The productivity of the agricultural sector and its effects on economic growth: a CGE analysis

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Abstract

Productivity is the essence of medium- and long-term economic growth in any economy; in this sense, we quantify the effects on economic activity in Colombia of productivity growth in the agricultural sector under a computable general equilibrium framework capable of incorporating determinants of its sources of growth of a more .endogenous"nature. The above, within an economy with heterogeneous firms and imperfect competition under the Melitz model, has 12 branches, 17 products, 16 trading partners, skilled and unskilled labor (informal or not), a fiscal framework with exogenous government and 20 types of households (half urban and half rural) that allows for distributional analysis. This analytical and quantitative tool is very useful for economic policy makers in that it allows them to contemplate a fairly complete picture of the Colombian economy in computational terms and to evaluate the different scenarios of the effects of the design and implementation of economic policies aimed at achieving productivity growth in the agricultural sector and its impact on the path of output over the next decade.

Additionally, it is one of the first steps in the integration of computable general equilibrium models with modern techniques such as those in the field of artificial intelligence, which allows for quantitative and analytical improvements in macroeconomic policy decision making, so necessary in times of crisis.

Key word: Productivity, Economic Growth, Computable General Equilibrium (CGE), Melitz model, Artificial Intelligence (AI). **JEL**: C68,C45,E24,F12.

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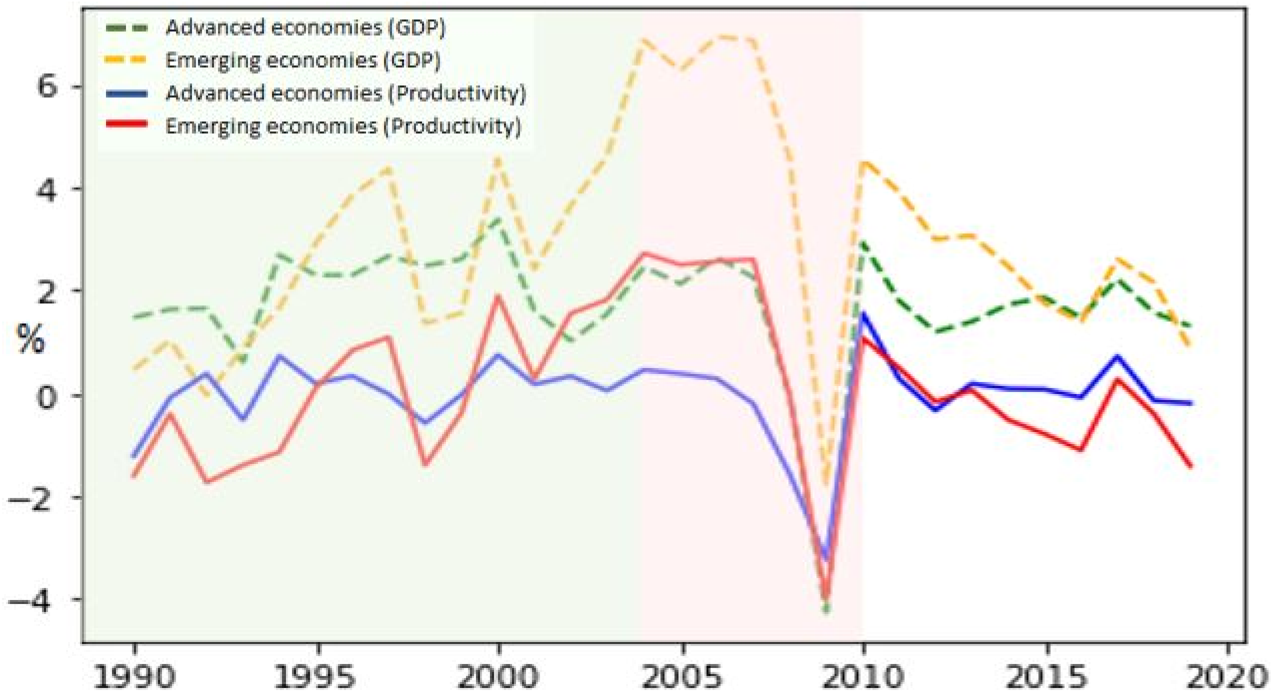
1. Introduction

Productivity is the essence of medium- and long-term economic growth in any economy; ensuring its sustained increase is undoubtedly one of the most important challenges facing economic authorities in alleviating poverty and satisfying the growing aspirations of the world's citizens. Empirical studies show that productivity growth is closely associated with technological change (Echavarría-Soto et al., 2019), the innovation (Cirera and Maloney, 2017), firm growth (Grover Goswami et al., 2019) and well-targeted public policies. (Cusolito and Maloney, 2018). However, its sources of growth have always been the subject of long discussion and questioning because of its distinctive complexity, despite multiple attempts it has not had a conclusive answer until today.

The debate on productivity growth has resurfaced worldwide, as evidenced by the recent launch of the World Bank's Productivity Project, which has initiated a series of summits, conferences, forums and published three very interesting studies: Grover Goswami et al. (2019); Cirera and Maloney (2017) and that of Cusolito and Maloney (2018); which seek to bring frontier thinking on the measurement and determinants of productivity to global policymakers. William Maloney, the project's chief economist, explains that each conference and volume in the series explores a different aspect of the topic through dialogue with academics and policymakers, and through sponsored empirical work in member countries. This is in light of the fact that productivity growth worldwide has stagnated over the last decade in both developed and emerging economies, as shown in Figure 1, where the red band shows a period of deterioration due to the financial crisis and the gray band shows a period of stagnation. It should be added that Figure 1 also shows economic growth, all series with annual frequency and in a period from 1990 to 2019.

It is evident that the world economy has not yet been able to fully recover from the 2008 financial crisis, at least in terms of productivity, interrupting periods of outstanding expansion by emerging economies, reaching rates of over 2 % recorded consecutively in the 2004-2007 period. It is important to note that in the last decade, both for advanced and emerging economies, the contribution of productivity to GDP growth has been in negative territory. In summary, in the period 1990-2019, the average economic growth was 2,95 % in emerging economies, with the contribution of productivity to GDP growth of 0,11 %; while advanced economies presented lower records of 1,68 % y -0,04 % respectively. However, as shown in Figure 1, the outlook for the period 2009-2019 is worrisome, with an average GDP growth of 2.17 % for emerging economies and 1.19 % for advanced economies, both of which are lower than in the previously analyzed period. Making the same calculations for the contribution of productivity to GDP growth in the last decade, we find a contraction of -0.59 % for emerging economies and stagnation for advanced economies, with a TFP variation of -0,09 %. Thus, it is evident that productivity growth at the global level has stagnated in the last decade, a phenomenon that has been exacerbated in recent years by the impact of Covid-19.

Figure 1: **Contribution of TFP to economic growth in emerging [4] and advanced economies[5], 1990-2019**



Source: Own elaboration and calculations with data from the Conference Board in Python.

However, from a regional perspective, the stagnation of productivity growth is generally and notably present in Latin America, although with some regional differentiation. Colombia is no exception. Moreover, according to one of the measures that have been most widely used in the literature to approximate the level of productivity of an economy, "Total Factor Productivity (TFP)", which allows evaluating the productive performance of a country, based on how much its product and productive resources grow, in the last 40 years there is evidence of stagnation in relative terms of the Colombian economy with respect to the main and similar economies of the region, as illustrated in Figure 2.

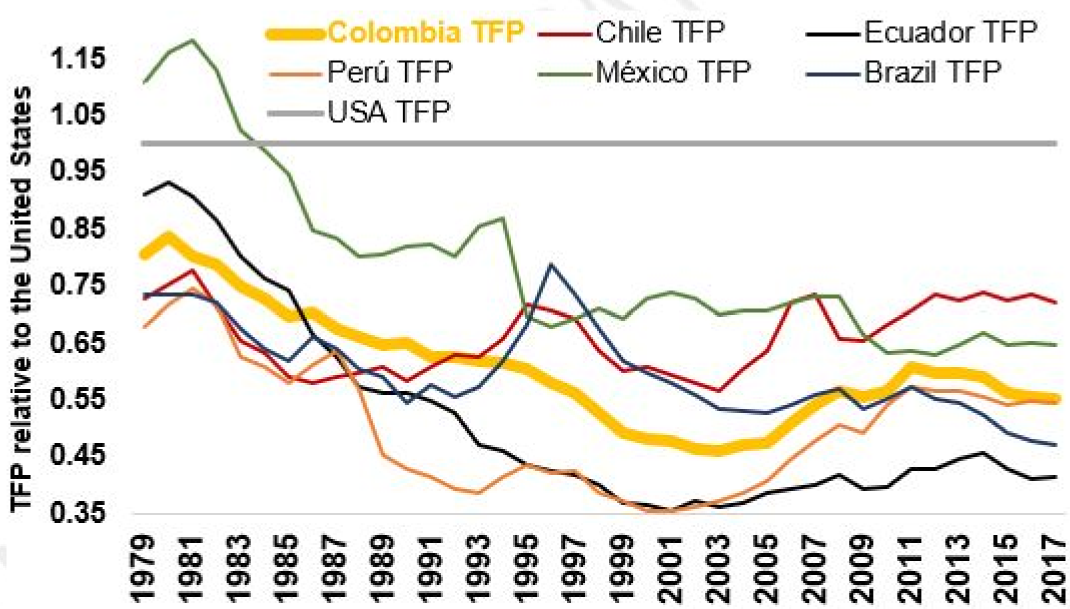
It is important to recognize that the relevance of TFP as a proxy for productivity has been debated among economists, given that the estimation process using production functions with productive factors generally presents endogeneity problems. Consequently, methods have been developed to correct these problems, among which the seminal studies of Olley and Pakes (1992); Levinsohn and Petrin (1999), and more recently,Wooldridge (2009), De Loecker (2011) and Ackerberg et al. (2015). The authors have developed

[4]Following the International Monetary Fund (IMF) classification, the average of GDP growth and the contribution of productivity to GDP of a set of countries is taken: Algeria, Angola, Egipto y Sudáfrica (Región de Africa); China, Indonesia, Malasya, Filipinas, Corea del Sur, Sri Lanka, Taiwan y Thailandia (Región de Asia); Azerbaijan, Belarus, Bulgaria, Croacia, República Checa, Grecia, Hungría, Kazajistán, Lituania, Polonia, Portugal, Rumania, Russia, Eslovaquia, Turquía y Ucrania (Región europea); Argentina, Brazil, Chile, Colombia, Dominicana, Ecuador, México, Perú, Uruguay y Venezuela (Región Latinoamérica).

[5]Following the IMF and the World Bank, the average GDP growth and the contribution of productivity to GDP of the economies of: Estados Unidos, Alemania, Japón, United Kingdom y Francia.

techniques to correct the bias, based on variables related to the production process that are used as a proxy for productivity. The former use investment demand as a proxy, while the latter use demand for intermediate inputs. Both papers establish semiparametric estimation techniques, based on firm-level information, while the latter use the demand for intermediate inputs Wooldridge (2009) shows how the semi-parametric estimators established in these methodologies can be implemented using the generalized method of moments (GMM). Ackerberg et al. (2015) extend the model of Olley and Pakes (1992), model by incorporating the estimation of the labor coefficient of the production function in the second stage of estimation in order to eliminate possible identification and multicollinearity problems (Echavarría-Soto et al., 2019).

Figure 2: **Evolution of Productivity (TFP) in Colombia and a set of economies in the region in 1977-2017**



Source: Own elaboration with data from Penn World Table version 9.1 (2020).

