Structure of GTAP*

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I. INTRODUCTION AND OVERVIEW

The purpose of this chapter is to develop the basic notation, equations, and intuition behind the GTAP model of global trade. The computer program documenting the basic model, GTAP94.TAB, is available in electronic form via the Internet (see Chapter 6). It provides complete documentation of the theory behind the model, and when converted to executable files using the GEMPACK software suite (Harrison and Pearson, 1994), it forms the basis for implementing the applications outlined in Part III of this book.

The organization of this chapter is as follows. We begin with an overview of the Global Trade Analysis Project (GTAP) model. Next, we develop the basic accounting relationships underpinning the data base and model. This involves tracking value flows through the global data base, from production and sales to intermediate and final demands. Careful attention is paid to the prices at which each of these flows is evaluated, and the presence of distortions (in the form of taxes and subsidies). The relationship between these accounting relationships and equilibrium conditions in the model is then developed. This leads naturally into a discussion of the implications of alternative "partial equilibrium" closures whereby these equations are selectively omitted and the associated complementary variables are fixed. The chapter then turns to the linearized representation of these accounting relations. This is the form in which they are implemented in GEMPACK, which solves the nonlinear equilibrium problem via successive updates and relinearizations.

Section VI of this documentation turns its attention to the equations underpinning economic behavior in the model. We deal in turn with production, consumption, global savings, and investment. There is also a special discussion of macroeconomic closure in the GTAP model. This material is reinforced in the closing section of the chapter by means of a numerical example using a three-region, three-commodity aggregation, in which there is a shock to a single bilateral protection rate.

II. OVERVIEW OF THE MODEL[1]

Closed Economy Without Taxes

Figure 2.1 offers an overview of economic activity in a simplified version of the GTAP model [see Brockmeier (1996) for a more comprehensive, graphical overview]. In this first figure, there is only one region, so there is no trade. There is also no depreciation, and no taxes or subsidies are present. At the top of this figure is the regional household. Expenditures by this household are

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governed by an aggregate utility function that allocates expenditure across three broad categories: private, government, and savings expenditures.[2] The model user has some discretion over the allocation of expenditures across these types of final demand. In the standard closure, the regional household's Cobb–Douglas utility function assures constant budget shares are devoted to each category. However, real government purchases and savings can also be dictated exogenously (i.e., fixed or shocked), in which case private household expenditure will adjust to satisfy the regional household's budget constraint.

This formulation for regional expenditure has some distinct advantages, as well as some disadvantages. Perhaps the most significant drawback is the failure to link government expenditures to tax revenues. Cutting taxes by no means implies a reduction in government expenditures in the GTAP model. Indeed, to the extent that these tax cuts lead to a reduction in excess burden, regional real income will increase and real government expenditure will likely also rise. This lack of fiscal integrity is dictated by the fact that the GTAP data have incomplete coverage of regional tax instruments. Therefore the model cannot accurately predict what will happen to total tax revenue, and the user who is interested in focusing on government expenditure effects would be required to make some exogenous assumptions in any case.

The greatest advantage of the formulation of regional expenditure displayed in Figure 2.1 is the unambiguous indicator of welfare offered by the regional utility function. A particular simulation might lead to lower relative prices for savings and the composite of government purchases, and higher prices for the private household's commodity bundle. If real private purchases fall, while savings and government consumption rise, is the regional household better off? Without a regional utility function we cannot answer this question.

An alternative approach to this problem of welfare measurement involves fixing the level of real savings and government purchases, and focusing solely on private household consumption as an indicator of welfare. However, private consumption is only slightly more than 50% of final demand in some regions. Forcing all the adjustment in the regional economy's final demand into private consumption seems rather extreme. We believe that the assumption of fixed expenditure shares dictated by the Cobb–Douglas regional expenditure function is more acceptable empirically. That is, a rise in income implies an increase in savings and government expenditures, as well as private consumption.

Since Figure 2.1 assumes the absence of taxes, the only source of income for regional households is from the "sale" of endowment commodities to firms. This income flow is represented by *VOA* (*endw*) which denotes *Value of Output at Agents' prices* of endowment commodities. (A complete glossary of GTAP notation is provided at the end of this book.) Firms combine these endowment commodities with intermediate goods (*VDFA = Value of Domestic purchases by Firms at Agents' prices*) in order to produce goods for final demand. This involves sales to private households (*VDPA = Value of Domestic purchases by Private households at Agents' prices*), government households (*VDGA = Value of Domestic purchases by Government household at Agents, prices*), and the sale of investment goods to satisfy the regional household's demand for savings (*REGINV*). This completes the circular flow of income, expenditure, and production in a closed economy without taxes.

Open Economy Without Taxes

Figure 2.2 [also taken from Brockmeier (1996)] introduces international trade by adding another region, *Rest of the World* (ROW), at the bottom of the figure. This region is identical in structure to the domestic economy, but details are suppressed in Figure 2.2. It is the source of imports into the regional economy, as well as the destination for exports (*VXMD* = *Value of eXports at Market prices by Destination*). It is important to note that imports are traced to specific agents in the domestic economy, resulting in distinct import payments to ROW from private households (*VIPA*), government households (*VIGA*), and firms (*VIFA*). This innovation departs from most models of global trade, and was adopted from the SALTER model (Jomini et al. 1991). It is especially important for the analysis of trade policy in regions where import intensities for the same commodity vary widely across uses.

In moving from a closed to an open economy, we also require the introduction of two global sectors, one of which is displayed in Figure 2.2. The *global bank*, shown in the center of this figure, intermediates between global savings and regional investment. As will be discussed in more detail below, it assembles a portfolio of regional investment goods, and sells shares in this portfolio to regional households in order to satisfy their demand for savings.

The second global sector (not shown in Figure 2.2) accounts for international trade and transport activity. It assembles regional exports of trade, transport, and insurance services and produces a composite good used to move merchandise trade among regions. The value of these services precisely exhausts the differences between global *fob* exports, and global imports, evaluated on a *cif* basis.

III. ACCOUNTING RELATIONSHIPS IN THE "LEVELS"

Distribution of Sales to Regional Markets

The basic accounting relationships in the data base/model are best understood in the context of a flow chart. For example, Table 2.1 portrays the sources of sectoral receipts in the global data base. (In the data and the model all sectors produce a single output. Thus there is a one-to-one relationship between producing sectors and commodities.) At the top of the figure, *VOA*(*i,r*) refers to the *Value of Output at Agents' Prices.* (The general explanation for this choice of notation is as follows: value/type of transaction/type of price. See the appendix to this chapter for an exhaustive listing of variables used in the model and their description.) *VOA*(*i,r*) represents the payments received by the firms in industry *i* of region *r*. As we will see, these payments must be precisely exhausted on costs, under the zero pure profits assumption. The terms *PS*(*i,r*) and *QO*(*i,r*) to the right of *VOA* represent the price and quantity indices that make up *VOA*. They will be discussed in more detail below.

If one adds back the producer tax (or deducts the subsidy) denoted by *PTAX*(*i,r*), then we arrive at the *Value of Output at Market prices*, *VOM*(*i,r*). This may be seen to be the sum of the *Value of Domestic sales at Market prices, VDM*(*i,r*) and the exports to all destinations, denoted as *Value of eXports of i from r evaluated at domestic Market prices (in r), and Destined for s, VXMD*(*i,r,s*). In addition, we must take account of possible sales to the international transport sector, denoted *VST*(*i,r*). These sales are designed to cover the international transport margins. They are evaluated at market prices and face no further (border) taxes. Similarly, since domestic sales do not cross a border, they do not face such taxes either.

In order to convert exports to *fob* values, it is necessary to add the export tax, denoted *XTAX*(*i*,*r*,*s*). Note that these taxes are written in a form that is destination—not the data base exhibits destination/source-specific trade policy measures at the level of *disaggregated* regions and commodities (this varies by type of policy intervention), once the data base has been aggregated over either commodities or regions, bilateral rates of taxation will vary due to compositional differences. Therefore, it is important to maintain this bilateral detail in the modeling framework. Once the export taxes are added in, we obtain the *Value of eXports at World prices by Destination*, *VXWD*(*i*,*r*,*s*). The difference between this and the *cif*-based *Value of Imports at World prices by Source, VIWS*(*i*,*r*,*s*), is the international transportation margin: *VTWR*(*i*,*r*,*s*) refers to the *Value of Transportation at World prices by Route* for commodity *i*, shipped from *r* to *s*.

At this point we have taken commodity *i* from its sector of origin in region *r* to its export destination in region *s*. In order to evaluate these sales at internal domestic prices in *s*, it is necessary to add import taxes, *MTAX*(*i*,*r*,*s*) to get *VIMS*(*i*,*r*,*s*), the *Value of Imports at Market prices by Source*. These imports from alternative sources may then be combined into a single composite, *VIM*(*i*,*s*), the *Value of Imports of i into s at Market prices*. Just as sales in the rth market had to be distributed across various destinations, so composite imports of *i* into *s* must be distributed across sectors and households in the sth market. Possible uses of imports include: *VIPM*(*i*,*s*) — the *Value of Imports by Private households, evaluated at Market prices: VIGM*(*i*,*s*) — the *Value of Imports by the Government, evaluated at Market prices*; and *VIFM*(*i*,*j*,*s*) — the *Value of Imports by Firms in industry j, at Market price*. In a similar fashion, domestic sales, denoted *VDM*(*i*,*r*), must be distributed across private household, government, and firms' uses, as shown at the bottom of Table 2.1.

Sources of Household Purchases

Having distributed sales across various markets and taken full account of intervening taxes and transport margins, we are now in a position to consider household and firms' purchases within each of these individual markets. Table 2.2 outlines the distribution of household purchases of tradeable commodities. The top half of this figure pertains to private household purchases, denoted *VPA*(*i*,*s*), to represent the *Value of Private household purchases at Agents' prices.* This represents the sum of expenditures on domestically produced goods, *VDPA*(*i*,*s*), and composite imports, evaluated at agents' prices, *VIPA*(*i*,*s*). Once private household commodity taxes, *IPTAX*(*i*,*s*), are deducted, this brings us to the *Value of Imports by the Private household at Market prices*, *VIPM*(*i*,*s*), which is the point where we left Table 2.1. Similarly, deducting domestic commodity taxes, *DPTAX*(*i*,*s*), from *VDPA*(*i*,*s*) yields *VDPM*(*i*,*s*), the *Value of Domestic purchases by the Private household, at Market prices.* Thus we have completed the link between industry sales at agents' prices (top of Table 2.1) and private household purchases at agents' prices (top of Table 2.2). The bottom half of Table 2.2 is completely analogous, only *P* is replaced by *G* in order to represent purchases by the government household.

Sources of Firms' Purchases and Household Factor Income

Next, turn to firms' purchases of intermediate and primary factors of production. The top of Table 2.3 tackles the intermediate inputs, starting with the *Value of Firms' purchases of i, by sector j, in region s at Agents' prices, VFA*(*i,j,s*). This may be broken into the domestic and imported components, *VDFA*(*i,j,s*) and *VIFA*(*i,j,s*). Deducting intermediate input taxes, *DFTAX*(i,j,s) and *IFTAX*(*i,j,s*), reduces these values to market prices, *VDFM*(*i,j,s*) and *VIFM*(*i,j,s*), which are the same as the values reported at the bottom of Table 2.1.

Firms also purchase services of nontradeable commodities, which in this model are termed *endowment commodities*. (In the current data base, these include: agricultural land, labor, and capital.) The next part of Table 2.3 traces the value flows from the firms employing these factors of production, back to the households supplying them. Note that by deducting taxes on endowment *i* used in industry *j*, *ETAX*(*i,j,s*), we can move from the *Value of Firms' purchases at Agents' prices, VFA*(*i,j,s*), to the *Value of Firms' purchases at Market prices, VFM*(*i,j,s*). The final section of Table 2.3 makes the link between firms' receipts [i.e., *VOA*(*j,s*)], as developed in Table 2.1, and firms' expenditures [i.e., *VFA*(*i,j,s*)], as shown in Table 2.3. Zero pure economic profits means that revenues must be exhausted on expenditures, once accounting for all tradeable (i.e., intermediate) inputs and endowment (i.e., primary) factors of production.

Table 2.4 details the sources of household factor income. Here, it is necessary to distinguish between endowment commodities that are perfectly mobile, and therefore earn the same market return (*ENDWM_COMM*), and those that are sluggish to adjust and that therefore sustain differential returns in equilibrium (*ENDWS_COMM*). In the former case, we may simply sum over all usage of the factor ─ since market prices are equal ─ thereupon deducting the tax on households' supply of primary factor *i* in region *s*, *HTAX(i,s)*, in order to obtain the *Value of this endowment's "Output" at Agents' prices* (*VOA*). The latter is the amount actually received by the private household supplying the factor in question.

In the case of the sluggish endowment commodities (e.g., land), shocks to the model will introduce differential price changes across sectors. This is reflected in the presence of an industry index (*j*), in the price component of *VFM*(*i,j,s*). These differential prices are then combined into a composite return to the sluggish endowment, at market prices, via a unit revenue function. The resulting *Value of endowment Output at Market prices, VOM*(*i,s*), is then handled in the same way as for mobile commodities, deducting household income taxes to arrive at the *VOA*(*i,s*).

Disposition and Sources of Regional Income

When taxes are present, the computation of disposable income for the regional household in Figures 2.1 and 2.2 becomes much more complex. At the top of Table 2.5, we have the condition that expenditures on private, government, and savings commodities must precisely exhaust regional income. This is followed by the expression that decomposes income by source. We begin by adding up endowment income (recall Figures 2.1 and 2.2). Note that all such income earned within a region accrues to households in that same region. From this, we must deduct depreciation expenses required to maintain the integrity of the initial capital stock, *VDEP*(*r*), thereupon adding net tax receipts and rents associated with any quantitative restrictions.

Rather than keeping track of individual tax/subsidy flows in the model, the approach taken here is to compare the value of a given transaction, evaluated at agents', market, or world prices. If there is a discrepancy between what households receive for their labor supply and the value of this supply at market prices, then the difference must equal *HTAX*(*i*,*r*), as shown in Table 2.4. Alternatively, this tax revenue could be rewritten in terms of an explicit *ad valorem* tax rate, $\tau(i,r)$, by noting that the household's supply price of endowment *i* is given by:

$$PS(i,r) = (1 - \tau(i,r))PM(i,r) = TO(i,r)PM(i,r),$$

where *TO*(*i*,*r*) is referred to as the *power* of the *ad valorem* tax. Therefore:

$$VOM(i,r) - VOA(i,r) = (1 - TO(i,r))\ PM(i,r)\,QO(i,r) = \tau(i,r)\,PM(i,r)\,QO(i,r).$$

Thus, the fiscal implications of all tax/subsidy programs may be captured by comparison of the value of a given transaction at agents' *versus* market (or market *versus* world) prices. We assume that *taxes levied in region r always accrue to households in region r.*

The remaining terms in the income expression given in Table 2.5 account for all the other possible sources of tax revenues/subsidy expenditures in each regional economy. These include: primary factor taxes on firms, commodity taxes on households', and firms' purchases of tradeable goods and trade taxes.[3]

Figures 2.3 and 2.4, taken from Brockmeier (1996), offer graphical depictions of border interventions in GTAP. The two panels in Figure 2.3 refer to the case of export interventions. (Because there are many export destinations, we can interpret the supply curve as representing supply, net of sales to domestic uses and other export markets.) In the first panel, the domestic price exceeds the world price (*PM*(*i*,*r*) > *PFOB*(*i*,*r*,*s*)), indicating the presence of a subsidy, so that *XTAX*(*i*,*r*,*s*) = *VXWD*(*i*,*r*,*s*) - *VXMD*(*i*,*r*,*s*) < 0. In the second panel, the opposite case is presented. Here, the world price is above the market price and their difference contributes positively to regional income. This will be the case regardless of the source of discrepancy in *VXWD* and *VXMD*. For example, if this difference arises due to export restraints, as opposed to taxation, then the resulting income flow is due to quota rents. Nevertheless, it still accrues to the region of origin (*r*).

The two panels in Figure 2.4 refer to the income consequences of import interventions. Because GTAP adopts the Armington approach to import demand, differentiating products by origin, there is no domestic supply of the imported good. Therefore the demand schedule in these panels is conditional on aggregate demand for commodity *i* in region *s*, as well as the prices of competing imports and the domestic market price of *i* in region *s*. The excess supply schedule for imports of *i* from *r* to *s* depends on supply conditions in *r* as well as demand for this commodity in region *s*.

When the market price exceeds the world price, *PMS*(*i*,*r*,*s*) > *PCIF*(*i*,*r*,*s*), then *MTAX*(*i*,*r*,*s*) > 0 and this term contributes positively to regional income. This can arise if there is a tariff on imports, *or* it could be due to an import quota. In the case of a binding quota on imports of *i* into *s* from *r*:

$$\begin{aligned} &VIMS(i,r,s) - VIWS(i,r,s) \\ &\qquad = (TMS(i,r,s) - 1)\,PCIF(i,r,s)\,QXS(i,r,s) > 0 \end{aligned}$$

represents the associated quota rents. In this instance, the closure must be modified so that *QXS*(*i*,*r*,*s*) is *exogenous* and the tax equivalent, *TMS*(*i*,*r*,*s*) is *endogenous*. Again, these quota rents are assumed to accrue to the region administering the quota.

Global Sectors

In order to complete the model, it is necessary to introduce two global sectors. The global transportation sector provides the services that account for the difference between *fob* and *cif* values for a particular commodity shipped along a specific route: *VTWR*(*i*,*r*,*s*) = *VIWS*(*i*,*r*,*s*) - *VXWD*(*i*,*r*,*s*). Summing over all routes and commodities gives the total demand for international transport services shown at the top of Table 2.6. The supply of these services is provided by individual regional economies, which export them to the global transport sector [*VST*(*i*,*r*)]. We do not have information that would permit us to associate regional transport services exports with particular commodities and routes. Therefore, all demand is met from the same pool of services, the price of which is a blend of the price of all transport services exports.

The other required global sector is the global banking sector. This intermediates between global savings and investment, as described in Table 2.7. It creates a composite investment good (*GLOBINV*), based on a portfolio of net regional investment (gross investment less depreciation), and offers this to regional households in order to satisfy their savings demand. Therefore, all savers face a common price for this savings commodity (*PSAVE*). A consistency check on the accounting relationships described up to this point involves separately computing the supply of the composite investment good and the demand for aggregate savings. If (1) all other markets are in equilibrium, (2) all firms earn zero profits (including the global transport sector); and (3) all households are on their budget constraint, then global investment must equal global savings by virtue of Walras' Law.

Finally, the value of the beginning of period capital stock, *VKB*(*r*), is updated by regional investment, *REGINV*(*r*), less depreciation, *VDEP*(*r*). This yields the value of ending capital stocks, *VKE*(*r*). This relationship is shown at the bottom of Table 2.7.

