Export, Horizontal FDI, or Export-Platform FDI with Heterogeneous Firms

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Abstract

Besides the firm's strategic choice between exporting (motivated by lower fixed costs) or horizontal FDI (motivated by lower trade costs) to serve a foreign market, there is another choice driven by lower factor costs. Export-platform FDI is characterized by investing and producing in a foreign country, while the final output is sold not in the home- or host-country markets but exported to third markets. This paper presents a numerical model that explains the decision of heterogeneous firms, which differ in productivity levels, to serve foreign markets either through exporting from the home country, local subsidiary sales (horizontal FDI), or exporting from a host country (export-platform FDI), in the three-country framework. Experimental simulations with the model clearly show that export-platform productions complement horizontal affiliate productions while direct exporting substitutes. In addition, the firms' operational strategy whether to choose the horizontal FDI responds quite sensitive to the changes in factor prices, and affects the other choices of the substitutable direct exporting and the complementary export-platform FDI.

Keywords: foreign direct investment, export platform, heterogeneous firms.

JEL Classification Numbers: C68, F12, F23, L11.

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1. Introduction

There has been extensive research on the strategic choice of firms to access foreign markets whether via exporting or via foreign direct investment (FDI) in a certain economic environment (Markusen, 1984; Brainard, 1997; Helpman, Melitz, and Yeaple, 2004). While those studies mainly focus on the proximity-concentration trade-off between (horizontal) FDI, which is motivated by lower trade costs, and exporting, motivated by lower fixed costs with intensive production at home, there is another choice driven by lower factor costs. The third choice, export-platform FDI, is characterized by investing and producing in a foreign country, while the final output is sold not in the home- or host-country markets but exported to third markets (Motta and Norman, 1996; Greenaway and Kneller, 2007; Ekholm, Forslid, and Markusen, 2007, Ito, 2013). Since the export-platform FDI accompanies both establishment of foreign affiliates and exporting, the fixed setup costs as well as trade costs tend to be high.

Our previous study with an extended knowledge-capital model based on Markusen (1997) and Zhang and Markusen (1999), which includes four countries (two market and two non-market countries) and six types of (homogeneous) firms' operational strategies, clarified that a non-market country, in which plenty of cheap production factors are available and it is possible to suppress trade costs to access destination market because of the close location or the existence of free trade agreement (FTA) with a market country, may host export-platform FDI (Oyamada, 2017). From a different point of view, this paper presents a numerical model that explains the decision of heterogeneous firms, which differ in productivity levels, to serve foreign markets either via exporting from the home country, via local subsidiary sales (horizontal FDI), or via exporting from a host country (export-platform FDI), extending the ideas by Helpman *et al.* (2004) and introducing it to the three-country framework (two market and one non-market countries) following Ekholm *et al.* (2007). For the purpose of focusing on factor costs as the third motivation, we consider as the first step how firms may access to one destination market, unlike the case discussed by Helpman, Melitz, and Yeaple (2003:32) that all countries are at the same levels of factor prices and there are many destination markets. If we increase the number of destination markets in an extended version of this study, the characteristics discussed by Helpman *et al.* (2003) may enter the model.

In the present model, a firm established in a market country chooses an operational strategy based on its productivity level. The least productive firms go out of business since they are incapable of generating positive operating profits. Other poorly productive firms serve only the domestic market. The more productive firms serve both the domestic and foreign markets, while their channel to have access to a foreign market differs. Although the economic conditions in fixed costs to setup production facilities, transportation costs or tariffs on international trade, and costs to hire local production factors affect, the most productive firms tend to choose investing abroad whereas the less productive group chooses exporting from their home country. Note that no firm engages in multiple activities to serve the same foreign market in an equilibrium.

The reminder of this paper is organized as follows. Sections 2 and 3 present notes on the main assumptions and parameterization process of the analytical model used in this study. In Section 4, we perform experimental simulations to clarify the behavioral characteristics of the model and verify the results. Then, Section 5 concludes this paper.

2. The Model

In this section, we overview the model used in this study for numerical simulations. The model is calibrated to an artificial data set assumed by the author.

2.1 Environment

Suppose there are two countries $M1$ and $M2$ that respectively have markets for the differentiated commodities produced by imperfectly competitive heterogeneous firms, which differ in productivity levels. Let those market countries be indexed r (home country where the headquarters of firms are located) and s (destination market where the products are sold). The elements included in r and s are identical ($M1$ and $M2$). In addition, there is one (low-cost) non-market country $N1$, in which production process of multinational productions may take place while the finished products are not sold locally but exported to a destination market. Non-market countries are indexed t. Three countries also produce a homogeneous good, which is our numéraire and internationally traded as perfect substitutes. Let us call the sector that produces differentiated commodities, "increasing-returns-to-scale (IRTS) sector," and the one that produces homogeneous good, "constant-returns-to-scale (CRTS) sector," for convenience.

Both IRTS and CRTS sectors use one kind of production factor, say labor, to produce goods. While fixed unit input of factor is required to produce one unit of output in the CRTS sector, input levels differ among firms based on their productivity in the IRTS sector.

As the first step, we do not consider intermediate inputs in this study. To enter the IRTS industry in country r, a firm bears the fixed entry costs F_r^B, measured in factor units. A successful entrant then draws an input coefficient $\hat{\varphi}_{ar}$ from a distribution $G(\hat{\varphi}_{ar})$. Index a denotes a member of the set of all firms successfully entered in country r. Upon observing this draw, a firm may decide whether to produce or to exit without producing. If a firm chooses to be active, it bears additional fixed costs F_r^D to make sales at least in the domestic market. While there is no fixed cost required when the firm remains in the domestic market, it bears additional fixed costs F_{rs}^E, F_{rs}^H, or F_{rst}^X to have access to a foreign market on the r-s link via exporting, horizontal FDI, or export-platform FDI through the non-market third country t. We regard F_{rs}^E as the costs of forming a distribution and servicing network in the foreign market. Similar costs for the home market are included in F_r^D. F_{rs}^H and F_{rst}^X include the costs of establishing an affiliate in a foreign country in addition to the costs of network formation. F_{rst}^X include the network formation costs both in the host and destination market countries.

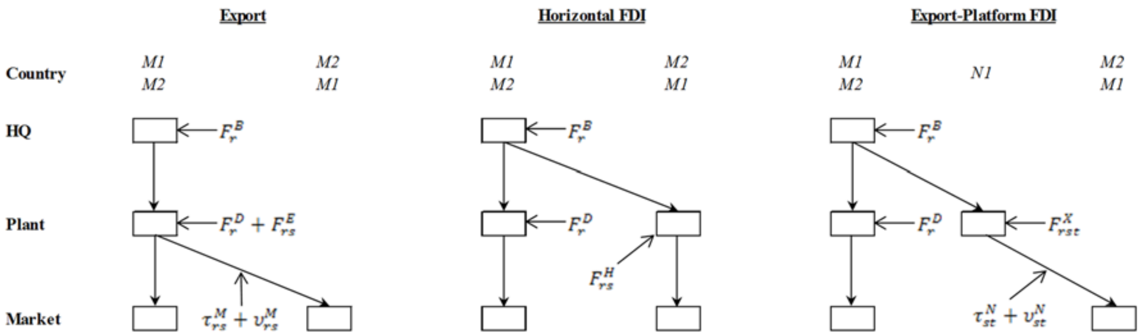
The differentiated products sold on the r-s link between market countries are subject to trade costs, transportation margin τ_{rs}^M and import tariff υ_{rs}^M. In a similar manner, the differentiated products sold on the s-t link between either of the market countries and a non-market country are subject to transportation margin τ_{st}^N and import tariff υ_{st}^N. We assume that a fraction of the homogeneous good produced in the CRTS sector is supplied as the international shipping service for transportation, while the tariff revenue is a part of income in the destination country. For simplicity, shipping the homogeneous products does not generate any trade costs. Since export-platform FDI bears highest fixed costs as well as trade costs, it requires the low-cost production factor in the host country. If the transportation costs are low because of the close location to the destination market, or import tariff is low because of FTA between host and destination market, a firm may operate via export-platform FDI under favorable conditions. The cost burdens in the three alternatives to have access to a foreign market are summarized in Table 1.

Table 1: Cost Burdens by Operational Strategy

	Export	Horizontal FDI	Export-Platform FDI
Fixed Costs	Low	High	High
Trade Costs	High	Low	High
Factor Costs	High	High	Low

Finally, Figure 1 shows schematic images of three alternative strategies of firms operating in the IRTS sector to access foreign market. In each pattern, the headquarters of a firm is located in the country placed on the left-hand side of the image, whereas the destination market is on the right-hand side. The process of firm entry, production, and sales is shown from top to bottom. In each image includes the points where fixed and trade costs are generated.

Figure 1: Operational Strategies to Access Foreign Market



2.2 Final Demand for the Differentiated Commodities

Suppose a is a member of the set of firms established in country r, *i.e.*, $a \in A(r)$. Similarly, d is a member of the set of firms that sell products at least domestically, *i.e.*, $d \in D(r)$. The set D is a subset of A so that $D \subseteq A$. Among the firms belonging to the set D, firm e, h, and x, respectively is a member of the set of firms that may have access to a foreign market on the r-s link, *i.e.*, $e \in E(rs)$, $h \in H(rs)$, and $x \in X(rst)$. The sets E, H, and X are subsets of D so that $E \subseteq D$, $H \subseteq D$, and $X \subseteq D$. Firm e sells its products not only in the domestic market but also in a foreign market using direct exporting. Firm h builds production facilities both in the home and foreign countries to supply its products to local markets. Firm x builds production facilities in a non-market third country, in addition to the home country, to export final products to the target market. Note that no firm engages in multiple activities to serve the same foreign market so that the sets E, H, and X are disjoint to each other. Then, total sales quantity by firms d, e, h, and x, as well as the aggregated total of the products supplied by all of those firms in market s are assumed to be given by the following functions:[1]

$$\widetilde{D}_s = \left\{\delta_s^D \sum_d \widehat{D}_{ds}^{(\sigma-1)/\sigma}\right\}^{\sigma/(\sigma-1)}, \tag{1}$$

$$\tilde{E}_{rs} = \left\{\delta_{rs}^E \sum_e \hat{E}_{ers}^{(\sigma-1)/\sigma}\right\}^{\sigma/(\sigma-1)}, \tag{2}$$

$$\widetilde{H}_{rs} = \left\{\delta_{rs}^H \sum_h \widehat{H}_{hrs}^{(\sigma-1)/\sigma}\right\}^{\sigma/(\sigma-1)}, \tag{3}$$

$$\tilde{X}_{rst} = \left\{\delta_{rst}^X \sum_x \hat{X}_{xrst}^{(\sigma-1)/\sigma}\right\}^{\sigma/(\sigma-1)}, \tag{4}$$

and

$$Q_s = \theta_s \left\{ \begin{matrix} \widetilde{D}_s^{(\sigma-1)/\sigma} + \sum_r \tilde{E}_{rs}^{(\sigma-1)/\sigma} \\ + \sum_r \widetilde{H}_{rs}^{(\sigma-1)/\sigma} + \sum_r \sum_t \tilde{X}_{rst}^{(\sigma-1)/\sigma} \end{matrix} \right\}^{\sigma/(\sigma-1)}, \tag{5}$$

where

$\widehat{D}_{ds}$ is quantity of the commodity produced in country s and sold domestically by firm d,