Figure 2 shows a stagnation in the TFP performance of most countries in the region over the last 40 years, with respect to the TFP relative to the United States; a worrisome scenario for the Colombian economy (red line), considering that productivity is the essence of medium and long term economic growth, the force that allows progress and improvement in per capita income. Following the World Bank's line, productivity accounts for half of the differences in GDP per capita between countries(Cusolito and Maloney, 2018), o it can be stated that regional divergences in the evolution of GDP are largely explained by the behavior of the TFP presented in Figure 2. It should be added that there are works suwhere they find that TFP explains about 90 percent of the divergence in per capita output growth across countries. This point is vital, since it highlights the importance of understanding the sources of productivity growth, which, to cite a few papers, claim is closely associated with technological change (Juan José et al., 2006), the innovation (Cirera and Maloney, 2017), the high growth (Grover Goswami et al., 2019) and police publics (Cusolito and Maloney, 2018). Therefore, it becomes a transcendental commitment for both Colombia and the region to join efforts to ensure sustained productivity growth that will allow greater per capita income for poverty alleviation, satisfying the growing and emerging aspirations of the region's citizens.

As far as Colombia is concerned, researchers, economic policy makers and the academic community in general have tried to explain the foundations of the latter. Considering the bibliographic review of what has been worked on in the country, not many works have been found, the most outstanding being that by (Echavarría-Soto et al., 2019) which find that policies and institutions that fall on firms in aggregate do not fully explain why Colombia's TFP is half that of the United States and that oligopolistic power (high Herfindahl) negatively affects total factor productivity growth. Other studies affirm the relevance of geographic and climatic factors in emerging economies (Juan José et al., 2006), but in contrast, other seminal studies for the country show evidence that the latter do not have a strong impact on the country (Clavijo and Banco, 2003). Traditionally, the analysis of productivity growth in Colombia has been studied using econometric methodologies and international rankings based on perception surveys; however, attempts have been made to use a new methodology, such as the study by Echeverry et al. (2005) based on direct work with businessmen through focus groups, finding that low productivity arises at the sectoral level. Finally, one of the most relevant lines of research in recent years has to do with a series of documents published by the CEDE of the Universidad de los Andes, such as the works of Eslava and Meléndez (2009) more recently, the work of Eslava and Haltiwanger (2020), which address productivity growth from another approach, in general terms expressing that Colombia has a high concentration of small companies compared to other countries. Following this line we find the study by Becerra et al. (2019) where they find that an enormous fraction of Colombian productive activity, much more than in developed countries, is concentrated in tiny productive units. The studies address how the sales growth of the less productive companies is driven by external factors, while the more productive ones, on the contrary, face brakes to their growth. The authors suggest that, to solve this problem, Colombia will need public policies that discourage the creation of companies with little vocation for growth and, on the contrary, facilitate their growth and formalization, as well as boost entrepreneurial capacity through measures such as the promotion of management skills among Colombians.

In this order of ideas, it can be inferred that, for the Colombian economy, there is a kind of consensus regarding the importance of studying the sources of productivity growth, however, there are multiple ways of approaching it, essentially because it is an interdependent and complex economic problem, which requires an analytical framework of general equilibrium. In this way it depends on a complexity of factors, actions, institutions and contexts, so it is necessary to identify and investigate various approaches and their sources. Thus, this research work aims to present a general equilibrium view of the effects of different scenarios of sustained productivity growth on short and medium term economic growth, taking into account what are considered to be its relevant sources, seeking to serve as a policy generator for the future of economic activity in this aspect, and to reverse the unfortunate trajectory presented in the country of the TFP in the last 40 years.

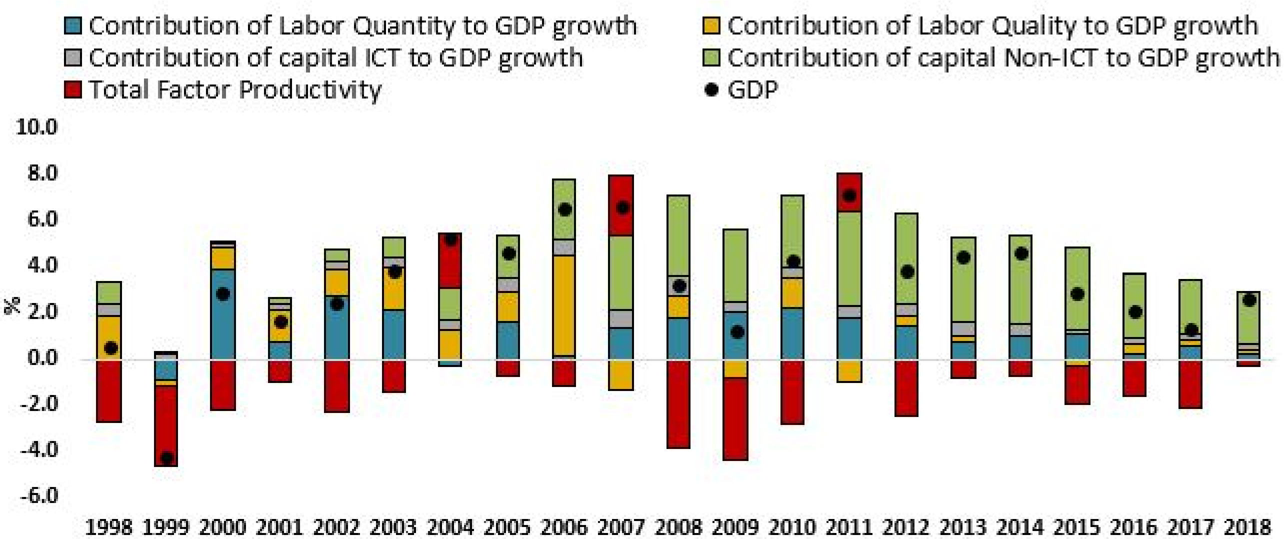
It is clear that an economy progresses when it achieves substantial improvements in productivity: when it produces more value from the resources at its disposal, either because it uses its resources more efficiently, or because it generates a greater quantity of goods and services with them, or because the goods and services it produces are better valued by consumers.

In this sense, the panorama of productivity growth in Colombia is lamentable and shameful. In order to demonstrate the importance of this type of studies in the country, we will make use of data from the well-known "Total Economy Database.ºf the Conference Board, which is used par excellence in the analysis of productivity at a global and country level. The distinctive novelty lies in the fact that it provides a view of the contribution of the main factors to the economic growth of a country, disaggregating the contribution of the quantity of work and quality of work, the contribution of total capital, which is broken down into Information and Communication Technology (ICT) capital and non-ICT capital, and finally, the contribution of total factor productivity (TFP).

The evolution of these contributions measured in annual growth rates for Colombia over the last two decades is presented in Figure 3. The red bars show the contribution of "total factor productivity (TFP)"to the country's economic growth, while the black dots represent economic growth (GDP) on an annual basis, which, in addition to being composed of TFP, contains four other bars that show, for the Colombian economy, the contribution to growth, in the period from 1999 to 2019, of productive factors, classified into four broad categories: capital with ICT (gray) and without ICT (green), quantity of labor (blue) and quality of labor (light color).

In Colombia, according to data from the Conference Board, it is estimated that the economy has grown an average of 3.3 % between 1999-2019; however, this growth incorporates a 0.6 % reduction in productivity. It should be added that, making similar calculations, but for the last ten years, the picture does not improve significantly, with GDP growth averaging 3.4 % between 2009-2019, and in this case such growth incorporates a 0.2 % reduction in productivity, despite experiencing periods of GDP growth averaging more than 5 % between 2010-2013. Undoubtedly, with the current crisis caused by the pandemic, the productivity will decrease even more.

Figure 3: **Contribution of TFP to economic growth in Colombia 1998-2018**



Source: Own elaboration with data from the Total Economy Database of the Conference Board.

It is important to highlight the critical situation of the contribution of the TFP to the growth of the Colombian economy, unfortunately, it can be stated that in the last two decades productivity has been persistently in negative territory, only except for 2004, 2007, 2011, 2013 and 2014. It should be added that, although in the last published data its contribution to economic growth had a significant rebound, it continues without reaching records higher than 1 %, which implies that in the last decade the contribution of productivity to GDP growth has been null (even in several years it could be considered negative).

In short, understanding, analyzing and modeling the sources of productivity growth in the agricultural sector is an unavoidable challenge for the design of development policies, the construction of possible scenarios and their implications for economic growth in post-pandemic times and for the next decade. It is important to highlight the cross-cutting nature and level of incidence that the sources of growth have on the economy as a whole. Hence the need to address these issues in a general equilibrium framework, since the latter is concerned with analyzing the interrelationships between markets for goods and services (as in macroeconomics) through economic agents making their own decisions (as in microeconomics). Thus, a computable general equilibrium (CGE) model analytically captures the problem, since it represents in numerical form the aggregate equilibrium conditions and in each of the markets of an economy in which producers and consumers intervene with behaviors established by means of production and consumer utility functions that depend on relative prices. In essence, they represent the theoretical structure of Walrasian general equilibrium in a computer language, in our case, GAMS, using information on the various markets with data from the real economy, and taking advantage of some mathematical algorithm for the search for equilibrium in the face of changes in economic policy.

Its importance lies in the fact that it is intended to .endogenize"to some degree the productivity analysis of the agricultural sector, with the exploration of modules through CGE modeling that start from the sources of its growth. The distinctive novelty of the research lies in the simulation of scenarios that already include the possible economic effects of the pandemic through Melitz-type modeling that allows endogenizing the growth of agricultural productivity through changes in 'iceberg' costs, average productivity and the limit of the Pareto distribution, in addition, The Melitz model also includes scenarios that contemplate human capital growth that endogenizes the effect of the sector's productivity on the country's economic growth in the next decade, with the above being an analysis that goes beyond the traditional exogenous shocks of the TFP in general equilibrium models and in the analysis of productivity in the agricultural sector in Colombia.

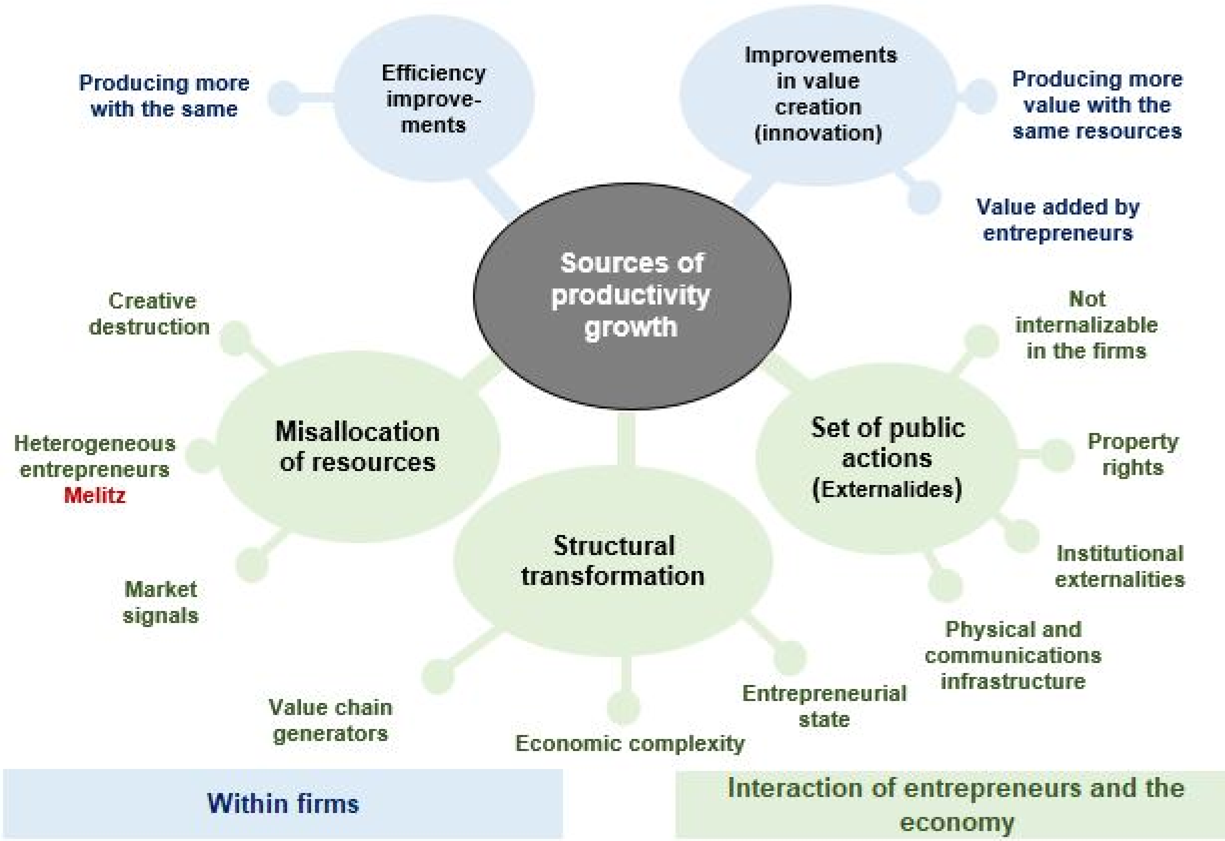
It should be added that it will have a robust and consistent nature, due to multiple advantages such as the creation of a module that replicates the fiscal rule in the country and an inclusion of 20 types of households that allows quantifying the distributional effects, in an analytical framework of heterogeneous agents and imperfect competition and, by the use both in the calibration of relevant parameters (elasticities of substitution) and in the training of the key macro time series that feed the CGE model and that affect the performance of the main economic variables in the country in the future path, by means of the most advanced techniques, in this case, the field of artificial intelligence will be used, mainly Machine Learning, being another of the distinctive novelties of the research integrating the field of artificial intelligence with the computable general equilibrium models that I have called 'AICGE'. All together will allow to better address the analysis and perspectives of the sources of agricultural productivity growth that will make it possible to understand the prolonged stagnation presented in recent decades and sources of agricultural productivity growth in the economy.

