasticity of substitution) functions. Suppressing the sector and time subscripts, the two functions can be written symmetrically, using the same form,

[14] The regional macroeconomic aggregates are available from World Bank World Development Indicators.

[15] Given bigger divergences between the 2011 GTAP SAM information and the World Bank World Development Indicators (WDI) macro data and to achieve an optimal convergence in the 2011 rebalanced SSA SAM procedure, we had to allow bigger macro target standard deviations with size of 0.05 the macro aggregate prior mean values, i.e., the symmetric macro aggregate error support covers a range of ± 0.15 the macro aggregate prior mean value.

(20) $X = \bar{A}\,[\sum \delta_i \cdot (\lambda_i \cdot x_i)^{\rho}]^{\frac{1}{\rho}}$.

In the two factor case, X is the CES or CET composite of factor x_1 and x_2, $\bar{A}$ the shift parameter, δ_i the CES share or distribution parameter and $\delta_1 + \delta_2 = 1$, λ_i a factor augmenting or biased productivity parameter for x_i, and ρ the exponent: $X = F(x_1, x_2; \delta_1, \delta_2, \rho, \bar{A}, \lambda_1, \lambda_2)$. The CES substitution elasticity σ and CET transformation elasticity Ω are given by $\sigma = 1/(1-\rho)\,;\ -\infty < \rho < 1$ in the CES case and $\Omega = 1/(\rho - 1); 1 < \rho < \infty$ in the CET case.

Below is the first order condition of the CES case, which is expressed in value terms of factor input per unit of output:

(21) $\frac{w_i \cdot x_i}{P \cdot X} = \bar{A}^{\ \sigma-1} \cdot \lambda_i^{\ \sigma-1} \cdot \delta_i^{\sigma} \left[\frac{P}{w_i}\right]^{\sigma-1}$.

where P is the price of the composite good X and w_i is the price of the input x_i. Alternatively, the value ratio of factor inputs for a homothetic aggregation function is the familiar:

(22) $\frac{w_1 \cdot x_1}{w_2 \cdot x_2} = \left[\frac{\lambda_2}{\lambda_1}\right]^{1-\sigma} \cdot \left[\frac{\delta_1}{\delta_2}\right]^{\sigma} \cdot \left[\frac{w_2}{w_1}\right]^{\sigma-1}$.

If there are only two components in the CES function, an index may summarize the relative factor-augmenting productivity, that is, $\lambda = \lambda_2/\lambda_1$. Since both the CET and CES functions exhibit constant returns to scale, the allocation of the composite good depends only on the relative prices of the individual components.

When data problems prevent traditional econometric estimation of the elasticities, they are often assumed or taken from other studies as extraneous estimates. The normal CGE calibration commonly uses a single and most recent SAM. Accordingly, prices in the model are initialized to 1.0, and values for the parameters, $\bar{A}$ and δ_i, are then derived based on the information in the SAM and the assumed values of the elasticities. However, if there are older historical SAMs, the priors can be improved with our approach and it is a mistake to discard the older historical SAMs, which contain vital information to improve extraneous numbers.

Furthermore, unless the prior estimates are initially close to the values of the unobserved parameters, the CGE-SAM-CE method will mostly improve the priors with additional SAMs; and the revised estimates can deviate significantly from the initial guesses. Even a single price deflator will generally bring about improved estimates over the priors. It is easy to see why from the two first-order conditions that are expressed in value shares above, equations (21) and (22).

In the absence of good price indices, the right-hand side clearly contains an error by a factor of $\left[\frac{P}{w_i}\right]^{\sigma-1}$ or $\left[\frac{w_2}{w_1}\right]^{\sigma-1}$ if nominal values of the SAMs are used in the left-hand side of the equations for the implicit derivation of the elasticities. Clearly, if relative price changes (between the SAMs) are stable, the error will be small; otherwise, they can be significant. Even so, the Bayesian method will make use of whatever implicit price indices are available and allows for errors and prior probability to be assigned and the posterior probability to be computed from the Kullback-Leibler objective function. In the SAM estimation step, we give more weight or certainty to known national accounts aggregates or values that are deflated by appropriate prices. Likewise, in the CGE calibration and estimation our method can also give more weight to a more recent SAM or to a particular SAM that is constructed with greater reliability. These steps will almost certainly improve the prior estimates.

Table 4 shows the results of the cross-entropy estimation for the Armington elasticities of South Korea under models 1, 2 and 3 above mentioned while table 5 shows the respective results for Sub-Saharan Africa under model 1, 2 and 4. Models 1 to 4 assume that there is additional information on the macroeconomic aggregates, i.e., more information than only an aggregated general GDP deflator. The difference between the four models relies on the dynamics of the CES-CET function parameters.

Model 1 considers constant delta-share parameters and time varying elasticities, model 2 contemplates constant elasticities and time varying delta-share parameters for the period 1990-2011 in the case of South Korea and 2004-2011 in the regional case of Sub-Saharan Africa. For the same periods, the South Korean applied model 3 contemplates a) constant delta-share parameters in all the CES/CET bundles (see figure 1), b) constant elasticities in the CES bundles of value added-factors and national output-value added-intermediates, and c) dynamic elasticities with linear trends in the CET domestic production-domestic sales-exports side and in the Armington CES national consumption-domestic purchases-imports hub. As the South Korean model 3 application, the 2004-2011 range Sub-Saharan African model 4 application consist on a) constant delta-share parameters in all the CES/CET bundles (figure 1), with the main difference that b) the dynamic behavior with linear trends is for the elasticities of three bundles: the CES value added-factors, the CET, and to Armington CES bundles, and c) there is constant 2004-2011 behavior for the elasticity of the CES national consumption-domestic purchases-imports hub.

For the dynamics of the elasticities and delta-share parameters of the CES and CET functions please refer to equations (25), (26) and (27). Likewise, in the South Korea and Sub-Saharan African cases, table 6 and table 7 summarize some results for the CET functions, table 8 and table 9 present the outcomes for the value added-factors CES bundle and table 10 and table 11 the estimates on the national output – value added – intermediates CES functions.

Additionally, note that the method is subject to the amount of parameters we want to recover simultaneously. By construction the prior error terms have zero mean (see appendix 2), then using the Kullback-Leibler objective function as in (1), the posterior error distributions tend to be as close as the prior ones. Thus, as we increase the number of parameters to estimate – as we increase the degrees of freedom – this minimal KL target produces posterior parameter error distributions that tend to have zero mean.

The same methodology may also be applied to derive parameters of other parts of the nested CES production structure in the standard CGE model, e.g., between value added and intermediate inputs in the output of each sector or between labor and capital in the value added of each sector. The method is also flexible enough to add more nested structure and can be applied to more flexible behavioral specifications such as the translog functions. In any case, the CES formulation may be viewed as local approximation of a more flexible form (Perroni, C. and T. F. Rutherford 1995).

