# The Global Trade and Environment Model: A projection of non-steady state data using Intertemporal GTEM

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# OBJECTIVES

- Derive an intertemporal version of GTEM
- Demonstrate that an intertemporal general equilibrium model can be solved in GEMPACK with a single period non-steady-state database.
- Examine the steady state properties of the model.

# Basic Assumptions

- investment is fully bond financed
- local bonds are perfectly substitutable with foreign bonds and hence earn the same global rate of return.
- the regional capital stock is fully owned by identical regional households.
- We do not impose the arbitrage condition that local bonds yield the same rate of return as the physical capital.

# Assumptions…

- (The rate of return in physical capital is allowed to vary across the regions)
- Regional households supply factors inelastically to the market
- The population and the labor supply grow exponentially at the rate of n
- Production functions are characterised by constant returns to scale

Given this environment, the only problem left to a household is –

the allocation of income between current consumption and accumulation of assets to maximise the sum of periodic utilities.

# The problem of the household

$$\max \int_0^{\infty} \ln c(t) e^{-\theta t} dt$$

Subject to

$$c_t + db_t^h / dt + db_t^f / dt + n(b_t^h + b_t^f)$$
$$= [R_t k_t - \rho_t b_t^h] + W_t + \rho_t (b_t^h + b_t^f)$$

where lower cases are for per capita variables

With the intertemporal budget constraint the problem becomes:

$$\max \int_0^\infty \ln c(t) e^{-\theta t} dt \qquad \text{subject to}$$

$$\int_0^\infty c_t \exp\{-\int_0^t (\rho_v - n) dv\} dt = b_0^f + \omega_0^k + \omega_0^h$$

where

$$\omega_0^k = b_0^h + \int_0^\infty [R_t k_t - \rho_t b_t^h] \exp\{-\int_0^t (\rho_v - n) dv\} dt$$

$$\omega_0^h = \int_0^\infty W_t \exp\{-\int_0^t (\rho_v - n) dv\} dt$$

The solution of the above problem can be obtained as

$$c_t = c_0 \exp\{\int_0^t (\rho_v - n - \theta) dv\}, \qquad \text{and}$$

$$c_0 = \theta(b_0^f + \omega_0^k + \omega_0^h).$$

To make the above solution operational we need to specify the expectational rules determining human and non-human wealth

# Assumptions on expectations:

- Households have static expectations regarding their income, prices and population growth
- They believe that the global interest rate is fixed for ever

Under these assumptions we have

$\omega_0^h = W_0 / \overline{\rho}$ and $\omega_0^k = R_0 k_0 / \overline{\rho}$

Therefore, in place of $c_0 = \theta(b_0^f + \omega_0^k + \omega_0^h)$

We can write $c_0 = [\theta / \overline{\rho}][R_0 k_0 + W_0 + \overline{\rho}\, b_0^f\,]$

Given that $\theta$ and $\overline{\rho}$ are fixed we obtain that the current

consumption is a fixed fraction of current income

# How about investors?

# The optimal investment

$$I_t^r = \delta^r K_t^r \exp\{\beta_r(\rho_t^{re} - \rho_t^g)\}$$

where,

$$\rho_t^{re} = [R_t^r + (1-\delta^r)\Pi_t^r]/\Pi_t^r - 1$$ - static expectations

$$\rho_t^{re} = [R_{t+1}^r + (1-\delta^r)\Pi_{t+1}^r]/\Pi_t^r - 1$$ - rational expectations

$$\rho_T^{re} = \rho_{T-1}^{re}$$ - terminal condition

# Calibration issue

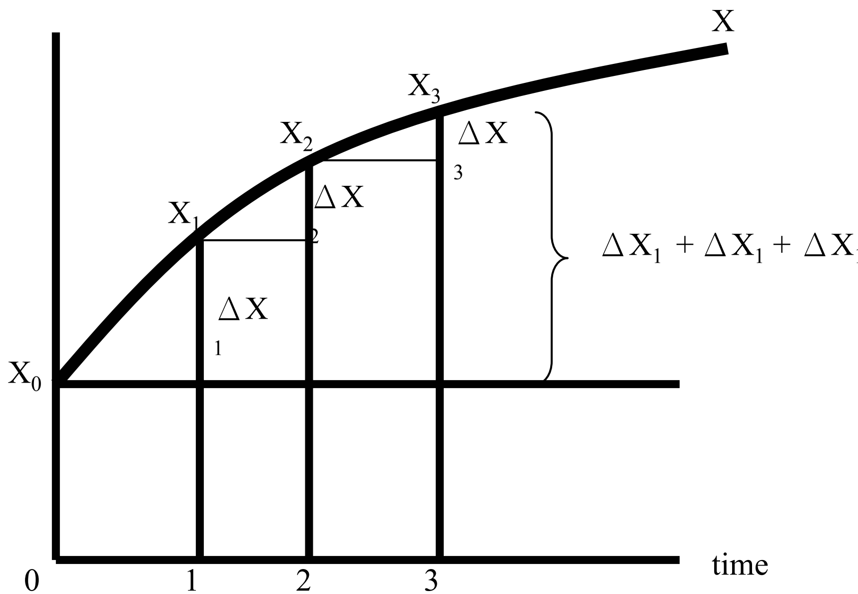
$F(Y_t, X_t) = 0$ - contemporaneous (static)

$F(Y_t, Y_{t-1}, X_t) = 0$ - recursively dynamic

$F(Y_t, Y_{t+1}, X_t) = 0$ - Intertemporal

# How do we calibrate an intertemporal model?

# Link between time t value and the base year value




$$X_t = X_0 + \sum_{k=1}^{t} \Delta X_k$$

Define

$$cc\_X_t = \sum_{k=1}^{t} \Delta X_k$$

Then, we can initialize

$$X_t = X_0$$

Update(change)

