A New Look at the Impact of Migration on the Wage Differential

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Abstract

The impact of migrants on the host country’s wages has been an important issue for policy makers and has attracted much research attention. Since no consistent conclusion has been reached regarding this impact, the debate over immigration policy continues in many countries. This paper constructs a theoretical dynamic general equilibrium model by which the transitional and long-run effects of a migration shock on endogenous variables, including the wage differential, can be illuminated. Consistent results from different scenarios are obtained: first, the wage differential is boosted by permanent migrants in both the short and the long run; second, if migrants stay temporarily, the wage differential increases in the short run and the effect dies out in the long run; third, permanent migrants impel domestic workers to upgrade skills by demanding an increased amount of education.

*JEL classification:* C61; C68; D10; D91; J31
1. Introduction

Migration has been a significant phenomenon around the world over the last two centuries. At the end of the 20th century, about 140 million persons- or roughly 2% of the world's population- resided in a country where they were not born (Borjas 1999). From the mass migration in the eighteenth century, which brought millions of Europeans to the America, to the recent rapidly growth of immigrants within Asia, this labour movement alters the labour endowment in both the source and host countries. The impact of the labour movement on the host country’s wages and unemployment levels has attracted much attention in the existing research. Due to the fact that no consistent conclusion has been provided, there is ongoing debate over immigration policy in many countries. This paper focuses on the issue of the effect of migration on the wage differential between skilled and unskilled labour in the host country. By taking the Borjas’ (1994) suggestion that we do not understand the “dynamic process through which natives respond to these supply shocks and reestablish labor market equilibrium” into account, this paper adopts a new methodology, Dynamic Intertemporal General Equilibrium (DIGE) modeling, and constructs a theoretical model of a closed economy, which blocks other effects but migration from outside of the host country, to reinvestigate the issue of the wage differential. By using the DIGE modeling framework, we can study the transitional dynamics following shocks to the system. A relatively comprehensive framework with one-good, two-labour (skilled and unskilled) and three-agent (firms, households, and government) is established. In consideration of temporary migration and permanent migration, simulations of this theoretical model illustrate an overall picture of the effect of migration on the wage differential in a dynamic process. Important insights and implications for immigration policies are discussed. Since it is a theoretical model, it can be applied to a range of countries only by changing the appropriate parameters.

A large amount of research investigating the effect of immigration on native wages and the findings have mixed results. One viewpoint is that there is little evidence of an adverse wage effect of immigration, such as Butcher and Card (1991) and Heckman et al (1998). The other viewpoint suggests that immigration adversely affects less-skilled native wages (Lalonde and Topel 1991, Altonji and Card 1991, Kuhn and Wooton 1991, Borjas et al 1992, Borjas et al 1996 and Friedberg and Hunt 1995). Almost all of the empirical studies in this literature using the spatial correlations approach have produced a confusing array of results (Borjas 1999). Substantial efforts regarding methodology innovation have been made. Altonji and Card (1991) and Schoeni (1997) use instrumental variable estimations; however, they end with very different estimates (Borjas 1999). Heckman et al (1998) and Kuhn and Wooton (1991) use the dynamic and the static general equilibrium approach respectively to look at the issues of wage inequality and native wages. Inconsistent conclusions result from their models. In contrast to these existing general equilibrium models, this paper constructs a relatively comprehensive dynamic framework in which government plays an important role of supplying education and wages are determined by labour supply and demand.

The following section outlines the theoretical framework of the model. Section three presents the simulation results from considering a range of scenarios of migration and section four discusses the conclusions reached.

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1 The model follows the general approach of the G-Cubed model (McKibbin and Wilcoxen 1999). The endogenized skill formation and education production follow the model in Chang (1999) with a novel application on the migration issue.
2 The spatial correlation is the relationship between labour market outcomes in a locality and the extent of immigrant penetration. The sign of the relevant coefficient changes erratically over time.
3 Heckman et al (1998) is the only existing research using a dynamic general equilibrium model. However, in their model, government plays a limited role in subsidizing human capital and is put aside in their analysis. In addition, the wage equation in their research is in an ad hoc form.
2. The Model

