Demographic Change, National Saving and International Capital Flows: A Tractable OLG Model

Weifeng Larry Liu

Crawford School of Public Policy, Australian National University
and
Australian Research Council Centre of Excellence in Population Ageing Research

GTAP 25th Annual Conference

8-10 June 2022

Overview

1. Introduction

- Fact 1: Global domographic change
- Fact 2: Global current account imbalances
- Theory 1: Real interest rate parity
- Theory 2: Life cycle hypothesis

1.1 Global demographic change

Fig.1: Fertility rate and life expectancy

(a) Total fertility rate

(b) Life expectancy

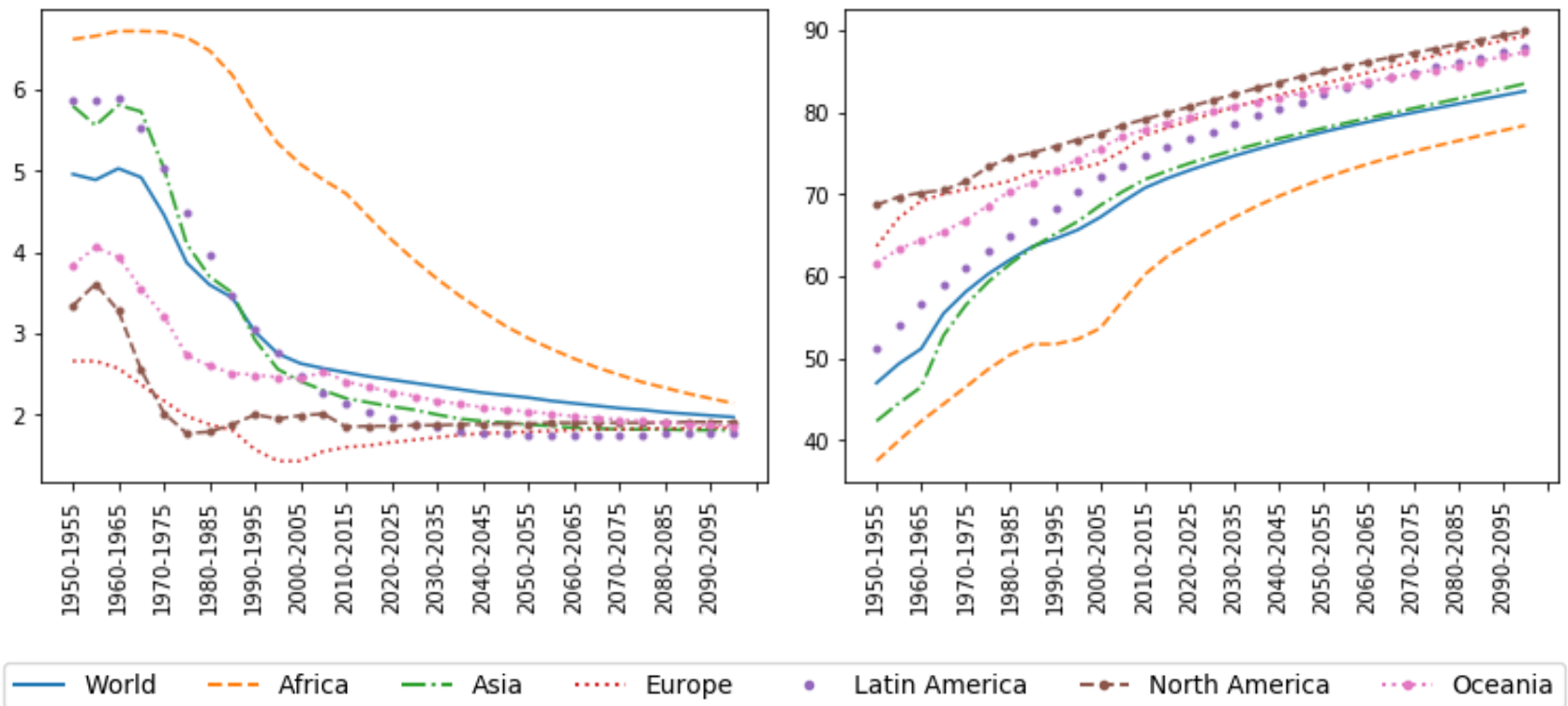
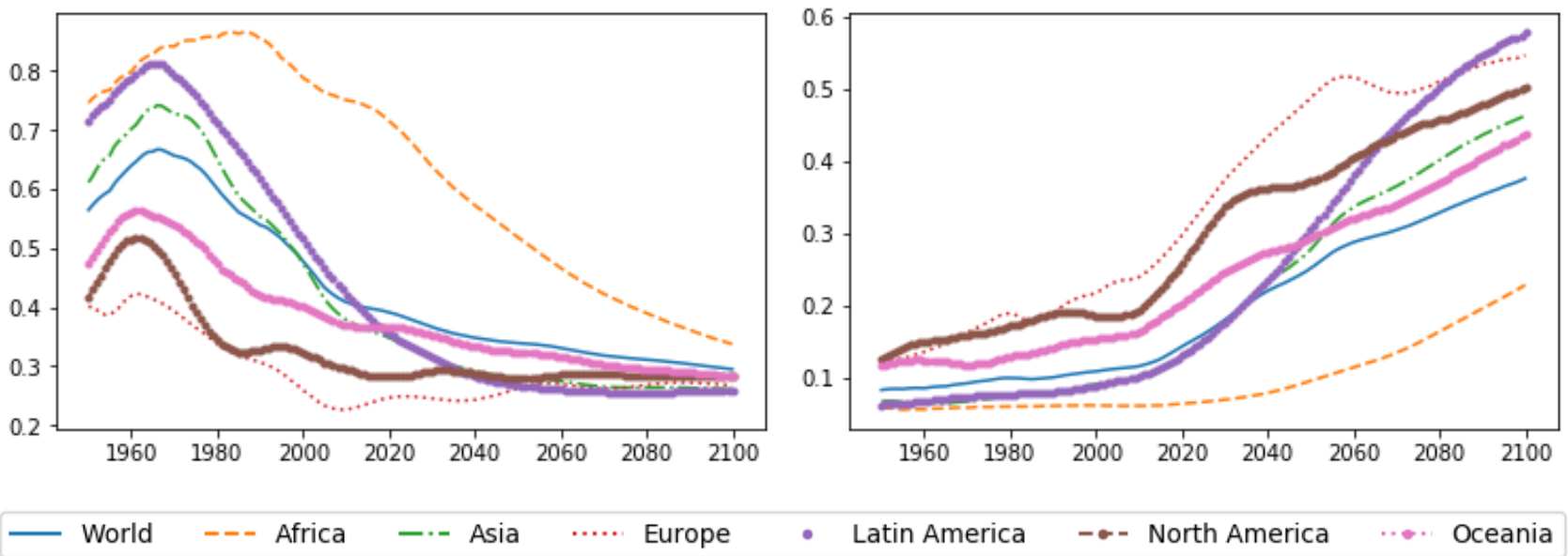