$\hat{E}_{ers}$ is quantity of the commodity produced in country r and exported to country s by firm e,

$\widehat{H}_{hrs}$ is quantity of the commodity produced in country s and sold locally by firm h headquartered in country r,

$\hat{X}_{xrst}$ is quantity of the commodity produced in country t and exported to country s by firm x headquartered in country r,

$\widetilde{D}_s$ is overall quantity of the commodity produced in country s and sold domestically by firms belonging to the set D,

$\tilde{E}_{rs}$ is overall quantity of the commodity produced in country r and exported to country s by firms belonging to the set E,

$\widetilde{H}_{rs}$ is overall quantity of the commodity produced in country s and sold locally by firms belonging to the set H,

$\tilde{X}_{rst}$ is overall quantity of the commodity produced in country t and exported to country s by firms belonging to the set X,

Q_s is final demand for the differentiated commodity in country s,

σ is the elasticity of substitution between varieties from different sources (firm d,

[1] The hat "^" and tilde "~" symbols respectively indicate the variable is at the firm level and at the aggregate level with respect to the firms. The variables without those symbols we will see later are those at the average firm level.

firm e, firm h, firm x, country r, and country t) such that $\sigma > 1$,

δ_s^D is the weight parameter that reflects preference of the representative consumer in country s for the commodity sold by firms belonging to the set D,

δ_{rs}^E is the weight parameter that reflects preference of the representative consumer in country s for the commodity imported from country r by firms belonging to the set E,

δ_{rs}^H is the weight parameter that reflects preference of the representative consumer in country s for the commodity sold by firms belonging to the set H,

δ_{rst}^X is the weight parameter that reflects preference of the representative consumer in country s for the commodity imported from country t by firms belonging to the set X, and

θ_s is the scaling factor of measuring units.

The representative consumer in country s determines the levels of $\hat{D}_{ds}$, $\hat{E}_{ers}$, $\hat{H}_{hrs}$, and $\hat{X}_{xrst}$ to minimize the total purchase value of the differentiated commodities subject to Equations (1) through (5). This problem can be expressed as

$$\begin{aligned} \min \quad & \textstyle\sum_d \hat{p}_{ds}^D \hat{D}_{ds} + \sum_r \sum_e (1+\upsilon_{rs}^M)(1+\tau_{rs}^M)\hat{p}_{ers}^E \hat{E}_{ers} \\ & + \textstyle\sum_r \sum_h \hat{p}_{hrs}^H \hat{H}_{hrs} + \sum_r \sum_t \sum_x (1+\upsilon_{st}^N)(1+\tau_{st}^N)\hat{p}_{xrst}^X \hat{X}_{xrst} \\ \text{s.t.} \quad & Q_s = \theta_s \left\{ \begin{matrix} \delta_s^D \sum_d \hat{D}_{ds}^{(\sigma-1)/\sigma} + \sum_r \delta_{rs}^E \sum_e \hat{E}_{ers}^{(\sigma-1)/\sigma} \\ + \sum_r \delta_{rs}^H \sum_h \hat{H}_{hrs}^{(\sigma-1)/\sigma} + \sum_r \sum_t \delta_{rst}^X \sum_x \hat{X}_{xrst}^{(\sigma-1)/\sigma} \end{matrix} \right\}^{\sigma/(\sigma-1)}, \end{aligned} \tag{6}$$

where

$\hat{p}_{ds}^D$ is the differentiated sales price of the commodity produced in country s and sold domestically by firm d,

$\hat{p}_{ers}^E$ is the differentiated sales price of the commodity produced in country r and exported to country s by firm e,

$\hat{p}_{hrs}^H$ is differentiated sales price of the commodity produced in country s and sold locally by firm h headquartered in country r,

$\hat{p}_{xrst}^X$ is differentiated sales price of the commodity produced in country t and exported to country s by firm x headquartered in country r,

τ_{rs}^M is the rate of transportation margin on the commodity sold on the r-s link,

τ_{st}^N is the rate of transportation margin on the commodity sold on the s-t link,

υ_{rs}^M is the rate of import tariff levied on the commodity sold on the r-s link, and

υ_{st}^N is the rate of import tariff levied on the commodity sold on the s-t link.

Equation (6) is derived by substituting (1) through (4) into (5). Setting the Lagrange multiplier for (6) as p_s, we get the following first-order conditions (FOCs):

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_s^D \left(\frac{Q_s}{\hat{D}_{ds}}\right)^{1/\sigma} = \hat{p}_{ds}^D, \tag{7}$$

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_{rs}^E \left(\frac{Q_s}{\hat{E}_{ers}}\right)^{1/\sigma} = (1+\upsilon_{rs}^M)(1+\tau_{rs}^M)\hat{p}_{ers}^E, \tag{8}$$

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_{rs}^H \left(\frac{Q_s}{\hat{H}_{hrs}}\right)^{1/\sigma} = \hat{p}_{hrs}^H, \tag{9}$$

and

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_{rst}^X \left(\frac{Q_s}{\hat{X}_{xrst}}\right)^{1/\sigma} = (1+\upsilon_{st}^N)(1+\tau_{st}^N)\hat{p}_{xrst}^X. \tag{10}$$

Since the value of a Lagrange multiplier can be interpreted as the shadow price at the optimal solution, p_s represents the price index for composite units of the differentiated commodity inclusive of transportation margin and import tariff.

2.3 Profits and Markup Prices

In the model, a firm established in a market country r chooses an operational strategy based on its productivity level. The least productive firms go out of business since they are incapable of generating positive operating profits. Other poorly productive firms serve only the domestic market. The more productive firms serve both the domestic and foreign markets, while their channel to have access to a foreign market differs. Although the economic conditions in fixed costs to setup production facilities, transportation costs or tariffs on international trade, and costs to hire local production factor affect, the most productive firms tend to choose investing abroad whereas the less productive group chooses exporting from their home country. Note that no firm engages in multiple activities to serve the same foreign market in an equilibrium.

Then, aggregated total profit of all firms operating in the IRTS sector in country r, π_r, can be expressed as

$$\pi_r = \sum_d \hat{\pi}_{dr}^D + \sum_s \sum_e \hat{\pi}_{ers}^E + \sum_s \sum_h \hat{\pi}_{hrs}^H + \sum_s \sum_t \sum_x \hat{\pi}_{xrst}^X - \sum_a w_r F_r^B, \tag{11}$$

where

$\hat{\pi}_{dr}^D$ is the contribution of firm d to the total profit from its domestic sales in country r,

$\hat{\pi}_{ers}^E$ is the contribution of firm e to the total profit from its export sales in country s,

$\hat{\pi}^{H}_{hrs}$ is the contribution of firm h to the total profit from its local sales in country s,

$\hat{\pi}^{X}_{xrst}$ is the contribution of firm x to the total profit from its export sales in country s of the commodity produced in non-market country t,

w_r is the rental price of the production factor in market country r, and

F^{B}_{r} is the fixed entry costs, measured in factor units, required to establish a firm in country r.

In the environment where all of the differentiated commodities are imperfect substitutes for the representative consumer, and aggregated based on the constant elasticity of substitution (CES) function as expressed by Equation (6), imperfectly competitive firms with the IRTS technology set the differentiated sales prices of their products based on the following markup rule:

$$\hat{p}^{D}_{dr} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_r}{\hat{\varphi}^{D}_{dr}}, \tag{12}$$

$$\hat{p}^{E}_{ers} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_r}{\hat{\varphi}^{E}_{ers}}, \tag{13}$$

$$\hat{p}^{H}_{hrs} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_s}{\hat{\varphi}^{H}_{hrs}}, \tag{14}$$

and

$$\hat{p}^{X}_{xrst} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w^{N}_{t}}{\hat{\varphi}^{X}_{xrst}}, \tag{15}$$

where

$\hat{\varphi}^{D}_{dr}$ is the productivity of firm d operating in country r,

$\hat{\varphi}^{E}_{ers}$ is the productivity of firm e exporting from country r to country s,

$\hat{\varphi}^{H}_{hrs}$ is the productivity of firm h that has local affiliate in market country s,

$\hat{\varphi}^{X}_{xrst}$ is the productivity of firm x that has local affiliate in non-market country t and exporting to market country s, and

w^{N}_{t} is the rental price of the production factor in non-market country t.

With the markup prices shown above, profit by firm type, $\hat{\pi}^{D}_{dr}$, $\hat{\pi}^{E}_{ers}$, $\hat{\pi}^{H}_{hrs}$, and $\hat{\pi}^{X}_{xrst}$, can be expressed as follows:

$$\hat{\pi}^{D}_{dr} = \left(\hat{p}^{D}_{dr} - \frac{w_r}{\hat{\varphi}^{D}_{dr}}\right)\hat{D}_{dr} - w_r F^{D}_{r}, \tag{16}$$

$$\hat{\pi}^{E}_{ers} = \left(\hat{p}^{E}_{ers} - \frac{w_r}{\hat{\varphi}^{E}_{ers}}\right)\hat{E}_{ers} - w_r F^{E}_{rs}, \tag{17}$$

$$\hat{\pi}^H_{hrs} = \left(\hat{p}^H_{hrs} - \frac{w_s}{\hat{\varphi}^H_{hrs}}\right)\hat{H}_{hrs} - w_s F^H_{rs}, \tag{18}$$

and

$$\hat{\pi}^X_{xrst} = \left(\hat{p}^X_{xrst} - \frac{w^N_t}{\hat{\varphi}^X_{xrst}}\right)\hat{X}_{xrst} - w^N_t F^X_{rst}, \tag{19}$$

where

F^D_r is the additional fixed costs to make sales at least on the domestic market in country r,

F^E_{rs} is the additional fixed costs to have access to a foreign market on the r-s link via exporting,

F^H_{rs} is the additional fixed costs to have access to a foreign market on the r-s link via horizontal FDI, and

F^X_{rst} is the additional fixed costs to have access to a foreign market on the r-s link via export-platform FDI through the non-market third country t.

Every kind of fixed costs are measured in factor units. F^D_r is required to activate the firm entered in country r. F^E_{rs} can be regarded as the costs of forming a distribution and servicing network in the foreign market. Similar costs for the home market are included in F^D_r. F^H_{rs} and F^X_{rst} include the costs of establishing an affiliate in a foreign country in addition to the costs of network formation. F^X_{rst} include the network formation costs both in the host and destination market countries.

