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The Global Trade and Environment Model:

A Projection of Non-Steady State Data Using Intertemporal GTEM

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The dynamic feature of the Global Trade and Environment Model (GTEM) has been enhanced. It is now based on intertemporal optimisation with investors having perfect foresight and savers having myopic expectations. In this context, it has been shown that a fixed savings rate out of current income (GNP) is consistent with intertemporally optimising behavior of the households. This result provides theoretical justification for the use of a Cobb-Douglas utility function at the top-level of the household utility tree in CGE models like GTEM and GTAP. The other significant innovation is that the model has been calibrated using a single period non-steady state database and implemented with GEMPACK.

Because the model is calibrated with non-steady state data, the model was run for some exploratory simulations covering a century to examine whether the model, in the absence of other shocks, arrives at a steady-state equilibrium by the end of the simulation period (momentum simulation). In addition, a test is performed on the effects on the model solution of the assumption that agents form rational expectations compared with the assumption of static expectations. The main objective in this paper is to describe the derivation of intertemporal GTEM and present the results of these initial exploratory simulations with the intertemporal version of the model.

With momentum simulation, we have observed that the model displays the property of a neo-classical growth model in the aggregate and the growth rate of each regional economy tends to converge toward the growth rate of its exogenously supplied factors. The assumption about expectations formation did not affect the trajectory noticeably.

Key words: *Intertemporal CGE modelling, GTEM, GEMPACK, steady state, temporary equilibrium, simulation, neo-classical growth model and convergence.*

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Introduction

This paper has three purposes. The first is to derive an intertemporal version of GTEM by imposing intertemporally optimising behavior on households to determine the optimal allocation of income into savings and consumption and on investors to determine the optimal level of capital accumulation. The second purpose is to demonstrate the applicability of the method described in Pant (2002b) of implementing an intertemporal general equilibrium model in GEMPACK with a single period non-steady-state database. The third objective is to examine the steady state properties of the model. In particular, we examine whether the global economy tends to a steady state equilibrium over time on its own given the initial non-steady state and explore any difference on the steady state and the path leading to the steady state associated with differing assumptions about the expectations of investors.

To achieve the first objective a Ramsey type model of household choice has been developed and a consumption function has been derived that bears a proportional relation with the GNP of a region under the assumption that the households have static expectations. This result justifies the use of the consumption side of the recursive GTEM (Pant 2002a) in intertemporal GTEM. To mimic the optimal behavior on the part of investors, an investment demand function has been specified that can be viewed as a solution to the stockholder's value maximising problem. The investment demand function is based on the expected rate of future return on investment and under the assumption of rational expectations the model becomes truly intertemporal. The implementation of the model under GEMPACK becomes a challenge in its own right.

To achieve the second goal, the GTEM code has been revised to accommodate the time dimension on the variables and coefficients and the update statements and the accumulation relations were re-written as described in Pant (2002b) so that the model calibrates itself overtime. The basic idea here is that all future values of a coefficient, which are changing and unobservable because of the non-steady state initial data, can be viewed as a deviation from the initial data of the base year. This deviation can be identified with the changes (and percentage changes) in variables of the model and therefore updateable by the model solution. The first paper that dealt with this issue of calibration with non-steady state data is Wendner (1999). The method developed by Wendner involves the use of initial data for all future periods and calculating the discrepancies on the state variables and applying the negatives of the discrepancies as additional shocks so that the state variables follow the dynamic path specified by their equation of motion. The method adopted in this paper is different from the one developed by Wendner in that it is based on updating the initial database by model solution and thus accords well with intuition.

The model has been simulated for 23 periods, 3 one-year and 20 five-year intervals, covering the period 1997 to 2100 using a six region, 24 commodity aggregation of the GTAP v5 database and GTEM emission database. In this exercise, the population module of GTEM has been removed and population and labor supply changes are taken as exogenous.

The results show convergence to the growth rates for the aggregates of each region as predicted by neoclassical growth theory. The growth rates of real GDP, GNP, capital stock, real investment all converge to the exogenously supplied uniform growth rates of labor supply, land and natural resources. In terms of the convergence test there appears to be no difference as a consequence of the expectations assumptions.

The paper is organised as follows. Section 2 summarises the existing version of recursive GTEM and section 3 describes the modifications introduced on the investment function. Section 4 describes the model closure and shocks. Section 5 reports simulation results and compares the results with and without rational expectation and conclusion are drawn in section 6.

2 The model

The Global Trade and Environment Model (GTEM) is a multisectoral and multiregional dynamic model of the global economy. As it was originally derived from GTAP (Hertel 1997). At its core GTEM looks very much like the GTAP model; many of the coefficient and variable names and data headers are the same and it is run using the same software, GEMPACK (Harrison and Pearson 2000). GTEM differs from the original GTAP model mainly in the following four aspects: GTEM uses a technology-bundle approach to energy-intensive industries; GTEM has a population module that generates endogenous changes in population and labor supply; GTEM has a greenhouse module that tracks emissions from production of various commodities and from the use of fossil fuels; and finally, GTEM contains accumulation relationships for capital stock, debt and population that make it dynamic. Agents in the standard version are not forward looking and so the model is recursively solved on a year-on-year basis.

Briefly, the main features of the model can be described as follows. It contains five basic types of agents: a representative consumer, regional production sectors, importers, an international transportation sector, and a global financial centre. All agents behave competitively and take prices as given. Supply of natural resources, land, government policies, technology and tastes are exogenous. All factors are owned by the regional household, which receives all factor incomes, all tax revenues and makes and receives transfer payments to and from the rest of the world. A representative consumer decides on the allocation of income of the regional household.

The current gross national income of each region is allocated to savings (units of global bonds) and to the consumption of commodities produced everywhere to maximise the utility of the representative consumer. This is done in three stages: first, a Cobb-Douglas utility function, defined over the private consumption of goods, government consumption of goods and real savings is maximised. This implies that a fixed share of gross national income is allocated to each of the three categories. The budget allocated to private and government consumption is further allocated to individual commodity composites using a Cobb-Douglas utility function for government consumption and a CDE function for private consumption. In the third stage, using the Armington assumption of imperfect substitution between sources and assuming rationality both government and private demand of each composite commodity is met from domestic and foreign sources so that the cost of each composite is minimized.

Production sectors use a combination of the four types of factors of production, capital, labor, land and the natural resource, and combine them with other energy and non-energy material inputs to produce their output.[1] Production technologies contain nests that allow intra-energy commodity, intra-factor and energy-factor substitution in response to relative price changes and are characterised by constant returns of scale. Each sector minimises cost in choosing the optimal input demand and industry output levels are chosen to maximise profit. Competitive conditions imply price taking behavior and satisfaction of zero profit conditions in equilibrium. Input demands for commodities are met from domestic as well as foreign sources. The Armington assumption of imperfect substitution between sources and the process of cost minimisation again determine the allocation of input demand between sources.

Aggregation of input and final demand for each commodity identified by source determine a region's imports by commodity and by region. This aggregation also yields a region's export of a commodity by destination and thus bilateral trade. Shipping of commodities from a source to its destination region is done by an international transport sector, which has a Leontief production technology. This sector buys inputs of transports (margin commodities) from various regions minimising the unit cost of the transport aggregate. Importers buy the transport services and the cost of transport creates the wedge between the *fob* and *cif* prices of commodities. Both the transport sector and importers satisfy zero profit conditions in equilibrium because of competition.