$$X_t = cc\_X_t$$

for all t = 0, 1, 2, …

# Equilibrium?

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# Steady state

**To stabilize debt and capital stock we need:** $\sigma^r P_t^r Y_t^r = \Pi_t^r I_t^r = \delta^r K_t^r \Pi_t^r$

In linearised form

$$p_t^r + y_t^r = \pi_t^r + i_t^r = k_t^r + \pi_t^r$$

When prices are constant, we have

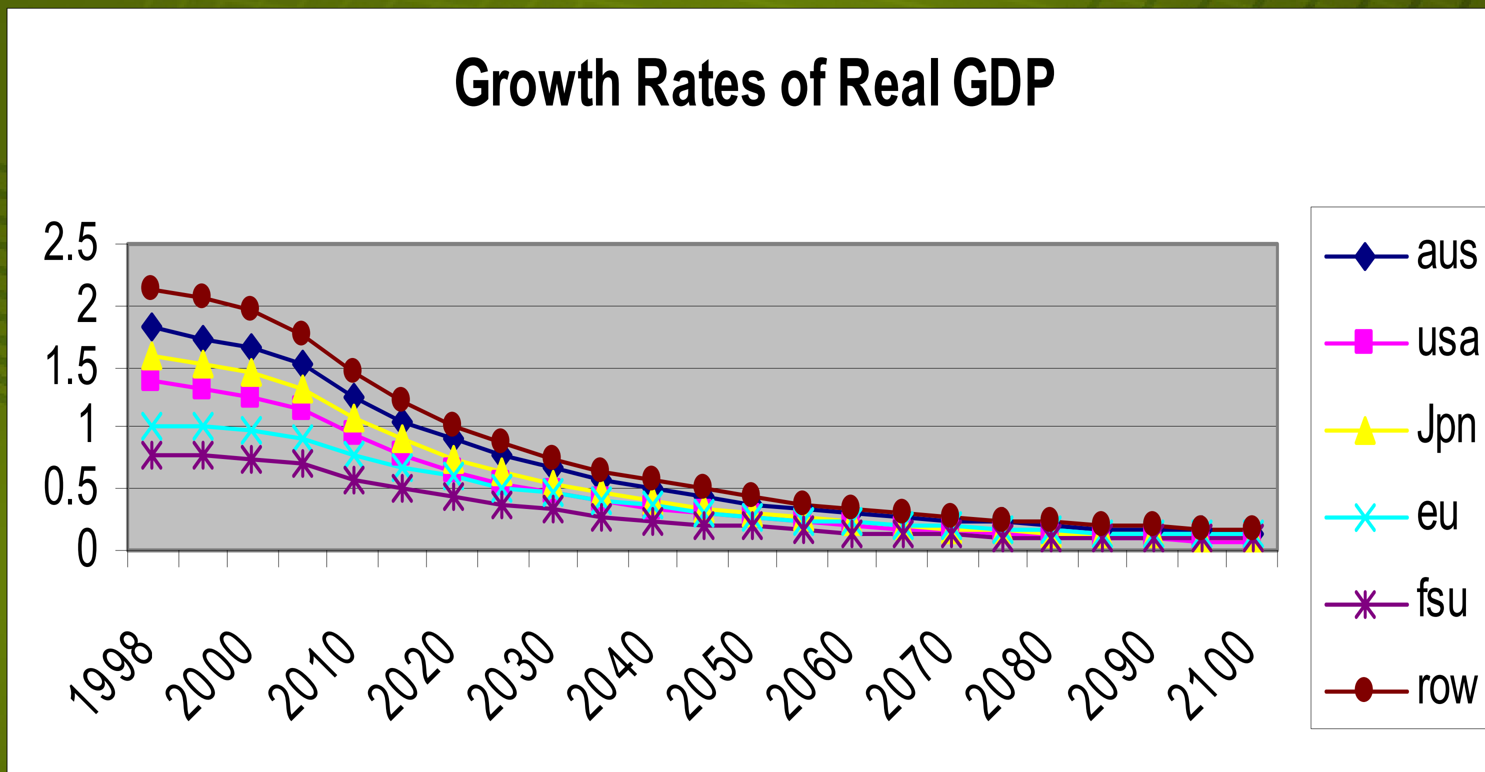
$$y_t^r = i_t^r = k_t^r$$

# Three test simulations

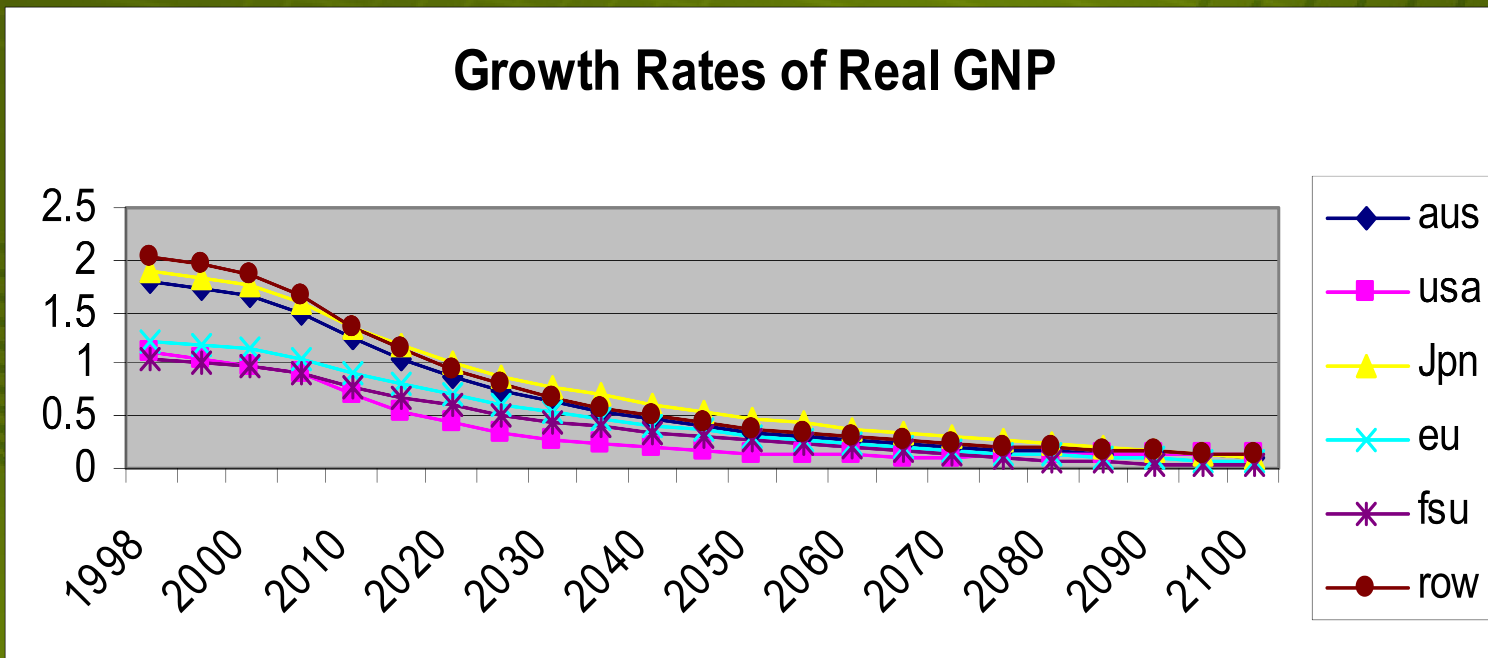
- to examine the convergence property of the model under initial data conditions (momentum simulation);
- to compare the behavior of trajectories under rational and static expectations, and
- To examine the convergence property of the model when all exogenous factors grow at 2 per cent per anum everywhere.