Fig. 2: Dependency ratios

(c) Youth dependency ratio

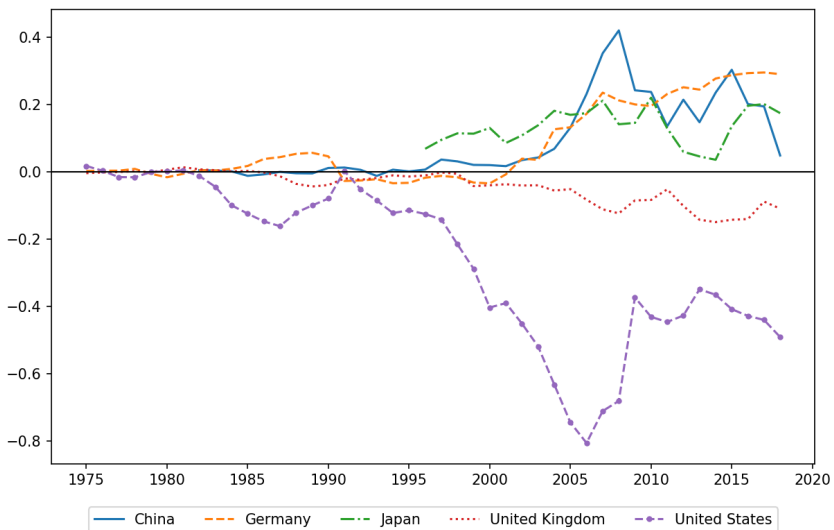
(d) Elderly dependency ratio



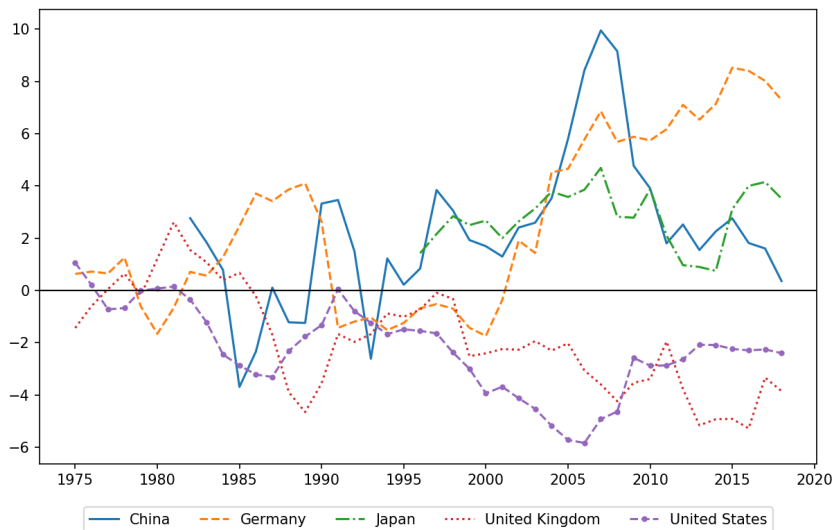
1.2 Global current account imbalances

Fig. 4: Current account balances

(a) Current account (Trillion USD)



(b) Current account (% GDP)