The framework of this model is as follows: firms produce the single good by hiring physical capital, skilled labour and unskilled labour, they then sell this good to the households for consumption, to the government for education capital investment and to themselves for physical capital investment. The objective of each firm is to maximize its intertemporal profit. The ownership of firms belongs to households. Households supply unskilled labour to firms and skilled labour to both firms and the government in order to earn wages which, together with the dividends from renting physical capital to firms, finance the purchase of goods and education. Leisure is consumed by the households with an opportunity cost of not working. The objective of households is to maximize utility by an optimal distribution of consumption between the good and leisure. The government buys the good from firms and transforms it into education capital. This capital is combined with skilled labour hired by the government to produce education. The role of government as an education supplier is essential. This model captures the reality of government supplying education in consideration of the associated beneficial externalities. The government balances its budget by collecting labour income tax and selling education to the household. The accumulation of physical capital, skill formation, education capital and financial assets drives the dynamic evolution of the economy over time.

2.1 Firms

The production function of the representative firm is assumed to be constant elasticity of substitution (CES) as follows

(1) \[ Q_t = \rho \left[ K_t^\rho + (L_{s,t}^F \alpha L_{u,t}^{1-\alpha})^{\frac{1}{\alpha}} \right]^{\frac{\alpha}{\rho}} \]

where \( Q \) is the output and \( K, L_s^F, L_u \) are respectively the physical capital stock, the skilled labour and the unskilled labour hired by the firm. \( \rho \) and \( \alpha \) are parameters.

Capital accumulation depends on the rate of fixed capital formation \( J \) and the rate of depreciation \( \delta \).

(2) \[ \frac{dK}{dt} = J_t - \delta K_t, \]

Under the assumption of rising marginal costs of installation in the investment process, the total investment expenditure \( I \) is

(3) \[ I_t = J_t \cdot [1 + ((\phi/2)(J_t/K_t)^2], \]

where \( \phi \) is a positive parameter, and \( (\phi/2)(J_t/K_t) \) is the unit cost of adjustment, which is assumed to be a linear function of the rate of capital formation.

The current value Hamiltonian function is employed to solve the above autonomous one state variable system with \( \lambda \) as the shadow price of capital. By solving the first-order differential equation and applying the transversality condition, the shadow price of capital becomes

(4) \[ \lambda(t) = \int_t^\infty [Q_K + (\phi/2)(J/K)^2] e^{-(r+\delta)s} ds, \]

\(^4\) To avoid unnecessary complexities, a subsidy rate on investment is assumed to be offset by a tax rate on financial dividends.
where \( Q_K \) is the marginal product of capital and \( (\phi/2)(J/K)^2 \) is the marginal product of capital in reducing adjustment costs in investment at each point in time. Therefore, \( \lambda \) is the increment to the real value of the firm from a unit increase in its investment at time \( t \).

### 2.2 Households

The aim of the household is to maximize its intertemporal utility subject to several constraints.

Max. \( \int_0^\infty U(C_t, l_t) \cdot e^{\theta t} \cdot dt \)

Subject to

\( dF/dt = r \cdot F_t + (1 - \tau) \cdot (W_{s,t} \cdot L_{s,t} + W_{u,t} \cdot L_{u,t}) - C_t - P_{E,t} \cdot S_{E,t} , \)

\( dL_s/dt = J_{s,t} - \delta_s \cdot L_{s,t} , \)

\( S_{E,t} = J_{s,t} \cdot \frac{\Phi}{2} \cdot \frac{J_{s,t}}{L_{s,t}} , \)

\( F_t = \lambda_t \cdot K_t , \)

where \( C_t \) is the consumption of the good, \( l_t \) is the leisure taken, \( \theta \) is the rate of time preference, \( r \) is the interest rate, \( F_t \) is financial assets, \( \tau \) is the tax rate, \( W_s \) and \( W_u \) are, respectively the wage rates of skilled and unskilled labour, \( P_{E,t} \) is the price of one unit of education, \( S_{E,t} \) is the amount of education bought, \( J_{s,t} \) is fixed skill formation, \( \delta_s \) is the depreciation rate of skill, \( L_{s,t} \) is the amount of skilled labour, \( \Phi \) is the adjustment cost parameter and \( \lambda_t \) is the shadow price of capital.

