HUMBOLDT-UNIVERSITÄT ZU BERLIN

Stochastic Simulation with Informed Rotations of Gaussian Quadratures

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24th Annual Conference on Global Economic Analysis

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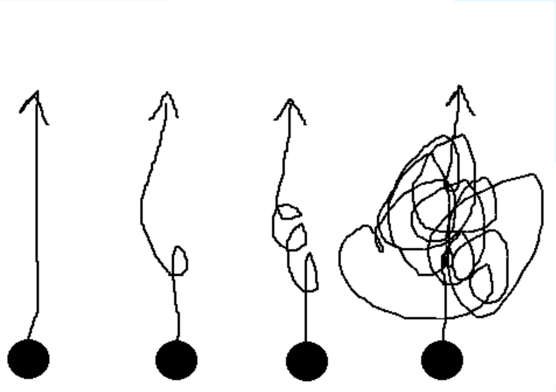
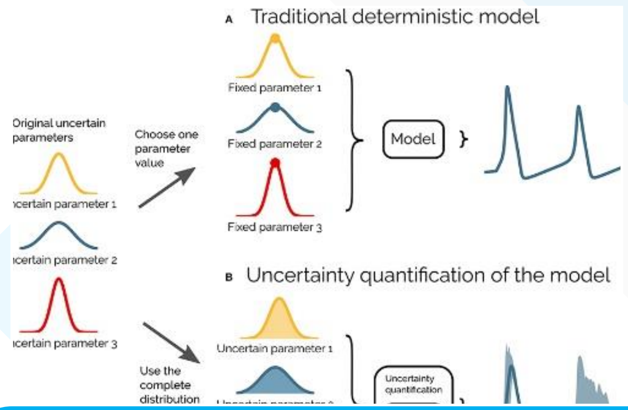
UA is becoming a good modeling practice



Increase in computational power

Expansion of simulation models in detail and complexity



SSA/UA becoming a standard/good modeling practice

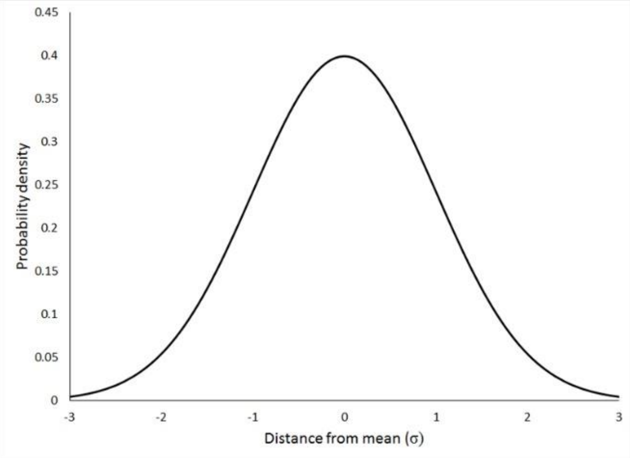
Increased uncertainty associated with model results

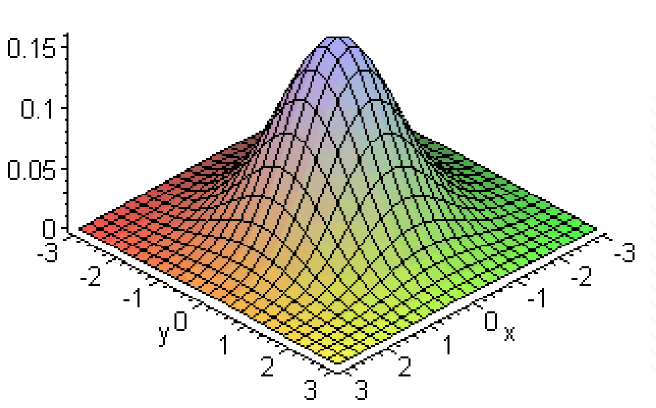
Why is UA a numerical integration problem?

1. Model variable/parameter: x (e.g., yields, prices, elasticities, etc.)
2. Pdf describing the uncertainty around x: g(x)
3. Model function: f(x)

$$(1) \quad E[f(x)] = \int_a^b f(x)g(x)dx$$

$$(2) \quad \tilde{E}[f(x)] = \sum_{k=1}^{n} f(x_k)w_k$$





Categorization of the methods according to Haber (1970)

Probabilistic

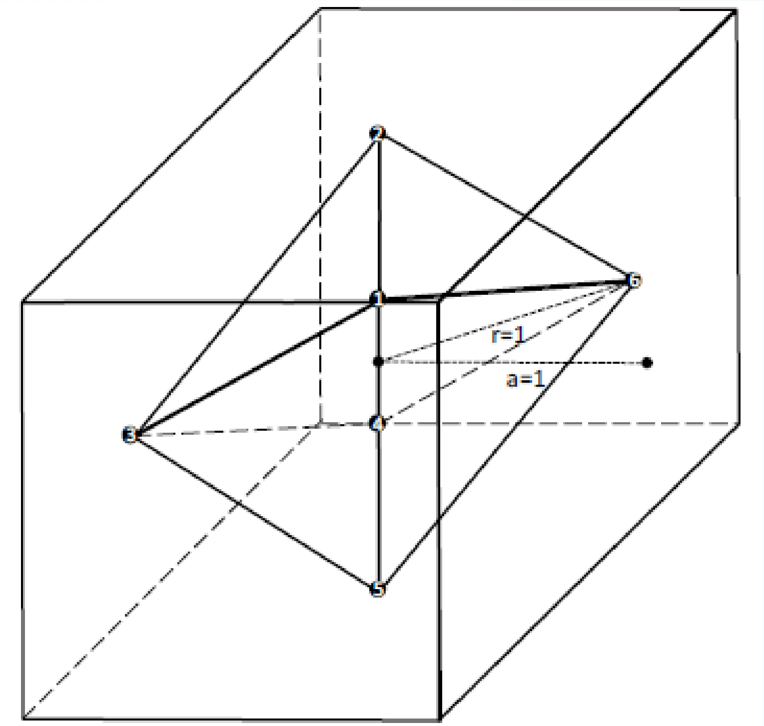
1. Monte Carlo (MC) (Metropolis and Ulam 1949)
2. Stratified sampling (e.g., Saltelli et al. 2008; Norton 2015)
3. Latin hypercube sampling (LHS) (Helton and Davis 2003)