IV. EQUILIBRIUM CONDITIONS AND PARTIAL EQUILIBRIUM CLOSURES

Thus far, we have said nothing about the behavior of individual firms and households. Neoclassical restrictions on such behavior are not necessary to obtain full general equilibrium closure. Rather, it is the exhaustive accounting relationships outlined above that make our model general equilibrium in nature. If any one of them is not enforced, Walras' Law will fail to hold. Since most economists are accustomed to seeing equilibrium conditions written in terms of quantities, not values, it is useful to demonstrate that the accounting relationships provided above do indeed embody the customary general equilibrium relationships. Consider, for example, the market clearing condition for tradeable commodity supplies:

$$VOM(i,r) = VDM(i,r) + VST(i,r) + \sum_{s \in REG} VXMD(i,r,s). \tag{2.1}$$

This may be rewritten in terms of quantities and a common domestic market price for *i* in region *r*:

$$PM(i,r) * QO(i,r) = PM(i,r) * \left[QDS(i,r) + QST(i,r) + \sum_{s \varepsilon REG} QXS(i,r,s) \right]. \tag{2.2}$$

Upon dividing by *PM*(*i,r*) we obtain the usual form of the tradeable commodity market clearing condition:

$$QO(i,r) = QDS(i,r) + QST(i,r) + \sum_{s \varepsilon REG} QXS(i,r,s). \tag{2.3}$$

A similar exercise may be applied to the market clearing conditions for nontradeable commodities. In sum, any market clearing condition can be converted to value terms by multiplying by a *common* price. In so doing, we circumvent the need to partition value flows into prices and quantities. This has the added benefit of vastly simplifying the problem of model calibration, as we will see below.

Having verified that the accounting relationships embody all the necessary general equilibrium conditions, we turn to the problem of creating special closures in which some of these conditions are dropped. This, in turn, permits one to fix certain variables exogenously, as is done *implicitly* in partial equilibrium analysis. The problem lies in ascertaining which variables are associated with which equilibrium conditions. This is akin to identifying the complementary slackness conditions associated with the general equilibrium model.

Perhaps the most obvious complementarity is that between prices and market clearing conditions. Clearly if the latter are to hold, prices must be free to adjust to resolve any imbalance between supply and demand. Therefore, if we fix the price of a tradeable commodity, we must eliminate the associated market clearing condition, equation (2.3). A common partial equilibrium closure for the analysis of farm and food issues involves fixing the prices of all nonfood commodities. In order to implement this closure in our model, all nonfood market clearing conditions must be dropped. (The "dropping" of individual equations is achieved by endogenizing slack variables in the equations to be eliminated. We must always retain equal numbers of endogenous variables and equations if the model is to provide a unique equilibrium solution.)

It is also common in partial equilibrium analyses to assume that the opportunity cost of nonspecific factors is exogenous. For example, in the case of agriculture, one might assume that the labor wage and capital rental rates are fixed. If this is to be done, then the associated regional market clearing conditions for these nontradeable primary factors must be dropped. Similarly, income may be fixed, provided the income computation equation is eliminated.

But what about quantities? Should any of them be fixed? Having fixed the price of nonfood commodities, for example, it hardly makes sense to permit their supplies to be determined endogenously. Any sector experiencing a rise in costs would be driven out of business altogether under such circumstances. For this reason it makes sense to fix nonfood output levels and drop the associated zero profit conditions. These partial equilibrium assumptions, for the example of a food policy shock, may be summarized as follows:

- *Nonfood output levels and prices are exogenous.*
- *Income is exogenous.*
- *Nonspecific primary factor rental rates are exogenous.*

V. LINEARIZED REPRESENTATION OF ACCOUNTING EQUATIONS

Solution via a Linearized Representation: While the accounting relationships detailed in Figures 2.1 and 2.2 and Tables 2.1–2.5 are most conveniently expressed in value terms, it is attractive to write the behavioral component of the model in terms of *percentage changes* in prices and quantities.[4] Indeed, we are usually most interested in these percentage changes, as opposed to their levels values. Expressing this nonlinear model in percentage changes does not preclude solution of the true nonlinear problem. Solution of nonlinear AGE models via a *linearized representation* (Pearson 1991)[5] involves successively updating the value-based coefficients via the formula:

$$dV/V = d(PQ)/PQ = p + q,$$

where the lowercase p and q denote percentage changes in price and quantity.

Figure 2.5 provides a graphical exposition of one method of solving a nonlinear model via its linearized representation. For simplicity, the entire model is given by a single equation $g(X,Y) = 0$, where X is exogenous and Y is endogenous. The initial equilibrium is represented by the point (X_0, Y_0). Our counterfactual experiment involves shocking the exogenous variable to X_1, and computing the resulting endogenous outcome Y_1. If we simply evaluated the linearized representation of the model at (X_0, Y_0) the equations would predict the outcome $B_J = (X_1, Y_J)$. This is the *Johansen approach*, and it is clearly in error, since $Y_J >> Y_1$. This type of error has led to criticism of the individuals using linearized Computable General Equilibrium (CGE) models.

However, note that the accuracy of the linearized model can be considerably enhanced by breaking the shock to X into two parts and updating the equilibrium after the first shock. This approach takes us from point A to C_2 to B_2. It is termed *Euler's method* of solution via linearized representation. By increasing the number of steps, one obtains an increasingly accurate solution of the nonlinear model.

Since Euler's contribution, this approach of relinearizing the model has been considerably refined to yield more rapid convergence to (X_1, Y_1). [See Harrison and Pearson (1994), section 2.5, for more details.] The default method used for solving the GTAP model is *Gragg's method*, with extrapolation. In this case the model is solved several times, each time with a successively finer grid. An extrapolated solution is formed based on these results. As illustrated in Harrison and Pearson (1994, pp. 2–24ff.), this yields good results.

Form of Accounting Equations: Linearization of the accounting equations involves total differentiation so they appear as a linear combination of appropriately weighted price and quantity changes. For example, the tradeable market clearing condition [equation (2.3)] becomes:

$$\begin{aligned} QO(i,r)\ qo(i,r) \ = \ & QDS(i,r)\ qds(i,r) + QST(i,r)\ qst(i,r) \\ & + \sum_{s \varepsilon REG} QXS(i,r,s)\ qxs(i,r,s) \end{aligned} \tag{2.4}$$

where the lowercase variables are again percentage changes. Multiplying both sides by the common price $PM(i,r)$ yields equation (1) in Table 2.8. Here the coefficients are now in *value* terms. [It is never necessary actually to compute price and quantity *levels* (P and Q) under this approach, although this can be done if one chooses to define initial units by choosing, for example, $PM(i,r) = 1$] . Also, note that a slack variable has been introduced into this equation in Table 2.8. It is indexed over all tradeable commodities and regions. By endogenizing selected components of this variable (which appear only in this equation), we are able to eliminate selectively market clearing for individual products. In this case, with the associated tradeable price fixed exogenously (*pm*(*i,r*) = 0), the endogenous change in *tradslack*(*i,r*) accounts for the excess of supply over demand in the new equilibrium (as a percentage of output in the initial equilibrium).

The next two equations in Table 2.8 enforce equilibrium in the *domestic market* for tradeable commodities, either that which is imported from region *r* in the case of equation (2) or that which is produced domestically in the case of equation (3). Therefore, the common price is once again a domestic market price. We do not include slack variables in these equations, since they refer to the same commodity treated in equation (1). To achieve a partial equilibrium closure, it is sufficient to fix the price of this good at one place in the model.

Equations (4) and (5) in Table 2.8 refer to market clearing for the nontradeable, endowment commodities. As noted above, the model distinguishes between primary factors that are perfectly mobile across sectors, and those which are "sluggish" in their adjustment. The latter class of endowment commodities can exhibit differential equilibrium rental rates across uses. In the case of mobile endowments, equation (4), the presence of a common market price permits the equilibrium relationship to be written in terms of values at domestic market prices. A slack variable is introduced to permit us selectively to eliminate the market clearing equations and fix rental rates on the respective endowment commodities. In the case of sluggish commodities, no such common price exists and sectoral demands are equated to sectoral supplies. The latter are generated from a Constant Elasticity of Transformation (CET) revenue function, which transforms one use of the endowment into another.

Equation (6) in Table 2.8 is the zero pure profit condition. Since firms are assumed to maximize profits, the quantity changes drop out when the expression at the bottom of Table 2.3 is totally differentiated in the neighborhood of an optimum (e.g., Varian 1978 p. 267). This leaves an equation relating input prices to output prices, where these percentage changes are weighted by values at *agent's* prices. For computational convenience we use different variables to refer to firms' prices for composite intermediate inputs (*pf*) and endowment commodities (*pfe*). The presence of *profitslack*(*j,r*) permits us to fix output and eliminate the zero profit condition for any sector *j* in any region *r*. In a similar fashion, equation (7) is the zero profit condition for the international transport sector. Here, the total value of transport services (*VT*) is constrained to equal the total value of services exports to this sector/use (*VST*), as described in Table 2.6.

Equation (8) in Table 2.8 assures the complete disposition of regional income (recall Table 2.5). This is done by first deducting savings and government spending (each of which *may* be exogenously specified under some closures) from disposable regional income, thereupon allocating the remainder to private household expenditures *PRIVEXP*(*r*). It is followed by equation (9), which generates available income in each region. This is the most complicated equation in the model. It must take account of changes in the value of regional endowments, as well as changes in the net fiscal revenues owing to the *ad valorem* taxes/subsidies. Even if these tax rates do not change, revenues will change due to changes in market prices and quantities. Therefore, in differential form, each of the values must be postmultiplied by the percentage change in both the price and quantity components of the value flow.

Note that in Table 2.8 the quantity change is common for each of the transactions taxes in equation (9). For example, in the case of the tax on firms' use of primary factors, the percentage change in firms' derived demands, *qfe*(*i,j,r*), enters both terms. This is simply a reflection of the fact that the tax refers to a particular transaction in quantities. In contrast, the prices faced by firms are: (1) potentially different from market prices, and (2) free to change at different rates when the tax rate dividing them is changed. This is reflected by the fact that $VFA(i,j,r)$ is post-multiplied by $pfe(i,j,r)$, while $VFM(i,j,r)$ changes according to $pm(i,r)$.

Before going through (Table 2.8) equation (9) in more detail it is useful to consider explicitly the taxes associated with each of these price differences. These are revealed in the price linkage equations given in Table 2.9; for example, equation (15) shows the role of income/output taxes that drive a wedge between $VOM(i,r)$ and $VOA(i,r)$. The *power of the ad valorem tax* in this case is given by $TO(i,r) = VOA(i,r)/VOM(i,r)$. Therefore, when $TO(i,r) > 1$, firms/households are actually receiving a *subsidy* on the commodity supplied. Similarly, if $dTO(i,r)/TO(i,r) = to(i,r) > 0$ then the subsidy is increased. This choice of notation may seem odd, but it gives rise to a useful pattern across tax instruments. In particular, we adopt the rule that tax rates are always defined as the ratio of agent's prices to market prices (or market prices to world prices in the case of border taxes).

Turning to the next price linkage relationship, equation (16) in Table 2.9, we note that an increase in $TF(i,j,r)$, that is, $tf(i,j,r) > 0$, will cause an *increase* in tax revenues. This is because in this case the firms in sector *j* of region *r* purchasing mobile endowment commodity *i* will be forced to pay more, relative to the market price, that is, $pfe(i,j,r) > pm(i,r)$. Owing to the fact that there is not a unique market price for the sluggish endowment commodities purchased by firms, we require a separate price linkage, equation (17) in this case.

Equations (18)–(20) in Table 2.9 describe the linkages between domestic market prices and agents purchasing *domestically produced*, tradeable commodities. These commodity transaction taxes can potentially vary not only across commodities and regions, but also across firms and households in each region. Similarly, equations (21)–(23) describe the linkage between the domestic market price of imports of *i*, by source *r*, and diverse agents in region *s*.

Equation (24) in Table 2.9 establishes the percentage change in the domestic market price for tradeable commodity *i* in region *s*, based on the change in the border price of that product, $pcif(i,r,s)$ as well as two types of border interventions. Both are *ad valorem* import tariffs. The first, $tms(i,r,s)$, is bilateral in nature, while the second, $tm(i,s)$, is source-generic. The latter may be used to insulate the domestic economy from world price changes. This is done by endogenizing $tm(i,s)$ and establishing some domestic price target. In this model, we choose to fix the ratio of the domestic market price for *i* to the price of the import composite. This is conveniently defined in the next price linkage, equation (25). In the normal closure, $tm(i,s)$ is exogenous and $pr(i,s)$ is endogenous. However, we imitate the European Union's variable import levy on food products by permitting $tm(i,s)$ to vary so as to fix $pr(i,s)$. In this circumstance, domestic consumers have no incentive to substitute imports for domestic food.

Equation (26) (in Table 2.9) links $pcif(i,r,s)$ and $pfob(i,r,s)$. Its derivation is based on the assumption that revenues must cover costs on all *individual* routes, for all commodities. Thus the change in the *cif* price is a weighted combination of the change in the *fob* price and the change in a general transport cost index, *pt*, where the weights refer to the shares of *fob* costs [$FOBSHR(i,r,s)$] and transport costs [$TRNSHR(i,r,s)$] in *cif* costs. To the extent that firms engage in cross-subsidization or the costs of transport services on different routes move independently, this equation will be inaccurate. It is also important to note the implications of equation (26) for price transmission across markets. The greater the transport margin along a given route (i.e., $TRNSHR(i,r,s)$ larger), the weaker the link between a change in the price of *i* in the export market *r* and the corresponding change in destination market *s*.

Equation (27) completes the "circle" of price linkages in Table 2.9 by connecting $pfob(i,r,s)$ and domestic market price, $pm(i,r)$. As was the case on the import side, there are two types of export taxes. The first, $txs(i,r,s)$ is destination-specific, while the second, $tx(i,r)$, is destination-generic. The latter may be "swapped" with the normally endogenous change in sectoral output, in order to insulate domestic producers from the vagaries of world markets. For example, this variable export tax/subsidy has been used in modeling the European Union's (EU) common agricultural policy. Note that since these export taxes refer to the ratio of domestic market prices to world prices, an increase in $TXS(i,r,s)$ results in a fiscal outflow, that is, a subsidy on exports.

Having established the linkage between prices in this model, we are in a position to return to the income computation equation (9) in Table 2.8. In particular, consider the effect of omitting some component of this complicated equation, say, income taxes. How will this affect, for example, a welfare analysis of trade policy reform? Given the presence of income taxes in the initial equilibrium data base, $VOM(i,r) > VOA(i,r)$, if the experiment in question does not alter the *rate* of income taxation, then $to(i,r) = 0$ and $\alpha = ps(i,r) = pm(i,r)\ \forall i \varepsilon ENDW$. This means the two terms in square brackets [*] change at the same rate. If this change is positive, then omission of this term will lead to an *understatement* of income tax revenues and a subsequent *understatement* of disposable income and household welfare in the new equilibrium. In sum, even when distortions are not affected by a given policy experiment it is important to acknowledge their presence in the economy if an accurate welfare analysis is to be provided.

The final group of accounting equations in Table 2.8 refer to global savings and investment. Because this is a comparative static model, current investment does not augment the productive stock of capital available to firms. The latter is constrained by beginning-of-period capital stock which is exogenous. Therefore, there is only a limited role for investment in our simulations. When investment (and savings) is specified exogenously it will facilitate accumulation of the targeted end-of-period capital stock [see equation (10)]. When investment is endogenous it adjusts in order to accommodate the global demand for savings. (More discussion of these macroeconomic closure issues is provided below.) Equation (11) aggregates regional *gross* investment into global net investment. Equation (13) aggregates regional savings, and equations (12) and (14) permit us either to force the two to be equal (*walraslack* is exogenous) or verify Walras' Law (*walraslack* is endogenous and should be found equal to zero in the solution).

VI. BEHAVIORAL EQUATIONS

Firm Behavior

The "Technology Tree": Figure 2.6 provides a visual display of the assumed technology for firms in each of the industries in the model. This kind of a production "tree" is a convenient way of representing separable, constant returns-to-scale technologies. At the bottom of the inverted tree are the individual inputs demanded by the firm. For example, the primary factors of production are: land, labor, and capital. Their quantities are denoted $QFE(i,j,s)$, or, in percentage change form, $qfe(i,j,s)$. (For the time being, please ignore the terms in brackets [.] in Figure 2.6. They refer to rates of technical change, to which we will turn momentarily.) Firms also purchase intermediate inputs, some of which are produced domestically, $qfd(i,j,s)$, and some of which are imported, $qfm(i,j,s)$. In the case of imports, the intermediate inputs must be "sourced" from particular exporters, $qxs(i,r,s)$. Recall from Figure 2.1 that this sourcing occurs at the border, since information on the composition of imports by sector is unavailable; hence the dashed line between the firms' production tree and the *constant elasticity of substitution* (CES) nest combining bilateral imports.

The manner in which the firm combines individual inputs to produce its output, $QO(i,s)$, depends largely on the assumptions that we make about *separability* in production. For example, we assume that firms choose their optimal mix of primary factors *independently* of the prices of intermediate inputs. Since the level of output is also irrelevant, owing to our assumption of constant returns to scale, this leaves only the relative prices of land, labor, and capital as arguments in the firms' conditional demand equations for components of value-added. By assuming this type of separability, we impose the restriction that the elasticity of substitution between any individual primary factor, on the one hand, and intermediate inputs, on the other, is equal. This is what permits us to draw the production tree, for it is this common elasticity of substitution that enters the fork in the inverted tree at which the intermediate and primary factors of production are joined. It also represents a significant reduction in the number of parameters that need to be provided in order to operationalize the model.

Within the primary factor branch of the production tree, substitution possibilities are also restricted to one parameter. This CES assumption is quite general in those sectors that employ only two inputs: capital and labor. However, in agriculture, where a third input, land, enters the production function, we are forced to assume that all pairwise elasticities of substitution are equal. This is surely not true, but we do not have enough information to calibrate a more general specification at this point.

In general, the behavioral parameters at each level in the production tree can be specified by the user of the model. However, as will be seen below when we turn to the specific form of the equations used to represent firm behavior, we impose the restriction of nonsubstitution between composite intermediates and primary factors. The fact that this is a very common specification in applied general equilibrium (AGE) models is a poor justification for incorporating it into the GTAP model. Indeed, there is evidence of significant substitutability between some intermediate inputs and primary factors. For example, during the energy price shocks of the 1970s firms demonstrated considerable potential for conserving fuel via the purchase of new, more energy-efficient equipment. Similarly, farmers have shown considerable potential for altering the rate of chemical fertilizer applications in response to changes in the relative price of fertilizer to land. However, these substitution possibilities are not characteristic of *all* intermediate inputs, and their proper treatment requires a more flexible production function than that portrayed in Figure 2.6.[6]

Turning to the intermediate input side of the production tree in Figure 2.6, it can be seen that the separability is symmetric, that is, the mix of intermediate inputs is also independent of the prices of primary factors. Furthermore, imported intermediates are assumed to be separable from domestically produced intermediate inputs. That is, firms first decide on the sourcing of their imports; then, based on the resulting composite import price, they determine the optimal mix of imported and domestic goods. This specification was first proposed by Paul Armington in 1969 and has since become known as the "Armington approach" to modeling import demand. However, it has been widely criticized in the literature. For example, Winters (1984), and Alston et al. (1990) argue that the functional form is too restrictive and that the nonhomothetic, AIDS specification is preferable. Although we agree that more flexible functional forms are preferable, this critique could apply just as well to every other behavioral relationship in the model. The question is: can it be estimated/calibrated and operationalized in the context of a disaggregated global model? At this point the answer is "no," although progress has been made in the context of one-region models (e.g., Robinson et al. 1993).

