

DEVELOPING A COST OF CAPITAL MODULE FOR COMPUTABLE GENERAL EQUILIBRIUM MODELLING

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Abstract

Computable general equilibrium (CGE) models are ideal tools for policy analysis. However, work conducted by the Centre of Policy Studies on recent reforms to business taxation policy in Australia has highlighted the limitations of CGE models in this specific area. This paper outlines the ongoing development of a more detailed and theoretically rigorous cost of capital module for a CGE model. Following King and Benge, we develop a model in which the firm maximises the value of its shareholder equity, taking account of: various company and personal income tax regimes; various capital-gains taxes regimes (including a treatment of realisation-based capital-gains taxation); depreciation allowances; investment allowances; and the costs of issuing various securities.

The model is developed in two stages:

Firstly, we develop an expression for the value of the firm to its shareholders. This approach assumes a given level of before-tax profit and seeks to determine how changes in various tax rates and allowances might impact on the firm's value and, thus, the rate of return available to those holding its equity. With the before-tax income streams fixed, movements in the value of the firm and the after-tax income flows tell us something about this rate of return. As well as gaining some insights into the effect of policy changes on the value of equity, we can readily infer from this the effect of policy adjustments on the willingness of investors to provide funds as stakeholders.

Secondly, we construct and solve a constrained optimisation problem for all of the firm's choice variables, using the expression for the value of the firm as the objective function. We develop a set of expressions to constrain the firm's ability to maximise the after-tax return on equity to its shareholders, and solve for the cost of capital and, thus, the level of investment, for an optimising firm. This approach enables us to generate a time path for investment and the outcomes of the firm's other choice variables (that is, a complete solution to the "producer problem"). In this framework, we can see how taxation can influence both the choices the firm makes and the outcomes it can expect *given* those choices. By embedding this in a dynamic CGE model like MONASH, we can simulate the effects of tax changes on the user-cost of capital and thus on investment.

Keywords: Cost of Capital, Investment, Taxation, Computable General Equilibrium Modelling.

1. Introduction

In the last few years, the Australian taxation system has undergone significant reform, and part of this process was a review of the business taxation system. Among the changes under consideration were the rates of company tax and capital gains tax, the question of capital gains indexing and averaging, and changes to the deductions available on the purchase and use of various types of capital. Such policy reforms are difficult to model with CGE (computable general equilibrium) models currently, because typically they do not take account of such things as the rate of company tax, or the interplay between the tax system and the behaviour of firms. This paper reports on the development of an investment structure for a CGE model that explicitly incorporates business taxation. Motivated by a desire to enable a direct analysis within a CGE model of the effect of changes in a corporation's tax environment, this paper attempts to contribute to the existing approach in two ways.

Following an approach developed by King (1974, 1977), and applied more recently in Australia by Benge (1997, 1999), we develop an expression for the value of the firm to its shareholders. This approach assumes a given level of before-tax profit and seeks to determine how changes in various tax rates and allowances might impact on the firm's value and, thus, the rate of return available to those holding its equity. With the before-tax income streams fixed, movements in the value of the firm and the after-tax income flows tell us something about this rate of return. As well as gaining some insights into the effect of policy changes on the value of equity, we can readily infer from this the impact of policy adjustments on the willingness of investors to provide funds as stakeholders.

The second approach is to solve a constrained optimisation problem for all of the firm's choice variables, using the expression for the value of the firm as the objective function. We develop a set of expressions to constrain the firm's ability to maximise the after-tax return on equity to its shareholders, and solve for the cost of capital and, thus, the level of investment, for an optimising firm. This approach enables us to generate a time path for investment and the level of demand for the firm's other factors of production (provide a solution to the "producer problem"). In this framework, we can see how taxation can influence both the choices the firm makes and the outcomes it can expect *given* those choices.

The aim is to apply these two methodologies to a dynamic computable general model such as the MONASH model, to enable detailed analysis of changes in business taxation on the user-cost of capital and thus on investment. Part of the focus in developing these models is to incorporate them into a CGE model in such a way that makes a sensible and tractable CGE analysis of such policy issues available to the user without specific expertise in financial economics or investment modelling.

2. The Value of the Firm

In this section, we derive an expression for the value of the firm to its shareholders. This can then be used to answer questions such as: how does a change in a taxation parameter affect the value of the firm, *given* knowledge of the value of dividend payments in each period? That is, if we assume that the firm's behaviour is exogenous and known, we can determine how changes in tax rates or a regime switch will change the firm's value to its shareholders. This could be used to inform an analysis of how some factors underlying the firm's cost of capital are influenced by taxation provisions. If we know, for example, that a policy reform of some type will increase the value to shareholders of the firm's fixed income streams, we can infer something about the willingness of potential investors and lenders to provide funds to the firm and, thus, the required rate of return needed to convince them to do so. In this sense, the required rate of return that is implied by such an analysis is a qualitative measure of a change in the firm's cost of capital.

2.1. Basics

Firstly, let's deal with some definitions. We allow firms the option of issuing new equity to acquire capital in any period, and so we need to allow for the dilution of pre-existing owners' equity that this entails. If V is total equity in the firm, VO is the total value of pre-existing equity and VN is the total value of new share issues, then

$$V_t^j = VO_t^j + VN_t^j \quad (1)$$

All values are measured at the beginning of the period. Subscript t refers to the period and superscript j refers to an industry. New share issues occur ex-dividend, and the variable denoting them includes issue costs.

Let's now define D as the dividend payable at the beginning of period t on the previous period's operations and κ as the firm's distributable earnings before tax or earnings *after* interest but before tax (EAIBT)¹. In the absence of any taxes, shareholders' earnings E_t at the beginning of period t are denoted by

$$E_t^j = \kappa_t^j = D_t^j \quad (2)$$

Earnings are defined in this model as after-tax net receipts by shareholders. This includes capital gains tax liabilities where applicable (at this stage we abstract from them), but not unrealised capital gains. We also assume that un-retained profits generated in period t are distributed as dividends at the beginning of period $t+1$. As we are abstracting from taxes at this stage, equation (2) simply says that shareholder earnings in period t are equal to the dividends distributed at the beginning of period t stemming from period $t-1$'s operations, and that in the absence of taxes, dividends are equal to EAIBT.

In inter-temporal modelling, we need to choose an arbitrage condition. In order to attract equity capital, the representative firm needs to provide some minimum rate of return that is related to the investor's marginal rate of time preference or discount rate. If i is the interest rate on one-period bonds in period t (which reflects a riskless required rate of return)², arbitrage behaviour in financial markets ensures that a two-period equilibrium is characterised by

$$i_{t+1}V_t^j = E_{t+1}^j + (VO_{t+1}^j - V_t^j) \quad (3)$$

Extending the time horizon to T periods, (1), (2) and (3) provide

$$V_0^j = \sum_{t=1}^T \left[\frac{D_t^j - VN_t^j}{\prod_{s=1}^t (1+i_s)} \right] + \frac{V_T^j}{\prod_{s=1}^T (1+i_s)} \quad (4)$$

Equation (4) defines the value of the firm at the beginning of period zero in a discrete time, multi-period setting with perfect foresight. At the beginning of period zero, the value of the firm is equal to the present value of all future dividend payments plus the present value of holding the firm at time T .

¹ In the terminology of finance, this is EBIT (earnings before interest and tax) minus *net* interest cost (that is, the net-of-tax value of interest).

² Because it is riskless, arbitrage behaviour in financial markets should drive this into equality with the investor's marginal rate of time preference.

It is common in inter-temporal problems to remove the terminal constraint by the application of a transversality condition:

$$\lim_{t \rightarrow \infty} \left[\frac{V_t^j}{\prod_{s=1}^t (1+i_s)} \right] = 0 \quad (5)$$

This states that, as long as the present value of V remains bounded as t approaches infinity, the right-most term of (4) approaches zero. This can probably best be understood by recognising what behaviour it rules out. According to the arbitrage condition implied by (3), for the firm's value to grow at a rate faster than i requires the payment of negative dividends³. Thus, the transversality condition rules out the possibility of a firm growing faster than i forever while paying negative dividends⁴. Efficient capital markets place significant constraints on the ability of firms to behave in this manner for even a single period let alone forever.

Applying the transversality condition to (5) we obtain

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{D_t^j - VN_t^j}{\prod_{s=1}^t (1+i_s)} \right] \quad (6)$$

Equation (6) says that, in equilibrium, the present value of equity in the firm is equal to the sum of all future dividend streams minus any new share issues, appropriately discounted. The assumptions underlying the application of the transversality condition ensure that this value is always finite.

One final issue to deal with at this stage is the question of uncertainty. By defining the value of the firm as a function of the present value of future resource flows and changes in stock values as we did in expression (6), we assume these values can be known today. Given that the future cannot be known with certainty, there is a role for investor expectations in valuing a firm. We introduce now a basic form of expectations in a general form.

In valuing the stock of a particular firm, shareholders will need to form an expectation of both the return on holding equity and the return on the alternative. It seems sensible, therefore, to replace all of the future values of dividends, new equity issues and the bond rate in equation (6) with *expected* values. If Ω is the information set available at time t with which investors form an expectation of the future values of these variables, then (6) becomes

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{D_t^j [\Omega_0^j] - VN_t^j [\Omega_0^j]}{\prod_{s=1}^t (1+i_s [\Omega_0])} \right] \quad (7)$$

Expectations can refer to an information set regarding the prospects of a single industry (those with superscript j) or to economy-wide factors (no superscript). Notice that the discount rate is a

³ I.e. if $(V_{t+1}^O - V_t) > i_{t+1}V_t$ then $D_{t+1} < 0$.