Equation (5) is the household’s budget constraint. Equation (6) shows that net skill accumulation is skill depreciation subtracted from fixed skill formation. Equation (7) states that education investment depends on fixed skill formation and an adjustment cost function. The adjustment cost relies on the ratio of fixed skill formation to skilled labour. When an economy has more skilled labour, the adjustment cost becomes smaller. This is plausible due to the spillover effect among labour. The marginal adjustment cost of skill formation is positive and follows an increasing rate. This states that, in a resource limited economy, resources become more expensive when more resources are drawn into use for skill formation.

The current value Hamiltonian function is employed to solve the above autonomous two-state variables system with \( \mu_1 \) and \( \mu_2 \) as the respective shadow prices for the financial asset and skill. The shadow price of skill is greater than the shadow price of the financial asset because the total cost of forming a unit of skill is greater than that of accumulating one unit of financial asset, due to the adjustment cost of skill formation. If the shadow price of skill is not greater than that of the financial asset, the household would prefer to defer spending on skill formation and instead accumulate financial assets for future consumption.

Applying the transversality condition to the shadow price of skill, \( \mu_2 \), results in
Equation (9) states that the shadow price of skill is equal to the present discounted value of future marginal utility. The first component of the shadow price of skill contains the marginal utility of consuming goods, the after-tax skilled wage, and the reduction of the adjustment cost in education investment. It provides the gross increment of utility the household can get from supplying one additional unit of skilled labour. The second part is the marginal disutility of offering one unit of skilled labour. Combining these two gives the net utility the household can achieve by supplying one unit of skilled labor.

2.3 Government

Government is a supplier of education. The education production function is

\[ E_t = f(K_{E,t}, L_{s,t}^G) = (K_{E,t}^\xi + L_{s,t}^G)^{1\xi}, \]

where \( E \) is the education supply, \( K_E \) is the education capital, \( L_s^G \) is skilled labour working for government and \( \xi \) is a parameter. The interpretation of \( K_E \) could be the hardware associated with schooling, e.g., classroom, equipment, etc. \( L_s^G \) could be the software associated with schooling, e.g., teachers, administrators, etc.

Education capital accumulates via governmental investment in education as follows

\[ \frac{dK_E}{dt} = I_{E,t}^G - \delta_E \cdot K_{E,t}, \]

where \( I_E^G \) is the government investment in education, and \( \delta_E \) is the depreciation rate of education capital.

2.4 The Steady-State

A full model in a steady state is presented in Appendix 1. This model produces the following relationship between skilled and unskilled wage in the steady state\(^5\),

\[ W_s = W_u + \frac{P_E}{(1 - \tau)} \cdot \frac{\Phi}{2} \cdot (\theta + \Phi \cdot \delta_s + \delta_s + \frac{1}{2} \cdot \Phi \cdot \delta_s^2). \]

The expression of equation (12) is independent of the functional form of both the utility and production functions\(^6\). It provides a rigorous theoretical result for the wage differential. The relationship between the skilled and unskilled wage depends on the rate of time preference, the depreciation rate of skill, the skill adjustment cost parameter, the tax rate and the price of education. A higher skill adjustment cost, skill depreciation rate, or time preference all tend to raise the wage differential. The reason why higher skill adjustment costs and higher skill depreciation rates raise the wage differential is straightforward. The reason for a larger time preference having this effect is that the rate of time preference counts because an

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\(^5\) Due to the complicated framework, it is not possible to solve for a reduced form of the wage differential.

\(^6\) A detailed proof is available from the author.
investment in skill formation takes time to repay. A larger time preference involves a larger adjustment cost for skill formation, therefore a patient household will expect a higher skilled wage.

The government’s migration policy plays an important role in the wage differential via its impact on the education price and taxation. An intuition of the effect of migration on the wage differential is that migrants as unskilled labour participate in the production of goods and therefore pressure the domestic unskilled workers to upgrade their skills. This boosts the price of education, resulting in an increased wage differential. More details on policy implications obtained by the simulation results are discussed in the next section.