To model the difficulties in choosing three kinds of operational strategy to get access to a foreign market, we place the following assumption on the productivity levels of firms that choose different strategies:

$$\hat{\varphi}^E_{ers} < \hat{\varphi}^H_{hrs} < \hat{\varphi}^X_{xrst}. \tag{20}$$

2.4 Distribution of Heterogeneous Firms and Productivity Levels

As explained previously, a firm bears the fixed entry costs F^B_r to enter the IRTS industry in a market country r. A successful entrant then draws an input coefficient $\hat{\varphi}_{ar}$ from a distribution $G(\hat{\varphi}_{ar})$. Following Melitz (2003) and Helpman *et al.* (2004), we assume the firm-specific productivity $\hat{\varphi}_{ar}$ is drawn from a Pareto distribution with the following properties:

Cumulative Distribution Function: $$G(\hat{\varphi}_{ar}) = 1 - \left(\frac{\varphi^*}{\hat{\varphi}_{ar}}\right)^{\gamma}, \tag{21}$$

and

Probability Density Function: $$g(\hat{\varphi}_{ar}) = \frac{\gamma}{\hat{\varphi}_{ar}}\left(\frac{\varphi^*}{\hat{\varphi}_{ar}}\right)^{\gamma}, \quad (22)$$

where

γ is the shape parameter such that $\gamma > \sigma - 1$, and

φ^* is the scale parameter.

φ^* defines the minimum possible level of the productivity that a successful entrant may draw. In this study, we set $\varphi^* = 1$ for simplicity.

$G(\varphi)$ defines the probability (proportion) to take a certain value less than or equal to φ. Then, the probability for a successful entrant to have a productivity level greater than or equal to the cut-off level of the productivity $\tilde{\varphi}^o$ $(o = D, E, H, X)$ required to have access to the domestic or a foreign market with an operational type becomes $1 - G(\tilde{\varphi}^o)$. As defined by Melitz (2003), let us assume the average levels of productivity $\bar{\varphi}^o$ $(o = D, E, H, X)$, with which a firm has "potential" to make sales on the domestic or a foreign market, is determined by the following function:

$$\bar{\varphi}^o(\tilde{\varphi}^o) = \left\{\frac{1}{1-G(\tilde{\varphi}^o)}\int_{\tilde{\varphi}^o}^{\infty} \varphi^{\sigma-1} g(\varphi)\mathrm{d}\varphi\right\}^{\frac{1}{\sigma-1}}. \quad (23)$$

Substituting Equations (21) and (22) into (23) and after some manipulation, we obtain:

$$\bar{\varphi}^o(\tilde{\varphi}^o) = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}^o. \quad (24)$$

Since we assume $\sigma > 1$ as well as $\gamma > \sigma - 1$, $\gamma > \gamma - \sigma + 1 > 0$. $\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}} > 1$ then derives $\bar{\varphi}^o > \tilde{\varphi}^o \geq 1$.

The proportion of firms, which have "potential" to make sales on the domestic or a foreign market with a productivity level above the cut-off level $\tilde{\varphi}^o$, accounted for in the successful entrants $\bar{\eta}^o$ $(o = D, E, H, X)$ are defined as follows:

$$\bar{\eta}^o = (\tilde{\varphi}^o)^{-\gamma} \quad (25)$$

Since $\varphi^* = 1$, $\bar{\eta}^o \in (0, 1]$ is ensured by $\tilde{\varphi}^o \in [1, \infty)$. On the other hand, the "actual" proportion η^o $(o = D, E, H, X)$ may deviate from its "potential" level, because of the assumption that firms may not engage in multiple activities to have access to the same foreign market. Under this assumption, firms must choose one strategy from direct exporting (E), horizontal FDI (H), or export-platform FDI (X) in an equilibrium. Then, the "actual" proportion for each type of activity becomes

$$\eta_r^D = \bar{\eta}_r^D = (\tilde{\varphi}_r^D)^{-\gamma}, \quad (26)$$

$$\eta_{rs}^E = \bar{\eta}_{rs}^E - \bar{\eta}_{rs}^H = (\tilde{\varphi}_{rs}^E)^{-\gamma} - (\tilde{\varphi}_{rs}^H)^{-\gamma}, \quad (27)$$

$$\eta_{rs}^H = \bar{\eta}_{rs}^H - \bar{\eta}_{rst}^X = (\tilde{\varphi}_{rs}^H)^{-\gamma} - (\tilde{\varphi}_{rst}^X)^{-\gamma}, \quad (28)$$

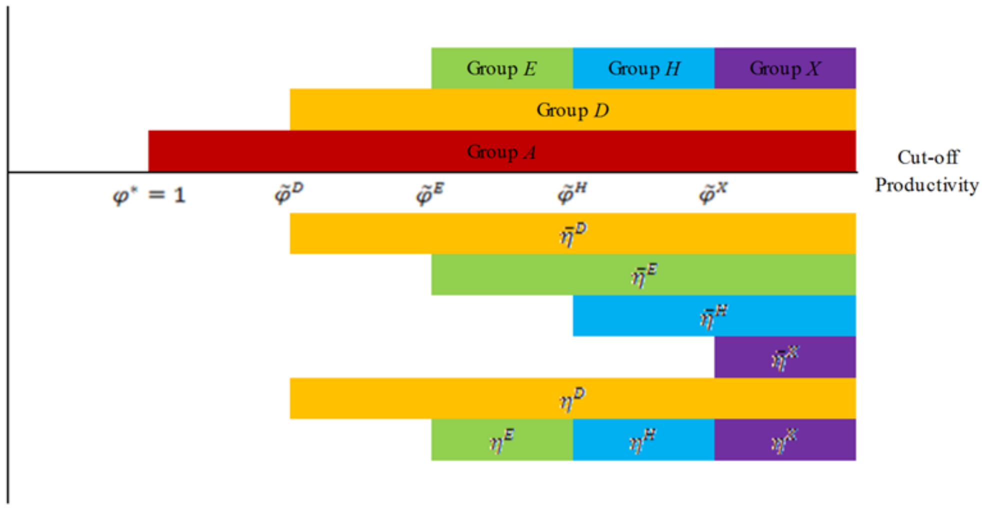
and

$$\eta_{rst}^{X} = \bar{\eta}_{rst}^{X} = (\tilde{\varphi}_{rst}^{X})^{-\gamma}. \tag{29}$$

Note that a non-market country, which can be approached with the lowest level of productivity, will be chosen for the subscript "t" of $\bar{\eta}_{rst}^{X}$ and $\tilde{\varphi}_{rst}^{X}$ in Equation (28). Since we currently consider only one non-market country, this kind of selection problem that might require special treatments in the program code will not arise in this study.

An important point is that Equation (24) does not calculate the average productivity level of firms in the "actual" activities choosing one strategy to have access to a foreign market, but the average of all of the firms, which are "capable" of taking the corresponding strategy. For instance, a firm that has a sufficient level of productivity to take the export-platform strategy also is a "potential" firm, which "can" take the horizontal strategy. In the model, firms are assumed to "actually" take the strategy, which requires higher productivity level, from the "potential" choices. In consequence, the average productivity levels of both "actual" and "potential" firms coincide with each other when the firms belong to the sets D and X (not necessarily at the same time), while some attentions to distinguish two kinds of average productivity are needed for the firms belonging to the sets E and H. Figure 2 shows the relationships between the "potential" choices of operational strategy and the "actual" activities.

Figure 2: Correspondence between the "Potential" and "Actual" Choices



2.5 Average Productivity and Average Sales Quantity

Let us start with deriving the average productivity level of "potential" firms, which are "capable" of taking each strategy. First, we consider the firms belonging to the set D, which can sell products in the domestic market. To handle two variables that are not directly related to the individual level of productivity of each firm, the cut-off productivity level for entry and the average productivity level of the whole group, let us purge the firm index d from the endogenous variables and equations. Specifically, $\hat{p}_{dr}^D$ and $\widehat{D}_{dr}$ are eliminated from Equations (7), (12), and (16). Substituting (12) into (7) and (16), respectively, we obtain

$$\widehat{D}_{dr} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} {\theta_r}^{\sigma-1} (\delta_r^D)^{\sigma} {p_r}^{\sigma} Q_r \left(\frac{\hat{\varphi}_{dr}^D}{w_r}\right)^{\sigma}, \tag{30}$$

and

$$\hat{\pi}_{dr}^D = \frac{1}{\sigma-1}\left(\frac{w_r}{\hat{\varphi}_{dr}^D}\right)\widehat{D}_{dr} - w_r F_r^D. \tag{31}$$

Plugging (30) to (31), we get

$$\hat{\pi}_{dr}^D = \sigma^{-\sigma}(\sigma-1)^{\sigma-1} {\theta_r}^{\sigma-1} (\delta_r^D)^{\sigma} {p_r}^{\sigma} Q_r \left(\frac{\hat{\varphi}_{dr}^D}{w_r}\right)^{\sigma-1} - w_r F_r^D. \tag{32}$$

The cut-off level of productivity $\tilde{\varphi}_r^D$ required for a firm in country r to be active at least in the domestic market is determined at the level that satisfies $\hat{\pi}_{dr}^D = 0$. Using (32), we obtain

$$\tilde{\varphi}_r^D = \frac{1}{(\sigma-1)\theta_r}\left(\frac{\sigma}{\delta_r^D}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{w_r}{p_r}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{F_r^D}{Q_r}\right)^{\frac{1}{\sigma-1}}. \tag{33}$$

Based on Equation (24), the relationship between the cut-off level $\tilde{\varphi}_r^D$ and the average productivity of all of the firms ("capable" of) serving the domestic market (entire firms belonging to the set D) $\bar{\varphi}_r^D$ is defined as

$$\bar{\varphi}_r^D = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_r^D. \tag{34}$$

Plugging (33) to (34) derives the following relation:

$$\bar{\varphi}_r^D = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}} \frac{1}{(\sigma-1)\theta_r}\left(\frac{\sigma}{\delta_r^D}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{w_r}{p_r}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{F_r^D}{Q_r}\right)^{\frac{1}{\sigma-1}}. \tag{35}$$

Since Equation (30) holds even with the average productivity of firms $\bar{\varphi}_r^D$ and the average sales quantity of a firm $\overline{D}_r$, respectively belonging to the set D, instead of $\hat{\varphi}_{dr}^D$ and $\widehat{D}_{dr}$, we erase w_r and p_r in Equation (35). Let us first rewrite (30) as follows:

$$\frac{w_r}{p_r} = \left(\frac{\sigma-1}{\sigma}\right) {\theta_r}^{\frac{\sigma-1}{\sigma}} \delta_r^D \left(\frac{Q_r}{\overline{D}_r}\right)^{\frac{1}{\sigma}} \bar{\varphi}_r^D.$$

Substituting this equation to (35) and after some manipulation, we get $\bar{\varphi}_r^D$:

$$\bar{\varphi}_r^D = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{D}_r}{F_r^D}\right), \tag{36}$$

where

$\bar{\varphi}_r^D$ is the average productivity level of ("potential") firms ("capable" of) serving the domestic market, and

$\bar{D}_r$ is the average sales quantity of a ("potential") firm ("capable" of) serving the domestic market.