The savings of the regional households is pooled by the global financial centre and then lent to investors residing in all regions. The allocation responds to the differential of the expected rate of return with the global rate of return that clears the market. The market clearing rate is used to service the debt or pay the savers, which guarantees that the global financial centre satisfies its zero-profit condition as well. Regions may differ in their risk characteristics and policy regimes, therefore it is maintained that different regions may have different expected rates of return in equilibrium. The equilibrium condition simply requires that changes in the expected rate of return be the same across all regions, which equals the changes in the global rate of return. In this sense,

[1] One of the production sectors, called the capital goods sector, assembles capital goods from material inputs without using primary factors and its production technology is characterised by a Leontief function. There are two sectors – electricity and iron and steel – that have technology bundles. Electricity is produced by seven technologies and iron and steel is produced using two different technologies. Each technology employs a Leontief production function. Industry output is allocated between different technologies to minimise the average cost of producing the CES aggregate of their outputs.

the allocation of investment in GTEM is inefficient. There is scope for another allocation of investment (and hence the global capital stock), from a low return region to a high return region, which may raise global income and welfare. However, despite the mobility of investible funds, it is maintained in GTEM that the global capital market does not equalize the expected rates of return to investment.

GTEM is built around the Walrasian tradition. Therefore for each commodity and factor there is a competitive market. It is maintained that with fully flexible prices, markets for all goods and factors clear in each period. Commodities are distinguished by source and sold globally. Thus, they have a global market clearing condition. Capital and labor are region specific, but freely mobile across activities in search of a higher return; land is mobile within agricultural industries and natural resources are specific to each resource based industry such as coal, oil, gas, forestry and fishing. Factors are inelastically supplied and their prices are determined by the respective market demand conditions.

The savings of a regional household does not bear any relationship with the amount of regional investment, it is possible for each region to have its capital account in imbalance. A surplus leads to an accumulation of foreign debt, which needs servicing from the next period. This mechanism sets the dynamics of accumulation of net debt in GTEM. As there is a restriction on the amount of investment that a region can undertake in any period via the competition for limited global savings and it cannot borrow for consumption, there is no Ponzi game problem in GTEM.[2] Capital at the start of a period is given by the depreciated stock of the previous period and the gross investment undertaken over the previous period. As long as the amount of gross investment is different from the depreciation requirement, the capital stock of a region keeps on changing.

In its greenhouse module, GTEM accounts for three gases: carbon dioxide; methane; and nitrous oxide. In calculating CO_2 emissions GTEM accounts for combustion, fugitive emissions and industrial processes. In the case of methane and nitrous oxide, it accounts for emissions from livestock and farming activities, fugitive emissions, transport, and chemical industries. The main assumption used is that the combustion emission is proportional to the use of fossil fuels and other emissions are proportional to the activity level and are identified with a non-fossil fuel Leontief input of the sector. The built-in emission response functions contain provision for assessing the impact of technical change in the production process and input uses in response to economic incentives such as an emission tax and energy prices.

[2] In an intertemporally optimising model, if a restriction is not imposed on the accumulation of debt, then it becomes rational for the household to borrow and consume until its marginal utility is zero in each period. It will borrow to service the debt and hence the amount of debt rises ‘astronomically’, which is called the Ponzi game. As we will see below, in a simplified model of intertemporal choice a no Ponzi game condition has been imposed via a transversality condition in the intertemporal version of GTEM.

GTEM also contains a population module that projects age and sex-specific changes in the population of each region in response to economic development. This module in turn is used to determine the size of the regional household and the aggregate supply of labor in the region. For the purpose of this paper, the population module has been removed and labor supply and the total population of each region are taken as exogenous.

3 Modifications

We take the recursive version of the model, as summarised above and described in Pant (2002a), and introduce a minimal change to make it consistent with the inter-temporal optimising behavior of the agents. We first show that a fixed share of savings out of regional gross income is consistent with intertemporally optimising behavior of the household under strict assumptions. We maintain, for the purpose of this paper, that these assumptions hold. We then modify the investment demand equation to allow for the investors to have either static or rational expectations about the future rate of return.

The Household

A simple model of household choice based on intertemporal considerations is presented here. It is based on a Ramsey model of decentralised economy where investment decisions are separated from saving decisions (Abel and Blanchard, 1983). We extend the Abel and Blanchard's closed economy Ramsey model to the global economy where regions are linked by trade and investment and in each region, investment is fully bond financed. From the point of view of the regional households, local bonds are perfectly substitutable with foreign bonds and hence earn the same global rate of return. Consequently, we assume that households buy all domestic bonds issued by the investors and trade with the world (via the global financial centre) to maintain their optimal bond holdings. We do not impose the arbitrage condition that local bonds yield the same rate of return as the physical capital. The rate of return in physical capital is allowed to vary across the regions. It is, however, maintained that the regional capital stock is fully owned by identical regional households.

We consider an economy where the regional households supply factors inelastically to the market (and government policies are given), all agents behave competitively and thus take all prices as given. The population and the labor supply grow exponentially at the rate of *n*; and production functions are characterised by constant returns to scale. Given this environment, the only problem left to a household is the allocation of its income between current consumption and accumulation of assets domestic and foreign bonds (savings) so that the discounted sum of periodic utilities is maximised.

In each period, an average (representative) regional household would be allocating its income to either consumption or savings, and so we must have

(1) $y_t = c_t + s_t$

where y_t, c_t and s_t are per person income, consumption and savings of a regional household.[3] The households receive factor income from the ownership of labor and capital, and interest incomes from the ownership of the stock of domestic and foreign bonds. So the per person income is given by

(2) $$y_t = R_t k_t + W_t + \rho_t b_t^f + \rho_t b_t^h$$

where R_t is the rental rate of capital, W_t is the wage rate, ρ_t is the global real rate of interest paid on bonds, b_t^f is the current holdings of foreign bonds and b_t^h is the current holdings of domestic bonds at time t.

Let i_t be the optimal per person investment of the of the region, which is chosen by the investors independently, then as the investment is bond financed we must have

(2) $$db_t^h / dt + nb_t^h = i_t .$$

Equation (2) simply states that change in per person holdings of domestic bonds is equal to per person investment. As the savings is fully allocated into domestic and foreign bonds, we have

(3) $$s_t = db_t^f / dt + nb_t^f + db_t^h / dt + nb_t^h .$$

Equation (3) simply states the fact that changes in the bond holdings is equal to the total household savings in per person terms.

It follows from (2) and (3) that

(4) $$db_t^f / dt + nb_t^f = s_t - i_t .$$

Equation (4) neatly states that changes in the foreign bond holdings reflect the current changes in the capital account of the region. If the region is saving more than it is investing in any given period, then it accumulates additional foreign bonds. It is worth noting that the holding of domestic bonds is not a choice variable of the regional household; it is determined by the behavior of the investors. Given the holdings of domestic bonds, the household optimises by choosing the amount of foreign bonds it holds.

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[3] In this section, we describe the model for a region and suppress the index identifying the region to avoid cluttering. For simplicity, we assume that total population is equal to labor supply and capital and labor are the only two factors of production. Quantity variables in lower case represent per person quantities, rather than per cent change of the variables.