2. Literature review

Productivity, the capacity to produce economic value from the use of productive resources, is the fundamental key to the development of societies. A country progresses when it achieves substantial improvements in productivity: when it produces more value from the resources at its disposal, either because it uses its resources more efficiently, or because it generates a greater quantity of goods and services with them, or because the goods and services it produces are better valued by consumers.

The Colombian economy can progress when it achieves substantial productivity improvements. According to (Botero García et al., 2005) there are five sources of productivity growth in an economy, which can be grouped into two main categories: those that reflect the efforts made by firms to be more productive (within firms, those in blue); and those that refer to the allocation of resources in the aggregate economy (interaction of businessmen and the economy as a whole, those in green). As far as the sources within firms are concerned, it basically consists of two approaches: i) improvements in efficiencies and ii) improvements in value creation. When it comes to the sources of productivity growth from the interaction of entrepreneurs and the economy as a whole, it can be addressed from three approaches: iii) mis-allocation of resources; iv) structural transformation; and v) the set of public actions. All of the above is summarized in Figure 4.

Figure 4: **Conceptual approach to productivity**



Source: Own elaboration according to the systematic review of the literature.

At a more detailed level, what refers to sources within firms, consists basically of two standard approaches in the literature: the first can be called .efficiency improvements", which basically deals with how to produce the same, with fewer resources, which is achieved through the appropriate use of the factors of production; and the second can be called "value creation improvements", which consists of how to produce more value with the same resources, which is achieved through creative innovation, which generates products and services that are more valued by consumers (García et al., 2020).

However, when dealing with the sources of productivity growth from an aggregate perspective, it can be approached from three approaches that have been worked on in recent literature. The first has to do with the better allocation of resources among firms in a sector, making the most productive firms participate in a greater proportion in the use of resources, which is achieved by improving access to credit or eliminating distortions that inhibit the growth of the most productive firms, known as 'Misallocation', which has to do with many heterogeneous entrepreneurs, market signals (Melitz, which will be the object of research) and creative destruction of the Shumpeter type. The second approach is based on structural transformation under the idea of a better allocation of resources among productive sectors, making the most efficient sectors participate more in the product, which is achieved by encouraging investment in sectors with greater productive potential, generally associated with export activity, which revolves around economic complexity (Hidalgo and Hausmann, 2009), entrepreneurial state (Mazzucato, 2011) and value chain generators that distinguish between the companies that organize them and those that are part of the link in the chain, enabling the creation of high-tech value chains (McMillan and Rodrik, 2011).

However, productivity analysis has advanced considerably based on the methodologies of decomposition of productivity growth, between factors internal to the firms, and resource allocation ("Misallocation"), among others, as has been shown. Going into detail, the key items in this analysis are Baily et al. (1992); Olley and Pakes (1992); the seminal work of Hsieh and Klenow (2009); McMillan and Rodrik (2011); where they demonstrate the divergences in the TFP by misallocations in Restuccia and Rogerson (2013) and Melitz and Polanec (2015); with addition of heterogeneous production units Restuccia and Santaeulalia-Llopis (2017); others that add international trade to the analysis as theBerthou et al. (2018); heterogeneity of the agents in Sandoz-Dit-Bragard (2018); where they propose the optimal solution Shao and Tang (2020); implemented in a general equilibrium model in Baqaee and Farhi (2020) and using the growth of income in Bils et al. (2020).

From the modeling point of view, the analysis of productivity and resource allocation is linked to what is called the 'new trade theory', associated with the works of Krugman (1980) and Melitz (2003) which introduce imperfect competition, the entry and exit of firms to the markets, the economies of scale and fixed costs, and the heterogeneity of firms in the sectors that will be fundamental in the model to determine towards which strategic sectors the public policies that have an impact on the growth of productivity in the country should be directed. The integration of these issues in general equilibrium modeling is part of the current research agenda in this type of modeling, as shown in the works of Zhai (2008) and Dixon et al. (2016) and more recently in a work by Dixon et al. (2018) from which the mathematical analysis described in the appendix will be based, being the chapter entitled "Trade Theory in Computable General Equilibrium Models"published in 2018 which is part of a compendium of articles that make up the book 'Advances in Applied General Equilibrium Modeling' that will substantially expand the possible uses of the model in the analysis of public policies. In this vein, the CGE model has become an important tool used by multilateral entities such as the World Trade Organization (WTO), the Organization for Economic Cooperation and Development (OECD), the Inter-American Development Bank (IDB) and the World Bank to measure the impact of shocks or changes in policies (Meng and Siriwardana, 2017). This type of modeling is able to link the aggregate level of the economy with different levels of disaggregation, in particular, specific regions, disaggregated population, and investment projects. They attempt to capture the essentials of the economic system using the mathematical and statistical structure of the Social Accounting Matrix (SAM), in our case which is fed by the most recent national accounts (Input-Output Matrix, Supply and Use Table and the integrated economic accounts) published by DANE (Colombian Statistical Entity) for 2018.

There are CGE models developed by some international organizations, World Bank, OECD, WTO at a global level, for Latin America there are the IDB manuals of the IDB of Giordano et al. (2013) and a little more applied to the specific case of climate change of the same IDB of Chisari et al. (2012); documents published by central banks in the world, such as that of the Nicaraguan central bank by Gámez (2018), without counting the infinity of practical applications in different regions of the world, to mention some relevant ones concerning environmental policy and agricultural productivity, such as the work of Pradhan and Ghosh (2019); fiscal policy, the works of Lora (1996), those of García (2007), García (2011) and Ríos Arredondo (2016) for Colombia; on international trade of Dixon et al. (2016) and furthermore, integrating and modeling Melitz of Jafari and Britz (2018), improving calibration in Dixon et al. (2018); the agricultural economy and productivity policies in Shikur (2020), an analysis of this sector for Colombia by Perfetti et al. (2017); more specifics such as the labor market by Botero-García (2012) and a study of gender in productivity labor market by Zidouemba (2020)); the design of policies in Colombia by Suescún et al. (2017); heterogeneous firms by Bekkers and Francois (2018); infrastructure by Montaud et al. (2020); an analysis to decrease oil dependence from Peter Mgeni et al. (2019); Foreign Direct Investment from Latorre et al. (2018); productivity growth at the regional level and pollution Cui et al. (2017); tourism and construction policies Meng and Siriwardana (2017); among others; but in general terms, they show how the complexity and calibration have improved substantially, allowing a better analysis of the real economy and its impacts with increasingly reliable quantifiable results.

Currently, there has been a strong resurgence of what has been called the new generation of CGE models worldwide, taking into account the enormous growth, allowing the transition to more modern models that are considerably superior to the traditional ORANI (generic model for a country) and those published by GTAP, such as those involving Bayesian calibration (Go et al., 2015), behavior economics (Ahmed et al., 2018), advanced Melitz calibration (Dixon et al., 2018), which integrate micro simulations as presented in several papers in the recent book published at the end of 2018 entitled "The New Generation of Computable General Equilibrium Models: Modeling the Economy.$^{\text{ed}}$ited by Perali and Scandizzo (2018). It is important to highlight the enormous growth of artificial intelligence that will take greater strength in economics in the coming years when this type of modeling is integrated and complemented, hence the importance of starting to create links as we will try to do in the paper. To recapitulate, all this CGE hype is due to the multiplicity of advantages compared to existing economic modeling, in simple terms, they are flexible, versatile, practical and easy to understand models that allow synthesizing the real economy in computational terms.

After this summary and exciting tour of CGE modeling, the most relevant aspects of how productivity analysis has been conceptually approached in the literature and its respective integration with general equilibrium modeling are presented. First, from the perspective of the approach that refers to productivity as seen from the entry and exit of firms, according to the World Bank's productivity project it has to do with "the entry of more productive firms and the exit of less productive firms in a period of normal economic activity is an important contributor to productivity growth", and it is an approach that has gained popularity in recent years. The pioneering work entitled "Productivity Dynamics in Manufacturing Plants"by Baily et al. (1992) which is based on the premise of an oligopolistic market structure, which largely explains the productivity gap between firms from a purely static view, although the seminal work entitled "The dynamics of productivity in the telecommunications equipment industry"by Olley and Pakes (1992) which analyzes plant level productivity in a specific industry is also often identified. Several years had to pass until the work of Levinsohn and Petrin (1999) which involves a study of the dynamics of productivity, dabbling in modern methodological techniques and the dynamic part, now, following Olley's productivity study, the work of (Melitz and Polanec, 2015) performs an extension of the decomposition of productivity. Finally, it is important to highlight in recent years the project on productivity as a mission of the World Bank, where two manuals have been published by the authors Grover Goswami et al. (2019) and Cusolito and Maloney (2018) that incorporate the experience of developed and some emerging economies in terms of combating the phenomenon of stagnation of productivity at the global level and the new trends and their sources. Preliminarily, this is the most attractive approach to analyze productivity from the firm and to achieve .endogeneity.of this variable in a CGE context with imperfect competition and heterogeneous firms, i.e., the incorporation of the Melitz model to our CGE.

Secondly, with regard to aggregate productivity, the first works that attempt to approach a conceptual definition are those of Foster et al. (2001), and later, more recent theoretical developments are presented, such as those of the theory Nübler (2014). However, it is important to insist that currently there is a slowdown in productivity growth as shown in Figures 1, 2 and 3. Being a worldwide phenomenon, in recent years the World Bank has joined efforts and published works that, either by means of a transversal analysis of productivity, such as the work of Cusolito and Maloney (2018) or the analysis of aggregate productivity from the point of view of innovation with the work of Cirera and Maloney (2017) broaden the understanding of the problem accompanied by an analysis of innovation in emerging countries.