2.5 Migration in the Model

Migrants are an exogenous unskilled work force in goods production after their arrival. Migrants are assumed to pay the same tax rate and to receive the same wage rate as local workers. Three cases of the migrants’ status are considered: first, migrants who stay in the country temporarily, i.e., they only stay for a period of time and then they leave the country; second, migrants who stay in the country permanently, but they work as unskilled labour all the time, i.e., they do not take part in education; third, migrants who become permanent residents and start to take part in education to upgrade skills after an initial settlement period. The first and second cases represent a temporary and permanent shock for unskilled labour in goods production. The third case states that migrants can also work as skilled labour in goods production after the first period of arrival. In the first period, migrants create an exogenous shock for unskilled labour and after that, migrants become permanent additional skilled and unskilled workers. The migrants working as skilled or unskilled labour are endogenous in the economy.

3. Simulation Results

The production of this one-type of good could be skilled-labour intensive or unskilled-labour intensive. The simulations take into account different labour intensities in goods production and three cases of the migrants’ status stated previously. To carry out these different situations, parameters and exogenous variables are calibrated as shown in Appendix 2 and an experiment for each case is demonstrated by giving an increased quantity of migrated unskilled labour equal to 1 per cent of the domestic unskilled labour. The calibration is based on good empirical knowledge albeit subjective. For solving this nonlinear system, this calibration provides reasonable time frames of variables’ transitions between the initial and the new steady state after shocks. Table 1 summarizes the results of the simulations and Appendix 3 shows the dynamic paths.

3.1 Skilled-Labour Intensive Good

3.1.1 Temporary Migration

The wage differential increases in the short run with a jump to 0.42 per cent immediately after the shock and then drops back to the original level in the long run. In the short run, the new unskilled workers step into the production of goods, which boosts the marginal productivity of skilled labour and therefore, raises the skilled wage. This motivates domestic unskilled workers to obtain more education and become ordinary.

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7 The cases of foreign workers receiving less pay than local workers are also tested. They result in a consistent conclusion with the case of receiving the same wage rate for the impact of foreign workers on the wage differential.
skilled labour. Because skill formation takes time, the extra demand for skilled labour in the education sector is initially drawn from the goods sector. Firms prefer to substitute unskilled labour for skilled labour due to the increase in the skilled wage. This induced demand for unskilled labour increases the unskilled wage and motivates households to work harder by reducing leisure. The result for the short-run growth in the wage differential comes from the increased size of the skilled wage being larger than the increased size of the unskilled wage.

After the removal of migrants from goods production, the skilled wage drops below the original level due to the glut of skilled labour, then gradually increases back to the benchmark. The unskilled wage also gradually decreases back to the original level. Therefore, the wage differential drops after the departure of the migrants, then in the long run, it adjusts back to the original level.

3.1.2 Permanent Migration without Subsequent Education

The wage differential increases in both the short and long run. The increased extent in the long run is larger than that in the short run, i.e. the wage differential increases by 0.21 per cent in the first period after the shock and jumps to 1.25 per cent in the new steady state.

In the short run, the effect of these increased unskilled migrants on the wage differential is similar to the case of a temporary stay, i.e. the skilled wage increases to a larger extent than the unskilled wage does. However, unlike the case of a temporary shock, when migrants work in goods production permanently, domestic unskilled labour is crowded out in the short run and takes more leisure. The reason for more leisure being affordable for households is that the increases in both skilled and unskilled wages result in the income effect dominating the substitution effect. However, over time as the domestic workers confront the competition from the migrants as unskilled labour, more are motivated to become skilled labour and also tend to reduce leisure. One important point is that production is raised in the short run due to more labour being available and more demand created by migrants and households, but it decreases in the long run because more resources in the economy are withdrawn from the goods sector to the education sector and less physical capital is accumulated. This result shows an interesting issue that migrants who stay in a host country as unskilled labour permanently could actually affect its physical capital formation and slow down its economic growth. In the long run, the skilled wage is increased further than that in the short run, in line with more skilled labour demanded in the education sector. This is the reason why in the long run the wage differential is larger than that in the short run. This experiment sheds light on the importance of adopting a policy to ensure the growth of the capital when more unskilled migrants move into an economy.