Following similar steps, $\bar{\varphi}_{rs}^E$, $\bar{\varphi}_{rs}^H$, and $\bar{\varphi}_{rst}^X$ can be derived:

$$\bar{\varphi}_{rs}^E = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{E}_{rs}}{F_{rs}^E}\right), \tag{37}$$

$$\bar{\varphi}_{rs}^H = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{H}_{rs}}{F_{rs}^H}\right), \tag{38}$$

and

$$\bar{\varphi}_{rst}^X = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{X}_{rst}}{F_{rst}^X}\right), \tag{39}$$

where

$\bar{\varphi}_{rs}^E$ is the average productivity level of "potential" firms "capable" of serving a foreign market via direct exporting,

$\bar{\varphi}_{rs}^H$ is the average productivity level of "potential" firms "capable" of serving a foreign market via horizontal FDI,

$\bar{\varphi}_{rst}^X$ is the average productivity level of ("potential") firms ("capable" of) serving a foreign market via export-platform FDI,

$\bar{E}_{rs}$ is the average sales quantity of a "potential" firm "capable" of serving a foreign market via direct exporting,

$\bar{H}_{rs}$ is the average sales quantity of a "potential" firm "capable" of serving a foreign market via horizontal FDI, and

$\bar{X}_{rst}$ is the average sales quantity of a ("potential") firm ("capable" of) serving a foreign market via export-platform FDI.

Tedious but keep in mind that $\bar{\varphi}_r^D$, $\bar{\varphi}_{rs}^E$, $\bar{\varphi}_{rs}^H$, $\bar{\varphi}_{rst}^X$, $\bar{D}_r$, $\bar{E}_{rs}$, $\bar{H}_{rs}$, and $\bar{X}_{rst}$ may not necessarily be the variables for the firms "actually" taking each operation strategy, but just the ones for the "potential" firms "capable" of taking each action.

The cut-off level of productivity to enable firms to take each operational strategy can be derived substituting Equations (36) through (39) respectively to (34):

$$\tilde{\varphi}_r^D = \frac{1}{\sigma-1}\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\bar{D}_r}{F_r^D}\right), \tag{40}$$

$$\tilde{\varphi}_{rs}^E = \frac{1}{\sigma-1}\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\bar{E}_{rs}}{F_{rs}^E}\right), \tag{41}$$

$$\tilde{\varphi}_{rs}^H = \frac{1}{\sigma-1}\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\bar{H}_{rs}}{F_{rs}^H}\right), \tag{42}$$

and

$$\tilde{\varphi}_{rst}^X = \frac{1}{\sigma-1}\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\bar{X}_{rst}}{F_{rst}^X}\right), \tag{43}$$

where

$\tilde{\varphi}_r^D$ is the lowest level of productivity required to serve the domestic market,

$\tilde{\varphi}_{rs}^E$ is the lowest level of productivity required to serve a foreign market via direct exporting,

$\tilde{\varphi}_{rs}^H$ is the lowest level of productivity required to serve a foreign market via horizontal FDI, and

$\tilde{\varphi}_{rst}^X$ is the lowest level of productivity required to serve a foreign market via export-platform FDI.

Next, let us derive the indices for the average productivity level and sales quantity with respect to the firms "actually" taking each operation strategy. First, the average productivity levels φ_r^D, φ_{rs}^E, φ_{rs}^H, and φ_{rst}^X can be expressed by the following four equations using the indices with respect to the "potential" firms:[2]

$$\varphi_r^D = \bar{\varphi}_r^D, \tag{44}$$

$$\varphi_{rs}^E = \frac{\bar{\eta}_{rs}^E\bar{\varphi}_{rs}^E - \bar{\eta}_{rs}^H\bar{\varphi}_{rs}^H}{\bar{\eta}_{rs}^E - \bar{\eta}_{rs}^H}, \tag{45}$$

$$\varphi_{rs}^H = \frac{\bar{\eta}_{rs}^H\bar{\varphi}_{rs}^H - \bar{\eta}_{rst}^X\bar{\varphi}_{rst}^X}{\bar{\eta}_{rs}^H - \bar{\eta}_{rst}^X}, \tag{46}$$

and

$$\varphi_{rst}^X = \bar{\varphi}_{rst}^X. \tag{47}$$

As in Equation (28), a non-market country, which can be approached with the lowest level of productivity, will be chosen for the subscript "t" of $\bar{\eta}_{rst}^X$ and $\bar{X}_{rst}$ in Equation (46).

[2] Although there may still be a question on the appropriateness to handle productivity index as the one, which is operable through making a total mass of productivity multiplying the average productivity level by the number of firms, taking differences between those total masses of productivity for different groups, or deriving new levels of average productivity dividing the differences by the number of firms, we tentatively use these indices in this study. When we find problems in using these indices, we will try to find better approach. For the average sales quantity, problems are expected to less than the case of productivity.

In a similar manner, the average sales quantity per firm in each "actual" activity D_r, E_{rs}, H_{rs}, and X_{rst} can be expressed as follows:

$$D_r = \bar{D}_r, \tag{48}$$

$$E_{rs} = \frac{\bar{\eta}_{rs}^{E}\bar{E}_{rs} - \bar{\eta}_{rs}^{H}\bar{H}_{rs}}{\bar{\eta}_{rs}^{E} - \bar{\eta}_{rs}^{H}}, \tag{49}$$

$$H_{rs} = \frac{\bar{\eta}_{rs}^{H}\bar{H}_{rs} - \bar{\eta}_{rst}^{X}\bar{X}_{rst}}{\bar{\eta}_{rs}^{H} - \bar{\eta}_{rst}^{X}}, \tag{50}$$

and

$$X_{rst} = \bar{X}_{rst}. \tag{51}$$

The treatment of the subscript "t" of $\bar{\eta}_{rst}^{X}$ and $\bar{X}_{rst}$ in Equation (50) also follows the cases of (28) and (46).

2.6 Number of Firm Entry

In this subsection, we derive a conditional expression to determine the number of successful entrant N_r, which implies firms belonging to the set A, in country r. Respectively substituting Equations (12) through (15) into (16) through (19), we get the following four equations:

$$\hat{\pi}_{dr}^{D} = \left(\frac{1}{\sigma}\right)\hat{p}_{dr}^{D}\hat{D}_{dr} - w_r F_r^D, \tag{52}$$

$$\hat{\pi}_{ers}^{E} = \left(\frac{1}{\sigma}\right)\hat{p}_{ers}^{E}\hat{E}_{ers} - w_r F_{rs}^E, \tag{53}$$

$$\hat{\pi}_{hrs}^{H} = \left(\frac{1}{\sigma}\right)\hat{p}_{hrs}^{H}\hat{H}_{hrs} - w_s F_{rs}^H, \tag{54}$$

and

$$\hat{\pi}_{xrst}^{X} = \left(\frac{1}{\sigma}\right)\hat{p}_{xrst}^{X}\hat{X}_{xrst} - w_t F_{rst}^X. \tag{55}$$

Plugging these (52) through (55) to (11), we obtain

$$\pi_r = \frac{1}{\sigma}\begin{pmatrix} \sum_d \hat{p}_{dr}^{D}\hat{D}_{dr} \\ +\sum_s\sum_e \hat{p}_{ers}^{E}\hat{E}_{ers} \\ +\sum_s\sum_h \hat{p}_{hrs}^{H}\hat{H}_{hrs} \\ +\sum_s\sum_t\sum_x \hat{p}_{xrst}^{X}\hat{X}_{xrst} \end{pmatrix} - \begin{pmatrix} \sum_a w_r F_r^B \\ +\sum_d w_r F_r^D \\ +\sum_s\sum_e w_r F_{rs}^E \\ +\sum_s\sum_h w_s F_{rs}^H \\ +\sum_s\sum_t\sum_x w_t F_{rst}^X \end{pmatrix}.$$

The number of successful entrant N_r is determined at the level that satisfies $\pi_r = 0$, although N_r does not show up in the expression. To derive the equation which is included in the model, the variables related to each individual firm, $\hat{p}^{D}_{dr}$, $\hat{p}^{E}_{ers}$, $\hat{p}^{H}_{hrs}$, $\hat{p}^{X}_{xrst}$, $\widehat{D}_{dr}$, $\hat{E}_{ers}$, $\widehat{H}_{hrs}$, and $\hat{X}_{xrst}$ are respectively replaced by the average sales price set by the firms "actually" taking each operation strategy, p^{D}_{r}, p^{E}_{rs}, p^{H}_{rs}, and p^{X}_{rst}, and the average sales quantity of those "actual" firms, D_r, E_{rs}, H_{rs}, and X_{rst}. Furthermore, the summation parts are expressed with the proportion of firms that "actually" take each operational strategy in the entire entrants, η^{D}_{r}, η^{E}_{rs}, η^{H}_{rs}, and η^{X}_{rst}. Then, we obtain

$$w_r(F^B_r + \eta^D_r F^D_r + \textstyle\sum_s \eta^E_{rs} F^E_{rs}) + \sum_s \eta^H_{rs} w_s F^H_{rs} + \sum_s \sum_t \eta^X_{rst} w^N_t F^X_{rst}$$

$$= \frac{1}{\sigma}(\eta^D_r p^D_r D_r + \textstyle\sum_s \eta^E_{rs} p^E_{rs} E_{rs} + \sum_s \eta^H_{rs} p^H_{rs} H_{rs} + \sum_s \sum_t \eta^X_{rst} p^X_{rst} X_{rst}). \quad (56)$$

2.7 Representative Consumer and Final Demand

The representative consumer in market country s maximizes his/her welfare U_s subject to the budget constraint, given as the total factor income and tariff revenue transferred from the national authority. In this setting, we presume that the current account is always balanced for simplicity. This problem can be expressed as follows:

$$\max \quad U_s = \rho_s {Q_s}^{\alpha_s} {C_s}^{1-\alpha_s}$$

$$\text{s.t.} \quad p_s Q_s + p^C C_s = w_s \bar{\bar{L}}_s + T_s, \quad (57)$$

where

C_s is final demand for the homogeneous good in market country s,

p^C is the international price of the homogeneous good produced in every country,

α_s is the share of the differentiated commodity in total consumption,

ρ_s is the scaling factor of measuring units,

$\bar{\bar{L}}_s$ is the given factor endowment of market country s, and

T_s is the tariff revenue, defined as

$$T_s = \textstyle\sum_r v^M_{rs}(1+\tau^M_{rs})\, N_r \eta^E_{rs} p^E_{rs} E_{rs} + \sum_r \sum_t v^N_{st}(1+\tau^N_{st})\, N_r \eta^X_{rst} p^X_{rst} X_{rst}.$$

Setting the Lagrange multiplier for (57) as λ_s, we get the following FOCs:

$$\alpha_s \rho_s \left(\frac{Q_s}{C_s}\right)^{\alpha_s - 1} = \lambda_s p_s, \quad (58)$$

and

$$(1-\alpha_s)\rho_s \left(\frac{Q_s}{C_s}\right)^{\alpha_s} = \lambda_s p^C. \quad (59)$$

Note that λ_s can be regarded as the total change in consumption given a unit increase of income.