Structural change and productivity

CGE models are increasingly applied to economic scenarios over a long-term horizon. With more SAMs, the approach provides the information basis to anticipate the economic transformation over the long-term in conjunction or consistent with the elasticity estimation. By economic transformation, we include both shifts in the economic structure and changes in productivity. A growth rate r may be estimated for the shift or scale parameter in the CES and CET functions as in (23). However, the growth in A_t affects all CES factor inputs, but one of the CES factor input may have a factor-specific productivity change which might incorporate dynamics as stated in (24); the growth rate of the factor augmenting productivity, g, can also be estimated from our cross-entropy methodology.

(23) $$A_t = A_{t_0}(1+r)^{t-t_0} .$$

(24) $$\lambda_t = \lambda_{t_0}(1+g)^{t-t_0} .$$

Recent literature seems to suggest that technical change in advanced countries appears to be net factor or labor augmenting, e.g., Jorgenson, D. W. (2001), Krusell, P., et al. (1997), Carraro, C. and E. De Cian (2009), etc. The factor augmenting productivity change may be estimated for various CES nested level of the supply side. Table 12 and table 14 show the results of our estimates for TFP and the labor augmenting productivity in the factors and value added function under setups 1, 2 and 3 for the South Korean case. Table 13 and table 15 also show the estimates of the above mentioned TFP and labor augmenting productivity for the Sub-Saharan Africa analysis under models 1, 2 and 4.

In our approach, it is also possible to consider that economic transformation will bring about less rigidness in the production and demand behavior of a developing country. The assumption of a constant elasticity in the Armington and CET functions may not hold over long-run simulations and can be relaxed. One way to handle this is to allow the elasticities to rise over time. If data are insufficient to estimate a flexible functional form, it may still be possible to incorporate changes in the elasticities if there are sufficient SAMs to cover several growth episodes using linear trends.

(25) $\boldsymbol{\sigma_t = \sigma_{t_0} + a \cdot t}$.

(26) $\Omega_t = \Omega_{t_0} + b \cdot t$.

Alternatively, foreign trade shares can be allowed to change to capture the effects of globalization, effects that are difficult to capture in a homothetic function like the CES or CET specification. A different approach is needed to capture rising trade shares as output expands. One option is to calibrate the change in the CES delta-share or distributional parameter. In our approach, a delta-share parameter corresponding to each year of SAM can be computed simultaneously with the other parameters and its linear and quadratic trend factors can be derived:

(27) $\delta_t = \delta_{t_0} + a_0 \cdot t + a_1 \cdot t^2$.

$$\delta_t = \delta_{t_0} + a_0 \cdot t + a_1 \cdot t^2.$$

Importantly, in our system approach there will be constraints in the number of parameters to be estimated simultaneously due to the degrees of freedom reflected in the behavioral relationships and in the limited amount of data. The estimates are therefore linked intrinsically. For example, the posterior estimates of the elasticities will tend to match their prior values if many of the other parameters are set free and/or are capturing much of the structural change story in the economy. The trade-offs can be exploited in the following ways. If there are good prior estimates of the elasticities, then the entropy method can be used to gain better estimates of the productivity and other parameters. If new estimates of the elasticities are important, then no many of the other parameters can be set free. This is illustrated with the estimated elasticities via the model variations on the CES and CET functions of the CGE; from table 4 through table 11 we present model results that are consistent with what is being allowed to be estimated with respect to the other parameters in the CGE model.

In addition, under models 1, 2, 3 and 4, we allow for the estimation of the labor augmenting productivity. In the first year, one reference sector, manufacturing in South Korea and SSA region, is set to 0.67[16] and to 1.0, respectively, allowing other sectors to vary relative to it so that there are relative differences in the sector productivity to start with. In subsequent years, labor augmenting productivity in all sectors will vary relative to their initial values.

In table 8 and table 9 the estimates of the elasticity of substitution between factors (capital and labor) in the value added – factors CES bundle are presented. This type of elasticity measures the percentage change variation in the capital-labor ratio (K/L) relative to a percentage change variation in the ratio of their prices (w_L/w_K), i.e., $\sigma_{K,L} = \frac{\%\Delta(K/L)}{\%\Delta(w_L/w_K)} = \frac{d\left(\frac{K}{L}\right)/(\frac{K}{L})}{d\left(\frac{w_L}{w_K}\right)/(\frac{w_L}{w_K})} = \frac{d\left(\frac{K}{L}\right)}{d\left(\frac{w_L}{w_K}\right)} \frac{w_L L}{w_K K}$. Under model 1, $\sigma_{K,L}$ increases its value in the agriculture, mining, utilities, construction and services sectors, while manufacturing remains at the same level during the period 1990-2011. On the other side, considering the assumptions of model 2 (dynamics on the delta-share parameters of the CES/CET functions and static elasticities of substitution),

[16] Based on the World Bank CGE LINKAGE model sources.

the delta-share index defined as the ratio of the posterior to the prior mean times one hundred shows that the agriculture, mining and utilities sectors experience a concave behavior in the period 1990-2011, the construction sector presents a convex index path. In contrast, manufacturing declines its ratio delta-share index while services does the opposite from 1990 to 2011.

In the same value added – factors CES bundle, two more parameters are estimated: total factor productivity and labor augmenting productivity. Under model specifications 1, 2, 3 and 4 we present posterior and prior TFP values in table 12 and table 13. For the Korean case, in model 1 and 2, mining and construction present a downward slope linear behavior, whereas agriculture, manufacturing, utilities and services describe a positive linear trend. Our results suggest that under this setup 1, manufacturing is the sector with the biggest total factor productivity in 2011 which is consistent with our priors, while under setup 2, is the second lowest TFP holder. Besides, in table 14 our results show that with the exception of services, all the South Korean sectors present a labor augmenting productivity with upward trend, being agriculture the one with the biggest change during the period 1990-2011 under setup 1. However, the LAP coefficients behave different for manufacturing, mining and services under setup 2. LAP for mining and manufacturing present downward sloping trends whereas services an upward path.

Model selection and parameter assessment

In a context of ambiguous results we would like to differentiate the model setups, and the CGE-SAM-CE procedure automatically fits this task. First, for data validation, we make use of equations (17) and (18) to estimate the adjusted R^2 and the AIC statistics under models 1, 2 and 3. The South Korean aggregated adjusted R^2 and the AIC of table 16 indicate that between 1990 and 2011, case 1 works better to explain and validate the SAMs' information, while in table 17, the SSA regional results imply that model 2 explain and validate better the CGE model and data for the period 2004-2011.

For the specific case of South Korea, the Kullback-Leibler (KL) statistic in (19) shows that among setup 1 and 2, there is a bigger reduction of data noise in results of model 1 during the period 1990-2011. Nonetheless, model 1 has bigger Kullback-Leibler statistic in 1995 for the prediction side than model 2; this implies that the standard CGE fits better in 1995 under the assumptions of dynamic delta-shares and fixed elasticities.