Efficient

1. Gaussian quadratures
   - Degree 3 quadrature formulae by Stroud (1957)

Convergence evaluation

Stroud's (1957) theorem for degree 3 quadrature formulae



Source: Artavia et al. (2015)

$$\gamma_{k,2j-1} = \sqrt{\frac{2}{3}}\cos\left(\frac{(2j-1)k\pi}{n}\right),$$

$$\gamma_{k,2j} = \sqrt{\frac{2}{3}}\sin\left(\frac{(2j-1)k\pi}{n}\right),$$

$$\gamma_{k,n} = \frac{(-1)^k}{\sqrt{3}}$$

For k = 1,…,2n,

j = 1,…,$\left[\frac{n}{2}\right]$

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Convergence evaluation

Efficient

1. Gaussian quadratures
   - Degree 3 quadrature formulae by Stroud (1957)
   - Arndt (1996) adapted Stroud's formulae for integrals with MSND as a weight function

$$GQ = A\Gamma + \vec{\mu}\,[1 \ldots 1]$$

$$\Sigma = AA^T \qquad \Gamma_k = (\gamma_{k,1}, \gamma_{k,2}, \ldots, \gamma_{k,n})$$

$$\gamma_{k,2j-1} = \sqrt{2}\cos\left(\frac{(2j-1)k\pi}{n}\right),$$
$$\gamma_{k,2j} = \sqrt{2}\sin\left(\frac{(2j-1)k\pi}{n}\right),$$
$$\gamma_{k,n} = (-1)^k$$

Current state of art

Arndt (1996)

Introduced Stroud's (1957) order 3 GQ in the GTAP model.

Preckel et al. (2011a)

Research question: The impact of limited sampling interval of the GQ method.

Finding: Sampling breadth is important for highly nonlinear models.

Preckel et al. (2011b)

Research question: The impact of using different linear transformation methods.

Finding: Noncrucial for the quality of the results.

Artavia et al. (2015)

Research question: Do rotations of Stroud's octahedron matter?

Finding: Yes, they do!

Villoria and Preckel (2017)

Research question: Evaluating the quality of GQ results compared to MC results.

Finding: Large differences in the first three moments of the pdfs of the results.

Environmental Modelling and Software 136 (2021) 104929

Contents lists available at ScienceDirect

Environmental Modelling and Software

journal homepage: http://www.elsevier.com/locate/envsoft

Multiple rotations of Gaussian quadratures: An efficient method for uncertainty analyses in large-scale simulation models

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ARTICLE INFO

Keywords:
Uncertainty analysis
Systematic sensitivity analysis
Stochastic modeling
Multiple rotations of Gaussian quadratures
Monte Carlo sampling
Computable general equilibrium models
Partial equilibrium models

ABSTRACT

Concerns regarding the impact of climate change, food price volatility, and weather uncertainty have motivated users of simulation models to consider uncertainty in their simulations. One way to do this is to integrate uncertainty components in the model equations, thus turning the model into a problem of numerical integration. Most of these problems do not have analytical solutions, and researchers, therefore, apply numerical approximation methods. This article presents a novel approach to conducting an uncertainty analysis as an alternative to the computationally burdensome Monte Carlo-based (MC) methods. The developed method is based on the degree three Gaussian quadrature (GQ) formulae and is tested using three large-scale simulation models. While the standard single GQ method often produces low-quality approximations, the results of this study demonstrate that the proposed approach reduces the approximation errors by a factor of nine using only 3.4% of the computational effort required by the MC-based methods in the most computationally demanding model.

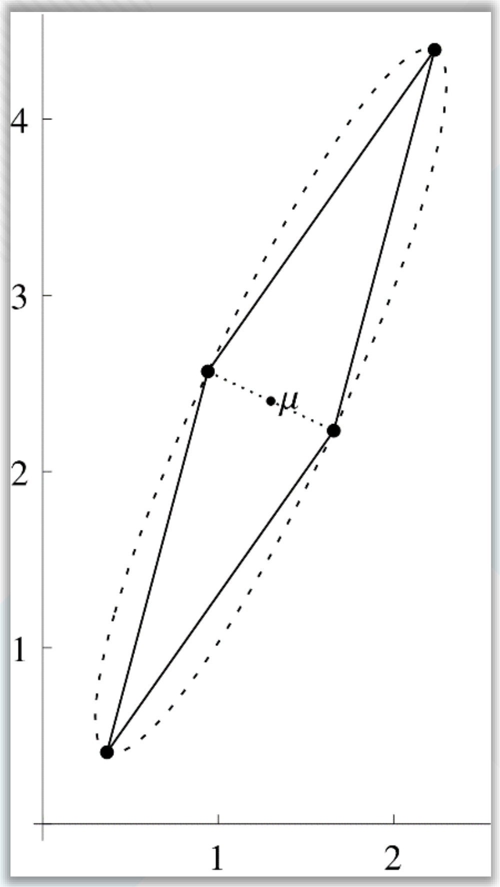
Identification of good rotations

- **Campolongo et al. (2007)**
  - Improving the EE method (Morris 1991) by maximizing the dispersion of the sampling points in the input space.
- **Dispersion of a GQ family**