A more fundamental critique of the Armington approach is provided by the literature on industrial organization, imperfect competition, and trade. Here, product differentiation is *endogenous* and it is associated with individual firms' attempts to carve out a market niche for themselves. Early work along these lines is offered by Spence (1976), and Dixit and Stiglitz (1979). It is now the preferred approach for introducing imperfect competition into AGE models (e.g., Brown and Stern 1989), and it can have significant implications for the effects of trade policy liberalization (Hertel and Lanclos 1994). Also, Feenstra (1994) shows that the failure to account for endogenous product differentiation may be part of the reason import demands appear to be nonhomothetic. This is due to the correlation of income increases with the entry of new exporters and the subsequent increase in import varieties. Even at constant prices, this would dictate an increasing market share for imports.

In sum, although we are not particularly happy with the Armington specification, it *does* permit us to explain cross-hauling of similar products and to track bilateral trade flows. We believe that, in many sectors, an imperfect competition/endogenous product differentiation approach would be preferable. However, those models require additional information on industry concentration (firm numbers) as well as scale economies (or fixed costs), which is not readily available on a global basis. Clearly this is an important area for future work. Indeed, a number of authors have used aggregated versions of the GTAP data base to implement models with imperfect competition (Harrison, Rutherford, and Tarr 1995; Hertel and Lanclos 1994, Francois, McDonald, and Nordstrom 1995).

Behavioral Equations: The equations describing the firm behavior portrayed in Figure 2.6 are provided in Tables 2.10 and 2.11. Each group of equations refers to one of the "nests" or branches in the technology tree discussed above. For each nest there are two types of equations. The first describes substitution among inputs within the nest. Its form follows directly from the CES form of the production function for that branch. (Details are provided later in this section.) The second type of equation is the composite price equation that determines the unit cost for the composite good produced by that branch (e.g., composite imports). (It takes the same form as the sectoral zero profit condition given in Table 2.8.) The composite price then enters the next higher nest in order to determine the demand for this composite.

There are several approaches to obtaining the CES-derived demand equations. Here, we opt for an intuitive exposition that begins with the definition of the elasticity of substitution. Indeed, this is the way the CES functional form was invented (Arrow et al. 1961). Consider the two input-case, where the elasticity of substitution is defined as the percentage change in the ratio of the two cost minimizing input demands, given a 1 percent change in the inverse of their price ratio:

$$\sigma \equiv (Q_1 \hat{/} Q_2) / (P_2 \hat{/} P_1) \tag{2.5}$$

(Here, the "hats" denote percentage changes.) A familiar benchmark is the Cobb–Douglas case, whereby σ equals 1. In this case cost shares are invariant to price changes. For larger values of σ, the rate of change in the quantity ratio exceeds the rate of change in the price ratio and the cost share of the input that becomes more expensive actually falls. Expressing equation (2.5) in percentage change form (lowercase letters), we obtain:

$$(q_1 - q_2) = \sigma\ (p_2 - p_1). \tag{2.6}$$

In order to obtain the form of demand equation used in Table 2.10, several substitutions are necessary. First, note that total differentiation of the production function, and use of the fact that firms' pay factors their marginal value product, gives the following relationship between inputs and output (i.e., the composite good):

$$q = \theta_1 q_1 + (1 - \theta_1) q_2 , \tag{2.7}$$

where θ_1 is the cost share of input 1 and $(1 - \theta_1)$ is the cost share of input 2. Solving for q_2 gives:

$$q_2 = (q - \theta_1 q_1) / (1 - \theta_1) . \tag{2.8}$$

which may be substituted into (2.6) to yield:

$$q_1 = \sigma (p_2 - p_1) + [q - \theta_1 q_1] / (1 - \theta_1) . \tag{2.9}$$

This simplifies to the following derived demand equation for the first input:

$$q_1 = (1 - \theta_1) \sigma (p_2 - p_1) + q . \tag{2.10}$$

Note that this conditional demand equation is homogeneous of degree zero in prices, and the compensated cross-price elasticity of demand is equal to $(1 - \theta_1) * \sigma$.

The final substitution required to obtain the CES demand equation introduces the percentage change in the composite price:

$$p = \theta_1 p_1 + (1 - \theta_1) p_2 . \tag{2.11}$$

As noted above, this is identical to the zero profit condition (6) in Table 2.8, only we have divided both sides by the value of output at agents' prices. Since revenue is exhausted on costs, the resulting coefficients weighting input prices are the respective cost shares. From here, we proceed in an manner analogous to that explored above, first solving for p_2 as a function of p_1 and p, then substituting this into (2.10) to obtain:

$$q_1 = (1 - \theta_1) \sigma\{[p - \theta_1 p_1] / (1 - \theta_1) - p_1\} + q . \tag{2.12}$$

This simplifies to the following, final form of the derived demand equation for the first input in this CES composite:

$$q_1 = \sigma (p - p_1) + q . \tag{2.13}$$

The beauty of equation (2.13) is the intuition it offers, and the fact that *its form is unchanged when the number of inputs increases beyond two*. This equation decomposes the change in a firm's derived demand, q_1, into two parts. The first is the substitution effect. It is the product of the (constant) elasticity of substitution and the percentage change in the ratio of the composite price to the price of input 1. The second component is the expansion effect. Owing to constant returns to scale, this is simply an equiproportionate relationship between output and input.

We are now in a position to return to Tables 2.10 and 2.11 and consider the individual equations more closely. As noted above, each CES "nest" in Figure 2.6 contains two types of equations: a composite price equation and the set of conditional demand equations. For example, equation (28), at the top of Table 2.10, explains the percentage change in the composite price of imports *pim*(*i,s*). In contrast to the sectoral price equation (6) in Table 2.8, we use a cost share, *MSHRS*(*i,k,s*) which is the share of imports of *i* from region *k* in the composite imports of *i* in region *s* (recall that this composite is the same for all uses in the region). The next equation determines the sourcing of imports, according to their individual market prices, *pms*(*i,r,s*), relative to the price of composite imports, *pim*(*i,s*).

The first set of equations in Table 2.11 describes the composite intermediate inputs nest. This is specific to the individual sector in question. Here, *FMSHR*(*i,j,r*) refers to the share of imports in firms' composite tradeable commodity *i* in sector *j* of region *r*. Note that our choice of notation requires separate conditional demand equations for imported [equation (31)] and domestic [equation (32)] goods. Otherwise, the structure of these demands follows the usual format.

Equations (33) and (34) in Table 2.11 describe the value-added nest of the producers' technology tree. In particular, they explain changes in the price of composite value-added (*pva*) and the conditional demands (*qfe*) for endowment commodities in each sector. Here, the coefficient *SVA*(*i,j,r*) refers to the share of endowment commodity *i* in the total cost of value-added in sector *j* of *r*. In addition to the price variables, *pfe*(*i,j,r*), these equations include variables governing the rate of primary factor—augmenting technical change, *afe*(*i,j,r*). More specifically, this is the rate of change in the variable *AFE(i,j,r)*, where *AFE*(*i,j,r*)**QFE*(*i,j,r*) equals the *effective* input of primary factor *i* in sector *j* of region *r*. Therefore, a value of *afe*(*i,j,r*) > 0 results in a decline in the effective price of primary factor *i*. For this reason it enters the equations as a deduction from *pfe*(*i,j,r*). This has the effect of: (1) encouraging substitution of factor *i* for other primary inputs via the right-hand side of equation (34), (2) diminishing the demand (at constant effective prices) for *i* via the left-hand side of equation (34) and (3) lowering the cost of the value-added composite via equation (33) — thereby encouraging an expansion in the use of all primary factors.

Finally, we have the top-level nest, which generates the demand for composite value-added and intermediate inputs. Since we have assumed no substitutability between intermediates and value-added, the relative price component of these conditional demands drops out, and we are left with only the expansion effect. Furthermore, there are three types of technical change introduced in this nest. The variables *ava*(*j,r*) and *af*(*i,j,r*) refer to input augmenting technical change in composite value-added and intermediates, respectively. The variable *ao*(*j,r*) refers to Hicks-neutral technical change. It uniformly reduces the input requirements associated with producing a given level of output. Finally, we have restated the zero profits condition (6′), which serves to determine the price of output in this sector. This revised equation reflects the effect of technical change on the composite output price for commodity *j* produced in region *r*.

Implications for Tariff Reform: At this point it is useful to employ the linearized representation of producer behavior provided in Table 2.11 to think through the effects of a trade policy shock. Consider, for example, a reduction of the bilateral tariff on imports of *i* from *r* into s, *tms*(*i,r,s*). This lowers *pms*(*i,r,s*) via price linkage equation (24) in Table 2.9. Domestic users immediately substitute away from competing imports according to equation (29) in Table 2.10. Also, the composite price of imports facing sector *j* falls via equations (28) and (23), thereby increasing the aggregate demand for imports through equation (31) in Table 2.11. Cheaper imports serve to lower the composite price of intermediates through equation (30), which causes excess profits at current prices, via equation (6). This in turn induces output to expand, which in turn generates an expansion effect via equations (35) and (36) in Table 2.11. (Of course, in a partial equilibrium model whereby nonfood sectors' activity levels are exogenous, the latter effect will be present only when *j* refers to a food sector.)

The expansion effect induces increased demands for primary factors of production via equation (34) in Table 2.11. In a partial equilibrium closure, labor and capital might be assumed forthcoming in perfectly elastic supply from the nonfood sectors, so *pfe*(*i*,*j*,*r*) is unchanged for *i* = labor, capital. However, in the general equilibrium model, this expansion generates an excess demand via the mobile endowment market clearing condition equation (4), thereby bidding up the prices of these factors, and transmitting the shock to other sectors in the liberalizing region.

Now turn to region *r*, which produces the goods for which *tms*(*i*,*r*,*s*) is reduced. Equation (29) in Table 2.10 may be used to determine the implications for total sales of *i* from *r* to *s*, given the responses of agents in region *s* to the tariff shock. Equation (1) dictates the subsequent implications for total output: *qo*(*i*,*r*) (unless this market clearing condition has been eliminated, and *pm*(*i*,*r*) fixed, under a particular PE closure). At this point, the equations in Table 2.11 again come into play, with equations (35) and (36) transmitting the expansion effect back to intermediate demands and to region *r*'s factor markets.

Household Behavior

Theory: As shown in Figures 2.1 and 2.2, regional household behavior is governed by an aggregate utility function, specified over composite private consumption, composite government purchases, and savings. The motivation for including savings in this static utility function derives from the work of Howe (1975), who showed that the intertemporal, extended linear expenditure system (ELES) could be derived from an equivalent, atemporal maximization problem, in which savings enters the utility function. Specifically, he begins with a Stone–Geary utility function, thereupon imposing the restriction that the subsistence budget share for savings is zero. This gives rise to a set of expenditure equations for current consumption that are equivalent to those flowing from Lluch's (1973) intertemporal optimization problem.[7] In the GTAP model we employ a special case of the Stone–Geary utility function, whereby *all* subsistence shares are equal to zero. Therefore, Howe's result, linking this specification with a well-defined intertemporal maximization problem, is applicable.

The other feature of our regional household utility function requiring some explanation is the use of an index of current government expenditure to proxy the welfare derived from the government's provision of public goods and services to private households in the region. Here, we draw on the work of Keller (1980) (chap. 8), who demonstrates that if (1) preferences for public goods are separable from preferences for private goods, and (2) the utility function for public goods is identical across households within the regional economy, then we can derive a public utility function. The aggregation of this index with private utility in order to make inferences about regional welfare requires the further assumption that the level of public goods provided in the initial equilibrium is optimal. Users who do not wish to invoke this assumption can fix the level of aggregate government utility, letting private consumption adjust accordingly.

Equations: The behavioral equations for regional households in the model are laid out in Table 2.12. As previously noted, this household disposes of total regional income according to a Cobb–Douglas per capita utility function specified over the three forms of final demand: private household expenditures, government expenditures, and savings [equation (37)]. Thus in the standard closure, the claims of each of these areas represent a constant share of total income. This may be seen from equations (38) and (39), which determine the changes in real expenditures on savings and government activities as a function of regional income and prices. These equations also include slack variables that may be swapped with the quantities of savings and government composites, *qsave* and *ug*, if the user wishes to specify the latter variables exogenously. In order to assure the exhaustion of total regional income under these closures, equation (8) computes the change in private household spending as a residual. Both private and government demands are composite goods that require further elaboration. We turn first to the disaggregate government demands.

Government Demands: Once the percentage change in real government spending has been determined, the next task is to allocate this spending across composite goods. Here, the Cobb–Douglas assumption of constant budget shares is once again applied. This is implemented via equations (40) and (41) in Table 2.12. In the first of these equations an aggregate price index for all government purchases, *pgov*(*r*), is established. This in turn provides the basis for deriving the conditional demands for composite tradeable goods, *qg*(*i*,*r*). Note the similarity between equation (41) and the CES production nests in Table 2.11. [Since we restrict the elasticity of substitution among composite products in the government's utility function to be unitary, this parameter does not appear in equation (41).]

Once aggregate demand for the composite is established, the remainder of the government's utility "tree" is completely analogous to that of the firms represented in Figure 2.6 and Table 2.11. First, a price index is established, equation (42), then composite demand is allocated between imports and domestically produced goods. Finally the sourcing of imports occurs at the border, via the equations in Table 2.10. Due to the lack of use-specific Armington substitution parameters, σ_D is also assumed to be equal across all uses, that is, across all firms and households. Therefore, the only thing that distinguishes firms and households' import demands are the differing import shares. However, this is not an insignificant difference. Some sectors/households are more intensive in their use of imports. Consequently, they will be more directly affected by a change in, for example, a tariff on the imported goods. This is why the effort expended to establish the detailed mapping of imports to sectors is warranted.

Private Demands: The nonhomothetic nature of private household demands necessitates a somewhat different treatment. First of all, the computation of the utility of private household consumption must now take explicit account of the rate of population growth. Therefore the percentage change in private utility, *up*(*r*), is defined on a per capita basis. The particular method for calculating the percentage change in the utility of private consumption is dictated by the assumed form of private household preferences. For practical reasons, we have chosen to employ the constant difference of elasticities (CDE) functional form, first proposed by Hanoch (1975). It lies midway between the nonhomothetic CES on the one hand, and the fully flexible functional forms on the other. For our purposes, its main virtue is the ease with which it may be calibrated to existing information on income and own-price elasticities of demand. (For an exhaustive treatment of the calibration and use of the CDE functional form in AGE models, see Hertel et al. 1991.) The CDE implicit expenditure function is given by (2.14):

$$\sum_{i \varepsilon TRAD} B(i,r) * UP(r)^{\beta(i,r)\gamma(i,r)} * [PP(i,r) / E(PP(r))]^{\beta(i,r)} \equiv 1. \qquad (2.14)$$

Here, ($E(\cdot)$) represents the minimum expenditure required to attain a prespecified level of private household utility, $UP(r)$, given the vector of private household prices, $PP(r)$. Minimum expenditure is used to normalize individual prices. These scaled prices are then raised to the power $\beta(i,r)$ and combined in an *additive* form. Unless β is common across all commodities in a given region, minimum expenditure cannot be factored out of the left-hand side expression and (2.14) is an *implicitly additive* expenditure function. The calibration problem involves choosing the values of β to replicate the desired compensated, own-price elasticities of demand, then choosing the γ's to replicate the targeted income elasticities of demand. (The shift term $B(i,r)$ is a scale factor embodied in the budget share, CONSHR(i,r), in the linearized representation of these preferences.)

Total differentiation of (2.14) and use of Shephard's lemma permits us to derive the relationship between minimum expenditure, utility, and prices that is given in equation (45) of Table 2.12 (see also Hertel, Horridge, and Pearson 1992). Equation (46) determines per capita private household demands for the tradeable composite commodities: $qp(i,r)$ - $pop(r)$. As long as $EY(i,r)$ departs from unity, the $pop(r)$ term does not cancel out, as it did in the case of homothetic government and savings demands. Finally, in Table 2.12 we have a block of equations that develop the mix of composite consumption of tradeable commodities, based on domestic and composite imported goods.

As noted in the previous paragraph, the parameters of the CDE function are initially selected (i.e., calibrated) to replicate a prespecified vector of own-price and income elasticities of demand. However, with the exception of some special cases of the CDE (e.g., the Cobb–Douglas), these elasticities are not constants. Rather, they vary with expenditure shares/relative prices. [See Hertel et al. 1991 for derivations and more detailed discussion of these formulas. Chapter 4 also provides illustrations of how the income elasticities of demand vary over expenditure levels.] For this reason we need some supplementary formulas describing how the elasticities are updated with each iteration of the nonlinear solution procedure.

The formulas for the uncompensated price and income elasticities of demand, $EP(i,k,r)$ and $EY(i,r)$, are reported in Table 2.13. (These are not assigned equation numbers, as they are merely used to compute parameter values to be used in the system of equations representing the model. Therefore, they are given the prefix "F.") The first of these simply defines a parameter, α, that is equal to one minus the CDE substitution parameter. (This simplifies some of the other formulas.) Formulas (F2) and (F3) compute the own– and cross–price allen partial elasticities of substitution in consumption. (The latter are symmetric.) These are simply a function of α and the consumption shares. It may be seen that when $\beta(i,r) = \beta \ \forall i$, then the cross-price elasticities of substitution are all equal to $1 - \beta = \alpha$ and the CDE simplifies to a CES function. Furthermore, when $\beta = 1$, there is no substitution in consumption and when $\beta = 0$, preferences are Cobb–Douglas. When premultiplied by $CONSHR(i,r)$, formula (F3) yields the compensated, own-price elasticity of demand for commodity i. Once these have been specified, this linear system of equations may be solved for the "calibrated" values of α, and hence β, via (F1). (See Chapter 4 for a more extensive discussion of calibration procedures.)

Formula (F4) shows how the income elasticities of demand are computed as a function of consumption shares, the income expansion parameters, γ's, and the α's. Because of this, calibration of the own-price elasticities of demand must precede calibration of the income elasticities. Finally, the two may be combined to yield the uncompensated, own-price elasticities of demand reported in (F5).

Imperfect Factor Mobility

The two equations in Table 2.14 describe the responsiveness of imperfectly mobile factors of production to changes in the rental rates associated with those sectors in which these sluggish factors are employed. The mobility of these endowments is described with a CET revenue function (Powell and Gruen 1968), which is completely analogous to the CES cost functions used above, except the revenue function is *convex in prices*. Thus the elasticity of transformation is nonpositive, $\sigma_T < 0$. As σ_T becomes larger in absolute value, the degree of sluggishness diminishes and there is a tendency for rental rates across alternative uses to move together. As with the CES nests discussed above, the first equation (50) introduces a price index and the second equation (51) determines the transformation relationships. Note also that equation (51) is where we introduce the slack variable, to be used in those cases where the user wishes to fix the market price of a sluggish endowment commodity.

