

Solving an intertemporal CGE model without an intertemporal database using GEMPACK - the GTEM way

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Given a one-period standard general equilibrium *non-steady state* database, this paper provides a systematic description of the process for solving an intertemporal CGE model using GEMPACK without requiring an intertemporal database. To illustrate the process, a stylised neoclassical growth model of a small open economy with forward-looking expectations is developed and implemented with a hypothetical *one-period* temporary equilibrium (*non-steady state*) database, similar to databases used by static CGE models. One particularly attractive feature of this approach is that the Tablo input file via the update statements handles the calibration process and therefore there is no need to create an intertemporal database before the model is solved. The intertemporal model can be solved directly by using WinGEM or by using the RunDynam interface of the GEMPACK software as if one is simulating a static or a recursively dynamic model. It is hoped that this paper will dispel the daunting myth that GEMPACK needs an equilibrium intertemporal database in the first place to solve an intertemporal general equilibrium model. Once mastered, this approach can be applied to convert the Tablo code of any working static or recursively dynamic CGE model, such as GTAP, in a short time into the code of an intertemporal model that can be implemented in GEMPACK.

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I hear and I forget. I see and I remember. I do and I understand.

- Confucius

Introduction

Though it is possible, implementing an intertemporal CGE model using GEMPACK under existing methods of calibration is a daunting task, which perhaps explains why there are not many intertemporal models and so many static or recursively dynamic models in use. The main reason for this could be the following conundrum. In order to simulate an intertemporal model, GEMPACK needs a ‘base case’ dataset covering the model horizon in the first place to calibrate the intertemporal model itself. This database must be consistent with both the contemporaneous and the intertemporal equations of the model. Unless the model is used to generate it, such a database is difficult to obtain. At best, we have a database for a single period, which is used to calibrate and solve static or recursively dynamic models and most likely reflects a non-stationary state of the economic system. If model consistent forward-looking behavior is represented, then the model contains equations that relate current values of some variables with model solutions for *future periods*. Thus, we need a complete solution of the intertemporal model itself to calibrate the model and without first calibrating the model we cannot solve the model. So, we have a conundrum. In addition to this, there is one more challenge in making sure that in the ‘base case’ the base year data forms a part of the solution trajectory. For example, if investment depends on expected future rates of return then with model consistent expectations, it is necessary that the base year level of investment, which is given, is consistent with the future rates of return and the investment demand function of the model.

Codsi, Pearson and Wilcoxon (1992), Malakellis (1999) and Wendner (1999) resolved this conundrum in their own way. The approach developed in Codsi, Pearson and Wilcoxon (1992) and Malakellis (1999) is limited to calibrating intertemporal models with static expectations (or recursively dynamic models). Wendner (1999) refined their approach further and devised a method of calibrating an intertemporal model of a closed economy containing forward looking expectations using a two-stage simulation procedure. In each of these cases, a ‘base case’ solution of the model is required covering the horizon of the model and then the intertemporal model is calibrated using the base case solution.

It is obvious that the main reason for running the two-stage simulation of the model using the replicated base-year data is to make sure that the resulting database, which is a solution of the model, is consistent with the dynamic equations of the model. It is, therefore, natural to think that the solution thus obtained must contain a special property that the databases of any two consecutive periods are intimately linked by what happens in the earlier period of the two. Using this property as a requirement, this paper

develops an alternative approach of solving an intertemporal model using GEMPACK that takes away the need to create an intertemporal database (the base case) satisfying the model equations to calibrate the intertemporal model. In this approach all future period data are viewed as base year data plus cumulative changes from the base year to period t . By doing so, the problem of calibrating an intertemporal model is shifted from generation of a base case to the problem of updating the base year data by the endogenous cumulative changes in model variables, which can be easily handled in the Tablo code. There are several advantages of this approach over the existing ones. They are: (a) the calibration of an intertemporal model no longer needs an intertemporal database; (b) there is no need to run a two-stage model simulation just to calibrate the model; (c) the model solutions can be read directly as year-on-year changes, rather than changes from the base case; and (d) the calibration procedure is transparent, intuitively clear and simple. Moreover it provides the same calibration for a closed economy as one would obtain following the procedure described in Wendner (1999). The illustrative simulations reported in this paper demonstrate these facts.

The paper is organised as follows. Section 2 reviews existing approaches of calibrating an intertemporal model and section 3 describes the new approach proposed in this paper. In section 4 a stylised growth model for a closed economy is described, which is calibrated using existing as well as the proposed approach based on updating the database by cumulative changes. The simulation results show that both approaches yield the same result. In section 5 the growth model is modified to include forward looking expectations and to allow foreign debt to accumulate. This model is simulated using data for the base period and tested to determine whether the model displays the properties of neoclassical growth model. Conclusions are drawn in section 6.

2 Review of existing approaches to the construction of dynamic databases

Ignoring the need to validate the base year data, the problem of obtaining a model consistent initial solution was first discussed in Codsì, Pearson and Wilcoxon (1992). They distinguished between the calibration of the model using a steady-state or a non-steady-state base year data.

In the first case, the base year data are adjusted to make them consistent with the steady state of the model, given the base year values of the exogenous variables. These data are then replicated for all future periods assuming that the exogenous variables do not change. As this dataset satisfies the model equations, it can be replicated for all future periods and then used to calibrate the model. Since this method requires adjustment to the base year data, it is not very interesting.

In the second case, which is more interesting, two methods are proposed, irrespective of whether exogenous variables are assumed to be fixed or changing along a given path. The first method involves finding a value of exogenous variable(s), using a one-period (or static) version of the model, for the base year so that the base year data, modified for the value of exogenous variables, reflects the steady-state equilibrium.¹ These data are replicated for the model horizon and the intertemporal model is calibrated. Next the intertemporal model is then simulated again to bring exogenous variables back to their original level of the base year. The intertemporal database thus obtained has been named the '*true base case*' by Codsì, Pearson and Wilcoxon (1999). Needless to say, this method also changes the base year data of the economy.

The second method, which they attribute to their colleague Mark Horridge (referred to from here as the MH method), is the most interesting one. It has also been used in Malakellis (1999) and in Wendner (1999) to generate the base case of an intertemporal model. In addition to employing the MH method, Wendner has employed a method of adjusting the intertemporal parameters to tackle the problem of validation of the base year data.

The MH method can be described as follows. Given a non-stationary state equilibrium database for the base year, such as any typical database for a static or a recursively dynamic CGE model, replicating it for all future periods satisfies all intraperiod equations of the model, but not necessarily the dynamic equations. For example, the capital accumulation equation can be written as:

$$(1) \quad K_{t+1} = (1 - \delta)K_t + I_t$$

where, K_t and I_t represent capital stock and gross investment respectively at time t and δ represents the depreciation rate. Given that $I_t \neq \delta K_t$, and therefore $K_{t+1} \neq K_t$; the economy will not be in a stationary state. Hence, replication of the base year database will not satisfy (1) and therefore cannot be a solution to an intertemporal model containing (1).

¹ It is important to note that not all steady state equilibrium values can be replicated as they may be growing over time by the steady state growth rate. To eliminate this possibility, the steady state referred to by Codsì, Pearson and Wilcoxon (1992) should contain constant levels of all exogenous variables. This, in other words, means that the base year data must reflect the stationary-state-equilibrium of the economic system.

