MODELLING INDUSTRY-SPECIFIC POLICY USING TREASURY’S INDUSTRY MODEL (TIM): A DYNAMIC GENERAL EQUILIBRIUM MODEL OF THE AUSTRALIAN ECONOMY

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2 The views expressed in this paper are those of the authors and do not necessarily reflect those of The Australian Treasury or the Australian Government.
ABSTRACT

Treasury’s Industry Model (TIM) is a new dynamic general equilibrium model of the Australian economy developed by the Australian Treasury for analysing the effects of industry policy. The model extends previous Australian models with many production sectors (see for example, Adams et al., 2015) by incorporating forward looking agents and a well-defined balanced growth path. The two main advantages of this approach over its predecessors are: (1) agents realistically can respond to anticipated shocks/policy changes; and (2) because decisions are derived from intertemporal optimisation the framework provides a welfare measure useful for evaluating the effects of policy. Compared to similar models developed internationally (see for example Hornstein et al. (1997), Hornstein (2000) and Horvath (2000)) the models includes significantly more production complexity, such as including margins and separately identifying commodities and industries. The resulting model provides a powerful tool for analysing and understanding the propagation of industry-specific shocks through the economy.

1. INTRODUCTION

The production side of any modern economy is complex in a way that presents challenges for policy analysis. It is a system that comprises a web of interconnected parts: a large number of firms that produce an almost infinite array of goods; production processes that vary substantially from one good to another; goods from some firms that feed into the production process of others; and firms that, in turn, interact with households, government and the rest of the world.

To make this system tractable for policy analysis, a common approach is to sort firms into industries grouped according to their principal output. Though a simplification, this approach still provides production heterogeneity and linkages. Analysis at the industry-level can therefore capture how a policy change affects different industries. But policy analysis often requires an assessment of not just industry-level effects, but also effects on the aggregate economy - a task complicated by the economy’s interconnectedness. To understand the net effects of policy change, it is therefore important to have a model that captures a number of dimensions: the differences of production processes, inter industry linkages and the inter-relatedness of the economy as a whole.

This paper describes such a model developed by the Australian Treasury. Treasury provides advice to the Government on a broad range of policy issues, including industry specific policies. It often provides advice on both granular impacts and on the aggregate economy. To inform this advice, Treasury needs to draw from a suite of policy analysis models, with the choice of model depending on the type of policy and its size and scope. To inform its advice where industry detail is important, Treasury therefore requires a model that captures inter-industry linkages and whole-of-economy effects.

Treasury’s Industry Model (TIM) is a dynamic general equilibrium model of the Australian economy for analysing the economy-wide effects of industry specific policies. At its core, TIM is a small open economy version of the well-known neoclassical growth model, also known as the Ramsey Cass

3 In addition to TIM, Treasury’s current suite of macroeconomic models includes an overlapping generations model of the Australian economy (OLGA) which is intended to be Treasury’s workhorse model for fiscal policy analysis (see Cai, et al (2019) for further detail on OLGA).
Koopmans model\textsuperscript{4}. Given its purpose to inform policy advice, the model includes a sufficient level of detail about key economic agents and their interactions. In contrast to typical neoclassical growth models, TIM incorporates 114 firms that represent the 114 Australian industries defined by the Australian Bureau of Statistics Input-Output (IO) tables. To give a sense of the level of detail this provides, the IO tables separately identify the electricity generation and transmission industries but do not identify generation industries by technology such as coal, gas or renewables. In addition to firms, TIM has three agents: households, government and the rest of the world (ROW).

TIM extends previous Australian models with many industries (see for example Dixon P. B. et al. (1982)) by incorporating forward looking agents and a well-defined balanced growth path. This approach overcomes three limitations of existing models. First, variables which can have a significant bearing on the results such as the long-run level of net foreign liabilities, household savings and long-run labour supply are now determined within the model. Second, agents can realistically respond to anticipated policy changes. This is an important issue in industry policy analysis because industry specific policies are typically phased in over time or announced well ahead of their implementation to avoid potentially significant costs of adjustment faced by firms and households (in their role as workers). Finally, because decisions are derived from well-defined optimisation problems TIM provides a model-consistent welfare measure that is useful for evaluating the benefits and costs of policy.

TIM captures the behaviour of the economy as a whole by modelling stylised representations of the key agents. The behaviour of all Australian households is represented by a single infinitely lived representative household. The representative household chooses labour supply and consumption to maximise lifetime welfare. Each of the 114 forward-looking infinitely-lived firms choose investment, labour and intermediate inputs to maximise the net present value of shareholder dividends. Firms can produce one or more of the 114 commodities used in the economy. Domestic production competes with differentiated goods produced by the ROW. Household and government final use is assumed to be non-durable consumption, while firms undertake industry specific investment of durable capital goods.

In TIM the economy incurs distribution costs captured by so called margins which are separately identified from more general intermediate goods and services consumed in production. All domestic production prices are endogenous, with producers facing a downward sloping foreign demand curve for exports. Financial capital is assumed to be perfectly mobile with marginal investment undertaken by the ROW. In TIM the general government sector incorporates detailed modelling of expenditure taxes (import duties, excise and goods and services tax (GST)) and production taxes but has basic modelling of income taxes, government spending and simplified fiscal rules.

The resulting model provides a powerful tool for analysing and understanding the propagation of industry-specific changes through the economy. For example, a policy induced increase in demand for an industry’s output will in turn increase the demand for its inputs, which raises the price of investment/capital, intermediate goods or wages. This will in general raise the production costs of other industries, which reduces their supply. For some industries the directly affected industry’s increased use of intermediate inputs and capital may also raise the demand for their output. Working out the net effect of these competing forces would be impossible without the aid of a model like TIM.

Despite the significant detail in TIM it is worth noting that analysis of certain policies may require further model development. This is not considered a general limitation. Rather, we consider ensuring the model is fit for purpose for each policy analysed is a strength of our approach. It is also worth

noting that TIM does not explicitly include unemployment. We also acknowledge that the welfare measure, while being internally consistent, is only for the representative household and does not directly provide information on the distributional effects of policy. That said, the labour market and firm effects by industry provide a guide on effects on households involved with different industries.

In many ways TIM resembles multisector models developed in the United States in the late 1990s/early 2000s (see for example Hornstein et. al. (1997), Hornstein (2000) and Horvath (2000)). The focus of these models was business cycle analysis while the intended purpose of TIM is analysing the effects of policies that are aimed at raising the long-run efficiency of the economy. Readers interested in Australian business cycle analysis are well served by multisector models of, albeit with less sectors by Rees, et al. (2016), and Kulish and Rees (2017)).

The competing objectives that guide Treasury’s macroeconomic modelling development programme are capability readiness and that models should be fit-for-purpose. Balancing these objectives is difficult. It is made more difficult by the fact that while the latter implies that the specification of the model should be sufficient to analyse the scenario, but no more complicated, to meet the needs of disparate stakeholders Treasury’s modelling will in general need to be far more detailed than general equilibrium models used in academia. To meet these objectives models are developed using a modular approach. Following the approach used in developing information technology applications, models advance over time with successive versions incorporating new features and a working version always available for policy analysis.

Again following the information technology industry development is marked by changes in version number. The remainder of the paper describes TIM Version 1.0: Section 2 provides an overview of TIM Version 1.0 for non-technical readers; Section 3 is aimed at technical readers and developers and describes in detail the theoretical structure of the model; Section 4 describes model calibration; Section 5 discusses welfare analysis; and Section 6 concludes with a list of planned development modules and a summary of the key design features of TIM.

2. NON-TECHNICAL OVERVIEW OF MODEL

Economic models are tools to better understand key relationships in the economy. Whole of economy models, such as TIM, can be thought of as simplified representations of the whole economy. By removing some of the detail and complexity we can focus on the aspects we want to better understand. TIM captures a large amount of the Australian economy’s production detail while also including representations of other key agents.

There are four agents in TIM: households, firms, government and the rest of the world (ROW). These agents are highly interconnected and interdependent as demonstrated in Figure 1. TIM is a general equilibrium model. This means the model results show the net effect of policy. That is, TIM captures the economy’s interconnectedness. The model results show the outcome of all the agents acting in their own best interest.

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5 There are a range of players in the development and use of economic models in public policy. All have important roles to play, from the design and build to using model findings to inform policy analysis. Recognising the importance of different roles and the benefit of a broader audience gaining insights into how the model works, this section provides an overview of the structure of TIM Version 1.0 for non technical readers. A more detailed and technical explanation follows in Section 3 of the paper.
Households

There is one household in TIM. It represents the aggregated behaviour of all Australian households. The household chooses how much it will work or enjoy leisure, how much of its income it will spend or save, and what commodities it will buy. These choices determine household welfare. The household prefers: more consumption and leisure to less; an equal distribution of consumption and leisure over time; and consuming a mixed bundle of goods.