⁴ A "negative" dividend involves a cash flow from the shareholder to the firm based on the shareholder's proportional ownership of the firm. This is uncommon, but is possible, for example, in the case of contributing shares. In most cases, equities are issued with the protection of limited liability intact in all senses.

function of an expected interest rate that is related to economy-wide factors and not to specific industry prospects.

2.2. Income Taxation

2.2.1. The Classical Company Income Tax System

Under the classical system of company-income taxation, dividends are taxed at both the full rates of company and personal income tax without any transfer of tax credits. If τ is the company income tax rate, DC is a dividend distributed in a classical system and all profits are distributed, the dividend paid in a classical company taxation system is

$$DC_t^j = (1 - \tau_t) \kappa_t^j \quad (8)$$

Equation (8) simply says that the firm pays company tax on its EAIBT and distributes the net result as a dividend. Taxes, of course, are not industry specific. On receiving this dividend, the shareholder is liable for the full marginal rate of personal income tax on its value. If θ is the marginal personal income tax rate of the representative shareholder, shareholder earnings are

$$E_t^j = (1 - \theta_t) DC_t^j = (1 - \theta_t)(1 - \tau_t) \kappa_t^j \quad (9)$$

Following this, (7) becomes

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{(1 - \theta_t) DC_t^j [\Omega_0^j] - VN_t^j [\Omega_0^j]}{\prod_{s=1}^t (1 + i_s [\Omega_0]) (1 - \theta_s)} \right] \quad (10)^5$$

We also now make the tax rates contingent on expectations. If shareholders or potential equity investors believe that rates of taxation on their distributions are likely to change, then the value of the firm will adjust accordingly. We now define the information set Ω for each period to contain information regarding the after-tax value of dividends and capital gains payments. In this way, this information set can also contain information that the investor can use to form an expectation of tax policy changes, and tax parameters take on expected values.

2.2.2. The Dividend Imputation CIT System

The second broadly defined type of company taxation is a dividend imputation system. Dividend imputation refers to the transfer of tax credits to the shareholder for tax paid at the company level. The objective of dividend imputation is to subject all distributed income to one rate of taxation – the personal income tax rate of the shareholder –and remove biases between sources of external finance.

Under the dividend imputation system, shareholders can receive franked or unfranked dividends. *Unfranked dividends* are distributed with no tax credits, and the shareholder is liable for the full marginal rate of personal income tax on each dollar of the distribution. *Franked dividends* are distributed with tax credits equal to some proportion of the tax paid at the company level, and the shareholder is liable for the difference between this amount and the amount that would have to be paid if the entire pre-company tax dividend was taxed solely at the personal tax rate.

⁵ Notice that we have also added a tax coefficient containing the personal income tax rate to the bond rate in the discounting terms. Interest earnings are typically deductible for personal income tax purposes, and so the after-tax value of interest income now becomes the discounting factor for future dividend streams.

Firstly, let's look at franked dividends. To simplify the analysis, let's begin by assuming a full dividend imputation system. Company income tax is paid on the EAIBT, and the after-company-tax profit is paid as a franked dividend. Assuming for now that all profits are distributed as franked dividends, denoting these payments by **DF** we can define them as

$$DF_t^j = \kappa_t^j (1 - \tau_t) \quad (11)$$

that carry tax credits worth $\tau_t \kappa_t^j$, or $\frac{\tau_t}{(1 - \tau_t)} DF_t^j$, so the shareholder receives κ_t^j in full:

$$DF_t^j + \tau_t \kappa_t^j = \kappa_t^j (1 - \tau_t) + \tau_t \kappa_t^j = \kappa_t^j \quad (12)$$

For tax credits to be distributed, the firm must have paid some tax. We can relax the assumption that all earnings are distributed as franked dividends by adding some deductions to (11) and (12). If **TD** denotes the total value of deductions available to the firm from any potential source, it follows that

$$DF_t^j = (1 - \tau_t) (\kappa_t^j - TD_t^j) \quad (13)$$

and

$$DF_t^j + \tau_t \kappa_t^j = (1 - \tau_t) (\kappa_t^j - TD_t^j) + \tau_t (\kappa_t^j - TD_t^j) = \kappa_t^j \quad (14)$$

Therefore, shareholder after-tax earnings with full imputation are

$$E_t^j = (1 - \theta_t) (\kappa_t^j - TD_t^j) = \left[\frac{1 - \theta_t}{1 - \tau_t} \right] DF_t^j \quad (15)$$

We can also account for partial dividend imputation systems. To allow us to choose the “degree” of dividend imputation, we introduce a new variable, γ , that denotes the proportion of total tax paid at the company level in period t that can be claimed as tax credits for personal income taxation purposes. The expression for shareholder earnings then becomes

$$E_t^j = \frac{(1 - \theta_t) [1 - (1 - \gamma_t) \tau_t]}{1 - \tau_t} DF_t^j = (\kappa_t^j - TD_t^j) (1 - \theta_t) [1 - (1 - \gamma_t) \tau_t] \quad (16)$$

With full imputation, the firm generates κ , pays $\tau\kappa$ tax, pays a franked dividend and passes on $\tau\kappa$ to shareholders as tax credits (again, assuming for now that all profits are distributed). Shareholders then pay personal income tax on the sum of these two amounts. With partial imputation, the firm generates κ and pays $\tau\kappa$ tax, but only passes on $\gamma\tau\kappa$ in tax credits. Therefore, $\gamma_t = 1$ denotes full dividend imputation and $0 < \gamma_t < 1$ denotes partial dividend imputation.

Next, we come to unfranked dividends. We assume that because firms have incentives to distribute tax credits when they are available (or, at least, have little incentive to distribute unfranked dividends when tax credits are available), a firm will have unfrankable earnings available when it has paid *no* company tax on some portion of its earnings. If it has paid company tax, it will distribute a franked dividend. If this is true, the two types of dividends are equivalent from the point of view of the company's EAIBT. To see why, consider, firstly, that the unfranked dividend is a dividend that carries no tax credits because it is untaxed at the point that it is received by the shareholder. This portion of the firm's earnings can only have been untaxed due to tax deductions. The unfranked dividend, denoted by **DU**, is therefore simply defined as

$$TD_t^j = DU_t^j \quad (17)$$

and shareholder earnings are

$$E_t^j = (1 - \theta_t)TD_t^j = (1 - \theta_t)DU_t^j \quad (18)$$

It is true that the after-tax value of a dollar of franked dividends is higher than the after-tax value of a dollar of unfranked dividends, but a dollar of EAIBT has the same value to the shareholder *regardless* of whether it franked or unfranked. If we now assume that the firm is paying both types of dividends, shareholder earnings are

$$\begin{aligned} E_t^j &= \left[\kappa_t^j \left[1 - (1 - \gamma_t)\tau_t \right] + TD_t^j (1 - \gamma_t)\tau_t \right] (1 - \theta_t) \\ &= \left[\frac{(1 - \theta_t) \left[1 - (1 - \gamma_t)\tau_t \right]}{1 - \tau_t} \right] DF_t^j + (1 - \theta_t)DU_t^j \end{aligned} \quad (19)$$

As such, it is not strictly correct to say that unfranked dividends are equivalent to those paid in a classical system of company taxation. While it is true that a dollar of dividends in a classical system is taxed in the same way at the personal level as a dollar of unfranked dividends in a dividend imputation system, it is *not* true to say that a dollar of EAIBT is taxed identically as a classical and unfranked dividend. Simply put, if there are incentives to pay franked dividends when franking credits are available, unfranked dividends are *not* double-taxed. If we focus on the firm's dividend policy rather than its EAIBT, we can dispense with the distinction between *DC* and *DU*, and define the value of the firm as

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{(1 - \theta_t)D_t^j \left[\Omega_0^j \right] + \left[\frac{(1 - \theta_t) \left[1 - (1 - \gamma_t)\tau_t \right]}{1 - \tau_t} \right] DF_t^j \left[\Omega_0^j \right] - VN_t^j \left[\Omega_0^j \right]}{\prod_{s=1}^t \left[1 + i_s \left[\Omega_0 \right] (1 - \theta_s) \right]} \right] \quad (20)$$

In applying this expression to the analysis of a company in a classical system of taxation, we would simply set *DF* at zero.

2.3. Capital Gains Taxation

2.3.1. Taxing Accrued Capital Gains

Accrual-basis capital gains tax (CGT) systems require an asset holder to pay CGT at the end of every period, on gains accrued during that period. This type of CGT system is usually assumed in theoretical modelling due to its convenience, largely because, in comparison to a realisation-basis system, endogenous timing issues do not affect the accrual-basis system. Shareholders have no discretion over the timing of capital gains tax payments, and thus the important realisation-system issue of deferral⁶ plays no role.

Denoting the effective rate of the capital gains tax under an accrual-basis system by *ca*, the expression for shareholder earnings becomes

$$E_t^j = (1 - \theta_t)D_t^j + \frac{(1 - \theta_t) \left[1 - (1 - \gamma_t)\tau_t \right]}{1 - \tau_t} DF_t^j - ca_t (VO_t^j - V_{t-1}^j) \quad (21)$$

⁶ I.e. delaying the sale of an asset to push the capital gains tax payment into the future and reduce its present value.