# Some results…

# Momentum simulation…



# Momentum simulation…

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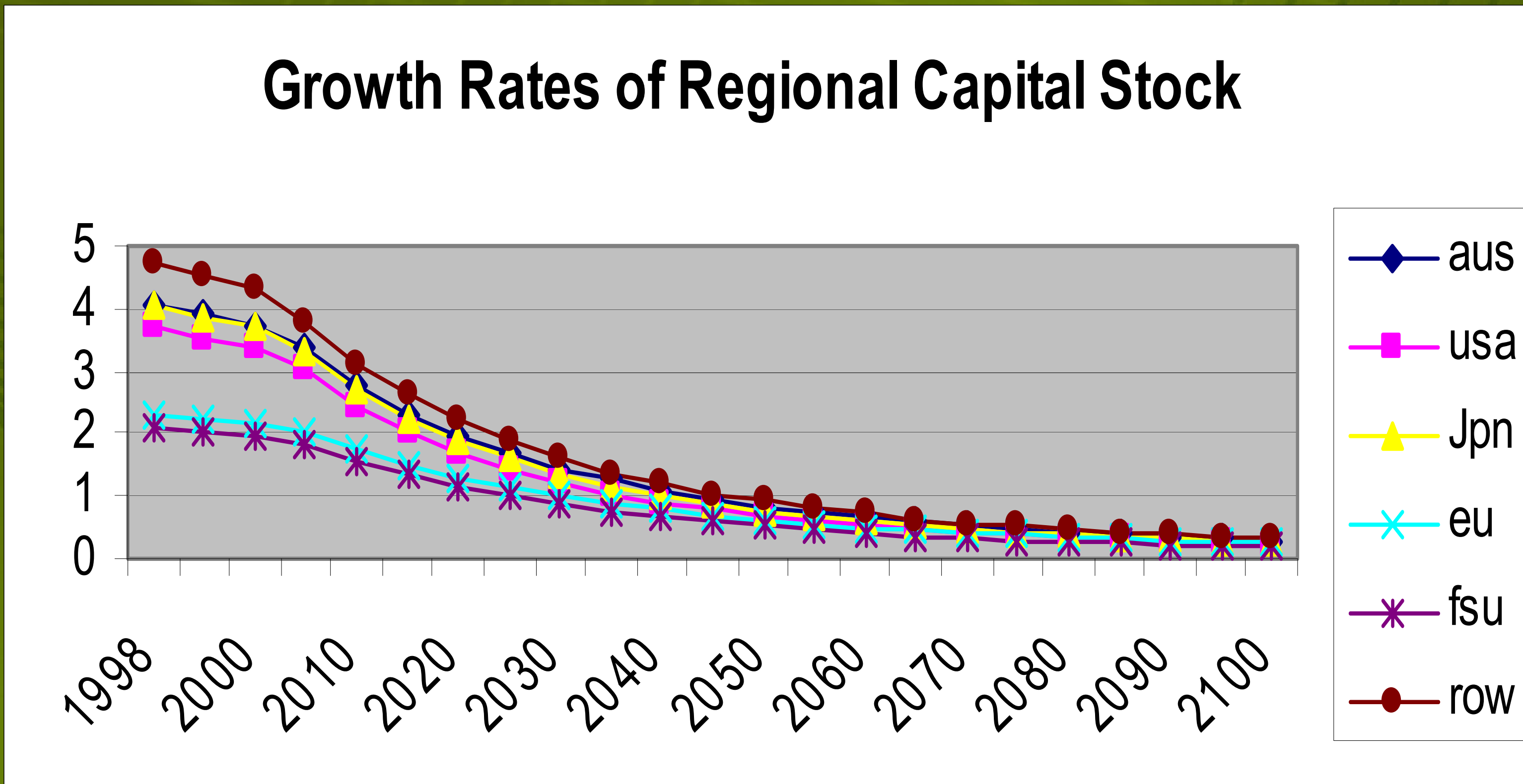