Considering the works involving CGE, there have been articles incorporating imperfect competition of the labor market and the relationship with the specific factor of labor productivity in the work of Flaig et al. (2013),and following the line of the labor market in CGE but incorporating foreign trade through trade policies to understand the behavior of wage productivity through simulation, there is the work of Ciuriak and Xiao (2016). There have also been works from the perspective of human capital, where, based on productive transformation, the aspect of knowledge and its fields are dealt with, specifically in the article developed by Balduzzi and Rostan (2016). Finalmente, y teniendo en cuenta análisis recientes, se encuentran los CGE desarrollados con metodologías como la micro simulación, como el de Pradhan and Ghosh (2019) for an economy that has presented a high growth average in the last decade. Another type of methodology that is being used although it has not yet ventured into productivity analysis has to do with a heuristic method (Yamazaki et al., 2018). Hence the importance of making a model with these integrated characteristics for an emerging economy like Colombia, of great interest in the economic agenda in post-pandemic times and to define the course of economic activity in the next decade.

Now, from the perspective of productivity among firms, the seminal work is entitled: ORANI, a multi-sectorial model of the Australian economy by Dixon (1982), being a significant contribution to the economy. ORANI models have been widely used in terms of productivity. Subsequently, the work of Challies and Murray

(2006) appears, which disaggregates the analysis with a comparative analysis of productive transformation but from a regional perspective. In recent years, research on sectoral productivity has increased considerably with the work of Sun and Anwar (2019) where the steel industry is analyzed. What is important in this work is to analyze the effects of FDI and inter-firm productivity returns taking into account research and development. Work like that of Yamazaki et al. (2018) incorporates calculations of elasticities recursively, additionally planning a dynamic model that would be helpful for policy forecasting for more years, combining to some extent the firm entry and exit approach. Finally, the work of Boratyński et al. (2019) incorporating a CGE that analyzes wage cost effects and aggregate productivity improvement in countries (Poland, Romania, Czech Republic and Hungary) with the tradable and non-tradable goods model as its axis. Undoubtedly, considerable progress has been made, especially in recent years, due to the growing importance of sectoral and sector-specific productivity studies.

Finally, and this is another point I was trying to get at, CGE modeling requires large amounts of detailed sector and product level data to simulate an economy. Although the tables that the SAM is fed provide a large amount of data for CGE modeling, other data are also needed. For example, a CGE model needs numerous behavioral parameters such as elasticities of substitution, expenditure elasticities, demand and export elasticities, and various share values, macro time series (oil prices, coal) for economic projection of key variables. Data collection for a CGE model is a labor-intensive task and increases the cost of modeling. The extensive data requirements of CGE modeling increase both the cost of modeling and the possibility of having to make assumptions, hence the need to introduce modern techniques into the calibration taking advantage of the impressive growth of "Big Data.ªnd machine learning models.

In this sense, in the emerging literature, there is a growing interest in integrating this type of techniques to significantly reduce prediction errors, calibration of large models such as our CGE that allows closing part of the lags in the publication of macro information, taking advantage of the high volumes of "Big Data", in this line of immediate prediction is putting all its efforts in the work of the International Monetary Fund (IMF) as those of Bolhuis and Rayner (2020), those of Tiffin (2016), also the Atlanta Federal Reserve with its GDPNOW, the New York Federal Reserve with its Nowcasting Report, and in the Colombian case, entities such as Bancolombia and the Grupo Coyuntura de la Universidad EAFIT with its NowCast techniques, the former from daily bank transfers and the latter with more than 80 predictors, and the Banco de la República from the daily frequency payment record, all with the purpose of predicting economic variables in the most timely manner and thus obtain more reliable results of the exogenous time series that feed the CGE model.

Within the new generation of CGE, there is a paper in the literature for an African economy entitled .ᴬ CGE Model for Productivity and Investment in Kenya"by Scandizzo et al. (2018), where they develop a model for the agricultural sector evaluating the impacts on economic growth, sustainability and food security in that region; the study by Kilimani et al. (2018) applies productivity shocks to agricultural industries, after which the resulting impacts on the rest of the sectors of the economy are obtained, and the study by Das (2019) implements a CGE to assess productivity spillovers.

In summary, taking into account the interesting versatility of the CGE modeling type, the fundamental objective refers to the exploration and simulation of some of the sources of agricultural productivity growth, specifically what has to do with heterogeneous firms by means of Melitz's model[6] which involves the sources of efficiency improvements and value creation, as well as being part of the resource misallocation approach; other sources of growth such as externalities will not be investigated, since the discussion and evaluation for the country of infrastructure, public-private partnerships and productivity are found extensively in our work of García et al. (2020); nor the structural transformation including economic complexity is beyond the scope of the research, but a brief module on human capital is incorporated. Productivity analysis is one of the most relevant topics on the country's economic agenda and is The analysis of productivity is one of the most relevant topics in the economic agenda of the country and the world, and in recent decades it has stagnated in both advanced and emerging economies (see Figure 1). In the regional case, as we have seen, it has stagnated in the last 40 years (see Figure 2), and in the country its contribution has been practically null in economic growth (see Figure 3), therefore, it is essential to design and analyze public development policies that boost productivity in a sustained manner, allowing an increase in per capita income to alleviate poverty in part, and for a better economic performance of the country, especially in times of crisis.

3. Description of the CGE model on agricultural productivity

3.1. General structure

The CGE model[7] it has become an important tool used by multilateral entities such as the World Trade Organization (WTO), the Organization for Economic Cooperation and Development (OECD) and the World Bank to measure the impact of shocks or policy changes (Meng and Siriwardana, 2016). A CGE[8] is a type of economic model that can reveal information about the entire economy and about detailed industries. A CGE simulation usually starts with a general equilibrium situation. A shock (sources of agricultural productivity growth) is introduced and the CGE model can generate a new equilibrium situation.

The CGE model structure is actually a system of equations that mimic the economic interconnectedness in a real economy. A CGE database includes all the data that must be fed into the CGE model structure to obtain simulation results (Meng and Siriwardana, 2017). Because of the versatility and the different ways in

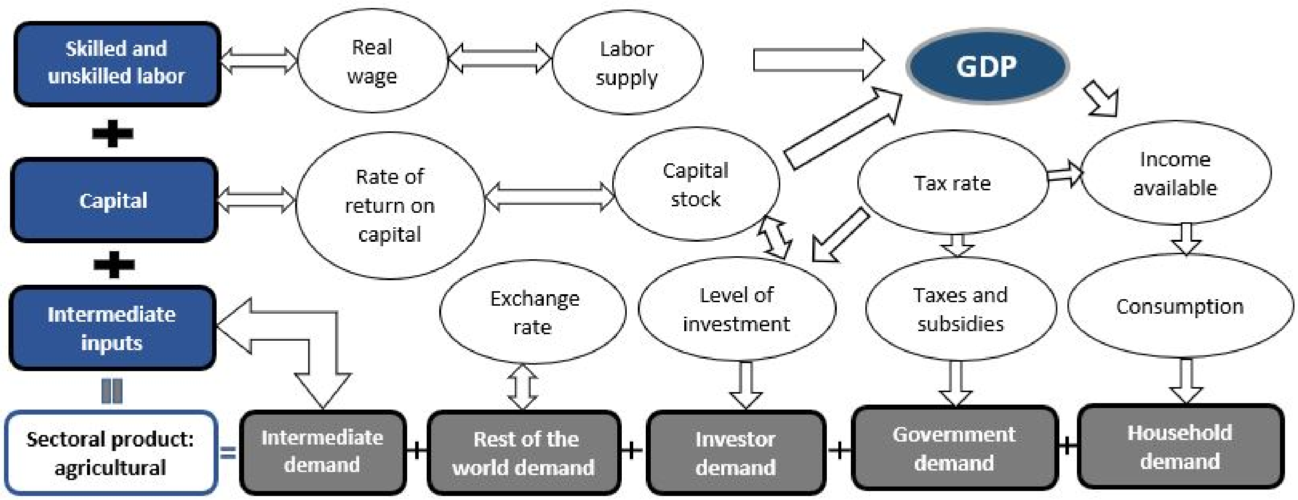
[6]The basic reference of this type of modeling, which can be called "heterogeneous agents"modeling, is Krugman (1980), Melitz (2003) y Zhai (2008).

[7]The term 'computable' in CGE modeling indicates that it is capable of quantifying the effect of a shock, hence the importance of its use in our particular analysis (Hosoe et al., 2010)

[8]The term 'computable' also implies that a computer is involved in the CGE modeling, in this case we will make use of the GAMS (General Algebraic Modeling System) program, one of the most widely used in the empirical literature.

which issues can be addressed in CGE models, applications have been made that transcend exclusively macro issues, for example, the case involving competitive policy issues, the basic reference continues to be (Devarajan and Rodrik, 1991). It is worth mentioning that the interest in integration with economic complexity issues, as addressed byr (Hidalgo and Hausmann, 2009); still persists; it is worth adding that there are books dedicated exclusively to it, such as the one written by Hosoe et al. (2010) entitled "Textbook of computable general equilibrium modeling: programming and simulations", even has a second version of the year 2016, and with a greater level of detail in a compendium of Dixon and Jorgenson (2012), developed by a joint work in the two volumes. Figure 5 below captures the essence of a CGE.

Figura 5: **An illustration of the economic system in a CGE model**



Source: Own elaboration adapted from Hosoe et al. (2010) and Dixon and Jorgenson (2012).

The three key elements on the left side of Figure 5, i.e., skilled and unskilled labor, capital and intermediate inputs, are the resources (or inputs) to produce a good or service (i.e., output). In an equilibrium, this good or service must be purchased by different users (demands), e.g., intermediate demand, rest of the world demand, investor demand, government demand, and household demand. These different users (demands) are the key elements of the economic system. The system represents the circular flow of the economy.

Thus, the EAFIT modeling group's general equilibrium model will be reviewed and updated, in which recent research has allowed refining its calibration, generating the possibility of using the model for medium and long term simulations, in addition to the traditional comparative statics exercises, for which it had already been enabled. The model was used to evaluate policies aimed at promoting the bioeconomy, in simulations with a time horizon of 15 years. It was also used to evaluate labor informality in Colombia in Botero-García (2012) for which reason we do not go into this point in more detail. An example of comparative statics, on the other hand, is the use of the model to evaluate the agricultural protection policy, in Perfetti et al. (2017). Currently, it is used for the evaluation of the 4G road program by means of a dynamic recursive model, which explains in detail the functioning of the Colombian economy, incorporating 12 productive[9], 17 basic types of products[10], 16 commercial allies also under Melitz type modeling, that is, with heterogeneous companies and monopolistic competition and three types of productive resources: skilled labor, unskilled labor (formal labor and informal labor) and capital that is specified in García et al. (2020).The model considers formal and informal markets, taking into account a fundamental characteristic of the Colombian economy García (2011), and one of its distinctive novelties lies in the inclusion of 20 types of households, which allows for distributive analysis. For all of the above, the CGE presents the structure of Colombia's real economy in computational terms. Now, the challenge is to adapt it to the needs of the analysis of issues related to the sources of productivity growth, fundamentally that which has to do with the misallocation of resources through the use of the Melitz model for heterogeneous firms and its effect on the growth of the Colombian economy in the next decade together with its also novel implementation of the fiscal rule, accompanied by a calibration and estimation of the exogenous time series through automatic learning that provides greater reliability to its results.

Thus, our computable general equilibrium model, in summary, presents the structure of the Colombian economy in computational terms. It is a recursive dynamic model (dynamics is achieved through period-by-period simulation, in our case year by year from the SAM calibrated to 2018), with 20 types of households, formal and informal labor (skilled and unskilled) and capital as productive factors, foreign trade modeled to Melitz type[11] which details its mathematical development in Annex1, has a quasi-ideal demand system proposed by Deaton and Muellbauer (1980) known as the AIDS which specifies its mathematical development in Annex2 and finally with Central National Government. Capital dynamics is one of the key driving forces in the model. The above and the large database used, makes it a model that includes detailed information at the micro level of an industry, as detailed in the model equations in Annex 3 of the mathematical appendix.