Macroeconomic Closure

Having described the structure of final demand, as well as factor market closure in the GTAP model, it remains to discuss the determination of aggregate investment. Like most comparative static AGE models, GTAP does not account for macroeconomic policies and monetary phenomena, which are the usual factors explaining aggregate investment. Rather, we are concerned with simulating the effects of trade policy and resource-related shocks on the medium term patterns of global production and trade. Because this model is neither an intertemporal model (e.g., McKibbin and Sachs 1991), nor sequenced through time to obtain a series of temporary equilibria (e.g., Burniaux and van der Mensbrugghe 1991), investment does not come "on-line" next period to affect the productive capacity of industries/regions in the model. However, a reallocation of investment across regions *will* affect production and trade through its effects on the profile of final demand. Therefore, it is important to give this some attention. Also, a proper treatment of the savings–investment link is necessary in order to complete the global economic system, thereby assuring consistency in our accounting.

Because there is no intertemporal mechanism for determination of investment, we face what Sen (1963) defined as a problem of *macroeconomic closure* [see also Taylor and Lysy (1979)]. Following Dewatripont and Michel (1987), we note that there are four popular solutions to the fundamental indeterminacy of investment in comparative static models. The first three are nonneoclassical closures in which investment is simply fixed and another source of adjustment is permitted. In the fourth closure investment is permitted to adjust; however, rather than including an independent investment relationship, it simply accommodates any change in savings.

In addition to adopting a closure rule with respect to investment, it is necessary to come to grips with potential changes in the current account. Many multiregion trade models have evolved as a set of single-region models that are linked via bilateral merchandise trade flows [e.g., early versions of the SALTER model, which evolved from the ORANI model of Australia; see also Lewis, Robinson, and Wang (1995)]. These models have no *global closure* with respect to savings and investment, but instead impose the macroeconomic closure at the regional level. Here it is common to force domestic savings and investment to move in tandem, by fixing the current account balance. To understand this, it is useful to recall the following accounting identity, e.g., Dornbusch 1980, which follows from equating national expenditure from the sources and uses sides:

$$S - I \equiv X + R - M \tag{2.15}$$

which states that the national savings (S) minus investment (I) is identically equal to the current account surplus, where R is international transfer receipts. (In the GTAP data base we do not have observations on R, so it is set equal to zero and S is derived as a residual, which reflects national savings, net f the unobserved transfers.) By fixing the right-hand side of identity (2.15) one also fixes the difference between national savings (including government savings) and investment. This may be accomplished in the GTAP framework by fixing the trade balance [DTBAL(r) = 0, see equation (98) in Table 2.18] and freeing up either national savings [endogenize *saveslack*(r) in equation (38)] or investment [endogenize *cgdslack*(r) in equation (11′)].

If global savings equals global investment in the initial equilibrium, then the summation over the left-hand side of equation (2.15) equals zero and the sum of all current account balances must initially be zero (provided *cif/fob* margins are accounted for in national exports). Furthermore, by fixing the right-hand side of (2.15) on a regional basis, each region's share in the global pool of net savings is fixed. In this way, equality of global savings and investment in the new equilibrium is also assured, in spite of the fact that there is no "global bank" to intermediate formally between savings and investment on a global basis. Finally, since investment is forced to adjust in line with regional changes in savings, this approach clearly falls within the "neoclassical" closure, as identified by Dewatripont and Michel (1987).

The exogeneity of the current account balance embodies the notion that this balance is a macroeconomic, rather than microeconomic, phenomenon: to a great extent, the causality in identity (2.15) runs from the left side to the right side. It also facilitates analysis by forcing all adjustment to external imbalance onto the current account. If savings does not enter the regional utility function (as is the case in most multiregion AGE models outside of GTAP), this is also the right approach to welfare analysis because an arbitrary shift away from savings toward current consumption and increased imports would otherwise permit an increase in utility to be attained, even in the absence of improvements in efficiency or regional terms of trade.

For some types of experiments, however, modelers may wish to endogenize the balances on either side of identity (2.15). For example, some trade policy reforms raise returns to capital and/or lower the price of imported capital goods. In this case, we would expect the increased rate of return on new investment to result in an increase in regional investment and, *ceteris paribus*, a deterioration in the current account. In other cases one might wish to explore the implications of, for example, an *exogenous* increase in foreign direct investment, which would also dictate a deterioration in the current account. Once the left-hand side of (2.15) is permitted to adjust, a mechanism is needed to ensure that the global demand for savings equals the global demand for investment in the postsolution equilibrium. The easiest way to do so is through the use of a "global bank" to assemble savings and disburse investment. This is the approach that we adopt here.

The global bank in the GTAP model uses receipts from the sale of a homogeneous savings commodity to the individual regional households in order to purchase (at price *PSAVE*) shares in a portfolio of regional investment goods. The size of this portfolio adjusts to accommodate changes in global savings. Therefore, the *global closure* in this model is neoclassical. However, on a regional basis, some adjustment in the mix of investment is permitted, thereby adding another dimension to the determination of investment in the model.

Fixed Capital Formation and Allocation of Investment Across Regions

We have incorporated two alternative investment components into the model. The user may choose which "theory" to employ, depending on her or his individual needs and the simulation being conducted. The first investment component enforces a close link between regional rates of return on capital across regions. This component is described in equations (2.16) – (2.26) below. It draws on the formulation used to allocate investment across *sectors* in the ORANI model (Dixon et al. 1982). The second investment component is based on the assumption that the regional composition of global capital stock will be left unaltered in the simulation, and it is described in equations (2.26) and (2.27) below. At the end of this section we incorporate these two alternative investment components into a single set of composite equations, and explain how the user may specify which is to be used.

We begin by assuming that the productive capacity of capital declines geometrically over time, with depreciation rate *DEPR*(*r*). As a result the end-of-period capital stock, *KE*(*r*), is equal to the beginning-of-period capital stock, *KB*(*r*), multiplied by [1 - *DEPR*(*r*)] and augmented by gross investment, *REGINV*(*r*). This accounting relationship is shown in the lower part of Table 2.7 and it is reproduced below:

$$KE(r) = KB(r) * [1 - DEPR(r)] + QCGDS(r). \tag{2.16}$$

We differentiate both sides of accounting relationship (2.16) to obtain:

$$dKE(r) = [1 - DEPR(r)] * dKB(r) + dQCGDS(r), \tag{2.17}$$

which may be rewritten in terms of percentage changes as:

$$\begin{aligned} ke(r) = {} & [1 - DEPR(r)] * [KB(r) / KE(r)] * kb(r) \\ & + [QCGDS(r) / KE(r)] * qcgds(r), \end{aligned} \tag{2.18}$$

where variables in lowercase represent the percentage change of the corresponding level variables in uppercase.

Let us now define the ratio of investment to end-of-period capital stock, $INVKERATIO(r)$, as:

$$\begin{aligned} INVKERATIO(r) &= PCGDS(r) * [KB(r) / KE(r)] * kb(r) \\ &= REGINV(r) / VKE(r) \end{aligned}$$

and note that

$$\begin{aligned} [1 - DEPR(r)] * [KB(r) / KE(r)] &= \{VKB(r)\ [1 - DEPR(r)] \\ &\quad + REGINV(r) - REGINV(r)\} / VKE(r) \\ &= \{VKE(r) - REGINV(r)\} / VKE(r) \\ &= 1 - INVKERATIO(r). \end{aligned}$$

We substitute this into (2.18) to obtain the following relation:

$$ke(r) = [1 - INVKERATIO(r)] * kb(r) + INVKERATIO(r) * qcgds(r). \qquad (2.19)$$

This is equation (10) in Table 2.8.

We then define the current net rate of return on fixed capital in region r, *RORC(r)*, as the ratio of the rental for capital services, *RENTAL*(*r*), to the purchase price of capital goods, *PCGDS*(*r*), less the rate of depreciation, *DEPR*(*r*):

$$RORC(r) = RENTAL(r) / PCGDS(r) - DEPR(r). \qquad (2.20)$$

Expressing equation (2.20) in percentage change terms, we obtain:

$$rorc(r) = [RENTAL(r) / (RORC(r) * PCGDS(r))] * [rental(r) - pcgds(r)]. \qquad (2.21)$$

We note that

$$RENTAL(r) / [RORC(r) * PCGDS(r)] = [RORC(r) + DEPR(r)] / RORC(r), \qquad (2.22)$$

and we define the ratio of gross returns, [i.e., *RORC*(*r*) + *DEPR*(*r*)] to net returns as:

$$GRNETRATIO(r) = [RORC(r) + DEPR(r)] / RORC(r). \qquad (2.23)$$

We substitute equations (2.22) and (2.23) into equation (2.21) to obtain equation (57) in Table 2.15.

For our rate-of-return investment component, we assume that investors are cautious in assessing the effects of net investment in a region. They behave as if they expect that region's rate-of-return in the next period, *RORE*(*r*), to decline with positive additions to the capital stock. The rate at which this decline is expected is a function of the flexibility parameter *RORFLEX*(*r*)>0:

$$RORE(r) = RORC(r)[KE(r) / KB(r)]^{-RORFLEX(r)}. \qquad (2.24)$$

Therefore, the elasticity of *RORE(r)* with respect to *KE(r)* is equal to minus *RORFLEX(r)*. Equation (2.24) in percentage change terms is given by equation (58) in Table 2.15. We then assume that investors behave in such a way that changes in regional rates of return are equalized across regions:

$$rore(r) = rorg, \qquad (2.25)$$

where *rorg* is the percentage change in a global rate of return. Thus, the model will distribute a change in global savings across regions in such a way that all *expected* regional rates of return change by the same percentage. A small value for *RORFLEX*(*r*), say. *RORFLEX*(*r*)=0.5, implies that a 1% increase in *KE*(*r*) is expected to reduce the rate of return on capital by 0.5%. (For example, if the current rate of return were 10%, the expected rate of return on a net investment equal to 1% of *KE*(*r*) would be 9.95%, i.e., little change.) In this case the supply of new capital goods is *very* sensitive to the expected rate of return. In order to maintain equal changes in *RORE* across regions, the model will produce large changes in regional investment.

However, a large value for *RORFLEX*(r), say, *RORFLEX*(r) = 50, implies that a 1% increase in *KE*(r) is expected to cut the rate of return on capital in half. In this case the supply of new capital goods is not very sensitive to changes in the expected rate of return. Therefore, equal changes in *RORE* across regions can be accommodated with small changes in regional investment. In other words, if the user believes that the experiment under consideration will not have a great impact on regional investment (or wishes to abstract from such effects) large values of *RORFLEX*(r) should be chosen.

Relatively high values for the coefficient *RORFLEX*(r) are supported by the work of Feldstein and Horioka (1980). They correlated the share of gross domestic investment to gross domestic product with the share of gross domestic savings to gross domestic product (see Feldstein and Horioka 1980; Feldstein 1983). They found a close correlation between savings and investment, and they concluded that even between industrialized countries, international capital mobility may be limited.

The second investment component adopts an extreme position in which we assume that the regional composition of capital stocks will not change at all so that regional and global net investment move together:

$$\begin{aligned} globalcgds = & [REGINV(r) \; / \; NETINV(r)] * qcgds(r) \\ & - [VDEP(r) \, / \, NETINV(r)] * kb(r), \end{aligned} \tag{2.26}$$

where *globalcgds* is the percentage change in global supply of new capital goods. In this case, the percentage change in the global rate of return on capital variable, *rorg*, is computed as a weighted average of regional variables (the latter being now wholly unrelated):

$$rorg = \sum_{r \varepsilon REG} [NETINV(r) \, / \, GLOBINV] * rore(r)$$

where

$$NETINV(r) = (REGINV(r) - VDEP(r)) \tag{2.27}$$

To summarize, under the rate-of-return component, investment behavior is determined by equations (2.25) above and equation (11) in Table 2.8. Under the alternative component, investment behavior is determined by equations (2.26) and (2.27). Both systems are summarized in Table 2.16.

We have combined these two systems in equations (2.28) and (2.29), employing the parameter *RORDELTA*: this is a binary parameter that takes the values 0 and 1. For *RORDELTA*=1 we obtain the rate-of-return model, and for *RORDELTA*=0 we obtain the alternative model.

$$\begin{aligned} RORDELTA \; & * rore(r) + (1 - RORDELTA) * [REGINV(r)/NETINV(r)] \\ & * qcgds(r) - [VDEP(r)/NETINV(r)] * kb(r) \\ = RORDELTA \; & * RORG + (1 - RORDELTA) * globalcgds \end{aligned} \tag{2.28}$$

and

$$
\begin{aligned}
& RORDELTA * \ globalcgds \ + \ (1 - RORDELTA) \ * \ rorg \\
& = \ RORDELTA * \sum_{r \varepsilon REG} \{[REGINV(r) / GLOBINV]\} \\
& * \ qcgds(r) - [VDEP(r)/GLOBINV] * kb(r) \\
& + (1 - RORDELTA) * \sum_{r \varepsilon REG} [NETINV(r) / GLOBINV] * rore(r)
\end{aligned}
\tag{2.29}
$$

Equation (2.28) is shown in Table 2.15 as equation (59), and equation (2.29) is shown in Table 2.15 as equation (11´). It replaces equation (11) in Table 2.8.

Once the level of investment activity in each region has been determined, it remains only to generate the mix of expenditures for domestic and imported inputs used in the production of fixed capital in region *r*: *VDFA*(*i,"cgds",r*) and *VIFA*(*i,"cgds",s,r*), respectively. This is completely analogous to the production of tradeable commodities. In fact, the same equations are used to generate these derived demands. We assume that a unit of capital for investment in region r is created by assembling composite intermediate inputs in fixed proportions [equation (36) in Table 2.11]. The composite intermediate input is, in turn, a CES combination of domestic and foreign imported inputs [equations (31) and (32) in Table 2.11 and equation (29) in Table 2.10]. However, in contrast to the production of tradeable commodities, capital creation requires no services of primary factors. This is because it is a fictitious activity that merely assembles goods destined for fixed investment in region r. In other words, the use of land, labor, and capital associated with capital formation is already embodied in the intermediate inputs assembled by this investment sector.

Global Transportation

In addition to the global bank, another global activity is required in this model in order to intermediate between the supply of, and demand for, international transport services. These services are provided via a Cobb–Douglas production function that demands, as inputs, services exports from each region. Lacking the data to link exports of transport services with specific routes, we simply combine these services into a single composite international transport good, the value of which is *VT* = *QT* * *PT*. The percentage change equation for the composite price index was given in equation (7) of Table 2.8. For convenience, it is repeated as equation (7´) in Table 2.17. Recall that this is akin to a zero profit condition for the aggregate transport sector. The next equation (61) in Table 2.17 derives the conditional demands for the inputs to the shipping services sector, assuming that the share of each region in the global industry is constant, that is, Cobb–Douglas technology. Therefore, this equation includes an expansion effect (*qt*) and a substitution effect, whereby the elasticity of substitution is assumed to be unitary.

The next two equations in Table 2.17 refer to the uses of the composite international shipping service. We assume that this composite is employed in fixed proportions with the volume of a particular good shipped along a particular route, *QXS*(*i,r,s*). In other words: *ATR*(*i,r,s*) * *QTS*(*i,r,s*) = *QXS*(*i,r,s*), where *QTS*(*i,r,s*) is the amount of the homogeneous product *QT* used in shipping commodity *i* from *r* to *s*, and *ATR*(*i,r,s*) is a technical coefficient. Equilibrium in the global transport services market therefore requires that:

$$
\sum_{i \varepsilon TRAD} \sum_{r \varepsilon REG} \sum_{s \varepsilon REG} QTS(i,r,s) \ = \ QT \ . \tag{2.30}
$$

Proportionately differentiating this equation gives:

$$\sum_{i\varepsilon TRAD}\ \sum_{r\varepsilon REG}\ \sum_{s\varepsilon REG} QTS(i,r,s) \ * \ qts(i,r,s) \ = \ QT \ * \ qt \,. \tag{2.31}$$

Multiplying both sides by the common price of the composite transport service, and substituting [*qxs*(*i*,*r*,*s*) - *atr*(*i*,*r*,*s*)] for *qts*(*i*,*r*,*s*) gives equation (62) in Table 2.17. The presence of *atr*(*i*,*r*,*s*) in this formulation permits the user to introduce commodity/route-specific technical change in international transport services. This also requires us to modify the *fob/cif* price linkage equation (26) in Table 2.9 to reflect the fact that an increase in efficiency along a particular route will lower *cif* values, for a given *fob* price. This revision is reported in (26´) of Table 2.17.

Summary Indices

This section discusses the summary indices computed in the GTAP model. These equations do not play a role in determining the equilibrium solution. Indeed, all these indices *could* be computed after the fact. However, it is convenient to include them in the model so that their rates of change are reported along with the other results. Table 2.18 shows aggregate indices of prices received [*psw*(*r*), equation (64)] and prices paid [*pdw*(*r*), equation (65)] for products sold and purchased by each region (inclusive of savings and investment, which represent transactions with the global bank). The difference between *psw*(*r*) and *pdw*(*r*) measures the percentage change in each region's terms of trade, *tot*(*r*).

GTAP also computes regional equivalent variation measures, *EV*(*r*), which arise due to the simulation under consideration. The values for *EV*(*r*) are in 1992 $US million, and they are computed as:[8]

$$EV(r) = u(r) * INC(r)/100.$$

Since *u*(*r*) reports per capita welfare, equation (67) in Table 2.18 also includes the rate of change in population on the right–hand side so that the *EV* reported by the model represents total regional welfare. The worldwide equivalent variation (WEV) is then computed as the simple summation of the regional *EV*s, equation (68). This is followed by an equation generating the percentage in the region-specific consumer price index, *ppriv*(*r*).

Other useful price and quantity indices included in GTAP refer to trade, regional gross domestic product (GDP), and income magnitudes. To obtain quantity indices, it is necessary to compute the corresponding value and price indices first, because we are aggregating over different commodities. For example, variable *qgdp*(*r*), equation (72) of Table 2.18, is a quantity index for domestic product.[9] Table 2.18 shows that we first compute a value index, *vgdp*(*r*), in equation (70), which accounts for changes in prices and quantities, and a price index, *pgdp*(*r*), in equation (71), which accounts for changes in prices only. The quantity index, *qgdp*(*r*), is then computed as the difference between *vgdp*(*r*) and *pgdpr*(*r*). For simulations of trade and domestic policy changes, the solution value for *qgdp*(*r*) will typically be small, reflecting only shifts in the economy's production possibilities frontier owing to the improved allocation of a fixed resource base. But for simulations of endowment growth, the solution value for *qgdp*(*r*) will provide a summary measure of growth for the region.

We next turn to a set of equations defining changes in aggregate trade values, prices and quantity indices. Equations (73)–(78) compute the percentage change in export and import values: (1) by commodity *and* region, (2) by region for all traded commodities, and (3) by commodity for all regions in the world. Equation (79) computes the percentage change in the value of total world trade, and equation (80) computes the percentage in value of world output, by commodity.[10] These are followed by eight analogous equations, (81)–(88), which compute the associated price indices, after which we are able to extract pure volume changes for aggregate trade and output [equations (89)–(96)].