The household receives wage income for its hours worked where extra time spent working means less leisure time. The household’s total disposable income is the sum of its after tax labour income, the value of previously purchased equity, dividends from this equity and payments from government. This total income can either be used to buy consumption goods or saved by buying equity.\(^6\)

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\(^6\) The only saving vehicles in the model are claims to the equity of domestic firms. While this may appear at first glance to be a major abstraction from reality it is not when you consider that the modelling assumes debt and equity are perfect substitutes and that all goods and services are produced by firms. For instance, firms in the Ownership of Dwelling industry produce rental services using capital and fixed factors so the household’s claim to the equity of these firms is actually a claim to their capital which is houses and their fixed factors which is land. In other words, the model assumes households own houses and land.
The behaviour of the representative household matches attributes of aggregate consumption and labour supply data and intuition on the behaviour of individuals. The household is forward looking and makes decisions in line with its preferences. The household responds to a higher income by increasing both consumption and leisure. However, if the higher income comes from higher wages then this inducement to work more can either partly or more than offset the preference for more leisure. If the household anticipates higher income in the future it will consume more now and save less. An increase in the price of a consumption commodity will lead the household to reduce the consumption of that commodity to a degree but not completely. This choice reflects the household’s preference for a mixture of different goods.

Firms

There are 114 firms in TIM, with each firm representing one of the 114 industries that the Australian Bureau of Statistics identifies in its input-output tables. Each firm represents the aggregated behaviour of all Australian firms operating in the industry. Shareholders own the firms – the shareholders are the household and investors from the ROW. The firm’s goal is to maximise shareholder value.

Firms use labour, capital, intermediate commodities, and fixed factors (representing land and mineral resources) to produce their outputs. Firms buy labour services from households. A firm owns its capital and fixed factors; firms can undertake investment to increase capital over time, but cannot purchase additional fixed factors. Firms buy intermediate commodities from other Australian firms or the ROW to help produce other commodities—for example, a coal mine buys some imported diesel to extract coal. The firms face costs from undertaking rapid investment or changes in their labour inputs.

Firm behaviour in TIM matches characteristics of Australian firms. Firms choose the inputs of capital, labour and intermediate inputs to minimise the cost of production. The firms respond to higher returns by increasing investment until the expected return equals the required return on investment. If a firm expects higher future profits it will undertake some additional investment and bring on some workers early to integrate them into its production processes. Conversely, a firm that expects investment to be cheaper in the future may delay investment until then.

Rest of the world

Given the small and open nature of the Australia economy, it is important to capture interactions with the Rest of World. In TIM, the household, government, and firms can buy commodities from the ROW and Australian firms can sell commodities to the ROW. Australia does not impact global prices for any of the commodities it imports because it is too small. Further, the commodities sold by Australian firms differ from global goods, so they have their own prices and demand. That is, the quantity of exports ROW is willing to buy depends on the price exports are sold at.

Investors from the ROW can buy equity in Australian firms. The modelling assumes Australia is a small open economy with internationally perfectly mobile financial capital. That is, investors from the ROW are willing to buy shares in Australia while the expected return matches that available elsewhere. This implies that the required return on investment for Australian firm equals the world interest rate.

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7 The representative household in TIM does not make savings decisions with an explicit saving goal such as saving for retirement. Instead its saving decision reflect its desire to smooth income. As such, it broadly maintains equity that grows with the economy. If it did not, it would either reach a point where it would be forced to cut consumption or end up very wealthy but with low consumption.
Government

The government in TIM collects taxes that fund its spending. The government spending is exogenous to the model. That is, in the absence of policy change the government buys the same bundle of commodities every year. TIM incorporates detailed expenditure taxes (import duties, excise and GST) and taxes on production. It has limited details of income tax, government spending and transfers. For example, TIM version 1.0 assumes a simple fiscal strategy where the government balances its budget in every period by adjusting the level of transfers to households.

Industry supply and commodity use

In the Australian economy there is a degree of complexity regarding how output from firms reaches users; these being the household, government, other firms or ROW. As TIM captures the production structure of each industry in significant detail it also tracks the flow of outputs from producer to end user.

Firstly, firms (industries) in TIM can produce multiple commodities. In the Australian economy, firms often produce multiple commodities that do not align with their industry classification. For example, a firm in the ‘Structural Metal Product Manufacturing’ industry may also produce some ‘Iron and Steel Manufacturing’ commodities. TIM assumes that each firm can produce some subset of the 114 commodities but shifting production to other commodities comes at a cost, as shown in Figure 2.

![Figure 2: Domestic supply of each commodity](image)

Figure 2: Domestic supply of each commodity

In Australia, consumers and firms implicitly choose between domestic goods and imported goods with the choice often based on the decision to buy a good without explicitly knowing its source. In TIM the shares of imported and domestic commodities are consistent with the historical preferences of users. The shares respond to prices changes by increasing the share of the cheaper source. The total supply of each commodity to each user is given by the combination of the domestic commodity and the imported commodity as shown in Figure 3. Figure 3 also shows that import duties add to the price of imports with duty rates specific to the user and the commodity.

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8 These modelling choices reflect the fact that Treasury’s suite of models includes an overlapping generations model of the Australian (OLGA) economy which incorporates detailed modelling of Australia’s: income tax system; government spending; transfer, pension and means testing system; and allows for complex fiscal rules. See Cai, et al. (2019) for further detail of OLGA.

9 The shares are consistent with those observed in the IO table data and the magnitude of the responses are consistent with literature.

10 The detail on import shares and duty rates by commodity and user compensates for some of the commodity aggregation in TIM. While we have 114 commodities, petrol and diesel are combined in the ‘petroleum manufacturing’ commodity. For example, capturing different duty rates and different domestic and imports shares reflects that mining and households use different shares of petrol and diesel.
TIM explicitly models the distribution costs involved in transferring output from firms to users. In general users do not buy commodities directly from firms. For example, when a loaf of bread is transported from the bakery to supermarket where it is purchased for consumption, both the transport and the retail trade add to the costs of the loaf of bread paid by the user. TIM includes these distribution costs known as margins. TIM includes 11 margins which are a subset of the 114 commodities. Figure 4 shows the end use commodity is a combination of the actual commodity and the margins required in transferring the commodity plus sales and excise tax. Some of the margins good and services such as road transport can also be consumed directly, by catching a bus, or as margins by buying bread that has been transported from a factory.

**Figure 4: End use supply by use and commodity**

3. **Detailed Description of Model**

This section provides a technical description of the model. It provides details on the modelling of the four agents: households, firms, the government and the ROW; and the model’s equilibrium definition.
Household

The representative household chooses paths for consumption and leisure to maximise its expected lifetime utility. The household has a time endowment and receives wage income for the time it supplies to the labour market. It can choose to spend this income on goods and services or save it for future consumption. Savings earn a rate of return in the following period which provides another source of income for the household to consume goods and services and save.

Endowments

At time $t$, the model is assumed to be inhabited by a single representative household with a total endowment of time $H$ that is allocated between working (labour input to production) $N$, and leisure $L$, such that:

$$N_t = H_t - L_t$$

(1)

Total hours are assumed to grow at rate $\gamma^h$:

$$H_{t+1} = (1 + \gamma^h)H_t$$

(2)

The household is also endowed with human capital in the form of labour augmenting technical progress denoted by $\xi$, which is assumed to grow at rate $\gamma^\xi$:

$$\xi_{t+1} = (1 + \gamma^\xi)\xi_t$$

(3)

In line with definitions above trend growth rate non-labour variables are equal to the growth rate of total effective hours $\gamma^e$:

$$H_{t+1}\xi_{t+1} = (1 + \gamma^h)(1 + \gamma^\xi)H_t\xi_t$$

$$H_{t+1}\xi_{t+1} = (1 + \gamma^e)H_t\xi_t$$

(4)

Without loss of generality it is convenient to describe the stationary version of the model, so from here on lower case non-labour variables reflect actuals normalised by total effective hours.\(^{11}\) That is, quantities per unit of effective hours are given by:

$$X_t = \frac{X_t}{\xi_tH_t}$$

Actuals can always be recovered by inflating the normalised variable by the total effective hours.