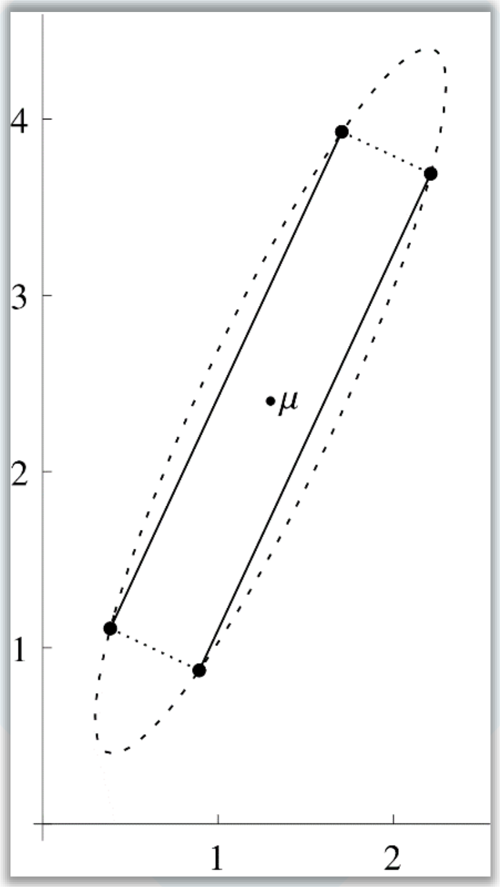
$$disp\left(\overrightarrow{GQ_k}\right) = \min_{j \neq i} \left\| x_j - x_i \right\|$$

$$\max(disp\left(\overrightarrow{GQ_k}\right))$$

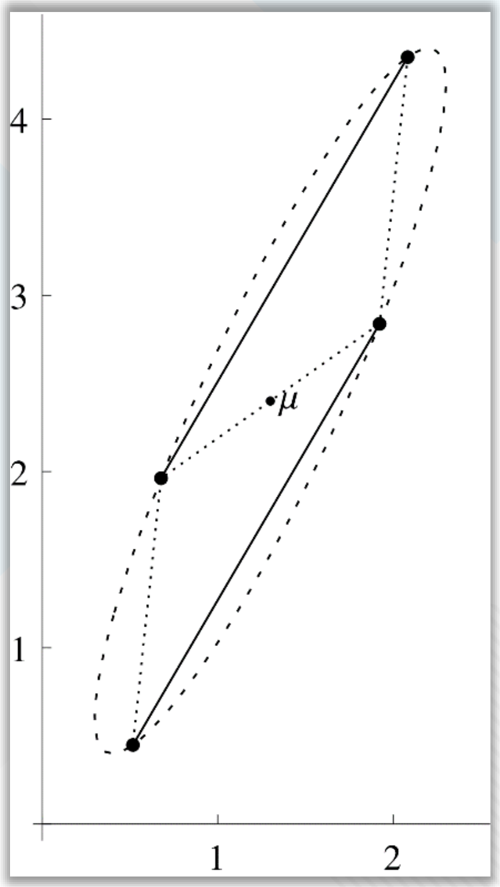
The IRGQ method in R2



(a)

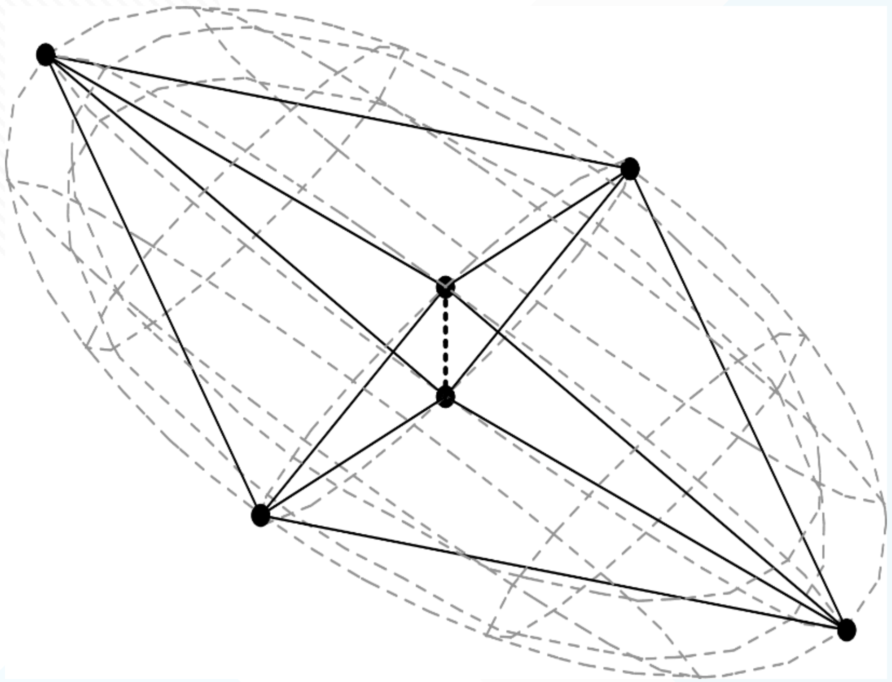


(b)

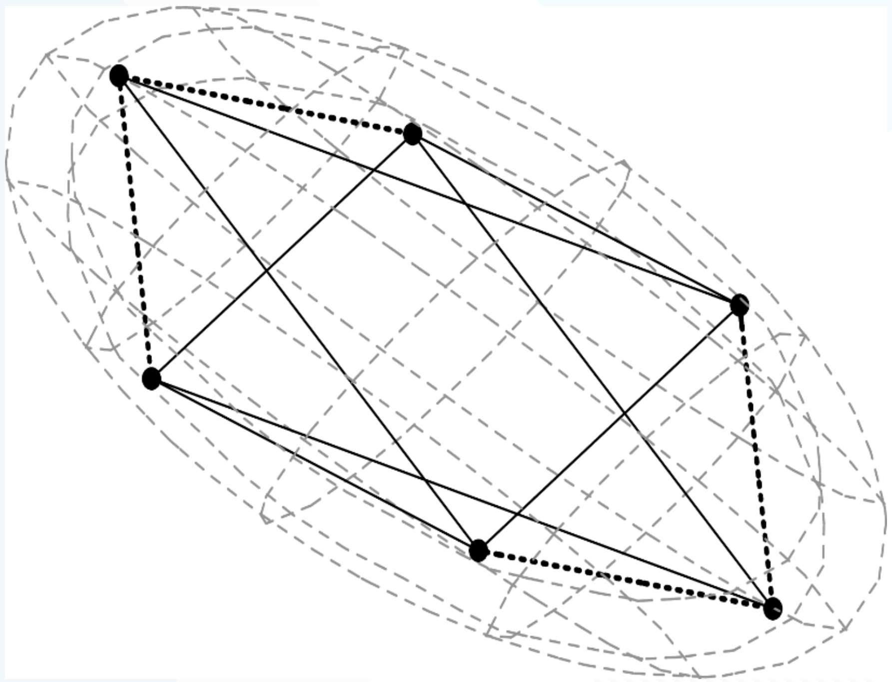


(c)