The last term on the right-hand side of (21) takes account of the periodical outflow of funds required to pay the accrued CGT liability. The effective rate of the capital gains tax will be defined differently for each capital gains system, but for an accrual-basis system we define this rate as

$$ca_t = c_t \psi_t \quad (22)$$

where c is the statutory rate of the capital gains tax, and ψ is the proportion of the total capital gain that is taxable⁷. With this definition of shareholder earnings, the value of the firm over an infinite horizon is

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{\left[\frac{1-\theta_t}{1-ca_t} \right] D_t^j [\Omega_0^j] + \left[\frac{(1-\theta_t)[1-(1-\gamma_t)\tau_t]}{(1-\tau_t)(1-ca_t)} \right] DF_t^j [\Omega_0^j] - VN_t^j [\Omega_0^j]}{\prod_{s=1}^t \left[\frac{1+i_s [\Omega_0^j](1-\theta_s) - ca_s}{1-ca_s} \right]} \right] \quad (23)$$

The product of these CGT terms in the denominator of the discount factor for all periods prior to period t acts to dilute the value of the firm (by increasing the value of the discount factor) to take account of the way in which capital gains tax payments in prior periods dilute the value of the shareholder's equity. Every dollar paid in capital gains tax is equivalent to not receiving one dollar of discounted future earnings. The capital gains tax term in the numerator of the discount rate acts to subtract the base-value of the firm for the calculation of the capital gain that occurs in any period.

2.3.2. Taxing Realised Capital Gains

The essential difference between the accrual-basis and realisation-basis CGT systems arises from the timing of CGT payments, and therefore their present value. With a realisation-basis CGT, the payment of the CGT liability is delayed until the asset is realised, and thus the liability is discounted at the rate applicable to the period of the sale, which may or may not be the period in which the capital gain was incurred.

Our approach, which has a basis in the work of King (1977), can be understood by starting with a question: From the perspective of an individual in period t , what is the difference between the value of shares held at time t but purchased in $t-1$ and those held and purchased at time t ? The answer is the additional capital gains payments that will be due on the shares acquired in period $t-1$ (assuming that the shares appreciated in value). Let's denote the proportion of the shareholder's total equity in the firm that is realised in some period by ε , let cr be the effective rate of capital gains tax under a realisation-basis system, and let Z denote the present value of the additional future capital gains taxes payable on the capital gain received in period t . The arbitrage condition for this model, inclusive of all dividend and capital gains, becomes

$$\begin{aligned} (i_t [\Omega_{t-1}^j](1-\theta_t))V_{t-1}^j &= (1-\theta_t)D_t^j [\Omega_{t-1}^j] + \frac{(1-\theta_t)[1-(1-\gamma_t)\tau_t]}{1-\tau_t} DF_t^j [\Omega_{t-1}^j] \\ + \varepsilon_t (VO_t^j [\Omega_{t-1}^j] - V_{t-1}^j) - cr_t (VO_t^j [\Omega_{t-1}^j] - V_{t-1}^j) &+ (1-\varepsilon_t)(VO_t^j [\Omega_{t-1}^j] - V_{t-1}^j) - Z_t^j [\Omega_{t-1}^j] \end{aligned} \quad (24)$$

⁷ We include ψ because only a proportion of capital gains are taxed under some CGT regimes. Two examples are Australia (50% of the nominal gain) and Canada (75% of the nominal gain). Further, while in some countries we see CGT rates reflect personal income tax rates, this is not universally true and so we denote the CGT rate as a separate parameter of the model.

where

$$cr_t = \varepsilon_t \psi_t c_t \quad (25)$$

$$Z_t^j [\Omega_{t-1}^j] = (VO_t^j [\Omega_{t-1}^j] - V_{t-1}^j) \sum_{s=t+1}^{\infty} \left[\frac{\varepsilon_s \prod_{h=t}^{s-1} (1 - \varepsilon_h) \psi_s c_s}{\prod_{z=t+1}^s (1 + i_z [\Omega_{t-1}^j] (1 - \theta_z))} \right] \quad (26)$$

The price received by the seller for realising a proportion ε of shares is defined as the value of these shares to the purchaser.⁸ This makes sense, because in determining the value of ε to be exogenous, we make the supply curve for shares perfectly inelastic and, thus, the market price is determined by demand. This defines the value of these shares to the seller as $VO_t^j - Z_t^j$. Substituting equation (26) into (24), we can define an accrual-equivalent rate of capital gains tax under the realisation system⁹:

$$cr_t = \varepsilon_t \psi_t c_t + \sum_{z=t+1}^{\infty} \left[\frac{\varepsilon_z \prod_{k=t}^{z-1} (1 - \varepsilon_k) \psi_z c_z}{\prod_{s=t+1}^z (1 + i_s [\Omega_{t-1}^j] (1 - \theta_s))} \right] \quad (27)$$

Notice that switching between the accrual- and realisation-basis capital gains tax systems can be accomplished by setting the value of ε in (27). If we set ε at one, all of the terms relating to subsequent years become zero and fall out of the expression, transforming (27) into equivalency with (22). We can therefore simply use c as the capital gains tax parameter, and switch between CGT systems by setting ε appropriately.¹⁰ The value of the firm over an infinite horizon becomes

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{\left[\frac{1 - \theta_t}{1 - c_t} \right] D_t^j [\Omega_0^j] + \left[\frac{(1 - \theta_t) [1 - (1 - \gamma_t) \tau_t]}{(1 - \tau_t)(1 - c_t)} \right] DF_t^j [\Omega_0^j] - VN_t^j [\Omega_0^j]}{\prod_{s=1}^t \left[\frac{1 + i_s [\Omega_0^j] (1 - \theta_s) - c_s}{1 - c_s} \right]} \right] \quad (28)$$

which is identical in structure to (23).

⁸ i.e. V_t^o is the *realisable* value of the original shareholder's equity, which is given by their value to a new shareholder.

⁹ Following King (1977).

¹⁰ This formulation implicitly allows the immediate deduction of capital losses from taxable income at the rate of the capital gains tax. The ability to offset capital losses against capital gains is not universally applied to capital taxation, and it is not uncommon that there is no provision at all for capital loss offset. When offset is allowed, it can be for all or part of the capital loss or it can be against only current capital gains, future capital gains, or both. Therefore, as specified, equation (28) will overestimate the value of the firm in the presence of capital losses because it will (a) allow an immediate deduction when it is not allowed in practise or (b) not appropriately discount the value of a capital loss offset if it is carried forward.

2.3.3. Indexing Capital Gains

As it stands, equation (28) embodies an implicit assumption that *nominal* capital gains are taxed, or alternatively, there is no inflation. Allowing for inflation indexing in this model is quite simple, and we use a method that follows Bengt (1997 and 1998). The intuition behind this adjustment can be found in recognising that we need to index the base value of shares for the capital gains tax calculation to take account of changes in the value of the unit of measurement – money. We modify shareholder earnings to be partly a function of real or nominal capital gains, depending on the value of a dummy variable that allows us to activate a price index. If we define π as a price deflator, the two period arbitrage condition becomes

$$(i_t[\Omega_{t-1}](1-\theta_t))V_{t-1}^j = (1-\theta_t)D_t^j[\Omega_{t-1}^j] + \frac{(1-\theta_t)[1-(1-\gamma_t)\tau_t]}{1-\tau_t} DF_t^j[\Omega_{t-1}^j] + (1-c_t)(VO_t^j[\Omega_{t-1}^j] - [1+\alpha\pi_t[\Omega_{t-1}]]V_{t-1}^j) \quad (29)$$

With the dummy variable α set at zero, π falls out of the expression and the nominal change in the value of equity is taxed. When α is set at one, the value of the firm's equity in the base period is inflated to take account of the change in the price level. The value of the firm now becomes

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{\left[\frac{1-\theta_t}{1-c_t} \right] D_t^j[\Omega_0^j] + \left[\frac{(1-\theta_t)[1-(1-\gamma_t)\tau_t]}{(1-\tau_t)(1-c_t)} \right] DF_t^j[\Omega_0^j] - VN_t^j[\Omega_0^j]}{\prod_{s=1}^t \left[\frac{1+i_s[\Omega_0](1-\theta_s) - c_s[1+\alpha\pi_s[\Omega_0]]}{1-c_s} \right]} \right] \quad (30)$$

In introducing inflation indexing, we know that this reduces the tax payable at any point in time in comparison to taxing the raw nominal gain (as long as π is positive), and will increase the after-tax gain received. This, therefore, is equivalent to reducing the discount rate on before-CGT earnings streams, and therefore increases the value of the firm.