The representative consumer in non-market country t maximizes his/her welfare

U_t^N subject to the budget constraint, given as the total factor income. As in the case of market countries, the current account is always balanced. This problem can be expressed as follows:

$$\max \quad U_t^N = C_t^N$$
$$\text{s.t.} \quad p^C C_t^N = w_t^N \bar{\bar{L}}_t^N, \tag{60}$$

where

C_t^N is final demand for the homogeneous good in non-market country t, and

$\bar{\bar{L}}_t^N$ is the given factor endowment of non-market country t.

Setting the Lagrange multiplier for (60) as λ_t^N, we get the following FOC:

$$1 = \lambda_t^N p^C. \tag{61}$$

2.8 Market Equilibrium

To close the model, we introduce three equilibrium conditions for the markets of the production factor and the homogeneous commodity. The factor market equilibrium for market country r is given by

$$\sum_d \frac{\hat{D}_{dr}}{\hat{\varphi}_{dr}^D} + \sum_s \sum_e \frac{\hat{E}_{ers}}{\hat{\varphi}_{ers}^E} + \sum_s \sum_h \frac{\hat{H}_{hsr}}{\hat{\varphi}_{hsr}^H} + L_r^C$$
$$= \bar{\bar{L}}_r - \left(\sum_a F_r^B + \sum_d F_r^D + \sum_s \sum_e F_{rs}^E + \sum_s \sum_h F_{sr}^H\right),$$

where

L_r^C is factor input for the homogeneous good production in market country r.

As noted previously, fixed costs to establish a firm, start making sales at least in the domestic market, and start serving a foreign market via direct exporting or FDI are all regarded as the ones that consume the production factor. Since we presume that the fixed costs are generated in the country where the headquarters of a firm or local affiliate is located, the costs related to export-platform FDI does not enter in the equation shown above, but in the following factor market equilibrium condition for non-market country t:

$$\sum_r \sum_s \sum_x \frac{\hat{X}_{xrst}}{\hat{\varphi}_{xrst}^X} + L_t^{NC} = \bar{\bar{L}}_t^N - \sum_r \sum_s \sum_x F_{rst}^X,$$

where

L_t^{NC} is factor input for the homogeneous good production in non-market country t.

The equilibrium conditions for the factor markets actually included in the model are derived by replacing the variables related to each individual firm, $\hat{D}_{dr}$, $\hat{E}_{ers}$, $\hat{H}_{hrs}$, $\hat{X}_{xrst}$, $\hat{\varphi}_{dr}^D$, $\hat{\varphi}_{ers}^E$, $\hat{\varphi}_{hrs}^H$, and $\hat{\varphi}_{xrst}^X$ with the average sales quantity and average productivity level of firms "actually" taking each operation strategy, D_r, E_{rs}, H_{rs}, X_{rst}, φ_r^D, φ_{rs}^E, φ_{rs}^H, and φ_{rst}^X, and using the proportion of those firms in the entire entrants, η_r^D, η_{rs}^E, η_{rs}^H, and η_{rst}^X along with the number of successful entrants N_r:

$$N_r \left\{ \begin{matrix} F_r^B + \eta_r^D \left(\frac{D_r}{\varphi_r^D} + F_r^D \right) \\ + \sum_s \eta_{rs}^E \left(\frac{E_{rs}}{\varphi_{rs}^E} + F_{rs}^E \right) \end{matrix} \right\} + \sum_s N_s \eta_{sr}^H \left(\frac{H_{sr}}{\varphi_{sr}^H} + F_{sr}^H \right) + L_r^C = \bar{\bar{L}}_r, \tag{62}$$

and

$$\sum_r \sum_s N_r \left(\frac{X_{rst}}{\varphi_{rst}^X} + F_{rst}^X \right) + L_t^{NC} = \bar{\bar{L}}_t^N. \tag{63}$$

Finally, the market equilibrium for the homogeneous commodity is defined as

$$\sum_s C_s + \sum_t C_t^N + \frac{1}{p^C} \sum_r \sum_s \left(\begin{matrix} \tau_{rs}^M N_r \eta_{rs}^E p_{rs}^E E_{rs} \\ + \sum_t \tau_{st}^N N_r \eta_{rst}^X p_{rst}^X X_{rst} \end{matrix} \right) = \sum_r \psi_r L_r^C + \sum_t \psi_t^N L_t^{NC}, \tag{64}$$

where

ψ_r is given factor productivity to produce one unit of homogeneous good in market country r, and

ψ_t^N is given factor productivity to produce one unit of homogeneous good in non-market country t.

Notice that the homogeneous commodity is supplied as the international shipping service for transportation. Equation (64) also implies that the homogeneous commodity is internationally traded as perfect substitutes. Since we handle the homogeneous commodity as our numéraire, (64) is dropped from the model instead of setting p^C, which level is determined by (64), to unity, because *Walras' law* holds in this general equilibrium framework.

2.9 System Equations

Based on the equations explained above, the system of equations to be included in the model can be summarized as follows:

$$Q_s = \theta_s \left\{ \begin{matrix} \delta_s^D N_s \eta_s^D D_s^{(\sigma-1)/\sigma} + \sum_r \delta_{rs}^E N_r \eta_{rs}^E E_{rs}^{(\sigma-1)/\sigma} \\ + \sum_r \delta_{rs}^H N_r \eta_{rs}^H H_{rs}^{(\sigma-1)/\sigma} + \sum_r \sum_t \delta_{rst}^X N_r \eta_{rst}^X X_{rst}^{(\sigma-1)/\sigma} \end{matrix} \right\}^{\sigma/(\sigma-1)}, \tag{65}$$

$$p_r^D = \left(\frac{\sigma}{\sigma-1} \right) \frac{w_r}{\varphi_r^D}, \tag{66}$$

$$p_{rs}^{E} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_r}{\varphi_{rs}^{E}}, \tag{67}$$

$$p_{rs}^{H} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_s}{\varphi_{rs}^{H}}, \tag{68}$$

$$p_{rst}^{X} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_t^N}{\varphi_{rst}^{X}}, \tag{69}$$

$$N_r \begin{Bmatrix} F_r^B + \eta_r^D \left(\frac{D_r}{\varphi_r^D} + F_r^D\right) \\ + \sum_s \eta_{rs}^E \left(\frac{E_{rs}}{\varphi_{rs}^E} + F_{rs}^E\right) \end{Bmatrix} + \sum_s N_s \eta_{sr}^H \left(\frac{H_{sr}}{\varphi_{sr}^H} + F_{sr}^H\right) + L_r^C = \bar{\bar{L}}_r, \tag{62}$$

$$\sum_r \sum_s N_r \left(\frac{X_{rst}}{\varphi_{rst}^X} + F_{rst}^X\right) + L_t^{NC} = \bar{\bar{L}}_t^N, \tag{63}$$

$$\alpha_s \rho_s \left(\frac{Q_s}{C_s}\right)^{\alpha_s - 1} = \lambda_s p_s, \tag{58}$$

$$(1-\alpha_s)\rho_s \left(\frac{Q_s}{C_s}\right)^{\alpha_s} = \lambda_s p^C, \tag{59}$$

$$1 = \lambda_t^N p^C, \tag{61}$$

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_s^D \left(\frac{Q_s}{D_s}\right)^{1/\sigma} = p_s^D, \tag{70}$$

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_{rs}^E \left(\frac{Q_s}{E_{rs}}\right)^{1/\sigma} = (1 + \upsilon_{rs}^M)(1 + \tau_{rs}^M) p_{rs}^E, \tag{71}$$

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_{rs}^H \left(\frac{Q_s}{H_{rs}}\right)^{1/\sigma} = p_{rs}^H, \tag{72}$$

$$p_s {\theta_s}^{(\sigma-1)/\sigma} \delta_{rst}^X \left(\frac{Q_s}{X_{rst}}\right)^{1/\sigma} = (1 + \upsilon_{st}^N)(1 + \tau_{st}^N) p_{rst}^X, \tag{73}$$

$$\psi_r p^C = w_r, \tag{74}$$

$$\psi_t^N p^C = w_t^N, \tag{75}$$

$$w_r(F_r^B + \eta_r^D F_r^D + \sum_s \eta_{rs}^E F_{rs}^E) + \sum_s \eta_{rs}^H w_s F_{rs}^H + \sum_s \sum_t \eta_{rst}^X w_t^N F_{rst}^X$$

$$= \frac{1}{\sigma}(\eta_r^D p_r^D D_r + \sum_s \eta_{rs}^E p_{rs}^E E_{rs} + \sum_s \eta_{rs}^H p_{rs}^H H_{rs} + \sum_s \sum_t \eta_{rst}^X p_{rst}^X X_{rst}), \tag{56}$$

$$\eta_r^D = \bar{\eta}_r^D, \tag{26}$$

$$\eta_{rs}^E = \bar{\eta}_{rs}^E - \bar{\eta}_{rs}^H, \tag{27}$$

$$\eta_{rs}^H = \bar{\eta}_{rs}^H - \bar{\eta}_{rst}^X, \tag{28}$$

$$\eta_{rst}^X = \bar{\eta}_{rst}^X, \tag{29}$$

$$\varphi_r^D = \bar{\varphi}_r^D, \tag{44}$$

$$\varphi_{rs}^{E} = \frac{\bar{\eta}_{rs}^{E}\bar{\varphi}_{rs}^{E} - \bar{\eta}_{rs}^{H}\bar{\varphi}_{rs}^{H}}{\bar{\eta}_{rs}^{E} - \bar{\eta}_{rs}^{H}}, \tag{45}$$

$$\varphi_{rs}^{H} = \frac{\bar{\eta}_{rs}^{H}\bar{\varphi}_{rs}^{H} - \bar{\eta}_{rst}^{X}\bar{\varphi}_{rst}^{X}}{\bar{\eta}_{rs}^{H} - \bar{\eta}_{rst}^{X}}, \tag{46}$$