Our recursive dynamic model that is calibrated with artificial intelligence as the other distinctive novelty from the social accounting matrix (SAM) 2018. Now, the programming elements of the model in the GAMS language are separated into three major groups. First, a code that solves the model corresponding to the intra-period, i.e., this agglomerate of codes forms the static version of the model in 2018. Secondly, the codes of the inter-period module are shown which add to the previous one the 2018 SAM update to estimate the model up to the most current year (2020) taking into account an exogenous shock how the new corona virus is taken and the strong ravages on the fiscal accounts. Finally, the third code makes the projections starting from the inter-period module to shape the dynamics of the model up to the year 2040, although our interest will only be up to 2030.

[9]The 12 standard productive branches of the classification in the DANE national accounts.

[10]Agriculture, mining, oil, food, light, heavy, energy, inter, construction, commerce, transport, tics, finance, real estate, government services, health and the rest of services

[11]The "Melitz"model assumes conditions of monopolistic competition in which firms produce a variety of goods and draw their productivity from a fixed probability distribution function. There are fixed production costs and fixed and variable entry costs in export markets, therefore, the firm's productivity and the expected probability of entering the foreign market are positively related. See Melitz (2003).

The structure of our CGE model is actually a system of equations that mimic the economic interconnectedness in a real economy. The model uses combinations of functions as shown in Figure 7 which can be CES (Constant Elasticity of Substitution), CET (Constant Elasticity of Transformation), Cobb-Douglas, Leontief, which describe technology for producers and preferences for consumers, and input and consumption demands are derived from the optimization process.

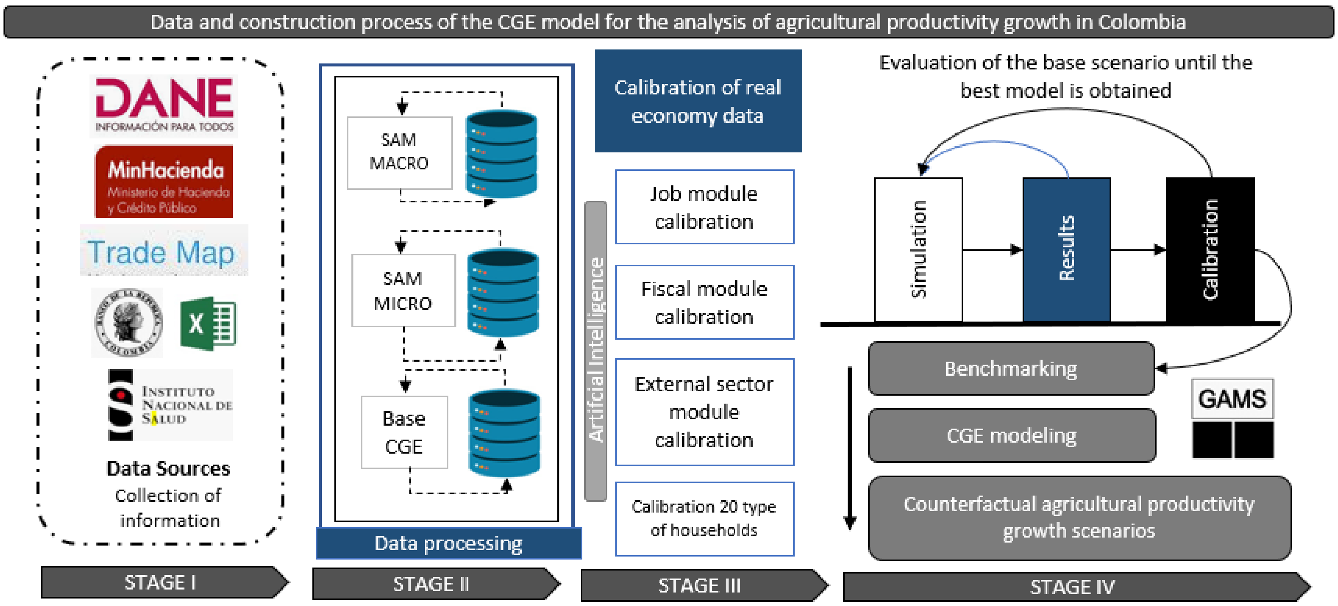
3.2. Data for the CGE model of agricultural productivity

The data base for a CGE model consists of two parts: parameter values and income and expenditure flows in an economy, as illustrated in Figure 6. For calibration, the first step is to collect data from the national accounts, mainly from the Supply Matrix (Input-Output), Utilization Table, Integrated Economic Accounts (CEI), and GDP series, both from the supply and demand side, all published by DANE.

Secondly, for foreign trade, the Balance of Payments and terms of trade data published by 'Banco de la República' are important. Since there is a Melitz-type module (monopolistic competition, differentiated productivity and heterogeneous agents), it is necessary to download the country's trade links with the rest of the world, recorded in the TRADE MAP platform, which is obtained at a 4-digit disaggregated level, and compacted for the main 16 trading partners.

Additionally, in order to analyze the intervention of the National Government and more in times of pandemic, knowing the quite important fiscal effort in social programs, it is necessary to collect figures from the 'Ministerio de Hacienda', both from the Central National Government (CNG) and the Medium Term Fiscal Framework. Finally, the last input is the Gran Encuesta Integrada de Hogares (GEIH), to model the labor market, both skilled and unskilled, salaried and non-salaried employment, and labor indicators for 20 types of households in Colombia, which allows for distributive analysis accompanied by counterfactual scenarios. The process is summarized in Figure 6 from the collection of data from the real economy to the simulations with the base scenario that includes what I consider the quantitative effects of Covid-19 and with the endogenous productivity shock given by the Melitz model that will yield results for all the detailed variables up to the year 2030.

Figure 6: **Simplified diagram of the construction of the CGE for the analysis of agricultural productivity**



Source: Own elaboration.

In summary, it is a dynamic recursive model (the dynamics is achieved through sequential simulation year by year, updating the relevant balances of the stock variables in the model from, in our case, the SAM calibrated to 2018), with 20 types of households, foreign trade modeled 'Melitz', consumption modeled through a linear expenditure system and the expenditures of the Central National Government (GNC) and the rest of the government fixed exogenously. All of the above, added to the broad data base used, makes it an adequate model to analyze in detail the micro-level functioning of the productive branches considered.

In this order of ideas, the CGE model has Melitz-type modeling as the cornerstone of the productivity analysis, with the 16 main trading partners of the Colombian economy: United States, European Union, United Kingdom, Pacific Alliance, Australia, Canada, New Zealand, Singapore, Rest of ASEAN, China, Japan, South Korea, Mercosur, Panama, Turkey and the rest of the world at a 4-digit level of detail thanks to TradeMap (Map, 2017). This implies that it is an open economy model, which can be analyzed from Melitz and Armington. Importantly, it contains Dixit-Stiglitz style monopolistic competition which allows us to find productivity distribution functions being the heart of our analysis, with agriculture the only sector that is not modeled by Mark up. In this case perfect competition is assumed, so its prices are perfectly flexible.

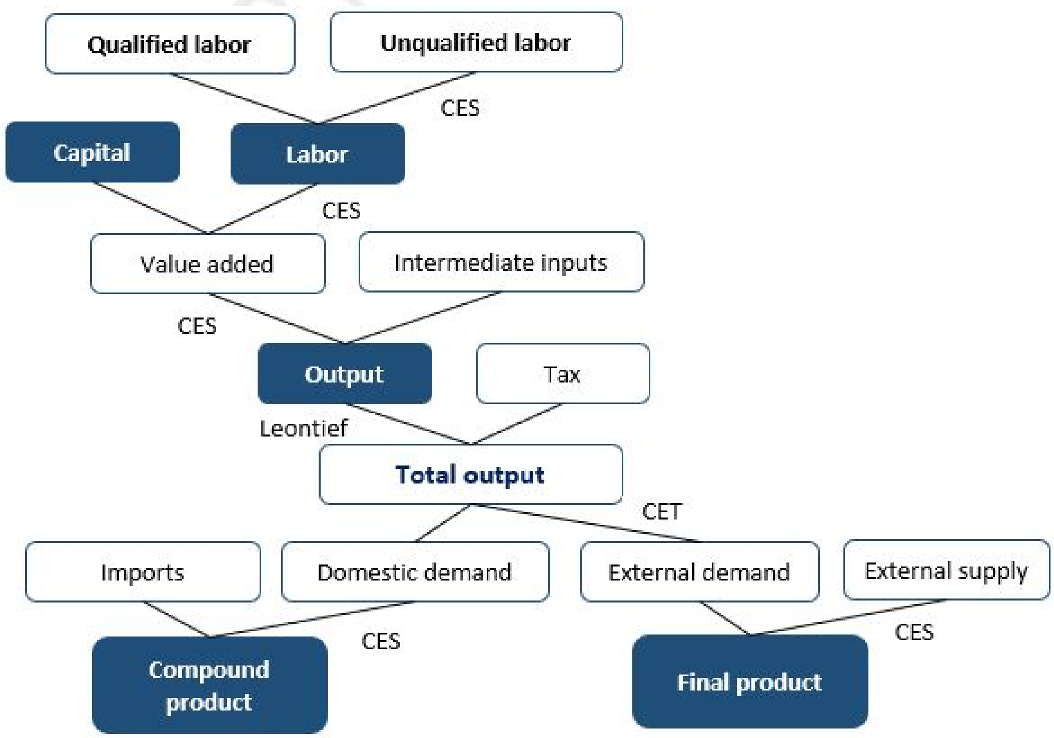
The labor market, which is one of the productive factors, is disaggregated between formal and informal labor, and the latter between skilled and unskilled labor. The latter is paid with mixed income, has a low capital intensity, low productivity and minimum wage regulations do not apply to it (Botero-García, 2012) and (García, 2011). Likewise, the labor supply is distributed between the salaried and informal markets, the former being a rigid price market and the latter a flexible price market. The model's closure takes into account the surplus of the rest of the world and household savings.

Both the theory and application of CGE modeling have grown rapidly, partly as a consequence of the advancement of knowledge in the fields of social accounting, mathematical programming and the development of computational power (Meng and Siriwardana, 2017). With the increasing implementation of this type of modeling, there have been many recent papers that, broadly speaking, show how the complexity and calibration have improved substantially, allowing better analysis of the real economy and its impacts with increasingly reliable quantifiable results.

3.3. CGE modeling for the analysis of productivity growth in the agricultural sector

A CGE model can also be labeled according to a research area, in our case, it is a CGE model for the analysis of agricultural productivity. Its essence is schematized in a simplified form along with its respective mathematical behavior function in Figure 7.

Figure 7: **Simplified CGE scheme for agricultural productivity analysis in Colombia**



Source: Authors' elaboration based on Perfetti et al. (2017) y García et al. (2020).

Model closure consists of choosing a particular set of exogenous variables in a way that allows for a consistent and possibly unique set of solutions. Because a problem of overdetermination may arise when the number of equations implied by the model exceeds the number of endogenous variables, four basic closure rules are handled in the literature: the neoclassical, the Keynesian, the Johansen and the New-Keynesian Scandizzo et al. (2018) the option selected in our model is investment-led, mainly by the installed capacity index. Finally, CGE modeling needs a wide range of detailed sectoral and commodity-level data to simulate an economy. Although a considerable information set described in Figure 6 is available, exogenous time series influencing economic performance is sometimes critical, and a large number of behavioral parameters such as elasticities of substitution, expenditure elasticities, demand elasticities and export elasticities are required. In this sense, data collection for a CGE model is a labor-intensive task and increases the cost of modeling. The extensive data requirements of CGE modeling increase both the cost of modeling and the possibility of having to make assumptions. Hence the importance of calibrating both the key time series and the elasticities of substitution using artificial intelligence to further refine the results and lower modeling costs.