3. Constraining the Firm

3.1. Cash Flow Constraint

The firm pursues what is effectively a distributed-profit maximisation objective. The firm's ability to distribute profits is constrained (in one respect) by a cash flow constraint. If we take the view that the firm seeks to maximise the present value of dividend payments over time, then we need to recognise that the dividends that the firm pays are the net result of the relative size of its sources and uses of funds:

$$D_t^j + DF_t^j = p_t^j Y_t^{i,j} + B_t^j + VN_t^j - \sum_{m=1}^M w_t^m L_t^{m,j} (1+\nu_t) - \sum_{i=1}^G P_t^i Q_t^{g,j} - (1+i_t)B_{t-1}^j - \sum_{n=1}^N q_t^n I_t^{n,j} - T_t \quad (31)$$

and

$$Y_t^{i,j} = A_t^j \left[a_1^j (L_t^j)^{-\rho_j} + a_2^j (K_{t-1}^j)^{-\rho_j} + a_3^j (Q_t^j)^{-\rho_j} \right]^{-\frac{1}{\rho_j}} \text{ is the firm's CES production function for}$$

producing good i ,

A is an efficiency parameter (always positive), which says something about the state of technology,

a_i is a distributional factor (positive) denoting input shares,

ρ	is a parameter taking a value greater than or equal to -1 but not equal to zero ¹¹ , embodying a constant elasticity of substitution $\sigma = 1/[1 + \rho]$,
w^m	is the price of labour of type m ,
L	is the firm's total <i>effective</i> usage of labour, being some function of labour of types 1 to M via a CES nest,
L^m	denotes labour of type m ,
I^n	is the level of real investment in new capital goods of type n ,
q^n	is the asset price of capital goods of type n ,
K	is the firm's effective capital stock, being some function of capital of types 1 to N distinguishable by tax status (allowances and deductions), via a CES nest,
Q^g	is the firm's total purchases of intermediate good g ,
P^g	is the purchaser's price of intermediate good g ,
Q	is the firm's effective total intermediate usage of goods, being some function of intermediate goods of type 1 to G via a CES nest,
p^i	is the producer or basic price of the firm's output of good i ,
B	is total <i>net</i> issues of one period bonds,
ν	is the rate of payroll tax, and
T	is corporate income tax payable.

Equation (31) defines the firm's sources of funds as operating revenue and funds received from selling claims over fixed (debt) and variable (new equity) amounts of future cash flows. The firm's uses of funds are related to its wage bill, usage of intermediate goods, debt servicing, capital expenditures and company income tax. This serves to define the relationship between the variables in the maximand and the firm's activities.

In comparison to using the value functions derived previously as stand-alone analytical tools, we can see here that dividend policy becomes a part of a wider optimisation problem. This incorporates the possibility of the firm choosing to retain earnings to finance investment expenditures in order to maximise the value of shareholder wealth in a fully inter-temporal optimisation problem. Retained earnings are implicit in (31), most directly through the relationship between the firm's revenue and the level of investment expenditure. To see how, let's simplify (31) for illustrative purposes to

$$D_t^j = f(K_{t-1}^j) + B_t^j + \nu N_t^j - q_t I_t^j \quad (32)$$

Expression (32) highlights the relationship between the alternative sources of finance and investment expenditure. If we now assume that the firm uses no external sources of finance, every dollar of investment expenditure reduces the payment of dividends by a dollar – that is, the firm is retaining earnings instead of distributing them. If the firm issues stock or debt, each dollar raised reduces the need for retained earnings by a dollar, and thus allows the firm to increase dividend payments by a dollar.

¹¹ As ρ approaches zero, the CES production function approaches the form of a Cobb- Douglas production function with an elasticity of substitution equal to 1.

3.2. Taxation

Taxable income in period t is equal to the firm's revenue minus variable costs and deductions. Above, we have grouped all of the firm's deductions into a single illustrative variable we called TD , whereas now we make the components of this explicit. If we define TD to recognise that the firm can deduct interest, a proportion of capital expenditures and an ongoing statutory rate of depreciation from taxable income,

$$TD_t^j = i_t B_{t-1}^j + \sum_{n=1}^N \Gamma_t^n q_t^n I_t^{n,j} + \sum_{s=-\infty}^t \sum_{n=1}^N \Delta_{s,t-s}^n q_s^n I_s^{n,j} \quad (33)$$

then the firm's tax bill is

$$T_t^j = \tau_t \left[p_t^i Y_t^{i,j} - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - i_t B_{t-1}^j - \sum_{n=1}^N \Gamma_t^n q_t^n I_t^{n,j} - \sum_{s=-\infty}^t \sum_{n=1}^N \Delta_{s,t-s}^n q_s^n I_s^{n,j} \right] \quad (34)$$

and

Γ_t^n is the (rate of) deductible investment allowance on capital goods of type n in period t , and

$\Delta_{s,t-s}^n$ is the deductible depreciation allowance on a dollar of capital of life n , purchased in period s , payable $t-s$ periods later (that is, the depreciation allowance deductible in period t on an asset held since period s).

Equation (34) defines the company tax that the firm must pay in any period t . Taxable income equals operating revenue minus labour costs, intermediate goods costs, interest payments and capital allowances (both capital expenditure allowances and depreciation allowances).

3.3. Evolution of the Capital Stock

The firm's revenues are positively related to its capital stock, and so we need to know how it evolves over time. With an eye on making this model suitable for short-run, as well as steady state, analysis, we could impose some adjustment costs on the firm in adjusting its capital stock. A brief discussion of this is provided below, including our reasoning for adopting an alternative approach to anchoring this model for short-run simulations.

In any period t , the firm's stock of capital of type n will increase in size by the level of real investment in that period, and depreciate at a constant geometric rate δ .

$$K_t^{n,j} = I_t^{n,j} + (1 - \delta^n) K_{t-1}^{n,j} \quad (35)$$

Depreciation in this context is *real economic* depreciation, and differs from the accounting principle defined in (34) above. With the firm using the capital stock available to it at the end of period $t-1$ in production in period t , this specification implies that there are time or "gestation" costs involved in installing capital. Investment decisions in period t are based in part on an analysis of the revenue streams produced by the marginal unit of capital starting from period $t+1$ – the period in which the new capital comes on-line.

The firm's total effective capital stock for use in production is a CES nest of all capital of types 1 to M :

$$K_t^j = \Lambda_t^j \left[\sum_{n=1}^M s^{n,j} (K_t^{n,j})^{-\sigma^j} \right]^{-\frac{1}{\sigma^j}} \quad (36)$$

and so, substituting (35) into (36),

$$K_t^j = \Lambda_t^j \left[\sum_{n=1}^M s^{n,j} \left(I_t^{n,j} + (1 - \delta^n) K_{t-1}^{n,j} \right)^{-\sigma^j} \right]^{-\frac{1}{\sigma^j}} \quad (37)$$

Equation (37) defines the constraint on the firm's capital stock.

Installation costs have been applied in dynamic general equilibrium models as a means of damping investment responses in short-run analyses. However, in the MONASH model – to which the cost of capital functions derived hereafter will be applied – an alternative approach is used in which the supply curve for investment funds is less than infinitely elastic. This will be the approach taken here, to be discussed briefly below.

3.4. Output Market Conditions

The CES production function in the cash flow constraints above exhibits diminishing short-run returns but, like all linearly homogenous functions, embodies constant returns to scale. It is possible that a change in scale could occur without bound in this setting because the sources of finance and the prices of all the factor inputs are fixed. One partial solution to this problem is to make the firm face a less than perfectly elastic demand curve. The effect of this is to make the firm's total revenue function embody a positive first derivative with a *negative* second derivative. That is, the firm must accept positive but decreasing marginal revenue as it increases output.

We now define the price of the firm's output as

$$p_t^i = \left(Y_t^{i,j} \right)^{-\eta^j} \quad (38)$$

where η is a parameter determining the own-price elasticity by the function $ED_t = -\frac{1}{\eta}$.

Redefining the revenue functions embodied in equations (31) and (34) in light of (38) provides

$$p_t^i Y_t^{i,j} = \left[A_t^j \left[a_1^j \left(L_t^j \right)^{-\rho^j} + a_2^j \left(K_{t-1}^j \right)^{-\rho^j} + a_3^j \left(Q_t^j \right)^{-\rho^j} \right]^{-\frac{1}{\rho^j}} \right]^{1-\eta^j} \quad (39)$$

Equation (39) is a total revenue function for the firm with factor inputs explicit. Therefore, taking the derivative of this with respect to L , K , or Q provides the marginal revenue product of that factor.

Substituting (39) into equations (31) and (34) we obtain

$$D_t^j + DF_t^j = \left[Y_t^j \right]^{1-\eta^j} + B_t^j + VN_t^j - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^i Q_t^{i,j} - (1 + i_t) B_{t-1}^j - \sum_{n=1}^N q_t^n I_t^{n,j} - T_t \quad (40)$$

and

$$T_t^j = \tau_t \left[\left[Y_t^j \right]^{1-\eta^j} - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^i Q_t^{i,j} - i_t B_{t-1}^j - \sum_{n=1}^N \Gamma_t^n q_t^n I_t^{n,j} - \sum_{s=-\infty}^t \sum_{n=1}^N \Delta_{s,t-s}^n q_s^n I_s^{n,j} \right] \quad (41)$$

We have now imposed on the firm a monotonically decreasing long-run marginal revenue function. The firm will expand its scale in response to positive pure profits until the price of its product falls enough to equate long-run marginal revenue and long-run marginal cost.