Further, along the balance growth path average hours adjusted from population growth do not change over time. Given the assumption of homogeneity the model can usefully be described in the context of a stationary representative household. Total labour and leisure per hour can be found by dividing aggregate labour and leisure hours by total hours $n_t \equiv N_t/H_t$ and $l_t \equiv L_t/H_t$, respectively, where

\(^{11}\) Hours variables such as the time endowment and labour supply are only normalised for time not productivity.
lower case variables reflect the transformed variables. This implies that the household’s total time
constraint is:

\[ n_t = 1_t - l_t \]  

(5)

**Preferences**

The household’s lifetime utility is given by the sum of current and discounted future utility, given by:

\[ \sum_{t=0}^{\infty} \beta^t u(ch_t, l_t) \]  

(6)

where \( \beta^t \geq 0 \) is the discount factor and \( ch_t \) is aggregate household consumption at time \( t \). Utility within each period is derived from an aggregate consumption good and leisure with decreasing marginal return benefit from each consistent with a constant relative risk aversion (CRRA) function:

\[ u(ch_t, l_t) = \left[ \left( ch_t \right)^{\alpha_t} \left( l_t \right)^{(1-\alpha_t)} \right]^{-1-\sigma} \]  

(7)

where \( \sigma \) is the inverse of the intertemporal elasticity of substitution and \( \alpha_t \in (0, 1) \) is the parameter measuring the household’s relative preference over consumption and leisure.

**Consumption of commodities**

The aggregate consumption good is a composite of commodities which are aggregated according to a Constant Elasticity of Substitution function (CES):

\[ ch_t = \left( \sum_{c \in C} \theta^{ch}_{c,t} \left( ch_{c,t} \right)^{\eta^{ch}_{c,t}} \right)^{\frac{1}{\eta^{ch}_{c,t}}} \]  

(8)

where \( \eta^{ch}_{c,t} > 0 \) is the elasticity of substitution between different types of consumption commodities and \( \theta^{ch}_{c,t} \in (0, 1) \) is the weighting parameter for consumption commodity \( c \). \( C \) is the set of goods and services available in the economy (hereafter referred to as commodities).

Total household consumption expenditure is:

\[ p_{t}^{ch} ch_t = \sum_{c \in C} p_{c,t}^{ch} c_{c,t} \]  

(9)

where \( p_{c,t}^{ch} \) is the price of consumption commodity \( c \) and \( p_{t}^{ch} \) is the price of aggregate consumption.

**Asset allocation**

TIM assumes that households allocate a fixed share of their savings, \( \theta^v \), to equity holdings in each industry. As such the value of each industry’s equity held by households, \( V^v_{i,t} \), in period \( t \) is given by:
where $v^h_i$ is the total value of the household’s assets at the end of the period and the asset allocation shares sum to one

$$\sum_{i \in I} \theta^i = 1.$$ 

In the case the households demand for a particular asset exceeds supply then the demand for that asset type is adjusted according to

$$\text{if } \theta^i v^h_i > v_{i,t} \text{ then } \theta^i_{i,t} = \frac{v_{i,t}}{v^h_i}.$$ 

The asset allocation shares for other industries are adjusted proportionately so that they continue to sum to one. While there are numerous other ways the asset allocation could be specified only the initial allocation affects welfare and this is determined by the calibration. When there are no unexpected policy changes the return on assets equals the internationally required rate, this is described in more detail below in the global funds market section. As the policy changes are realized in the first period only the initial allocation of assets affects welfare impacts.

The household’s assets generate an aggregate return $r^h_i$ which includes both dividends, $d_{i,t}$, and any capital gains given by

$$r^h_i = \frac{\sum_i (v_{i,t} + d_{i,t}) v^h_{i,t-1} / v_{i,t-1}}{\sum_i v^h_{i,t-1}} - 1.$$ 

**Household’s budget constraint**

The household begins each period with the assets they purchased in the previous period $v^h_{i,t-1}$, these are valued in the previous period asset prices. The household receives labour income which is subject to tax at the rate $\tau^n$. The household also receives a lump sum transfer from the government $\tau^n$. The household can choose to spend its income on current consumption of goods and services or save it for future consumption.

The household’s budget constraint is:

$$v^h_{i,t} = \frac{p_v^h}{1 + r_i^h} c h_t = (1 + r_i^h) v^h_{i,t-1} + (1 - \tau^n) w_i n_i + \tau^n.$$ 

The household chooses consumption, leisure and saving to maximise its lifetime utility, equation (6), subject to its budget constraint, equation (10), and its initial level of savings $v^h_{i,t-1}$. The household’s expectations for future prices and wages, transfers are consistent with their policy beliefs and they take prices as given.

**Firms – production sector**

Each of the model’s industries is represented by a single price taking firm which uses an industry specific technology to produce output $y_{i,t}$ where $i$ denotes the industry from the set of industries, $I$. Each firm is assumed to make decisions to maximise its market value on behalf of its shareholders.
Production technology

Firms combine labour $n_{i,t}$, capital $k_{i,t}$, a fixed factor $f_{i,t}$ and an industry specific aggregate of intermediate inputs $z_{i,t}$, to produce output using a CES production technology:

$$\Gamma(n_{i,t}, k_{i,t}, z_{i,t}, f_{i,t}, \lambda_{i,t}^n, \lambda_{i,t}^k, \lambda_{i,t}^z, \lambda_{i,t}^f) = \lambda_{i,t}^y \left( \frac{(\theta^n_{i,t})^{\frac{1}{\eta}} (\lambda_{i,t}^n n_{i,t})^{\eta - 1}}{\eta^n} + (\theta^k_{i,t})^{\frac{1}{\eta}} (\lambda_{i,t}^k k_{i,t})^{\eta - 1}}{\eta^k} + (\theta^z_{i,t})^{\frac{1}{\eta}} (\lambda_{i,t}^z z_{i,t})^{\eta - 1}}{\eta^z} + (\theta^f_{i,t})^{\frac{1}{\eta}} (\lambda_{i,t}^f f_{i,t})^{\eta - 1}}{\eta^f} \right)$$

(11)

where: $\theta^n_{i,t}$, $\theta^k_{i,t}$, $\theta^z_{i,t}$ and $\theta^f_{i,t}$ are the CES weights for labour, capital, intermediate inputs and fixed factor, respectively; $\eta^n > 0$ is the elasticity of substitution for factors of production.

Given that (10) has been normalised for trend population and labour augmenting technical progress, the terms $\lambda_{i,t}^n$, $\lambda_{i,t}^k$, $\lambda_{i,t}^z$ and $\lambda_{i,t}^f$ are the total factor, labour, capital, intermediate input and fixed factor productivity shifters, respectively.\(^\text{12}\)

**Labour input**

The labour market is assumed to be perfectly competitive and labour is perfectly substitutable across production activities so households face a single wage. Total labour units supplied by the household must equal to the aggregate labour demand of each firm\(^\text{13}\).

$$n_t = \sum_{i=1}^{I_t} n_{i,t}$$

(12)

While labour is mobile across industries, firms face a cost of adjusting their labour inputs. For example, new workers are typically less productive and existing workers are less productive while they are training new workers. Following Jaimovich and Rebelo (2008) and Aretzki, et. al, (2015), the firm incurs a quadratic adjustment cost when its labour use does not grow in line with trend growth.

$$\Omega^n(n_{i,t}, n_{i,t-1}) = \frac{\zeta^n_{i,t}}{2} \left( \frac{n_{i,t}}{n_{i,t-1}} - 1 \right)^2 n_{i,t}$$

(13)

where $\zeta^n_{i,t} \geq 0$ is the labour adjustment cost parameter.

**Capital input**

Productive capital $k_{i,t}$ represents durable physical inputs such as machines, buildings and plants. Firms accrue capital by investing in an industry specific capital good $ib_{i,t}$ and the capital stock of firm $i$ evolves according to the following accumulation identity:

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\(^{12}\) It is not necessary to specify the stochastic processes for these productivity shifters because the model is deterministic.

\(^{13}\) Labour is assumed to be internationally immobile implying that the household cannot migrate in response to economic conditions.
\[(1 + \gamma_i)k_{i,t+1} = (1 - \delta_i)k_{i,t} + ib_{i,t}\]  

(14)

where \(\delta_i \geq 0\) is the industry specific depreciation rate depreciation rate. Each firm’s investment \(ib_{i,t}\) is an industry-specific composite of a variety of investment commodities which are aggregated according to the following CES aggregation function:

\[ib_{i,t} = \left( \sum_{c \in C} \left( \theta_{c,i,t} \right)^{\frac{1}{\eta}} \left( \frac{ib_{c,i,t}}{\theta_{c,i,t}} \right)^{\eta - 1} \right)^{\frac{1}{\eta - 1}}\]  

(15)

where \(ib_{c,i,t}\) is the investment in commodity \(c\) for firm \(i\), \(\theta_{c,i,t} \in (0,1)\) is the CES weight and \(\eta_{ib} > 0\) is the elasticity of substitution between different types of investment.

This approach captures the fact that each industry has a capital stock composed of different goods and as such the industry specific investment goods are composed of different commodities.

The total value of investment at time \(t\) for firm \(i\) given by:

\[p_{i,t}^{ib}ib_{i,t} = \sum_{c \in C} p_{c,i,t}^{ib}ib_{c,i,t}\]  

(16)

where \(p_{c,i,t}^{ib}\) is the industry \(i\) price of investment commodity \(c\) and \(p_{i,t}^{ib}\) is the industry-specific price of aggregate investment.