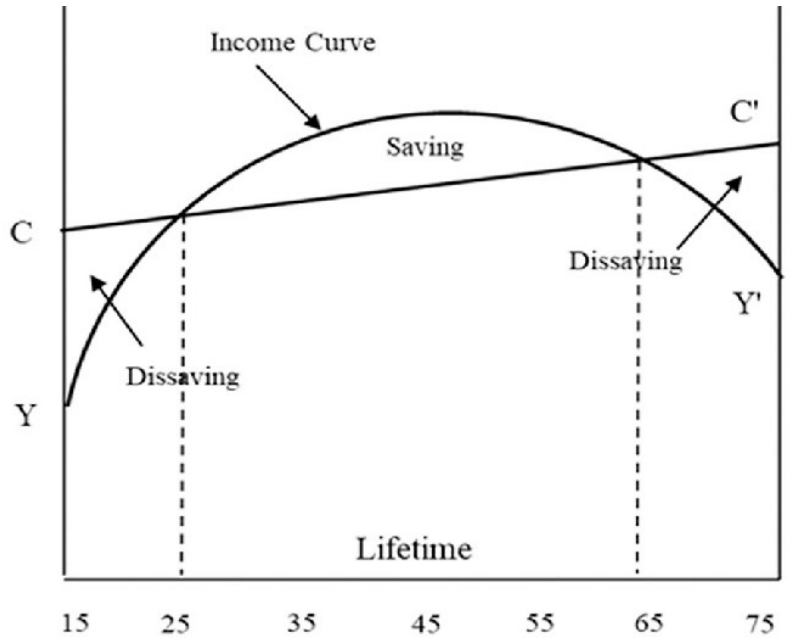
1.3 Real interest rate parity

$$r_i - \epsilon_i = r_j - \epsilon_j + \frac{\dot{e}_{ij}}{e_{ij}}$$

- The real interest rate, adjusted by country risks and expected changes of the real exchange rate, tends to be equal across countries unless there are capital controls.
- The real interest rate parity holds well in the long run.
- If the real exchange rate remains unchanged (e.g., assuming a single composite good), the condition reduces to

$$r_i - \epsilon_i = r_j - \epsilon_j$$

1.4 Life cycle hypothesis



- Individuals smooth consumption over life cycle.
- Borrow when young, save as prime workers, and dis-save when retired.

Literature

- Global demographic change is linked to global imbalances.
- Many factors can drive capital flows across countries especially in the short term.
- Demographic change contributes to medium- and long-term low-frequency capital flows.
- Two broad strands of literature from a time perspective:
  - Project the impacts of future demographic change and hypothetical pension reforms.
  - Focus on historical current account imbalances and include demographics as an explanatory factor or focus on demographics alone.

Contributions

- Tractable model with analytic solutions
- Tractable analysis of various shocks

2. Model

A small open economy with real interest rate given.

- National saving:
  - HH finite periods with different productivity over life cycle
  - HH smooth consumption over time to maximize lifetime utility
  - Saving = - Y-worker borrowing * Y-worker size + O-worker saving * O-worker size - retiree dis-saving * retiree size
- National investment:
  - Firms hire capital and labor for production to maximize profits
  - The marginal product of capital = real interest rate
  - The marginal product of labor = wage
- Stationary population:
  - National saving = 0
  - National investment = 0 (constant labor)
- Aging population:
  - National saving ?
  - National investment ?
- Productivity growth across generations
- Wealth transfer across generations via pension system

- Environment
- Demographics
- Pension system
- Consumers
- Firms
- Equilibrium
- Saving and investment
- Current account

2.1 Environment

- Time is discrete and infinite, $t = 0, 1, 2, ...$
- A small open economy.
- A single composite good.
- A perfect international bonds market.
- A perfect domestic capital market.
- A perfect international goods market.
- A perfect domestic labor market.
- Labor is not mobile across countries.
- A balanced PAYG system financed by labor income taxes.
- Perfect foresight except households face mortality risks.
- A perfect annuity market insures away unintended bequests.
- The world interest rate is constant at r.

2.2 Demographics

- Each individual lives up to four stages of life: child (C), young worker (Y), old worker (O), retiree (R).
- Young workers give birth to children, children turn into young and then old workers with certainty, but survive into retirement with a probability of p_{t+1}.

$$N_t^C = n_t N_t^Y, \quad N_{t+1}^Y = N_t^C, \quad N_{t+1}^O = N_t^Y, \quad N_{t+1}^R = p_{t+1} N_t^O$$

where N_t^C: children; N_t^Y: young workers; N_t^O: old workers; N_t^R: retirees; n_t: fertility rate; p_{t+1}: survival rate.

- Children do not work and depend on young workers.
- The labor productivity ratio of old to young workers is represented by $e > 1$.

2.3 Pension system

- The PAYG system is financed by taxing labor income at τ_t.
- A retiree in period t receives pensions, $\rho_t e w_{t-1}$, depending on the past labor income $e w_{t-1}$ and the replacement rate ρ_t, where e is effective labor and w_{t-1} is wage.
- The system runs a balanced budget each period.

$$N_t^Y \tau_t w_t + N_t^O \tau_t e w_t = N_t^R \rho_t e w_{t-1} \tag{1}$$

Equivalently,

$$\rho_t = \tau_t \frac{n_{t-2}(n_{t-1}+e)}{p_t e} \frac{w_t}{w_{t-1}}, \quad \tau_t = \rho_t \frac{p_t e}{n_{t-2}(n_{t-1}+e)} \frac{w_{t-1}}{w_t}$$

- Defined contributions (DC) if $\tau_t = \bar{\tau}$.
- Defined benefit (DB) if $\rho_t = \bar{\rho}$.
- Completely pre-funded if $\tau_t = \rho_t = 0$.