3.5. Inequality Constraints

Finally, we add a few more constraints on the firm's behaviour. Firstly, we assume that the level of investment, unfranked dividends, franked dividends and new equity issues must be non-negative,

$$I_t^{n,j} \geq 0 \quad (42)$$

Constraining investment in this way stops the firm from liquidating its productive capital in order to finance any use of funds (paying dividends, servicing debt or funding new capital). This sign restriction also limits the rate at which the firm's capital stock can shrink to the rate of depreciation, δ .

$$D_t^j \geq 0 \quad (43)$$

$$DF_t^j \geq 0 \quad (44)$$

These sign restrictions constrain the firms in this model to be profitable. Firms that are making a loss before tax are still notionally able to “claim” depreciation and investment allowances, but they will have no taxable income to which to apply the deductions. In addition, because the firm's behaviour and its consequences are constant with a given set of parameter values, a firm that is making a loss in one period will make a loss forever or until its environment changes. Such firms can be assumed away as those that “shut-down” – in this model, capital stocks can respond to changes in a firm's environment in one period, and thus the definition of the long run for the purposes of determining firm entry and exit is one period. The “payment” of negative dividends is also constrained by elements of the corporations law. For example, incorporation and limited liability for public companies caps the liability of shareholders for company debts to, at most, their paid-up capital¹².

$$VN_t^j \geq 0 \quad (45)$$

It seems sensible to assume that firms are not interested in share buy-backs. We are trying to define the cost of capital for firms wishing to finance productive activities in order to maximise shareholder wealth, rather than those wishing to manipulate their balance sheets in pursuit of the same goal. This is quite apart from the fact that a firm's ability to re-purchase its own equity is usually restricted by corporations law in a way that makes it difficult to model in a systematic fashion.

$$B_t^j \geq 0 \quad (46)$$

The non-negativity constraint on debt is to stop the firm from behaving like a bank. If retained earnings were a cheaper source of finance than debt, the firm could potentially retain earnings and lend the proceeds to others issuing bonds. Likewise, under certain circumstances, it could be possible for the firm to increase shareholder wealth by issuing equity to fund the purchase of debt. We want to focus on firms seeking to finance expenditures on physical capital, precisely for the reason that the cost-of-capital functions we seek are intended to inform investment behaviour in a general equilibrium model. Balance sheet substitutions seem of little relevance to such an analysis.

Under a dividend imputation regime, a firm must maintain a franking account. This is not part of, and is independent of, the company's financial accounts, and holds direct benefits only to shareholders. Credits to a franking account can arise from:

- Company tax payments; these increase the franking account balance by $((1 - \tau_t)/\tau_t)$ for each dollar of tax paid, and

¹² Excluding the instance of contributing shares, which represent a small part of equity markets.

- Franked dividends received from holdings in other firms, which increase the franking account balance by the total value of the franked dividend.

Firms can pay a dollar of franked dividends for every dollar available in their franking account, and it is illegal for a firm to issue franked dividends over and above this balance. Firms are also (generally) required to distribute franking credits when they are available. This constraint is effectively brought to bear when tax deductions cause a dichotomy between the statutory rate of CIT and the actual amount paid.

$$\left[\left(\frac{1 - \tau_t}{\tau_t} \right) T_t - DF_t^j \right] \geq 0, \quad \forall t \quad (47)$$

This simply imposes the requirement that the firm have a dollar in its franking account sourced during period t for every dollar of franked dividends it issues at time t .

4. The Cost of Capital

Below we derive a set of functions defining capital costs that cover all of the possible financial policies that a corporate enterprise can pursue (in the world assumed by the model). With four potential sources of finance, the firm has open to it many potential financial policies. However, the cost of capital at the margin is only affected by the *infra*-marginal financial sources that the firm uses to the extent that they have an affect on the amount of tax that it pays and, therefore, its ability to distribute franking credits and/or the opportunity cost of foregone income streams. It is important to remember that we are interested in the *marginal* cost of capital, and thus the net opportunity cost of the marginal dollar of finance. Measuring capital costs with, for example, a weighted average cost of capital (WACC) formulation ignores that marginal cost determines behaviour. Therefore, other than for the qualification mentioned above, we need to only know about the source of the marginal dollar of finance.

This allows us to focus on finding seven potential cost of capital expressions. These comprise four expressions (one for each source of finance) when the franking account constraint not binding, and three for when it is binding. A binding franking account constraint says that the firm is paying the maximum amount of franked dividends because it is distributing the maximum amount of franking credits. Therefore, when we analyse the case of a firm retaining frankable earnings, and therefore not distributing all franking credits, we can ignore the case of the franking account constraint being binding.

The cost-of-capital expressions are found via the solution to a Kuhn-Tucker optimisation problem. The method is outlined in the appendix. Below we present and discuss the form of the expressions for all of the firm's potential financial policies. The expressions will be stated in two forms – one for variable expected tax parameters (which would be used for modelling changes in expectations or advance announcements of policy shifts) and constant expected tax parameters.

The equations we derive are based on a manipulation of the first-order condition for capital:¹³

¹³ Hereafter we will denote the first derivative of the capital stock with respect to a specific type of capital as k^n , so that

$$k_t^n = \left[\frac{s^{n,j} (1 - \delta^n)}{(\Lambda_{t+1}^j)^{\sigma^j}} \right] \left[\frac{K_{t+1}^j}{K_t^{n,j}} \right]^{1+\sigma^j}.$$

$$\left[\lambda_{t+1}^1 - \lambda_{t+1}^2 \tau_{t+1} \right] k_t^{n'} \left[\frac{(1-\eta)a_2}{A_{t+1}^\rho} \right]^{-\eta} \left[\frac{Y_{t+1}}{K_t} \right]^{-\eta(1+\rho)} + \lambda_{t+1}^3 k_{t+1}^{n'} (1-\delta) - \lambda_t^3 k_t^{n'} \phi_{t+1} = 0 \quad (48)$$

λ_t^1, λ_t^2 and λ_t^3 are the Lagrangian multipliers on the cash flow, tax and capital stock constraints respectively (see the appendix). Equation (48) says that the firm should increase the size of its capital stock until the benefits and costs derived from doing so are driven to equality at the margin. The benefits to shareholders of the marginal unit of capital are the after-tax marginal revenue product and the residual value at the end of the period's productive activities (after physical depreciation) – these are the first two terms on the left-hand side respectively. The cost is given by the third term, which can be interpreted as the cost of purchasing the asset. We know that, in equilibrium, the firm will equate the marginal revenue product of capital with its marginal cost, and so we know that

$$k_t^{n'} \left[\frac{(1-\eta)a_2}{A_{t+1}^\rho} \right]^{-\eta} \left[\frac{Y_{t+1}}{K_t} \right]^{-\eta(1+\rho)} - \delta^n = \frac{\lambda_t^3 k_t^{n'} \phi_{t+1} - \lambda_{t+1}^3 k_{t+1}^{n'} (1-\delta^n)}{\left[\lambda_{t+1}^1 - \lambda_{t+1}^2 \tau_{t+1} \right]} - \delta^n \quad (49)$$

We deduct depreciation from both sides because the firm's net marginal benefit from this unit of capital is found after deducting depreciation from the value of its product. Our task now is to evaluate the Lagrangian multipliers on the right-hand side of (49) to find an expression for the user cost of capital. These multipliers will contain taxation parameters, the exact format being determined by the firm's financial policy (that is, its sources of finance). For notational convenience, we'll replace the left hand side with "COC", and denote the present value of all future depreciation allowances available on an asset as

$$\Xi_t^n = \Delta_{t,0}^n + \sum_{h=t+1}^{\infty} \left[\frac{\Delta_{t,h}^n}{\prod_{s=t}^{t+r} \phi_s} \right] \quad (50)$$

4.1. Financing with Retained Unfrankable Earnings at the Margin

4.1.1. A Slack Franking Account Constraint

Variable Expected Tax Parameters

$$COC_t^j = \left[\frac{1-c_{t+1}}{(1-\theta_{t+1})(1-\tau_{t+1})} \right] \cdot \left\{ \left[\frac{1+i_{t+1}[\Omega_t](1-\theta_{t+1})-c_{t+1}[1+\alpha\pi_{t+1}[\Omega_t]]}{1-c_{t+1}} \right] \left[\frac{1-\theta_t}{1-c_t} \right] \left[1-\tau_t(\Gamma_t^n + \Xi_t^n) \right] \frac{k_t^{n'}}{K_t^{j'}} \right\} - \delta^n \quad (51)^{14}$$

$$\left\{ -\frac{q_{t+1}^n}{q_t^n} \left[\frac{1-\theta_{t+1}}{1-c_{t+1}} \right] \left[1-\tau_{t+1}(\Gamma_{t+1}^n + \Xi_{t+1}^n) \right] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1-\delta^n) \right\}$$

¹⁴ We also now denote the first derivative of the stock of capital of type n with respect to a change in investment in capital goods of type n as $K_t^{j'}$.

Constant Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - \tau(\Gamma^n + \Xi^n)}{1 - \tau} \right] \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] \frac{k_t^{n'}}{K_t^{j'}} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (52)$$

These two expressions tell us about the user-cost of capital to the firm. Hereafter we will interpret each result for the alternative financial policies primarily with reference to the constant expected tax rate case. In deriving these expressions, we divide both sides by the initial asset price of the unit of capital to give us the net cost of the marginal dollar's worth of the asset. Equation (52) says that the net cost of the marginal dollar of capital is a function of the present value of that portion of the capital stock that is physically annihilated during the period in production (the second bracketed term on the right-hand side), multiplied by a tax coefficient that deducts the value of investment- and capital-related allowances.