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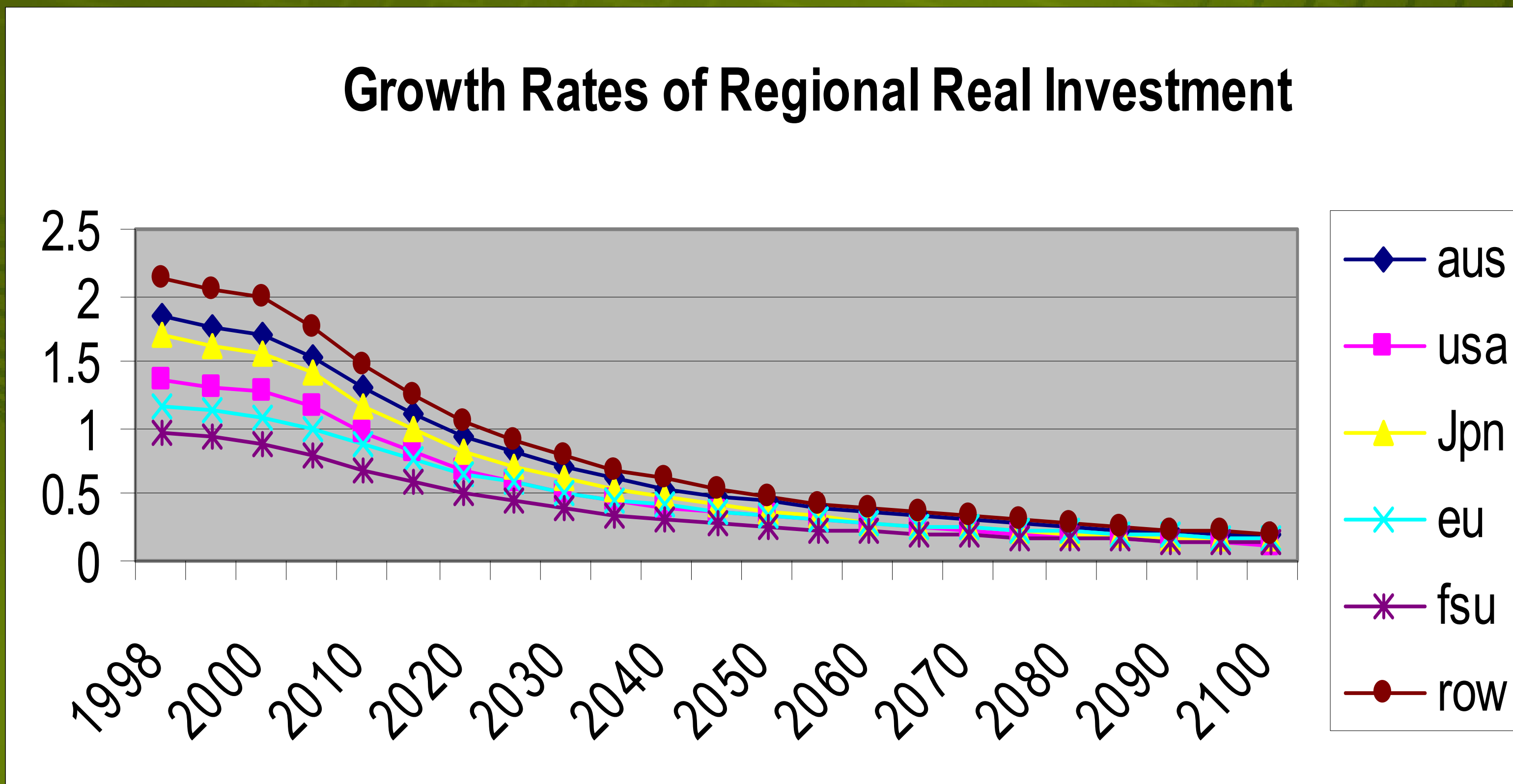
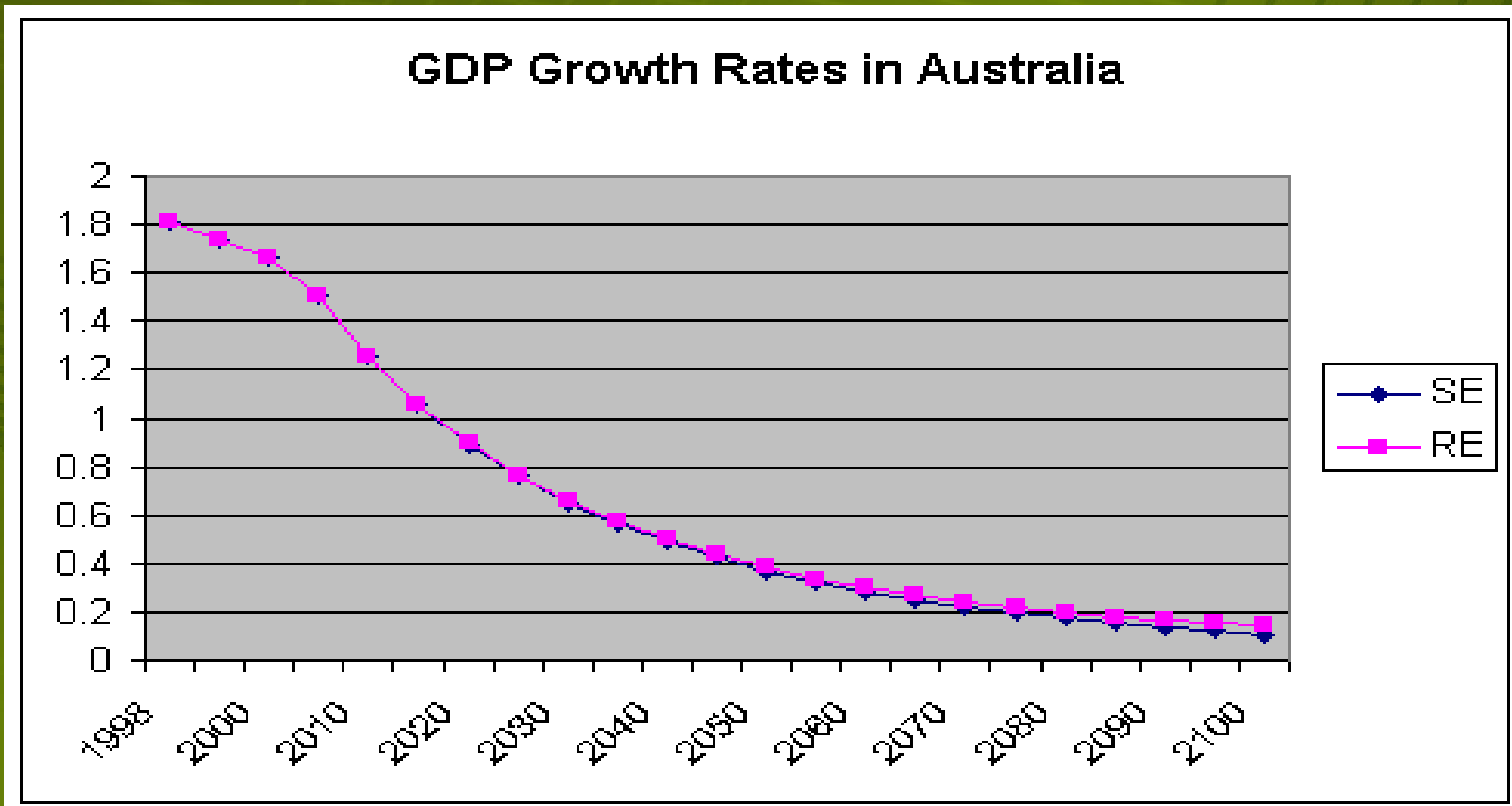
# Momentum simulation…