The IRGQ method in R3



(a)



(b)

Simulation models used

ESIM

- Partial equilibrium (PE) (Grethe 2012)
- GLOBAL (16 regional groups, 3 crops)
- 42 stochastic variables
- Benchmarks by (Artavia et al. 2015)

GLOBIOM

- Partial equilibrium (PE) (Havlík et al.2011; Havlík et al. 2014)
- Bottom-up
- Recursive-dynamic
- Global (31 regions, 18 crops)

Dynamic CGE

- Single-country
- Recursive-dynamic CGE model (Diao and Thurlow 2012)
- linked to the IMPACT modeling system (Robinson et al. 2015)
- 2012 SAM for the Sudan (Siddig et al. 2018)

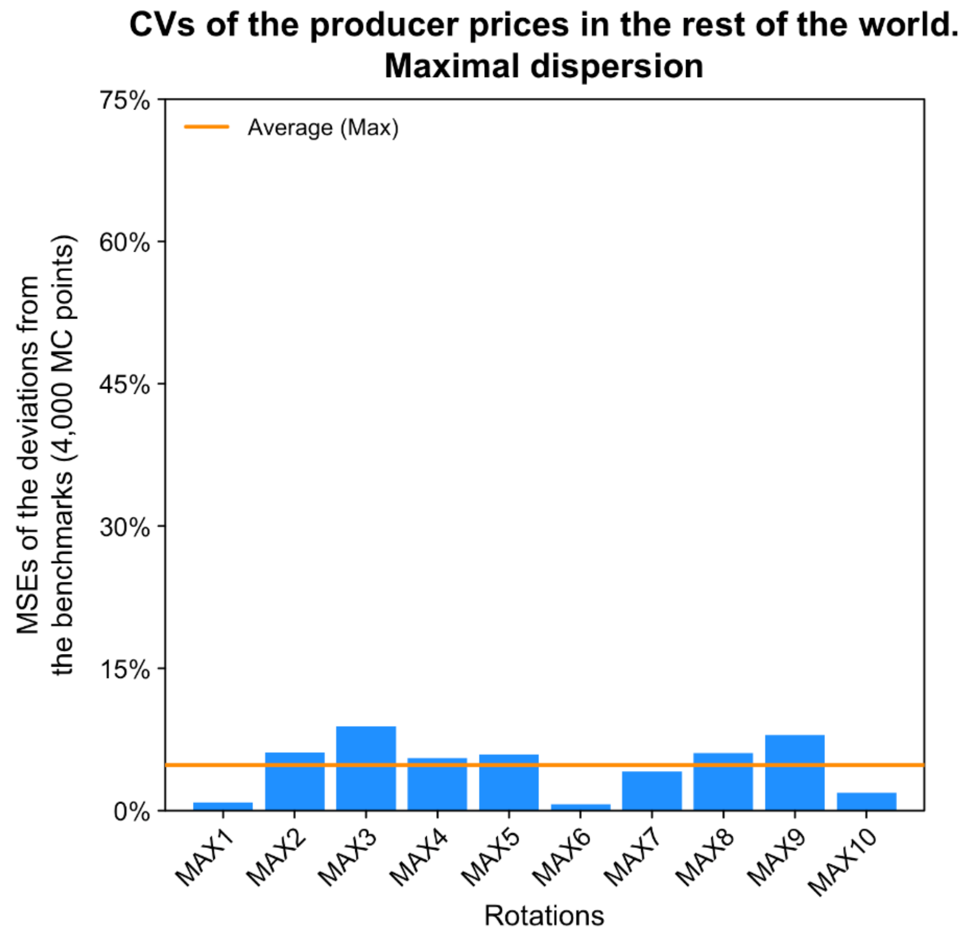
Experimental design

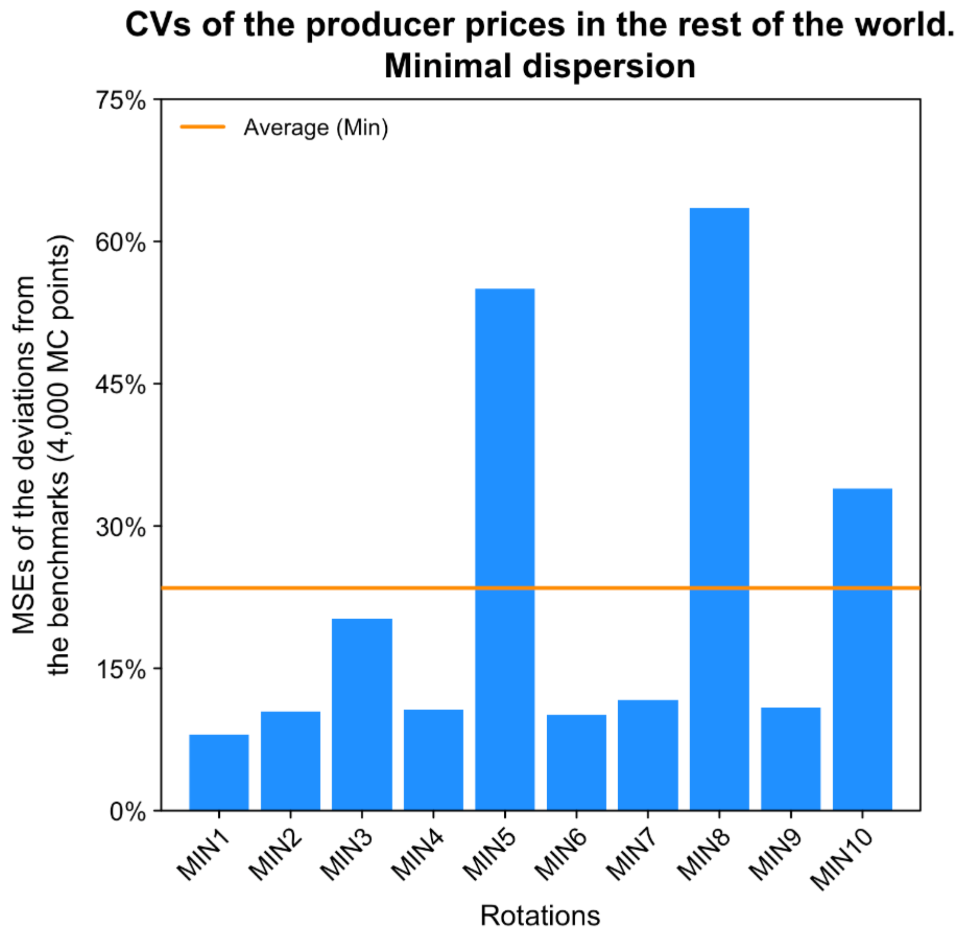
- 10 rotations with maximal dispersion for each model
- 10 rotations with minimal dispersion for each model
- Compare with benchmarks obtained by LHS
- Analyzing yield uncertainty