To solve this problem, the MH method employs a slack variable, which is also called the homotopy variable or calibration variable, F_t , to modify equation (1) as

$$(2) \quad K_{t+1} = [(1 - \delta)K_t + I_t]F_t$$

and, calibrates F_t as

$$(3) \quad F_t = K_0 / [(1 - \delta)K_0 + I_0].$$

It is clear that with F_t given by (3), the dynamic equation (2) of the modified intertemporal model will be satisfied by the base year database and thus the replication of the database provides a basis for the calibration of the model. Now if F_t takes the value of one, then equation (2) becomes equation (1) and the modified model becomes the original intertemporal model, whose solution is what we are looking for. By forcing the slack variable, F_t , to take the value of one, that is shocking it by adding $[1 - K_0 / \{(1 - \delta)K_0 + I_0\}]$ in the value of F_t given by (3), and simulating the model we will obtain the intertemporal solution that satisfies all equations of the model. This has been the approach followed in Malakellis (1999) as well.

With this hindsight, Wendner (1999), however, works it out slightly differently. He defines a slack coefficient $\hat{K}_t$ as

$$(4) \quad \hat{K}_t = K_{t+1} - [(1 - \delta)K_t + I_t]$$

and a slack variable (the so-called homotopy variable) F to modify the capital accumulation equation (1) as

$$(5) \quad K_{t+1} = [(1 - \delta)K_t + I_t] + \hat{K}_t F.$$

Naturally, with $F = 1$, equation (5) becomes an identity and thus this equation is also satisfied with other contemporaneous equations of the model when the base year data is replicated year after year as a working solution of the model. Using this approach to modify all dynamic equations of the intertemporal model, one can easily obtain a modified intertemporal model that can be temporarily calibrated using the base year data only. Now by forcing F to go to zero, by shocking a change in F by -1 , in the modified intertemporal model we can generate all future period databases (the model solution) that satisfies all the contemporaneous as well as intertemporal equations of the model. Most importantly, these data will not necessarily be a replication of the base year data on a year-after-year basis and the procedure does not involve a modification of the base year database either. This approach is thus quite interesting.

It is however important to note that the base year values of variables that are determined by forward looking agents may not necessarily be consistent with the newly created data for the future periods. For example, if investors choose their optimal level of current investment based on expected future rates of return, which depend on future prices, then it is quite likely that in each case the base year level of investment will turn out to be inconsistent with the investment demand function and the newly created values of all future prices. A solution to this problem could be to calibrate a parameter of the investment demand function so that the base year level of investment becomes consistent with the future rates of return. To make sure that this happens, Wendner (1999) runs an additional simulation using the newly created database in which (n-1) of the n inconsistent forward-looking variables that have values in the base year database are treated as exogenous while the same number of corresponding intertemporal parameters are treated as endogenous.² These two simulations together create a database that is fully consistent with all equations of the model for all periods and includes the base year data in the initial solution trajectory.

It is worth mentioning that Wendner's approach, though similar to the one described in Codsì, Pearson and Wilcoxon (1992) and applied in Malakellis (1999), has an additional advantage in automating the calibration procedure. It does not require an outside calculation of the shocks to the homotopy variables. In each case, we set the initial value of the homotopy variable, F , equal to 1 and always shock the change in homotopy variable, c_F , by -1 , as a default. In the method suggested by Codsì, Pearson and Wilcoxon (1992) the value of the slack variable that satisfies the base year data may be different for different dynamic equations and therefore to determine the appropriate size of the shock to the homotopy variables we need outside calculations. Moreover, Wendner has correctly identified the problem of validating the base year data as a part of the solution of the intertemporal model and suggested the two-stage simulation procedure to complete the calibration of an intertemporal model. In the absence of forward-looking behavior, that is, if all agents hold static expectations, then both approaches lead to the same solution and the same database. In what follows, the focus is directed to Wendner's approach as it includes the method suggested by Codsì, Pearson and Wilcoxon as a special case and it is referred to as the MH-RW approach.

² To allow this closure change, the intertemporal parameters, such as the discount rates if the households are forward looking, and some parameter of the investment demand function, should be declared as updatable coefficients, NOT parameters. It is handy to declare such parameters as level variables because level variables are automatically updated by the associated change variable determined endogenously.

The method outlined above yields a database that is consistent with all equations of the model. The intuition behind this solution, however, has remained a mystery. It is not obvious to the general reader, what in fact we are doing by forcing the homotopy variable to take a certain value, other than that it generates a consistent solution.

We know that in recursively dynamic models we can always obtain solutions by shocking the stock variables, such as the capital stock, debt, and population, in each period by their historically determined changes on a year-on-year basis (known as *momentum simulation*). If the base year data imply that the economy is not in a stationary state, then this process can be used to generate the trajectories of all variables that can be explained by the ‘momentum’ contained in the original database and the model itself.

If the same recursive model is solved as an intertemporal model using the method outlined above, that is by shocking to the homotopy variable (MH approach), we can also obtain a solution that satisfies all contemporaneous and intertemporal equations of the model. If the model has a unique solution, then this solution must be identical to the one we can obtain from its momentum simulation.³ If this is the case, then, it can easily be hypothesised that, at least in the case of recursive models the MH solution must represent the momentum solution of the model, given the initial database.

In the case of an intertemporally dynamic model, that is, in a model with at least one forward-looking variable, running a momentum simulation is not that straightforward. We can conjecture, however, that if a momentum simulation can be obtained, then this solution must also be identical to that of the MH-RW solution.⁴ Where, it must be noted once again that, the momentum simulation is a simulation of the model in which all the stock variables, which are determined by dynamic relationships, of the model are allowed to change according to their equations of motion from their base year value while all other exogenous variables are held fixed. The problem is to find a technique, not known so far, that allows us to do so. If we can run a momentum simulation of an

³ In the documentation of the MONASH model, Dixon and Rimmer (2000) define momentum as “The change in net foreign liability that would have occurred in the absence of changes in GNP and K. ... momentum is recognized in MONASH simulations via movements from zero to one in the homotopy variable, U.” (p. 16). Thus, Dixon and Rimmer clearly indicate that the MH-RW solution provides the effects of the initial momentum, defined broadly. I thank Ken Pearson for drawing my attention to this document, which is forthcoming as a book from North-Holland.

⁴ This is a testable proposition.

intertemporal model with a single period equilibrium database, and show that the results are identical to that obtained from the MH-RW approach, we obtain an intuitive explanation of the MH-RW approach to calibration. The MH-RW approach could then provide a check against the new approach to generating the momentum simulation of an intertemporal model. We shall accomplish this task in the next section.

3 A generic approach to the calibration of an intertemporal model

In this section, we describe a generic approach to the calibration of an intertemporal model from a single period non-steady state temporary equilibrium database that is intuitively clear and simple. It is based on the fact that the level values of any variable at any two consecutive periods are connected by the changes that takes place over the former period. For example, the value of a variable X at the start of period 2 is equal to the sum of the value of X at the start of period 1 and the change in X that takes place over period 1. By extending this relationship over all time periods we can see that the value of any variable at the start of any period t is the sum of its value in the base year and the cumulative change that takes place from the base year to year t . As the model determines the cumulative changes endogenously, the only information needed in calibrating a model, viewed this way, is the database for the base year.

