

Developing a Cost of Capital Module for Computable General Equilibrium Modelling

Ashley Winston
Centre of Policy Studies
Monash University

Motivation.

- Applied CGE models typically do not incorporate business taxes explicitly.
 - ◆ Example: Recent Review of Business Taxation in Australia
 - ◆ MONASH.
 - ◆ MM303.
- Investment outcomes are central in determining the behaviour of dynamic CGE models.
 - ◆ Changes in the value of the firm and the cost of capital, due to changes in tax rates and taxation regimes, are important for understanding investment outcomes.
- The ultimate aim is to enable CGE modelling of the economy-wide effects of reforms to business taxation .

Modelling Capital Gains Taxation.

- Theory and reality diverge.
 - ◆ Accrual systems dominate theoretical work.
 - ◆ Realisation systems dominate policy applications.
- Accrual vs. Realisation systems.
 - ◆ Timing issues.
 - ◆ Realisation-basis capital gains taxation drives a wedge between buyer and seller valuations.
- A solution.
 - ◆ Determine the rate of share realisations.
 - ◆ Accrual Equivalency.

The Value of the Firm.

- We are interested in the value of the firm to a prospective investor.
- We need to account for the dilution of pre-existing shareholder equity over time due to new share issues.

$$V_t^j = VO_t^j - VN_t^j$$

- V = total value of the firm in industry j .
- VO^j = value of pre-existing shareholders equity in a firm industry j .
- VN^j = value of new share issues by firm in industry j .

- We base the valuation on an arbitrage condition.
- Assume no taxes and perfect foresight,

$$i_{t+1}V_t^j = E_{t+1}^j + (VO_{t+1}^j - V_t^j)$$

- E^j = shareholder earnings (cash flows) received on holding equity.
- i = discount factor.

The Value of the Firm...

- The value of the firm in period 0 is the sum of all future income streams and its terminal value in period T , minus new share issues, appropriately discounted.

$$V_0^j = \sum_{t=1}^T \left[\frac{D_t^j - V N_t^j}{\prod_{s=1}^t (1 + i_s)} \right] + \frac{V_T^j}{\prod_{s=1}^T (1 + i_s)}$$

- D_t^j is the dividend paid to shareholders of a firm in industry j .

- A transversality condition allows us to assume away the firm's terminal value.

$$\text{Let } \frac{V_T}{\prod_{s=1}^T (1 + i_s)} \rightarrow 0 \quad \text{as } T \rightarrow \infty$$

- Assuming an infinite horizon...

$$V_0 = \sum_{t=1}^{\infty} \left[\frac{D_t - V_t^N}{\prod_{s=1}^t (1 + i_s)} \right]$$

- Adding one generic income tax...

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{D_t^j (1 - \theta_t) - VN_t^j}{\prod_{s=1}^t (1 + i_s (1 - \theta_s))} \right]$$

- θ is the rate of the income tax.

- Adding one generic accrual-basis capital gains tax...

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{D_t^j \left[\frac{1 - \theta_t}{1 - c_t} \right] - VN_t^j}{\prod_{s=1}^t \left[\frac{1 + i_s (1 - \theta_s) - c_s}{1 - c_s} \right]} \right]$$

- c is the rate of the capital gains tax.

The Value of the Firm...

- V = total value of firm.
- VN = value of new share issues.
- D = unfranked dividends.
- DF = franked dividends.
- i = riskless interest rate.
- Ω = information set for determining expectations.

- θ = personal income tax rate.
- τ = company income tax rate.
- γ = degree of dividend imputation.
- c = capital gains tax rate.
- ϕ = proportion of capital gain taxable.
- ε = rate of equity realisations.
- π = inflation rate.
- α = switch on capital gains tax indexing.
 - (1=real CGT, 0=nominal CGT).

$$V_0^j = \sum_{t=1}^{\infty} \left[\frac{\left[\frac{1-\theta_t}{1-c_t} \right] D_t^j [\Omega_0^j] + \left[\frac{(1-\theta_t)[1-(1-\gamma_t)\tau_t]}{(1-\tau_t)(1-c_t)} \right] DF_t^j [\Omega_0^j] - VN_t^j [\Omega_0^j]}{\prod_{s=1}^t \left[\frac{1+i_s [\Omega_0](1-\theta_s) - c_s [1+\alpha\pi_s [\Omega_0]]}{1-c_s} \right]} \right]$$

and

$$c_t = \varepsilon_t \psi_t c_t + \sum_{z=t+1}^{\infty} \left[\frac{\varepsilon_z \prod_{k=t}^{z-1} (1-\varepsilon_k) \psi_z c_z}{\prod_{s=t+1}^z (1+i_s [\Omega_{t-1}](1-\theta_s))} \right] - \text{is the effective, accrual equivalent rate of the capital gains tax.}$$

The Cost of Capital.