Following Lucas (1967) and Uzawa (1969), we assume that the firm incurs a quadratic capital adjustment cost when its investment deviates from that implied by trend growth in real GDP. This capital adjustment cost is given by:

\[\Omega^{k} (ib_{i,t}, k_{i,t}) = \frac{\zeta_i}{2} \left( \frac{ib_{i,t}}{k_{i,t}} - (\gamma_i + \delta_i) \right)^2 k_{i,t}\]  

(17)

where the capital adjustment cost parameter, \(\zeta_i \geq 0\) governs the size of the adjustment costs. The capital adjustment cost reflects the disruption that is caused by bringing new capital into the production process.

**Intermediate input**

Production requires the consumption of intermediate inputs in the period in which they are produced. While intermediate inputs are directly sourced from each firm, for ease of exposition the model assumes that firms purchase a specific composite of intermediate commodity.

The aggregate intermediate input in each firm is sourced according to the following CES technology

\[z_{i,t} = \left( \sum_{c \in C} \left( \theta_{c,i,t}^{z} \right)^{\frac{1}{\eta}} \left( \frac{z_{c,i,t}}{\theta_{c,i,t}^{z}} \right)^{\eta - 1} \right)^{\frac{1}{\eta - 1}}\]  

(18)
where $\eta^z > 0$ is the elasticity of substitution between intermediate inputs in the production of aggregate intermediate input used by firm $i$ and $\lambda^z_{c,i,t}$ is intermediate input productivity shifter. The total value of intermediate input used by firm $i$ is:

$$p^z_{i,t} z_{i,t} = \sum_{c \in C} p^z_{c,i,t} z_{c,i,t} \tag{19}$$

where $p^z_{c,i,t}$ is the industry $i$ price of intermediate commodity $c$ and $p^z_{i,t}$ is the industry-specific price of aggregate intermediate inputs.

**Fixed factor input**

The model assumes that in some industries the production process requires fixed factors, such as land. Further the fixed factors are owned by the firm and fixed factor augmenting technical progress is assumed to grow at the same rate as labour augmenting technical progress and the population. This ensures the contribution of fixed factors to the economy is stationary in the long run. In other words, there is balanced growth.

**Gross value of production**

**Gross output**

Along the transition path, capital and labour adjustment costs prevent firms from achieving their potential level of output, with actual output in each industry given by $y_{i,t}$, where:

$$y_{i,t} = \Gamma \left( n_{i,t}, k_{i,t}, z_{i,t}, f_{i,t}, \lambda^y_{i,t}, \lambda^k_{i,t}, \lambda^z_{i,t}, \lambda^f_{i,t} \right) - \Omega^o \left( n_{i,t}, n_{i,t-1} \right) - \Omega^k \left( b_{i,t}, k_{i,t} \right) \tag{19}$$

The adjustment costs can be thought of as the penalty that occurs when firms change capital and labour out of line with trend growth. This penalty restricts firms from achieving the level of output they would produce using the same inputs if it were not for the adjustment costs.

**Domestic supply of commodities**

Each firm can produce multiple commodities. Following the literature, this is modelled using a constant elasticity of transformation (CET) function:

$$y_{i,t} = \left( \sum_{c \in C} \left( \theta^y_{c,i,t} \right)^{\frac{1}{\nu^y}} \left( y_{c,i,t} \right)^{\frac{\nu^y-1}{\nu^y}} \right)^{\frac{1}{\nu^y-1}} \tag{19}$$

where $y_{c,i,t}$ is the amount of commodity $c$ produced by industry $i$, $\theta^y_{c,i,t} \in (0,1)$ is the CET weight of commodity $c$ and $\nu^y$ is the elasticity of transformation which determines the ease with which firms can transform their output mix in response to changes in the relative prices of commodities.

The gross value of production for industry $i$ is:

---

14 see for example (van der Mensbrugghe & Peters, 2016, p. 3)
where $p^y_{c,t}$ is the [producer] price of commodity $c$.

The total quantity of commodity $c$ produced domestically is:

$$y_{c,t} = \sum_{i=1}^i y_{c,i,t}$$

(21)

**Market value of the firm**

The firm owns its capital inputs and fixed factor inputs. It makes production and investment decisions in order to maximise its market value. The firm’s shareholders are the domestic household or investors in the ROW.

The no-arbitrage condition requires that expected return on assets available to foreign investors equals the available world interest rate or:

$$E\left[ \frac{(d_{i,t+1} + v_{i,t+1})(1 + \gamma_{t+1}) - v_{i,t}}{v_{i,t}} \right] = r_{t+1}$$

(21)

Where $d_{i,t+1}$ is the net cash flow from firms to shareholders, $\gamma_t$ is trend growth rate, $r_{t+1}$ is world interest rate paid in period $t+1$ on bonds bought in the period $t$, and $v_{i,t}$ is the ex-dividend value of the firm’s equity in period $t$. This implies that:

$$v_{i,t} = E\left[ \frac{(d_{i,t+1} + v_{i,t+1})(1 + \gamma_{t+1})}{(1 + r_{t+1})} \right]$$

Forward substitution of this relationship implies the market value of the firm’s equity is equivalent to the net present value of current and all future net cash flow to shareholders:

$$v_{i,t} = E\left[ \sum_{s=t}^{\infty} \left( \prod_{u=t}^{s} \frac{1 + \gamma_{u+1}}{1 + r_{u+1}} \right) d_{i,s+1} \right]$$

(22)

For the value maximising this implies the firm’s discount rate is equal to the global required rate of return as foreigners are the marginal source of funds, this is discussed further below in the global funds market section.

**Production taxes**

The government levies taxes on factors of production: fixed factors, capital and labour. These correspond to land tax, payroll tax and other production taxes, including motor vehicles registration taxes and licencing taxes. The government also pays subsidies to production. Total production tax paid by industry $i$ is the sum of land, capital and labour input tax paid minus subsidies:

$$tax^y_{i,t} = tax^f_{i,t} + tax^k_{i,t} + tax^p_{i,t} - tax^{sub}_{i,t}$$

(23)
Land tax is levied on the value add from the fixed factor. The rate varies by industry accounting for both treatment of land tax by industry and also that the fixed factor includes land and also mineral resources. The value add from the fixed factor is calculated using the marginal product of the fixed factor multiplied by the stock. The revenue is then given by

\[ \text{tax}_{i,j}^{f} = \tau_{i,j}^{f} \cdot \text{MPF}_{i} \cdot \frac{\partial y_{i,t}}{\partial f_{i,t}} \]  

Payroll tax is levied on the compensation of employees and is applied as an effective industry wide rate:

\[ \text{tax}_{i,j}^{py} = \tau_{i,j}^{py} \cdot w_{i,t} \cdot n_{i,j} \]

Production taxes on capital are applied to current replacement value of capital, given by the investment price.

\[ \text{tax}_{i,j}^{ky} = \tau_{i,j}^{ky} \cdot p_{i,j}^{ib} \cdot k_{i,t} \]

Subsidies are received ad valorem to the value of output and given by

\[ \text{tax}_{i,j}^{sub} = \tau_{i,j}^{sub} \cdot p_{i,j}^{y} \cdot y_{i,j} \cdot k_{i,t} \]

Firm’s budget constraint

The firm’s budget constraint can be written as:

\[ d_{i,j} + p_{i,j}^{ib} \cdot b_{i,t,j} = (1 - \tau_{i}^{k}) \left( p_{i,j}^{y} \cdot y_{i,j} \cdot n_{i,t} - p_{i,j}^{z} \cdot z_{i,j} - \text{tax}_{i,j}^{y} \right) + \tau_{i}^{k} \cdot \delta \cdot p_{i,j}^{ib} \cdot k_{i,t,j} \]

where \( \tau_{i}^{k} \) is the effective corporate income tax rate levied on the firm’s net operating surplus, that is, the gross value of output, less compensation of employees, cost intermediate inputs, depreciation allowance and production taxes \( \text{tax}_{i,j}^{y} \). The modelling assumes the firm can deduct depreciation at the economic rate of depreciation and current investment price. The net cash flow from firms to shareholders is the residual of the firm’s after-tax net operating surplus less the cost of investment. Net cash flow to shareholders can be positive or negative. A negative value implies the implicit issuance of shares is greater than the distribution of profits.