Percentage deviations of the CVs of producer prices in the "ROW" from the benchmarks. ESIM

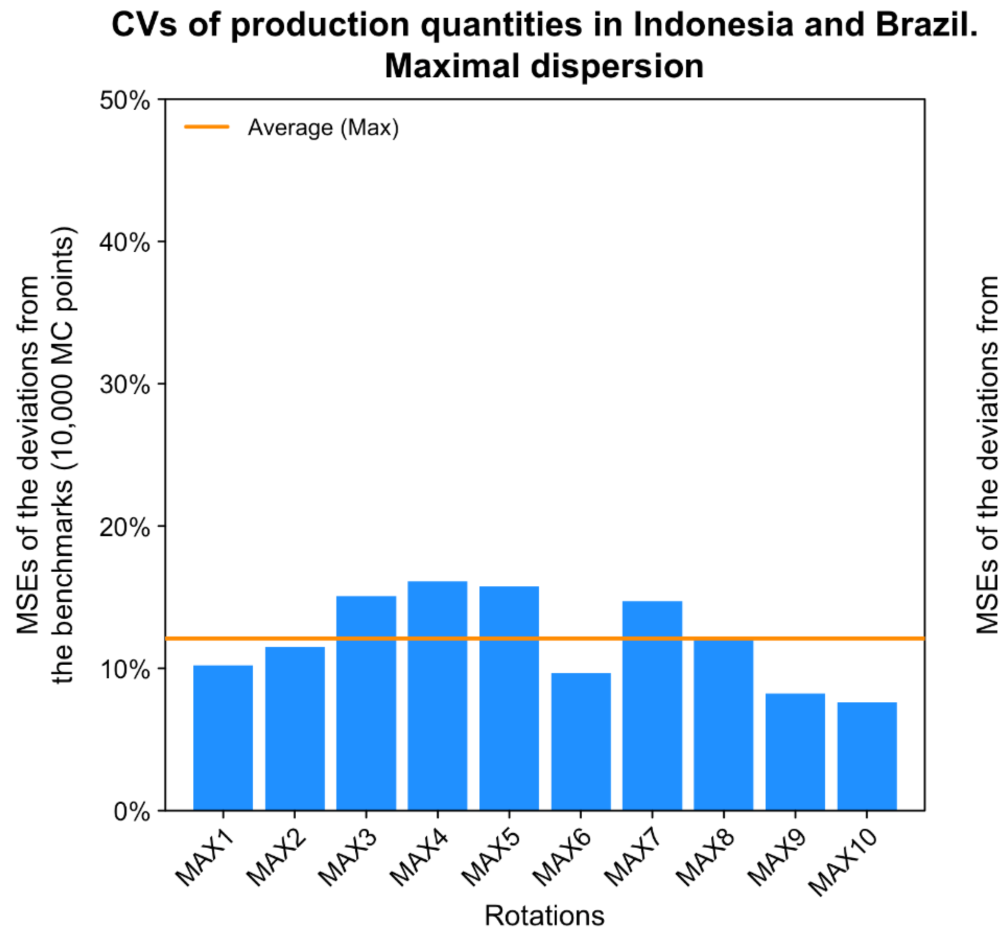
Results obtained by the GQs with maximal dispersion					Results obtained by the GQs with minimal dispersion				
	Barley	Rapeseed	Wheat	MSE		Barley	Rapeseed	Wheat	MSE
MAX1	1.388	0.133	-0.799	0.861	MIN1	3.672	2.832	-1.627	8.050
MAX2	-3.340	0.405	-2.654	6.120	MIN2	-3.508	0.790	-4.305	10.490
MAX3	-4.052	0.855	-3.103	8.928	MIN3	-3.885	0.161	-6.760	20.272
MAX4	2.058	1.523	3.163	5.520	MIN4	-5.283	0.414	-1.956	10.637
MAX5	-3.573	0.806	-2.097	5.938	MIN5	-9.431	1.245	-8.638	55.034
MAX6	1.109	0.811	-0.262	0.652	MIN6	5.387	0.931	0.685	10.117
MAX7	-3.416	0.745	-0.310	4.107	MIN7	-5.034	1.592	2.686	11.695
MAX8	-3.773	0.069	-1.987	6.064	MIN8	13.512	0.582	-2.776	63.539
MAX9	-3.909	0.035	-2.937	7.970	MIN9	-4.608	-0.572	-3.337	10.899
MAX10	0.183	0.818	-2.245	1.914	MIN10	-5.607	-2.424	-8.037	33.972