# Static vs rational expectations



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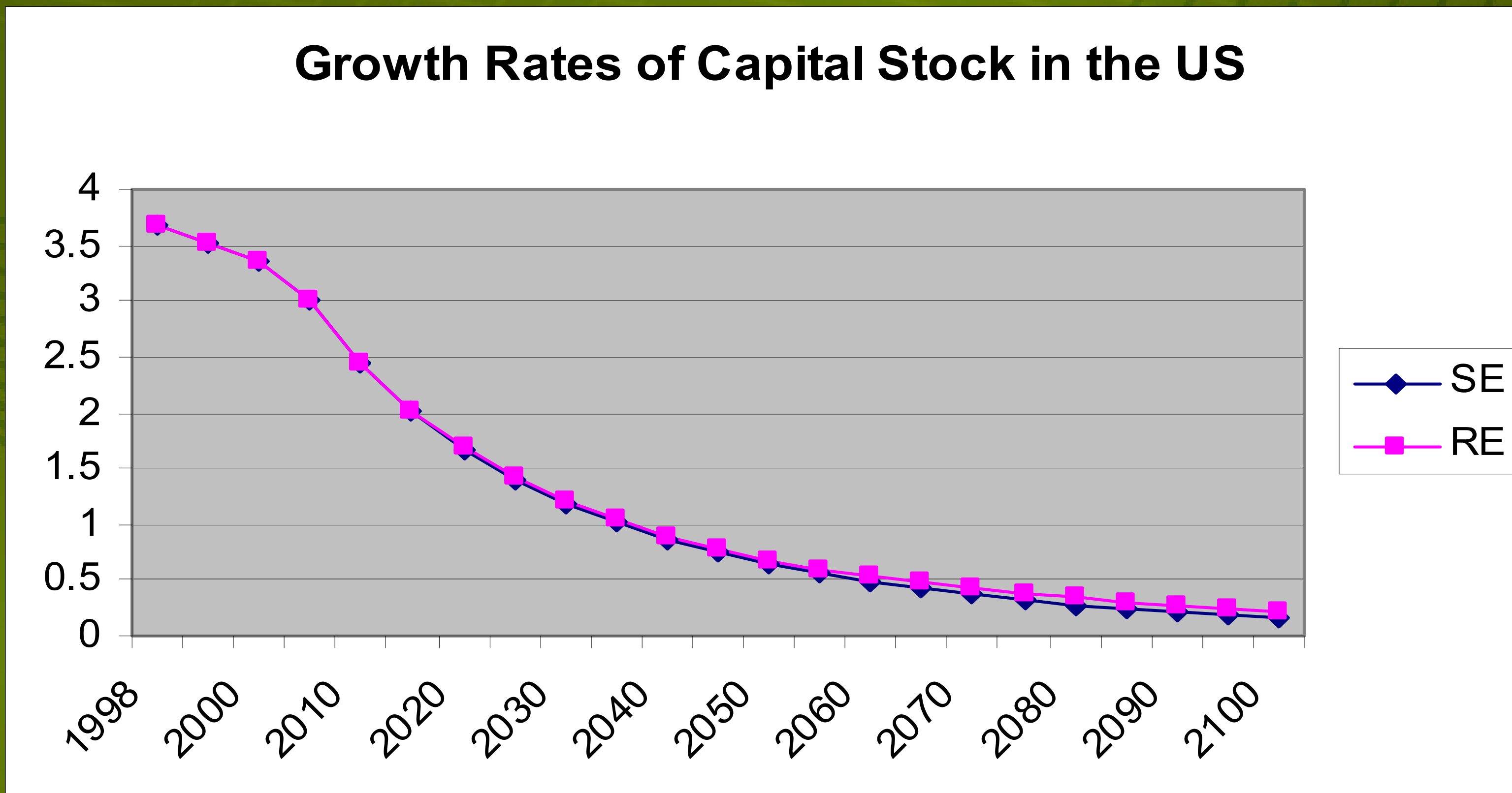


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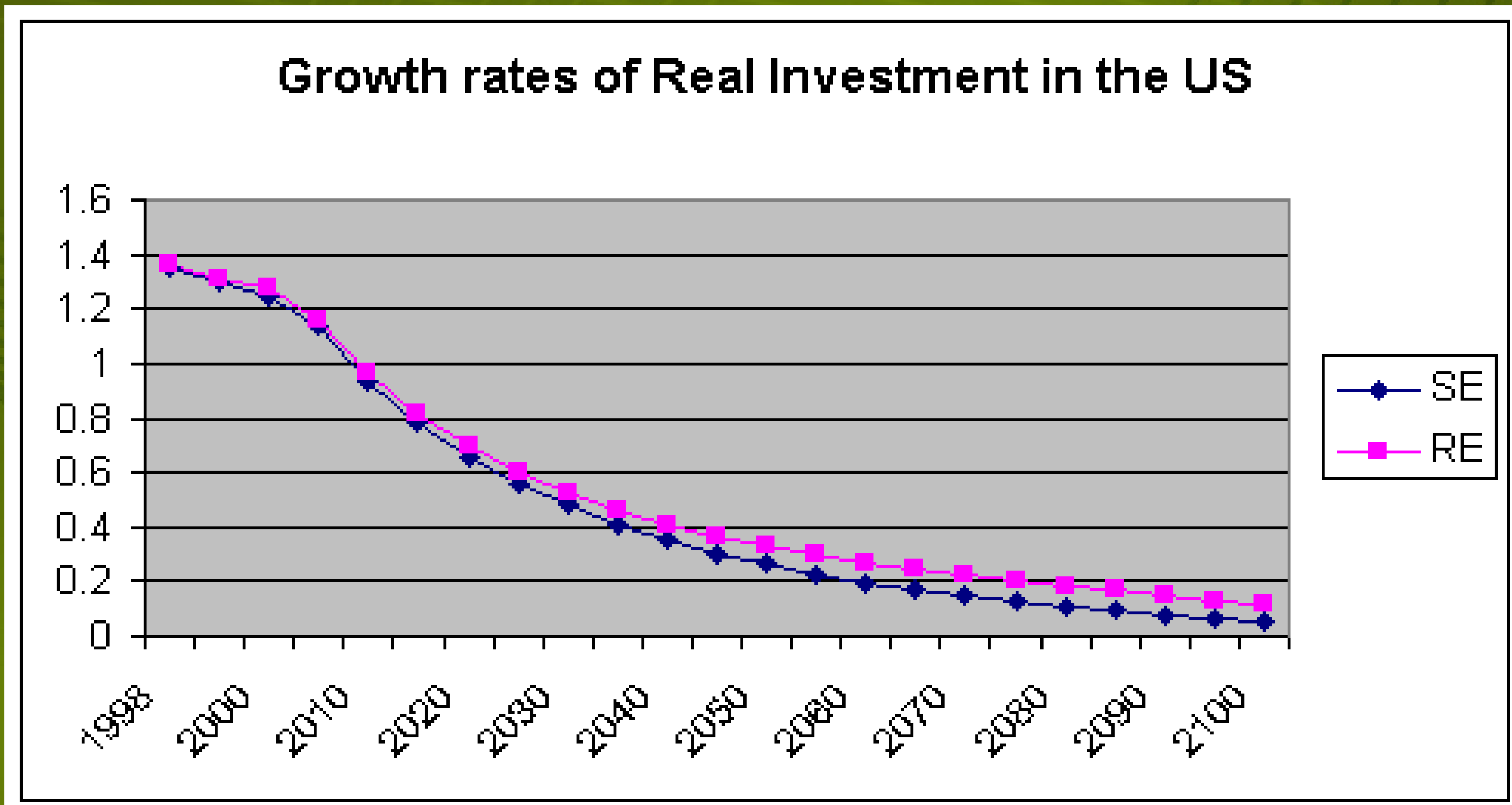
# Static vs rational expectations