From equations (1), (2) and (3) we obtain the dynamic budget constraint of the representative household as

(5) $$c_t + db_t^f / dt + db_t^h / dt + n(b_t^f + b_t^h) = R_t k_t + W_t + \rho_t (b_t^f + b_t^h) .$$

Equation (5) simply states that per person consumption and changes in per person bond holdings (which equals per person savings) should be equal to per person factor income plus interest income from per person holdings of domestic and foreign bonds.

Given the dynamic budget constraint (5), a representative household solves the following problem:

(6) $$\max\ U_s = \int_0^\infty u(c_t) e^{-\theta t} dt$$

where, θ is the subjective discount rate of the household.

It is worth noting that the right hand side of the dynamic budget constraint (5) is the per person regional GNP and the left hand side is per person consumption and per person savings in real terms, measured in units of the consumer good.

It is assumed that the following transversality condition (NPG) holds:

(7a) $$\lim_{t \to \infty} b_t^f \exp\{-\int_0^t (\rho_v - n) dv\} \geq 0 \text{ and}$$

(7b) $$\lim_{t \to \infty} b_t^h \exp\{-\int_0^t (\rho_v - n) dv\} \geq 0 .$$

The condition (7a) implies that a region cannot keep on borrowing forever. Eventually it must have non-negative present value of foreign assets. Since, the regional holdings of foreign assets should always sum to zero, it is impossible for any region to hold a positive amount of foreign asset without some other region holding a foreign debt, appropriately discounted. Hence, condition (7a) imposes the requirement that the holding of foreign assets be stabilized so that its present value is asymptotically zero for all regions. Similarly, condition (7b) implies that the discounted present value of investment and thus domestic bonds should asymptotically be zero. This condition will be met if $\rho_t > n$ for sufficiently large t and regional investments are not growing 'astronomically' for ever.

Following steps outlined in Blanchard and Fisher (1994), the dynamic budget constraint can be integrated using (7) to obtain the intertemporal budget constraint:

(8) $$\int_0^\infty c_t \exp\{-\int_0^t (\rho_v - n) dv\} dt = b_0^f + \omega_0^k + \omega_0^h$$

where,

(9) $$\omega_0^k = b_0^h + \int_0^\infty R_t k_t \exp\{-\int_0^t (\rho_v - n)dv\}dt$$

(10) $$\omega_0^h = \int_0^\infty W_t \exp\{-\int_0^t (\rho_v - n)dv\}dt$$

The intertemporal budget constraint (8) implies that the discounted sum of current and future consumption should be equal to the total wealth of the representative consumer. The wealth consists of the current holdings of foreign bond, b_0^f , value of the physical capital owned and the value of human capital. Equation (9) defines the value of physical capital, ω_0^k, as shown in Abel and Blanchard (1983), as the sum of the current liabilities of the firms, given by b_0^h, and the discounted flow of all current and future dividends, the term under the integral in (9). Similarly, equation (10) defines the value of human capital as the sum of all discounted current and future labour income, all expressed in per person terms.

Let us define

(11) $$U(c_t) = \ln(c_t)$$

as the explicit form of the periodic utility function, which implies that the intertemporal elasticity of substitution is unity.[4]

Then the Hamiltonian of the problem (6) subject to (5) can be written as:

(12) $$H(c_t, b_t^f) = \ln(c_t)\exp(-\theta\, t) + \mu_t[R_t k_t + W_t + (\rho_t - n)(b_t^f + b_t^h) - c_t]$$

where μ_t is the costate variable, representing the shadow price of foreign bonds.

The first order conditions for the maximisation are:

(13) $1/c_t \exp\{-\theta t\} - \mu_t = 0$, and

(14) $d\mu_t / dt = -\mu_t(\rho_t - n)$.

Differentiating (13) with respective to t, and using the result together with equation (13) in equation (14) we get

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[4] It is quite common in applied intertemporal CGE models to assume a the form of intertemporal utility function time-separable and the periodic utility function is logarithmic. See, for example, Diao and Somwaru (2000), Wendner (1999), CPB (1999), and McKibbin and Wilcoxen (1999).

(15) $\quad (1/c_t)dc_t/dt = \rho_t - n - \theta.$

Equation (15) constitutes Euler's equation for this problem.

Integrating (15) we obtain

(16) $\quad c_t = c_0 \exp\{\int_0^t (\rho_v - n - \theta)dv\}.$

Substituting the expression for c_t from (16) into the intertemporal budget constraint (8) we obtain, upon simplification,

(17) $\quad c_0 = \theta(b_0^f + v_0^k + v_0^h).$

Equation (16) and (17) together provide the optimal path of per person consumption chosen by the representative household at time 0 and for all future period t. These expressions are not operational at this stage. The key to operationalise the consumption trajectory is the expectation of various prices held by the representative household that determines the value of its wealth in various forms.

Assume that households have static expectations regarding their income, prices and population. That is, households believe that $n = 0$, $k_t = k_0$, $W_t = W_0$ and $R_t = R_0$ for all t, while revising their expectations of prices and endowment every period and take a longer-term view on the real interest rate expecting that the real interest rate is fixed forever at $\rho_t = \overline{\rho}$. Then, it follows from (9) and (10) that

(18) $\quad v_0^h = W_0 / \overline{\rho}$ and $v_0^k = R_0 k_0 / \overline{\rho}$

Making use of equation (18), equation (17) is rewritten as

(19) $\quad c_0 = [\theta / \overline{\rho}][R_0 k_0 + W_0 + \overline{\rho} b_0^f]$.

Equation (19) determines the consumption out of current income, with consumption plans for the future given by (16), which will be revised in the next period when prices and incomes are known for the next period and so on. Hence, for each period, we take (19) as the aggregate consumption function for each region of the global economy.

As $[\theta / \overline{\rho}]$ is a constant, it follows that the representative household allocates a fixed proportion of the regional per person income to per person consumption and hence the per person savings rate of the region is fixed. Provided that $\theta < \overline{\rho}$, that is, the subjective discount rate is less than the expected fixed real interest rate, the marginal propensity to consume out of current income is less than unity and the system remains convergent.[5]

Given that the saving rates of the households are fixed; production functions satisfy constant returns of scale and there are some exogenously supplied factors, such as land, natural resources and labor supply, the fundamental characteristics of the model is close to that of the neo-classical growth model (Solow 1956). Hence, we can expect that in the steady state the ‘golden rule’ prevails. That is, in the context of GTEM described in section 2, if we assume that labor supply, population, supply of land and the supply of natural resources grow at an exogenous rate n, then in the steady state capital stock, investment, real GNP, and real GDP are all expected to grow at rate n in the regional economy.

The Investment Demand Function

In principle, investors are expected to choose the level of regional investment to maximise the value of the regional capital stock subject to the economic structure specified in the model and the closure rules. Without specifying the problem formally and solving for the solution, we write the investment demand function for region directly, which can be taken as the integral solution of the appropriate Euler equations.

For our purpose, the investment demand function for an arbitrary region r is written as

(20) $$I_t^r = \delta^r K_t^r \exp\{\beta_r(\rho_t^{re} - \rho_t^g)\}$$

where ,

β_r is a region specific positive parameter reflecting the speed of adjustment in the regional rates of return via changes in the quantity of regional investment;

ρ_t^{re} is the expected rate of return on investment in region r at time t; and

ρ_t^g is the global rate of return that clears the global financial market.