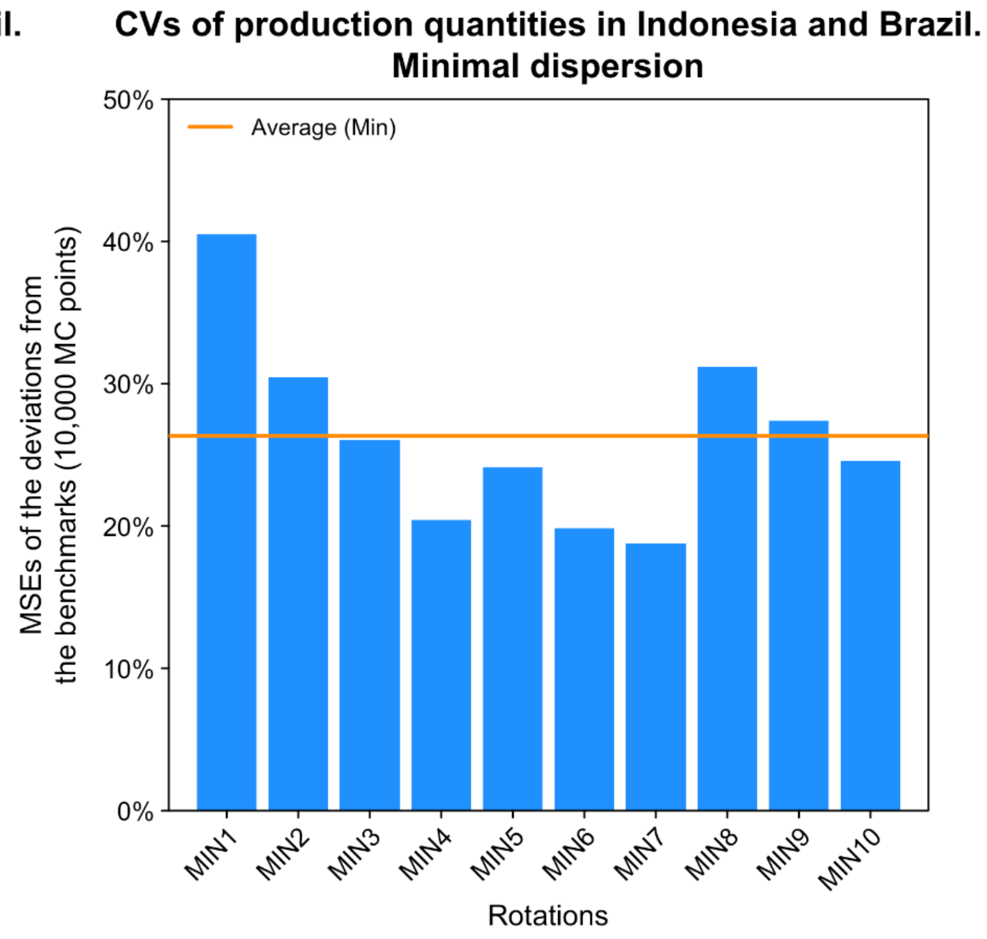
ESIM. Results obtained by GQs with max dispersion vs. min dispersion