2.4 Consumers

- Each worker inelastically supplies one unit of labor in a competitive labor market, and retires when she is old.
- Individuals freely invest in foreign assets and domestic capital.
- An annuity market insures away the risk of unintended bequests.
- The period budget constraint of an individual born in period t:

$$
\begin{aligned}
& n_t c_t^C + c_t^Y + b_{t+1}^O + k_{t+1}^O = (1-\tau_t) w_t \\
& c_{t+1}^O + b_{t+2}^R + k_{t+2}^R = (1+r)(b_{t+1}^O + k_{t+1}^O) + (1-\tau_{t+1}) e w_{t+1} \\
& c_{t+2}^R = \frac{(1+r)(b_{t+2}^R + k_{t+2}^R)}{p_{t+2}} + \rho_{t+2} e w_{t+1}
\end{aligned}
$$

where w_t: wage; c_t: consumption; s_t: saving; b_{t+1}: foreign assets; k_{t+1}: domestic capital.

- The intertemporal budget constraint is

$$n_t c_t^C + c_t^Y + \frac{c_{t+1}^O}{1+r} + p_{t+2}\frac{c_{t+2}^R}{(1+r)^2} = \omega_t \tag{2}$$

where ω_t denotes lifetime wealth of a young individual in period t defined as

$$\omega_t = (1-\tau_t)w_t + \frac{(1-\tau_{t+1})ew_{t+1}}{1+r} + p_{t+2}\frac{\rho_{t+2}ew_{t+1}}{(1+r)^2}$$

- Each young individual at time t makes a consumption plan $\{c_t^C, c_t^Y, c_{t+1}^O, c_{t+2}^R\}$ to maximize her lifetime utility

$$u_t = \lambda n_t \frac{(c_t^C)^{1-1/\sigma}}{1-1/\sigma} + \frac{(c_t^Y)^{1-1/\sigma}}{1-1/\sigma} + \beta\frac{(c_{t+1}^O)^{1-1/\sigma}}{1-1/\sigma} + \beta^2 p_{t+2}\frac{(c_{t+2}^R)^{1-1/\sigma}}{1-1/\sigma}$$

where $\sigma > 0$: the elasticity of intertemporal substitution, $0 < \beta < 1$: the subjective discount factor, and λ: the degree to which parents care about their children.

- The optimal consumption for each individual can be solved as

$$
\begin{aligned}
c_t^C &= \frac{\lambda^\sigma \omega_t}{n_t\lambda^\sigma + 1 + \beta^\sigma(1+r)^{\sigma-1} + p_{t+2}\left[\beta^\sigma(1+r)^{(\sigma-1)}\right]^2} \\
c_t^Y &= \frac{\omega_t}{n_t\lambda^\sigma + 1 + \beta^\sigma(1+r)^{\sigma-1} + p_{t+2}\left[\beta^\sigma(1+r)^{(\sigma-1)}\right]^2} \\
c_{t+1}^O &= \frac{\beta^\sigma(1+r)^\sigma \omega_t}{n_t\lambda^\sigma + 1 + \beta^\sigma(1+r)^{\sigma-1} + p_{t+2}\left[\beta^\sigma(1+r)^{(\sigma-1)}\right]^2} \\
c_{t+2}^R &= \frac{\left[\beta^\sigma(1+r)^\sigma\right]^2 \omega_t}{n_t\lambda^\sigma + 1 + \beta^\sigma(1+r)^{\sigma-1} + p_{t+2}\left[\beta^\sigma(1+r)^{(\sigma-1)}\right]^2}
\end{aligned}
$$

- The saving of each individual at different life stages is

$$
\begin{aligned}
s_t^Y &= (1-\tau_t)w_t - (1+n_t\lambda^\sigma) * \\
&\quad \frac{\omega_t}{n_t\lambda^\sigma + 1 + (1+r)^{\sigma-1}\beta^\sigma + p_{t+2}\left[(1+r)^{(\sigma-1)}\beta^\sigma\right]^2} \\
s_{t+1}^O &= r(1-\tau_t)w_t + (1-\tau_{t+1})ew_{t+1} - \left[r(1+n_t\lambda^\sigma) + \beta^\sigma(1+r)^\sigma\right] * \\
&\quad \frac{\omega_t}{n_t\lambda^\sigma + 1 + \beta^\sigma(1+r)^{\sigma-1} + p_{t+2}\left[\beta^\sigma(1+r)^{(\sigma-1)}\right]^2} \\
s_{t+2}^R &= -\frac{1}{p_{t+2}}\left((1+r)(1-\tau_t)w_t + (1-\tau_{t+1})ew_{t+1}\right) + \frac{1}{p_{t+2}}\left((1+r)(1+n_t\lambda^\sigma\right. \\
&\quad \frac{\omega_t}{n_t\lambda^\sigma + 1 + \beta^\sigma(1+r)^{\sigma-1} + p_{t+2}\left[\beta^\sigma(1+r)^{(\sigma-1)}\right]^2}
\end{aligned}
$$

where they satisfy

$$s_t^Y + s_{t+1}^O + p_{t+2}s_{t+2}^R = 0 \tag{5}$$

2.5 Firms

- An aggregate production sector.
- Firms use capital and labor to produce output.

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha} \tag{6}$$

where Y_t, K_t, L_t, A_t: output, capital, labor, TFP.