The last two equations in the model are given at the bottom of Table 2.18. They are used to compute the change in trade balance, by commodity and by region. This is a value-based concept, and *DTBAL*(*r*), equation (98) refers to the changes in the current account for each region.

VII. A SIMPLE NUMERICAL EXAMPLE

Perhaps the best way to understand how this model works is to perform a simple experiment and examine the resulting changes in endogenous variables of interest. (This is example 21 in the Hands-On document, referred to in Chapter 6, and available through the Web site.) In order to keep things simple, we will work with a three-commodity/three-region aggregation of the data base. The three commodities are: food, manufactures, and services. The three regions are: the United States (US), the European Union (EU) and the rest of the world (ROW). The experiment involves a bilateral reduction in the level of the EU's import tariff on US food products. In particular, *tms*(*food,usa,eu*) = -10%. This implies a cut of 10% in the *power of the ad valorem tariff*, which amounts to a 10% cut in the domestic price of US food exports to the EU, *ceteris paribus*. Furthermore, we begin by performing only the *first step* in a multistep solution of the model developed above. In terms of Figure 2.5, this means we are moving from (X_0, Y_0) to (X_1, Y_J), where Y_J is a Johansen approximation to Y_1 (the true solution). This is merely a pedagogical device to facilitate discussion of our example, since in the Johansen solution, the linearized form of the model in Tables 2.8–2.18 will *hold exactly*. For small shocks this may provide a reasonable approximation to the true, nonlinear price and quantity changes. However, *it is a very poor method for assessing welfare changes* [see Hertel, Horridge, and Pearson (1992) for an extensive discussion of these issues]. The reader can observe this approximation error by comparing the Johansen solution with the Gragg outcomes (given in brackets) in Tables 2.20, 2.22, and 2.23.

Tables 2.19 and 2.20 report selected changes in the EU, resulting from the bilateral tariff cut. We begin at the top of the table, with the market price of US food in the US. This market price rises by 0.140% due to increased demand. Since there is no change in the border tax, *pfob* rises by the same amount, via equation (27) in Table 2.9. The *cif* price of US food exports to the EU depends also on changes in the price index of international transport services, *pt*. This drops slightly due to the decline in the price of EU transport services [Table 2.19 and equation (7)]. Therefore, *pcif* increases by a slightly smaller amount.

The bilateral tariff instrument, which is the subject of this experimental shock, enters via equation (24) in Table 2.9. Its reduction serves to lower the domestic market price of EU food imports from the US by an additional 10%, so that *pms*(*food,usa,eu*) = -9.876%. This price cut has two immediate effects. First, it lowers the price of composite imports by 1.631% [equation (28) in Table 2.10], a value that is roughly equal to the share of US imports in total expenditures on imported food multiplied by -9.876%. The second immediate effect of this price cut is that it encourages agents in the EU to alter their sourcing of food imports in favor of US products [equation (29) in Table 2.10]. The responsiveness of this shift in the model is dictated by the elasticity of substitution among food imports (σ_M). Its value in the aggregated data base is 4.64. This figure is multiplied by the percentage change in the cost of food imports from the US, relative to composite import costs, or the difference in these two individual percentage changes. This equals 38.26%. If the level of imports *qim* were unchanged, this would be the end of the story. However, the impact of the bilateral cut in protection continues, since the cheaper imports result in a substitution of composite imports for domestic food. This effect varies by sector in this model, due to the differing importance of imports in the composite intermediate good. Since the substitution structure in each of these is very similar, we choose to focus on the EU food industry, which is the largest user, accounting for 52.7% of total food imports in that market. In this industry, aggregate imports increase by 3.18%. Thus the total increase in US food imports by the EU food industry is equal to 41.4%.

The numerical implementations of equations (30) and (31) in Table 2.11 describe the changes at the next level of the production tree (recall Figure 2.3). They account for the 3.0% increase in composite food imports by this sector. However, note from equation (31) that in this case the expansion and substitution effects work in opposite directions, since *qf*(*food,eu*) = *qo*(*food,eu*) < 0. That is, the food sector as a whole contracts, and with it, there is a decline in the demand for intermediate products, in this case food. Equation (32) shows that the demand for domestically produced intermediates actually falls. Finally, owing to a decline in the total demand for domestically produced food, the price of EU-produced food falls.

Table 2.20 reports selected price and quantity changes for the EU as a whole, owing to this bilateral tariff cut. The price of farmland falls, since this factor has no alternative uses outside of the food sector in our model, and output in that sector has declined. With labor and capital being released from the food sector, the nonfood sectors are able to expand. In general equilibrium, households increase their consumption of all nonsavings commodities due to the lower prices. The demand for savings falls, since its price is determined by a weighted combination of the capital goods prices from all regions, which tend to rise relatively more than other goods prices.

Now turn to the effects of the tariff cut on the US economy, which are reported in Tables 2.21 and 2.22. Equation (1) in Table 2.21 combines the increase in US–EU exports, together with changes in sales to other destinations/uses in order to estimate the change in food sector output in the US. The first figures in parentheses are the shares of sales to various uses. From this, it can be seen that exports to the EU account for only 1.3% of the value of total US food sector output (at domestic market prices). This considerably tempers the impact of the 41.4% increase in sales. Of course the importance of this market for selected, *disaggregated* producer groups can be much larger, and might warrant strategic disaggregation of the data base to capture such effects.

It is not surprising that the bulk of US food sales goes to the domestic market (92.6%). However, it is somewhat surprising that the tariff cut in the EU causes domestic sales of US food to *increase*. More insight into this result may be obtained by considering the numerical implementation of equation (3) in Table 2.21. This shows the changes in composition of domestic sales. As expected, sales to other industries and final demand fall, as US supply prices for food are bid up by the EU users. However, these declines are more than offset by an increase in *intermediate* demands for food in the US food sector. In other words, to meet the increased demand for food in the EU, domestic sales of intermediate goods must also increase.

Table 2.22 describes the economywide effects of the bilateral tariff cut on the US. Here, the land rental rate rises by more than the food price, and labor and capital wages rise by somewhat less, with the relative capital intensity of the food sector favoring capital over labor. Continuing the analogy, we see that the manufacturing sector must contract in order to make way for expansion of the US food sector. Finally, note that composite consumption of nonfood manufactures and services increases, as households substitute imported for domestic goods.

The final table, Table 2.23, summarizes the macroeconomic effects of the EU's bilateral tariff cut. The increase in demand for US products bids up US prices, relative to the prices of products supplied from the EU and ROW. Since the EU must export more products to pay for the increase in food imports, their export volume increases by .233%, in the case where *RORDELTA* = 0 and a simple Johansen solution is used. (See the top entry in the second column of Table 2.23.) Therefore, the EU supply prices must fall relative to other regions. This results in a terms-of-trade deterioration for the EU, as seen in the third row of Table 2.23. The terms-of-trade for ROW are marginally worsened, due to displacement by US exporters. This translates into a welfare loss for ROW. In the EU, the terms-of-trade decline is more than offset by the improved allocation of domestic resources, and aggregate regional welfare rises by $346 million. The US gains $778 million due to its improved terms-of-trade, following the preferential cut in border taxes on US food exports to the EU.

It is interesting to note that the trade balance hardly changes, $DTBAL(r) \approx 0$, in those simulations where *RORDELTA* = 0. This is a robust outcome that follows from equation (2.15) and the treatment of savings and investment in the model. The demand for savings is tied directly to income, which is little affected in this (and most other) policy reform experiments. Since regional savings doesn't change much, global savings, and hence global investment, are unaltered. Therefore, the only means of altering the left-hand side of (2.15), ($S - I$), and hence the trade balance, is to alter the regional allocation of investment. When *RORDELTA* = 0, this is not possible. Therefore, there can be little change in the right-hand side of (2.15), ($X - M$).

This is no longer true, however, when *RORDELTA* = 1, and the global bank's allocation of investment across regions is flexible. In the lower (parenthetical) entries of Table 2.23, we report results of the Johansen simulation with *RORFLEX* = 10 (the default setting for this parameter). Now changes in rate-of-return on investment come into play. From the second row of Table 2.23, we see that $rorc(eu) < 0$, since the rental rate on capital declines relative to the price of capital goods. Therefore, there is an incentive to divert some investment to other regions. Given S, the resulting decline in I requires an increase in ($X - M$) via identity (2.15). This is achieved by a slightly larger increase in export volume (.263% vs. .233%) from the EU, and a smaller increase in imports. Not surprisingly, this results in a stronger terms-of-trade deterioration and therefore a smaller gain in welfare, as opposed to the case where *RORDELTA* = 0.[11]

A comparison of the Johansen results with the nonlinear results (reported in brackets in Table 2.23) shows that the Johansen solution yields a poor approximation to the true welfare effects on the EU, even for this relatively small shock. This is because the change in EU utility reflects the difference between two larger changes, one of which is positive (efficiency gains) and one of which is negative (terms-of-trade effect). As can be seen from the third row of Table 2.23, the Johansen solution underestimates the true deterioration in the EU's terms of trade by a third. On the other hand, this solution procedure tends to *overstate* the gains from elimination of a distortion. Therefore, it is not surprising that the gain in EU welfare is *overstated* by more than five times ($346 million vs. the true gain of $62 million, reported in brackets). Indeed, it is not uncommon for such comparisons to yield sign reversals in some regions' welfare. In sum, use of the Johansen one-step solution procedure for purposes of decomposing small changes in prices and quantities (as in Tables 2.19 and 2.21) is very useful. However, *it is not an acceptable procedure for conducting* *welfare analysis of policy reforms*. For welfare analysis, the nonlinear solution procedures available in GEMPACK must be used.

VIII. SUMMARY

This completes our summary of the structure of the GTAP model. For your convenience, we have assembled a glossary of notation used in the model. This is provided at the back of this book. As noted, the electronic file, GTAP94.TAB, contains a complete listing of the model code. It is available on the FTP site. Access to this site is discussed in Chapter 6. The best way to become familiar with the model is to apply it to a particular problem of interest. After covering the data base, the parameters, aggregation, and computing issues, we will turn to a set of seven diverse applications of this model.

NOTES

1. The authors would like to thank Martina Brockmeier for her development of much of this material. For a more extensive graphical exposition of the GTAP model, see Brockmeier (1996).

2. The motivation for including savings in this atemporal utility function derives from Howe (1975) and is discussed at greater length below.

3. In some cases the initial data base does not include taxation in these markets. However, the possibility of introducing such taxation is available in the model, and it must therefore be accounted for in the computation of regional income.

4. The most natural way to implement this model would be via a *mixed* levels and percentage change representation. Indeed, this is possible in GEMPACK (Harrison and Pearson 1994). However, it is computationally more burdensome. Also, by linearizing the accounting equations some additional insights may be obtained.

5. This type of nonlinear solution procedure is now the default option in GEMPACK. For an exhaustive comparison of the linearized and levels approaches to AGE modeling, the reader is referred to Hertel, Horridge, and Pearson (1992).

6. For the user with an interest in applications for which intermediate–intermediate and intermediate–primary factor substitution is important, it will be necessary to modify the basic model, tailoring it to the specific needs at hand. However, this is not particularly difficult, as will be seen below.

7. Howe (1975) also shows that the savings share parameter in the atemporal utility function can be related to 1 one minus the ratio of consumer's rate of time preference to the rate of reproduction of capital.

8. The coefficient *INC*(*r*) reports <u>initial</u> equilibrium values for regional expenditure (which must equal income).

9. Values for the coefficient gross domestic product, GDP(r), are computed as follows:

$$GDP(r) = \sum_{i \varepsilon TRAD} [VGA(i,r) + VPA(i,r)] + VOA("CGDS", r)$$
$$+ \sum_{i \varepsilon TRAD} \sum_{s \varepsilon REG} VXWD(i,r,s) + \sum_{i \varepsilon TRAD} VST(i,r) - \sum_{i \varepsilon TRAD} \sum_{r \varepsilon REG} VIWS(i,r,s)$$

10. The coefficient *VOW*(*i*,*r*) measures the value of regional production at world prices, and its values are computed as follows: *VOW*(*i*,*r*) = *VDM*(*i*,*r*) * *PW_PM*(*i*,*r*) + $\sum_{s \varepsilon REG} VXWD(irs)$. The coefficient *PW_PM*(*i*,*r*) converts domestic use valued at market prices, *VDM*(*i*,*r*), to world prices, and it is computed as follows: $PW\_PM(i,r) = \sum_{s \varepsilon REG} VXWD(i,r,s) / \sum_{s \varepsilon REG} VXMD(i,r,s)$

11. The combination of *RORDELTA* = 1 and the Gragg nonlinear solution procedure actually gives a slight *decline* in EU welfare.

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Figure 2.1. One Region Closed Economy without Government Intervention

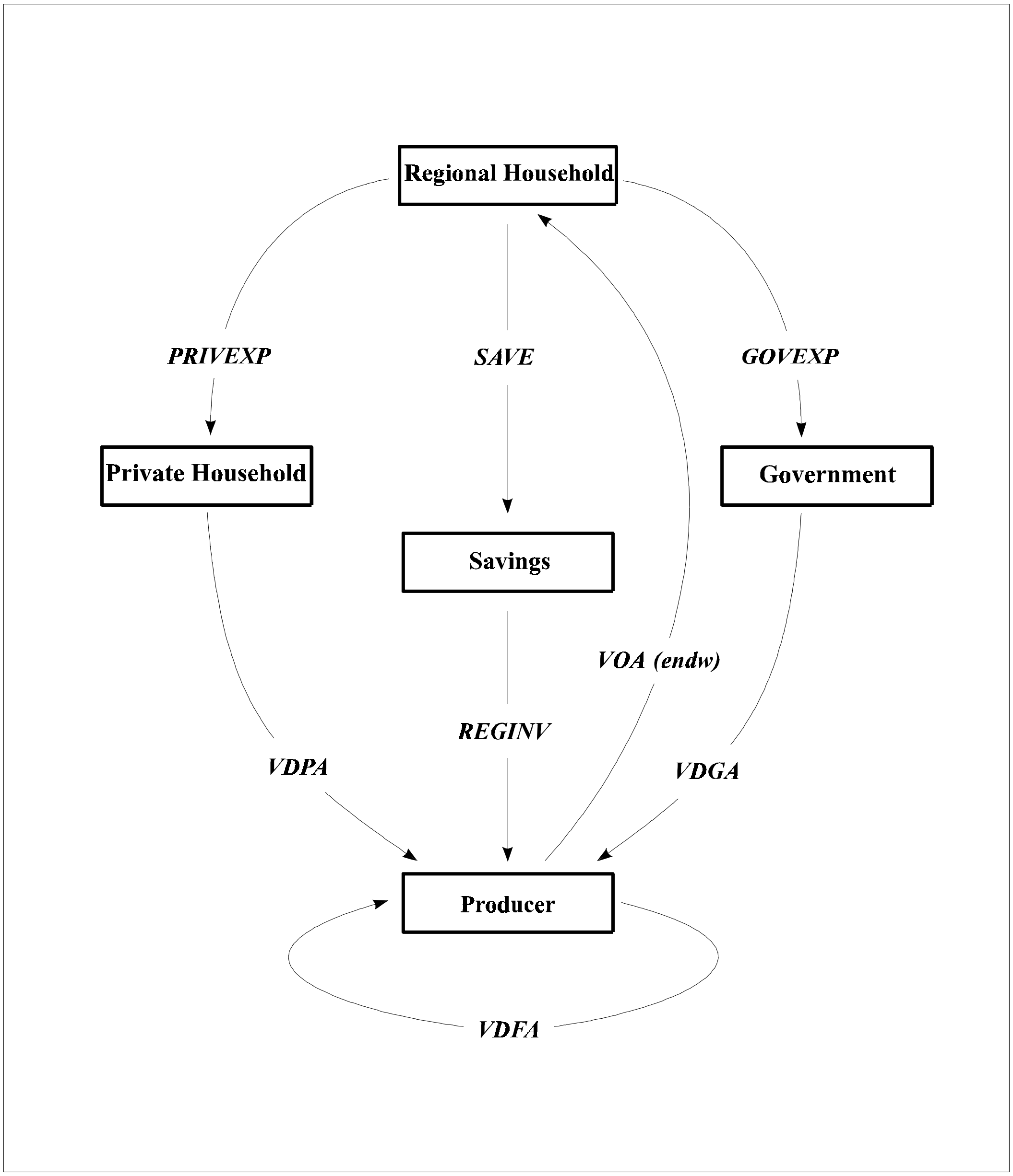


Figure 2.2. Multi Region Open Economy without Government Intervention

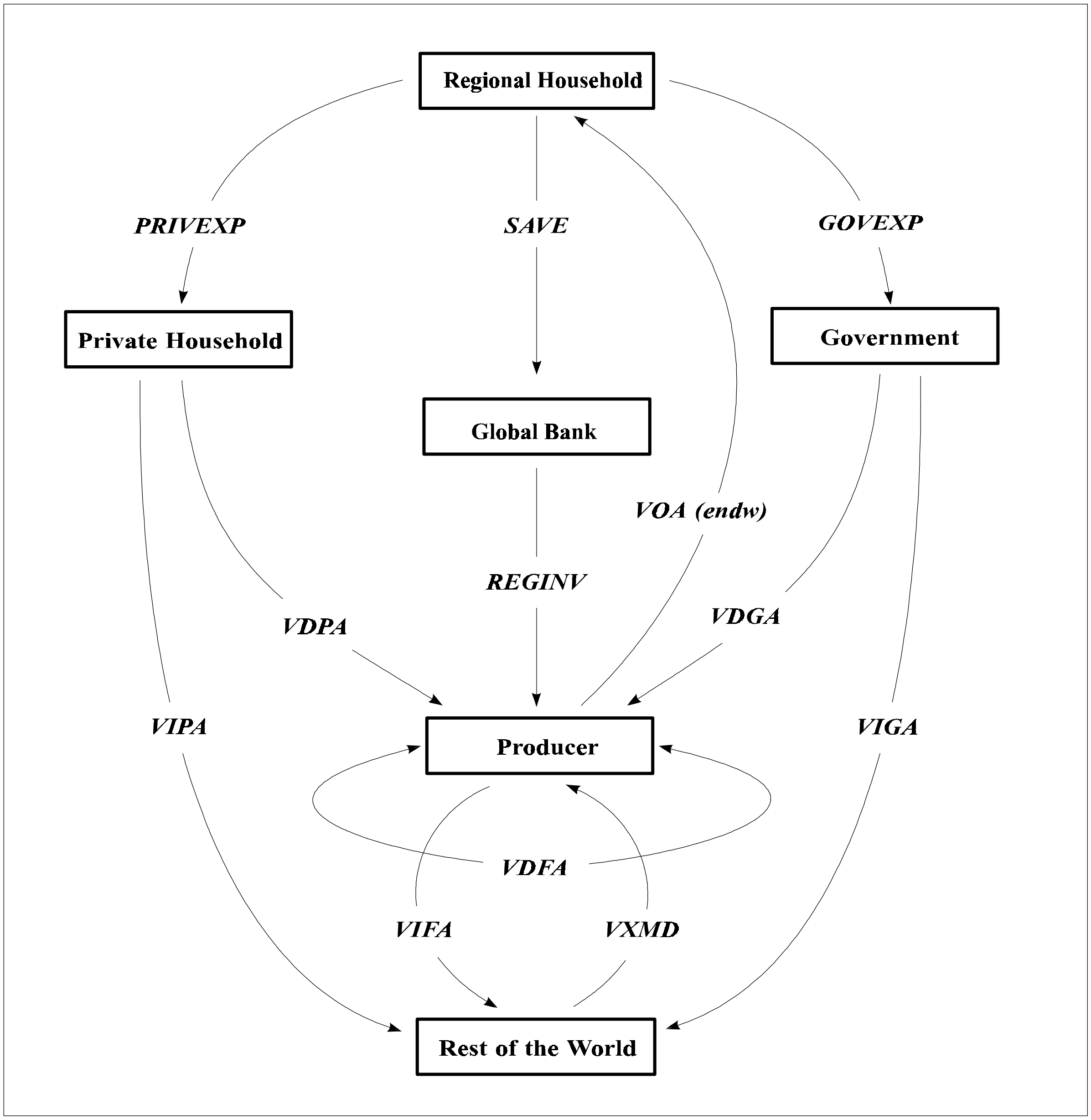
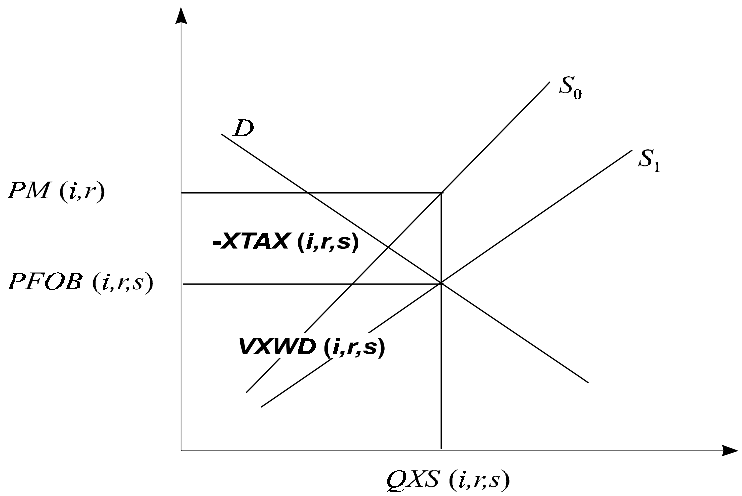