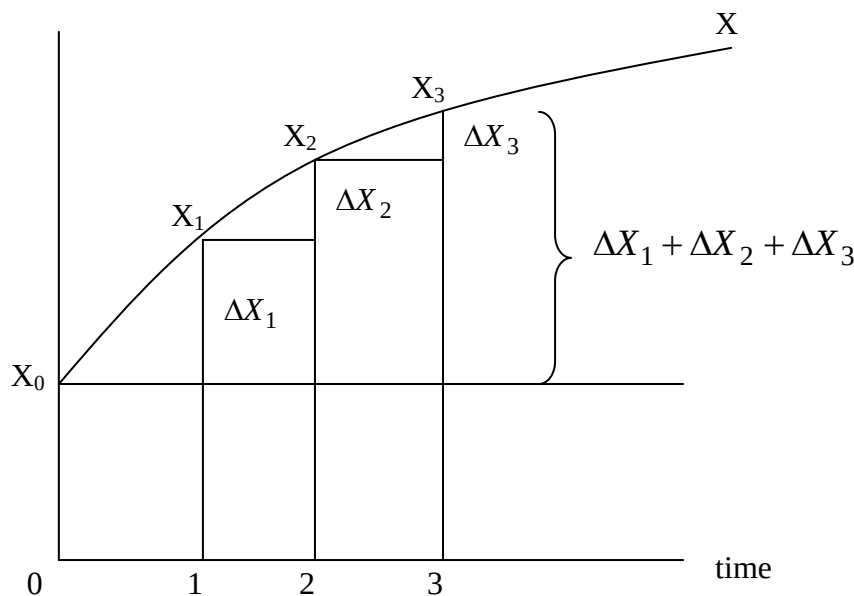


Figure 1: The link between the period- t values of a variable with its value at the base year

To help focus on the calibration process we use figure 1 to represent the linkages of the values of a variable at various points in time with its value at the base year. The x-axis represents time and the y-axis represents the level value of a variable X . At year 0, the value of X is X_0 . At the start of year 1, its value is X_1 and at the start of year 2 its value is X_2 and so on. ΔX_1 is the change in X that has taken place between year 0 and year 1 (over the year 0). Similarly, ΔX_2 and ΔX_3 represent the annual changes in X that occurred over year 1 and 2 respectively. Thus, we have:

$$(6a) \quad X_1 = X_0 + \Delta X_1$$

$$(6b) \quad X_2 = X_0 + \Delta X_1 + \Delta X_2, \text{ and}$$

$$(6c) \quad X_3 = X_0 + \Delta X_1 + \Delta X_2 + \Delta X_3,$$

and so on for all t in the model horizon.

Defining

$$(7) \quad cc_X_t = \sum_{k=1}^t \Delta X_k$$

as the cumulative changes in X that took place between the base year and the year- t , we can write equations like (6a) – (6c) as

$$(8) \quad X_t = X_0 + cc_X_t \quad \text{for all values } t \text{ in the model horizon.}$$

Noting that cc_X_t is endogenously determined by the model for each t , the value of the variable X at the start of each period t can always be written as the sum of the base year value X_0 and the cumulative change cc_X_t . It thus follows that by setting initially $X_t = X_0$ for all t and then updating X_t by cc_X_t we can calibrate an intertemporal model just with the database of the base year. It is essential that the simulation be run in multi-steps, which we do in any case as the model may contain many non-linear equations. The error caused by the initialisation of all future period values by the base year value becomes negligible, if simulations are run in sufficiently large number of steps.

By introducing a new class of variables representing cumulative changes, cc_X_t , for each coefficient (or level variable) X , which is initialised by its base year value X_0 , and updating X by its cumulative change variable, cc_X_t , the calibration process can be completed. The only problem left beyond this is validating the base year data as a model solution, which, following Wendner (1999), we do by calibrating appropriate parameters endogenously through closure changes.

To see how the initial dynamic disequilibrium can provide the momentum for the simulation let us rewrite the capital accumulation equation (1) as:

$$(9) \quad K_{t+1} - K_0 = (1 - \delta)[K_t - K_0] + [I_t - I_0] + [I_0 - \delta K_0] * one_k$$

where, K_0 is the level of capital stock at the beginning of the base year, I_0 is the investment undertaken over the base year; and one_k is a ‘homotopy’ variable that is always treated as an exogenous change variable and shocked by 1 during all dynamic simulations.

In terms of cumulative change variables equation (9) can be written as:

$$(10) \quad cc_K_{t+1} = (1 - \delta) * cc_K_t + cc_I_t + [I_0 - \delta K_0] * one_k.$$

It is worth noting that neither equation (10) nor equation (9) is a temporarily modified form of (1) as was the case with equation (2) or (5). As long as one_k is always shocked by one, they are just another way of writing equation (1). The homotopy variable one_k is merely a convenient device for converting the *momentum* coefficient $[I_0 - \delta K_0]$ into a variable. The use of such a variable is required by GEMPACK syntax if we would like to implement the capital accumulation process via equation (10), as it needs a variable in each term. Thus, by writing all dynamic equations as we did in (10) and shocking all associated homotopy variables, such as one_k , by 1 and holding all other exogenous variables constant at their base year level we can run a momentum simulation of an intertemporal model. The simulation starts from a one-period, possibly non-stationary state, database and finishes with a complete intertemporal solution of the model, calibrating the model at the same time. To show that this simulation yields the same result as the MH-RW approach we set up a stylised neo-classical growth model in the next section and solve it using both techniques.

4 Implementation of a stylised neoclassical growth model (SOLOW2) in GEMPACK

In this section, we write a simple neoclassical growth model of a one-sector closed economy, which we call SOLOW2. In this economy, we assume that households save a constant fraction of the national income. The assumption of market clearing implies that savings are equal to investment in each period.

The production of the single commodity is assumed to be described by a Cobb-Douglas function defined over capital and labor as

$$(11) \quad Y_t = L_t^\alpha K_t^{1-\alpha}, \quad \text{where } 0 < \alpha < 1.$$

The capital stock evolves according to (1) and the supply of labor is exogenous.

Given constant saving rates, σ , and market clearing condition for the single good produced in the closed economy, the investment level is given by

$$(12) \quad I_t = \sigma Y_t.$$

Equations (1), (11) and (12) describe a simple neoclassical growth model, SOLOW2. By augmenting equations (4) and replacing (1) by (5), this model can be calibrated according to the MH-RW approach.

Similarly, to implement this model as proposed in this paper we first linearise the equations of the model.

The production function can be linearised as:

$$(13) \quad y_t = \alpha l_t + (1 - \alpha) k_t$$

and the investment equation (12) can be written as:

$$(14) \quad i_t = y_t.$$

Thus equation (10), (13) and (14) characterise the same model, SOLOW2, in a linearised form. In the technique we have proposed here the level values of all variables I_t, K_t, Y_t and L_t are initialised by their base year values as in the MH-RW approach, but they are continuously updated by the cumulative changes of the respective variables. These updating equations are as follows:

$$(15a) \quad Y_t = \sum_{u \leq t} 0.01 * Y(u) * y(u)$$

$$(15b) \quad L_t = \sum_{u \leq t} 0.01 * L(u) * l(u)$$

$$(15c) \quad I_t = \sum_{u \leq t} 0.01 * I(u) * i(u)$$

$$(15d) \quad K_t = \sum_{u \leq t} cc_K(u).$$

The model is simulated for a shock of $one_k = 1$. A Tablo input file containing codes of the model for the two approaches described above and a command file to run the momentum simulation is provided in Appendix A. The results of the momentum simulation based on our approach and the results from shocking the homotopy variable by -1 in the MH-RW approach are given in table 1. The results of simulating the model with a 2 per cent per annum growth in labor supply in addition to the initial momentum is given in table 2, in which the variable WY represents the level of output in MH-RW

module and p_WY percentage change in WY relative to the base year data. Results shown in table 1 confirm that the MH approach to calibrating intertemporal models with static expectations is the same as running a momentum simulation of the model in which the data coefficients are updated by cumulative changes from the base year and vice versa. The results presented in table 2 show that both approaches generate the intertemporal database correctly, as they display the property of the neoclassical growth model. The growth rates of all real variables - income, investment and capital stock, converge to the exogenous growth rate of labor supply, which is a distinctive characteristic of the neoclassical growth model. Thus, these results show that the two approaches confirm each other and therefore the approach presented in this paper provides an intuitive explanation of the MH approach. It is that the MH solution to the calibration problem is identical to using the solution of the momentum simulation of an intertemporal model to initially calibrate it.