The firm’s objective is to maximise its cum dividend value of the firm, which is given by the equity value from equation (22) plus cash flow:

\[ \max d_{i,j} + v_{i,j} \]

Each firm maximises this objective by choosing labour, inputs of each intermediate commodity, the use of each investment commodity and the share of output allocated to each commodity. These choices determine gross output and output of each commodity via the production technologies in equations (11), (13), (14), (15), (17), (18), (19) and (19). This further determines the firm’s revenue, tax liabilities in equations (22), (23), (24) and (25), and the firm’s net cash flow and value. Firms take the prices of inputs and outputs of production as given when making these choices.
Firms – distribution sector

TIM captures key complexity regarding how output from firms and ROW reaches users. The distributional sector attempts to embody this complexity and simplify its exposition. The distribution sector is perfectly competitive. The distribution of goods to different uses is the same in its structure but the parameterization varies by commodity and use. Hence, it is convenient to define a set of end uses of commodities $A$ that is composed of: intermediate inputs by industry, household consumption, government consumption, firm investment by industry and exports.\textsuperscript{15} The set has 231 elements.

Margins

In an economy some goods and services are required to ensure other goods and services are available for end use. For example, when households purchase fresh produce then services such as transport, wholesale trade and retail trade are required to ensure the fresh produce is available to consumers. In the literature these are referred to as margins. The literature treats margins separately from intermediate inputs due to substitutability. TIM includes 11 margins which are wholesale trade; retail trade; restaurants, hotels and clubs; road transport; rail transport; pipeline transport; water transport; air transport; port handling; marine insurance; gas distribution; and electricity distribution.

Some commodities are both a margin and an end use commodity. For example, households may indirectly use road transport when they purchase fresh produce or they may directly purchase road transport when they catch a bus. As such the total quantity of each commodity $c$ used, $q_{c,a,t}$, is given by the amount directly used as end use $a_{c,t}$ plus the total amount used in transferring other commodities and is given by

$$q_{c,a,t} = a_{c,t} + \sum_{c' \in c} \psi_{c,c',a} a_{c',t}$$  \hspace{1cm} (29)

Firstly, here $a_{c,t}$ refers to quantity of the end use category. As such, $\sum_{c' \in c} \psi_{c,c',a} a_{c',t}$ is the total margin use in transferring other commodities $c'$. Following the literature, the demand for margins has been modelled as a fixed proportion of the volume of end use commodities. That is, the end use of fresh produce always embodies a fixed share of transport, retail and wholesale trade and so on where $\psi_{c,c',a}$ is the proportion of margin goods required. Hence, $\psi_{c,c',a} a_{c,t}$ is the requirement for the quantity of margin commodity $c'$ associated with commodity $c'$ for that use category. The majority of commodities are not used as margins and for these $\sum_{c' \in c} \psi_{c,c',a} a_{c',t} = 0$.

Total supply

The distribution sector combines domestic output with imports to form total supply of each commodity by type of use. In particular, total supply, $q_{c,a,t}$, of commodity $c$ to use $a \in A$ combines domestically produced commodity $y_{c,a,t}$ and imports $im_{c,a,t}$ according to a use-specific CES aggregation function:

\textsuperscript{15} The set of end uses differs from the set of end users who are the household, government, firms and ROW.
where $\theta^m_{c,a,t}$ indicates bias in favour of the domestically produced variety of commodity $c$ in use $a$, and $\eta^m$ is elasticity of substitution between domestic and imported varieties of commodity $c$.

The production price, $p^q_{c,a,t}$, of commodity $c$ for use $a$ satisfies:

$$p^q_{c,a,t}q_{c,a,t} = p^q_{c,a,t}y_{c,a,t} + p^m_{c,a,t}im_{c,a,t}$$

The price of imports here, $p^m_{c,a,t}$, is adjusted for duties applied to the imports in use category $a$.

Import duty is an ad valorem tax and the rates are specific to each use category and each commodity.

$$p^m_{c,a,t} = (1 + \tau^d_{c,a,t})p^w_{c,t}$$

where $p^w_{c,t}$ is the price at which the world supplies imports.

The total demand for domestically produced commodities is the sum of their use in intermediate inputs, household consumption, business investment, government spending or exports:

$$y_{c,t} = \sum_{a \in A} y_{c,a,t}$$

End use prices

The end use price of a commodity, commonly known as the purchaser’s price, depends on the production price, the margins required to transfer the good to final use and taxes on products (excise, sales taxes). Excise is levied per quantity unit at a rate $\tau^e_{c,a,t}$ which depends on the use category and the commodity. Sales tax represents a combination of GST, subsidies and other sales tax, and is levied on the combined basic value of the commodities, the associated margins duties and excise at a rate $\tau^s$. The purchaser’s prices of commodities are given by:

$$p^q_{c,t} = (1 + \tau^s_{c,a,t})(p^q_{c,a,t} + \sum_{m \in M} \psi_{m,c} p^q_{m,a,t} + \tau^e_{c,a,t})$$

Where $M$ is the set of margin commodities, which is a subset of the set of commodities. This approach assumes that sales taxes on products are levied on a base that includes the cost of the margin.

Government

Revenue

The government collects revenue by levying taxes on labour income, corporate income, factors of production and products.
Income taxes

TIM does not incorporate heterogeneous households or different savings vehicles, which limits the modelling of the personal income tax to stylised labour and capital income taxes.

Labour income tax is modelled as a flat rate $\tau^w_t$ applied to total labour income:

$$\text{tax}_t^w = \tau^w_t \sum_i w_i n_{i,t}$$  \hspace{1cm} (35)

Total capital income tax is simply the company income tax which is paid by firms on behalf of shareholders and is levied on corporate earnings after deductions for production taxes and depreciation:

$$\text{tax}_t^k = \tau_t^k \sum_i \left( p_{i,t} y_{i,t} - w_i n_{i,t} - p_{i,t} z_{i,t} - \text{tax}^w_{i,t} - \delta p^b_{i,t} k_{i,t} \right)$$  \hspace{1cm} (36)

The model ignores withholding tax or similar on passive income earned by to the ROW.

Taxes on products

The government collects excise and sales tax. Sales tax is modelled as an ad valorem commodity tax, on the value of household final consumption, government expenditure, exports and private gross fixed capital formation:

$$\text{tax}_t^s = \sum_{a \in A} \sum_{c \in C} \frac{\tau_{c,a,t} p_{c,a} a_{c,j}}{1 + \tau_{c,a,t}^s}$$  \hspace{1cm} (37)

The excise is levied on the volume of sales with total revenue given by:

$$\text{tax}_t^e = \sum_{c \in C} \sum_{a \in A} \tau_{c,a,t}^e a_{c,j}$$  \hspace{1cm} (38)

The import duties are levied on the value of imports with total revenue given by:

$$\text{tax}_t^{db} = \sum_{a \in A} \sum_{c \in C} \tau_{c,a,d}^{db} p_{c,d}^{win} m_{c,a,d}$$  \hspace{1cm} (39)

Expenditure

Tax revenue is used to finance general government spending and a lump-sum transfer to the household. Government spending does not affect the welfare of the household or the productivity of private factors of production. Real government spending is assumed to grow at the same rate as trend output. That is, the ratio of real government spending to real gross domestic product is constant. The total cost of government expenditure is given by:

$$p^s_t g_t = \sum_{c \in C} p^s_{c,t} s_{c,t}$$  \hspace{1cm} (40)

Government’s budget constraint

TIM’s fiscal rule is that the government’s budget is balanced in every period (including calibration) with lump sum transfers adjusting to offset any changes in revenue. As such government transfers are given by
Rest of the world

TIM has a Rest of the World (ROW) region that is assumed to interact with the Australian economy in three ways. First, the ROW purchases Australian exports $x_{ct}$ at the Australian export price $p_{ct}^x$. Second, the ROW produces commodities that are imported by Australians $im_{ct}$ at the import price of $p_{ct}^{im}$. Finally, the ROW supplies capital perfectly elastically while the required rate of return $r_t^f$ is expected to be met. In standard scenarios any unexpected change is realised in the first period and the realised return deviates from the required return only in this first period.

Global goods and services market

Imports

As a small open economy, TIM assumes domestic demand cannot affect the price at which the ROW supplies imports $p_{ct}^{im}$. In other words, the supply of imports is perfectly elastic in price.

Exports

Domestic exports of commodity $c$ compete with those produced in the rest of world. As such, domestic commodities are assumed to be imperfect substitutes for foreign goods. The degree to which the foreign demand for exports responds to the prices of exports is dictated by the degree of substitutability. In particular, the demand for exports is given by the following foreign demand relationship:

$$x_{ct} = \Theta_{ct}^x \left( \frac{p_{ct}^w}{p_{ct}} \right)^{\eta^x} y_{ct}^w$$

where for commodity $c$, $x_{ct}$ is the level of exports, $y_{ct}^w$ is total world use of each commodity, $p_{ct}^x$ is the price of Australian exports, $\Theta_{ct}^x \in (0,1)$ is the ROW CES weight for Australian exports, $p_{ct}^w$ is the world price of competing varieties ROW price, and $\eta^x > 0$ is the foreign elasticity of import demand. Notice, the ROW price against which domestic exports compete potentially varies from the price at which ROW supplies imports. Essentially TIM assumes that the commodities that ROW sells to Australia differ from those against which Australian exports compete.