Two potential approaches to analysing the firm's cost of capital:

1. Use changes in the value of the firm as a basis for analysis.
 - Firm behaviour given / solution to production problem assumed.
 - Analyse the effect of changes in tax provisions on the value of given income streams to shareholders.
2. Solve the firm's production problem and analyse its financial policy.
 - The firm seeks to maximise its value to shareholders subject to a set of constraints.
 - Analyse how changes in tax provisions affect firm behaviour and the size and timing of income streams.
 - Most importantly for cost of capital:
 - Investment plan.
 - Financial policy.

The Cost of Capital – Constraints.

■ Capital Accumulation.

- Primary factors and intermediate goods enter the firm's production function via CES nests.

e.g. The firm's stock of capital of type n evolves according to

$$K_t^{n,j} = I_t^{n,j} + (1 - \delta^n) K_{t-1}^{n,j}$$

...and augments the total effective capital stock via a CES nest

$$K_t^j = \Lambda_t^j \left[\sum_{n=1}^M s^{n,j} \left(K_t^{n,j} \right)^{-\sigma^j} \right]^{-\frac{1}{\sigma^j}}$$

Thus the firm's total effective capital stock is

$$K_t^j = \Lambda_t^j \left[\sum_{n=1}^M s^{n,j} \left(I_t^{n,j} + (1 - \delta^n) K_{t-1}^{n,j} \right)^{-\sigma^j} \right]^{-\frac{1}{\sigma^j}}$$

Constraints...

■ Product Demand.

- The firm faces a finite elasticity of demand for its output.

$$p_t^i = \left(Y_t^{i,j}\right)^{-\eta^j} \quad \text{with} \quad ED_t = -\frac{1}{\eta}$$

Applied to the firm's CES production function, the revenue function is

$$p_t^i Y_t^{i,j} = \left[Y_t^j\right]^{1-\eta^j} = \left[A_t^j \left[a_1^j \left(L_t^j\right)^{-\rho^j} + a_2^j \left(K_{t-1}^j\right)^{-\rho^j} + a_3^j \left(Q_t^j\right)^{-\rho^j} \right]^{\frac{1}{\rho^j}}\right]^{1-\eta^j}$$

- K^j = capital stock.
- $I^{n,j}$ = real investment.
- δ^n = physical rate of depreciation.
- p^j = producer price of firm's output.
- $Y^{i,j}$ = firm's level of output.
- η^j = parameter for setting own-price elasticity of demand for firm's product.

Constraints...

The nature of the firm's sources and uses of funds are captured in the following expressions:

■ Cash Flow

$$D_t^j + DF_t^j = \left[y_t^j \right]^{1-\eta^j} + B_t^j + VN_t^j - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - (1 + i_t) B_{t-1}^j - \sum_{n=1}^N q_t^n I_t^{n,j} - T_t$$

■ Tax

$$T_t^j = \tau_t \left[\left[Y_t^j \right]^{1-\eta^j} - \sum_{m=1}^M w_t^m L_t^{m,j} (1 + \nu_t) - \sum_{i=1}^G P_t^g Q_t^{g,j} - i_t B_{t-1}^j - \sum_{n=1}^N \Gamma_t^n q_t^n I_t^{n,j} - \sum_{s=-\infty}^t \sum_{n=1}^N \Delta_{s,t-s}^n q_s^n I_s^{n,j} \right]$$

- q^n = asset price of capital of type n .
- L^j = effective labour usage (CES nest).
- w = nominal wage rate.
- ν = payroll tax rate.
- Q^j = effective total intermediate good usage (CES nest).
- P = purchaser's price of intermediate goods.
- p^i = producer price of good i in period t .
- A = efficiency parameter (all-primary factor technical change).
- a_i = distributional parameter.
- ρ = determines elasticity of substitution.
- I = real investment.
- Δ = depreciation and investment allowances.
- δ = physical rate of depreciation.
- B = debt issues.

Constraints...