3.1.3 Permanent Migration with Subsequent Education

In this case migrants are treated in the same way as domestic workers after the first period of arrival. That is, when they just arrive in the host country, they work as unskilled labour. After this initial period they can choose to upgrade themselves to skilled labour by taking education.

Although the wage differential follows a similar transition to that in the case of permanent migration without subsequent education, i.e. the wage differential increases in both short and long run and the increase in the long run is larger than that in the short run, a range of other variables react differently. The wage differential increases by 0.28 per cent in the first period after the shock and grows to 0.41 per cent in the new steady state.
In the first period after more unskilled labour is injected into the economy, key variables such as the skilled wage and the unskilled wage follow a similar transition to that in the case without education. However, two points should be highlighted. First, for the case in which migrants can choose to become skilled labour, (in contrast to the case in which migrants remain in the category of unskilled labour) the domestic labour works harder by reducing leisure. This is because the pressure from migrants who can choose to become skilled labour is not only directly on the unskilled labour but also on the skilled labour. Second, the increased production in this case is larger than that in the case of not taking education. The intuition for this is that a subgroup of the migrants work as relatively more productive skilled labour than previously worked as unskilled labour. Therefore, allowing migrants to upgrade their skills can increase the capital stock in the short run, whereas it decreases for the case without education. However, in the long run, the capital stock drops below the benchmark in the same way as the case of migrants who do not take part in education.

The major differences for the endogenous variables emerge in the second period when immigrants are treated exactly the same as domestic labour. Both wages drop in the second period due to more skilled and unskilled labour being available. The unskilled wage even drops below the benchmark. The wage ratio declines in the second period compared to the first period, then gradually increases till the new steady state is reached. Overall, the effect of migrants with subsequent education on the wage differential is positive in both the short and long run.

Regardless of whether migrants are able to transform to skilled labour or remain as unskilled labour, the wage differential grows in both the short and the long run. However, the transition of wages for these two cases is different: for the case of migrants without subsequent education, both skilled and unskilled wage are boosted and for the case of migrants with subsequent education, the skilled wage is driven up and unskilled wage is driven down. The skilled wage in the case of those without subsequent education increases by 1.57 per cent more than that in the case of those with education, due to a larger shortage of skilled labour in line with an influx of more unskilled labour.

3.2 Unskilled-Labour Intensive Good

3.2.1 Temporary Migration

The wage differential increases in the short run with a jump to 0.21 per cent and then drops back to the original level in the long run. In the short run, the transitions of skilled wage and unskilled wage are similar to those in the skill-intensive case, and an increased wage differential is created by a larger increase in the skilled wage than an increase in the unskilled wage. However, unlike the case of skill-intensive good, these raised wages motivate households to reduce work by increasing leisure due to the income effect dominating the substitution effect.

After the removal of migrants from goods production, transitions reverse, i.e. the skilled wage drops below the original level due to the glut of skilled labour, especially with an unskilled-labour intensive production, then gradually increases back to the benchmark. The unskilled wage also decreases back to the original level. Therefore, the wage differential drops below the benchmark after the departure of migrants, then in the long run, it adjusts back to the original level.

3.2.2 Permanent Migration without Subsequent Education

The wage differential increases in both short and long run. The increase in the long run is larger than that in the short run, i.e. the wage differential increases by 0.37 per cent in the first period after the shock and
increases to 0.39 per cent in the new steady state. In comparison with the results from the same scenario when production is skill-intensive, the wage differential has less variation in the long run. This shows that unskilled migrants have a smaller effect on the wage differential while engaging in unskilled-labour intensive production than in skill-intensive production. The intuition for this is that the shock of unskilled migrants raises relatively less demand for skilled labour in the unskilled-labour intensive production than in the skill-intensive production and therefore, a smaller increase in the skilled wage.

In the short run, as in the temporary case, the wage differential enlarges and households consume more leisure. However, over time, the domestic workers face competition from the migrants as unskilled labour, more domestic workers are motivated to become skilled labour, and in so doing the education sector is boosted. Domestic households are also willing to reduce leisure to work more as unskilled labour in line with an increase in the unskilled wage. In the long run, the skilled wage is increased further than that in the short run, in line with more skilled labour being demanded in the education sector.