- TFP rises over time.

$$A_{t+1} = g_{t+1}^{1-\alpha} A_t \tag{7}$$

- Capital accumulates through investment without depreciation.

$$K_{t+1} = K_t + I_t \tag{8}$$

- The firm maximizes the present value of entire profits:

$$\Pi = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \frac{1}{1+r} \right) (Y_t - w_t L_t - I_t) \tag{9}$$

- The first-order conditions:

$$r = \alpha A_t (k_t)^{\alpha-1} \tag{10}$$

$$w_t = (1-\alpha) A_t (k_t)^{\alpha} \tag{11}$$

where $k_t = K_t / L_t$: the capital-labor ratio.

2.6 Equilibrium

- **A recursive equilibrium** is a set of policy functions $\{c_t^Y, c_t^O, c_t^R, b_{t+1}^O, k_{t+1}^O, b_{t+1}^R, k_{t+1}^R\}_{t=0}^{\infty}$ and wages $\{w_t\}_{t=0}^{\infty}$ such that
  1. Aggregate and individual behaviors are consistent:

$$C_t = N_t^C c_t^C + N_t^Y c_t^Y + N_t^O c_t^O + N_t^R c_t^R \quad (12)$$

$$S_t = N_t^Y s_t^Y + N_t^O s_t^O + N_t^R s_t^R \quad (13)$$

$$K_{t+1} = N_t^Y k_{t+1}^O + N_t^O k_{t+1}^R \quad (14)$$

$$B_{t+1} = N_t^Y b_{t+1}^O + N_t^O b_{t+1}^R \quad (15)$$

  2. Capital accumulates through investment:

$$K_{t+1} = K_t + I_t \quad (16)$$

  3. The labor market clears:

$$L_t = N_t^Y + eN_t^O \quad (17)$$

  4. The goods market clears:

$$Y_t = C_t + I_t + T_t \quad (18)$$

  where T_t represents trade balance in period t.
  5. The domestic capital market clears:

$$S_t = K_{t+1} - K_t + B_{t+1} - B_t = I_t + CA_t \quad (19)$$

- The capital-labor ratio and the wage rate:

$$k_t = \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}} \tag{20}$$

$$w_t = A_t \left(\frac{\alpha A_t}{r}\right)^{\frac{\alpha}{1-\alpha}} \tag{21}$$

2.7 National saving, investment and current account

$$
\begin{aligned}
\frac{CA_t}{Y_t} &= \frac{S_t^Y + S_t^O + S_t^R - I_t}{Y_t} \\
&= \frac{n_{t-1}(1-\alpha)}{n_{t-1}+e}\left\{1 - \frac{\rho_t p_t e}{g_t n_{t-2}(n_{t-1}+e)}\right. \\
&\quad \left. - (1+n_t\lambda^\sigma)\frac{\left(1-\frac{\rho_t p_t e}{g_t n_{t-2}(n_{t-1}+e)}\right) + \left(1-\frac{\rho_{t+1}p_{t+1}e}{g_{t+1}n_{t-1}(n_t+e)}\right)\frac{eg_{t+1}}{1+r} + \frac{p_{t+2}\rho_{t+2}eg_{t+1}}{(1+r)^2}}{n_t\lambda^\sigma + 1 + (1+r)^{\sigma-1}\beta^\sigma + p_{t+2}\left[(1+r)^{(\sigma-1)}\beta^\sigma\right]^2}\right\} \\
&\quad + \frac{1-\alpha}{n_{t-1}+e}\left\{\frac{r}{g_t}\left(1-\frac{\rho_{t-1}p_{t-1}e}{g_{t-1}n_{t-3}(n_{t-2}+e)}\right) + e\left(1-\frac{\rho_t p_t e}{g_t n_{t-2}(n_{t-1}+e)}\right)\right. \\
&\quad \left. - \left(r(1+n_{t-1}\lambda^\sigma) + \beta^\sigma(1+r)^\sigma\right)\frac{\left(1-\frac{\rho_{t-1}p_{t-1}e}{g_{t-1}n_{t-3}(n_{t-2}+e)}\right)\frac{1}{g_t} + \left(1-\frac{\rho_t p_t e}{g_t n_{t-2}(n_{t-1}+e)}\right)\frac{e}{1+r} + \frac{p_{t+1}\rho_{t+1}e}{(1+r)^2}}{n_{t-1}\lambda^\sigma + 1 + (1+r)^{\sigma-1}\beta^\sigma + p_{t+1}\left[(1+r)^{(\sigma-1)}\beta^\sigma\right]^2}\right\} \\
&\quad + \frac{1-\alpha}{n_{t-2}(n_{t-1}+e)}\left\{-\frac{1+r}{g_t g_{t-1}}\left(1-\frac{\rho_{t-2}p_{t-2}e}{n_{t-4}(n_{t-3}+e)g_{t-2}}\right) - \frac{e}{g_t}\left(1-\frac{\rho_{t-1}p_{t-1}e}{n_{t-3}(n_{t-2}+e)g_{t-1}}\right)\right. \\
&\quad + \left((1+r)(1+n_{t-2}\lambda^\sigma) + \beta^\sigma(1+r)^\sigma\right) * \\
&\quad \left. \frac{\left(1-\frac{\rho_{t-2}p_{t-2}e}{n_{t-4}(n_{t-3}+e)g_{t-2}}\right)\frac{1}{g_t g_{t-1}} + \left(1-\frac{\rho_{t-1}p_{t-1}e}{n_{t-3}(n_{t-2}+e)g_{t-1}}\right)\frac{e}{(1+r)g_t} + \frac{p_t\rho_t e}{(1+r)^2 g_t}}{n_{t-2}\lambda^\sigma + 1 + (1+r)^{\sigma-1}\beta^\sigma + p_t\left[(1+r)^{(\sigma-1)}\beta^\sigma\right]^2}\right\} \\
&\quad - \frac{\alpha}{r}\left(g_{t+1}n_{t-1}\frac{n_t+e}{n_{t-1}+e} - 1\right)
\end{aligned}
$$