5 Implementation of a stylised intertemporal growth model with forward-looking expectations (SOLOWRE)⁵

In this section, we relax the assumption that the economy is closed. We now assume that the economy is small and open. As a result, it is possible to borrow or lend at a fixed real interest rate with the rest of the world. We continue to make the assumption that households save a fixed proportion of their income and assume that investment depends on the expected rate of return relative to the cost of borrowing. As investment and savings in each period are determined independently, the capital account may be in imbalance and thus there may be an accumulation of foreign debt or assets over time. We take the economy described in the previous section and add these features onto it. We call this model SOLOWRE. As the economy is small and there is only one good, we continue to assume that the price of the good is unity in all periods.

Since the profit maximising competitive producers pay the marginal product to capital, the rental rate is given by

$$(16) \quad R_t = (1 - \alpha)[L_t / K_t]^\alpha.$$

Written in linearised form (16) becomes

$$(17) \quad r_t = \alpha[l_t - k_t].$$

⁵ This model can be viewed as a much-stylised form of intertemporal GTEM.

As the price of a unit of capital good is unity, the expected rate of return, R_t^e , can be defined as

$$(18) \quad R_t^e = [R_{t+1} + (1 - \delta)] - 1.$$

In linearised form, the change in expected rate of return can be written as

$$(19) \quad dR_t^e = dR_{t+1} = 0.01 * R_{t+1} * r_{t+1}.$$

Equation (19) states that the change in the expected future rate of return in period t is just the change in the rental rates in period $(t+1)$. It is so because we have excluded any capital gain or loss by the choice of the numeraire in this one good economy.

Assume that the optimal investment demand is given by the following function:

$$(20) \quad I_t = \delta K_t \exp(\rho[R_t^e - R^w])$$

where, ρ is a positive coefficient, and R^w is exogenous global real interest rate. Because of the presence of the expected rate of return in (20) that is determined by equation (18) the model becomes forward-looking. A linearised form of equation (20) is:

$$(21a) \quad i_t = k_t + 100 * \rho * dR_t^e \quad \text{for } t \geq 1 \text{ and}$$

$$(21b) \quad i_t = k_t + 100 * \rho * dR_t^e + \rho[R_t^e - R^w] * \hat{\rho} \quad \text{for } t = 0$$

where $\hat{\rho}$ is the percentage change form of ρ and dR_t^e is the change in expected rates of return, R_t^e . The purpose of writing equation (21) in two parts is to allow a swap between the variable $\hat{\rho}$ and i_0 so that the base year level of investment could be obtained as a solution of the model by calibrating the value of ρ endogenously. This method is similar to that of Wendner (1999) except that now we complete the calibration process in just one step, while he used two steps. To model investors holding static expectations in an open economy we revert to the standard model closure by setting $\hat{\rho}$ as an exogenous variable, which in effect means that we would treat ρ as a parameter.

As we have assumed that households save a fixed proportion of the national income, which includes incomes from foreign assets or a cost of debt servicing, the saving function needs to be respecified.

Allowing for the debt servicing out of the national income, the savings function can be written as⁶

$$(22) \quad S_t = \sigma[Y_t - R^w D_t]$$

where, σ is the propensity to save out of national income, and D_t is the net foreign debt accumulated so far, which will be serviced at the exogenously given real rate of interest, R^w , assumed to be fixed at 5 per cent.

A linearised form of (22) can be written as

$$(23) \quad S_t s_t = \sigma[Y_t y_t - 100 * R^w dD_t]$$

As the variable D_t could change sign, its linear form is expressed as a change variable, dD_t . Equation (23) simply states that the change in household savings is a constant fraction of the sum of changes in domestic income and net foreign transfers.

Now finally, the possible discrepancy between national savings and investment could give rise to the accumulation of foreign assets or debt. As the debt is regularly serviced, the accumulation of net debt can be written as:

$$(24) \quad D_t = D_{t-1} + (I_{t-1} - S_{t-1})$$

Equation (24) means that the level of foreign debt in period t is the sum of debt in period $t-1$ and the surplus in the capital account accrued in period $t-1$, which is the excess of investment over savings in period $t-1$. Noting dD_t is the change in the level of debt over period $t-1$, we can write

$$(25) \quad dD_t = [(I_{t-1} - I_0) - (S_{t-1} - S_0)] + [I_0 - S_0].$$

Including equation (25) into the equation system using the Tablo syntax is a bit tricky as levels of every variable would be treated as coefficients. The right-hand side of (25) includes a term with only coefficients. We solve this problem by adding an artificial (or a homotopy) variable *one_D*, as we did with the capital accumulation equation (10), and rewrite equation (25) as

⁶ An alternative way of modelling savings would be to exclude the debt servicing component from the household account and include in the debt accumulation equation (24). We have followed this approach in writing the Tablo code provided in Appendix B.

$$(26) \quad dD_t = [(I_{t-1} - I_0) - (S_{t-1} - S_0)] + [I_0 - S_0] * one_D$$

where *one_D* is always taken as an exogenous variable and given a value of one in every simulation. Note again that, as long as *one_D* is always unity, equation (26) is not a modification of equation (24), it is the same equation written in a more convenient form.

Using the notation defined in the previous section for cumulative changes, equation (26) can be written as:

$$(27) \quad dD_t = cc_I(t) - cc_S(t) + [I_0 - S_0] * one_D.$$

Finally, we specify a formula that relates the cumulative change in the capital stock given by equation (10) and the annual percentage change in the capital stock required by equation (13) as

$$(28) \quad k_t = 200 * [(K_t - K_{t-1}) / (K_t + K_{t-1})] d\mu_t + 400 * \mu_t [1 / (K_t + K_{t-1})^2] * [K_{t-1} * cc_K_t - K_t * cc_K_{t-1}].$$

where, μ_t is a continuously increasing variable in the interval [0, 1]. It takes the value of zero at the start and the value of unity at the end of the t^{th} period. The differential $d\mu_t$ measures the length of time elapsed. If the growth rate is defined for the whole period, then $d\mu_t$ equals unity. Therefore, in the implementation of the model we replace $d\mu_t$ by *one_k*.

Equation (28) provides a way of approximating the annual percentage change of a variable, such as K_t , from the updated values of the coefficients that are initialised by the base year values. For sufficiently fine multi-step simulations, equation (28) has been found to work well. The intuition behind the formula is as follows. With $d\mu_t$ equal to unity, and μ_t equal to zero, the formula gives the growth rate based on the mid-point value of K , which is the first term in the formula. The second term allows for changes in K quantities as μ_t moves from zero to one.

These 28 equations complete the derivation of the model. The model in level form is described by equations (5), (11), (16), (18), (20), (22) and (24) determining K , Y , R , I , R^e , S and D given the values of L and F (the slack variable) over time. In linearised form the model is represented by the corresponding equations (10), (13), (17), (19), (21), (23), (27) and (29). Leaving the updating statements and the associated equations determining the cumulative changes, the model in linearised form also contains seven equations (we leave out equation (10), as it determines the cumulative change in the

capital stock) determining the corresponding seven linearised variables, y, r, dR^e, i, s, dD and k given l, one_k, one_D and $\hat{\rho}$.