In this order of ideas, the contribution of artificial intelligence has to do with the calibration of relevant parameters (elasticities of substitution), training and improvement in the predictive capacity of the time series of key macro-financial variables in the evolution of economic activity in the next decade, such as the price and quantities produced of Brent oil, the price and quantities produced of coal, remittances and foreign direct investment, that strengthen the results of the computable general equilibrium model, remembering that in this type of modeling the economic agents act in a myopic way, i.e. Machine Learning is used as an ideal complement to solve this problem, but not as a substitute, taking advantage of the fact that this field is growing by leaps and bounds in the improvement of macroeconomic projections competing with traditional econometric models AR, MA, ARIMA, VAR, etc.

4. Results

This section presents the results from a variety of simulations that are performed with the recursive dynamic computed general equilibrium model for the totality of variables and equations in question, and that relate to aspects of how productivity increases and its effects on the path of output over the next decade.

First, a base scenario is simulated that presents the structure of Colombia's real economy in a detailed and computationally organized manner, which considers Covid-19 as a transitory supply shock that triggers demand effects, and the annual impact is constructed from shocks to the installed capacity index, taking into account the time of social isolation experienced by the country. To this is added simultaneously the oil shock in price (annual fall in the price of Brent oil 53.14% according to EIA) and quantities (annual fall in barrel production of 8.6% according to Corficolombia), the expected drop in Foreign Direct Investment (FDI) (10% according to our ML model), remittances (of 20% according to the World Bank) and the increase in public spending (an exogenous increase of 23 trillion according to the MFMP2020) disbursed in the measures implemented by the Government to mitigate and recover economic and social activity from the effects of the pandemic.

Table 1 below shows the results of what we will now call the base scenario, which indicates a drop of -7,68% of GDP in 2020 and a replica in 2019, as well as two variables that will be relevant in view of the strong impact on the labor market and the suspension of the fiscal rule in the country.

Table 1: **Base scenario of the Colombian economy in times of pandemic**

Variable	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
GDP growth	3.30%	-7.68%	4.71%	3.66%	3.47%	3.39%	3.46%	3.49%	3.42%	3.38%
Unemployment rate	9.21%	15.78%	12.15%	12.23%	12.21%	12.20%	12.12%	12.05%	11.99%	11.92%
Fiscal deficit	-2.52%	-6.71%	-4.63%	-3.94%	-3.62%	-3.27%	-2.80%	-2.35%	-1.93%	-1.54%

Source: Calculations with the CGE model in GAMS.

The narrative that supports the calculation of the base scenario is as follows: the spread of the new corona virus affects the management of companies, which are forced to take precautionary measures to avoid infecting their employees, and therefore experience reductions in their production that are reinforced by eventual breaks in supply chains. Given that, in principle, the theoretical production capacity is maintained, the best way to understand the phenomenon is to assimilate it to a shock in the installed capacity index of -12% and to close the model with investment.

These effects are exacerbated by the social distancing and isolation measures taken to contain the virus, to the extent that they limit people's mobility, imposing additional restrictions on business management. These measures have costs in themselves and, therefore, must be accompanied by impact mitigation measures, which make the burden on people bearable, and repair measures, which prevent the destruction of the productive apparatus reflected in the overflow of the fiscal deficit in the model accompanied by an annual drop in the tax rate of 20 %. In line with this, all together they form the base scenario for the analysis of agricultural productivity.

In essence the trajectory of an economy can change in four ways due to productivity issues in our CGE: starting with the most obvious one is that the TFP improves, which in our model is a scale parameter called (BV)[13] of value added of the production function. For the analysis, three sets that exogenously affect the TFP in the model are created as follows: primary sector[14], secondary sector[15] and tertiary sector[16] with the 12 branches of the DANE. In this sense, an exogenous increase of the TFP by sector of 1 % in the Colombian economy is simulated (see Figure 9). The scenario proposed consists of evaluating what would happen if we are able to stop decreasing by -0.6 % which has been the average for the last two decades according to Conference Board data.

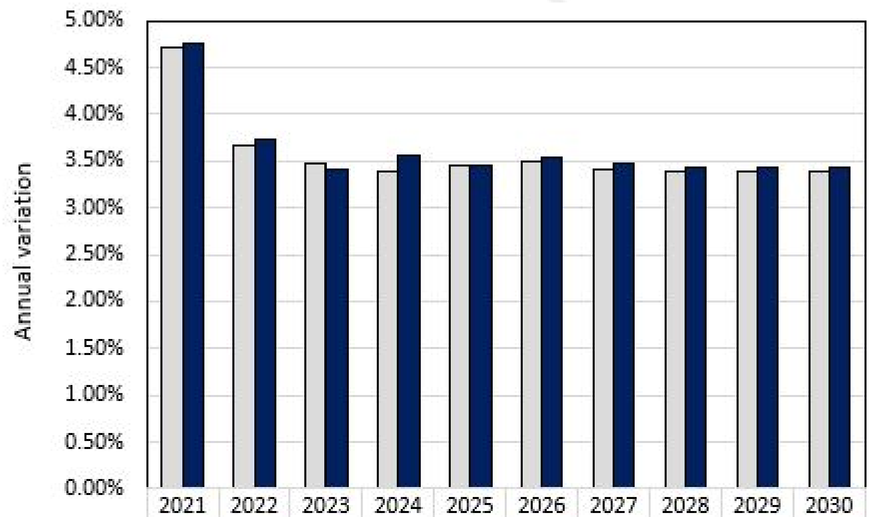
The narrative of these calculations lies in the design and implementation of public policies aimed at each entrepreneur to achieve productivity improvements, i.e., with the mechanisms of productivity school and the State that allows providing modern tools that have grown so much in recent years worldwide (Mazzucato, 2011). In this order of ideas, the second way would be with public development policies that allow a structural transformation and productivity growth, which, as explained above, will not be the object of analysis. The third would undoubtedly be human capital to boost productivity, greater investment in education, skilled labor and calculating how much it costs to educate people. The fourth mechanism can be grouped into the resource misallocation approach, the first being misallocation, i.e., firms are all very different and can gain market share, but if the state protects the bad ones, resources do not flow where they should flow, and the analysis is done by creating distribution functions. For example, there may be distortions in tariffs, taxes, marketing costs and/or capital costs, each of the sources being measurable, however, in practice three are measured and identified that are capable of calculating mis-allocation. Now, the second within this approach is the Melitz modeling which will be our main object of analysis.

Consequently, being the TFP a scale parameter in the production function in our model for each sector, the BV is increased and the results are compared in Figure 9 with the base scenario that includes the expected impacts of the new corona virus in the Colombian economy, therefore, the shock is performed as of 2021. The increase is 1 % of the productivity of the agricultural sector. Figure 8 summarizes the effects of each of the shocks on the future economic growth path. In the first place, the improvement of the counterfactual scenario with respect to the base scenario is evident, being the growth of the TFP exogenously, where on average in the base scenario between 2021-2030 there is a GDP growth of 3,57 % and in order of importance, making the

[13] See Annex 3 with the description of the central equations of the model
[14] Agriculture
[15] Industry, mining, construction and electricity, gas and water.
[16] Real estate, services, administration, financial, training, commerce and transportation.

same calculations, there is an improvement with the increase of the TFP of the agricultural sector with 3,62 %, all average growth rates between 2021-2030. This suggests, at least in terms of GDP growth, where public policy efforts on productivity should be coordinated.

Figure 8: **Agricultural productivity growth scenarios (TFP) by sector in the Colombian economy 2019-2030.**



	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Base scenario (includes covid-19)	4.71%	3.66%	3.47%	3.39%	3.46%	3.49%	3.42%	3.38%	3.38%	3.38%
Productivity growth in the agricultural sector	4.75%	3.74%	3.41%	3.56%	3.46%	3.53%	3.47%	3.43%	3.43%	3.44%

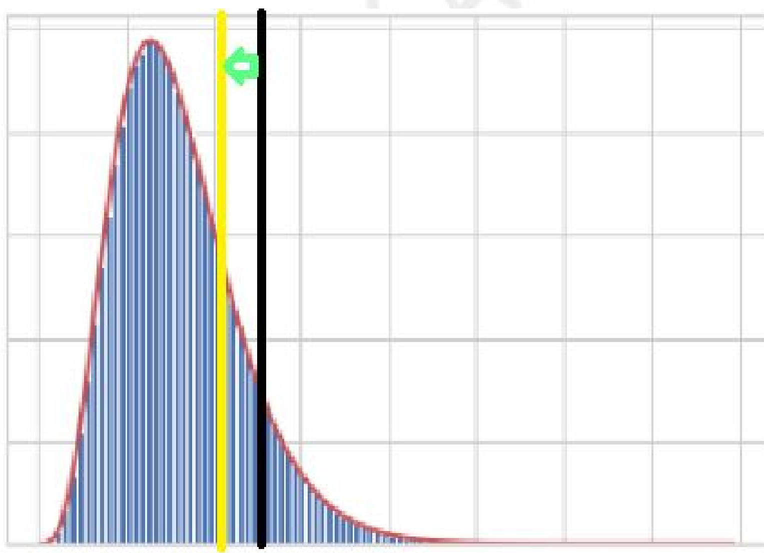
Source: Calculations with the CGE model in GAMS.

However, taking into account that the aim of the paper is to present at least one simulation that presents productivity growth of a more 'endogenous' nature, we will make use in this section of the Melitz (2003) con adaptaciones en lo que se refiere al modelado de Dixon et al. (2016) y Dixon et al. (2018) with adaptations to the modeling of the modern and trade models described extensively in Annex 1. In summary, the Melitz model incorporated in our CGE assumes conditions of monopolistic competition in which firms produce a variety of goods and draw their productivity from a fixed distribution. There are fixed production costs and fixed and variable entry costs in export markets, therefore, the productivity of the firm and the expected probability of entering the foreign market are positively related[10].The Melitz version is based on the Pareto specification being the distribution of productivities. Our Melitz firms face a fixed cost for each market in which they sell. Consequently, they may sell in some markets but not others, depending on whether or not they can find a price/quantity combination that generates a sufficient margin over variable costs to cover market-specific fixed costs (see Figure 9). For some companies there may be no markets in which they can cover market-specific fixed costs (Dixon et al., 2018).

[10]Further details along with the description of the variables in the mathematical appendix can be found in section 1 of the mathematical appendix at the end of the paper.

In this order of ideas, three simulations that endogenize productivity for the Colombian economy will be carried out using in the first instance the 'iceberg' costs or marketing costs, the average productivity of the Pareto productivity distribution and the lower limit of this distribution to evaluate the counterfactual states with respect to the base scenario for economic activity in the coming years. In order to provide a graphic idea for further illustration, Figure 9 is presented. Thus, there is the famous cost called 'iceberg cost' that limits entry (the red line) and only the companies after the black line can export, i.e., in the domestic markets all the companies in the distribution participate, but in the external market only the most productive companies after the red line participate. In this case, productive firms will enter the market in the external sector, which generates an endogenous productivity change.

Figure 9: **Pareto probability distribution of productive enterprises Melitz model**



Source: Own elaboration adapted from Melitz (2003).

In summary, what is being done in the model is to move from the black line to the yellow line, with the fall of the iceberg cost, therefore, all the companies that are in the green arrow will be able to export, this implies that there is a productivity effect in the country, but in the total production block the companies (between the yellow and black line) are gaining participation, which in the model would be giving us an endogenous productivity effect, consequently, there is a greater economic growth, as markets are opened to the good companies, with the fall of the marketing cost threshold and thus the aggregate productivity improves. It is important to note that in our CGE the external sector incorporates the new theory of international trade, i.e., that only the good ones sell abroad having as a set of choice the 16 trading partners described (it may be due to faster procedures, reduction of administrative obstacles and/or the installation of an office in the 16 trading partners of the model that represent the Colombians improving processes), opening the markets to the most productive ones due to their imperfect competition nature.