On the other side, the parameter evaluation in the CGE-SAM-CE setup is elaborated through the cross-entropy $\hat{R}^2$ statistic as in (16). In this regard, where correspondent, in tables of appendix 3 we show that generally under models 1 and 2 our prior parameter error Gaussian assumptions deviates from the total degree of ignorance assumption[17] in systematic way, i.e., cross-entropy $\hat{R}^2$ values of at least 0.30. There are also cases where the method identifies recovered parameters with more data noise and/or with bigger gain of information, for instance, in table 12, 13, 14 and table 15 we have examples that TFP growth rates and

[17] See the description of the uninformative prior from appendix 2.

labor augmenting productivity coefficients have bigger cross-entropy $\hat{R}^2$ values $(\hat{R}^2 \geq 0.30)$, i.e., bigger difference between the ignorance (uniform) distribution and the posterior-recovered one.

As previously commented, the method is subject to the amount of parameters we want to estimate simultaneously: the more parameters the more flexible estimation and the less likelihood to recover a posterior distribution far from the prior. In model 3 for the South Korean analysis we specify a setup that is an imperfect mirror of model 1, where the difference relies on fixed elasticities during the period 1990-2011 of the Armington and CET functions instead of the dynamic behavior with linear trend of the elasticity, i.e., model 3 means less parameters to estimate than model 1. Given model 3 rigidness, we observe in table 4 that the posterior mean of the CES Armington elasticity moves from a prior of 2.80 to an inelastic value of 0.10. Another result of the smaller flexibility of model 3 is observed in table 10, where the elasticity for manufacturing in the CES national output, value added and intermediates function changes from a prior of 1.5 to a posterior of 0.79 in 1990 and 0.10 in 2011. Finally, for the same South Korean example, on prediction side, table 16 also shows that model 3 gets worse scores (statistics) than model 1 and 2, meaning that model 3 is less reliable in terms of data validation. Same conclusions can be derived when we compare model 1 and 2 with model 4 for Sub-Saharan Africa analysis.

CONCLUSIONS AND FUTURE RESEARCH

In this paper, we define a formal Bayesian approach to estimate parameters of a CGE model under noisy and limited data environment. The cross-entropy estimation method described in the paper is potentially applicable to all calibrated CGE models and their SAM database.

Inferring information about parameters and elements of a country or regional CGE model could be problematic because of incomplete, infrequent, inaccessible, or uncertain data such as dated or poorly constructed SAMs and auxiliary information. To estimate behavioral parameters and structural change in CGE models, we implement a three-step cross-entropy estimation method. The data step adjusts the historical SAMs of South Korea and Sub-Saharan Africa to a common base year taking into account that the SAMs are measured with errors and relative price indices are scant. [18] Next, a parameter estimation step calculates (filters) parameters and structural change simultaneously within the specifications of the country/regional CGE model postulates. The last step provides some statistics that are useful to measure noise and information gains – possibly because of structural change and/or because of model specification – in recovered parameters as well as statistics that differentiate models through data validation measures.

The approach can be easily used to estimate how economic transformation will bring about structural change and less rigidness in the production and demand behavior of a developing country. There are, of course, trade-offs in the number of parameters to be estimated simultaneously due to the degrees of freedom and data constraints. As illustrations, we estimated the Armington and CET elasticities for South Korea and

[18] The selected base year was 2005.

Sub-Saharan Africa that were consistent with the case where labor augmenting productivity captured compositional changes in the sector value-added and indirectly, its effects on the final demand vectors. There are other possibilities, such as allowing for trade elasticities and trade shares to evolve in order to capture the effects of globalization and increasing trade.

We have a few suggestions for future research. One is to employ peripheral estimates of historical producitivty change and trade pattern in order to improve the estimates of the evolution of value added and trade elasticities. Second, a research focus on the combination of our current ex-post analysis method with ex-ante (prediction) techniques. Lastly, the approach seems ideal for regional or global CGE modeling where SAM and other auxliary data are still very limited; hence we hope to extend the estimation of parameters to various regional aggregation of multiple countries and outside of Sub-Saharan Africa, which are increasingly becoming available for global CGE modeling.

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APPENDIX I. CGE-SAM-CE MODEL NOTATION

k: subindex to represent the supports (outcomes).

m: subindex that characterizes the behavioral parameter on the $\boldsymbol{B}$ matrix; $m \in \{0,1,2,\dots,M: M \in \mathbb{N}_0\}$.

n: subindex to symbolize a SAM target on the $\boldsymbol{Y}$ matrix; $n \in \{0,1,2,\dots,N: N \in \mathbb{N}_0\}$

n_t^{SAM}: represents the number of selected SAM targets at period t.

s: subindex that denotes an unobserved parameter on the $\boldsymbol{Z}$ matrix; $s \in \{0,1,2,\dots,S: S \in \mathbb{N}_0\}$.

As consequence, k_m, k_n, and k_s are compound index that indicate outcomes of the m^{th} behavioral parameter, the n^{th} SAM target, and the s^{th} unobserved parameter, respectively; k_m, k_n, k_s $\in \{2,3,\dots,K: K \in \mathbb{N}_{>1}\}$ $\forall m,n,s$.

t: index to denote through t such that $t \in \{t_1,\dots,t_T: T \in \mathbb{N}_{>1}\}$.

$\boldsymbol{\delta}$: vector to denote the calibrated parameters.

$\boldsymbol{\Phi}$: vector of the CGE calibration procedure.

$\boldsymbol{B}$ **:** an M-dimensional vector of behavioral parameters such as the Armington and CET elasticities.

$\boldsymbol{F}$: an $(I * T)$-dimensional vector valued function.

$\boldsymbol{G}$: a $(L * T)$-dimensional vector valued function.

$\boldsymbol{V}$: a $(L * L * T)$-array representation of the SAMs.

$\boldsymbol{X_t}$: an I-dimensional vector of endogenous CGE variables such as prices and quantities.

$\boldsymbol{Y}$: this symbolizes a $\left(n_{t_1}^{SAM} + n_{t_2}^{SAM} + \dots + n_{t_T}^{SAM}\right)$-dimensional vector of SAM targets.

$\boldsymbol{Z}$: a $(S+R)$-dimensional vector of exogenous parameters such as endowments and tax rates; $\boldsymbol{Z}$ is partitioned in two components: $\boldsymbol{Z} = \{\boldsymbol{Z}_t^o, \boldsymbol{Z}_t^u\}$, such that, the first element, $\boldsymbol{Z}_t^o$ is an R-dimensional vector of observed exogenous parameters, which may consist of historical data elements such as tax rates, endowments, prices, government spending rates, household consumption or saving rates, etc. The second element, $\boldsymbol{Z}_t^u$ symbolizes an S-dimensional vector of unobserved parameters, which may contain labor augmenting productivity coefficients, growth rates of CES/CET scale-shifter parameters, implicit or unobserved tax or subsidy rates, and other items that are not available from the historical information.

$\boldsymbol{e}$: symbol to denote the error terms; $e \in \mathbb{R}$.

$\boldsymbol{v}$: indicates the vector of error outcomes (error support elements); each element of this vector belongs to the real numbers, i.e., $v_i \in \mathbb{R}$.

$\boldsymbol{w}$ and $\overline{\boldsymbol{w}}$: vectors of posterior and prior probabilities of specific outcome's vector $\boldsymbol{v}$, respectively; $w, \overline{w} \in [0,1]$.