If the expected rate of return is given by

(21) $$\rho_t^{re} = [R_t^r + (1-\delta^r)\Pi_t^r]/\Pi_t^r - 1$$

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[5] In so far as (19) is based on expected prices, the dynamic budget constraint (5) should also be understood to have been specified at expected prices. A somewhat similar approach has been followed in the WorldScan model (CPB 1999) in which households expect all prices to rise at a constant rate with a fixed real interest rate. The long-term real interest is fixed at 5 per cent in McKibbin and Wilcoxen (1999) as well.

where,
R_t^r is the rental rate of capital in period t,
δ^r is the depreciation rate of capital in region r, and
Π_t^r is the purchase price of a unit of physical capital at t in region r,

then, agents do not expect prices to change (static expectation). In this case all variables in the model are either contemporaneous or are given as data by history hence the model could be solved recursively, one period at a time.

However, if the expected rate of return on investment is given by:

(22) $$\rho_t^{re} = [R_{t+1}^r + (1-\delta^r)\Pi_{t+1}^r]/\Pi_t^r - 1$$

with $\rho_T^{re} = \rho_{T-1}^{re}$ where T is the end of the simulation horizon and we maintain that the rental rates and prices of capital goods that actually prevail in period (t+1) are correctly anticipated in period t by investors, then the expectation formed according to (22) is defined as the *rational expectation* (of the rate of return).

In simulating the intertemporal GTEM, we consider both forms of expectation as an alternative to examine whether perfect foresight on the part of investors makes any difference on the growth path of the regional economies and their steady states. To facilitate this implementation we combine both (21) and equation (22) into one equation:

(23) $$\begin{aligned}\rho_t^{re} = {} & \gamma\{[R_{t+1}^r + (1-\delta^r)\Pi_{t+1}^r]/\Pi_t^r - 1\} \\ & + (1-\gamma)\{[R_t^r + (1-\delta^r)\Pi_t^r]/\Pi_t^r - 1\}\end{aligned}$$

where, γ is the binary coefficient that takes the value of one when the investors are modelled as having rational expectation and of zero when modelled as holding static expectation.

Steady state of regional economies

The steady state of the model is achieved when the state variables of the model do not change. In the current version of GTEM, the state variables are foreign debt (assets) and the capital stock of each region. The level of foreign debt changes if the capital account of the region is out of balance and the capital stock changes if the gross investment is different from the depreciation of the current capital stock.

Given that the savings rate in region r is ϕ^r, which is given, to have steady state equilibrium we must have

(24) $$\phi^r Y_t^r = \Pi_t^r I_t^r = \delta^r K_t^r \Pi_t^r$$

where, K_t^r is the capital stock, δ^r is the depreciation rate, I_t^r is gross real investment and Y_t^r is the nominal GNP of region r in period t. The left side of the equality implies that current savings and investment in each region are equal hence the debt level does not change. The right side of the equality implies that the gross nominal investment in each region is just sufficient to meet the depreciation cost of the current stock of capital in the region.

Noting from (19) that $\phi^r = \theta^r / \overline{\rho}^r$ it follows from (24) that in the steady state, we will have

(25) $K_t^r \Pi_t^r / Y_t^r = \theta^r / (\bar{r}^r \delta^r)$.

Equation (25) shows that, in steady state, the regional capital-output ratios (in units of GNP composites) are uniquely determined by the respective ratios of their saving rates to the rates of depreciation. The steady state in GTEM does not require that the foreign debt or assets are zero in a finite time interval; it only requires that the current account, which also includes cost of debt servicing, of each region be in balance.

It follows from (20) that in order to satisfy the right-hand side equality in (24) we must have

(26) $\rho_t^{re} = \rho_t^g$

for all region r.

In linearised form, condition (26) can be written as

(27) $d\rho_t^{re} = d\rho_t^g$

This condition simply means that the changes in the expected regional rates of return, in equilibrium, are equalised with the change in the global rate of return. This means that if the regional rates of return are not equalised in the initial database, then the difference will remain in the steady state equilibrium as well. If (20) is linearised, it can be seen that, in the steady state equilibrium, growth rate of real investment will be equal to the growth rate of the regional capital stock.

Steady state of the global economy

The global economy is in steady state when all regions are simultaneously in steady state. From the above discussion we can infer that if the regional saving rates are fixed, then each regional economy will gravitate, in the steady state, toward the growth rate of its exogenously supplied factors, provided they have a uniform growth rate. This rate could be different for different regions. The time taken to reach a steady state may be different for different regions. As there will be a transmission of disequilibrium from one region to others, it may take longer for the global economy to arrive at its steady state equilibrium. The global equilibrium will reflect the average of the regional steady state values as we can see that the global growth rate of the exogenously supplied factors is the average of the regional growth rates. This conjecture can be examined by simulating the model with uniform and non-uniform regional growth rates for the exogenously supplied factors. Because of the time constraint, we will not examine this feature of the global model in this paper. Our focus will be limited to the examination of the steady state path of regional economies.

4 Model Closure

The basic model closure rules of GTEM as described in Pant (2002a) apply here with two exceptions. First, as mentioned in the above section, we select expectational behavior either (21) or (22) for investors via the choice of a binary coefficient γ, which takes the value of 1 to activate rational expectation and zero to maintain static expectations.

Second, we apply the following steps to replicate the base year data. For static expectations, we do not need to validate the base year data as a model solution, because changes in all endogenous variables will naturally be zero and hence the updated data for the base year will replicate the base year automatically. However, for rational expectations, the expected rate of return depends on future prices and hence the current level, an thus the base year level of investment depends on the course of future prices. As future prices change, the change in the base year investment will not necessarily be zero and hence we have the problem of replicating the base year data.

To resolve this issue we treat the coefficient β_r in equation (20) as an updatable one with a per cent change variable, p_β_r, attached to it. In a simulation with rational expectations, we hold the base year level of investment constant, that is, changes in the base year investment levels are exogenously set to zero, and make the per cent change of the adjustment coefficient, p_β_r, endogenous. This ensures that the parameter of the forward-looking investment demand function is calibrated consistently. We initially set the value of β_r equal to unity for all regions.

Shocks

In order to make the debt and capital accumulation process operational we shock the respective homoptoy variables by one.[6] The imbalance contained in the base year data between savings and investment on the one hand and between investment and capital depreciation on the other keeps the regional economies moving along the path of temporary equilibrium year after year.

Our objective is to see whether the regional economies and hence the global economy finally arrives at some sort of steady state or not. As the growth rates of labor supply, land and natural

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[6] For further discussion on the use of homotopy variables in implementing the stock accumulation process in GTEM see Pant (2002b).

resources are exogenously set to zero, initially and to 2 subsequently, the other interesting question is whether the model displays the golden rule property.

5 Simulation results

In this section, we present a selection of results from three types of simulations - that show the convergence property of the model under momentum simulation; compare the behavior of trajectories under rational and static expectations, and the convergence of the model when all exogenous factors grow at 2 per cent per anum everywhere. The results are presented as charts and listed at the end of the paper.