In the calibration process, we swap $\hat{\rho}$ with i_0 in the model closure and make $\hat{\rho}$ an endogenous variable. Using equation (20) for $t=0$, in equation (21b) we replace $\rho[R_0^e - R^w]$ by $\log_e[I_0 / \delta K_0]$, which frees the model solution from the value chosen to initialise R^e . However, we set $R_t^e = R_t$ initially and update by dR_t^e . From the very beginning, we have taken the global interest rate, R^w , as given and maintain this assumption throughout. To solve this model it is necessary to specify its boundary conditions. There are three such conditions. They are: K_0 given; D_0 given; and $R_T^e = R_{T-1}^e$ where T is the end of simulation horizon. In linearised form these conditions become $dK_0 = 0$; $dD_0 = 0$ and $dR_T^e = dR_{T-1}^e$. Of the three conditions, the last one is critical. It implies that at the end of the model horizon, the equilibrium expectations become static. It is therefore important to take a very long time horizon so that all prices will have converged to their steady state values by the terminal date, and thus the expected rate of return does not change over time.

The Tablo input file of the linearised model, described above, is given in Appendix B.⁷ In the code both ‘homotopy’ variables, which are just the switches in our case, one_k and one_D are given the same name $c_MU(t)$, and to obtain an intertemporal solution of the model this variable should be given a uniform shock of unity. In this section, we have provided the results of two simulations. The first one with a uniform shock of unity for $c_MU(t)$ alone to obtain a momentum simulation of the model. From this simulation, we can see whether the model converges to zero growth rates for all real variables. The results are presented in table 3. In the second simulation, we have solved the model with an additional uniform shock to labor supply of 2 per cent per annum, to see whether the economy converges to a 2 per cent per annum growth rate for all real variables. The results are presented in table 4. The simulation results presented in table 3 and 4 clearly show that the model displays the properties of a neoclassical growth model. If labor supply is fixed at the base year level, the growth rates of all real variables converge to zero and when the supply of labor grows uniformly at 2 per cent per annum over the model horizon, the growth rates of the real variables converge to 2 per cent per annum. Moreover, the updated database satisfies the model equations, as it is a solution of the model.

6 Conclusions

⁷ The code is also available for downloading from <http://www.abareconomics.com>

Following the lead of Codsí, Pearson and Wilcoxon (1992), Malakellis (1999) and Wendner (1999) in solving an intertemporal model using GEMPACK with a non-steady state database representing the base year, we have presented another simple and intuitive approach that can produce the same outcome without the need to produce an intertemporal database to calibrate the model. The task of model calibration was handed over to the Tablo code and the model closure. Our approach is based on the simple idea that any solution of an intertemporal model must display a specific property. It is that the level values of every variable at the beginning of any two consecutive periods are linked by the changes that take place during the first of the two periods. By using this simple rule for all periods covered by the model horizon, we were able to see a relation between the value of a variable at any future date T and its value in the base year. These two quantities are linked by a cumulative change in the variable that takes place endogenously over the time interval, $0 - T$.

Using the cumulative change as an instrument to update the 'coefficients' of the model, which were initialised using the base year data, we were able to replicate the solution that would have been obtained had we used the method suggested by previous authors for a closed economy. This result not only confirmed the validity of our approach, but also provided the rationale for the MH approach to the calibration of an intertemporal model by showing that it is equivalent to the momentum simulation of the dynamic model of a closed economy. The most interesting aspect of the approach developed in this paper is that we can now completely forget the database issue associated with the calibration problem of an intertemporal model irrespective of whether the economy is closed or open. Any static CGE model or a recursively dynamic model, therefore, can be converted into an intertemporal one without the need for additional data.

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The author wishes to thank Brian Fisher and Vivek Tulpule for helpful comments and Ronald Wendner for providing me with the Tablo code of his model as well as providing helpful comments on this paper. I have also benefited from many discussions with Ken Pearson, who also supplied the beta version of GEMPACK release 8.0.

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Table1: Simulation of the neoclassical growth model SOLOW2: Results from the momentum simulation and a shock to the homotopy variable of the MH-RW approach						
	Momentum Simulation			Shock to Homotopy Variable (MH-RW)		
Periods	Output	Capital Stock	Investment	Output	Capital Stock	Investment
T[0]	100	100	10	100	100	10
T[1]	101.76	106	10.18	101.76	106	10.18
T[10]	114.39	156.57	11.44	114.4	156.57	11.44
T[20]	123.85	204.03	12.39	123.85	204.03	12.39
T[30]	130.39	242.23	13.04	130.4	242.24	13.04
T[40]	135.05	272.3	13.51	135.06	272.31	13.51
T[50]	138.42	295.64	13.84	138.43	295.65	13.84
T[60]	140.9	313.59	14.09	140.9	313.6	14.09
T[70]	142.72	327.32	14.27	142.72	327.34	14.27
T[80]	144.07	337.78	14.41	144.08	337.79	14.41
T[90]	145.08	345.71	14.51	145.08	345.73	14.51
T[95]	145.48	348.93	14.55	145.49	348.94	14.55
T[96]	145.56	349.52	14.56	145.56	349.53	14.56
T[97]	145.63	350.09	14.56	145.63	350.11	14.56
T[98]	145.7	350.65	14.57	145.7	350.67	14.57
T[99]	145.76	351.2	14.58	145.77	351.21	14.58
T[100]	145.83	351.72	14.58	145.84	351.74	14.58

Table2: Simulation of SOLOW2: momentum plus a 2% pa growth in labor supply								
Periods	Level values of variables				Percentage change			
	WY	Y	K	I	p_Y	p_K	p_I	p_WY
T[0]	100.00	100.00	100.00	10.00	0.0000	0.0000	0.0000	0.00
T[1]	103.18	103.18	106.00	10.32	3.1833	5.9981	3.1833	3.18
T[10]	133.12	133.11	163.47	13.31	2.6754	4.2687	2.6754	33.12
T[20]	170.50	170.49	234.99	17.05	2.3944	3.3205	2.3944	70.50
T[30]	214.10	214.09	316.25	21.41	2.2414	2.8069	2.2414	114.10
T[40]	265.85	265.83	409.96	26.58	2.1517	2.5067	2.1517	165.85
T[50]	327.88	327.86	519.62	32.79	2.0969	2.3234	2.0969	227.88
T[60]	402.71	402.68	649.50	40.27	2.0625	2.2085	2.0625	302.71
T[70]	493.29	493.25	804.69	49.33	2.0406	2.1353	2.0406	393.30
T[80]	603.23	603.18	991.33	60.32	2.0264	2.0882	2.0264	503.23
T[90]	736.86	736.79	1216.78	73.68	2.0173	2.0576	2.0173	636.86
T[95]	814.16	814.09	1346.78	81.41	2.0140	2.0466	2.0140	714.16
T[96]	830.56	830.48	1374.31	83.05	2.0134	2.0446	2.0134	730.56
T[97]	847.27	847.19	1402.39	84.72	2.0128	2.0427	2.0128	747.27
T[98]	864.33	864.24	1431.01	86.42	2.0123	2.0411	2.0123	764.32
T[99]	881.71	881.63	1460.20	88.16	2.0118	2.0392	2.0118	781.71
T[100]	899.45	899.36	1489.95	89.94	2.0113	2.0377	2.0113	799.45