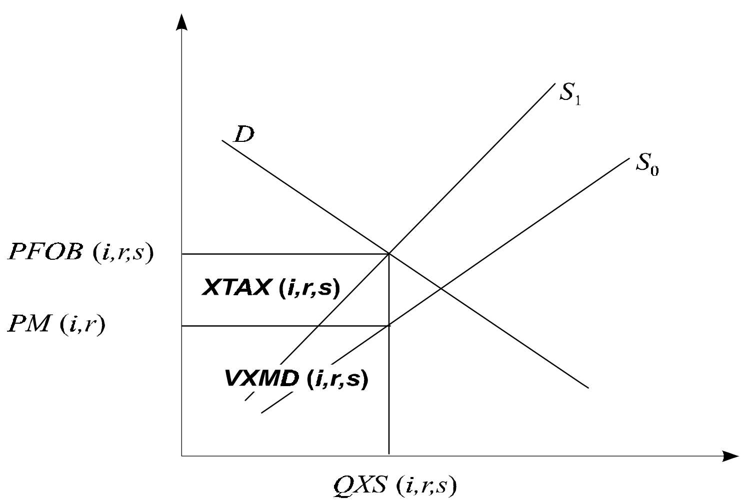
Figure 2.3. Export Subsidy or Tax in Region *r* on Sales to Region *s*

Export Subsidy



$$VXWD\ (i,r,s) = VXMD\ (i,r,s) - XTAX\ (i,r,s)$$

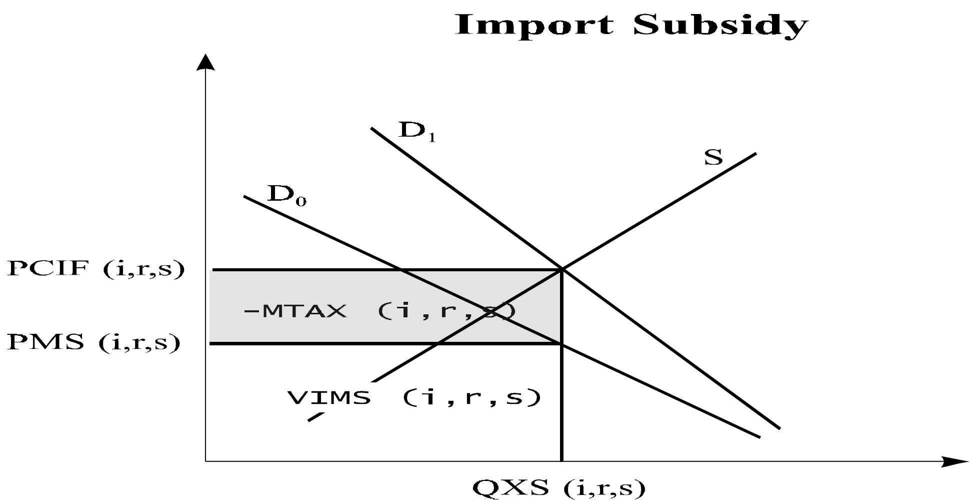
Export Tax



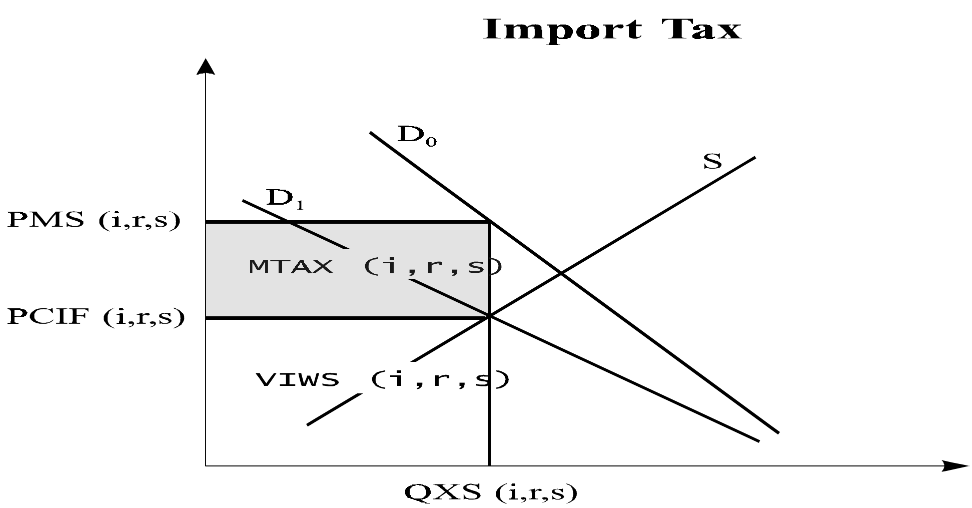
$$VXWD\ (i,r,s) = VXMD\ (i,r,s) + XTAX\ (i,r,s)$$

PM	Domestic price of commodity *i* in region *r*
PFOB	*FOB* price of commodity *i* supplied from region *r* to region *s*
QXS	Export of commodity *i* from region *r* to region *s*
VXWD	Exports of commodity *i* from region *r* to region *s*, valued at exporter's domestic price
VXWD	Exports of commodity *i* from region *r* to region *s*, valued at FOB price
XTAX	Tax revenues/Subsidy expenditures
D	Demand for imports of commodity *i* supplied from region *r* by region *s*
S_0	Pretax net supply of commodity *i* from region *r* in region *s*
S_1	Taxed net supply of commodity *i* from region *r* in region *s*,
	where: $QXS(i,r,s) = QO(i,r) - \sum_{k \neq s} QXS(i,r,k) - VST(i,r) = 1$ net supply of commodity *i* from *r* to *s*

Figure 2.4. Import Subsidy or Tax in Region *s* on Purchases from Region *r*



VIMS (i,r,s) = VIWS (i,r,s) - MTAX (i,r,s)



VIMS (i,r,s) = VIWS (i,r,s) + MTAX (i,r,s)

PMS	Importer's domestic price of commodity *i* supplied from region *r* to region *s*
PCIF	*CIF* price of commodity *i* supplied from region *r* to region *s*
QXS	Exports of commodity *i* from region *r* to region *s*
VIMS	Imports of commodity *i* from region *r* to region *s*, valued at importer's domestic price
VIWS	Imports of commodity *i* from region *r* to region *s*, valued at CIF price
MTAX	Tax revenues/Subsidy expenditures
D_0	Pretax demand for differentiated imports of commodity *i* from region *r* in regions
S_1	Net supply of commodity *i* from region *r* in region *s*
	where: $QXS(i,r,s) = QO(i,r) - \sum_{k \neq s} QXS(i,r,k) - VST(i,r) =$ 2 net supply of commodity *i* from *r* to *s*

Figure 2.5. Solving a Non-linear Model via its Linearized Representation

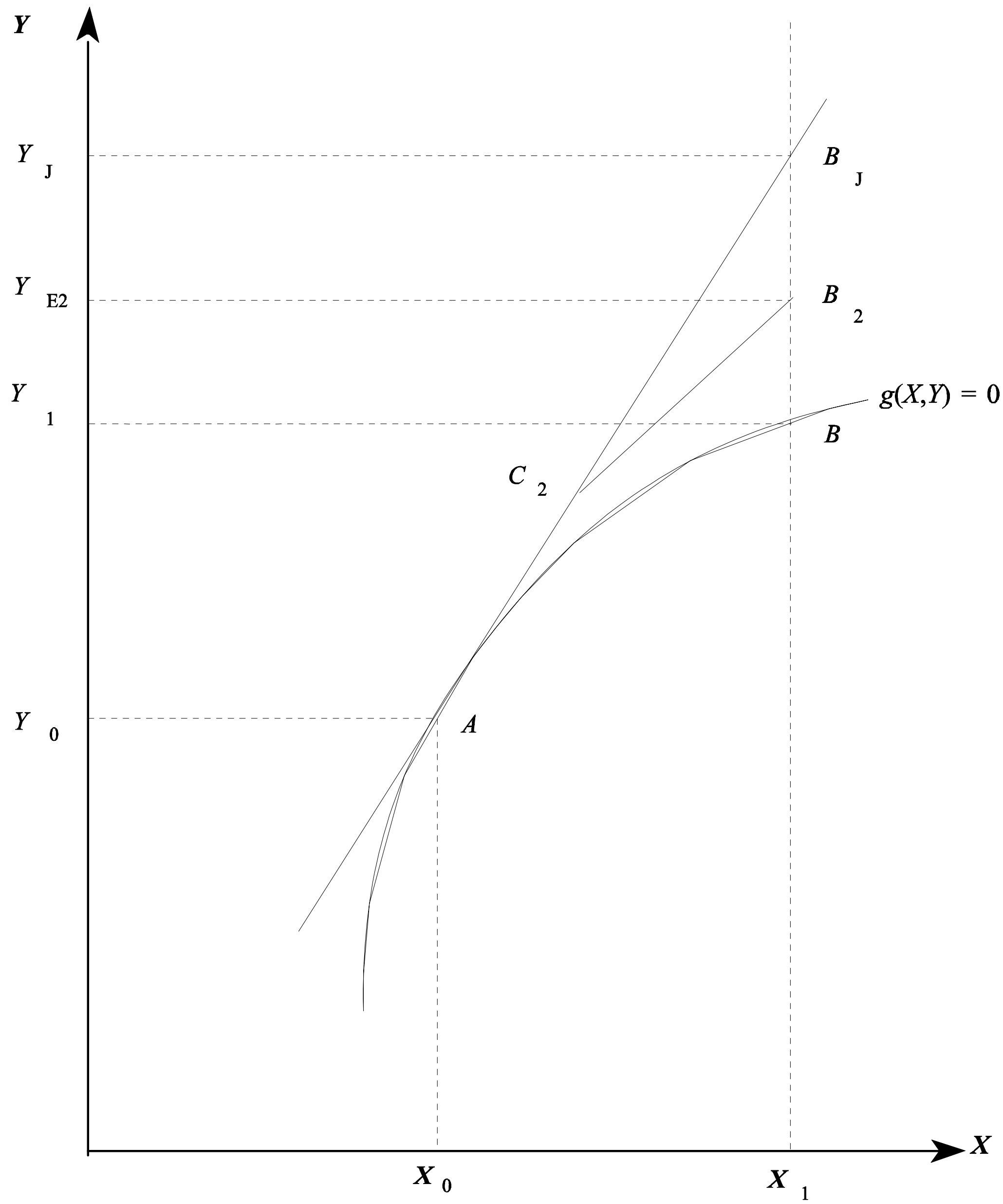


Figure 2.6. Production Structure

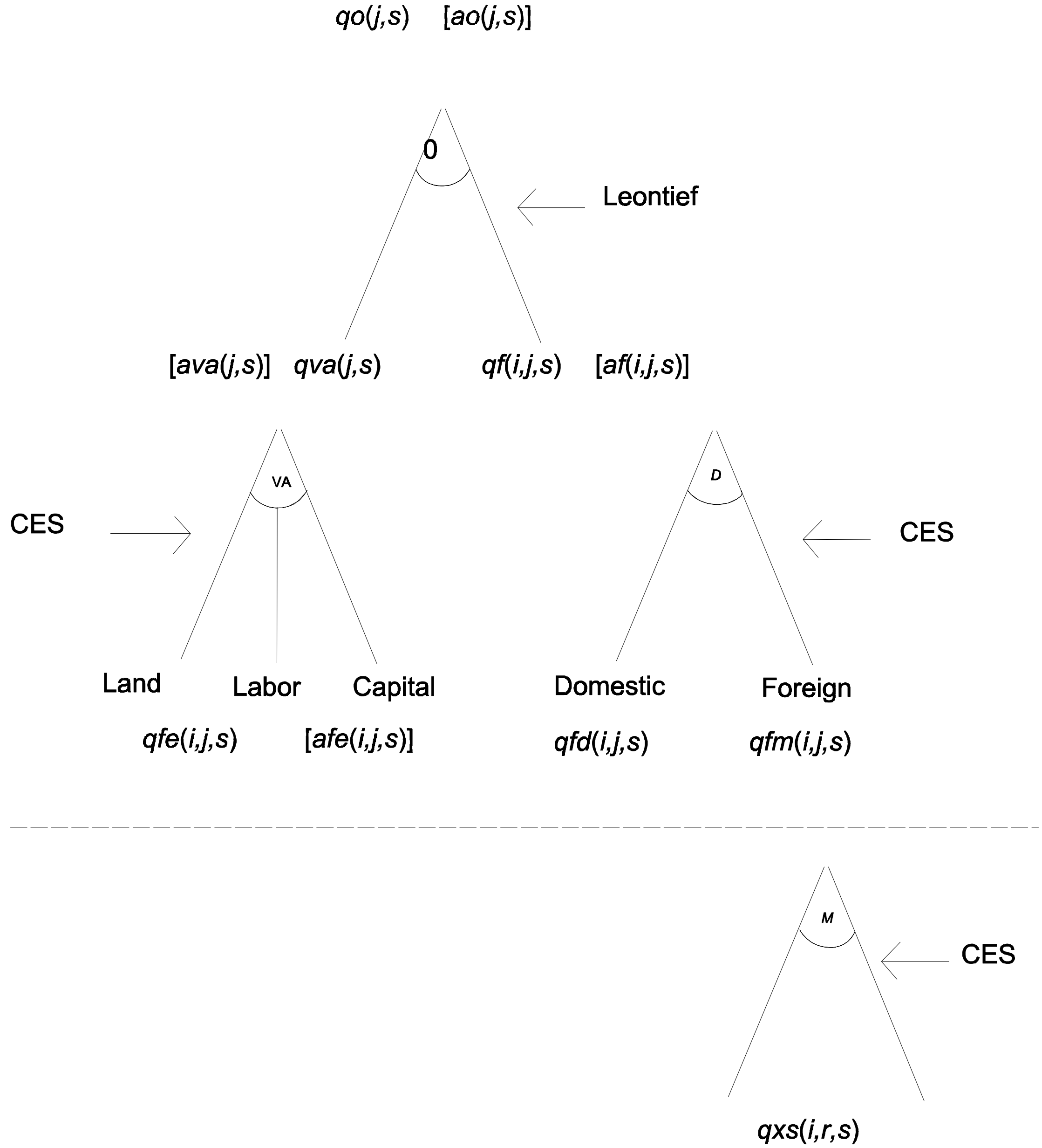


Table 2.1. ***Distribution of Sales to Regional Markets*** **(*i ε TRAD*)**

		$VOA(i,r)$	$: PS(i,r) * QO(i,r)$
		$\pm PTAX(i,r)$	
Domestic market *r*		$= VOM(i,r)$	$: PM(i,r) * QO(i,r)$
	$VDM(i,r)$		$VST(i,r)$
		$VXMD(i,r,s)$	$: PM(i,r) * QXS(i,r,s)$
		$\pm XTAXD(i,r,s)$	
		$= VXWD(i,r,s)$	$: PFOB(i,r,s) * QXS(i,r,s)$
World market		$\pm VTWR(i,r,s)$	
		$= VIWS(i,r,s)$	$: PCIF(i,r,s) * QXS(i,r,s)$
		$\pm MTAX(i,r,s)$	
		$= VIMS(i,r,s)$	$: PMS(i,r,s) * QXS(i,r,s)$
		$VIM(i,s)$	$: PIM(i,s) * QIM(i,s)$
Domestic market *s*		$= VIPM(i,s)$	$: PIM(i,s) * QPM(i,s)$
		$+ VIGM(i,s)$	$: PIM(i,s) * QGM(i,s)$
	$+ \sum_{j \varepsilon PROD}$	$VIFM(i,j,s)$	$: PIM(i,s) * QFM(i,j,s)$
Where:	$VDM(i,r)$	$= VDPM(i,r)$	$: PM(i,r) * QPD(i,r)$
		$+ VDGM(i,r)$	$: PM(i,r) * QGD(i,r)$
	$+ \sum_{j \varepsilon PROD\_COMM}$	$VDFM(i,j,r)$	$: PM(i,r) * QFD(i,j,r)$

Table 2.2. ***Sources of Household Purchases*** ($i \; \varepsilon \; TRAD$)

Private household

	$VPA(i,s)$		$: PP(i,s) * QP(i,s)$
$VDPA(i,s)$	$: PPD(i,s) * QPD(i,s)$	$VIPA(i,s)$	$: PPM(i,s) * QPM(i,s)$
$- DPTAX(i,s)$		$- IPTAX(i,s)$	
$= VDPM(i,s)$	$: PM(i,s) * QPD(i,s)$	$= VIPM(i,s)$	$: PIM(i,s) * QPM(i,s)$