Figures 1-4 show that the annual growth rates of real GDP, GNP regional capital stock and real investment converge to zero asymptotically. Their growth rates at 2100 are very close to zero. As the simulation was run under the assumption that there will be no growth in the labor supply, population, land supply and the supply of natural resources in all of the regions, this convergence displays the golden rule property of the model.

To examine whether the above results would have been any different had we assumed that the investors hold static expectations rather than rational expectations we run the momentum simulation again with static expectations. Figures 5-10 provide the comparison for a sample of countries; the results display the same pattern as described above.

It can be seen from the figures that although by 2100 the growth rates of each of the variables are slightly higher with rational expectations than with static expectations, the difference is not appreciable. The most pronounced effect, as expected, is on the growth rates of investment. This difference in the growth rate of investment, however, has not been significant enough to create a significant divergence in the trajectories of other variables, such as real GDP and real GNP. This accords well with the intuition that in the momentum simulation, the year-after-year changes in the values of the variables are diminishing as the model is converging to its steady state. Hence, the difference between the rates of return based on current prices and expected rates of return based on future prices would be vanishing thus the overlapping graphs of the trajectories.

To examine whether the model also behaves well when all exogenous factors grow uniformly at some rate, say by 2 per cent, we shock the annual growth rates of the supply of labor, land, natural resources and the population uniformly by 2 per cent and simulate the model under rational expectations. If the model is behaving well, we expect to observe that real GDP, real GNP, capital stock, real investment, etc, of each region grow at 2 per cent. The results are presented in the form of charts in figures 11-14.

Figures 11-14 show that the economy converges to the steady state growth rate of 2 per cent by 2050. Figures 12 and 14 also show that the growth rates of real GNP and real investment start diverging from the steady state path after 2060. The capital stock, however, keeps on growing at 2 per cent (figure 13).

Trade

In all simulations, it was found (not reported) that the structure of exports from each region remains unchanged throughout the century. The fob value share of a particular commodity in total exports of a region remains roughly constant. This shows that under uniform growth or under momentum simulation, the trade pattern remains unaffected by the convergent growth of regional economies.

Environment

In both momentum and uniform growth simulations, we have tracked the emissions of greenhouse gases. Figure 15 plots the total annual emission of greenhouse gases from all regions under momentum simulation. The annual emissions were about 28 billion tonnes of CO_2 equivalent in 1997 and are projected to be about 44 billion tonnes by 2100. Figure 16 plots the total annual global emissions over the years under the assumption that the supply of exogenously determined factors increases uniformly by 2 per cent per year from 1998 to 2100. In this scenario, the model projects total global emissions of greenhouse gases will rise from 28 billion tonnes to 187 billion tonnes of CO_2 equivalent. This is close to a five-fold increase in the annual rate of greenhouse gas emissions. This result, although preliminary, indicates that unless there is a rapid progress on the technological front, there is a real possibility of substantial emissions growth.

6 Conclusion

In this paper, we described the basic steps taken to convert the recursive version of the GTEM into an intertemporal model. In doing so, we showed that a fixed rate of saving out of current income is consistent with intertemporal optimisation. The model was calibrated using single period non-steady state data and solved using GEMPACK. The model was simulated to examine whether it can find its own steady state equilibrium. In this simulation, the supply of all exogenously supplied factors was held fixed at the initial (base year) level. This is called momentum simulation as the motive came from the non-steady state nature of the initial database. The simulation results showed clearly that the regional economies converged to zero growth rates by the end of the century. The convergence path was not noticably affected by the change of assumption about the way investors from their expectations. When the model was simulated with a uniform growth rate of 2 per cent for all exogenous factors, the regional economies converged to the two per cent growth rate by 2050 and they appear to start to deviate from the steady state path after 2060 onwards.

This exercise in itself represents a significant contribution to intertemporal CGE modelling on two fronts: there are very few intertemporal models that have been solved using GEMPACK, and this paper demonstrates that it is possible to calibrate an intertemporal model with a single period non-steady state data.

We are, however, mindful of the limitation of the approach adopted here. The calibration technique used here requires a multi-step solution at the finest level. We solved GTEM using a 1-2-4 step Euler solution and as a result our solution contains a significant amount of linearisation error.[7] But the choice was dictated by the time constraint as a set of 1-2-4 step solution alone requires about 11 hours on a PC with a 1GHz processor and about 5 hours on a PC with two 2GHz processors. Adopting, say 100-200-400 step solutions, would take weeks to complete one set of solutions. We also experienced PC memory problems, which prevented us from running yearly solutions for the 103 years. The problem mainly comes from the PC's inability to allocate more than two Gb of memory to any one application even if the machine has more memory available.

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[7] Change in the foreign debt level of regions, one of the state variables of the model, did not converge to zero in all simulations. The size of discrepancy as a proportion of regional investment is less than 1 per cent. Although we would have liked it to have converged to zero, we have taken this being the result of either numerical error, linearisation error or some bugs in the code. All of these possibilities are under investigation.

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Figure 1: Growth rates of real GDP under momentum simulation