5. Conclusions

Agricultural productivity in Colombia and globally has stagnated in recent decades. It is a phenomenon that does not seem to show signs of increasing disruptive changes and that is foreseen by the economic and social effects of the pandemic, which highlights the role of the economic authorities together with the business sector. To generate the desired rebound in productivity growth, a set of coordinated and permanent actions involving the national government, the private sector and the scientific and academic community is required. It is a commitment that will be sustained in the long term, when all the agents involved are directed towards the same goal of productivity growth, always keeping in mind that it is the essence of sustained economic growth in any economy and entails improvements for all. Being such a complex phenomenon, it is not so easy to approach it from a single perspective, therefore, this paper provided a more global and relevant vision for the achievement of the objective and scenario analyses, considering current and future challenges and proposals, which were mostly inspired by a framework of heterogeneous agents and imperfect competition.

One of the first reflections is that although there is still a long way to go to understand in a more complete way the determinants of productivity in Colombia, this document serves to fill part of that gap, and it is conclusive that it is more important to join efforts in the challenges we have in competitiveness in a post-pandemic context and we must not miss this unique opportunity in history that will mark our entire generation, for important changes such as the gradual decrease of labor informality, the productive diversification of the export profile towards products and services with high added value and greater growth and profitability for their sustainability, with sophisticated value chains that allow greater productive linkages and joining efforts for a redirection of resources to education as the support of any productive transformation, since, finally, knowledge is technology, the generation of knowledge is the basis for the growth of innovation and productivity.

Showing one of them. The first is by creating a set that exogenously affects the TFP of the agricultural sector. In this sense, an exogenous increase of the TFP of the agricultural sector of 1 % in the Colombian economy is simulated. The narrative of these calculations, lies in the design and implementation of public policies aimed at each entrepreneur to achieve productivity improvements, that is, with the mechanisms of productivity school and the State that allows to provide modern tools that have grown so much in recent years worldwide. There is an improvement with respect to the base scenario, where the average in the base scenario between 2021-2030 (in 2020, a drop of -7.3 % is estimated in all scenarios as a result of Covid-19), GDP growth will be in the order of 3.57 %. Making the same calculations, but including the impact on the agricultural sector, average growth rates between 2021-2030 would be in the order of 3.62 %. This suggests that, at least in terms of GDP growth, public policy efforts to increase productivity in this sector must be coordinated.

Another point to note is that the CGE model improves significantly with calibration on the parameters and exogenous macro series integrated with Machine Learning, suggesting that machine learning techniques could be an important part of the emerging countries macro forecasting toolkit and an excellent complement to computable models.

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6. Mathematical Annex: Specification of the CGE Model

This appendix provides the mathematical details necessary to understand the computable general equilibrium (CGE) model.

6.1. Appendix 1: Melitz Module (MM)

We assume when it is convenient that the possible productivity values of firms are continuous, rather than discrete. Following the CGE convention of Dixon et al. (2016) and Dixon et al. (2018), we assume that productivity values in the Colombian economy form a Pareto distribution:

$$g_s(\Phi) = \alpha \Phi^{-\alpha-1}; \Phi \geq 1 \tag{1}$$

Where α is a positive parameter. In (1), we assume that the lowest potential productivity value is 1. This assumption can be made without loss of generality through an appropriate choice of units for labor. From (1) we obtain:

$$\int_{\Phi_{min}}^{\infty} g_s(\Phi)\, d(\Phi) = (\Phi)_{min}^{-\alpha} \tag{2}$$

(2) It means that the proportion of productivity values in countries that are greater than any given level, Φ_{min} , es $(\Phi_{min}^{-\alpha})$. Therefore, the proportion of firms in country s with a productivity of at least $\Phi_{min(s,d)}$, i.e,the proportion of firms $(\frac{N_{sd}}{N_s})$ operating in the link is $(\Phi_{min(s,d)}^{-\alpha})$.
The following equation justifies the Melitz version, being $S(s,d) = \left\{k|\Phi_k \geq \Phi_{min(s,d)}\right\}$: All firms.

$$N_{sd} = N_s * \Phi_{min(s,d)}^{-\alpha}; (0)$$

Moreover, it is essential to define the productivity of each firm in a Dixit-Stiglitz fashion, which implies monopolistic competition:

$$\Phi_{s,d} = \left[\sum_{k\varepsilon S(s,d)} \Phi_k^{\sigma-1} \frac{N_s g_s(\Phi_k)}{N_{sd}}\right]^{\frac{1}{1-\sigma}}; (a)$$

Next, we apply (1) and (0) in a continuous version of (a). We obtain:

$$\Phi_{s,d}^{\sigma-1} = \int_{\Phi_{min(s,d)}^{\alpha}}^{\infty} \alpha\Phi^{-\alpha-1}\Phi^{\sigma-1}d(\Phi) \tag{3}$$

Simplified:

$$\Phi_{s,d}^{\sigma-1} = \left(\frac{\alpha}{\alpha-(\sigma-1)}\right)\Phi_{min(s,d)}^{\sigma-1} \tag{4}$$

Deriving, the following condition is assumed:

$$\alpha > (\sigma-1) \tag{5}$$

This has no obvious economic interpretation. However, without it, the integral of equation (3) is unbounded. From (4), we obtain:

$$\Phi_{s,d} = \beta\Phi_{min(s,d)} \tag{6}$$

Being:

$$\beta = \left(\frac{\alpha}{\alpha-(\sigma-1)}\right)^{1/(\sigma-1)} \tag{7}$$

In this way we proceed to the derivation of the Melitz version itself. From the AKME version of (T2.1), we can see that the price ratio for either firm is the ratio of their productivities raised to the power -1. Applying this idea in the AKME version (Armington, Krugman and Melitz as special cases of an Envelope):

$$\rho_d^{1-\sigma} = \sum_s \sum_{k\varepsilon S(s,d)} N_s\alpha\Phi_k^{-\alpha-1}\delta_\sigma^{s,d}\rho_{s,d}^{1-\sigma}\left(\frac{\Phi_k}{\Phi_{s,d}}\right)^{\sigma-1} \tag{8}$$

Which can be rewritten in continuous format as:

$$\rho_d^{1-\sigma} = \sum_s \left(N_s\alpha\Phi_{sd}^{1-\sigma}\right)\delta_\sigma^{s,d}\rho_{s,d}^{1-\sigma}\int_{\Phi_{min(s,d)}^{\alpha}}^{\infty}\alpha\Phi^{-\alpha-1}\Phi^{\sigma-1}d(\Phi) \tag{9}$$

Applying (3) we have:

$$\rho_d^{1-\sigma} = \sum_s (N_s\alpha\Phi_{sd}^{1-\sigma}\rho_{sd}^{1-\sigma}\delta_{sd}^{\sigma}\Phi_{min(s,d)}^{\sigma-1}\Phi_{s,d}^{\sigma-1}) \tag{10}$$

Via the Melitz version of (0), (10) reduces to:

$$\rho_d^{1-\sigma} = \sum_s (N_s\alpha\rho_{sd}^{1-\sigma}\delta_{sd}^{\sigma}) \tag{11}$$

This leads to the original Melitz version. The starting point for deriving the Melitz version is the AKME version. With γ_{ksd} equal to 1, we write this as:

$$Q_{sd}^{(\sigma-1/\sigma)} = \sum_s (N_s g_s\left(\Phi_k Q_{ksd}^{(\sigma-1)/\sigma}\right)) \tag{12}$$

Then continue assuming that $\gamma_{ksd} = 1$, we use the AKME versions together with (1) to obtain:

$$Q_{sd}^{(\sigma-1/)\sigma} = \sum_{k\varepsilon S(s,d)} N_s \alpha \left(\Phi_k^{-\alpha-1}\right) \left(\frac{\Phi_k}{\Phi_{s,d}}\right)^{\sigma-1} Q_{sd}^{(\sigma-1)/\sigma} \tag{13}$$

i.e,

$$Q_{sd}^{(\sigma-1)/\sigma} = N_s \Phi_{min(s,d)}^{-\sigma} Q_{sd}^{(\sigma-1)/\sigma} \Phi_{s,d}^{(\sigma-1)/\sigma} \sum_{k\varepsilon S(s,d)} \Phi_{min(s,d)}^{\alpha} \alpha \Phi_k^{-\alpha-1} \Phi^{\sigma-1} \tag{14}$$

Integrating (3) and (4) simplifies to:

$$Q_{sd}^{\frac{\sigma-1}{\sigma}} = N_s \Phi_{min(s,d)}^{-\sigma} Q_{s,d}^{\frac{\sigma-1}{\sigma}} \tag{15}$$

The version Melitz (2003) provides a further simplification, leading to the general Melitz version. To derive the general Melitz version, we begin by adopting the Krugman / Melitz assumption that equals -. Then, using the AKME versions together with (1), (2) and (0) we obtain:

$$\pi_{total_s} = \frac{1}{\sigma} \sum_d \sum_{k\varepsilon S(s,d)} N_s \alpha \Phi_k^{-\alpha-1} \left(\frac{\rho_{ksd}}{T_{ksd}}\right) Q_{ksd} - \sum_d N_{sd} F_{sd} W_s - N_{sd} F_{sd} W_s - N_s H_s W_s \tag{16}$$

A través de las versiones AKME y (6) se convierte en:

$$\pi_{total_s} = \frac{1}{\sigma} \sum_d \sum_{k\varepsilon S(s,d)} N_s \alpha \Phi_{sd}^{1-\sigma} \Phi_k^{\alpha-1} \Phi_k^{-\alpha-1} \left(\frac{\rho_{ksd}}{T_{ksd}}\right) Q_{ksd} - \sum_d N_{sd} F_{sd} W_s - N_{sd} F_{sd} W_s - N_s H_s W_s \tag{17}$$

Aplicando (3) y (0), simplificamos (17) a:

$$\pi_{total_s} = \frac{1}{\sigma} \sum_d N_{sd} \left(\frac{\rho_{sd}}{T_{sd}}\right) Q_{sd} - \sum_d N_{sd} F_{sd} W_s - N_{sd} F_{sd} W_s - N_s H_s W_s \tag{18}$$

The final task to justify the elimination of the firm dimension of the Melitz equations is to derive the general Melitz version. Apply (3) and (1) to the first term in the AKME version and simplify the second term finally found:

$$L_s = \sum_s \sum_{k\varepsilon S(s,d)} \alpha \Phi_k^{-\alpha-1} \Phi^{\sigma-1} N_{sd} \left(\frac{Q_{sd}}{\Phi_{sd}}\right)^{\sigma} + \sum_d N_{sd} F_{sd} - N_s H_s \tag{19}$$

Then, via (3) and (0), (19) reduces to the versión Melitz (2003) of the type Dixon et al. (2018).