Global funds market

TIM assumes Australia is a classic small open economy: Australia is assumed to have no influence in the global equity market and cannot influence the global required rate of return $r_t^i$; the supply of foreign capital is assumed to be perfectly elastic at the global required rate of return; and Australian equity is a perfect substitute for international equity, which implies that the required rate of return on domestic equity is equal to the international rate of return.

Under these assumptions the ROW is willing to buy any equity available in Australia as long as the expected rate of return equals the international rate of return. As such, the ROW is the marginal investor. Therefore, ROW determines the price of equity and the size of the capital stock. The value of net foreign assets is given by:

$$\text{tran}_t = \text{tax}_t^n + \text{tax}_t^k + \text{tax}_t^x + \text{tax}_t^c + \text{tax}_t^m - t^g_{ct} \text{g}_t$$ (41)
National budget constraint

Given the quantities and price of imports and exports, as well as the global required rate of return, the national budget constraint is as follows:

\[
v^f_t = \sum_i v_{i,t}^f - v^h_t\]

where \( \hat{r}_t^f \) is the ROW’s realised return on its Australian assets. The realised return equals the required return in expectation, however after an unexpected policy change it may not.

The trade balance is given by the value of exports minus imports.

\[
tb_t = \sum_{c e C} p^x_{c,t} x_{c,t} - \sum_{c e C a e A} \sum p^{wim}_{c,t} im_{c,a,t} \]

where \( p^{wim}_{c,t} \) is the price of at which the world supplies imports. The aggregate price of imports and exports are given by

\[
p^x_t = \frac{\sum_{c e C} p^x_{c,t} x_{c,t}}{x_t}\]

\[
p^{wim}_t = \frac{\sum_{c e C a e A} \sum p^{wim}_{c,t} im_{c,a,t}}{im_t}\]

The aggregate volume of exports and imports are calculated as:

\[
x_t = \sum_{c e C} x_{c,t}\]

\[
im_t = \sum_{c e C a e A} \sum im_{c,a,t}\]

The return paid to ROW is the negative net foreign income:

\[
-nfi_t = \hat{r}_t^f \frac{v^f_t}{1 + \gamma_t}\]

The capital account measures the ROW’s net acquisition of domestic assets are given by the sale of assets owned by the household:

\[
ca_t = v^f_t - \frac{v^f_{t-1}}{1 + \gamma_t}\]
more detail below, ensures we do not need to constrain foreign and domestic asset holdings as in many small open economy models, see Schmitt-Grohe & Uribe (2003) for an example.

**Equilibrium definition**

An equilibrium is defined by markets clearing such that the quantity demanded is equal to the quantity supplied in all markets. The model assumes that both the household and firms are price takers and firms accept zero economic profits.

Given the sequences of tax rates, government spending and transfers, the initial capital stock and land owned by each firm, the assets initially owned by the household and ROW, the household’s endowment of time, the productivity factors \( \lambda_{y,t}, \lambda^{n}_{t}, \lambda^{f}_{t}, \lambda^{z}_{t}, \lambda^{z}_{t} \) and foreign demand and international prices \( \{ p^{wm}_{c,t}, y_{t}^{w} \} \) a competitive equilibrium is given by a sequence of prices \( \{ p^{y}_{c,t}, w_{t}, p^{imp}_{c,t}, p^{g}_{c,t}, p^{d}_{t} \} \forall c \in C, a \in A, t \in T \), quantities, assets holdings and dividends and government revenue and expenditure for all commodities and firms that satisfies the following conditions:

1. The household maximises the discounted value of its expected utility, subject to the budget constraint, as per equations (6), (7), and (8);
2. Each firm maximises its equity value, subject to its resource constraint, as per equations (22) and (27);
3. Foreign demand for exports is satisfied as per equation (42);
4. The commodities markets clear such that total demand is equal to total supply per equations (29) and (30);
5. The labour market clears as per equation (12);
6. The balance of payments condition is satisfied in line with the trade balance, net factor income and the capital account as per equations (45), (50) and (51);
7. All equity is owned by either residents or foreigners as per equation (43); and
8. The government consumes its desired commodities and its budget is balanced as per equation (41); and
9. The aggregate value of production (GDP-P) equals aggregate value of expenditure (GDP-E) as given by

\[
\sum_{i=1}^{n} p^{y}_{i,t} y_{i,t} + tax^{y}_{t} + tax^{d}_{t} + tax_{t}^{n} - \sum_{i=1}^{n} \sum_{c \in C} p^{i}_{c,t} z_{c,i,t} = \\
\sum_{c \in C} p^{c}_{c,t} c^{c}_{t} + \sum_{c \in C} p^{g}_{c,t} g^{c}_{t} + \sum_{i=1}^{n} \sum_{c \in C} p^{ib}_{c,t} b^{c}_{i,c,t} + \sum_{c \in C} p^{x}_{c,t} x^{c}_{t} - \sum_{c \in C} p^{WIM}_{c,t} im_{c,t}.
\] (52)

It is worth noting this last market clearing condition is redundant as it is implied by the previous conditions.
4. Calibration

This model is calibrated to match Australian economic data. While most of the model parameters are calibrated to match Australian data, several parameters reflect consensus in the macroeconomic literature.

Version 1.0 of TIM assumes that the baseline economy is on its long run balanced growth path. To that end the growth rates of the model’s exogenous trends (that is, population and labour augmenting technical progress) are constant.

TIM is an annual model with annual trend output growth rate of 2.8 per cent where population contributes 1.3 per cent and labour augmenting progress 1.5 per cent.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.028</td>
<td>Trend output growth rate</td>
</tr>
<tr>
<td>$\gamma^h$</td>
<td>0.013</td>
<td>Population growth rate</td>
</tr>
<tr>
<td>$\gamma^i$</td>
<td>0.015</td>
<td>Growth rate of labour augmenting technical progress</td>
</tr>
</tbody>
</table>

Table 1: Growth parameters

When calibrating the model, is it necessary to make a decision about what time period the model is calibrated to. Due to data availability it is not always possible to calibrate all parts of the model to the same time period. TIM is designed to approximate the Australian economy using the most recent year of data published by the ABS’s Input-Output tables. TIM version 1.0 currently uses 2014-15 Input-Output tables (Australian Bureau of Statistics, 2017a).

The baseline scenario deviates from the Input-Output table data along three important dimensions. Firstly, the household’s consumption and income are consistent with a steady state balanced household budget. Secondly, investment is consistent with investment along the trend growth path. Finally, goods are produced and used in the same period so we do not include the change in inventories.

As part of model validation process, we check the model’s calibration and solution method by finding an equilibrium consistent with the initial Input-Output table calibration. We do so by allowing the household to spend beyond its budget constraint by including additional household borrowing to match spending observed in the Input-Output table. We adjust demand for investment using an over investment rate. Lastly, we include demand for inventories which are calibrated to match those observed in the data. This scenario exactly replicates the Input-Output table data. In baseline, without these replication adjustments, labour supply is 2.4 per cent higher than in the data and output is 1.4 per cent higher. This is due to the household balancing their budget in the baseline and working more to do so. The changes in the household budget along with changes in investment and inventories demand raise output from some industries by over 50 per cent and decrease output by around 20 per cent for other industries. The large changes are generally only observed for small industries, particularly certain manufacturing industries.
Household

Following the dynamic stochastic general equilibrium (DSGE) literature, we set the coefficient of risk aversion $\sigma = 2$ for the household (King, Plosser, & Rebelo, 1987). This implies an intertemporal elasticity of 0.5.

In Version 1.0, TIM’s calibrated labour supply targets a Frisch elasticity of labour supply, $\eta^s$, of 0.46. This elasticity is consistent with the weighted average calibrated Frisch elasticity within OLGA (Cai, Gustafsson, Kouparitsas, Smith, & Zhang, 2019). The Frisch elasticity is calculated using the following equation:

$$\eta^s = \frac{1 - N_d}{N_d} \left[ \frac{1 - \alpha_t (1 - \sigma)}{\sigma} \right]$$

where, $N_d$, is the value of labour hours in calibration data. This Frisch elasticity is consistent with the micro-econometric literature surveyed by Keane (2011). As a long term counterfactual model, this elasticity allows TIM to more accurately model the long run labour response. As TIM does not attempt to model business cycles it does not need a higher Frisch elasticity which is used in other general equilibrium macroeconomic models (Peterman, 2015).