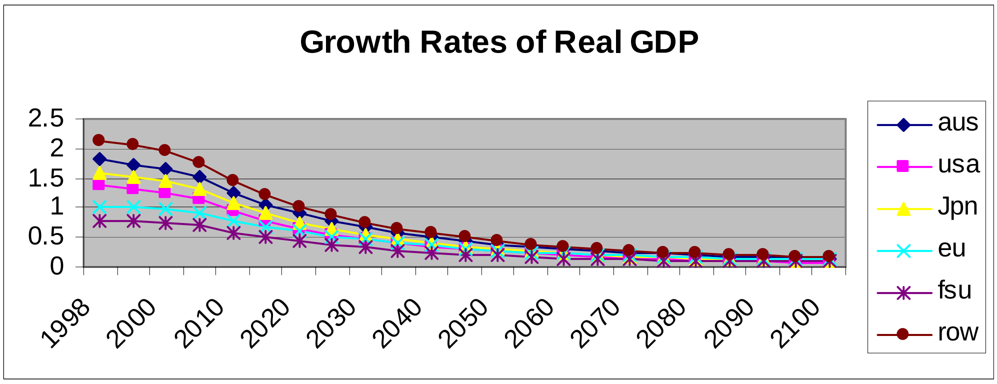

Figure 2: Growth rates of real GNP under momentum simulation

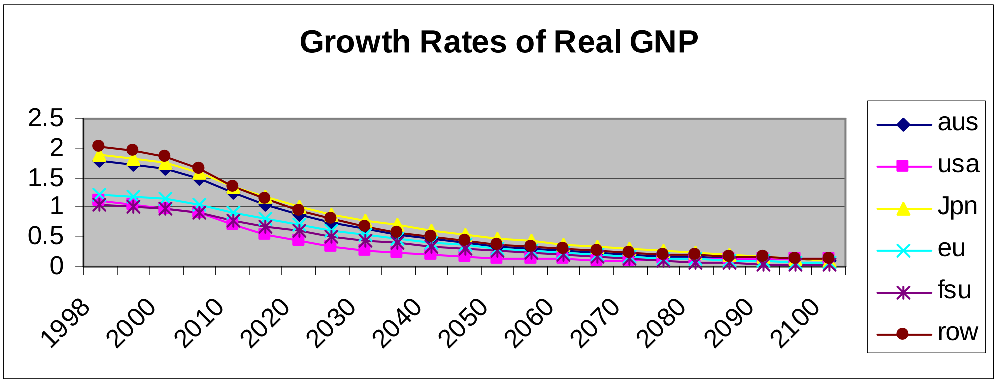

Figure 3: Growth rates of regional capital stock under momentum simulation

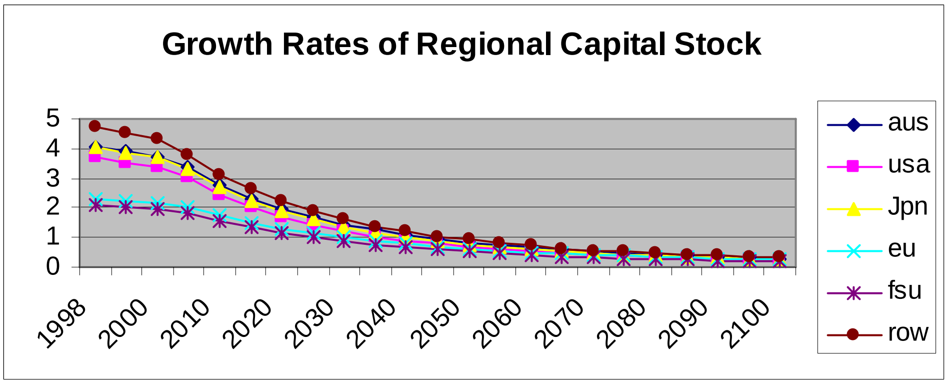

Figure 4: Growth rates of real investment under momentum simulation

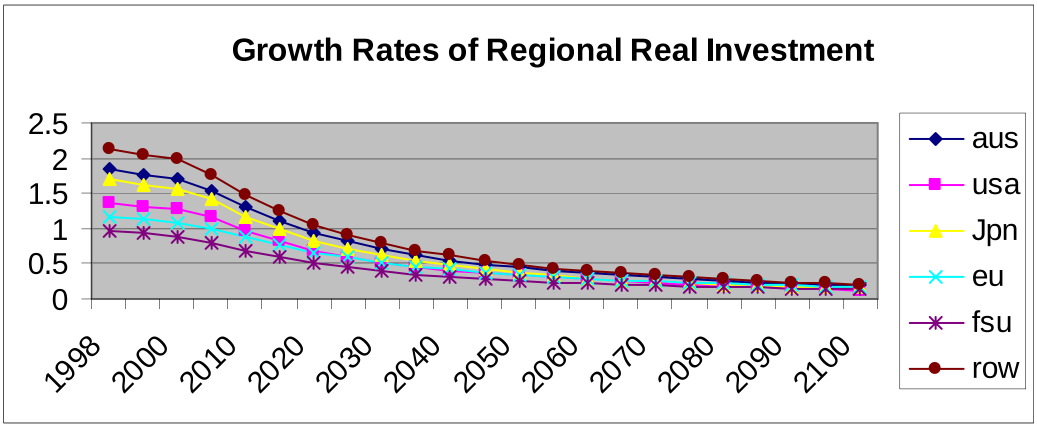

Figure 5: Growth rates of real GDP in the Australia under momentum with RE and with SE