6.2. Annex 2: Near-ideal demand system (AIDS))

Following Deaton and Muellbauer (1980),specifically, the function to be estimated is a system of equations, summarized in the following expression:

$$wsi_{it} = \propto si_i + \sum_{j=1}^{N} \gamma si_{ij} lnp_{ij} + \beta si_i ln\left(\frac{Xsi_t}{Psi_t}\right) + e_{ij}; Para\ i = 1,2,..N \tag{20}$$

Where wsi_{it} are the shares of the different goods in the total expenditure, Psi_t are the prices of the goods within the consumption basket, $\left(\frac{Xsi_t}{Psi_t}\right)$ is the total real consumer spending, βsi_t are the coefficients associated with spending, γsi_{ij} are the coefficients on prices, $_{ij}$ are the individual effects assigned to each share, and ξ_{ij} are the stochastic shocks. Furthermore Psi_t is a price index that is a function of the price of the analyzed goods and is expressed as:

$$lnP_t = \propto si_0 + \sum_{j=1}^{N} a_j lnp_j + \frac{1}{2} \sum_{I=1}^{N} \sum_{j=1}^{N} b_{ij} lnp_i lnp_j \tag{21}$$

To simplify the expression, Deaton and Muellbauer (1980) propose to approximate P_t from Stone's price index (Stone, 1953)[11] :

$$lnP_t^s = \sum_{i=1}^{N} wsi_{it} lnp_{ij} \tag{22}$$

6.3. Annex 3: Central equations of the model

Labor market block

Aggregation of unqualified jobs

$$TTA_{II} = TNCA_{II} + TC_{II} \tag{23}$$

Labor aggregation path:

$$\frac{TNCA_{II}}{TC_{II}} = K_{TT} * \left(\frac{WC_{II} * \delta_{AII}}{WNCA_{II} * (1 - \delta_{AII})}\right)^{\sigma_{AII}} \tag{24}$$

Formal market average price:

$$TTA_{II} * WA_{II} = TNCA_{II} * WNCA_{II} + TC_{II} * WC_{II} \tag{25}$$

Labor and capital aggregation:

$$VA_{II} = \left(\frac{k_{tp}}{k_{po}}\right)^{\varepsilon_{kp}} BVA_{II} * \left[\left(\delta_{VA(II)} * TTA_{II}\right)^{\frac{\delta_{VAII}-1}{\delta_{VAII}}} + (1 - \delta_{VAII}) * K_{II}^{\frac{\delta_{VAII}-1}{\delta_{VAII}}}\right]^{\frac{\delta_{VAII}}{\delta_{VAII}-1}} \tag{26}$$

Value added expansion path:

$$g_s(\Phi) = \alpha \Phi^{-\alpha-1}; \Phi \geq 1 \frac{TT_{II}}{K_{II}} = KCT_{II} * \left(\frac{R_{II} * \delta_{VAII}}{W_{AII} * (1 - \delta_{VAII})}\right)^{\sigma_{VAII}} \tag{27}$$

Identifying value added:

$$PVAA_{II} * VAA_{II} = TTA_{II} * WA_{II} + K_{II} * R_{II} \tag{28}$$

[11] This methodology is known as the LA-AIDS model since it calculates, prior to estimation, the price index from a linear approximation of the AIDS model.

Informal sector block

Informal sector:

$$VAI_{II} = KVAI_{II} * TNCI_{II} \tag{29}$$

Valor agregado total:

$$PVAA_{II} * VAA_{II} = TTA_{II} * WA_{II} + K_{II} * R_{II} \tag{30}$$

Modelando la competencia monpolística:

$$\lambda * A_{ICM} * Y_{ICM} = RPUA_{ICM} * K_{ICM} + WA_{ICM} * TTA_{ICM} + YA_{ICM} * \sum_{i=1}^{n} IO_{ICM} * PXC_{I} \tag{31}$$

Producción total independiente:

$$PYA_{ICM} = (1 + MARKUPA_{ICM})\lambda A_{ICM} \tag{32}$$

Beneficio de capital unitario:

$$GANA_{ICM} = PYA_{ICM} - \lambda_{ICM} * YA_{ICM} \tag{33}$$

Remuneración capital básica:

$$RCMA_{ICM} = \frac{GANA_{ICM}}{K_{ICM}} \tag{34}$$

Ganancias de capital:

$$R_{ICM} = RCMA_{ICM} + RPUA_{ICM} \tag{35}$$

Block of imperfect competition

Production of sectors with monopolistic competition:

$$PYA_{INCM} * YA_{INCM} = PVAA_{INCM} * VAA_{INCM} + \sum_{i=1}^{n} PXC * YA_{INCM} * IO_{INCM,I} \tag{36}$$

Demanda de insumos intermedios:

$$VAA_{II} = YA_{II} * (1 - \sum_{i=1}^{n} IO_{II}) \tag{37}$$

Demanda de insumos intermedios:

$$VAI_{II} = YI_{II} * (1 - \sum_{i=1}^{n} IO_{II}) \tag{38}$$

Valor de producción:

$$PYI_{INCM} * YI_{INCM} = PVAI_{INCM} * VAI_{INCM} + \sum_{i=1}^{n} PXC * YI_{INCM} * IO_{INCM,I} \tag{39}$$

Producción de agregación formal e informal:

$$YIMP_{II} = YIMPA_{II} + YI_{II} \tag{40}$$

Senda de expansión:

$$\frac{YI_{II}}{YIMPA_{II}} = \left(\frac{PYIMPA_{II} * \delta_{DII}}{PYI_{II} * (1 - \delta_{DII})} \right)^{\sigma_{DII}} \tag{41}$$

Valor agregado:

$$PYPIMP_{II} * YIMP_{II} = PYI_{II} * YI_{II} + PYIMPA_{II} * YIMPA_{II} \tag{42}$$

Total de exportaciones y demanda:

$$C_I + G_I + INV_I + V_I + ZINV_I = (M_1 + D_1) * (1 + IVA_1) \tag{43}$$

Senda de expansión de las expoetaciones-importaciones:

$$\frac{M_{IM}}{D_{IM}} = FEM * \left(PD_{IM} * \frac{\delta_{IM}}{(PM_{IM} * (1 + AVE_{IM}))^{(1-\delta_{IM})}} \right)^{\sigma_{MMI}} \tag{44}$$

Senda expansión:

$$\frac{M_{INM}}{D_{INM}} = {}_{.FEM*MDEXO_{INM}} \tag{45}$$

Identidad walrasiana:

$$PXC_I * (C_I + G_I + INV_I + V_I + ZINV_I) = (1 + IVA_I) * (PM_I * M_I + PD_I * D_I) \tag{46}$$

Exportaciones tipo CET:

$$YP_{INE} = \sum_{i=1}^{n} SE, X_{INE,SE} + D_{INE} \tag{47}$$

Precio de las exportacioness:

$$PWX_{INEX,SE} = PWXEXO_{INEX,SE} \tag{48}$$

Frontera de Posibilidades de producción:

$$\frac{X_{INEE,SE}}{D_{INEE}} = \delta_{CINEE} * \left(\frac{PD_{INEE}}{PX_{INEE,SE}} \right)^{\sigma_{CINEE}} \tag{49}$$

Valor de producción:

$$PP_{INE} * YP_{INE} = \sum_{i=1}^{n} SE, PX * X_{INE,SE} + PD_{INE} * D_{INE} \tag{50}$$

Block foreign markets

Exportaciones globales:

$$XXT_{INE,SE} = X_{INE,SE} + XT_{INE,SE} \tag{51}$$

Cantidades exportadas:

$$X_{INEX,SE} = XEXO_{INEX,SE} \tag{52}$$

Demanda de exportaciones totales:

$$XXT_{IE,SE} = X_{IE,SE} + XT_{IE,SE} \tag{53}$$

Senda de expansión exportaciones:

$$\frac{X_{IE,SE}}{XT_{IE,SE}} = \left(\frac{PW_{IE,SE} * \delta_{E(IE,SE)}}{PWX_{IE,SE} * \left(1 - \delta_{E(IE,SE)}\right)} \right)^{\sigma_{E(IE,SE)}} \tag{54}$$

Agregación de trabajos:

$$TNC_{II} = TNCA_{II} * TNCI_{II} \tag{55}$$

Identidad del trabajo:

$$TNC_{II} * WNC_{II} = TNCA_{II} * WNCA_{II} + TNCI_{II} * WNCI_{II} \tag{56}$$

Revenue block

Business factorial revenues:

$$YFAC_{EMP} = SHARE_{KI} * \sum_{i=1}^{n} K_{II} * R_{II} \tag{57}$$

Ingreso factorial gobierno:

$$YFAC_{gob} = SHARE_{gI} * \sum_{i=1}^{n} K_{II} * R_{II} \tag{58}$$

Ingreso factorial resto del mundo:

$$YFAC_{rm} = SHARE_{rm} * (\sum_{i=1}^{n} K_{II} * R_{II})/ER \tag{59}$$

Ingreso factorial GNCt:

$$YFAC_{gnc} = SHARE_{gnc} * (\sum_{i=1}^{n} K_{II} * R_{II}) \tag{60}$$

Impuestos de empresas:

$$IMPUES_{EMP} = TIMP_{EMP} * (YFAC_{EMP} * (1 - TINT_{EMP} - TINT_{RM}) \tag{61}$$

Saving capital:

$$SAVE_{RM} = FK * ER \tag{62}$$

Consumption by sector:

$$CS_{IS} = \propto si_i + \sum_{j=1}^{N} \gamma si_{i_j} lnp_{ij} + \beta si_i ln\left(\frac{Xsi_t}{Psi_t}\right) * (1 - MSAVE/PXCS)\,; Para\ i = 1, 2, ..N \tag{63}$$

Public expenditure by sector:

$$G_I = SHARE_{GI} * GPX_{GNC} + GPX_{GOB} \tag{64}$$

Investment by sector:

$$INV_I = SHARE_{II} * INVT_I + INVGNC + INVAPP \tag{65}$$

Other sectoral demands:

$$ZINV_I = TZINV_I * (C_I + G_I + INV_I + V_I) \tag{66}$$

Intermediate:

$$V_I = \sum_{i=\mathbf{1}}^{n} (YA_{II} + YI_{II}) * IO_{II,I} \tag{67}$$

External prices:

$$PM_I = PWM_I * (1 + ARAN_I) * ER \tag{68}$$

Export prices:

$$PX_{I,SE} = PWX_{I,SE} * ER \tag{69}$$

Equilibrium in the labor market:

$$TSC * (\mathbf{1} - DESEMC) = \sum_{i=\mathbf{1}}^{n} TC_{II,I} \tag{70}$$

Market equilibrium:

$$DESEMA = (TSNCA - \sum_{i=\mathbf{1}}^{n} TNCA_{II,I})/TSNCA \tag{71}$$

Labor equilibrium:

$$TSNCI = \sum_{i=\mathbf{1}}^{n} TNCI_{II,I} \tag{72}$$

Salary scale:

$$WNCA_{II} = WDISTNCA_{II} * WMNCAX_{II} \tag{73}$$

Number of informales:

$$WNCI_{II} = WDISTNCI_{II} * WMNCI_{II} \tag{74}$$

Independent qualifier:

$$WC_{II} = WDISTC_{II} * WMC_{II} \tag{75}$$

Savings-investment balance:

$$\sum_{i=\mathbf{1}}^{n} INST, SAVE_{INST} = \sum_{i=\mathbf{1}}^{n} INV_I + ZINV_I * PXC_I \tag{76}$$

price index:

$$P = IPC \tag{77}$$

Total product:

$$YP_I = \sum_{i=1}^{n} MP_{I,II} * YIMP_{II} \tag{78}$$

Walras' law:

$$YP_{I,j} = \sum_{i=1}^{n} MP_{II,IJ} * YIMP_{II} \tag{79}$$

Production taxes:

$$YIMPA_{II} = YA_{II} * (1 + IMPRA_{II}) \tag{80}$$

Price taxes:

$$YIMPA_{II} * PYIMPA_{II} = YA_{II} * PYA_{II} * (1 + IMPRA_{II}) \tag{81}$$

Installed capacity:

$$IUSO = KIUSO \tag{82}$$

Investment:

$$INVT = K * INVT \tag{83}$$

Interest on public debt:

$$INTEE = RME * DEUDAEXT * ER \tag{84}$$

Government interest:

$$INTEI = RMI * DEUDAINT \tag{85}$$

Interest:

$$RME = RMFR + * (K_{deuda} * DPIB)^{\propto_{deuda}} \tag{86}$$

Interest rate:

$$RMI = RME * RMEI \tag{87}$$

Objective function:

$$VAT = \sum_{i=1}^{n} VAA_{II} * VAI_{II} \tag{88}$$