The current account depends on:

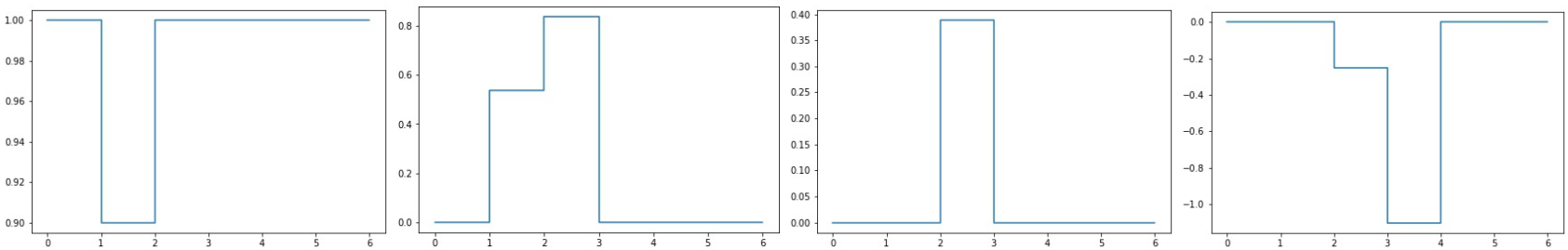
- past and current fertility rates,
- past, current and future mortality rates,
- past, current and future pension replacement rates,
- past, current and future productivity growth rates,
- real interest rate,
- elasticity of intertemporal substitution in consumption,
- elasticity of output with respect to capital and labor,
- subjective discount factor,
- life-cycle productivity growth rate,
- degree of parent caring about their children.

3. Theoretical results

- Demographic change
  - Temporary fertility shock
  - Permanent fertility shock
  - Permanent mortality shock
  - Persistent mortality shock
- Demographic change with pension system
- Demographic change with productivity growth

Figure 4: Temporary fertility decrease (no pension)

(a) Fertility (b) Young saving (c) Old saving (d) Retiree saving



(e) National saving (f) Investment (g) Current account (h) Foreign assets

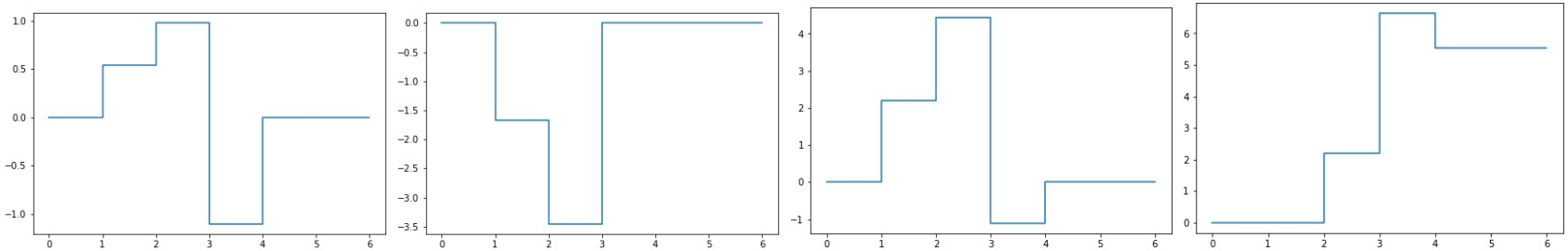
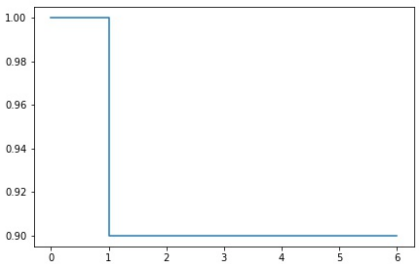
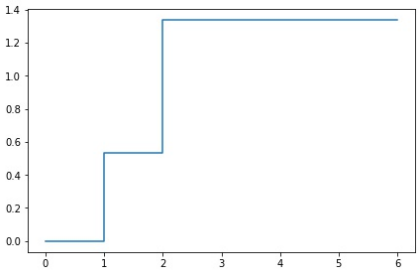


Figure 5: Permanent fertility decrease (no pension)