APPENDIX 2. DESCRIPTION OF THE PRIORS

Uninformative prior. An uninformative prior provides information only about the bounds between which the errors must be located. In Bayesian estimation and information theory, the most uninformative prior is the uniform distribution, which has maximum entropy. We will specify a discrete prior probability mass function that approximates the continuous uniform distribution between known upper and lower bounds. Assume that the upper and lower bounds on v are given by plus or minus 3s, where "s" is a specified constant. For the continuous uniform distribution between these bounds, the variance is: $\sigma^2 = \frac{(3s-(-3s))^2}{12}$ $= 3s^2$.

Specifying an evenly-spaced, 7-element support set (k = 7), : $v_1 = -3s$, $v_2 = -2s$, $v_3 = -s$, $v_4 = 0$, $v_5 = +s$, $v_6 = +2s$, $v_7 = +3s$, with identical (uniform) prior probability weights $\overline{w}_k = \frac{1}{7}$, then the variance of e is $\sigma^2 = \sum_k \overline{w}_k v_k^2 = \frac{s^2}{7}(18 + 8 + 2) = 4s^2$. A discrete uniform prior with 7-element support set is a conservative uninformative prior, with a prior variance of $4s^2$. Adding more elements to the support set would more closely approximate the continuous uniform distribution, reducing the prior variance toward the limit of $3s^2$. Note that the estimated posterior distribution will be essentially unconstrained.

Informative 2-parameter prior. Start with a prior on both the mean and standard deviation of a symmetric, two-parameter error distribution. The prior mean on the error is zero by construction and the prior standard deviation of e is specified as the prior on the standard error of measurement of the item. Specify an evenly- spaced support set with $s = \sigma$ so that the bounds are now $\pm 3\sigma$. Then, $v_1 = -3\sigma$, $v_2 = 0$, $v_3 = +3\sigma$ with probability priors $w_1 = \frac{1}{18}$, $w_2 = \frac{16}{18}$, $w_3 = \frac{1}{18}$, thus, the mean, $\mathbb{E}[e] = \sum_k \overline{w}_k v_K = 0$, and the variance, $var[e] = \sum_k \overline{w}_k v_k^2 = \sigma^2$. The discrete distribution specification is symmetric, $w_1 = w_3 = \frac{1}{18}$. Estimation of a posterior distribution in this case can retrieve information about essentially two moments of the error distribution, since the 3-element prior only allows two degrees of freedom in estimation (since the probability weights must sum to one). One can specify a more informative prior using a larger support set.

Informative 4-parameter prior. To recover more information about the error distribution the prior must include more moments—for example: mean, variance, skewness, and kurtosis. Assume a normal distribution with a prior for the mean and variance so that prior skewness is zero and kurtosis is a function of σ. $\mathbb{E}[e] = \sum_k \overline{w}_k v_K = 0$, $var[e] = \sum_k \overline{w}_k v_k^2 = \sigma^2$, $\mathbb{E}[e^3] = \sum_k \overline{w}_k v_k^3 = 0$, $\mathbb{E}[e^4] = \sum_k \overline{w}_k v_k^4 = 3\sigma^2$. This prior discrete distribution is obtained specifying an evenly-spaced 5-element support set: $v_1 = -3\sigma$, $v_2 = -1.5\sigma$, $v_3 = 0$, $v_4 = +1.5\sigma$, $v_5 = +3\sigma$. In this case, the prior values on the probability weights are $\overline{w}_1 = \overline{w}_5 = \frac{1}{162}$, $\overline{w}_2 = \overline{w}_4 = \frac{16}{81}$, $\overline{w}_3 = \frac{48}{81}$, which can be calculated from the known prior moments and the assumption of adding up as seen in the system of equations represented in (28). As with the other priors, the estimated posterior distribution is unconstrained.

$$(28) \qquad \begin{bmatrix} 0 \\ \sigma^2 \\ 0 \\ 3\sigma^4 \\ 1 \end{bmatrix} = \begin{bmatrix} -3\sigma & -1.5\sigma & 0 & 1.5\sigma & 3\sigma \\ 9\sigma^2 & 2.25\sigma^2 & 0\sigma^2 & 2.25\sigma^2 & 9\sigma^2 \\ -27\sigma^3 & -3.375\sigma^3 & 0\sigma^3 & 3.375\sigma^3 & 27\sigma^3 \\ 81\sigma^4 & 5.0625\sigma^4 & 0\sigma^4 & 5.0625\sigma^4 & 81\sigma^4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \bar{w}_1 \\ \bar{w}_2 \\ \bar{w}_3 \\ \bar{w}_4 \\ \bar{w}_5 \end{bmatrix} = \begin{bmatrix} mean \\ variance \\ skewness \\ kurtosis \\ additivity \end{bmatrix}.$$

APPENDIX 3. TABLES

Table 1. Percent Deviations of Cross-Entropy Estimates Versus Control Values of National Accounts, The Case of South Korea

Based on different (SAM) years, in constant 2005 prices.					
	1990	*1995*	*2000*	*2005*	*2011*
Personal consumption	-1.807	-2.849	-0.173	0.135	0.601
Government	-0.395	-0.551	-0.071	0.028	0.118
Investment	-0.772	-2.101	-0.065	0.078	0.228
Exports	0.057	0.048	0.053	0.007	0.005
Agriculture value added	0.049	0.043	0.022	-0.015	-0.036
Mining value added	0.031	0.038	0.027	0.001	-0.024
Manufacturing value added	0.052	0.058	0.027	-0.017	-0.041
Utilities value added	0.042	0.035	0.032	0.001	-0.038
Construction value added	0.024	0.004	0.011	-0.009	-0.028
Value added for all activities	2.597	2.987	0.330	-0.226	-1.397

Source: Authors' calculations.

Table 2. Percent Deviations of Cross-Entropy Estimates Versus Control Values of National Accounts, The Case of SSA region

Based on different (SAM) years, in constant 2005 prices.			
	2004	*2007*	*2011*
Government	0.018	-0.014	-2.192
Investment	0.001	-0.086	-10.315
Exports	0.715	0.674	-14.999
Imports	-0.563	-0.908	-11.215
Agriculture value added	0.044	-0.083	-6.952
Manufacturing value added	0.082	0.013	-0.443
Services value added	0.234	-0.119	-9.493
Value added for all activities	-0.544	0.103	-6.064

Source: Authors' calculations

Table 3. Model structure on the CES-CET functions

function/model	Model I	Model 2	Model 3 (only for South Korea)	Model 4 (only for SSA region)
CES: value added and factors bundle.	-dynamic TFP parameter with compound growth rate. -dynamic labor augmenting productivity coefficient with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.	-dynamic TFP parameter with compound growth rate. -dynamic labor augmenting productivity coefficient with compound growth rate. -dynamic delta-share parameter with linear and quadratic trend coefficients. -fixed elasticity.	-dynamic TFP parameter with compound growth rate. -dynamic labor augmenting productivity coefficient with compound growth rate. -fixed delta-share parameter. -fixed elasticity.	-dynamic TFP parameter with compound growth rate. -dynamic labor augmenting productivity coefficient with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.
CES: national output, value added and intermediates bundle.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.	-dynamic scale parameter with compound growth rate. -dynamic delta-share parameter with linear and quadratic trend coefficients. -fixed elasticity.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -fixed elasticity.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -fixed elasticity.
CET: domestic production, domestic sales and exports bundle	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.	-dynamic scale parameter with compound growth rate. -dynamic delta-share parameter with linear and quadratic trend coefficients. -fixed elasticity.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.
CES: National consumption, domestic purchases and imports bundle (Armington).	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.	-dynamic scale parameter with compound growth rate. -dynamic delta-share parameter with linear and quadratic trend coefficients. -fixed elasticity.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.	-dynamic scale parameter with compound growth rate. -fixed delta-share parameter. -dynamic elasticity with linear trend.