A discount rate is present because we evaluate all flows in period $t+1$ terms, and so the opportunity cost of resources measured in period t is partly a function of the shareholder's willingness to wait a period to receive the benefits of investment by the firm. This brings the opportunity cost of investment into play, which is a function of the bond rate, the personal income tax rate (because interest income from bonds is deductible on personal account) and the capital gains tax rate (which can be measured in nominal or real terms).

The rate of personal income tax is absent when we have constant tax rates (except for its presence in the discount rate). This is because the various post-tax flows bear personal income tax at the same rate – the marginal revenue product of capital on the left-hand-side and the after-tax cost of capital on the right. In the case of period-specific tax rates shown in expression (51), the after-tax value of these flows is dependent on these differing tax rates, and so the rates of taxation become part of the calculus. This is also true of the capital gains tax rate.

The company tax rate is only important to the extent that it determines the value of the various deductions to the shareholder when we assume constant tax rates. As the values of the various deductions increase, the coefficient on the right-hand side of (52) falls in value, thus reducing the cost of capital. If there are no deductions available, this simplifies to $1/(1 - \tau_c)$, which simply grosses up the right hand-side to take account of the company tax due on the marginal revenue product of capital – that is, an increase in company tax will reduce the after-tax value of the product of this new unit of capital, and so the rate of return falls. This occurs because the firm is not passing on any franking credits. By assuming the franking account constraint is slack, we are effectively saying (in this case) that the firm is not paying any franked dividends, which in turn, implies that this is an expression relevant to a classical taxation system¹⁵. Because no franking credits are available, the various deductions stemming from the asset purchase and its use in production will have a non-zero value to the shareholders. We might expect to see this term absent when we move next to look at the cost of capital when the franking account constraint is binding.

¹⁵ We have assumed that the firm is paying some unfranked dividends, which makes it illogical for the firm to be retaining frankable earnings if they are available. If it *did* have frankable earnings available, it would be in its interest to distribute the maximum amount of franking credits, thus rendering the franking account constraint binding. Assuming a slack constraint on the franking account is therefore equivalent to specifying this problem as relating to a system where no franking credits are available.

4.1.2. A Binding Franking Account Constraint

Variable Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - c_{t+1}}{(1 - \theta_{t+1})[1 - (1 - \gamma_{t+1})\tau_{t+1}]} \right] \cdot \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta_{t+1}) - c_{t+1}[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c_{t+1}} \right] \left[\frac{(1 - \theta_t)[1 - (1 - \gamma_t)\tau_t(\Gamma_t^n + \Xi_t^n)]}{1 - c_t} \right] \frac{k_t^{n'}}{K_t^{j'}} \right. \\ \left. - \frac{q_{t+1}^n}{q_t^n} \left[\frac{(1 - \theta_{t+1})[1 - (1 - \gamma_{t+1})\tau_{t+1}(\Gamma_{t+1}^n + \Xi_{t+1}^n)]}{1 - c_{t+1}} \right] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (53)$$

Constant Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - (1 - \gamma)\tau(\Gamma^n + \Xi^n)}{1 - (1 - \gamma)\tau} \right] \left\{ \frac{k_t^{n'}}{K_t^{j'}} \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (54)$$

Notice that, with full imputation, expression (54) collapses to

$$COC_t^j = \frac{k_t^{n'}}{K_t^{j'}} \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) - \delta^n \quad (55)$$

Notice firstly that setting the value of γ to zero in (53) and (54) (thus making all dividends paid by the firm unfranked) renders these equations identical to expressions (51) and (52). This lends support to the specification of the model.

Secondly, when the *total* amount of franking credits is passed on to the shareholder in a world of constant tax rates, the various taxes and allowances disappear almost completely (other than for those present in the discount rate). This result supports the discussion above about the irrelevance of deductions in a full dividend imputation system where the total amount of franked dividends is always paid. When the total amount is *not* paid, the rate of imputation partly determines the cost of capital – as it increases, the cost of capital falls. This occurs as the result of two competing forces: Firstly, an increase in the rate of imputation reduces the benefit of any tax deductions at the company tax level to the shareholder. At the limit as full imputation is imposed, they reduce to zero. Secondly, any tax that is paid at the company tax level is credited to the shareholder at the personal tax level. Because deductions constitute only a *proportion* of the tax paid at the company level, this “cost” of lost deductions due to imputation is overwhelmed by the benefit of imputation at the bottom line.

4.2. Financing with Retained Frankable Earnings at the Margin

Variable Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - c_{t+1}}{(1 - \theta_{t+1})[1 - (1 - \gamma_{t+1})\tau_{t+1}]} \right] \cdot \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta_{t+1}) - c_{t+1}[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c_{t+1}} \right] \left[\frac{(1 - \theta_t)[1 - (1 - \gamma_t)\tau_t]}{(1 - \tau_t)(1 - c_t)} \right] [1 - \tau_t(\Gamma_t^n + \Xi_t^n)] \frac{k_t^{n'}}{K_t^{j'}} \right. \\ \left. - \frac{q_{t+1}^n}{q_t^n} \left[\frac{(1 - \theta_{t+1})[1 - (1 - \gamma_{t+1})\tau_{t+1}]}{(1 - \tau_{t+1})(1 - c_{t+1})} \right] [1 - \tau_{t+1}(\Gamma_{t+1}^n + \Xi_{t+1}^n)] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (56)$$

Constant Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - \tau(\Gamma^n + \Xi^n)}{1 - \tau} \right] \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] \frac{k_t^{n'}}{K_t^{j'}} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (57)$$

Notice that equation (57) is identical to (52). This is because the shareholder never gains a benefit from the franking credits that are not distributed due to the retention of frankable earnings. Implicit in (57) is the assumption that this will never happen – if it does, we move to one of the other cost of capital equations, because in those circumstances the firm will be sourcing a different type of finance at the margin. In a sense, then, the marginal dollar of finance that is sourced from frankable earnings is exactly equivalent to a dollar of retained unfrankable earnings, because the shareholder never benefits from any franking credits they would have received if the franked dividend had been paid and not retained. Because the franking account constraint is implicitly never binding while equation (57) is relevant, this will never be passed back to the shareholder in a subsequent period. Without franking credits, tax deductions stemming from investment *do* have value to the shareholder, which is why these deductions appear in (57).

4.3. Financing with New Equity Issues at the Margin

4.3.1. A Slack Franking Account Constraint

Variable Expected Tax Parameters

$$COC_t^j = \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta_{t+1}) - c_{t+1}[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c_{t+1}} \right] \left[\frac{1 - \tau_t(\Gamma_t^n + \Xi_t^n)}{1 - \tau_{t+1}} \right] \frac{k_t^{n'}}{K_t^{j'}} \\ - \frac{q_{t+1}^n}{q_t^n} \left[\frac{1 - \tau_{t+1}(\Gamma_{t+1}^n + \Xi_{t+1}^n)}{1 - \tau_{t+1}} \right] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) - \delta^n \quad (58)$$

Constant Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - \tau(\Gamma^n + \Xi^n)}{1 - \tau} \right] \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] \frac{k_t^{n'}}{K_t^{j'}} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (59)$$

Expression (59) is identical to (52) and (57) above. To understand why, consider again the nature of this particular financial policy. A firm that is issuing equity with a slack franking account

constraint will be (a) issuing no dividends at all in a dividend imputation system¹⁶, or (b) paying maximum unfranked dividends or retaining all unfrankable earnings in a classical system. In either case, the payment of dividends to existing shareholders in the current period is not influenced by the equity issue. When dividends are being distributed, the present after-tax value of the claim over future dividends that will go to new shareholders from the next period onwards is captured in the current period by the value of the issue, and thus by V^N in the maximand. In the imputation system, if no dividends are ever paid, the relevance of dividend policy disappears again.

In the case above, the assumption has been made *a priori* that no dividends are being paid, which means that the firm is retaining all earnings infra-marginally. An equity issue of one dollar relaxes the cash flow constraint by one dollar, and reduces the wealth of current shareholders by the after CGT value of the dollar. The allowances that are permitted on the asset's purchase are deductible from company tax. With a slack franking account constraint, these deductions will have value to shareholders because they will either increase the value of distributed dividends or reduce the value of equity issues by increasing the availability of retainable earnings.

4.3.2. A Binding Franking Account Constraint

$$COC_t^j = \left[\frac{1 - c_{t+1}}{(1 - \theta_{t+1})[1 - (1 - \gamma_{t+1})\tau_{t+1}]} \right] \cdot \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta_{t+1}) - c_{t+1}[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c_{t+1}} \right] \left[1 - \left[\frac{\theta_t - c_t + \tau_t(1 - \theta_t)(1 - \gamma_t)}{1 - c_t} \right] (\Gamma_t^n + \Xi_t^n) \right] \frac{k_t^{n'}}{K_t^{j'}} \right\} - \delta^n$$

$$\left\{ - \frac{q_{t+1}^n}{q_t^n} \left[1 - \left[\frac{\theta_{t+1} - c_{t+1} + \tau_{t+1}(1 - \theta_{t+1})(1 - \gamma_{t+1})}{1 - c_{t+1}} \right] (\Gamma_{t+1}^n + \Xi_{t+1}^n) \right] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\}$$

$$COC_t^j = \left[\frac{1 - c - [\theta - c + \tau(1 - \theta)(1 - \gamma)](\Gamma^n + \Xi^n)}{(1 - \theta)[1 - (1 - \gamma)\tau]} \right] \cdot \left\{ \frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \frac{k_t^{n'}}{K_t^{j'}} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (61)$$

The value of deductions now appears in expression (61), even though full imputation and constant tax rates are assumed. This occurs because the use of a stock issue to finance investment can influence the availability of franking credits and the payment of franked dividends. Therefore, deductions can have a net positive or negative value to shareholders depending the relative magnitudes of c and θ . Again, notice that the company tax rate disappears as the imputation rate reaches one.