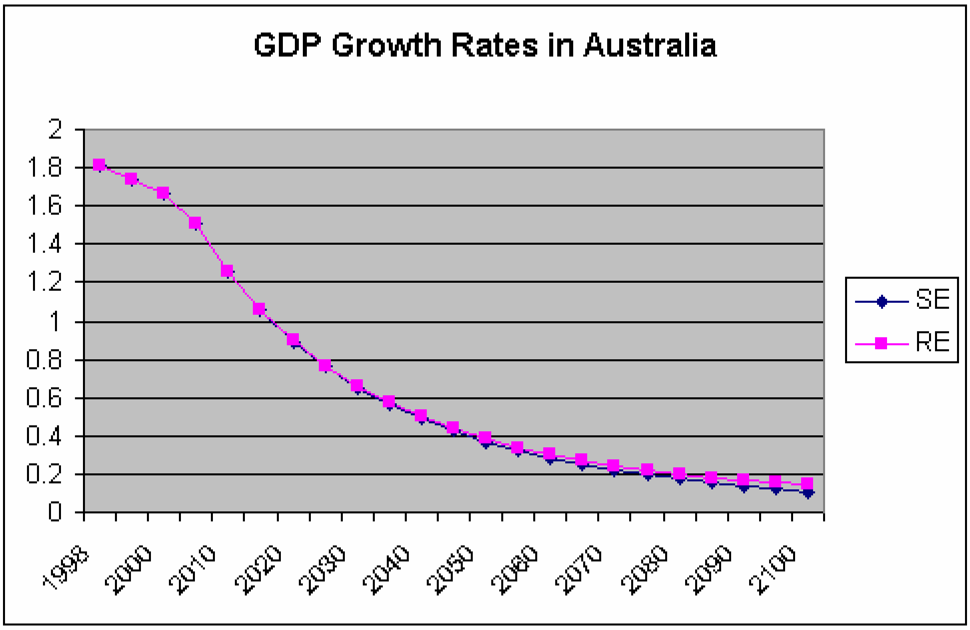

Figure 6: Growth rates of real GDP in RoW under momentum with RE and with SE

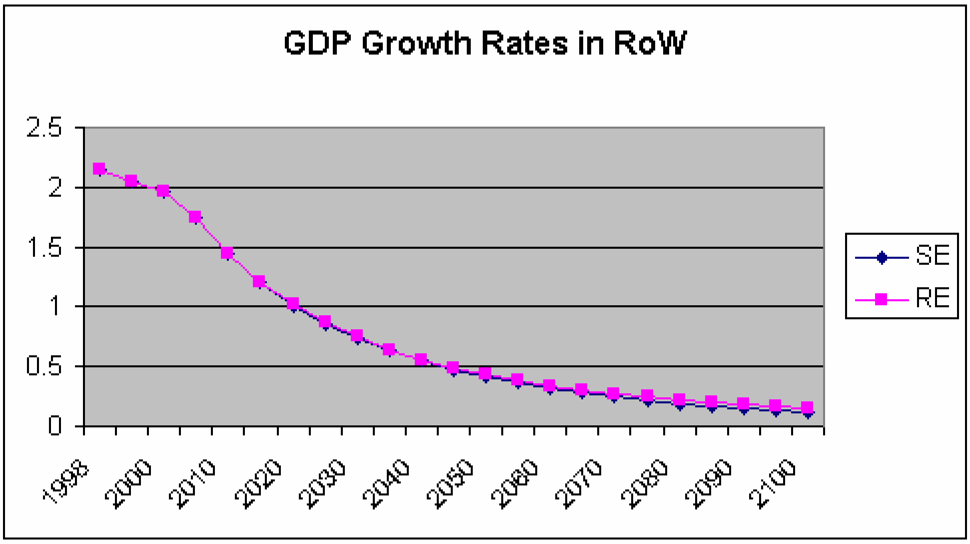

Figure 7: Growth rates of real GDP in Japan under momentum with RE and with SE

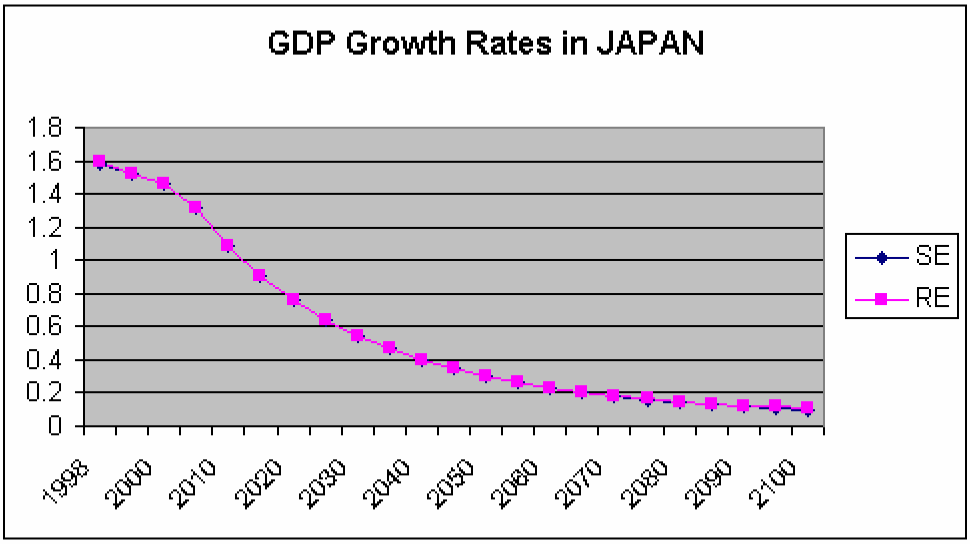

Figure 8: Growth rates of capital stock in the US under momentum with RE and with SE

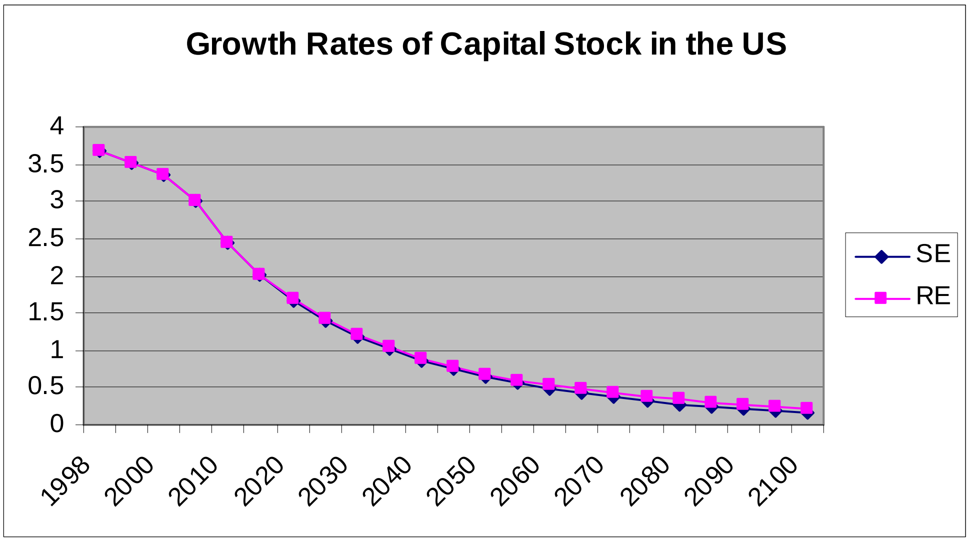

Figure 9: Growth rates of real investment in the US under momentum with RE and with SE

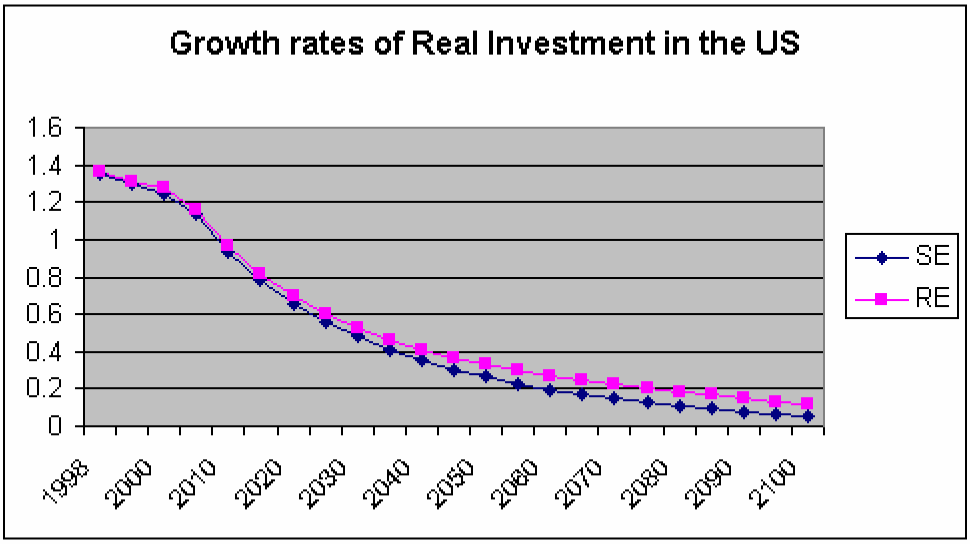

Figure 10: Growth rates of real investment in RoW under momentum with RE and with SE uniform 2 per cent growth in exogenous factor supply

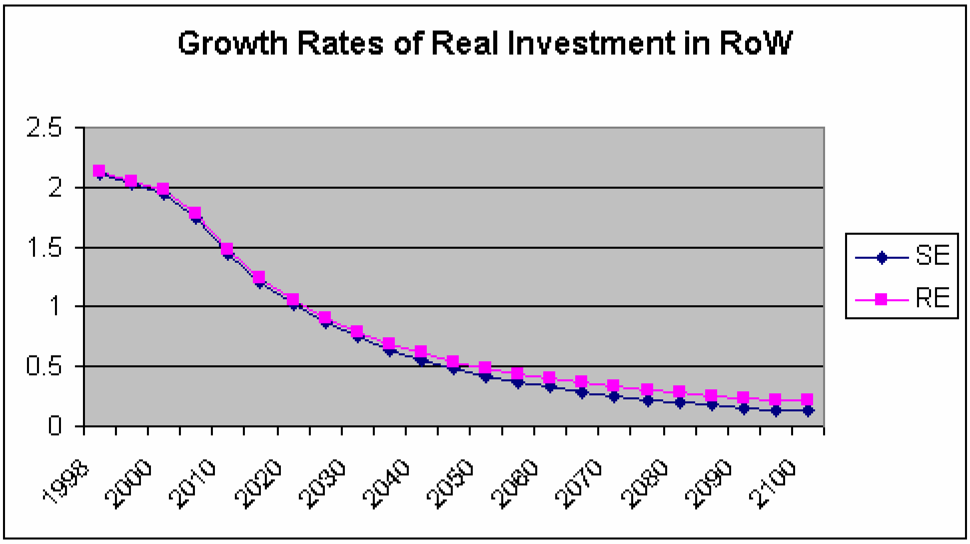

Figure 11: Growth rates of real GDP with a uniform 2 per cent growth in exogenous factor supply