Table 4. Armington elasticity, using SAMs with limited additional price data, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.							
CES Armington Elasticity	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Imports Delta-Share Index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	2.20	2.01*	2.11*	2.21	2.31	2.41	94.50
Mining	2.80	0.85	1.33*	1.81*	2.28*	2.76*	132.95
Manufacturing	2.20	1.43	1.23	1.03	0.84	0.64	58.99
Utilities	2.80	2.50	2.66	2.82	2.98	3.14	77.38
Construction	1.90	1.62	1.76	1.90	2.04	2.18	23.24
Services	1.90	1.83	1.88	1.94	1.99	2.05	93.60
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.							
Imports Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Posterior Mean CES Armington Elasticity (prior in parentheses)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.30	98.94	97.18	99.11	104.72	114.80	2.16 (2.20)
Mining	0.61	98.94	103.85	109.48	115.83	123.04	2.98 (2.80)
Manufacturing	0.23	75.38	103.51	123.33	134.80	136.18	1.69 (2.20)
Utilities	0.09	104.65	102.09	99.19	95.94	92.28	2.86 (2.80)
Construction	1.93E-04	68.93	117.02	133.62	118.56	65.32	1.82 (1.90)
Services	0.13	98.06	105.27	107.91	105.97	98.48	1.88 (1.90)
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.							
CES Armington Elasticity	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Imports Delta-Share Index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	2.20	2.13	2.13	2.13	2.13	2.13	98.16
Mining	2.80	0.10***	0.10***	0.10***	0.10***	0.10***	164.59
Manufacturing	2.20	2.62	2.62	2.62	2.62	2.62	115.44
Utilities	2.80	2.81	2.81	2.81	2.81	2.81	100.47
Construction	1.90	1.90	1.90	1.90	1.90	1.90	100.03
Services	1.90	1.87	1.87	1.87	1.87	1.87	97.18

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-*$\hat{R}^2$*, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-*$\hat{R}^2$ *greater than 0.40.*

*** *Posteriors with cross-entropy-*$\hat{R}^2$ *greater than 0.75.*

Table 5. Armington elasticity, using SAMs with limited additional price data, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.					
CES Armington Elasticity	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Imports Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	1.95	1.82	1.99	2.16	97.85
Mining	1.95	1.76	1.95	2.15	99.84
Manufacturing	2.25	2.07	2.27	2.47	99.31
Utilities	2.05	1.87	2.06	2.25	95.50
Construction	1.95	1.80	1.98	2.16	95.19
Services	1.95	1.83	1.99	2.14	98.86
Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.					
Imports Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Posterior Mean CES Armington Elasticity (prior in parentheses)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.17	101.44	89.27	107.45	1.87(1.85)
Mining	0.48	100.15	94.05	102.84	1.94(1.85)
Manufacturing	0.42	100.64	106.82	117.65	2.23(2.15)
Utilities	0.20	102.85	101.91	100.59	2.00(1.95)
Construction	0.13	90.92	119.41	106.46	1.75(1.85)
Services	0.19	98.67	111.45	106.61	1.83(1.85)
Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.					
CES Armington Elasticity	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Imports Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	1.95	1.83	1.99	2.15	98.76
Mining	1.95	1.76	1.96	2.15	99.85
Manufacturing	2.25	2.55	2.69	2.83	102.91
Utilities	2.05	1.86	2.05	2.24	94.64
Construction	1.95	1.81	1.98	2.16	96.00
Services	1.95	1.89	2.01	2.13	102.65

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

Table 6. CET elasticity, using SAMs with limited additional price data, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.							
CET Elasticity	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Exports Delta-Share Index: 100*(Posterior/Prior)
sector/period	1990-2011	1990	1995	2000	2005	2011	*1990-2011*
Agriculture	3.78	3.38	3.57	3.77	3.96	4.15	102.96
Mining	0.89	0.83	0.86	0.88	0.91	0.93	100.44
Manufacturing	0.39	0.56	0.47	0.38	0.29	0.21*	99.19
Utilities	1.10	0.98	1.06	1.14	1.22	1.30	100.29
Construction	1.10	1.05	1.07	1.09	1.12	1.14	100.08
Services	1.10	1.04	1.09	1.15	1.20	1.25	100.99
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.							
Exports Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Posterior Mean CET Elasticity (prior in parentheses)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.70	100.19	100.81	100.64	99.68	98.71	3.75 (3.78)
Mining	0.98	100.19	100.29	100.18	99.86	99.09	0.87 (0.89)
Manufacturing	0.999	100.01	100.06*	100.06*	100.01	99.91*	0.38 (0.39)
Utilities	0.99	99.88	100.01	100.09	100.11	100.20	1.14 (1.1)
Construction	0.997	99.94	100.01	100.04	100.03	99.99	1.14 (1.1)
Services	0.93	100.10	99.90	99.76	99.69	99.27	1.09 (1.1)
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.							
CET Elasticity	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Exports Delta-Share Index: 100*(Posterior/Prior)
sector/period	1990-2011	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	3.78	4.20	4.20	4.20	4.20	4.20	97.41
Mining	0.89	0.79	0.79	0.79	0.79	0.79	100.72
Manufacturing	0.39	0.48	0.48	0.48	0.48	0.48	99.69
Utilities	1.10	1.07	1.07	1.07	1.07	1.07	100.09
Construction	1.10	1.04	1.04	1.04	1.04	1.04	100.09
Services	1.10	1.10	1.10	1.10	1.10	1.10	100.06

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

Table 7. CET elasticity, using SAMs with limited additional price data, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.					
CET Elasticity	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Exports Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	2.25	2.11	2.30	2.48	100.45
Mining	2.45	2.26	2.46	2.66	97.57
Manufacturing	2.85	2.71	2.88	3.05	100.31
Utilities	2.45	2.22	2.42	2.62	101.60
Construction	2.25	2.23	2.38	2.52	99.28
Services	2.35	2.46	2.58	2.71	97.58
Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.					
Exports Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Posterior Mean CET Elasticity (prior in parentheses)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.71	101.38	100.11	89.10	2.04(2.15)
Mining	0.29	103.30	103.24	85.85	2.48(2.35)
Manufacturing	0.66	100.81	92.81	87.19	2.65(2.75)
Utilities	0.75	99.51	100.36	100.10	2.39(2.35)
Construction	0.92	101.67	98.98	97.69	1.97(2.15)
Services	0.78	100.08	98.35	95.43	2.24(2.25)
Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.					
CET Elasticity	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Exports Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	2.25	2.09	2.28	2.47	100.79
Mining	2.45	2.29	2.47	2.66	98.24
Manufacturing	2.85	2.82	2.97	3.12	99.41
Utilities	2.45	2.08	2.31	2.55	103.41
Construction	2.25	2.23	2.38	2.53	99.28
Services	2.35	2.37	2.46	2.55	98.54