To match the Frisch labour elasticity, a calibrated leisure share, $L_d / H$, of 0.35 is used. This gives the household’s disposable leisure share, that is, time spent outside of work and required resting. This Frisch elasticity calibration also determines the time endowment given that labour supply is given in the Input-Output table. This time endowment is shown in dollar value and this represents the total value of the household’s time spent on both labour and leisure over the entire year. The consumption share of utility, $\alpha_t$, is calibrated so that the share of labour supply and consumption match those in the Input-Output table.

The household’s discount rate, $\beta$, is calibrated so that consumption grows at trend of GDP in the steady state for the domestic return on assets implied by the Input-Output table data. This is calculated such that $\beta$ multiplied by the after tax return and economy growth rate is equal to one.

Consumption of different commodities

The consumption share of the commodity $c$, $\theta^{ch}_{ct}$, is calibrated so that the preferences match those indicated by the Input-Output table data. The consumption commodity elasticity, $\eta^{ch}$, is calibrated to 0.5. This value is consistent with OLGA (Cai, Gustafsson, Kouparitsas, Smith, & Zhang, 2019) and also similar to IECGE value of 0.6 (Independent Economics, 2013).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\sigma$</td>
<td>Coefficient of risk aversion</td>
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<tr>
<td>$\eta^s$</td>
<td>Frisch elasticity of labour supply</td>
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<tr>
<td>$L_d / H$</td>
<td>Households leisure share</td>
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<tr>
<td>$H$</td>
<td>Total hours endowment</td>
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### Parameter Description Value

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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>α</td>
<td>Consumption</td>
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<tr>
<td>β</td>
<td>Household’s discount rate</td>
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</tr>
<tr>
<td>$\eta^{ch}$</td>
<td>Consumption commodity elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta^{ch}_{c,i}$</td>
<td>Consumption share of commodity c</td>
<td>Range between 0 – 0.2044 for the 114 Industries.</td>
</tr>
</tbody>
</table>

#### Firms - Production Sector

Version 1.0 of TIM has 114 production industries corresponding to the industries published within the ABS Input-Output Table (Australian Bureau of Statistics, 2017a). The following section outlines the parameter and elasticity estimates used to calibrate all industries.

### Primary factor and intermediate input shares

The factor shares of production, $\theta^r_{i,j}$, $\theta^k_{i,j}$, $\theta^z_{i,j}$, and $\theta^f_{i,j}$ are calibrated using the shares of compensation of employees (COE), gross operating surplus and mixed income (GOSMI) and intermediate inputs from the Input-Output table. As Version 1.0 of TIM uses 114 industries there are 114 factor shares for $\theta^r_{i,j}$, $\theta^k_{i,j}$, $\theta^z_{i,j}$, and $\theta^f_{i,j}$.

The values for COE are adjusted by the ratio of employed to employee data, as published in Table 20 of the 2012-13 Input-Output table (Australian Bureau of Statistics, 2013) to account for labour income included within mixed income recorded in GOSMI. The labour share, $\theta^r_{i,j}$, is calculated by using this adjusted COE. Land’s share of output, $\theta^f_{i,j}$, is found by authors calculation using ABS multifactor productivity dataset ABS (2017a), ABS (2017b), and ABS (2016a). Capital’s share of output, $\theta^k_{i,j}$, is calculated by using the GOSMI value from ABS (2017a) net of the adjustment to COE and the land income. The aggregate intermediate input share, $\theta^z_{i,j}$, is obtained as a residual of the other three primary factor shares.

### Factor Substitution

The elasticity of substitution between capital, labour, land and intermediate inputs, $\eta^y$, is assumed to be 0.5, consistent with estimates of the elasticity of substitution between capital and labour reported by Hutchings and Kouparitas (2012).

### Input Substitution

The investment input elasticity of substitution, $\eta^b$, and the intermediate input elasticity of substitution, $\eta^z$, are set to 0.5 – consistent with the OLGA (Cai, Gustafsson, Kouparitsas, Smith, & Zhang, 2019).

### Investment / Adjustment Cost Parameters

The industry capital depreciation rate, $\delta$ of 0.054 is based on ABS National Accounts 5204.0 Table 65 (2018).
Following the broader DSGE literature, capital adjustment costs are set to 10 for quarterly models. Mendoza and Tesar (1998) argue that this value is consistent with the average rate of convergence to the long run growth path estimated by Barro et. al. (2004). Consistent with the methodology followed by OLGA (2019), an annual capital adjustment cost, $\kappa^k$, of 2.5 is used. This parameter is consistent with D’Erasmo et. al. (2016) which uses elasticity of capital tax base as the basis of their calibration.

The labour adjustment cost, $\kappa^n$, is set to 2, consistent with the calibration in Jaimovich & Rebelo (2008).

The rate of capital services from capital, $\omega^k$, is given by the ratio of GOS to capital. This data is obtained from ABS (2017b).

The firm’s investment share, $\theta^{ib}_{c,d}$, is calibrated using the ABS Input-Output tables. It is equal to after tax value of investment by firm divided by the sum of after tax value investment. Investment by industry is calculated combining various data sets. ABS National Accounts 5204.0 Table 64 provides private investment by capital type and by 2 digit industry. The Input-Output tables provide commodity use and taxation for private investment. This data and intuition (for example, the exploration and mining support services commodity is all located to mineral and petroleum exploration intellectual property) is used to allocate investment by commodity to each industry. Investment in each commodity is used to calculate each firm’s investment shares.

Table 3: Production parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^k_{t,d}$</td>
<td>Capital share of output</td>
<td>Range between 0– 0.5468 for the 114 Industries. Average is 0.1481</td>
</tr>
<tr>
<td>$\theta^l_{t,d}$</td>
<td>Land share of output</td>
<td>Range between 0 – 0.2504 for the 114 Industries. Average is 0.0135</td>
</tr>
<tr>
<td>$\theta^n_{t,d}$</td>
<td>Labour share of output</td>
<td>Range between 0.0000 – 0.8103 for the 114 Industries. Average is 0.2721</td>
</tr>
<tr>
<td>$\theta^z_{t,d}$</td>
<td>Aggregate intermediate input share</td>
<td>Range between 0.1128 – 0.9046 for the 114 Industries. Average is 0.5663</td>
</tr>
<tr>
<td>$\theta^c_{c,t,d}$</td>
<td>Intermediate input share of commodity c</td>
<td>Range between 0 – 0.2044 for the 114 Industries. Average is 0.0088</td>
</tr>
<tr>
<td>$\eta^v$</td>
<td>Elasticity of substitution between land, labour and capital</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta^z$</td>
<td>Intermediate input elasticity of substitution</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta^i$</td>
<td>Investment input elasticity of substitution</td>
<td>0.5</td>
</tr>
<tr>
<td>$\kappa^n$</td>
<td>Labour adjustment cost</td>
<td>2</td>
</tr>
<tr>
<td>$\kappa^k$</td>
<td>Capital adjustment cost</td>
<td>2.5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.0535</td>
</tr>
<tr>
<td>$\theta^{ib}_{c,d}$</td>
<td>Firms investment share</td>
<td>Average is 0.0083</td>
</tr>
</tbody>
</table>
Firms - Distribution Sector

Margins

The margin use coefficients for each final user $\psi_k^{ch}$, $\psi_k^{ib}$, $\psi_k^{g}$, $\psi_k^{z}$, and $\psi_k^{x}$ are based on the authors calculation using the ABS margin commodity Input-Output Tables 23-34 (2017a). It is calibrated by dividing the total basic price expenditure of margin commodity paid to transfer each commodity to end use, by the total basic value of expenditure plus duties of each commodity and for each end use.

Table 4: Margin parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_k^{ch}$</td>
<td>Margin use coefficient of households</td>
<td>Range between 0–0.3444 for the 114 Industries. Average is 0.0323.</td>
</tr>
<tr>
<td>$\psi_k^{ib}$</td>
<td>Margin use coefficient of investment</td>
<td>Range between -0.0792 – 1.1539 for the 114 Industries. Average is 0.0440</td>
</tr>
<tr>
<td>$\psi_k^{g}$</td>
<td>Margin use coefficient of government</td>
<td>Range between 0.0000 – 0.3784 for the 114 Industries. Average is 0.0296</td>
</tr>
<tr>
<td>$\psi_k^{z}$</td>
<td>Margin use coefficient of intermediate input sector</td>
<td>Range between 0.0 – 1.2 for the 114 Industries. Average is 0.0786</td>
</tr>
<tr>
<td>$\psi_k^{x}$</td>
<td>Margin use coefficient of exports</td>
<td>Range between 0 – 1.0674 for the 114 Industries. Average is 0.1189</td>
</tr>
</tbody>
</table>

Total supply of commodities

The domestic share of use share, $\theta^{im}_{c,a,t}$, is calculated by using the share of domestic production from the Input-Output Table (ABS, 2018b). The elasticity of substitution between domestic and imported varieties of commodity $c$, $\eta^{im}$, is calibrated to be 2.