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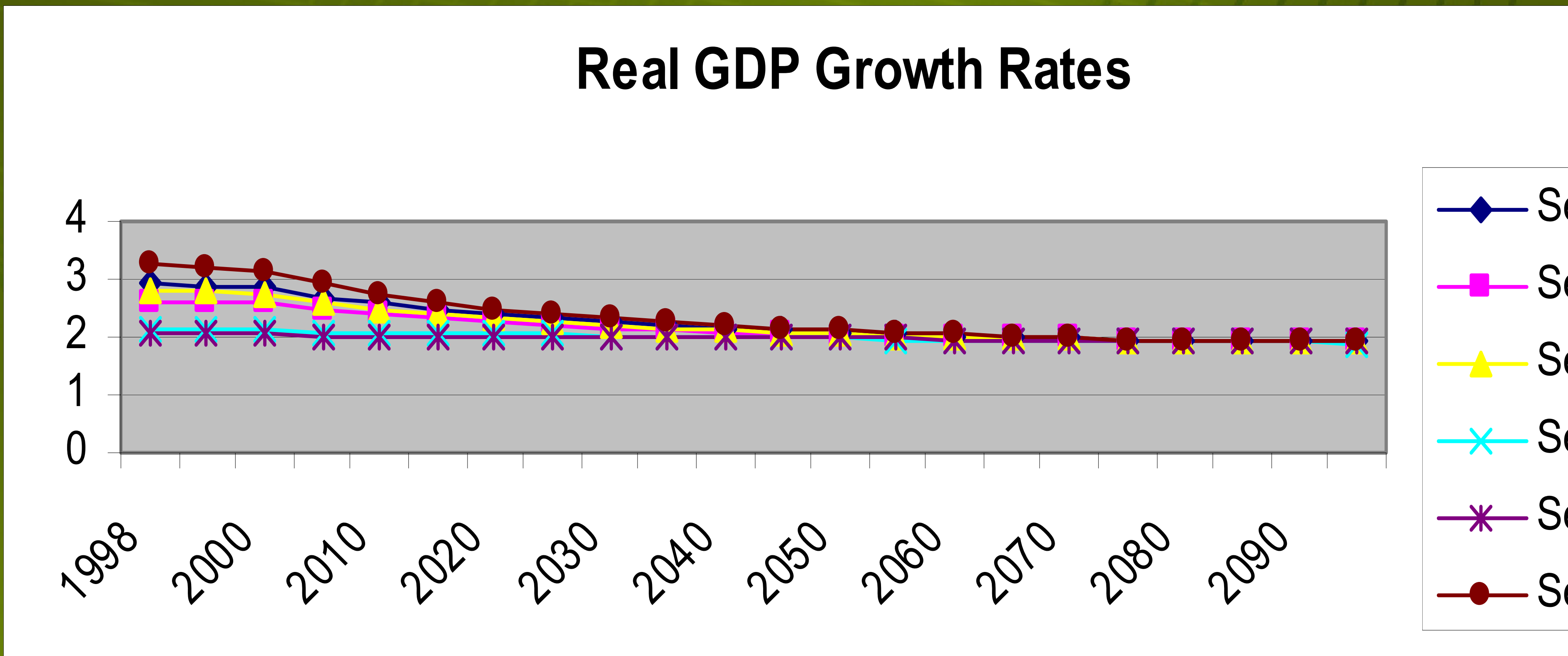
# Uniform growth of 2 per cent