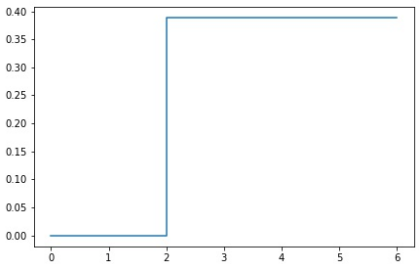
(a) Fertility



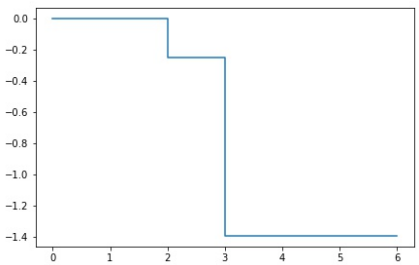
(b) Young saving



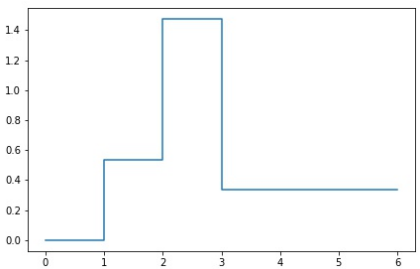
(c) Old saving



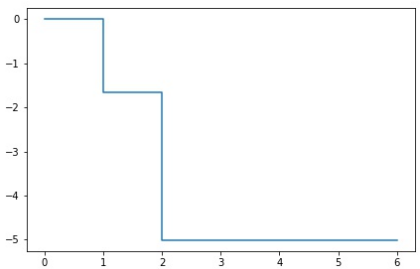
(d) Retiree saving



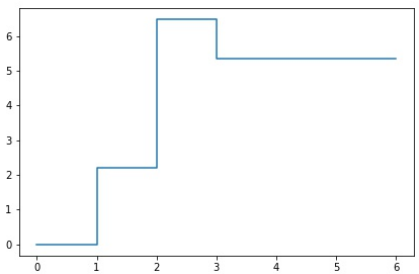
(e) National saving



(f) Investment



(g) Current account



(h) Foreign assets

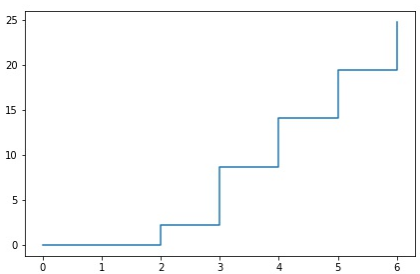
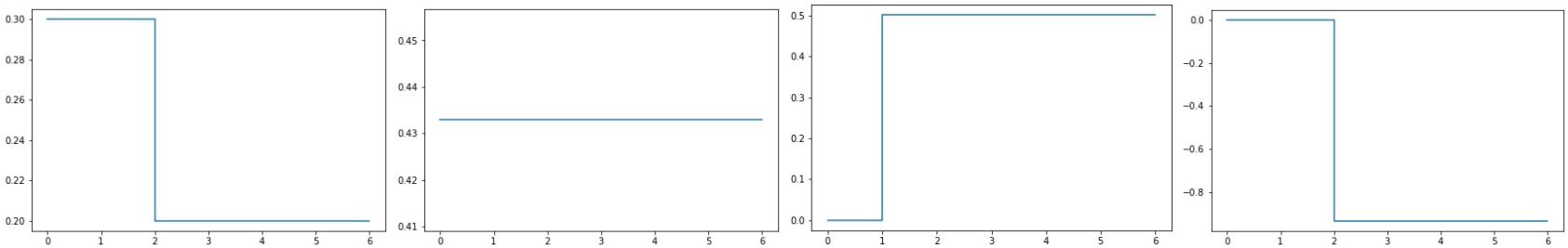


Figure 6: Permanent mortality decrease (no pension)

(a) Mortality (b) Young saving (c) Old saving (d) Retiree saving



(e) National saving (f) Investment (g) Current account (h) Foreign assets

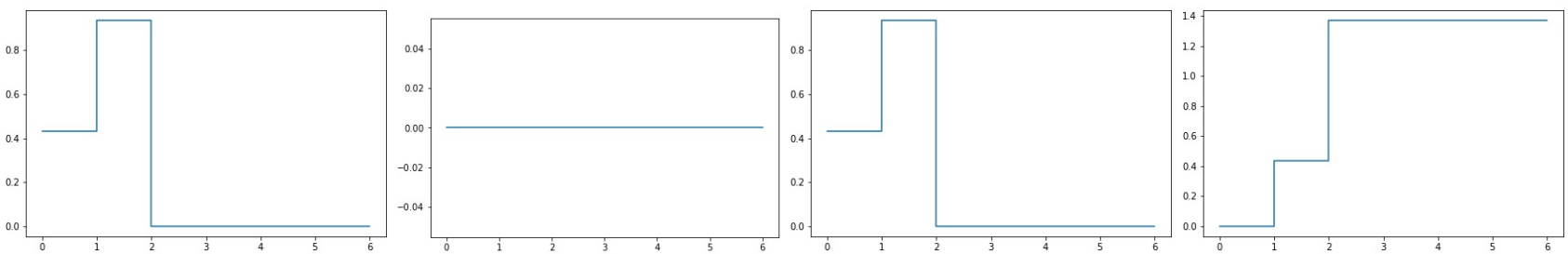
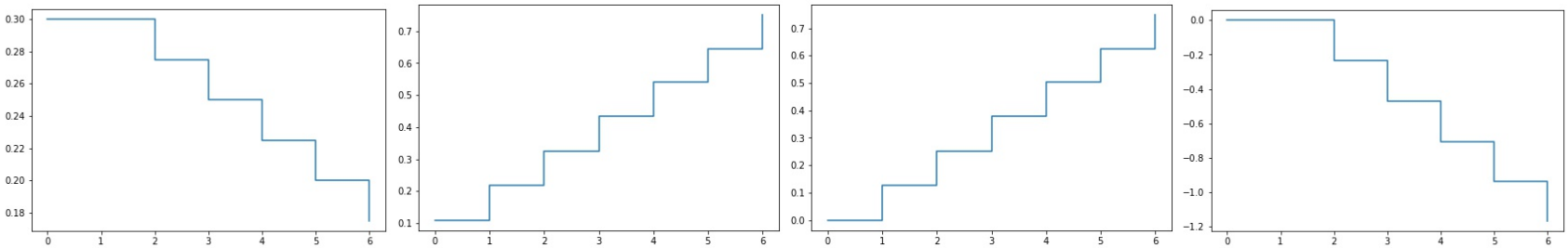