¹⁶ In a dividend imputation system, and given the behavioural implications of the structure of this model, if the firm is not retaining frankable earnings at the margin but has a slack franking account constraint, this must be because it is retaining all frankable earnings. Further, because unfranked dividends will never be paid when the firm is retaining a dollar of frankable earnings, the firm will also be retaining all unfrankable earnings.

4.4. Financing with Debt at the Margin

4.4.1. A Slack Franking Account Constraint

Variable Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - c_{t+1}}{(1 - \theta_{t+1})(1 - \tau_{t+1})} \right] \cdot \left\{ \left[\frac{1 - \theta_{t+1}}{1 - c_{t+1}} \right] (1 + r_{t+1} [1 - \tau_{t+1}]) [1 - \tau_t (\Gamma_t^n + \Xi_t^n)] \frac{k_t^{n'}}{K_t^{j'}} \right. \\ \left. - \frac{q_{t+1}^n}{q_t^n} \left[\frac{(1 - \theta_{t+2})(1 + r_{t+2} [1 - \tau_{t+2}])}{1 + i_{t+2} [\Omega_0](1 - \theta_{t+2}) - c_{t+2} [1 + \alpha \pi_{t+1} [\Omega_0]]} \right] [1 - \tau_{t+1} (\Gamma_{t+1}^n + \Xi_{t+1}^n)] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (62)$$

Constant Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - \tau (\Gamma^n + \Xi^n)}{1 - \tau} \right] \left\{ (1 + r_{t+1} [1 - \tau]) \frac{k_t^{n'}}{K_t^{j'}} \right. \\ \left. - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} \left[\frac{(1 + r_{t+2} [1 - \tau])(1 - c)(1 - \delta^n)}{1 + i_{t+2} [\Omega_t](1 - \theta) - c [1 + \alpha \pi_{t+2} [\Omega_t]]} \right] \right\} - \delta^n \quad (63)$$

This tax coefficient on expression (63) is equivalent to (52), (57) and (59), except that it contains a multiplier term that adjusts the cash flows for the repayment of principal and interest (with interest deductibility), and contains some slightly different timing issues. The second term on the right-hand side contains a discount rate for period $t+2$, because the value of the portion of the dollar of capital left over after one-period's production is calculated by assuming it is being financed by debt in period $t+1$ – which means that interest and principal would be due in period $t+2$. Its purchase price is calculated according to the net result of issuing debt in period t , and the resulting servicing obligations in period $t+1$, and so the discount rate on this term disappears (because the entire expression is evaluated in period $t+1$ terms). In basic terms, these expressions capture the idea that the cost of financing capital expenditures with debt is related to the cost of principal and interest to be repaid at some point in the future (in this model, one period later), with account taken of the deductions available and the nature of the tax system itself. The opportunity cost of funds used to service debt is related to the affect on the firm's cash flows, and thus its ability to distribute dividends.

4.4.2. A Binding Franking Account Constraint

Variable Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - c_{t+1}}{(1 - \theta_{t+1})[1 - (1 - \gamma_{t+1})\tau_{t+1}]} \right] \cdot \left\{ \left[\frac{1 - \theta_{t+1}}{1 - c_{t+1}} \right] [1 + r_{t+1} (1 - [1 - \gamma_{t+1}]\tau_{t+1})] [1 - (1 - \gamma_t)\tau_t (\Gamma_t^n + \Xi_t^n)] \frac{k_t^{n'}}{K_t^{j'}} \right. \\ \left. - \frac{q_{t+1}^n}{q_t^n} \left[\frac{(1 - \theta_{t+2})[1 + r_{t+2} (1 - [1 - \gamma_{t+2}]\tau_{t+2})] [1 - (1 - \gamma_{t+1})\tau_{t+1} (\Gamma_{t+1}^n + \Xi_{t+1}^n)]}{1 + i_{t+2} [\Omega_t](1 - \theta_{t+2}) - c_{t+2} [1 + \alpha \pi_{t+2} [\Omega_t]]} \right] \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n \quad (64)$$

Constant Expected Tax Parameters

$$COC_t^j = \left[\frac{1 - (1 - \gamma)\tau(\Gamma^n + \Xi^n)}{1 - (1 - \gamma)\tau} \right] \cdot \left\{ \begin{aligned} & \left[1 + r_{t+1}(1 - [1 - \gamma]\tau) \right] \frac{k_t^{n'}}{K_t^{j'}} \\ & - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} \left[\frac{[1 + r_{t+2}(1 - [1 - \gamma]\tau)](1 - c)}{1 + i_{t+2}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+2}[\Omega_t]]} \right] (1 - \delta^n) \end{aligned} \right\} - \delta^n \quad (65)$$

Full dividend imputation makes the deductibility of the interest rate on debt, depreciation and investment allowances and, indeed, the rate of company tax, irrelevant to the cost of capital to the shareholder. Because the various flows are measured after tax according to the same personal income tax rate, this parameter is also absent from this expression. Therefore, the cost of capital under this financial policy differs from those previous examples that assumed a binding franking account constraint (except for the new equity case) due to the relationship between the interest-and-principal multiplier and the fact that the cash flows required to service debt occur one period later.

5. Final Remarks

In this paper, we have outlined two approaches to incorporating business taxation into an investment problem for a corporate enterprise.

The first method allows us to analyse the role of taxes and allowances on the value of the firm, assuming that its investment decisions and before-tax profit streams are given. We saw that the interplay between taxes and the value of the firm can be quite complex, especially when we apply a dividend imputation system and a realisation-basis CGT system. Understanding the impact of a policy shift on the firm enables us to infer something about the required rate of return on investment and, thus, the level of investment undertaken.

The second tells us something about investment and the cost of capital to the firm under optimising assumptions. We set up a constrained optimisation problem and solved for the optimal level of investment and its associated cost, as well as the levels of all of its choice variables. In this way, we determine the optimal level of investment to maximise the firm's value to its shareholders, and generate an expression that tells us how the tax system affects the cost of funds to the firm in equilibrium. Compared to the first approach, this allows us to actually maximise the value of the expression that we discussed in the first section and then generate a time path for investment. For applied CGE modelling, this second method has a more intuitive appeal.

This paper outlines ongoing work. The next stage of this process is to embed these equations in the MONASH model, and to develop a database of the necessary parameter values. Implementation will involve developing a set of logical statements to allow the model to choose the correct cost of capital expression under different circumstances, and (for MONASH applications) the linearisation of the expressions themselves.

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Equation Section 1Appendix

In this appendix, we outline the method used to derive the cost of capital expressions by providing an example. Equations (53) and (54) are found as follows:

The Lagrangian function is

$$\begin{aligned}
L = & \Phi_t^{-1} \sum_{t=1}^{\infty} \left[\frac{1-\theta_t}{1-c_t} D_t^j [\Omega_0^j] + \frac{(1-\theta_t)[1-(1-\gamma_t)\tau_t]}{(1-\tau_t)(1-c_t)} DF_t^j [\Omega_0^j] - VN_t^j [\Omega_0^j] \right] \\
& + \sum_{t=1}^{\infty} \left[\lambda_t^1 \Phi_t^{-1} \left[A_t^j \left[a_1^j (L_t^j)^{-\rho^j} + a_2^j (K_{t-1}^j)^{-\rho^j} + a_3^j (Q_t^j)^{-\rho^j} \right]^{\frac{1}{\rho^j}} \right]^{1-\eta^j} + B_t^j + VN_t^j \right. \\
& \left. - \sum_{m=1}^M w_t^m L_t^{m,j} (1+\nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - (1+i_t) B_{t-1}^j - \sum_{n=1}^N q_t^n I_t^{n,j} - T_t - D_t^j - DF_t^j \right] \\
& + \sum_{t=1}^{\infty} \left[\lambda_t^2 \Phi_t^{-1} \left[T_t^j - \tau_t \left[A_t^j \left[a_1^j (L_t^j)^{-\rho^j} + a_2^j (K_{t-1}^j)^{-\rho^j} + a_3^j (Q_t^j)^{-\rho^j} \right]^{\frac{1}{\rho^j}} \right]^{1-\eta^j} \right. \right. \\
& \left. \left. - \sum_{m=1}^M w_t^m L_t^{m,j} (1+\nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - i_t B_{t-1}^j - \sum_{n=1}^N \Gamma_t^n q_t^n I_t^{n,j} - \sum_{s=-\infty}^t \sum_{n=1}^N \Delta_{s,t-s}^n q_s^n I_s^{n,j} \right] \right] \\
& + \sum_{t=1}^{\infty} \left[\lambda_t^3 \Phi_t^{-1} \left[\Lambda_t^j \left[\sum_{n=1}^M s^{n,j} \left(I_t^{n,j} + (1-\delta^n) K_{t-1}^{n,j} \right)^{-\rho^j} \right]^{\frac{1}{\rho^j}} - K_t^j \right] \right]
\end{aligned}$$

$$+\sum_{t=1}^{\infty}\left[\Phi_t^{-1}\left[\lambda_t^4 I_t^{n,j}+\lambda_t^5 D_t^j+\lambda_t^6 DF_t^j+\lambda_t^7 VN_t^j+\lambda_t^8 B_t^j+\lambda_t^9\left[\left(\frac{1-\tau_t}{\tau_t}\right)T_t-DF_t^j\right]\right]\right] \quad (A1)$$