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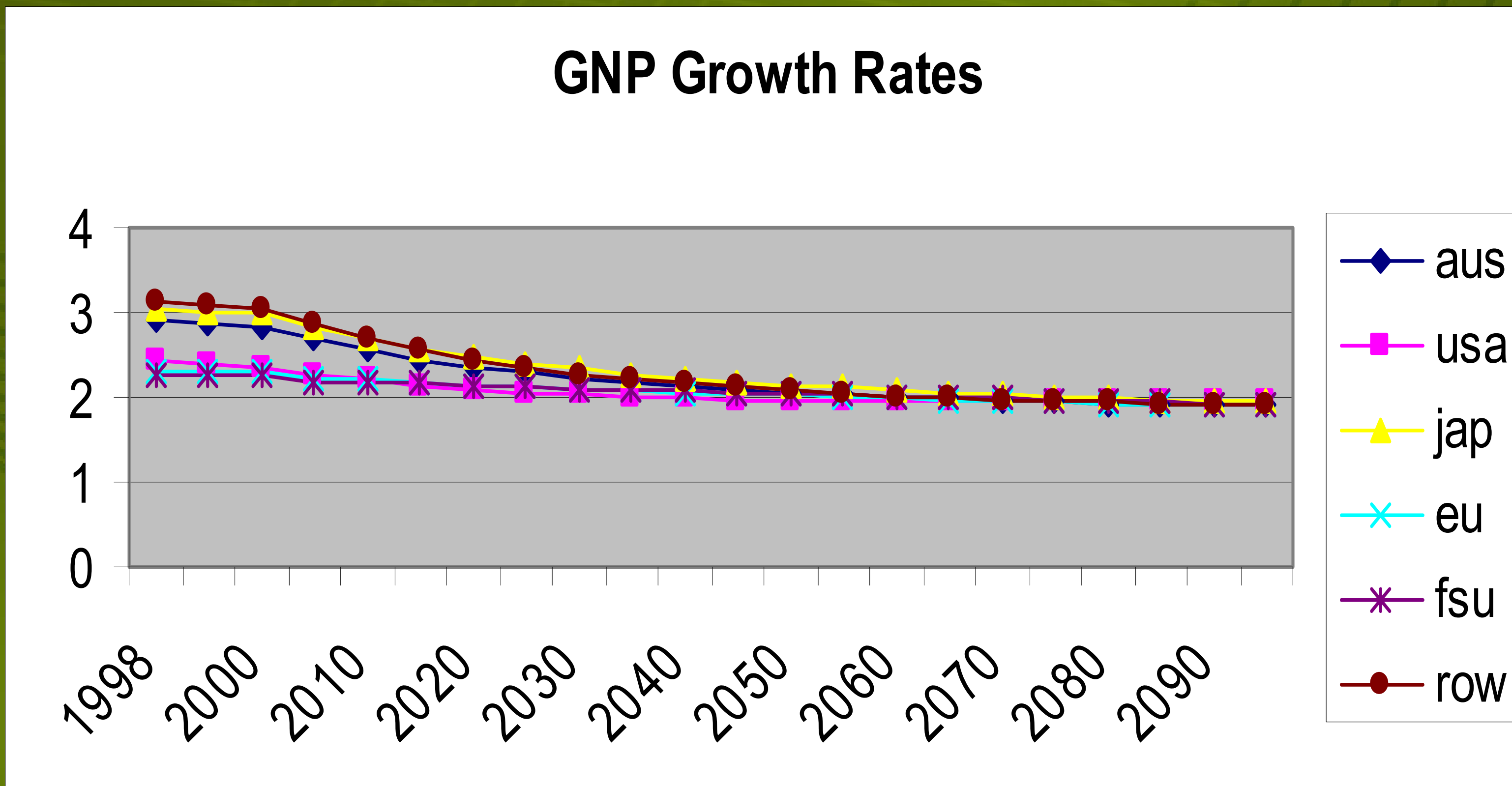


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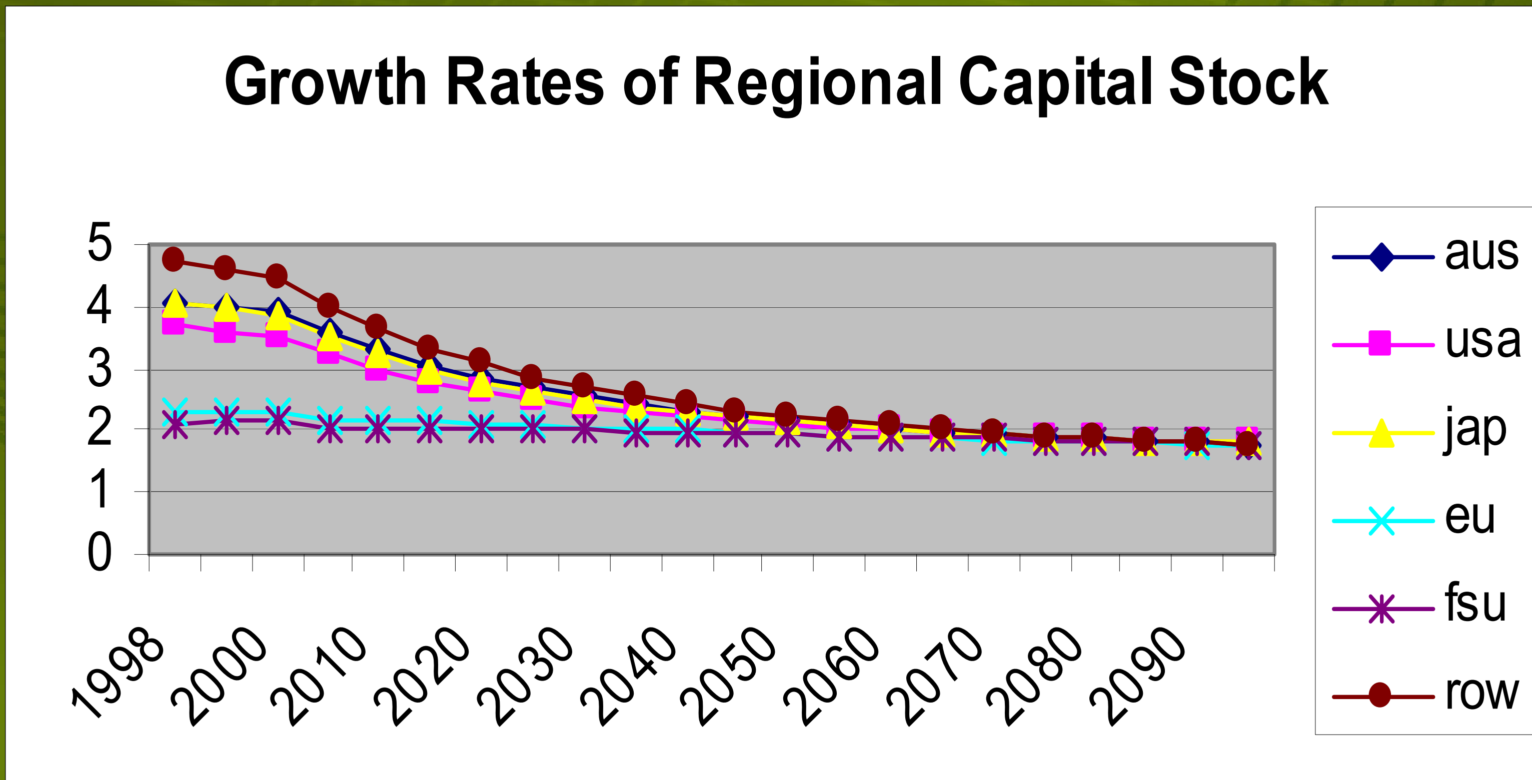