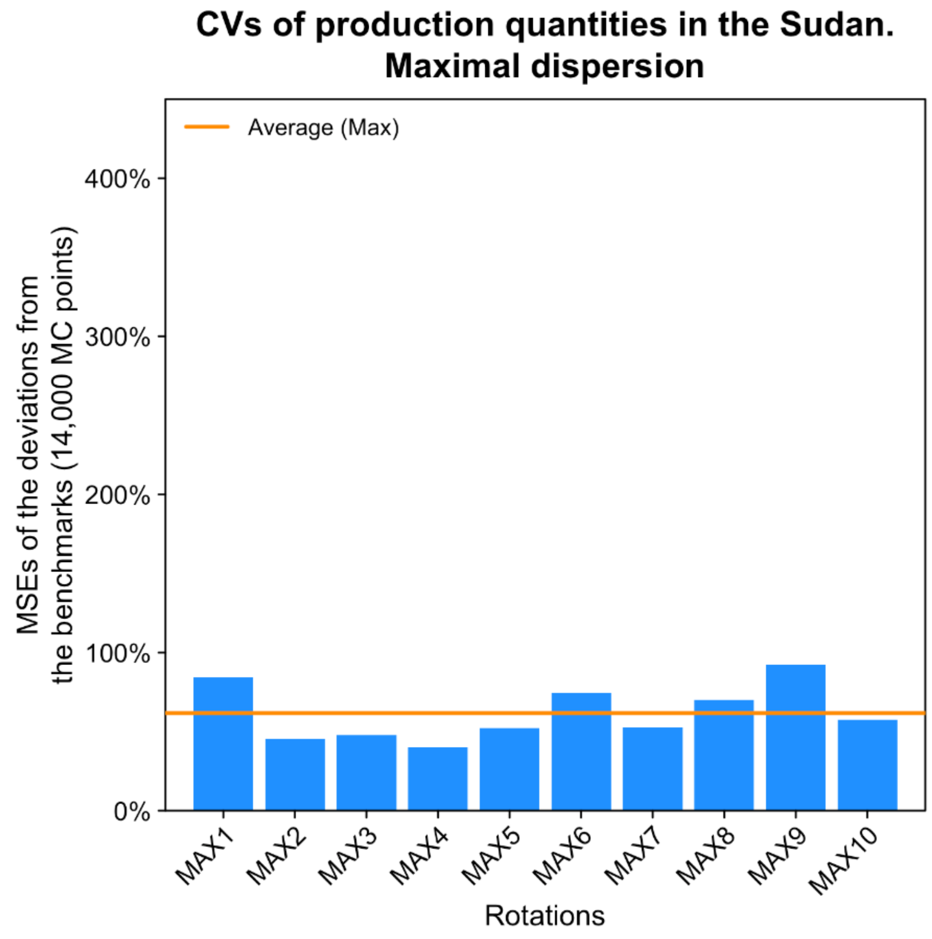


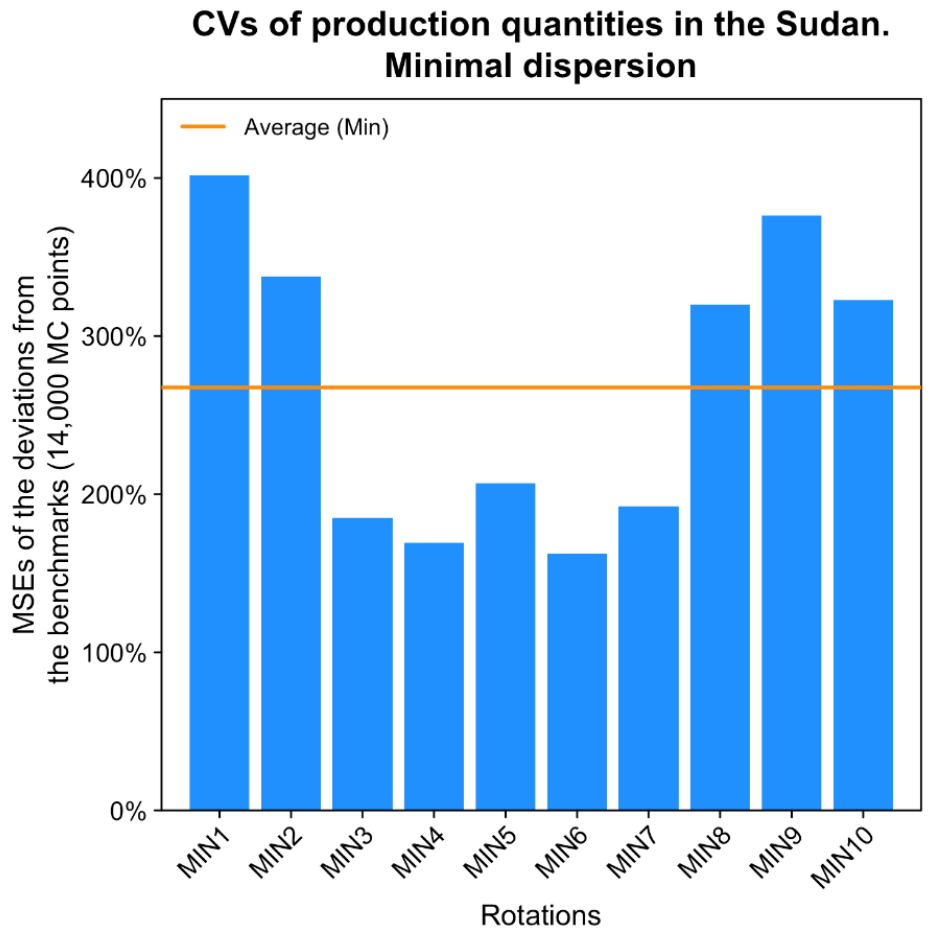
GLOBIOM. Results obtained by GQs with max dispersion vs. min dispersion





Dynamic CGE. Results obtained by GQs with max dispersion vs. min dispersion





Discussion

Limitations of the IRGQ method

- Extreme tails of the distributions are not captured
  - **Disadvantage:** if studying the impacts of extremes
    - Implement IRGQ along with Preckel's et al. (2011a) broader sampling approach
  - **Advantage:** many models are unable to handle large shocks
    - Truncate the distribution of shocks vs. IRGQ
- Restricted to approximating symmetric distributions
  - The central idea can be applied to other distributions

Future research agenda

- **Testing the proposed approaches to**
  - Widening the sampling intervals
  - Extending the methods to depict asymmetric distribution
- **Exploring other factors influencing the quality of the GQ method**
- **Reducing the dimensionality of the problem**
- **Applying and testing the proposed methods in other simulation models addressing policy-relevant questions**

THANK YOU

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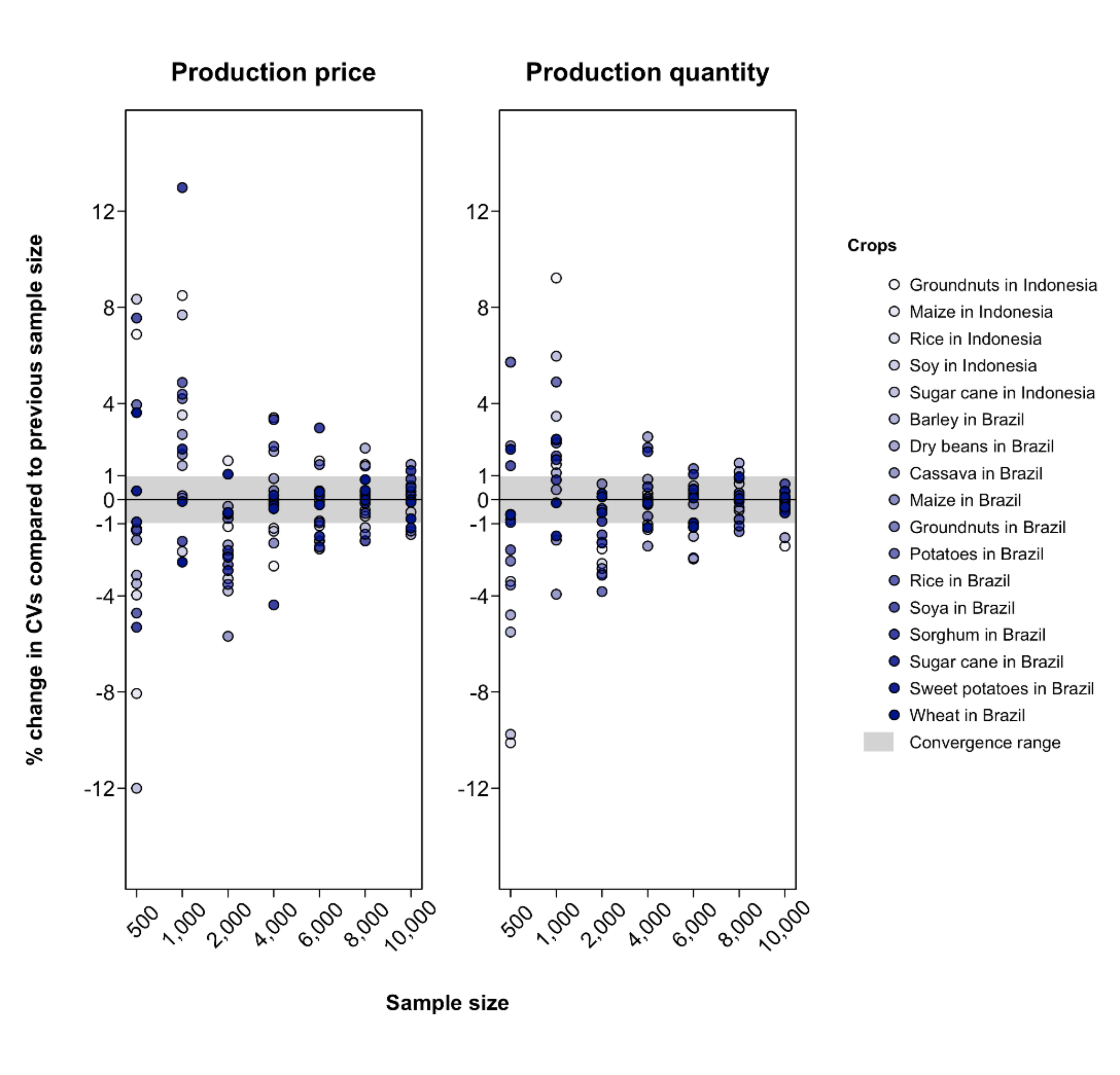
APPENDICES