Government household

	$VGA(i,s)$		$: PG(i,s) * QG(i,s)$
$VDGA(i,s)$	$: PGD(i,s) * QGD(i,s)$	$VIGA(i,s)$	$: PGM(i,s) * QGM(i,s)$
$- DGTAX(i,s)$		$- IGTAX(i,s)$	
$= VDGM(i,s)$	$: PM(i,s) * QGD(i,s)$	$= VIGM(i,s)$	$: PIM(i,s) * QGM(i,s)$ 3

Table 2.3. ***Sources of Firms' Purchases*** **(*j ε PROD*)**

***i ε TRAD*: Intermediate inputs**

	$VFA(i,j,s)$		$: PF(i,j,s) * QF(i,j,s)$
$VDFA(i,j,s)$	$: PFD(i,j,s) * QFD(i,j,s)$	$VIFA(i,j,s)$	$: PFM(i,j,s) * QFM(i,j,s)$
$-DFTAX(i,j,s)$		$-IFTAX(i,j,s)$	
$= VDFM(i,j,s)$	$: PM(i,s) * QFD(i,j,s)$	$= VIFM(i,j,s)$	$: PIM(i,s) * QFM(i,j,s)$

i ε ENDW: Primary factor services

$VFA(i,j,s)$ 4	$: PFE(i,j,s) * QFE(i,j,s)$ 5
$-ETAX(i,j,s)$ 6	
$= VFM(i,j,s)$ 7	$: PM(i,s) * QFE(i,j,s)$ 8

Zero pure profits

$$VOA(j,s) = \sum_{i \varepsilon TRAD} VFA(i,j,s) + \sum_{i \varepsilon ENDW} VFA(i,j,s) \quad 9$$

Table 2.4. ***Sources of Household Factor Service Income***

$i \; \varepsilon \; ENDWM$: Mobile endowments

$\sum_{j \varepsilon PROD} VFM(i,j,s)$	$= VOM(i,s)$	$: PM(i,s) \; * \; QO(i,s)$
	$- HTAX(i,s)$	
	$= VOA(i,s)$	$: PS(i,s) \; * \; QO(i,s)$

$i \; \varepsilon \; ENDWS$: Sluggish endowments

	$VFM(i,j,s)$	$: PMES(i,j,s) \; * \; QOES(i,j,s)$
	$VOM(i,s)$	$: PM(i,s) \; * \; QO(i,s)$
	$- HTAX(i,s)$	
	$= VOA(i,s)$	$: PS(i,s) \; * \; QO(i,s)$

Table 2.5. ***Disposition and Sources of Regional Income***

$$\text{EXPENDITURE}(r) = \sum_{i \varepsilon TRAD} [VPA(i,r) + VGA(i,r)] + SAVE(r) =$$

$$
\begin{aligned}
\text{INCOME}(r) \quad = \quad & \sum_{i \varepsilon ENDW} VOA(i,r) - VDEP(r) \\
+ \quad & \sum_{i \varepsilon NSAV} VOM(i,r) - VOA(i,r) \\
+ \quad & \sum_{j \varepsilon PROD} \sum_{i \varepsilon ENDW} VFA(i,j,r) - VFM(i,j,r) \\
+ \quad & \sum_{i \varepsilon TRAD} VIPA(i,r) - VIPM(i,r) \\
+ \quad & \sum_{i \varepsilon TRAD} VDPA(i,r) - VDPM(i,r) \\
+ \quad & \sum_{i \varepsilon TRAD} VIGA(i,r) - VIGM(i,r) \\
+ \quad & \sum_{i \varepsilon TRAD} VDGA(i,r) - VDGM(i,r) \\
+ \quad & \sum_{j \varepsilon PROD} \sum_{i \varepsilon TRAD} VIFA(i,j,r) - VIFM(i,j,r) \\
+ \quad & \sum_{j \varepsilon PROD} \sum_{i \varepsilon TRAD} VDFA(i,j,r) - VDFM(i,j,r) \\
+ \quad & \sum_{i \varepsilon TRAD} \sum_{s \varepsilon REG} VXWD(i,r,s) - VXMD(i,r,s) \\
+ \quad & \sum_{i \varepsilon TRAD} \sum_{s \varepsilon REG} VIMS(i,s,r) - VIWS(i,s,r)
\end{aligned}
$$

Table 2.6. ***The International Transport Sector***

	VT	$: PT * QT$
$= \sum_{i \varepsilon TRAD} \sum_{r \varepsilon REG} \sum_{s \varepsilon REG}$	$VTWR(i,r,s)$	$: PT * QS(i,r\ s)$
$= \sum_{i \varepsilon TRAD} \sum_{r \varepsilon REG}$	$VST(i,r)$	$: PM(i,r) * QST(i,r)$

Table 2.7. ***Demand for Regional Investment Goods***

$\sum_{\boldsymbol{r \varepsilon REG}} [REGINV(r)$	$: PCGDS(r) * QCGDS(r)$
$- \underline{VDEP(r)]}$	$: PCGDS(r) * KB(r)$
$= GLOBINV$	$: PSAVE * GLOBALCGDS$
$= \sum_{r \varepsilon REG} SAVE(r)$	$: PSAVE * QSAVE(r)$

Capital Stocks

$VKB(r)$	$: PCGDS(r) * KB(r)$
$+ REGINV(r)$	$: PCGDS(r) * QCGDS(r)$
$- \underline{VDEP(r)}$	$: PCGDS(r) * DEPR(r)\ KB(r)$
$= VKE(r)$	

Table 2.8. ***"Accounting" Relationships in the Model***

(1) $$VOM(i,r) * qo(i,r) = \qquad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG$$

$$VDM(i,r) * qds(i,r) + VST(i,r) * qst(i,r) + \sum_{s\varepsilon REG} VXMD(i,r,s) * qxs(i,r,s) + VOM(i,r) * tradslack(i,r)$$

(2) $$VIM(i,r) * qim(i,r) = \qquad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG$$

$$\sum_{j\varepsilon PROD} VIFM(i,j,r) * qfm(i,j,r) + VIPM(i,r) * qpm(i,r) + VIGM(i,r) * qgm(i,r)$$

(3) $$VDM(i,r) * qds(i,r) = \qquad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG$$

$$\sum_{j\varepsilon PROD} VDFM(i,j,r) * qfd(i,j,r,) + VDPM(i,r) * qpd(i,r) + VDGM(i,r) * qgd(i,r)$$

(4) $$VOM(i,r) * qo(i,r) = \qquad \forall i \varepsilon ENDWM \quad \forall r \varepsilon REG$$

$$\sum_{j\varepsilon PROD} VFM(i,j,r) * qfe(i,j,r,) + VOM(i,r) * endwslack(i,r)$$

(5) $$qoes(i,j,r) = qfe(i,j,r) \qquad \forall i \varepsilon ENDWS \quad \forall j \varepsilon PROD \quad \forall r \varepsilon REG$$

(6) $$VOA(j,r) * ps(j,r) = \qquad \forall i \varepsilon PROD \quad \forall r \varepsilon REG$$

$$\sum_{i\varepsilon ENDW} VFA(i,j,r) * pfe(i,j,r,) + \sum_{I\varepsilon TRAD} VFA(i,j,r) * pf(i,j,r) + VOA(j,r) * profitslack(j,r)$$

(7) $$VT * pt = \sum_{i\varepsilon TRAD\_COMM} \sum_{r\varepsilon REG} VST(i,r) * pm(i,r)$$

(8) $$PRIVEXP(r) * yp(r) = \qquad \forall r \varepsilon REG$$

$$\sum$$

Table 2.8. ***(Cont) "Accounting" Relationships in the Model***

(9) $$INCOME(r) * y(r) = 1 \qquad \forall r \varepsilon REG$$

$$\sum_{i\varepsilon ENDW} VOA(i,r)[ps(i,r)+qo(i,r)] - VDEP(r)*[pcgds(r)+kb(r)]$$
$$+ \sum_{i\varepsilon NSAV} VOM(i,r)*[pm(i,r)+qo(i,r)] - VOA(i,r)*[ps(i,r) + qo(i,r)]$$
$$+ \sum_{i\varepsilon ENDWM} \sum_{j\varepsilon PROD} VFA(i,j,r)[pfe(i,j,r) + qfe(i,j,r)] - VFM(i,j,r)*[pm(i,r)+qfe(i,j,r)]$$
$$+ \sum_{j\varepsilon ENDWS} \sum_{i\varepsilon PROD} VFA(i,j,r)[pfe(i,j,r)+qfe(i,j,r)] - VFM(i,j,r)*[pmes(i,j,r)+qfe(i,j,r)]$$
$$+ \sum_{j\varepsilon PROD} \sum_{i\varepsilon TRAD} VIFA(i,j,r)*[pfm(i,j,r)+qfm(i,j,r)] - VIFM(i,j,r)*[pim(i,r)+qfm(i,j,r)]$$
$$+ \sum_{j\varepsilon PROD} \sum_{i\varepsilon TRAD} VDFA(i,j,r)*[pfd(i,j,r,)+qfd(i,j,r)] - VDFM(i,j,r)*[pm(i,r)+qfd(i,j,r)]$$
$$+ \sum_{i\varepsilon TRAD} VIPA(i,r)*[ppm(i,r)+qpm(i,r)] - VIPM(i,r)*[pim(i,r)+qpm(i,r)]$$
$$+ \sum_{i\varepsilon TRAD} VDPA(i,r)*[ppd(i,r)+qpd(i,r)] - VDPM(i,r)*[pm(i,r)+qpd(i,r)]$$
$$+ \sum_{i\varepsilon TRAD} VIGA(i,r)*[pgm(i,r)+qgm(i,r)] - VIGM(i,r)*[pim(i,r)+qgm(i,r)]$$
$$+ \sum_{i\varepsilon TRAD} VDGA(i,r)*[pgd(i,r)+qgd(i,r)] - VDGM(i,r)*[pm(i,r)+qgd(i,r)]$$
$$+ \sum_{i\varepsilon TRAD} \sum_{s\varepsilon REG} VXWD(i,r,s)*[pfob(i,r,s)+qxs(i,r,s)] - VXMD(i,r,s)*[pm(i,r)+qxs(i,r,s)] \qquad 2$$
$$+ \sum_{i\varepsilon TRAD} \sum_{s\varepsilon REG} VIMS(i,s,r)*[pms(i,s,r)+qxs(i,s,r)] - VIWS(i,s,r)*[pcif(i,s,r)+qxs(i,s,r)]$$
$$+ INCOME(r)*incomeslack(r)$$

(10) $$ke(r) = INVKERATIO(r) * qcgds(r) + [1.0 - INVKERATIO(r)] * kb(r) \, 3 \, \forall r\varepsilon REG \; 4$$

(11)
$$globalcgds = \sum_{r\varepsilon REG} [REGINV(r) / GLOBINV] * qcgds(r) - [VDEP(r) / GLOBINV(r)] * kb(r) \; 5$$

(12) $$walras\_sup = globalcgds \; 6$$

(13) $$GLOBINV * walras\_dem = \sum_{r\varepsilon REG} SAVE(r) * qsave(r)$$

(14) $$walras\_sup = walras\_dem + walraslack$$

Table 2.9. ***Price Linkage Equations***

(15)	$ps(i,r) = to(i,r) + pm(i,r)$	$\forall i \varepsilon NSAVE$ $\forall r \varepsilon REG$
(16)	$pfe(i,j,r) = tf(i,j,r) + pm(i,r)$	$\forall i \varepsilon ENDWM$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$
(17)	$pfe(i,j,r) = tf(i,j,r) + pmes(i,j,r)$	$\forall i \varepsilon ENDWS$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$
(18)	$ppd(i,r) = tpd(i,r) + pm(i,r)$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$
(19)	$pgd(i,r) = tgd(i,r) + pm(i,r)$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$
(20)	$pfd(i,j,r) = tfd(i,j,r) + pm(i,r)$	$\forall i \varepsilon TRAD$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$
(21)	$ppm(i,r) = tpm(i,r) + pim(i,r)$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$
(22)	$pgm(i,r) = tgm(i,r) + pim(i,r)$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$
(23)	$pfm(i,j,r) = tfm(i,j,r) + pim(i,r)$	$\forall i \varepsilon TRAD$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$
(24)	$pms(i,r,s) = tm(i,s) + tms(i,r,s) + pcif(i,r,s)$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$ $\forall s \varepsilon REG$
(25)	$pr(i,s) = pm(i,s) - pim(i,s)$	$\forall i \varepsilon TRAD$ $\forall s \varepsilon REG$

Table 2.9. ***Price Linkage Equations (Continued)***

(26)	$pcif(i,r,s) = FOBSHR(i,r,s) * pfob(i,r,s) + TRNSHR(i,r,s) * pt$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$ $\forall s \varepsilon REG$
(27)	$pfob(i,r,s) = pm(i,r) - tx(i,r) - txs(i,r,s)$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$ $\forall s \varepsilon REG$

Table 2.10. ***Composite Imports Nest***

(28)	$pim(i,s) = \sum_{k \varepsilon REG} MSHRS(i,k,s) * pms(i,k,s)$	$\forall i \varepsilon TRAD$ $\forall s \varepsilon REG$
(29)	$qxs(i,r,s) = qim(i,s) - \sigma_M(i) * [pms(i,r,s) - pim(i,s)]$	$\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$ $\forall s \varepsilon REG$

Table 2.11. ***Behavioral Equations for Producers***

Composite intermediates nest:

(30) $pf(i,j,r) = FMSHR(i,j,r) * pfm(i,j,r) + [1 - FMSHR(i,j,r)] * pfd(i,j,r)$ $\forall i \varepsilon TRAD$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$

(31) $qfm(i,j,s) = qf(i,j,s) - \sigma_D(i) * [pfm(i,j,s) - pf(i,j,s)]$ $\forall i \varepsilon TRAD$ $\forall j \varepsilon PROD$ $\forall s \varepsilon REG$

(32) $qfd(i,j,s) = qf(i,j,s) - \sigma_D(i) * [pfd(i,j,s) - pf(i,j,s)]$ $\forall i \varepsilon TRAD$ $\forall j \varepsilon PROD$ $\forall s \varepsilon REG$

Value-added nest:

(33) $pva(j,r) = \sum_{k \varepsilon ENDW} SVA(k,j,r) * [pfe(k,j,r) - afe\ k,j,r)]$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$

(34) $qfe(i,j,r) + afe(i,j,r) = qva(j,r) - \sigma_{VA}(j) * [pfe(i,j,r) - afe(i,j,r) - pva(j,r)]$ $\forall i \varepsilon ENDW$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$

Total output nest:

(35) $qva(j,r) + ava(j,r) = qo(j,r) - ao(j,r)$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$

(36) $qf(i,j,r) + af(i,j,r) = qo(j,r) - ao(j,r)$ $\forall i \varepsilon TRAD$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$

Zero profits (revised):

(6´) $VOA(j,r) * [ps(j,r) + ao(j,r)] =$

$$\sum_{i \varepsilon ENDW\_COMM} VFA(i,j,r) * [pfe(i,j,r) - afe(i,j,r) - ava(j,r)]$$ $\forall j \varepsilon PROD$ $\forall r \varepsilon REG$

$$+ \sum_{i \varepsilon TRAD\_COMM} VFA(i,j,r) * [pf(i,j,r) - af(i,j,r)] + VOA(j,r) * profitslack(j,r)$$

Table 2.12. ***Household Behavior***

Aggregate utility

(37) $$INCOME(r) * u(r) = PRIVEXP(r) * up(r) + GOVEXP(r) * [ug(r) - pop(r)] + SAVE(r) * [qsave(r) - pop(r)] \quad \forall r \varepsilon REG$$

Regional savings:

(38) $$qsave(r) = y(r) - psave + saveslack(r) \quad \forall r \varepsilon REG$$

Government purchases:

(39) $$ug(r) = y(r) - pgov(r) + govslack(r) \quad \forall r \varepsilon REG$$

Demand for composite goods:

(40) $$pgov(r) = \sum_{i \varepsilon TRAD\_COMM} (VGA(i,r) / GOVEXP(r)) * pg(i,r) \quad \forall r \varepsilon REG$$

(41) $$qg(i,r) = ug(r) - [pg(i,r) - pgov(r)] \quad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG$$

Composite tradeables:

(42) $$pg(i,s) = GMSHR(i,s) * pgm(i,s) + [1 - GMSHR(i,s)] * pgd(i,s) \quad \forall i \varepsilon TRAD \quad \forall s \varepsilon REG$$

(43) $$qgm(i,s) = qg(i,s) + \sigma_D(i) * [pg(i,s) - pgm(i,s)] \quad \forall i \varepsilon TRAD \quad \forall s \varepsilon REG$$

(44) $$qgd(i,s) = qg(i,s) + \sigma_D(i) * [pg(i,s) - pgd(i,s)] \quad \forall i \varepsilon TRAD \quad \forall s \varepsilon REG$$

Table 2.12. ***(Cont) Household Behavior***

Private Household demands:

(45) $$yp(r) = \sum_{i \varepsilon TRAD} [CONSHR(i,r) * pp(i,r)] + \sum_{i \varepsilon TRAD} [CONSHR(i,r) * INCPAR(i,r)] * up(r) + pop(r)$$ $\forall r \varepsilon REG$

Composite demands:

(46) $$qp(i,r) = \sum_{k \varepsilon TRAD} EP(i,k,r) * pp(k,r) + EY(i,r) * [yp(r) - pop(r)] + pop(r)$$ $\forall i \varepsilon TRAD$ $\forall r \varepsilon REG$

Composite tradeables:

(47) $$pp(i,s) = PMSHR,(i,s) * ppm(i,s) + [1 - PMSHR(i,s)] * ppd(i,s)$$ $\forall i \varepsilon TRAD$ $\forall s \varepsilon REG$

(48) $$qpd(i,s) = qp(i,s) + \sigma_D(i,) * [pp(i,s) - ppd(i,s)]$$ $\forall i \varepsilon TRAD$ $\forall s \varepsilon REG$

(49) $$qpm(i,s) = qp(i,s) + \sigma_D(i) * [pp(i,s) - ppm(i,s)]$$ $\forall i \varepsilon TRAD$ $\forall s \varepsilon REG$

Table 2.13. ***Formulas for Private Households' Elasticities of Demand in the Presence of CDE Preferences***

(F1) $\alpha(i,r) = [1 - \beta(i,r)]$ $\quad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG$

(F2) $APE(i,k,r) = \alpha(i,r) + \alpha(k,r) - \sum_{m \varepsilon TRAD} [CONSHR(m,r) * \alpha(m,r)]$ $\quad \forall i \neq k \varepsilon TRAD \quad \forall r \varepsilon REG$

(F3)
$$APE(i,i,r) = 2.0 * \alpha(i,r) - \sum_{m \varepsilon TRAD} [CONSHR(m,r) * \alpha(m,r)] - \alpha(i,r) / CONSHR(i,r)$$
$\forall i \varepsilon TRAD \quad \forall r \varepsilon REG$