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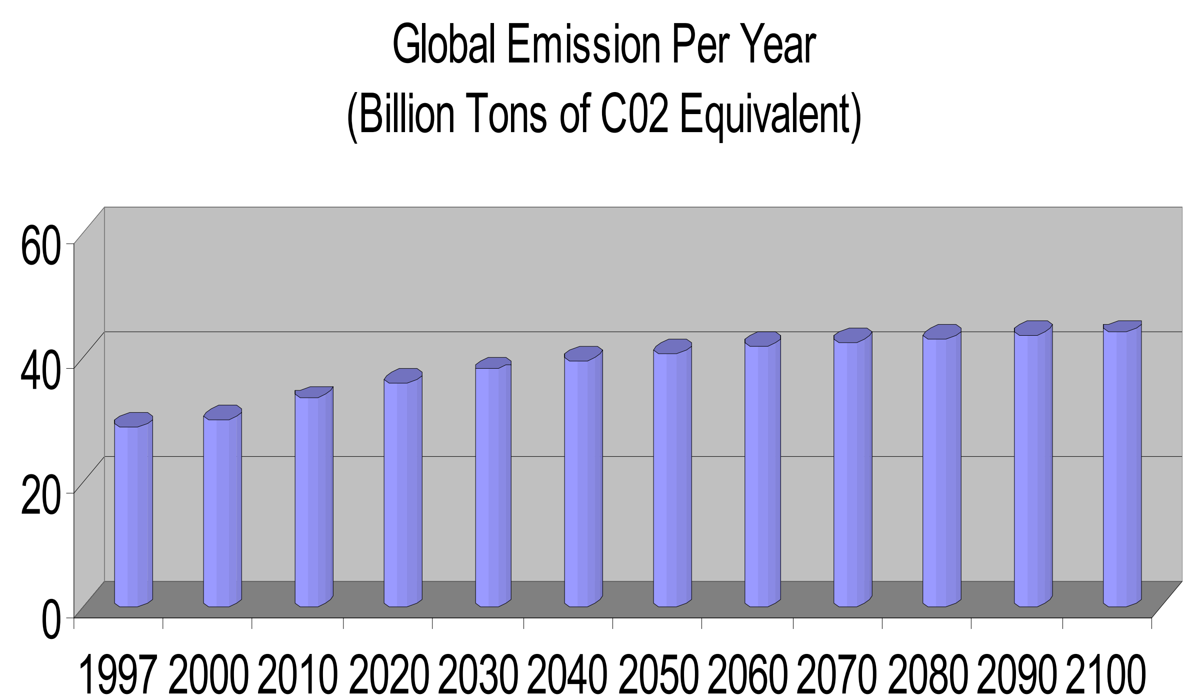
# Emission under the momentum

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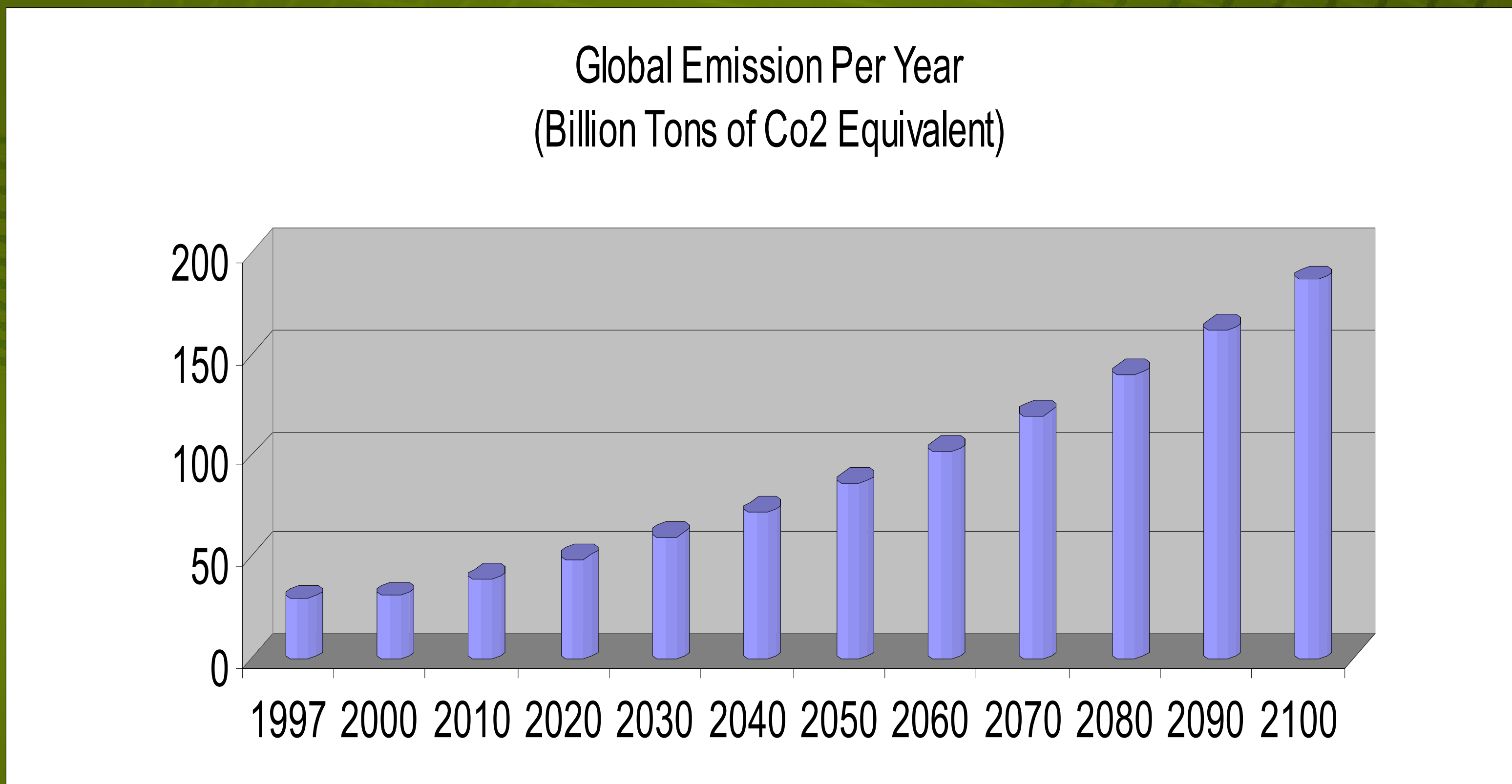
# Global emission under 2 percent per annum uniform growth

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# Conclusions

- A fixed savings rate out of current income (GNP) is consistent with intertemporally optimising behavior of the households.
- The model displays the property of a neo-classical growth model - the growth rate of regional economies tend to converge toward the growth rates of exogenously supplied factors
- The assumption about investor's expectations formation did not affect the trajectory noticeably
- An intertemporal CGE model can be solved by using a single period non-steady state database and implemented with GEMPACK.