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

Table 8. Value added-factors elasticity, using SAMs with limited additional price data, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.							
CES VA-Factors Elasticity	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Capital Delta-Share Index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.56	0.34	0.41	0.49	0.56	0.64	144.32
Mining	1.12	0.89	1.00	1.11	1.22	1.33	135.05
Manufacturing	1.26	1.05	1.05	1.05	1.05	1.05	102.12
Utilities	1.26	1.08	1.15	1.22	1.29	1.36	117.22
Construction	1.40	0.84	1.05	1.26	1.47	1.68	82.08
Services	1.26	1.02	1.14	1.26	1.38	1.51	105.18
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.							
Capital Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Posterior Mean VA-Factors Elasticity (prior in parentheses)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.57	93.42	111.22	119.74	118.93	106.86	0.50 (0.56)
Mining	0.39	95.97	113.73	124.58	128.48	124.00	1.16 (1.12)
Manufacturing	0.490	100.76	90.23	80.97	73.00	66.59	1.17 (1.26)
Utilities	0.51	100.87	113.72	116.02	107.73	86.65	1.25 (1.26)
Construction	0.567	100.07	89.85	83.65	81.48	84.18	1.45 (1.40)
Services	0.46	98.44	99.32	101.42	104.76	109.59	1.30 (1.26)
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.							
CES VA-Factors Elasticity	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Capital Delta-Share Index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.56	0.60	0.54	0.47	0.41	0.35	112.06
Mining	1.12	0.76	0.87	0.98	1.08	1.19	173.57
Manufacturing	1.26	1.09	1.18	1.27	1.35	1.44	101.62
Utilities	1.26	0.95	1.03	1.10	1.18	1.26	133.25
Construction	1.40	0.71*	0.95	1.20	1.45	1.69	75.81
Services	1.26	1.13	1.20	1.28	1.35	1.43	102.16

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

Table 9. Value added-factors elasticity, using SAMs with limited additional price data, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.					
CES VA-Factors Elasticity	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Capital Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.85	0.80	0.74	0.68	156.75
Mining	0.95	0.86	0.91	0.97	100.83
Manufacturing	0.95	0.89	0.79	0.69	125.78
Utilities	0.75	0.66	0.65	0.63	120.10
Construction	0.75	0.69	0.63	0.57	169.37
Services	1.15	1.06	1.15	1.24	102.61
Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.					
Capital Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Posterior Mean VA-Factors Elasticity (prior in parentheses)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.06	63.30	72.50	146.69	0.71(0.75)
Mining	0.87	101.28	99.43	99.22	0.87(0.85)
Manufacturing	0.27	93.45	119.50	126.38	0.84(0.85)
Utilities	0.08	91.68	97.81	116.35	0.64(0.65)
Construction	0.03	105.90	64.57	148.68	0.65(0.65)
Services	0.50	104.66	101.14	85.01	1.07(1.05)
Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.					
CES VA-Factors Elasticity	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Capital Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.85	0.78	0.73	0.67	134.24
Mining	0.95	0.87	0.92	0.97	101.40
Manufacturing	0.95	0.89	0.84	0.78	125.04
Utilities	0.75	0.67	0.65	0.63	123.29
Construction	0.75	0.69	0.64	0.58	174.49
Services	1.15	1.07	1.15	1.22	106.82

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-*$\hat{R}^2$*, greater than 0.30; see statistic definition in (16).*

Table 10. National output-value added-intermediates elasticity, using SAMs with limited additional price data, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.							
CES National Output-Value Added-Intermediates	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Intermediates Delta-Share Index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	1.50	1.23	1.35	1.47	1.59	1.71	100.29
Mining	1.50	1.26	1.38	1.50	1.62	1.73	100.47
Manufacturing	1.50	1.52	1.51	1.51	1.50	1.50	99.70
Utilities	1.50	1.29	1.40	1.51	1.62	1.72	102.81
Construction	1.50	1.38	1.43	1.49	1.55	1.61	100.96
Services	1.50	1.45	1.50	1.54	1.59	1.64	98.93
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.							
Intermediates Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Delta-Share Index: 100*(Posterior /Prior)	Posterior Mean National Output-Value Added-Intermediates Elasticity (prior in parentheses)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.51	100.02	102.28	107.88	116.84	129.84	1.47 (1.50)
Mining	0.51	100.00	100.42	102.33	105.73	110.93	1.50 (1.50)
Manufacturing	0.76	102.43	97.76	94.90	93.86	95.01	1.38 (1.50)
Utilities	0.61	99.49	96.74	99.10	106.58	120.26	1.54 (1.50)
Construction	0.56	99.80	97.47	96.85	97.94	101.11	1.53 (1.50)
Services	0.38	99.78	101.80	104.82	108.87	114.14	1.49 (1.50)
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.							
CES National Output-Value Added-Intermediates	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Intermediates Delta-Share Index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	1.50	1.17	1.26	1.34	1.42	1.51	100.36
Mining	1.50	1.21	1.28	1.35	1.43	1.50	100.61
Manufacturing	1.50	0.79*	0.62*	0.45*	0.27**	0.10***	118.27
Utilities	1.50	1.59	1.65	1.72	1.79	1.85	99.01
Construction	1.50	1.15	1.30	1.46	1.61	1.76	103.24
Services	1.50	1.55	1.55	1.55	1.54	1.54	101.02

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

** *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.50.*

*** *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.75.*

Table 11. National output-value added-intermediates elasticity, using SAMs with limited additional price data, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.

CES National Output-Value Added-	Elasticity Prior (average)	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Intermediates Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.95	0.74	0.94	1.14	89.64
Mining	1.05	0.94	1.08	1.21	98.56
Manufacturing	1.05	0.77	0.62*	0.47	105.54
Utilities	0.85	0.69	0.86	1.03	100.33
Construction	1.15	0.97	1.16	1.35	101.44
Services	1.25	1.06	1.23	1.41	99.08

Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.

Intermediates Delta-Share	Delta-Share Prior	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Delta-Share Index: 100*(Posterior/Prior)	Posterior Mean National Output-Value Added-Intermediates Elasticity (prior in parentheses)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.27	106.99	95.36	90.64	0.94(0.85)
Mining	0.21	104.50	125.41	82.17	0.99(0.95)
Manufacturing	0.82	100.24	100.35	94.43	0.94(0.95)
Utilities	0.52	99.56	105.59	94.87	0.84(0.75)
Construction	0.62	100.59	110.62	111.06	1.02(1.05)
Services	0.45	101.48	94.30	83.48	1.32(1.15)

Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.