$$\varphi_{rst}^{X} = \bar{\varphi}_{rst}^{X}, \tag{47}$$

$$D_r = \bar{D}_r, \tag{48}$$

$$E_{rs} = \frac{\bar{\eta}_{rs}^{E}\bar{E}_{rs} - \bar{\eta}_{rs}^{H}\bar{H}_{rs}}{\bar{\eta}_{rs}^{E} - \bar{\eta}_{rs}^{H}}, \tag{49}$$

$$H_{rs} = \frac{\bar{\eta}_{rs}^{H}\bar{H}_{rs} - \bar{\eta}_{rst}^{X}\bar{X}_{rst}}{\bar{\eta}_{rs}^{H} - \bar{\eta}_{rst}^{X}}, \tag{50}$$

$$X_{rst} = \bar{X}_{rst}, \tag{51}$$

$$\bar{\eta}_{r}^{D} = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\gamma}{\sigma-1}} (\bar{\varphi}_{r}^{D})^{-\gamma}, \tag{76}$$

$$\bar{\eta}_{rs}^{E} = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\gamma}{\sigma-1}} (\bar{\varphi}_{rs}^{E})^{-\gamma}, \tag{77}$$

$$\bar{\eta}_{rs}^{H} = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\gamma}{\sigma-1}} (\bar{\varphi}_{rs}^{H})^{-\gamma}, \tag{78}$$

$$\bar{\eta}_{rst}^{X} = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\gamma}{\sigma-1}} (\bar{\varphi}_{rst}^{X})^{-\gamma}, \tag{79}$$

$$\bar{\varphi}_{r}^{D} = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{D}_r}{F_r^D}\right), \tag{36}$$

$$\bar{\varphi}_{rs}^{E} = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{E}_{rs}}{F_{rs}^E}\right), \tag{37}$$

$$\bar{\varphi}_{rs}^{H} = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{H}_{rs}}{F_{rs}^H}\right), \tag{38}$$

$$\bar{\varphi}_{rst}^{X} = \frac{\gamma-\sigma+1}{\gamma(\sigma-1)}\left(\frac{\bar{X}_{rst}}{F_{rst}^X}\right), \tag{39}$$

$$p_s Q_s + p^C C_s = w_s \bar{\bar{L}}_s + T_s, \tag{57}$$

and

$$p^C C_t^N = w_t^N \bar{\bar{L}}_t^N. \tag{60}$$

To derive Equations (65) through (73) and (76) through (79), the variables related to each individual firm are replaced with the ones related to the "average" terms, as we did to obtain Equations (56), (62), and (63). Equations (74) and (75) are newly introduced to determine the consumption levels of homogeneous products. The levels of factor productivity in producing homogeneous commodity, ψ_r or ψ_t^N, affect the global economy through these equations. Then, given values of the exogenous variables and parameters, p^C, $\bar{\bar{L}}_r$, $\bar{\bar{L}}_t^N$, ψ_r, ψ_t^N, F_r^B, F_r^D, F_{rs}^E, F_{rs}^H, F_{rst}^X, τ_{rs}^M, τ_{st}^N, υ_{rs}^M, υ_{st}^N, γ, σ, δ_s^D, δ_{rs}^E, δ_{rs}^H, δ_{rst}^X, θ_s, α_s, and ρ_s, simultaneously solving the system of 39 equations shown above determines values of the 39 endogenous variables, p_s, p_r^D, p_{rs}^E, p_{rs}^H, p_{rst}^X, w_r, w_t^N, Q_s, C_s, C_t^N, D_s, E_{rs}, H_{rs}, X_{rst}, L_r^C, L_t^{NC}, N_r, η_r^D, η_{rs}^E, η_{rs}^H, η_{rst}^X, φ_r^D, φ_{rs}^E, φ_{rs}^H, φ_{rst}^X, $\bar{D}_r$, $\bar{E}_{rs}$, $\bar{H}_{rs}$, $\bar{X}_{rst}$, $\bar{\eta}_r^D$, $\bar{\eta}_{rs}^E$, $\bar{\eta}_{rs}^H$, $\bar{\eta}_{rst}^X$, $\bar{\varphi}_r^D$, $\bar{\varphi}_{rs}^E$, $\bar{\varphi}_{rs}^H$, $\bar{\varphi}_{rst}^X$, λ_s, and λ_t^N.

3. Parameterization

This section explains the procedure to set values of exogenous variables and behavioral parameters, which are required to program the analytical model as a tool for numerical simulations. Since information on initial values of endogenous variables is also necessary to parameterize the model, the parameterization procedure includes the process to obtain those initial values. In this study, we adopt the calibration procedure, which can be carried out with limited information and guarantees perfect consistency between data and the variables included in the model, for parameterization. On the other hand, the availability of data is not considered at all at this moment, since the present model remains a prototype under development. In the calibration process, all of the values of parameters, exogenous variables, and initial levels of endogenous variables are calculated from requisite minimum information.

3.1 Benchmark Data Set

The information to build a benchmark data set should include the Pareto shape parameter, γ, substitution elasticity for the CES aggregator related to the differentiated products from different sources, σ, *ad valorem* equivalent rates of transportation margin, τ_{rs}^M and τ_{st}^N, and import tariff, υ_{rs}^M and υ_{st}^N, consumption levels of homogeneous products, C_s and C_t^N, the value amount of the commodity produced in country r and sold on the domestic market, TF_r^D, the value amount of the commodity produced in market country r and exported to country s, TF_{rs}^E, the value amount of the commodity produced in market country s and sold locally by the firms headquartered in county r, TF_{rs}^H, and the value amount of the commodity produced in non-market country t and exported to country s by the firms headquartered in country r, TF_{rst}^X. This time, we assume that TF_r^D, TF_{rs}^E, TF_{rs}^H, and TF_{rst}^X are all exclusive of trade costs. All of the other information will be derived from these data or just set to unity following the usual cliché of calibrating a model (p^C, ψ_r, ψ_t^N, w_r, w_t^N, and N_r). While it may not be easy to obtain information on TF_{rs}^H or TF_{rst}^X from trade flow data, let us move forward assuming it was available by good fortune. The information on γ and σ can be found in empirical studies. Utilizing these data, 14 endogenous variables and parameters, $\bar{\bar{L}}_r$, $\bar{\bar{L}}_t^N$, F_r^B, F_r^D, F_{rs}^E, F_{rs}^H, F_{rst}^X, δ_s^D, δ_{rs}^E, δ_{rs}^H, δ_{rst}^X, θ_s, α_s, and ρ_s, as well as initial values of endogenous variables are calibrated.

3.2 Calibration Procedure

Suppose the four kinds of domestic and international trade flow data exclusive of trade costs, TF_r^D, TF_{rs}^E, TF_{rs}^H, and TF_{rst}^X, corresponds to the endogenous variables in the model as follows:[3]

$$TF_r^D = N_r \eta_r^D p_r^D D_r, \tag{80}$$

$$TF_{rs}^E = N_r \eta_{rs}^E p_{rs}^E E_{rs}, \tag{81}$$

$$TF_{rs}^H = N_r \eta_{rs}^H p_{rs}^H H_{rs}, \tag{82}$$

and

$$TF_{rst}^X = N_r \eta_{rst}^X p_{rst}^X X_{rst}. \tag{83}$$

Respectively plugging Equations (66) through (69) to (80) through (83), we obtain

$$D_r = \left(\frac{\sigma-1}{\sigma}\right) \frac{TF_r^D \varphi_r^D}{w_r N_r \eta_r^D}, \tag{84}$$

$$E_{rs} = \left(\frac{\sigma-1}{\sigma}\right) \frac{TF_{rs}^E \varphi_{rs}^E}{w_r N_r \eta_{rs}^E}, \tag{85}$$

$$H_{rs} = \left(\frac{\sigma-1}{\sigma}\right) \frac{TF_{rs}^H \varphi_{rs}^H}{w_s N_r \eta_{rs}^H}, \tag{86}$$

and

$$X_{rst} = \left(\frac{\sigma-1}{\sigma}\right) \frac{TF_{rst}^X \varphi_{rst}^X}{w_t^N N_r \eta_{rst}^X}. \tag{87}$$

Next, we obtain the following equation substituting (26), (44), and (48) into (84):

$$\bar{D}_r = \left(\frac{\sigma-1}{\sigma}\right) \frac{TF_r^D \bar{\varphi}_r^D}{w_r N_r \bar{\eta}_r^D} \tag{88}$$

In similar manners, we get three more equations substituting (27), (45), and (49), (28), (46), and (50), and (29), (47), and (51) into (85), (86), and (87), respectively.

[3] For the case one have to work with aggregated trade flow data, we do not exclude a possibility that r, s, or t is a region which consists of several countries and intra-regional international trade takes place within the region. In the numerical practice we will see later, we use artificial data that have positive numbers in the part $r = s$.

$$\bar{\eta}_{rs}^{E}\bar{E}_{rs}-\bar{\eta}_{rs}^{H}\bar{H}_{rs}=\left(\frac{\sigma-1}{\sigma}\right)\frac{TF_{rs}^{E}\left(\bar{\eta}_{rs}^{E}\bar{\varphi}_{rs}^{E}-\bar{\eta}_{rs}^{H}\bar{\varphi}_{rs}^{H}\right)}{w_r N_r\left(\bar{\eta}_{rs}^{E}-\bar{\eta}_{rs}^{H}\right)}, \tag{89}$$

$$\bar{\eta}_{rs}^{H}\bar{H}_{rs}-\bar{\eta}_{rst}^{X}\bar{X}_{rst}=\left(\frac{\sigma-1}{\sigma}\right)\frac{TF_{rs}^{H}\left(\bar{\eta}_{rs}^{H}\bar{\varphi}_{rs}^{H}-\bar{\eta}_{rst}^{X}\bar{\varphi}_{rst}^{X}\right)}{w_s N_r\left(\bar{\eta}_{rs}^{H}-\bar{\eta}_{rst}^{X}\right)}, \tag{90}$$

and

$$\bar{X}_{rst}=\left(\frac{\sigma-1}{\sigma}\right)\frac{TF_{rst}^{X}\bar{\varphi}_{rst}^{X}}{w_t^N N_r\bar{\eta}_{rst}^{X}}. \tag{91}$$