Dynamic CGE – closure rules

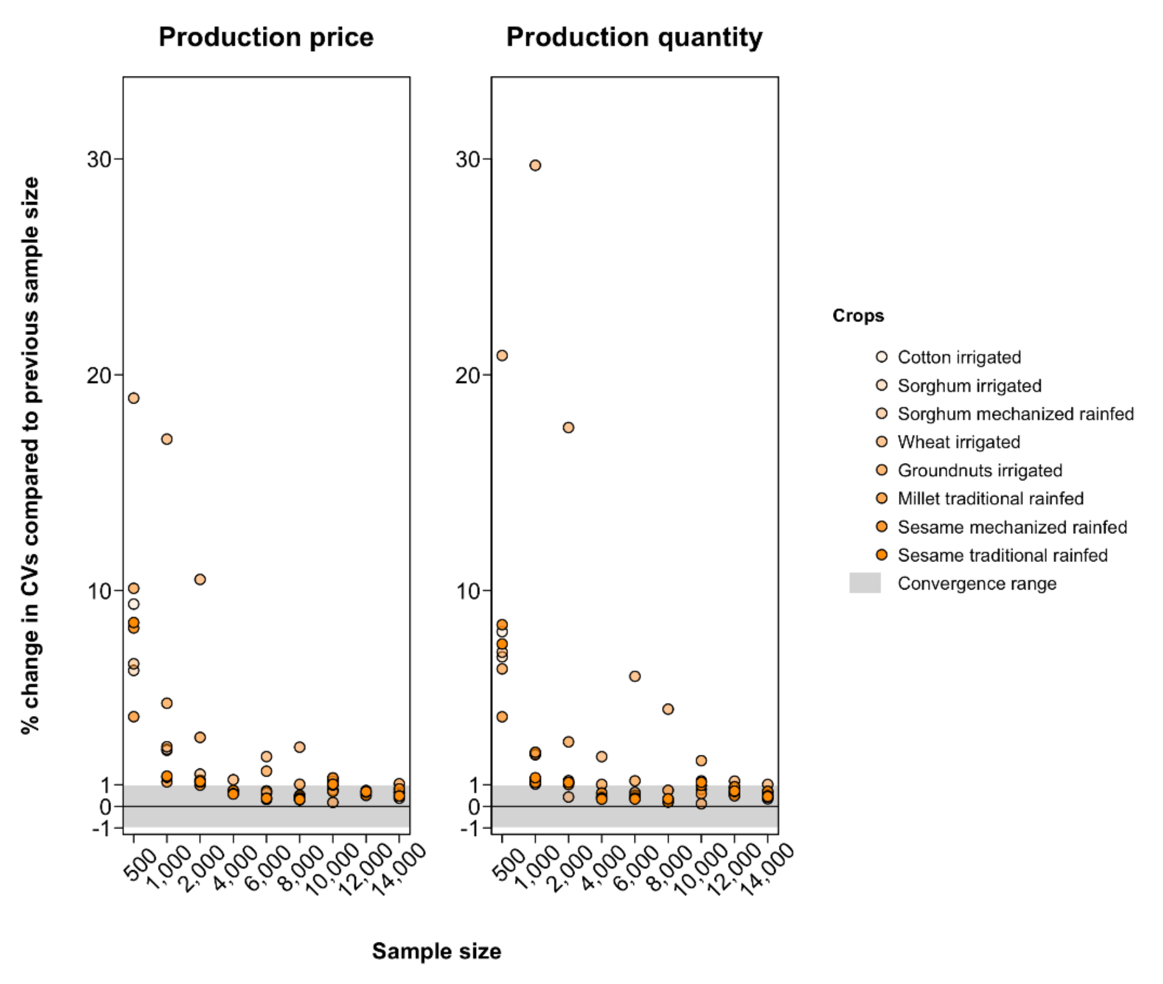
- Fixed investments as share of total absorption
- Flexible exchange rate, fixed foreign savings
- Flexible government savings and fixed tax rates
- Labour and Land - fully employed and mobile
- Capital - fully employed and activity-specific

GLOBIOM. Benchmark



Source: Stepanyan et al. 2021

Dynamic CGE. Benchmark



Source: Stepanyan et al. 2021

The process of identifying good rotations