CES National Output-Value Added-	Elasticity Prior	Posterior Mean Elasticity	Posterior Mean Elasticity	Posterior Mean Elasticity	Intermediates Delta-Share Index: 100*(Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.85	0.93	0.93	0.93	106.67
Mining	0.95	1.04	1.04	1.04	109.02
Manufacturing	0.95	1.12	1.12	1.12	95.62
Utilities	0.75	0.86	0.86	0.86	99.50
Construction	1.05	1.17	1.17	1.17	98.06
Services	1.15	1.39	1.39	1.39	101.94

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

Table 12. Total factor productivity, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.						
Total Factor Productivity (TFP)	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior TFP annual compound growth rate (in %)
sector/period	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.45	0.49	0.53	0.58	0.64	1.73*
Mining	1.22	1.15	1.07	1.01	0.93	-1.31**
Manufacturing	0.33	0.46	0.62	0.85	1.24	6.47**
Utilities	0.54	0.60	0.68	0.76	0.86	2.26**
Construction	1.10	1.08	1.05	1.02	1.00	-0.48**
Services	0.49	0.52	0.54	0.56	0.59	0.82*
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.						
Total Factor Productivity (TFP)	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior TFP annual compound growth rate (in %)
sector/period	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.31	0.31	0.36	0.43	0.52	3.55**
Mining	1.37	1.31	1.21	1.11	1.02	-1.66**
Manufacturing	0.33	0.33	0.40	0.47	0.56	3.55*
Utilities	0.60	0.59	0.65	0.71	0.78	1.85**
Construction	1.14	1.12	1.09	1.06	1.04	-0.51**
Services	0.55	0.53	0.56	0.59	0.62	1.10**
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.						
Total Factor Productivity (TFP)	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior TFP annual compound growth rate (in %)
sector/period	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.34	0.42	0.50	0.61	0.77	3.97**
Mining	1.31	1.36	1.41	1.47	1.53*	0.73**
Manufacturing	0.33	0.31	0.29*	0.27*	0.24*	-1.51
Utilities	0.50	0.60	0.72	0.86	1.07	3.71*
Construction	1.13	1.11	1.09	1.07	1.05	-0.38**
Services	0.52	0.55	0.58	0.61	0.65	1.12**
Prior Mean for Model 1, 2 and 3.						
Total Factor Productivity (TFP)	Prior TFP	Prior TFP	Prior TFP	Prior TFP	Prior TFP	Prior TFP annual compound growth rate (in %)
sector/period	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*
Agriculture	0.31	0.43	0.48	0.52	0.64	3.57
Mining	1.35	1.25	1.01	1.12	1.02	-1.35
Manufacturing	0.33	0.50	0.72	0.95	1.15	6.07
Utilities	0.60	0.60	0.53	0.69	1.01	2.50
Construction	1.08	1.11	0.98	1.01	0.97	-0.52
Services	0.51	0.61	0.60	0.61	0.66	1.26

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

** *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.50.*

Table 13. Total factor productivity, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.				
Total Factor Productivity (TFP)	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior TFP annual compound growth rate (in %)
sector/period	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.004	0.005	0.005	1.41
Mining	0.237	0.156	0.089	-13.00**
Manufacturing	0.014	0.012	0.011	-2.66*
Utilities	0.011	0.013	0.016	5.76**
Construction	0.006	0.007	0.011	9.83*
Services	0.024	0.023	0.021	-1.72**
Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.				
Total Factor Productivity (TFP)	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior TFP annual compound growth rate (in %)
sector/period	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.004	0.004	0.004	0.18
Mining	0.237	0.155	0.088	-13.21**
Manufacturing	0.010	0.012	0.016	6.48**
Utilities	0.010	0.012	0.015	6.03**
Construction	0.005	0.007	0.011	11.59**
Services	0.025	0.023	0.022	-1.65**
Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.				
Total Factor Productivity (TFP)	Posterior Mean TFP	Posterior Mean TFP	Posterior Mean TFP	Posterior TFP annual compound growth rate (in %)
sector/period	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.004	0.005	0.005	2.55*
Mining	0.238	0.155	0.088	-13.32**
Manufacturing	0.013	0.014	0.014	0.63
Utilities	0.011	0.013	0.016	5.49**
Construction	0.006	0.007	0.010	9.22*
Services	0.024	0.022	0.020	-2.03*
Prior Mean for Model 1, 2 and 4.				
Total Factor Productivity (TFP)	Prior TFP	Prior TFP	Prior TFP	Prior TFP annual compound growth rate (in %)
sector/period	*2004*	*2007*	*2011*	*2004-2011*
Agriculture	0.004	0.005	0.006	5.08
Mining	0.230	0.123	0.087	-12.90
Manufacturing	0.011	0.017	0.019	8.42
Utilities	0.009	0.015	0.016	7.33
Construction	0.005	0.008	0.013	15.90
Services	0.022	0.028	0.020	-1.47

Source: Authors' calculations.

† *Posteriors are estimated using a Gaussian prior error distribution with five elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

** *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.50.*

Table 14. Labor augmenting productivity, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.								
Labor Augmenting Productivity (LAP)	Prior LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Prior LAP annual compound growth rate (in %)	LAP annual compound growth rate index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*	*1990-2011*
Agriculture	0.69	0.21*	0.30	0.44	0.63**	0.97	10.00	75.25**
Mining	0.80	0.71**	0.74**	0.77**	0.80**	0.84**	10.00	7.77
Manufacturing	0.67	0.67**	0.68**	0.68**	0.69**	0.70**	10.00	2.01
Utilities	0.80	0.78**	0.79**	0.81**	0.83**	0.85**	10.00	4.35
Construction	0.80	0.71**	0.77**	0.84**	0.91**	1.01*	10.00	16.87
Services	0.80	0.82**	0.78**	0.73**	0.69**	0.65*	10.00	-11.06
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.								
Labor Augmenting Productivity (LAP)	Prior LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Prior LAP annual compound growth rate (in %)	LAP annual compound growth rate index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*	*1990-2011*
Agriculture	0.69	0.65**	0.69**	0.73**	0.77**	0.82*	10.00	11.19
Mining	0.80	0.87**	0.83**	0.79**	0.75**	0.71**	10.00	-9.34
Manufacturing	0.67	0.67**	0.62**	0.57**	0.53*	0.48*	10.00	-15.98
Utilities	0.80	0.83**	0.83**	0.83**	0.83**	0.84**	10.00	0.65
Construction	0.80	0.75**	0.79**	0.83**	0.87**	0.93**	10.00	10.46
Services	0.80	0.74**	0.77**	0.80**	0.83**	0.87**	10.00	7.72
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.								
Labor Augmenting Productivity (LAP)	Prior LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Prior LAP annual compound growth rate (in %)	LAP annual compound growth rate index: 100*(Posterior/Prior)
sector/period	*1990-2011*	*1990*	*1995*	*2000*	*2005*	*2011*	*1990-2011*	*1990-2011*
Agriculture	0.69	0.53	0.62	0.74	0.87	1.06	10.00	33.43
Mining	0.80	0.39	0.52	0.70	0.93	1.32	10.00	59.87*
Manufacturing	0.67	0.67	0.64	0.61	0.59	0.56	10.00	-8.55
Utilities	0.80	0.70	0.76	0.82	0.88	0.96	10.00	15.05
Construction	0.80	0.65	0.75	0.86	0.99	1.17	10.00	28.03
Services	0.80	0.76	0.78	0.79	0.81	0.83	10.00	4.00

Source: Authors' calculations.