### 3.2.3 Permanent Migration with Subsequent Education

Although the wage differential follows a similar transition to the case of permanent migration without education, i.e. the wage differential increases in both the short and long run and the increased extent in the long run is larger than that in the short run, a range of other variables react differently. The wage differential increases by 0.37 per cent in the first period after the shock and enlarges to 0.47 per cent in the new steady state. This shows a further enlargement of the wage differential in the long run compared to the case of migrants without subsequent education. The two highlighted points and the variations of wages for the same scenario with a skill-intensive production are still valid.

Similar to the results of simulations for a skill-intensive production, no matter whether migrants are able to transform to skilled labour or remain as unskilled labour, the wage differential grows in both the short and the long run. For the case of migrants without subsequent education, both the skilled and unskilled wages are raised and for the case of migrants with subsequent education, the skilled wage is driven up and the unskilled wage is driven down. The skilled wage in the case of migrants without subsequent education increases by 0.65 per cent more than that in the case of migrants with subsequent education.

### 3.3 Robustness Test on the Production Function

Table 2 shows the results for the wage differential simulated in different scenarios of migration by changing the production functions of goods and education from the CES to the Cobb-Douglas form.

[Insert Table 2 Here]

Both types of production functions provide fairly similar results. The only cells not matched are located in the category of the long-run effect with migrants without taking education. The similarity in all other cells suggests that the results of the simulations from the theoretical model in this paper are robust to the choice of production functions.

### 4. Conclusion

This paper constructs a theoretical model by which the transitional and long-run effects of a migration shock on endogenous variables, including the skilled wage and the unskilled wage, can be illustrated. To elucidate the investigated issue, i.e. the impact on the wage differential by an influx of migrants, a range
of scenarios are considered, such as whether the goods production is skilled-labour intensive or unskilled-labour intensive, whether migrants stay temporarily or permanently and whether permanent migrants take education to upgrade skills or not. Different forms for the goods and education production function are also tested to demonstrate that the results obtained are robust.

Consistent results from different scenarios may be summarized as follows, in order to provide policy implications. First, the wage differential is boosted by permanent migrants in both the short and the long run. The increased differential in the long run is larger than that in the short run. Second, if migrants only stay temporarily, the wage differential increases in the short run and the effect dies out in the long run. Third, permanent migrants impel domestic workers to upgrade skills by demanding an increased amount of education. In the long run, permanent migration causes a decumulation of the physical capital stock due to the limited resources in the economy being drawn away from the goods production to education production (by which skills or human capital are accumulated). A policy to ensure that the economy keeps growing should be addressed by the government while adopting an open policy of migration.

Most of the existing research suggests that there is little evidence of an impact of immigration on domestic wages or that immigration has a negative effect on less-skilled native wages. In contrast to these existing findings, this paper asserts that migration should have a positive effect on skilled wage in both the short and long run; migration has a positive effect on unskilled wage in both the short and long run if migrants stay as unskilled labour permanently; and migration has a positive effect on unskilled wage in the short run and a negative effect in the long run if migrants take education to upgrade skills. When both wages increase, the enlarged wage differential comes from a larger increase in the skilled wage compared to the unskilled wage. This shows that immigration may have an adverse effect on less-skilled native wages in terms of relativity, while having a positive effect in an absolute term.
### Table 1. The Effect of Migration on the Wage Differential (%)

<table>
<thead>
<tr>
<th></th>
<th>Skilled-Intensive Production</th>
<th>Unskilled-Intensive Production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temporary</td>
<td>No EDUN</td>
</tr>
<tr>
<td>Short-run</td>
<td>0.42</td>
<td>0.21</td>
</tr>
<tr>
<td>Long-run</td>
<td>0</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Note: No EDUN stands for the case that migrants who are permanent unskilled labour and take no education; EDUN stands for the case that migrants who stay permanently and can choose to take education to upgrade their skills. Numbers for Short-run are those in the first period following shocks. Numbers for Long-run are those in the steady state after shocks.