Table 3: Momentum simulation of SOLOWRE

	Percentage change in					
Time periods	Output	Capital Stock	Investment	Rental Rate	Change in Debt	Total Debt
T[0]	0	0	0	0	0	0
T[1]	1.76291	5.998176	-4.29464	-3.9956	0	0
T[2]	1.482525	5.02779	-3.63534	-3.37555	-0.60575	-0.60575
T[10]	0.534516	1.792856	-1.30512	-1.23618	-3.27134	-18.9736
T[20]	0.212286	0.709375	-0.50803	-0.49359	-5.05603	-62.0545
T[30]	0.09554	0.318821	-0.22676	-0.22257	-6.52566	-120.762
T[40]	0.045236	0.150867	-0.10646	-0.10547	-8.01047	-194.113
T[50]	0.021908	0.073047	-0.05164	-0.0511	-9.62329	-282.955
T[60]	0.010724	0.03575	-0.02481	-0.02502	-11.4218	-388.908
T[70]	0.005261	0.017538	-0.01228	-0.01228	-13.4451	-514.055
T[80]	0.002535	0.008452	-0.0063	-0.00592	-15.7262	-660.826
T[90]	0.001264	0.004212	-0.00286	-0.00295	-18.2963	-831.973
T[95]	0.000937	0.003124	-0.0018	-0.00219	-19.6996	-927.633
T[96]	0.000807	0.002691	-0.00175	-0.00188	-19.9902	-947.622
T[97]	0.000728	0.002426	-0.00245	-0.0017	-20.2842	-967.905
T[98]	0.000802	0.002674	-0.00145	-0.00187	-20.5817	-988.489
T[99]	0.000676	0.002252	-0.00164	-0.00158	-20.8827	-1009.37
T[100]	0.000638	0.002127	0.002127	-0.00149	-21.1872	-1030.56

Table 4: Simulation of SOLOWRE under a uniform 2 per cent growth in labor supply with the momentum

	Percentage change in					
Periods	Output	Capital Stock	Investment	Rental Rate	Change in Debt	Total Debt
T[0]	0	0	0	0	0	0
T[1]	3.183353	5.998176	-0.9128	-2.65554	0	0
T[2]	2.992661	5.346606	-0.45601	-2.23447	-0.40962	-0.40962
T[10]	2.313865	3.049978	1.225146	-0.71433	-2.45993	-13.7854
T[20]	2.096664	2.32257	1.780818	-0.22078	-4.13079	-47.8728
T[30]	2.034724	2.11579	1.945811	-0.07939	-5.69697	-97.7324
T[40]	2.015751	2.052513	2.000822	-0.03602	-7.41679	-163.982
T[50]	2.010202	2.034009	2.020317	-0.02333	-9.39286	-248.778
T[60]	2.008994	2.029985	2.02855	-0.02057	-11.6848	-355.027
T[70]	2.009066	2.030222	2.032314	-0.02074	-14.352	-486.219
T[80]	2.009492	2.031643	2.034492	-0.02171	-17.4508	-646.406
T[90]	2.009963	2.033215	2.036005	-0.02279	-21.0418	-840.23
T[95]	2.010205	2.034019	2.036711	-0.02334	-23.0427	-951.387
T[96]	2.010257	2.034196	2.03694	-0.02346	-23.4609	-974.848
T[97]	2.010294	2.034315	2.036929	-0.02354	-23.8851	-998.733
T[98]	2.010353	2.034515	2.037137	-0.02368	-24.3158	-1023.05
T[99]	2.010395	2.034652	2.037319	-0.02377	-24.7512	-1047.8
T[100]	2.010429	2.034767	2.034767	-0.02385	-25.1928	-1072.99

Appendix A: Calibration of a stylised neoclassical growth model (SOLOW2) of a closed economy using two alternative approaches

!
SOLOW2.TAB
Tablo codes written in two different ways, both implementing the same stylised neo-classical growth model of Solow (1956) type. To run this in Wingem correct for the length of some lines, which contain more than 80 characters!

```
FILE(for_updates) solowupd    #file holding intertemporal data#;

COEFFICIENT(integer) ninterval;
FORMULA ninterval =100;
SET(intertemporal) alltime maximum size 101 (T[0]-T[ninterval]);
SET(intertemporal) backtime maximum size 100 (T[1]-T[ninterval]);
SET(intertemporal) fwdtime maximum size 100 (T[0]-T[ninterval-1]);
SET(intertemporal) basetime maximum size 1 (T[0]);

SUBSET backtime is subset of alltime;
SUBSET fwdtime is subset of alltime;
SUBSET basetime is subset of alltime;

VARIABLE (change) (all,t,alltime) cc_K(t) #cumulative change in the capital stock#;
VARIABLE(change) (all,t,alltime) cc_I(t) #cumulative change in investment#;

Variable (all,t,alltime) p_I(t) #annual percent changes in investment#;
Variable (all,t,alltime) p_L(t) #annual percent changes in labor#;
Variable (all,t,alltime) p_k(t) #annual percent changes in capital stock#;
Variable (all,t,alltime) p_y(t) #annual percent changes in output#;

COEFFICIENT (parameter) I0 #level of investment in year t measured in physical units#;
COEFFICIENT (parameter) K0 #Units of physical capital stock at the start of the period#;
COEFFICIENT Y0;
COEFFICIENT L0;

!READ BASE YEAR DATA VIA FORMULA!
FORMULA Y0=100;
FORMULA L0=100;
FORMULA (initial) I0=10;
FORMULA (initial) K0=100;

!parameters!
COEFFICIENT (parameter) alpha #share of labor#;
COEFFICIENT (parameter) delta #depreciation rate#;
COEFFICIENT(parameter) sr #savings rate#;
FORMULA(initial) alpha=0.7;
FORMULA(initial) delta=0.04;
FORMULA(initial) sr=0.10;

!Declaration of coefficients- level values of the variables!
COEFFICIENT (all,t,alltime) I(t) #investment in year t#;
coefficient (all,t,alltime) K(t)#capital stock at the start of the period#;
COEFFICIENT (all,t,alltime) Y(t)#output#;
COEFFICIENT (all,t,alltime) L(t)#labor supply#;
!-----!
!INITIALIZATION and update OF DATA USING BASE YEAR VALUES!
!-----!
FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "I(t)") (all,t,alltime) I(t)= I0;
```

```

EQUATION E_CC_I (all,t,alltime) cc_I(t) = sum(u,alltime:u le t, 0.01*I(t)*p_I(u));
UPDATE(change) (all,t,alltime) I(t) = cc_I(t);

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "K(t)") (all,t,alltime) K(t)= K0;
UPDATE(change) (all,t,alltime) K(t)= cc_K(t);

FORMULA(initial, write updated value to file solowupd header "Y(t)") (all,t,alltime) Y(t)=Y0;
UPDATE(change)(all,t,alltime) Y(t)= sum(u,alltime:u le t,0.01*Y(t)*p_Y(u));

FORMULA(initial, write updated value to file solowupd header "L(t)") (all,t,alltime) L(t)=L0;
UPDATE(change)(all,t,alltime) L(t)= sum(u,alltime:u le t,0.01*L(t)*p_L(u));

VARIABLE(change) (all,t,alltime) one_k(t) #change in homotopy variable#;
COEFFICIENT (all,t,alltime) Mu(t) #Level of the homotopy variable#;
FORMULA(INITIAL) (all,t,alltime)Mu(t)=0;
UPDATE(change) (all,t,backtime)Mu(t) = one_k(t);

EQUATION E_p_y #EQUATION (13)#
(all,t,alltime) p_y(t) = alpha*p_l(t)+(1-alpha)*p_k(t);

EQUATION E_p_i #EQUATION(14)#
(all,t,alltime) p_i(t) = p_y(t);

EQUATION E_c_K #EQUATION (10)#
(all,t,fwdtime) cc_K(t+1)= (1-delta)*cc_K(t) + cc_I(t) +[I0-delta*K0]*one_k(t);
! initial condition:!
EQUATION E_K0 (all,t,basetime) cc_K(t) = 0;

!bonus: extraction of annual percentage changes in the capital stock!
EQUATION E_p_K
(all,t,backtime)
p_k(t) =200* {[K(t)-K(t-1)]/[K(t)+K(t-1)]}*one_k(t)
+ 400*mu(t)/[(K(t)+K(t-1))^2] * [K(t-1)*cc_K(t)-K(t)*cc_K(t-1)];

EQUATION E_p_K0
(all,t,basetime) p_k(t)= 0;

!-----!
!END OF TABLO CODE implementing the linearised NEOCLASSICAL GROWTH MODEL - SOLOW2
as derived in section 4!
!-----!

!-----!
! The MH-RW APPROACH BEGINS:
all variables and coefficients starting with W refer to level variables in the MH-RW module of the code
!-----!

VARIABLE (Levels) (all,t,alltime) WK(t) #capital stock at the start of t#;
!WK will update by percent change variable p_wk !
VARIABLE (Levels) (all,t,alltime) WI(t)#Level of real gross investment in period t#;
!WI will update by per cent change variable p_wi!
VARIABLE (Levels) (all,t,alltime) WL(t)#Labor supplied in period t#;
!WL will update by per cent change variable p_WL!
VARIABLE (Levels) (all,t,alltime) WY(t)#Level of regional output in period t #;
!WY will update by per cent change variable p_WY!

!-----!
! INITIALIZATION USING BASE YEAR DATA as read ABOVE!
!-----!

FORMULA(initial, write updated value to file solowupd header "WKKK") (all,t,alltime) WK(t)= K0;
FORMULA(initial, write updated value to file solowupd header "WIII") (all,t,alltime) WI(t) = I0;
FORMULA(initial, write updated value to file solowupd header "WYYY") (all,t,alltime) WY(t) =Y0;
FORMULA(initial, write updated value to file solowupd header "WLLL") (all,t,alltime) WL(t) =L0;