† *Posteriors are estimated using a prior error distribution with three elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

** *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.50.*

Table 15. Labor augmenting productivity, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.						
Labor Augmenting Productivity (LAP)	Prior LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Prior LAP annual compound growth rate (in %)	LAP annual compound growth rate index: 100* (Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*	*2004-2011*
Agriculture	1.0	1.11**	1.03**	0.92**	10.0	-2.71*
Mining	1.0	0.83*	0.97**	1.19*	10.0	5.31*
Manufacturin	1.0	1.00**	1.00**	1.00**	10.0	0.04
Utilities	1.0	0.92**	1.01**	1.14**	10.0	3.07
Construction	1.0	0.96**	1.03**	1.11**	10.0	2.06
Services	1.0	0.96**	0.97**	0.99**	10.0	0.32
Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.						
Labor Augmenting Productivity (LAP)	Prior LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Prior LAP annual compound growth rate (in %)	LAP annual compound growth rate index: 100* (Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*	*2004-2011*
Agriculture	1.0	0.91**	0.91**	0.89**	10.0	-0.32
Mining	1.0	0.86**	0.98**	1.17**	10.0	4.48*
Manufacturin	1.0	1.00**	1.02**	1.05**	10.0	0.74
Utilities	1.0	0.85**	0.96**	1.12**	10.0	4.02*
Construction	1.0	0.87**	0.97**	1.11**	10.0	3.61
Services	1.0	0.96**	0.99**	1.02**	10.0	0.83
Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.						
Labor Augmenting Productivity (LAP)	Prior LAP	Posterior Mean LAP	Posterior Mean LAP	Posterior Mean LAP	Prior LAP annual compound growth rate (in %)	LAP annual compound growth rate index: 100* (Posterior/Prior)
sector/period	*2004-2011*	*2004*	*2007*	*2011*	*2004-2011*	*2004-2011*
Agriculture	1.0	1.04**	1.03**	1.02**	10.0	-0.25
Mining	1.0	0.83*	0.96**	1.18**	10.0	5.25*
Manufacturin	1.0	1.00**	1.00**	1.00**	10.0	-0.06
Utilities	1.0	0.93**	1.01**	1.13**	10.0	2.76
Construction	1.0	0.97**	1.02**	1.11**	10.0	1.95
Services	1.0	1.12**	0.90**	0.68	10.0	-6.81**

Source: Authors' calculations.

† *Posteriors are estimated using a prior error distribution with three elements in its support.*

‡ *All the posteriors get cross-entropy-$\hat{R}^2$, greater than 0.30; see statistic definition in (16).*

* *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.40.*

** *Posteriors with cross-entropy-$\hat{R}^2$ greater than 0.50.*

Table 16. Goodness of fit statistics, using SAMs with limited additional price data, South Korea

Model 1. Constant delta-share parameter and time-varying elasticity across period 1990-2011.						
	1990	*1995*	*2000*	*2005*	*2011*	*1990-2011*
adjusted $\boldsymbol{R^2}$ for prediction side	1.00	0.9936	0.9964	0.9925	0.9941	0.9938
Akaike Information Criteria for prediction side	0.00	110.23	109.12	378.29	572.83	234.09
Kullback-Leibler divergence statistic for prediction side	0.00	1.58	0.69	0.56	0.40	3.23
Kullback-Leibler divergence statistic for prediction & precision sides	8.68	9.39	8.04	7.96	8.44	42.51
Model 2. Constant elasticity and time-varying delta-share parameters across period 1990-2011.						
	1990	*1995*	*2000*	*2005*	*2011*	*1990-2011*
adjusted $\boldsymbol{R^2}$ for prediction side	1.00	0.9963	0.9765	0.9725	0.9617	0.9690
Akaike Information Criteria for prediction side	0.00	64.52	720.85	1385.64	3701.00	1174.40
Kullback-Leibler divergence statistic for prediction side	0.00	0.81	1.49	0.81	1.63	4.74
Kullback-Leibler divergence statistic for prediction & precision sides	8.10	9.87	11.19	10.63	12.45	52.24
Model 3. Constant elasticity for Armington and CET functions and time-varying delta-share parameters across period 1990-2011.						
	1990	*1995*	*2000*	*2005*	*2011*	*1990-2011*
adjusted $\boldsymbol{R^2}$ for prediction side	1.00	0.9955	0.9667	0.9278	0.8484	0.8977
Akaike Information Criteria for prediction side	0.00	77.21	1020.34	3636.98	14656.29	3878.16
Kullback-Leibler divergence statistic for prediction side	0.00	2.06	6.03	7.67	12.80	28.56
Kullback-Leibler divergence statistic for prediction & precision sides	9.54	11.64	16.00	18.24	24.63	80.05

Source: Authors' calculations.

† *Posteriors for the prediction side are estimated using a prior error distribution with three elements in its support.*

Table 17. Goodness of fit statistics, using SAMs with limited additional price data, SSA region

Model 1. Constant delta-share parameter and time-varying elasticity across period 2004-2011.				
	2004	*2007*	*2011*	*2004-2011*
adjusted $\boldsymbol{R^2}$ for prediction side	1.0	0.993	0.982	0.991
Akaike Information Criteria for prediction side	0.0	1.7E-04	4.6E-04	2.1E-04
Kullback-Leibler divergence statistic for prediction side	0.0	0.39	0.35	0.74
Kullback-Leibler divergence statistic for prediction & precision sides	5.61	6.27	6.91	18.80
Model 2. Constant elasticity and time-varying delta-share parameters across period 2004-2011.				
	2004	*2007*	*2011*	*2004-2011*
adjusted $\boldsymbol{R^2}$ for prediction side	1.0	0.999	0.978	0.992
Akaike Information Criteria for prediction side	0.0	2.0E-05	5.7E-04	2.0E-04
Kullback-Leibler divergence statistic for prediction side	0.0	0.21	0.35	0.56
Kullback-Leibler divergence statistic for prediction & precision sides	2.54	3.39	4.59	10.53
Model 4. Constant elasticity for national output-VA-intermediate CES functions and time-varying delta-share parameters across period 2004-2011.				
	2004	*2007*	*2011*	*2004-2011*
adjusted $\boldsymbol{R^2}$ for prediction side	1.0	0.997	0.989	0.995
Akaike Information Criteria for prediction side	0.0	7.5E-05	2.7E-04	1.2E-04
Kullback-Leibler divergence statistic for prediction side	0.0	0.31	1.39	1.70
Kullback-Leibler divergence statistic for prediction & precision sides	5.94	6.25	7.80	19.99

Source: Authors' calculations.

† *Posteriors for the prediction side are estimated using a prior error distribution with three elements in its support.*

APPENDIX 4. FIGURES

Figure 1. CGE structure: production & consumption