The crucial first order conditions are:

$$\frac{\partial L}{\partial K_{t+1}^{n,j}}=\frac{\left[\lambda_{t+1}^1-\lambda_{t+1}^2\tau_{t+1}\right]\left[k_t^{n,j'}\left[\frac{(1-\eta)a_t^j}{A_{t+1}^{j,\rho^j}}\right]^{-\eta^j}\left[\frac{Y_{t+1}^j}{K_t^j}\right]^{-\eta^j(1+\rho^j)}\right]+\lambda_{t+1}^3k_{t+1}^{n'}(1-\delta^n)-\lambda_t^3k_t^{n,j'}\phi_{t+1}}{\Phi_{t+1}}=0 \quad (A2)$$

$$\frac{\partial L}{\partial T_t^j}=\frac{-\lambda_t^1+\lambda_t^2+\lambda_t^9\left[\frac{1-\tau_t}{\tau_t}\right]}{\Phi_t}=0 \quad (A3)$$

$$\frac{\partial L}{\partial I_t^{n,j}}=\frac{q_t^n\left[-\lambda_t^1+\lambda_t^2\tau_t\left(\Gamma_t^n+\Delta_{t,0}^n\right)\right]+\sum_{h=t+1}^{\infty}\left[\frac{\lambda_h^2\tau_h\Delta_{t,h}^nq_h^n}{\prod_{s=t}^{t+r}\phi_s}\right]+\lambda_t^3\frac{s^{n,j}}{\left(\Lambda_t^j\right)^\rho}\left[\frac{K_t^j}{I_t^{n,j}}\right]^{1+\rho^j}+\lambda_t^4}{\Phi_t}=0 \quad (A4)$$

$$\frac{\partial L}{\partial D_t^j\left[\Omega_0^j\right]}=\frac{\left[\frac{1-\theta_t}{1-c_t}\right]-\lambda_t^1+\lambda_t^5}{\Phi_t}=0 \quad (A5)$$

$$\frac{\partial L}{\partial DF_t^j\left[\Omega_0^j\right]}=\frac{\left[\frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-c_t)}\right]-\lambda_t^1+\lambda_t^6-\lambda_t^9}{\Phi_t}=0 \quad (A6)$$

$$\frac{\partial L}{\partial VN_t^j\left[\Omega_0^j\right]}=\frac{-1+\lambda_t^1+\lambda_t^7}{\Phi_t}=0 \quad (A7)$$

$$\frac{\partial L}{\partial B_t^j}=\frac{\phi_{t+1}\left(\lambda_t^1+\lambda_t^8\right)-\lambda_{t+1}^1\left(1+i_{t+1}\right)+\lambda_{t+1}^2\tau_{t+1}i_{t+1}}{\Phi_{t+1}}=0 \quad (A8)$$

In addition, the complimentary slackness conditions are:.

$$\lambda_t^4 \geq 0, \text{ and } \lambda_t^4 = 0 \text{ if } I_t^{n,j} > 0 \quad (A9)$$

$$\lambda_t^5 \geq 0, \text{ and } \lambda_t^5 = 0 \text{ if } D_t^j > 0 \quad (A10)$$

$$\lambda_t^6 \geq 0, \text{ and } \lambda_t^6 = 0 \text{ if } DF_t^j > 0 \quad (A11)$$

$$\lambda_t^7 \geq 0, \text{ and } \lambda_t^7 = 0 \text{ if } VN_t^j > 0 \quad (A12)$$

$$\lambda_t^8 \geq 0, \text{ and } \lambda_t^8 = 0 \text{ if } B_t^j > 0 \quad (A13)$$

$$\lambda_t^9 \geq 0, \text{ and } \lambda_t^9 = 0 \text{ if } \left[\left(\frac{1-\tau_t}{\tau_t}\right)T_t-DF_t^j\right] > 0 \quad (A14)$$

An optimisation problem with inequality constraints makes use of complimentary slackness conditions. This specification diverges from that of a traditional Kuhn-Tucker problem, in that the Lagrangian multipliers act as slack variables that play the role of more traditional complimentary slackness conditions. We associate

the Lagrangian multipliers $\lambda_t^4, \lambda_t^5, \lambda_t^6, \lambda_t^7, \lambda_t^8$ and λ_t^9 with the non-negativity constraints, and for each variable, the value of this multiplier is set at zero if the constraint is slack. The Kuhn-Tucker conditions include a set of complimentary slackness conditions that state:

- for any variable x subject to a non-negativity constraint, $x \geq 0$ and $x \frac{\partial L}{\partial x} = 0$.

This effectively means that the solution involves a non-negative stationary value of x **or**, as the alternative is a negative stationary value, the value of x must be zero. The addition of the slack variables means that

$$\frac{\partial L}{\partial x} = \lambda,$$

and thus $\lambda = 0$ when $x \geq 0$. Conversely, when $\lambda > 0$, $x = 0$.

These complimentary slackness conditions allow for the possibility that the stationary value of x might be negative, and so at the boundary (where $x=0$) the first derivative of L with respect to x might not be zero. The slack variables thus allow us to set-up all of the first-order conditions as equalities.

Equations (53) and (54) represent a firm that is retaining unfrankable earnings with a binding franking account constraint. Therefore, it will not be paying unfranked dividends, will pay the maximum amount of franked dividends, and investment is assumed to always be positive (by assumption). The relevant slack variables to set at zero are

$$\lambda_t^4 = \lambda_t^5 = \lambda_t^6 = 0, \quad \forall t \quad (\text{A15})$$

The cash flow affect of retaining unfrankable earnings at the margin has a cost for shareholders according to (A5) and (A15) of

$$\lambda_t^1 = \left[\frac{1 - \theta_t}{1 - c_t} \right] \quad (\text{A16})$$

With the maximum amount of franked dividends also being paid, and with this amount being partly determined by the franking credits available, a relaxation of the taxation constraint has an affect on the objective function that is related to the value of the slack variable on the franking account constraint. By (A6), (A15) and (A16), this equals

$$\lambda_t^9 = \left[\frac{(1 - \theta_t) \gamma_t \tau_t}{(1 - \tau_t)(1 - c_t)} \right] \quad (\text{A17})$$

which, along with (A3) provides

$$\lambda_t^2 = \left[\frac{(1 - \theta_t)(1 - \gamma_t)}{1 - c_t} \right] \quad (\text{A18})$$

Equation (A18) says that relaxing the constraint on the payment of franked dividends by one dollar is worth the after-personal income tax value of the franking credit that is passed to the shareholder. Expression (A6) implies that λ^9 tells us about the difference in the value of a dollar of franked dividends and the source of finance under consideration (when the firm is optimising). In this case, λ^9 tells us that increasing franked dividends at the expense of retaining unfranked earnings has a net value equal to the tax credit available at the personal tax level. If there are no franking credits, as would be the case in a classical system, expression (A17) falls to zero. From another point of view, (A17) indicates that a firm can increase its value to shareholders by retaining unfrankable earnings and paying the maximum amount of franked dividends as long as

$$(1 - \theta_t) \gamma_t \tau_t > 0 \quad (\text{A19})$$

which it will be as long as any degree of dividend imputation is possible. This highlights that – with any imputation available at all – retaining unfrankable earnings is cheaper than retaining frankable earnings.

Expression (A18) says that the net value of relaxing the taxation constraint by one dollar is equal to the after-tax affect of a reduction in net cash flow (due to the payment of tax) by one dollar *minus* the increase in the ability to distribute a franked dividend with a franking credit. This is equivalent to a one-dollar credit in the franking account. If full imputation is in force, this expression falls to zero because the entire reduction in cash flow due to the payment of tax at the company level is compensated for by the transfer of a credit to shareholders of the same amount, as credit for tax paid. If no imputation is allowed (as in a classical system), the paying of company tax has a cost equal to the after-personal income tax value of every dollar paid *in* company tax.

Substituting these values into (A4) to find λ_t^3 provides us with

$$\lambda_t^3 = q_t^n \left[\frac{(1-\theta_t) \left[1 - (1-\gamma_t) \tau_t (\Gamma_t^n + \Xi_t^n) \right]}{1-c_t} \right] \frac{1}{K_t^{j'}} \quad (\text{A20})$$

The multiplier λ_t^3 tells about the net cost of a unit of investment when the firm is optimising. In this case, the net cost of a marginal unit of investment is a function of the after-tax cost of a foregone unit of franked dividends at the personal tax level – a function of the personal tax rate – and the value of deductions. If full imputation is in force – that is, γ equals one – these deductions have no value to the shareholder, and the expression collapses to

$$\lambda_t^3 = q_t^n \left[\frac{1-\theta_t}{1-c_t} \right] \frac{1}{K_t^{j'}} \quad (\text{A21})$$

This expression highlights the effect of full imputation in making company level taxation irrelevant – in the end, regardless of the rate of company tax or the deductions available at that level, the shareholder is liable only for the personal income tax payable on a dollar of income. Making the appropriate substitutions in (49) we find (53) and (54).