(F4)
$$EY(i,r) = [\sum_{m \varepsilon TRAD} CONSHR(m,r) * \gamma(m,r)]^{-1} * \gamma(m,r) * [1.0 - \alpha(i,r)] + \sum_{m \varepsilon TRAD} CONSHR(m,r) * \gamma(m,r) * \alpha(m,r) + \{\alpha(i,r) - \sum_{m \varepsilon TRAD} [CONSHR(m,r) * \alpha(m,r)]\}$$
$\forall i \varepsilon TRAD \quad \forall r \varepsilon REG$

(F5) $EP(i,k,r) = [APE(i,k,r) - EY(i,r)] * CONSHR(k,r)$ $\quad \forall i \varepsilon TRAD \quad \forall k \varepsilon TRAD \quad \forall r \varepsilon REG$

Table 2.14. ***Supply of Sluggish Endowments***

(50) $pm(i,r) = \sum_{k \varepsilon PROD\_COMM} REVSHR(i,k,r) * pmes(i,k,r)$ $\quad \forall i \varepsilon ENDWS \quad \forall r \varepsilon REG$

(51) $qoes(i,j,r) = qo(i,r) - endwslack(i,r) + \sigma_T(i) * [pm(i,r) - pmes(i,j,r)]$ $\quad \forall i \varepsilon ENDWS \quad \forall j \varepsilon PROD \quad \forall r \varepsilon REG$

Table 2.15. ***Investment Equations***

Equations of notational convenience

$$(52)\ ksvces(r) = \sum_{h\varepsilon ENDWC} [VOA(h,r) / \sum_{k\varepsilon ENDWC} VOA(k,r)] * qo(h,r) \qquad \forall r\varepsilon REG$$

$$(53)\ rental(r) = \sum_{h\varepsilon ENDWC} [VOA(h,r) / \sum_{k\varepsilon ENDWC} VOA(k,r)] * ps(h,r) \qquad \forall r\varepsilon REG$$

$$(54)\ qcgds(r) = \sum_{h\varepsilon CGDS} [VOA(h,r) / REGINV(r)] * qo(h,r) \qquad \forall r\varepsilon REG$$

$$(55)\ pcgds(r) = \sum_{h\varepsilon CGDS} [VOA(h,r) / REGINV(r)] * ps(h,r) \qquad \forall r\varepsilon REG$$

$$(56)\ kb(r) = ksvces(r) \qquad \forall r\varepsilon REG$$

Rate of return equations

$$(57)\ rorc(r) = GRNETRATIO(r) * [rental(r) - pcgds(r)] \qquad \forall r\varepsilon REG$$

$$(58)\ rore(r) = rorc(r) - RORFLEX(r) * [ke(r) - kb(r)] \qquad \forall r\varepsilon REG$$

$$\begin{aligned}(11)\quad & RORDELTA * rore(r) + (1 - RORDELTA) \\ & * \{[REGINV(r)/NETINV(r)] * qcgds(r) - [VDEP(r)/NETINV(r)] * kb(r)\} \\ & = RORDELTA * rorg + (1 - RORDELTA) * globalcgds + cgdslack(r)\end{aligned} \qquad \forall r\varepsilon REG$$

$$\begin{aligned}(59)\quad & RORDELTA * globalcgds + (1 - RORDELTA) * rorg = \\ & RORDELTA * \sum_{r\varepsilon REG} \{[REGINV(r) / GLOBINV] * qcgds(r) - [VDEP(r)/GLOBINV] * kb(r)\} \\ & + (1 - RORDELTA) * \sum_{r\varepsilon REG} [NETINV(r) / GLOBINV] * rore(r)\end{aligned}$$

Price of Savings

$$(60)\ psave = \sum_{r\varepsilon REG} NETINV(r) / GLOBINV * pcgds(r)$$

Table 2.16. ***Regional Allocation of Investment Under Alternative Closures***

Rate-of-return component:

$$rore(r) = rorg$$

$$globalcgds = \sum_{r \varepsilon REG} \{[REGINV(r) \ / \ GLOBINV] * qcgds(r) - [VDEP(r) / GLOBINV] * kb(r)\}$$

Alternative component:

$$globalcgds = [REGINV(r) \ / \ NETINV(r)] * qcgds(r) - [VDEP(r) / NETINV(r)] * kb(r)$$

$$rorg = \sum_{r \varepsilon REG} [NETINV(r) \ / \ GLOBINV] * rore(r)$$

Table 2.17. ***The Global Shipping Industry***

(7□) $VT * pt = \sum_{i \varepsilon TRAD} \sum_{r \varepsilon REG} VST(i,r) * pm(i,r)$

(61) $qst(i,r) = qt + [pt - pm(i,r)]$ $\quad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG$

(62) $VT * qt = \sum_{i \varepsilon TRAD} \sum_{r \varepsilon REG} \sum_{s \varepsilon REG} VTWR(i,r,s) * [qxs(i,r,s) - atr(i,r,s)]$

(26□) $pcif(i,r,s) = FOBSHR(i,r,s) * pfob(i,r,s) + TRNSHR(i,r,s) * [pt - atr(i,r,s)]$ $\quad \forall i \varepsilon TRAD \quad \forall r \varepsilon REG \quad \forall s \varepsilon REG$

Table 2.18. ***Summary Indices***

(64) $$VWLDSALES(r) * psw(r) = \sum_{i \varepsilon TRAD} \sum_{s \varepsilon REG} VXWD(i,r,s) * pfob(i,r,s) + VST(i,r) * pm(i,r) + [REGINV(r) - VDEP(r)] * pcgds(r) \quad \forall r \varepsilon REG$$

(65) $$VWLDSALES(r) * pdw(r) = \sum_{i \varepsilon TRAD} \sum_{k \varepsilon REG} VIWS(i,k,r) * pcif(i,k,r) + SAVE(r) * psave \quad \forall r \varepsilon REG$$

(66) $$tot(r) = psw(r) - pdw(r) \quad \forall r \in REG$$

(67) $$EV(r) - [INC(r)/100] * [URATIO(r) * POPRATIO(r)] * [u(r) + pop(r)] = 0 \quad \forall r \in REG$$

(68) $$WEV - \sum_{r \in REG} EV(r) = 0 \quad \forall r \in REG$$

(69) $$PRIVEXP(r) * ppriv(r) = \sum_{i \in TRAD} VDA(i,r) * pp(i,r) \quad \forall r \in REG$$

(70) $$\begin{aligned} GDP(r) * vgdp(r) = & \sum_{i \in TRAD} VGA(i,r) * [pg(i,r) + qg(i,r)] \\ & + \sum_{i \in TRAD} VPA(i,r) * [pp(i,r) + qp(i,r)] + REGINV(r) * [pcgds(r) + qcgds(r)] \\ & + \sum_{i \in TRAD} \sum_{s \in REG} VXWD(i,r,s) * [pfob(i,r,s) + qxs(i,r,s)] + \sum_{i \in TRAD} VST(i,r) * [pm(i,r) + qst(i,r)] \\ & - \sum_{i \in TRAD} \sum_{r \in REG} VIWS(i,r,s) * [pcif(i,r,s) + qxs(i,r,s)] \end{aligned} \quad \forall r \in REG$$

(71) $$\begin{aligned} GDP(r) * pgdp(r) = & \sum_{i \in TRAD} VGA(i,r) * pg(i,r) \\ & + \sum_{i \in TRAD} VPA(i,r) * pp(i,r) + REGINV(r) * pcgds(r) \\ & + \sum_{i \in TRAD} \sum_{s \in REG} VXWD(i,r,s) * pfob(i,r,s) + \sum_{i \in TRAD} VST(i,r) * pm(i,r) \\ & - \sum_{i \in TRAD} \sum_{r \in REG} VIWS(i,r,s) * pcif(i,r,s) \end{aligned} \quad \forall r \in REG$$

(72) $$qgdp(r) = vgdp(r) - pgdp(r) \quad \forall r \in REG$$

(73) $$VXW(i,r) * vxwfob(i,r) = \sum_{s \in REG} VXWD(i,r,s) * [qxs(i,r,s) + pfob(i,r,s)] + VST(i,r) * [qst(i,r) + pm(i,r)] \quad \forall i \in TRAD, \ \forall r \in REG$$

Table 2.18. ***(cont.) Summary Indices***

(74) $VIW(i,s)* viwcif(i,s) = \sum_{r \in REG} VIWS(i,r,s)* [pcif(i,r,s) + qxs(i,r,s)]$ $\quad \forall i \in TRAD \quad \forall s \in REG$

(75) $VXWREGION(r)* vxwreg(r) = \sum_{i \in TRAD} VXW(i,r)* vxwfob(i,r)$ $\quad \forall r \in REG$

(76) $VIWREGION(s)* viwreg(s) = \sum_{i \in TRAD} VIW(i,s)* viwcif(i,s)$ $\quad \forall s \in REG$

(77) $VXWCOMMOD(i)* vxwcom(i) = \sum_{r \in REG} VXW(i,r)* vxwfob(i,r)$ $\quad \forall i \in TRAD$

(78) $VIWCOMMOD(i)* viwcom(i) = \sum_{s \in REG} viw(i,s)* viwcif(i,s)$ $\quad \forall i \in TRAD$

(79) $VXWLD * vxwwld = \sum_{r \in REG} VXWREGION(r)* vxwreg(r)$

(80) $VWOW(i)* valuew(i) = \sum_{r \in REG} VOW(i,r)* [pxw(i,r) + qo(i,r)]$ $\quad \forall i \in TRAD$

(81) $VXW(i,r)* pxw(i,r) = \sum_{s \in REG} VXWD(i,r,s)* pfob(i,r,s) + VST(i,r)* pm(i,r)$ $\quad \forall i \in TRAD \quad \forall r \in REG$

(82) $VIW(i,s)* piw(i,s) = \sum_{r \in reg} VIWS(i,r,s)* pcif(i,r,s)$ $\quad \forall i \in TRAD \quad \forall r \in REG$

(83) $VXWREGION(r)* pxwreg(r) = \sum_{i \in TRAD} VXW(i,r)* pxw(i,r)$ $\quad \forall r \in REG$

(84) $VIWREGION(s)* piwreg(s) = \sum_{i \in TRAD} VIW(i,s)* piw(i,s)$ $\quad \forall s \in REG$

(85) $VXWCOMMOD(i)* pxwcom(i) = \sum_{r \in REG} VXW(i,r)* pxw(i,r)$ $\quad \forall i \in TRAD$

(86) $VIWCOMMOD(i)* piwcom(i) = \sum_{s \in REG} VIW(i,s)* piw(i,s)$ $\quad \forall i \in TRAD$

(87) $VXWLD * pxwwld = \sum_{r \in REG} VXWREGION(r)* pxwreg(r)$

(88) $VWOW(i)* pw(i) = \sum_{r \in REG} VOW(i,r)* pxw(i,r)$ $\quad \forall i \in TRAD$

Table 2.18. ***(cont.) Summary Indices***

(89) $qxw(i,r) = vxwfob(i,r) - pxw(i,r)$	$\forall i \in TRAD$ $\forall r \in REG$
(90) $qiw(i,s) = viwcif(i,s) - piw(i,s)$	$\forall i \in TRAD$ $\forall s \in REG$
(91) $qxwreg(r) = vxwreg(r) - pxwreg(r)$	$\forall r \in REG$
(92) $qiwreg(s) = viwreg(s) - piwreg(s)$	$\forall s \in REG$
(93) $qxwcom(i) = vxwcom(i) - pxwcom(i)$	$\forall i \in TRAD$
(94) $qiwcom(i) = viwcom(i) - piwcom(i)$	$\forall i \in TRAD$
(95) $qxwwld = vxwwld - pxwwld$	
(96) $qow(i) = valuew(i) - pw(i)$	$\forall i \in TRAD$
(97) $DTBALi(i,r) = [VXW(i,r)/100] * vxwfob(i,r) - [VIW(i,r)/100] * viwcif(i,r)$	$\forall i \in TRAD$ $\forall r \in REG$
(98) $DTBAL(r) = [VXWREGION(r)/100] * vxwreg(r) - [VIWREGION(r)/100] * viwreg(r)$	$\forall r \in REG$

Table 2.19. ***Impact of a 10% Cut in the Power of the Ad Valorem Tariff on EU Imports of US Foods on EU Food Sector in a Standard GE Closure using Johansen Solution Method and Fixed Investment Portfolio*** **(*RORDELTA* = 0)**

VARIABLE *PERCENTAGE CHANGE*	*EQUATION #*
$pm(\ food, usa) = .140$	
$pfob(\ food, usa, eu\ ) = .140$ (tx, txs exogenous)	(27)
$pcif\ (\ food,\ usa,\ eu\ ) = .124 = (.893)\ *\ (.140) + (.107)\ *\ (-.008)$	(26)
$pms(\ food,\ usa,\ eu\ ) = -9.876 = .124 - 10.0$	(24)
$pim(\ food,\ eu\ ) = -1.631 = (.164)*(-9.876) + (.000)*(-.121) + (.836)*(-.016)$	(28)
$qxs(\ food,\ usa,\ eu\ ) = 41.433 =\ 3.18 -\ (4.64)\ *\ [\ -9.876 - (\ -1.631)]$	(29)
$pf\ (food,\ \ food,\ eu) = -.259 = .092\ *\ (-1.631) + .908\ *\ (\ -.121)$	(30)
$qfm(\ food,\ \ food,\ eu\ ) = 3.002 = -.288 - (2.40)\ *\ [\ -1.631 - (-.259)]$	(31)
$qfd\ (\ food,\ \ food,\ eu\ ) = -0.621 = -.288 - (2.40)\ *\ [\ -.121 - (-.259)]$	(32)
$ps(\ food,\ eu\ ) = -0.121$	(1)

[a] Equation numbers refer to GTAP model equations presented in earlier tables.

Table 2.20. ***Economywide Effects in EU of a 10% Cut in the Power of the Ad Valorem Tariff on EU Imports of US Food in a Standard GE Closure Using Johansen Solution Method and RORDELTA*** **= 0 (*Nonlinear solution in brackets*)**

COMMODITY	VARIABLE (PERCENTAGE CHANGE)					
	pm(i, eu)[a]		qo(i, eu)		qp(i, eu)	
Land	-.414	[-.515][b]	0.0	[0]	na	[na]
Labor	-.029	[-.041]	0.0	[0]	na	[na]
Capital	-.028	[-.041]	0.0	[0]	na	[na]
Food	-.121	[-.154]	-.288	[-.355]	.036	[.042]
mnfrs	-.030	[-.041]	.064	[.086]	.012	[.007]
Services	-.030	[-.042]	.012	[.012]	.011	[.007]
cgds	-.026	[-.037]	-.003	[-.004]	na	[na]

[a] All price changes are relative to the price of the numeraire, which is savings.
[b] Nonlinear solution obtained by applying the Gragg, 2-4-6, method.

Table 2.21. ***Impact of a 10% Cut in the Power of the Ad Valorem Tariff on EU Imports of US Food on Total Food Sales in the US in a Standard GE Closure Using Johansen Solution Method and RORDELTA* = 0**

VARIABLE	*PERCENTAGE CHANGE*	*EQUATION #*
$qo(food, usa) = 0.688$		(1)[a]

$$= SHRODM(food, usa) * qds(food, usa) \_ (.926) * (.207)$$

$$SHROTM(food, usa) * qst(food, usa) \_ (.000) * (.000)$$

$$\sum_j SHROXMD(food, usa, s) * qxs(food, usa, s)$$

$$s = usa \_ (.000) * (.- .133)$$

$$s = eu \quad \_ (.013) * (41.433)$$

$$s = row \_ (.060) * (-.634)$$

where:

$qds(food, usa) = .207$ (3)

$$= \sum_j SHRDFM(food, j, usa) * qfd(food, j, usa)$$

$$j = food \quad \_ (.334) * (.662)$$

$$j = mnfcs \_ (.010) * (-.143)$$

$$j = svcs \quad \_ (.121) * (-.022)$$

$$j = egds \quad \_ (.000) * (-.042)$$

$$+ SHRDPM(food, usa) * qpd(food, usa) \_ (.517) * (-.019)$$

$$+ SHRDGM(food, usa) * qgd(food, usa) \_ (.018) * (-.031)$$

[a] Equation numbers refer to GTAP model equations presented in earlier tables.

Table 2.22. ***Economywide Effects in the US of a 10% Cut in the Power of the Ad Valorem Tariff on EU Imports of US Food in a Standard GE Closure Using Johansen Solution Method and RORDELTA* = 0 (*Nonlinear solution in brackets*)**

COMMODITY	VARIABLE (Percentage Change)					
	pm(i, usa)[a]		*qo(i, usa)*		*qp(i, usa)*	
land	1.066	[1.378][b]	0	[0]	na	[na]
labor	.109	[.141]	0	[0]	na	[na]
capital	.125	[.162]	0	[0]	na	[na]
food	.140	[.181]	.688	[.886]	-.000	[-.000]
mnfrs	.100	[.129]	-.120	[-.155]	.037	[.048]
services	.111	[.144]	-.001	[-.001]	.009	[.011]
cgds	.095	[.123]	-.001	[-.002]	na	[na]

[a] All price changes are relative to the price of the numeraire, which is savings.
[b] Nonlinear solution obtained by applying the Gragg, 2-4-6, method.

Table 2.23. ***Macroeconomic Effects of a 10% Cut in the Power of the Ad Valorem Tariff on EU Imports of US Food in a Standard GE Closure: Fixed (RORDELTA = 0) and Variable (RORDELTA = 1) Portfolios, and Johansen and Nonlinear Solution Methods Compared***

VARIABLE	US		EU		ROW	
	PERCENTAGE CHANGE					
qxwreg(*r*)	.138 (.057)[a]	[.178][b]	.233 (.263)	[.317]	-.007 (.007)	[-.006]
rorc(*v*)	.045 (.051)	[.059]	-.003 (-.005)	[-.006]	-.003 (-.004)	[-.003]
tot(*v*)	.110 (.128)	[.142]	-.043 (-.049)	[-.060]	-.007 (-.008)	[-.008]
up(*r*)	.013 (.016)	[.017]	.015 (.014)	[.013]	-.003 (-.004)	[-.004]
ug(*r*)	.013 (.015)	[.016]	-0.007 (-.008)	[-.014]	-.005 (-.006)	[-.006]
qsave(*r*)	.118 (.138)	[.153]	-.037 (-.042)	[-.056]	-.006 (-.007)	[-.006]
u(*r*)	.015 (.018)	[.019]	.006 (.004)	[.001]	-.004 (-.005)	[-.004]
	$US Million					
EV(*r*)[c]	778 (941)	[1004]	346 (251)	[62]	-347 (-410)	[-396]
DTBAL(*r*)	-8 (-663)	[-9]	0 (297)	[-22]	7 (366)	[31]

[a] Flexible investment portfolio, *RORDELTA* = 1, and Johansen solution method reported in parentheses.
[b] Fixed investment portfolio, *RORDELTA* = 0, and *nonlinear* solution obtained via Gragg, 2-4-6, method in brackets.
[c] Equivalent variation refers to the Cobb–Douglas, superutility function for region *r*. It is computed in equation (67) of Table 2.18.