### Table 2. Robustness Test

<table>
<thead>
<tr>
<th></th>
<th>CES Production</th>
<th>Cobb-Douglas Production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Skilled-Int</td>
<td>Unskilled-Int</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>NoED</td>
</tr>
<tr>
<td>SR</td>
<td>0.28</td>
<td>0.21</td>
</tr>
<tr>
<td>LR</td>
<td>0.41</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Reference


Appendix 1

Model in the Steady State

Equations

\[ Q_t = \left[ K_t^\rho + (L_{s,t}^\alpha L_{u,t}^{1-\alpha} \right]^{\frac{1}{\rho}} \]
\[ J_t = \delta K_t \]
\[ I = J \cdot (1 + \phi \delta) / 2 \]
\[ Q_{L_s} = W_s/P \]
\[ Q_{L_u} = W_u/P \]
\[ \lambda = 1 + \phi \delta \]
\[ Q_K = (r + \delta) \lambda - \phi \delta^2 / 2 \]
\[ 0 = r_t \cdot F_t + (1 - \tau_t) \cdot (W_s \cdot L_{s,t} + W_u \cdot L_{u,t}) - C_t - P_{E,t} \cdot S_{E,t} \]
\[ J_{s,t} = \delta K_{L_s} \]
\[ F_t = \lambda K_t \]
\[ S_\varepsilon = J_{s,t} \cdot (1 + \Phi \delta^2 / 2) \]
\[ l_t = T - L_{u,t} - L_{s,t} \]
\[ U_C = \mu_1 \]
\[ U_{L_u,t} = -\mu_1 \cdot (1 - \tau) \cdot W_u \]
\[ \mu_2 = \mu_1 \cdot P_{E} \cdot (1 + \Phi \delta^2) \]
\[ r_t = \theta \]
\[ U_{L_s} = (\theta + \delta) \cdot \mu_2 - \mu_1 \cdot [(1 - \tau) \cdot W_s + P_{E} \cdot (\Phi \cdot \delta^2 / 2)] \]
\[ L_s = L_{s,t}^G + L_{s,t}^E \]
\[ S_\varepsilon = (K_{E,t}^\varepsilon + L_{s,t}^{G \varepsilon})^{\frac{1}{2}} \]
\[ I_{E}^G = \delta K_{E} \]
\[ I_{E}^G + W_s \cdot L_s^G = \tau \cdot (W_s \cdot L_s + W_u \cdot L_u) + P_{E} \cdot S_{E} \]
\[ Q_t = C + I_{E}^G + I \]

Notation:

Q: Production; \( L_{s,t}^E \): Skilled labor hired by firms; \( L_{s,t}^G \): Skilled labor hired by government; \( L_s \): Total skilled labor; \( L_u \): Unskilled labor; \( L_m \): Foreign Workers; K: Capital; J: Fixed capital formation; I: Capital investment; W_s: Skilled wage; W_u: Unskilled wage; P: Goods price (defined as 1); F: Financial asset; C: Consumption; l: Leisure; SE: Amount of education buying; \( J_{s,t} \): Fixed skill formation; T: Time constraint; I_E: Household’s education investment; U_Z: Marginal utility of Z; P_E: Price of education; r: Interest rate; K_E: Education capital; I_{E}^G: Government education investment; \( \tau \): Tax rate; \( \alpha, \beta \): Input shares in goods production function; \( \lambda \): Shadow price of capital; \( \delta \): Depreciation rate of capital; \( \phi \): Adjustment cost parameter of capital investment; \( \Phi \): Skill adjustment cost parameter; \( \mu_1 \): shadow price of financial asset; \( \mu_2 \): shadow price of skill; \( \eta \): Wage proportion of foreign workers to local unskilled wage; \( \theta \): Rate of time preference; \( \xi \): Input share in education production function; \( \delta_\varepsilon \): Depreciation rate of education capital; \( \delta_s \): Depreciation rate of skill.
Appendix 2

\[ T = 8760; \quad \tau = 0.25; \quad I^G_E = 80; \quad \rho = 2; \]
\[ \alpha = 0.6; \quad \delta = 0.2; \quad \delta_k = 0.1; \quad \delta_E = 0.2; \]
\[ \phi = 10; \quad \theta = 0.1; \quad \xi = -1; \quad \gamma = 0.5; \]
\[ \Phi = 10; \]
Appendix 3

**Skilled-Labour Intensive**

CES Production

**Unskilled-Labour Intensive**

CES Production