Table 5: Commodity supply parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta^{im}$</td>
<td>Elasticity of substitution between domestic and imported varieties of commodity $c$</td>
<td>2</td>
</tr>
<tr>
<td>$\theta^{im}_{c,a,t}$</td>
<td>Domestic share of use share</td>
<td>Range between 0.0472 – 1.0012 for the 114 Industries. Average is 0.7942.</td>
</tr>
</tbody>
</table>

Rest of the world

The initial asset holdings are calculated from the ABS Economic Activity of Foreign Owned Business Activity dataset. The share owned by foreigners is calculated from this data which give the domestic ownership and the share of assets allocated to each industry.

The rate of return required by foreign investors, $r$, is calculated so that the rate of return is consistent with observed GOS, private capital stock and depreciation. In practice, the required rate of return is set
equal to the rate of capital services from capital, \( \omega^k \), minus the depreciation rate, \( \delta \), adjusted for company income tax \( r = (1 - \tau^k)(\omega^k - \delta) \).

Australia is also close to being a price taker for exports, as such for the export price elasticity \( \eta^x \) we use a standard value of -1.2. For the following industries, where Australia has some market power or product differentiation (e.g. tourism services) a lower value of -6 is used: Sheep; grains; beef; dairy; Coal; Iron ore; Accommodation; Food and beverage service; Air and space transport; and Education. This is consistent with the calibration in (Independent Economics, 2013).

The export demand per commodity, \( \theta_{x,w} \), based on export observed in the Input-Output table. The model uses a composite parameter \( \theta_{x,w} = \theta_{v,w} \theta_{x,c} \), where this parameter calibrated from exports in the Input-Output table.

### Table 6: Rest of the world parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_i )</td>
<td>Share of household asset allocated to each industry</td>
<td>Sums to 1 across all industries, foreigners own 29 per cent of initial assets.</td>
</tr>
<tr>
<td>( \eta^x )</td>
<td>Export demand elasticity</td>
<td>6 - 12</td>
</tr>
<tr>
<td>( r_t )</td>
<td>World interest rate</td>
<td>0.0471</td>
</tr>
<tr>
<td>( \theta_{x,w} )</td>
<td>World export demand</td>
<td>Sums to 2,698,749</td>
</tr>
</tbody>
</table>

**Government**

**Revenue**

The government collects revenue by levying tax on labour income, company income, the consumption of commodities and production.

**Income Taxes**

Total company income taxes for 2014-15 can be obtained from the ATO’s Taxation statistics 2014–15 Company Table 1A (Australian Tax Office, 2017). To calculate the tax rate on capital, \( \tau^k \), company income taxes are divided by total of GOS and Land value add whilst subtracting depreciation on private capital obtained from Table 2.

Labour income taxes for 2014-15 can be obtained from the ATO’s Taxation statistics 2014–15 Individuals Table 1A (Australian Tax Office, 2017). To calculate the labour tax rate, \( \tau^x \), Labour income taxes is divided by total taxable income (obtained from the same table).

**Taxes on Production**

Payroll tax, the tax on the return to fixed factors, the tax on variable capital and production subsidies are calculated from taxes on production which are obtained from 2014-15 ABS Input-Output Table 2 (2017a). These taxes are allocated to industries consistent with aggregate taxes on production and industry data from ABS industry dataset and the government revenue data and the labour account.
Taxes on Products

Taxes on products are obtained from 2014-15 ABS Input-Output Tables 35-40 (2017a). This provides the information on duty, GST tax, subsidies, excise and other taxes collected on intermediate inputs, consumption, investment, government and exports. The implied tax rates were calculated as follows:

Table 7: Implied tax rates

<table>
<thead>
<tr>
<th>Implied tax rate</th>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of duty</td>
<td>Duty revenue divided by competing imports</td>
</tr>
<tr>
<td>Excise</td>
<td>Excise revenue divided by basic values of use including duties</td>
</tr>
<tr>
<td>Sales Tax</td>
<td>Sales tax revenue divided by the sum of basic values of use plus duty revenue, excise revenue and margins</td>
</tr>
</tbody>
</table>

Expenditure

Government spending is obtained from 2015-16 ABS Input-Output Table 4 (2017a). As mentioned in Section 3, lump sum transfers are adjusted to ensure that the government budget balances.

5. Welfare criterion for policy analysis

We use the equivalent variation to report both the welfare change of a scenario both in dollar terms and the consumption equivalent percentage welfare change.

If utility in the baseline is given by $u_0$ and $u_{1,t}$ is utility in each period in the counterfactual scenario then equivalent variation for a given period is given by

$$EV_t = P^*_0 u_{1,t} - P^*_0 u_0$$

For a measure of the lifetime equivalent welfare change we convert the lifetime welfare in the counterfactual into per period welfare under initial prices which is given by

$$u^*_t = \left(\frac{(1-\sigma)U_1}{1-\beta}\right)^{1/(1-\sigma)}$$

The life time equivalent variation in dollar terms is then

$$EV_l = P^*_0 u^*_t - P^*_0 u_0$$

We also report the welfare change as a consumption equivalent percentage change. Percentage change relative to consumption is more relatable than percentage change in welfare that includes leisure. The consumption equivalent welfare change is the change in consumption that delivers the same welfare as in the counterfactual scenario. If $c_0, l_0$ are consumption and leisure in the baseline and $u_1, c_1, l_1$ are consumption leisure and utility in the counterfactual the level of consumption $c^*_1$ that gives the same utility as the counterfactual holding leisure constant is given by
The consumption equivalent percentage change in welfare is then given by

\[ \frac{c_{1,t}^*}{c_0} - 1 \]

To calculate the consumption equivalent life time percentage welfare change use the per period utility that gives lifetime utility equal to that in the counterfactual as given by

\[ \frac{c_t^*}{c_0} - 1 \]

As above, the consumption equivalent percentage change in lifetime welfare is then given by

6. Conclusion

The Australian Treasury provides advice to government on a broad range of policy issues including specific industry policies. To inform this advice Treasury can draw from a suite of policy analysis models with the choice of model depending on the type of policy and its size and scope. In this paper we introduce a new dynamic general equilibrium model of the Australian economy for counterfactual analysis of industry specific policy.

TIM extends previous Australian models with many industries by incorporating forward looking agents and a well-defined balanced growth path. This approach overcomes many of the limitations of existing models by incorporating the latest mainstream economic theory into an industry rich general equilibrium model.

Despite the significant detail in TIM the model remains a representation of the Australian economy, not a replication. As such we acknowledge the following limitations of the model. First, the welfare measure, while being internally consistent, is only for the representative household and does not directly provide information on the distributional effects of policy. That said, the labour market and firm effects by industry provide a guide on effects on households involved with different industries.

A second limitation of the model is that we do not explicitly include unemployment in the model. Multisector models that include unemployment often do so through reduced form expression mapping unemployment to changes in industry labour demand.\(^{16}\) Essentially these relationships capture labour adjustment costs and these are included in TIM. Explicitly modelling employment in a way that captures the mechanisms would require a more micro founded approach such as modelling matching between employers and employees. This would add significant complexity to an already large model.

\(^{16}\) See Adams, Dixon and Horridge (2015) for example.
A number of model development decisions and assumptions also warrant explanation. The standard closure for TIM assumes there is no government debt and that the government balances its budget every period. Including government debt in a model with an infinitely lived representative household and perfect capital mobility would add no information as Ricardian equivalence holds. That is, including government debt would not change the model’s results.

TIM assumes agents have perfect foresight and does not include uncertainty. When government announces a future policy change there exists potential for uncertainty about the implementation. The degree to which Australians will incorporate announced policy changes into their decision making is beyond the scope of this modelling. In these cases, modelling both anticipated and unanticipated policy changes can provide a range for the estimates of the policy impacts.

There are limits to the degree detail that can feasibly be included in TIM. As such, the model will at times need to be linked with models that capture additional detail. Previous Treasury modelling linked a domestic many-sector CGE models with a range of other models. These included models with more detailed representation of the global economy, the domestic electricity market, the transport sectors and the land use sectors. TIM has been designed to allow similar linking between models.

Nonetheless, analysis of certain policies will be facilitated by additional model detail and development. This is not considered a general limitation. Rather, we consider ensuring the model is fit for purpose for each policy analysed is a strength of our approach. The version of the model presented in this paper is referred to version 1.0. This version is intended to serve as a bridge to previous Treasury industry policy modelling and has been designed so that it can be enhanced through discrete development modules. To meet the needs of our stakeholders, we will develop modules which will build on version 1.0 as either a major or minor change designated by Version Major.Minor. A structural change baseline version (version 1.1) will depart from a stationary baseline to a calibration with changing industry composition, known as structural change. An energy module (version 2.0) will expand the electricity sector calibration of version 1.1, which enhances the modelling of energy analysis.
7. REFERENCES