```

!As the depreciation rate $\delta = 4\%$, $I_0 = 10$, and $K_0 = 100$ the initial data base does not imply a stationary state equilibrium. Therefore, we use the slack variable approach to track it!

! Calculation of the slack coefficient!

COEFFICIENT (parameter)(all,t,backtime) WK_hat(t) # EQUATION (4):NSS discrepancy in capital stock data#;
FORMULA(initial) (all, t, backtime) WK_hat(t)= WK(t)-0.96*WK(t-1)-WI(t-1);
VARIABLE(LEVELS,change) (all,t,alltime) slack(t) #homotopy variable#;
FORMULA(initial, write updated value to file solowupd header "WSLK") (all,t,alltime) SLACK(t)= 1;
!This variable will update by the change variable c_slack!

! The modified growth model:!

EQUATION(LEVELS) E_WK #EQUATION(5)# (all,t,backtime)
 $WK(t) = 0.96 * WK(t-1) + WI(t-1) + WK_HAT(t) * SLACK(t);$
EQUATION(LEVELS) E_WY #EQUATION (11)#(all,t,alltime)
 $WY(t) = [(WK(t))^{(1-\alpha)}] * [WL(t)]^{\alpha};$
EQUATION(LEVELS) E_WI #EQUATION (12)#(all,t,alltime) WI(t)= sr*WY(t);
!END of the MH-RW module of the Tablo code!
![[=====
!the command file :
! SOLOW2.cmf
!-----
check-on-read all = warn;
auxiliary files =solow2; ! make sure that the name given here matches with your Table file name
method = euler;
steps = 1 2 4 ;
subintervals=100; !Try different values ...
verbal description = Simulation of a neoclassical growth model using both approaches;
updated file solowupd = solowupd.har;
Exogenous
p_l
one_k
!-----exogenous for the MH-RW module----
p_wl
p_wk(basetime)!this is the initial condition on capital stock
c_slack;

rest endogenous;
!
! Just the momentum simulation:
!----Shock for the momentum simulation-----
shock one_k = uniform 1.0;
!---shocks for the MH-RW module -----
shock c_slack = uniform -1.0;
!-----
!If running with a 2%pa growth in labor supply use the
!following shocks
!-----!
shock p_l(backtime) = uniform 2.0;
shock one_k = uniform 1.0;
!-----
shock p_wl(backtime) = select from file 101p_wl.shk;
shock c_slack = uniform -1.0;

!NOTE: the user needs to create the shock file 101p_wl.shk
!other growth rates for labor supply can be tried in the !same way.
!send me an e-mail if in trouble: hom.pant@abare.gov.au
!]]!

Appendix B: Tablo Input file for SOLOWRE

!
SOLOWRE.TAB

A Tablo code implementing a stylised neo-classical growth model of Solow (1956) type with rational expectation. The code contains relevant equations determining the solution of the general equilibrium model of a small open economy.

The model and the code written by:

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e-mail: hom.pant@abare.gov.au - for any comments and suggestions

!

FILE(for_updates) solowupd #file holding the intertemporal data created#;

COEFFICIENT(integer) ninterval #number of time intervals in the model horizon#;

FORMULA ninterval =100;

!This means there are 101 time grids (dates), 0 represents the database year!

SET(intertemporal) alltime maximum size 101 (T[0]-T[ninterval]);

SET(intertemporal) backtime maximum size 100 (T[1]-T[ninterval]);

SET(intertemporal) fwdtime1 maximum size 100 (T[0]-T[ninterval-1]);

SET(intertemporal) basetime maximum size 1 (T[0]);

SET(intertemporal) Endtime maximum size 1 (T[ninterval]);

SUBSET backtime is subset of alltime;

SUBSET fwdtime1 is subset of alltime;

SUBSET basetime is subset of alltime;

SUBSET endtime is subset of alltime;

!BASE YEAR DATA!

COEFFICIENT(parameter) Y0 #Units of output #;

COEFFICIENT(parameter) L0 #Units of Labor employed #;

COEFFICIENT (parameter) K0 #Units of capital stock #;

COEFFICIENT(parameter) S0 #level of savings #;

COEFFICIENT(parameter) I0 #level of investment #;

FORMULA(initial) Y0=100;

FORMULA(initial) L0=100;

FORMULA(initial) S0=10;

FORMULA(initial) I0=10;

FORMULA(initial) K0=100;

COEFFICIENT(parameter) sr # saving rates#;

FORMULA(initial) sr=0.10;

COEFFICIENT(parameter) alpha #share parameter of labor in a C-D prod. function#;

FORMULA(initial) alpha=0.7;

COEFFICIENT(parameter) delta # fixed depreciation rate#;

FORMULA(initial) delta=0.04;

!derivatives of Base year data -----!

!Calibration of the base year Rental Rate!

COEFFICIENT(parameter) R0

#level of Rental Rate = marginal product of capital in the base year#;

FORMULA(initial) R0=(1-alpha)*(L0/K0)^alpha;

! this number equals 0.3 as L0=K0=Y0 and alpha = 0.7!

COEFFICIENT(parameter) RE0

#level of expected rate of return on investment#;

FORMULA(initial) RE0=0.26;

! this number, which is 26%, equals approximately R0 - delta,

but can be chosen arbitrarily around this value !

COEFFICIENT(parameter) RW0

#Global rate of return or the cost of borrowing from the global market#;

FORMULA(initial) RW0=0.05;

!-----!

! ANNUAL CHANGE OR PERCENTAGE CHANGE VARIABLES !

!-----!

Variable (all,t,alltime) p_I(t) #annual percent changes in investment#;

Variable (all,t,alltime) p_L(t) #annual percent changes in labor#;

Variable (all,t,alltime) p_k(t) #annual percent changes in capital stock#;

Variable (all,t,alltime) p_y(t) #annual percent changes in output#;

Variable (all,t,alltime) p_s(t) #annual percent changes in savings#;

Variable (all,t,alltime) p_r(t) #annual percent changes in rental rates#;

Variable(change) (all,t,alltime) c_re(t) #annual change in expected rate of return at t#;

Variable(change) (all,t,alltime) c_D(t) # capital account imbalance in period t-1 = change in debt in period t#;

!VARIABLES REPRESENTING CUMULATIVE CHANGES!