Using (76) through (79), Equations (88) through (91) can be rewritten to

$$\bar{D}_r=\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}}\frac{TF_r^D\left(\bar{\eta}_r^D\right)^{-\frac{1}{\gamma}}}{w_r N_r\bar{\eta}_r^D}, \tag{92}$$

$$\bar{\eta}_{rs}^{E}\bar{E}_{rs}-\bar{\eta}_{rs}^{H}\bar{H}_{rs}=\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}}\frac{TF_{rs}^{E}\left\{\left(\bar{\eta}_{rs}^{E}\right)^{\frac{\gamma-1}{\gamma}}-\left(\bar{\eta}_{rs}^{H}\right)^{\frac{\gamma-1}{\gamma}}\right\}}{w_r N_r\left(\bar{\eta}_{rs}^{E}-\bar{\eta}_{rs}^{H}\right)}, \tag{93}$$

$$\bar{\eta}_{rs}^{H}\bar{H}_{rs}-\bar{\eta}_{rst}^{X}\bar{X}_{rst}=\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}}\frac{TF_{rs}^{H}\left\{\left(\bar{\eta}_{rs}^{H}\right)^{\frac{\gamma-1}{\gamma}}-\left(\bar{\eta}_{rst}^{X}\right)^{\frac{\gamma-1}{\gamma}}\right\}}{w_s N_r\left(\bar{\eta}_{rs}^{H}-\bar{\eta}_{rst}^{X}\right)}, \tag{94}$$

and

$$\bar{X}_{rst}=\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}}\frac{TF_{rst}^{X}\left(\bar{\eta}_{rst}^{X}\right)^{-\frac{1}{\gamma}}}{w_t^N N_r\bar{\eta}_{rst}^{X}}. \tag{95}$$

On the other hand, we get the following four equation respectively plugging (36) through (39) to (76) through (79):

$$\bar{D}_r=(\sigma-1)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{\sigma-1}}\left(\bar{\eta}_r^D\right)^{-\frac{1}{\gamma}}F_r^D, \tag{96}$$

$$\bar{E}_{rs}=(\sigma-1)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{\sigma-1}}\left(\bar{\eta}_{rs}^E\right)^{-\frac{1}{\gamma}}F_{rs}^E, \tag{97}$$

$$\bar{H}_{rs}=(\sigma-1)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{\sigma-1}}\left(\bar{\eta}_{rs}^H\right)^{-\frac{1}{\gamma}}F_{rs}^H, \tag{98}$$

and

$$\bar{X}_{rst}=(\sigma-1)\left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{\sigma}{\sigma-1}}\left(\bar{\eta}_{rst}^X\right)^{-\frac{1}{\gamma}}F_{rst}^X. \tag{99}$$

Substituting (96) through (99) into (92) through (95), and after some manipulation, the following four equations can be derived:

$$w_r N_r\bar{\eta}_r^D F_r^D=\left(\frac{\gamma-\sigma+1}{\gamma\sigma}\right)TF_r^D, \tag{100}$$

$$w_r N_r \left\{ (\bar{\eta}_{rs}^{E})^{\frac{\gamma-1}{\gamma}} F_{rs}^{E} - (\bar{\eta}_{rs}^{H})^{\frac{\gamma-1}{\gamma}} F_{rs}^{H} \right\} = \left(\frac{\gamma-\sigma+1}{\gamma\sigma}\right) \frac{TF_{rs}^{E}\left\{ (\bar{\eta}_{rs}^{E})^{\frac{\gamma-1}{\gamma}} - (\bar{\eta}_{rs}^{H})^{\frac{\gamma-1}{\gamma}} \right\}}{\bar{\eta}_{rs}^{E} - \bar{\eta}_{rs}^{H}}, \quad (101)$$

$$w_s N_r \left\{ (\bar{\eta}_{rs}^{H})^{\frac{\gamma-1}{\gamma}} F_{rs}^{H} - (\bar{\eta}_{rst}^{X})^{\frac{\gamma-1}{\gamma}} F_{rst}^{X} \right\} = \left(\frac{\gamma-\sigma+1}{\gamma\sigma}\right) \frac{TF_{rs}^{H}\left\{ (\bar{\eta}_{rs}^{H})^{\frac{\gamma-1}{\gamma}} - (\bar{\eta}_{rst}^{X})^{\frac{\gamma-1}{\gamma}} \right\}}{\bar{\eta}_{rs}^{H} - \bar{\eta}_{rst}^{X}}, \quad (102)$$

and

$$w_t^N N_r \bar{\eta}_{rst}^{X} F_{rst}^{X} = \left(\frac{\gamma-\sigma+1}{\gamma\sigma}\right) TF_{rst}^{X}. \quad (103)$$

Then, $\bar{\eta}_{rs}^{E}$, $\bar{\eta}_{rs}^{H}$, and $\bar{\eta}_{rst}^{X}$ can be associated with $\eta_r^D = \bar{\eta}_r^D$ as follows referring to Hummels and Klenow (2005):

$$\bar{\eta}_{rs}^{E} = \eta_r^D \left\{ \frac{(1+\upsilon_{rs}^{M})(1+\tau_{rs}^{M})TF_{rs}^{E} + TF_{rs}^{H} + \sum_t (1+\upsilon_{st}^{N})(1+\tau_{st}^{N})TF_{rst}^{X}}{TF_r^D} \right\}^{\varepsilon}, \quad (104)$$

$$\bar{\eta}_{rs}^{H} = \eta_r^D \left\{ \frac{TF_{rs}^{H} + \sum_t (1+\upsilon_{st}^{N})(1+\tau_{st}^{N})TF_{rst}^{X}}{TF_r^D} \right\}^{\varepsilon}, \quad (105)$$

and

$$\bar{\eta}_{rst}^{X} = \eta_r^D \left\{ \frac{(1+\upsilon_{st}^{N})(1+\tau_{st}^{N})TF_{rst}^{X}}{TF_r^D} \right\}^{\varepsilon}, \quad (106)$$

where ε is the elasticity, for which an estimated value of the extensive margin is applicable. Assuming $\eta_r^D = 0.6$ as well as $\varepsilon = 0.6$ following Zhai (2008), initial values of $\bar{\eta}_r^D$, $\bar{\eta}_{rs}^{E}$, $\bar{\eta}_{rs}^{H}$, and $\bar{\eta}_{rst}^{X}$ can be calculated. We can then derive initial values of η_{rs}^{E}, η_{rs}^{H}, and η_{rst}^{X} using Equations (27) through (29).

Based on our previous study, we know that the choice of initial levels of N_r in the calibration process is perfectly neutral and will not affect simulation results. Thus, we set $N_r = 1$ for simplicity. In addition, p^C, ψ_r, ψ_t^N, w_r, and w_t^N also are set to unity following the usual custom in general equilibrium modeling. Utilizing the information shown above, F_r^D and F_{rst}^{X} can be calibrated first using Equations (100) and (103), then F_{rs}^{H} followed by F_{rs}^{E} using (102) and (101), respectively. Once the values of F_r^D, F_{rs}^{E}, F_{rs}^{H}, and F_{rst}^{X} are calculated, initial values of $\bar{D}_r$, $\bar{E}_{rs}$, $\bar{H}_{rs}$, and $\bar{X}_{rst}$ are obtained from (96) through (99). We then get initial values of $\bar{\varphi}_r^D$, $\bar{\varphi}_{rs}^{E}$, $\bar{\varphi}_{rs}^{H}$, and $\bar{\varphi}_{rst}^{X}$ using Equations (36) through (39), and substituting those estimated values to (44) through (47) provides φ_r^D, φ_{rs}^{E}, φ_{rs}^{H}, and φ_{rst}^{X}. After that, levels of p_r^D, p_{rs}^{E}, p_{rs}^{H}, and p_{rst}^{X} can be set using (66) through (69), and it enables us to determine initial values of D_r, E_{rs}, H_{rs}, and X_{rst} using (80) through (83). An initial value of F_r^B is then calculated from Equation (56).

Initial values of Q_s and p_s can be calculated by

$$Q_s = \frac{TF_s^D}{p_s^D} + \sum_r \left(\frac{TF_{rs}^E}{p_{rs}^E} + \frac{TF_{rs}^H}{p_{rs}^H} + \sum_t \frac{TF_{rst}^X}{p_{rst}^X} \right), \tag{107}$$

and

$$p_s = \frac{TF_s^D + \sum_r \{(1+\upsilon_{rs}^M)(1+\tau_{rs}^M) TF_{rs}^E + TF_{rs}^H + \sum_t (1+\upsilon_{st}^N)(1+\tau_{st}^N) TF_{rst}^X\}}{Q_s}. \tag{108}$$

$\bar{\bar{L}}_r$ and $\bar{\bar{L}}_t^N$ can then be calibrated from Equations (57) and (58), and we obtain initial values of L_r^C and L_t^{NC} using (62) and (63). Finally, δ_s^D, δ_{rs}^E, δ_{rs}^H, δ_{rst}^X, θ_s, α_s, and ρ_s can respectively be calibrated applying usual procedures to obtain parameters for CES or Cobb-Douglas type aggregator functions. Note that the values of δ_s^D, δ_{rs}^E, δ_{rs}^H, and δ_{rst}^X satisfy the following constraint:

$$\delta_s^D + \sum_r \delta_{rs}^E + \sum_r \delta_{rs}^H + \sum_r \sum_t \delta_{rst}^X = 1.$$

The artificial data set we assumed for this study is summarized in Table 2. As noted, the trade flow data, TF_{rs}^E, TF_{rs}^H, and TF_{rst}^X, include intra-regional international trade where $r = s$. Therefore, we hereinafter use "region" instead of "country" for $M1$, $M2$, and $N1$.

Table 2: Benchmark Data Set

		M1	M2
C_s		300.00	300.00
C_t^N		300.00	
TF_r^D		210.00	210.00
TF_{rs}^E	M1	10.00	20.00
	M2	20.00	10.00
TF_{rs}^H	M1	8.00	16.00
	M2	16.00	8.00
TF_{rst}^X	M1	1.00	2.00
	M2	2.00	1.00
τ_{rs}^M	M1	0.50	0.50
	M2	0.50	0.50
τ_{st}^N		0.50	0.50
υ_{rs}^M	M1	0.33	0.33
	M2	0.33	0.33
υ_{st}^N		0.33	0.33
γ		5.00	
σ		5.00	

4. Experiments

We now report on some results of experimental simulations performed with the model we introduced in the previous sections. The simulations, which may reveal some of the behavioral characteristics of the model, are categorized into two types. In the first type, we explore the effects of reducing trade costs on the differentiated products exported either from the market region $M2$ or the non-market region $N1$ to $M1$. In the second type, we examine the effects of raising factor productivity of the CRTS sector in either the market region $M1$ or the non-market region $N1$.

4.1 Effects of Reducing Trade Costs

We start with examining the effects of 25 percent reduction in transportation margin τ_{rs}^{M} on the differentiated products directly exported from the market region $M2$ to $M1$. Table 3 show the percentage deviations of some indices from their base case levels.