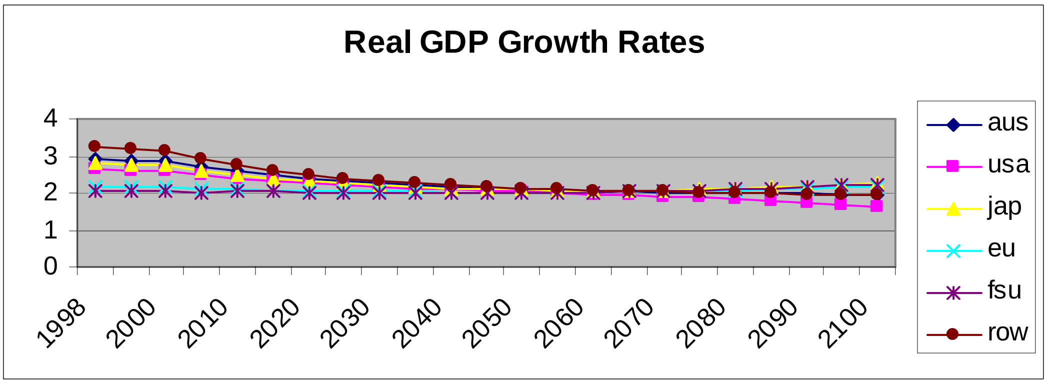

Figure 12: Growth rates of real GNP with a uniform 2 per cent growth in exogenous factor supply

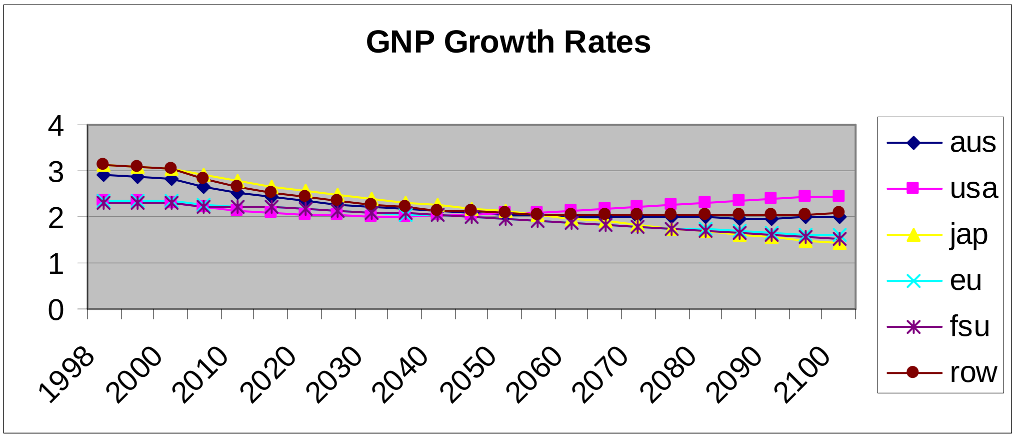

Figure 13: Growth rates of regional capital stock with a uniform 2 per cent growth in exogenous factor supply

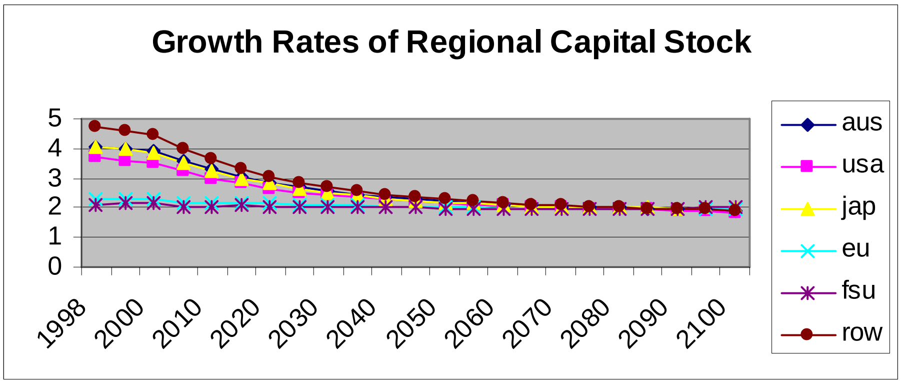

Figure 14: Growth rates of regional real investment with a uniform 2 per cent growth in exogenous factor supply

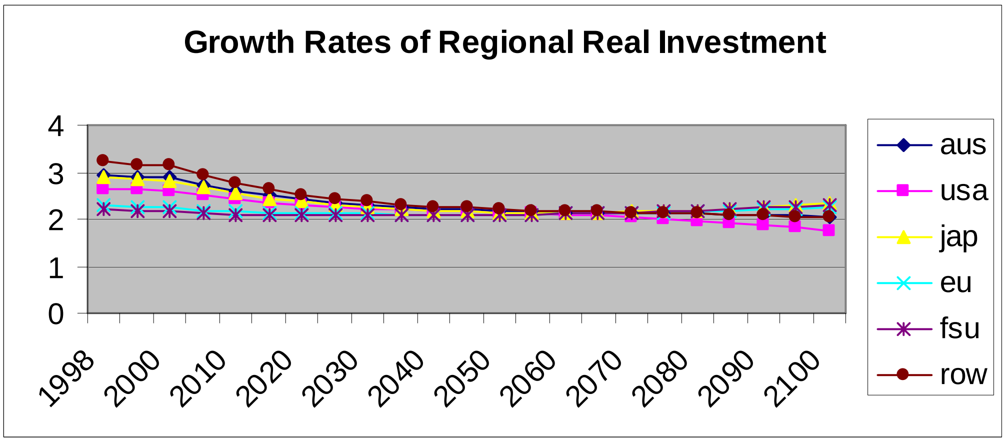

Figure 15: Global emission of GHGs under momentum simulation

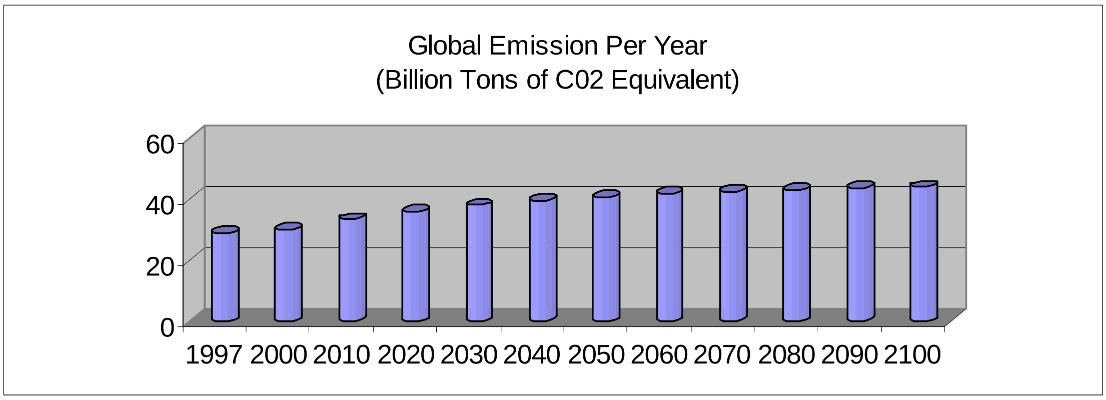

Figure 16: Global emission of GHGs under 2 per cent uniform growth assumption