VARIABLE(change) (all,t,alltime) cc_K(t) #cumulative change in capital stock from the base year level#;

VARIABLE(CHANGE) (all,t,alltime) cc_I(t) #cumulative change in real investment from the base year level#;

Variable(change) (all,t,alltime) cc_S(t) #cumulative changes in savings from the base year level#;

!-----!

!Initialisation AND UPDATING of the Intertemporal Database

!-----!

COEFFICIENT (all,t,alltime) I(t) # level of investment in year t #;

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "IIII") (all,t,alltime) I(t)= I0;

UPDATE(change) (all,t,alltime) I(t)= cc_I(t);

COEFFICIENT (all,t,alltime) K(t) #Units of physical capital stock at the start of the period t #;

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "KKKK") (all,t,alltime) K(t)= K0;

UPDATE(change) (all,t,alltime) K(t)= cc_K(t);

COEFFICIENT (all,t,alltime) S(t) # Savings in year t#;

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "SSSS") (all,t,alltime) S(t)= S0;

UPDATE(change) (all,t,alltime) S(t)= cc_S(t);

!The cumulative change variables can directly be incorporated in the update statements. This reduces number of variables and equations. Here is how we can do it:!

COEFFICIENT(all,t,alltime) Y(t);

FORMULA(initial, write updated value to file solowupd header "YYYY") (all,t,alltime) Y(t)=Y0;

UPDATE(change)(all,t,alltime) Y(t)= sum(u,alltime:u le t, 0.01*Y(t)*p_y(u));

COEFFICIENT(all,t,alltime) L(t);

FORMULA(initial, write updated value to file solowupd header "LLLL") (all,t,alltime) L(t)=L0;

UPDATE(change)(all,t,alltime) L(t)= sum(u,alltime:u le t, 0.01*L(t)*p_L(u));

COEFFICIENT (all,t,alltime) R(t) # level of the rental rates #;

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "RRRR") (all,t,alltime) R(t)= R0;

UPDATE(change) (all,t,alltime) R(t) = sum(u,alltime:u le t, 0.01*R(t)*p_r(u));

COEFFICIENT (all,t,alltime) RE(t) # level of expected rate of return #;

FORMULA (initial, WRITE UPDATED VALUE to FILE solowupd HEADER "EROR")

(all,t,alltime) RE(t)= RE0;

UPDATE(change) (all,t,alltime) RE(t) = sum(u,alltime:u le t, c_RE(u));

!Here we specify base year data directly via formula - initial!

COEFFICIENT (all,t,alltime) D(t) #outstanding foreign debt at time t#;

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "DEBT") (all,t,alltime) D(t) = 0.0;

!Life starts without having foreign debt or assets!

UPDATE(change) (all,t,alltime) D(t)=SUM(u,alltime: u le t, c_d(u));

!Here we declare a homotopy Variable, which is very handy in converting coefficients into variables as this is not allowed in GEMPACK!

VARIABLE(LEVELS,change) (all,t,alltime) Mu(t) # The homotopy variable #;

FORMULA(INITIAL,WRITE UPDATED VALUE to FILE solowupd HEADER "MU") (all,t,alltime)Mu(t)=0;

*!NOTE: MU(t) will automatically be updated by c_MU(t) for each period t.
When we want coefficients converted into variables, such as the one that arises
as we compute cumulative change the capital stock from the base year level,
we shock c_MU(t) by one. Then the level value of MU goes from 0 to one.!*

!-----!

!EQUATIONS:!

!-----!

!A: EQUATIONS computing the cumulative changes in variables specified!

EQUATION E_cC_S (all,t,alltime) cc_S(t)=sum(u,alltime:u le t, 0.01*S(t)*p_s(u));

EQUATION E_cC_I (all,t,alltime) cc_I(t)=sum(u,alltime:u le t, 0.01*I(t)*p_I(u));

EQUATION E_c_K (all,t,fwdtime1)

cc_K(t+1) = (1-delta)*cc_K(t) + cc_I(t) + [I0-delta*K0]*c_MU(t);

!initial condition:!

EQUATION E_K0

(all,t,basetime) cc_K(t) = 0;

!-----!

!B: EQUATIONS describing the NEO-CLASSICAL GROWTH MODEL - SOLOWRE

Labor supply has been taken as exogenous.!

!-----!

!C-D production function !

EQUATION E_p_y (all,t,alltime) p_y(t) = alpha*p_l(t)+(1-alpha)*p_k(t);

! percentage change in the capital stock!

EQUATION E_p_K (all,t,backtime)

p_k(t) = 200 * {[K(t)-K(t-1)]/[K(t)+K(t-1)]}*c_MU(t)

+ 400*mu(t)/[(K(t)+K(t-1))^2]

* [K(t-1)*cc_K(t)-K(t)*cc_K(t-1)];

EQUATION E_p_K0 (all,t,basetime) p_k(t)= 0;

!rental rate!

EQUATION E_p_r (all,t,alltime) p_r(t) = alpha*(p_l(t) - p_k(t));

!change in expected ror!

EQUATION E_c_RE (all,t,fwdtime1) c_re(t) = 0.01*R(t+1)*p_r(t+1);

EQUATION E_c_reT (all,t,endtime) c_re(t) = 0;

*!This is how a solution value of a forward looking variable
that has got value in base data can be made consistent with the base data:!*

VARIABLE(LEVELS) rho;

FORMULA(initial, WRITE UPDATED VALUE to FILE solowupd HEADER "RHO") RHO= 10.0;

!The Coefficient RHO will automatically be updated by the variable p_rho!

!Investment demand!

EQUATION E_p_i0 (all,t,basetime) p_i(t) = p_k(t) + 100*RHO*c_RE(t) + RHO*[RE(t)- RW0]*p_RHO;

!calibrates rho by determining p_rho so that p_i is zero at the base year.

So, in the model closure set p_rho endogenous and p_i(basetime) exogenous

!

EQUATION E_p_I (all,t,backtime) p_i(t) = p_k(t) + 100*RHO*c_RE(t) ;

!Determination of Savings: A fixed proportion of income Y is saved!

EQUATION E_p_S (all,t,alltime) S(t)*p_S(t) = sr*Y(t)*p_y(t) ;

!Accumulation of foreign DEBT: capital account imbalance!

EQUATION E_c_D (all,t,backtime) c_D(t) = [RW0*D(t-1)+I0-S0]*c_MU(t)+ [cc_I(t-1) - cc_S(t-1)];

!initial condition!

EQUATION E_c_D0 (all,t,basetime) c_D(t) = 0;

![[!

```

!alternative way of servicing foreign debt, which is more logical!
EQUATION E_p_S (all,t,alltime) S(t)*p_S(t) = sr*Y(t)*p_y(t) - if(D(t) gt 2, 100*sr*RW0*c_D(t));
!Accumulation of foreign DEBT: capital account imbalance!
EQUATION E_c_D (all,t,backtime) c_D(t) = [I0-S0]*c_MU(t)+ [cc_I(t-1) - cc_S(t-1)];
!]]!
!Command file
!solowRE.cmf
![[!=====
check-on-read all = warn;
auxiliary files =solowRE;
method = euler;
steps = 1 2 4 ;
subintervals=200;
verbal description = simulation of an inter-temporal model with RE;
updated file solowupd = solowupd.har;
Exogenous
p_l
c_MU
p_i(basetime) !p_rho
;
rest endogenous;
! effects of labor supply growth in a neo-classical growth model
shock p_l(backtime) = uniform 2.0;
shock c_MU = uniform 1.0;
!]]!

```