■ Sign Constraints

- The firm is unable to issue “negative dividends”.

$$D_t \geq 0$$

$$DF_t \geq 0$$

- The firm’s investment is always non-negative (i.e. it cannot liquidate productive capital).

$$I_t \geq 0$$

- The firm is constrained to generate revenue through productive activities.

$$B_t \geq 0$$

- The firm is denied the ability to pursue share repurchases.

$$VN_t \geq 0$$

The Cost of Capital – Theoretical basis.

- An optimising firm invests up to the point where $MRP_t^K = VMP_t^K$, or, in terms of our model,

$$k_t^{n'} \left[\frac{(1-\eta)a_2}{A_{t+1}^\rho} \right]^{-\eta} \left[\frac{Y_{t+1}}{K_t} \right]^{-\eta(1+\rho)} - \delta^n = \frac{\lambda_t^3 k_t^{n'} \phi_{t+1} - \lambda_{t+1}^3 k_{t+1}^{n'} (1 - \delta^n)}{[\lambda_{t+1}^1 - \lambda_{t+1}^2 \tau_{t+1}]} - \delta^n$$

- $k_t^{n'}$ = change in total capital stock due to a change in stock of capital of type n .
 - δ^n = physical rate of depreciation on a unit of capital of type n .
 - τ = company tax rate.
 - $\lambda_t^1, \lambda_t^2, \lambda_t^3$ = Lagrangian multipliers on cash flow, tax and capital stock constraints respectively.
- Our task now is to define the Lagrangian multipliers in terms of the parameters of the model – most importantly, tax-related parameters.
 - This involves an analysis of the financial incentives facing the firm to determine its financial policy – specifically, the source of the marginal unit of finance.

The Cost of Capital – Some Examples.

*Financing at the Margin with Retained Unfrankable Earnings - a **SLACK** franking account constraint.*

- $K^{j'}$ = change in stock of capital of type n due to a change in investment in capital of type n .
- $k^{n'}$ = change in total capital stock due to a change in stock of capital of type n .
- q^n = asset price of capital of type n .
- I^n = real investment in capital of type n .
- Ξ^n = present value of depreciation and investment allowances on a unit of capital of type n .
- Γ^n = investment expenditure allowances on capital of type n .
- δ^n = physical rate of depreciation on a unit of capital of type n .
- τ = company tax rate.
- θ = personal income tax rate.
- α = switch on capital gains tax indexing.
- π = inflation rate.

$$COC_t^j = \left[\frac{1 - \tau(\Gamma^n + \Xi^n)}{1 - \tau} \right] \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] \frac{k_t^{n'}}{K_t^{j'}} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n$$

The Cost of Capital – Some Examples

*Financing at the Margin with Retained Unfrankable Earnings - a **BINDING** franking account constraint*

- $K^{j'}$ = change in stock of capital of type n due to a change in investment in capital of type n .
- $k^{n'}$ = change in total capital stock due to a change in stock of capital of type n .
- q^n = asset price of capital of type n .
- I^n = real investment in capital of type n .
- Ξ^n = present value of depreciation and investment allowances on a unit of capital of type n .
- Γ^n = investment expenditure allowances on capital of type n .
- δ^n = physical rate of depreciation on a unit of capital of type n .
- τ = company tax rate.
- θ = personal income tax rate.
- γ = degree of dividend imputation (1 = full).
- α = switch on capital gains tax indexing.
- π = inflation rate.

$$COC_t^j = \left[\frac{1 - (1 - \gamma)\tau(\Gamma^n + \Xi^n)}{1 - (1 - \gamma)\tau} \right] \left\{ \left[\frac{1 + i_{t+1}[\Omega_t](1 - \theta) - c[1 + \alpha\pi_{t+1}[\Omega_t]]}{1 - c} \right] \frac{k_t^{n'}}{K_t^{j'}} - \frac{q_{t+1}^n}{q_t^n} \frac{k_{t+1}^{n'}}{K_{t+1}^{j'}} (1 - \delta^n) \right\} - \delta^n$$

Further Areas of .

- Further complicate the modelling of firm financial policy
 - ◆ New equity issues: Transaction costs, underwriting fees?
 - ◆ Retained earnings: Do dividends carry any non-cash benefits?
- Relaxing the assumption of the representative shareholder
- CGE Modelling tasks
 - ◆ Linearise the equations around a solution
 - ◆ Data set
 - ◆ Programming / Calibration tasks