Table 3: Effects of Reducing Trade Costs ($M2$-$M1$ Link, %)

		M1	M2
Consumption (IRTS)		10.79	5.33
Consumption (CRTS)		9.44	-1.59
Factor Input (CRTS)		73.07	-55.25
# of Firm Entry		-97.28	75.60
# of Firms (Type-D)		-97.14	22.54
# of Firms (Type-E)	M1	-97.14	-98.10
	M2	767.98	22.54
# of Firms (Type-H)	M1	-97.14	-98.10
	M2	84.87	22.54
# of Firms (Type-X)	M1	-97.14	-98.10
	M2	84.87	22.54
Total Variety		38.24	-9.38
Welfare Level		10.11	1.81

In this case, region $M1$ becomes capable of importing products of the IRTS sector with cheaper price, so that the region tends to shift to the homogeneous good productions exiting from the IRTS sector. It can be confirmed by the decrease in the number of firm entry (-97.28%) or by the increase in the factor input by the CRTS sector (73.07%). On the other hand, region $M2$ shifts to the IRTS sector to increase the differentiated good productions for the $M1$ market. Thus, the number of firm entry increases (75.60%)

whereas the factor input by the CRTS sector reduces (-55.25%).

The competition in the IRTS sector serving the $M2$ market becomes stiffer. In consequence, the required productivity level becomes higher and the proportion of active firms reduces. Hence, sizes of the firms serving the $M2$ market tend to be large, while the number of active firms in region $M2$ expands because of the increase in the firm entry. On the other hand, small firms in region $M2$ have access to the $M1$ market mainly by direct exporting (767.98% for type-E, 84.87% for type-H and type-X).

In the $M1$ market, the total number of variety increases (38.24%) based on the expanding entry by small firms in $M2$. Contrary, the number of variety reduces in the $M2$ market (-9.38%) since highly productive large-scale firms are dominant. Reflecting these changes in the number of variety and supply quantity per firm, welfare improves much in region $M1$ (10.11%) and a little in $M2$ (1.81%).

Note that the directions of these effects are perfectly similar in the case of reducing import tariff υ_{rs}^{M} by region $M1$ levied on the imports from $M2$. Therefore, we go proceed to the next case related to the non-market region $N1$.

Table 4: Effects of Reducing Trade Costs ($N1$-$M1$ Link, %)

		M1	M2
Consumption (IRTS)		-0.13	-0.03
Consumption (CRTS)		-0.09	0.02
Factor Input (CRTS)		-0.56	0.36
# of Firm Entry		0.72	-0.47
# of Firms (Type-D)		0.81	-0.22
# of Firms (Type-E)	M1	-3.38	0.97
	M2	-4.52	-0.22
# of Firms (Type-H)	M1	1.70	0.97
	M2	0.50	-0.22
# of Firms (Type-X)	M1	0.83	0.97
	M2	-0.36	-0.22
Total Variety		-0.26	0.09
Welfare Level		-0.11	-0.01

Table 4 captures the effects of 0.01 percent reduction in transportation margin τ_{st}^{N} on the differentiated products exported from the non-market region $N1$ to $M1$. The trade flow from $N1$ to $M1$ in focus includes the products made by both $M1$ and $M2$ firms. The reason why the magnitude of the given shock is so small is because the present model responds very sensitive to a certain kind of shock, which may affect factor prices. Since productions in both the IRTS and CRTS sectors are directly connected to the primary factor with simple linear functions, in addition to the fact that we presume the homogeneous good produced in every region is traded as perfect substitutes, adjustments among sectors might happen quite sensitive to the changes in factor prices and it sometimes goes beyond the absorbable level.

In this case, region $M1$ becomes capable of importing products of the IRTS sector with cheaper price, so that the region tends to expand the differentiated good production to increase exports to the other countries in the same region via the local affiliates in region $N1$. This can be confirmed by the increase in the number of firm entry (0.72%) or by the reduction in the factor input by the CRTS sector (-0.56%). In contrast, region $M2$ shifts to produce mainly the homogeneous good. It is reflected to the reduction in the number of firm entry (-0.47%) or the expansion of the factor input by the CRTS sector (0.36%).

In response to the shift of $M2$ to the CRTS sector, the competition in the IRTS sector in regions $M1$ and $M2$ serving the $M2$ market becomes weak, so that the required productivity level becomes lower and the average firm size shrinks. One interest point is that the number of type-H firms established in region $M2$ and serving the market in the other countries in the same region increases (0.50%), in spite of the fact that the total number of firm entry reduces in $M2$. It is because the firms in $M1$ tend to shift from direct exporting to the affiliate production in region $N1$ to serve the market in the other countries in the same region. This shift reduces factor demand in $M1$ so that the factor price depreciates in the region. Then, firms in the IRTS sector in region $M2$ change their strategy to start affiliate production in $M1$ to serve the local market. Thus, the export-platform FDI may substitute direct exporting while complementing the horizontal FDI.

The case of reducing import tariff υ_{st}^{N} by region $M1$ levied on the imports from $N1$ also presents a similar pattern.

4.2 Effects of Raising Factor Productivity of CRTS Sector

The second type experiment is to examine the effects of productivity growth in the CRTS sector. Table 5 shows the effects on selected indicators of raising the factor productivity of the CRTS sector ψ_r in region $M1$ by 0.01 percent.

In this case, region $M1$ is able to save the factor input for producing the homogeneous good so that the region tends to shift to the differentiated good production. It can be confirmed by the increase in the number of firm entry (3.86%) or by the reduction in the factor input by the CRTS sector (-2.98%). In response to this shift in $M1$, region $M2$ expands to the CRTS sector exiting from the IRTS production. It is captured by the reduction in the number of firm entry (-2.49%) or the expansion of the factor input by the CRTS sector (1.88%), followed by the increases in inter-regional trade flows from $M1$ (4.75% for type-E, 5.29% for type-H and type-X).

Table 5: Effects of Raising Productivity of CRTS Sector in $M1$ (%)

		M1	M2
Consumption (IRTS)		-0.72	-0.17
Consumption (CRTS)		-0.47	0.08
Factor Input (CRTS)		-2.98	1.88
# of Firm Entry		3.86	-2.49
# of Firms (Type-D)		4.49	-1.15
# of Firms (Type-E)	M1	-20.25	4.75
	M2	-23.55	-1.15
# of Firms (Type-H)	M1	8.19	5.29
	M2	1.58	-1.15
# of Firms (Type-X)	M1	4.55	5.29
	M2	-1.84	-1.15
Total Variety		-1.54	0.49
Welfare Level		-0.59	-0.05

The depreciation of the factor price in region $M1$, because of the relatively redundant supply of the production factor brought by the productivity growth, enables the firms in $M1$ to serve the domestic market (4.49%) and the other countries in the same region via the horizontal FDI (8.19%), as well as the firms in $M2$ to have access to the $M1$ market also via the horizontal FDI (1.58%). On the other hand, the direct exporting to $M1$ greatly shrinks (-20.25% for the intra-regional trade in $M1$, -23.55% for the imports from $M2$). These reductions in direct exports within/to region $M1$ decrease the number of total variety and therefor worsen the welfare in $M1$.

As in the previous case of reducing trade costs related to the $N1$- $M1$ link, the horizontal FDI enhancing effects of factor price depreciation are very large and tend to substitute the direct exporting.

Finally, let us see the effects of raising the factor productivity of the CRTS sector ψ_t^N

in region $N1$ by 2.50 percent. Table 6 shows those effects on the selected indicators.

Table 6: Effects of Raising Productivity of CRTS Sector in $N1$ (%)

		M1	M2
Consumption (IRTS)		5.36	5.36
Consumption (CRTS)		2.44	2.44
Factor Input (CRTS)		6.65	6.65
# of Firm Entry		-8.56	-8.56
# of Firms (Type-D)		-18.14	-18.14
# of Firms (Type-E)	M1	124.14	124.14
	M2	124.14	124.14
# of Firms (Type-H)	M1	-66.84	-66.84
	M2	-66.84	-66.84
# of Firms (Type-X)	M1	-29.85	-29.85
	M2	-29.85	-29.85
Total Variety		4.15	4.15
Welfare Level		3.89	3.89

In this case, the story might relatively be complicated. First, the non-market region $N1$ is able to save the factor input for producing the homogeneous good so that the factor price tends to depreciates. Then, firms in the IRTS sector in market regions $M1$ and $M2$ once try to shift to the affiliate productions via the export-platform FDI. The factor demand in $N1$ by those type-X firms expands at the level that it reduces the resource allocation to the CRTS sector. This eventually shifts regions $M1$ and $M2$ to produce the homogeneous good exiting from the IRTS sector. It is reflected by the reduction in the number of firm entry (-8.56%) or by the expansion in the factor input by the CRTS sector (6.65%)

Since the factor prices tend to remain high in regions $M1$ and $M2$, because of the expanded demand by the CRTS sector, FDI, particularly the horizontal type, is greatly substituted by the direct exporting (124.14% for type-E, -66.84% for type-H, -29.85% for type-X).

Based on the exploding expansions of direct exporting, firm entry to this category increases so that the welfare improves in both market regions $M1$ and $M2$ reflecting the large number of total variety.

Considering the simulation results we have seen above, it may be concluded that the firms' operational strategy whether to choose the horizontal FDI responds quite sensitive to the changes in factor prices, and affects the other choices of the substitutable direct exporting and the complementary export-platform FDI.

5. Concluding Remarks

To explain the decision of heterogeneous firms, which differ in productivity levels, to serve foreign markets either via exporting from the home country, via local subsidiary sales (horizontal FDI), or via exporting from a host country (export-platform FDI), we developed a numerical general equilibrium model with two market and one non-market countries/regions. Experimental simulations with the model clearly show that export-platform productions complement horizontal affiliate productions while direct exporting substitutes. In the case that trade frictions between market countries are small, by unilateral trade liberalization for example, or economies of scale are large, exporting directly from the home country increases more than two types of FDI. If there is a low-cost country, which may be insufficiently attractive as a market, adjacent to the final market (thus transportation costs are cheap), or such pair of low-cost (non-market) and final market countries conclude FTA (thus import tariff reduces), increase in export-platform FDI will be complementary to horizontal FDI. The experiments simulating productivity growth in the homogeneous good sector revealed that the firms' operational strategy whether to choose the horizontal FDI responds quite sensitive to the changes in factor prices, and affects the other choices of the substitutable direct exporting and the complementary export-platform FDI.

There are potentially important issues we have to tackle in the subsequent works. First, the present model shows quite sensitive response to a certain kind of shock, which sometimes makes computation difficult. Since the simple structure of the model with linear functions and the assumption of perfect substitutability might be the source of the problem, our next work is to introduce imperfect substitution in the trade part of the CRTS sector, intermediate transactions among sectors, etc. Second, reference to the role of the size distribution of firms in the firms' choices of operational strategy also is important, since the model introduces heterogeneity in the productivity levels of firms so that it is able to implement an analysis from such point of view.

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