Figure 7: Persistent mortality decrease (no pension)

(a) Mortality (b) Young saving (c) Old saving (d) Retiree saving



(e) National saving (f) Investment (g) Current account (h) Foreign assets

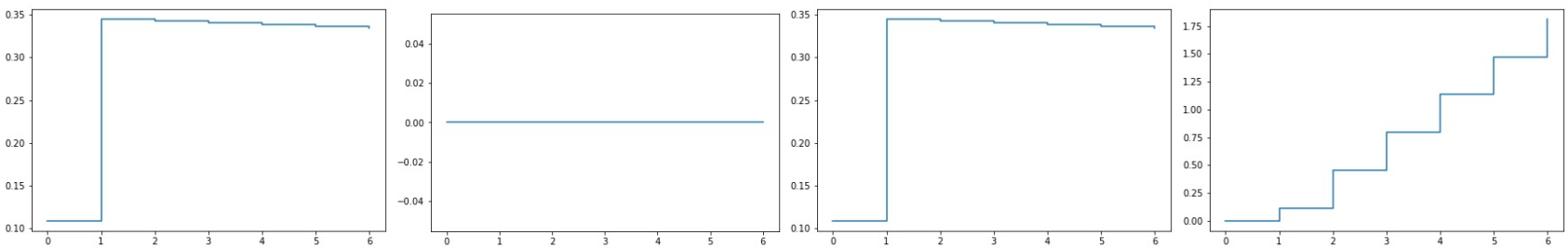
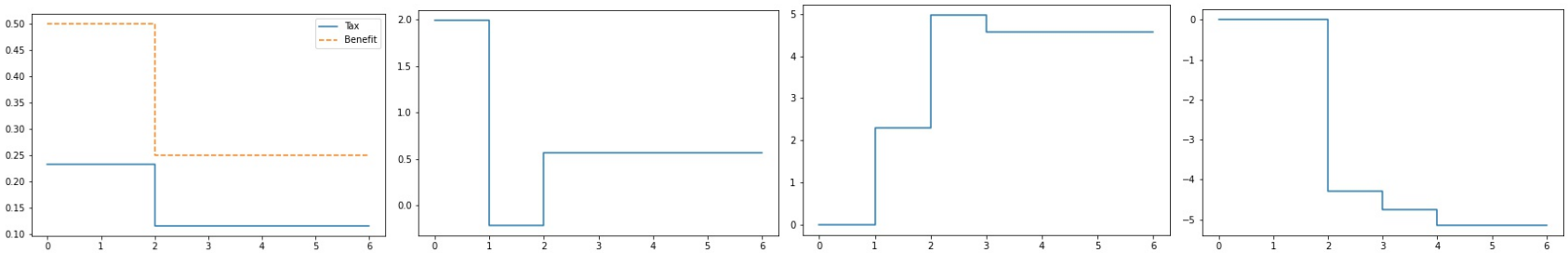


Table 1: Demographic effects on current accounts and foreign assets

Demographic driver	**Shock persistence**	**Population size**	**Current account**	**Foreign asset**
Fertility	Temporary	Permanent	Temporary	Permanent
	Permanent	Persistent	Permanent	Persistent
	Persistent	Persistent	Persistent	Persistent
Mortality	Temporary	Temporary	Temporary	Temporary
	Permanent	Permanent	Temporary	Permanent
	Persistent	Persistent	Permanent	Persistent

Figure 8: Pension reduction without demographic change

(a) Tax rate (b) Young saving (c) Old saving (d) Retiree saving



(e) National saving (f) Investment (g) Current account (h) Foreign assets

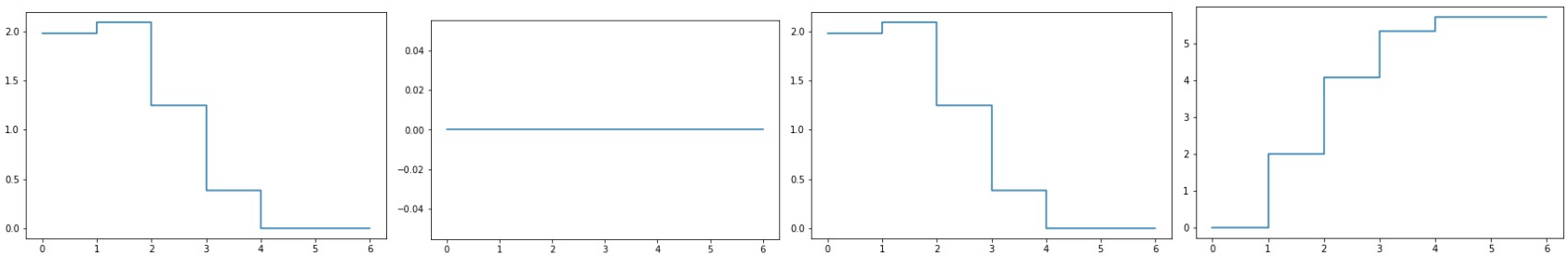
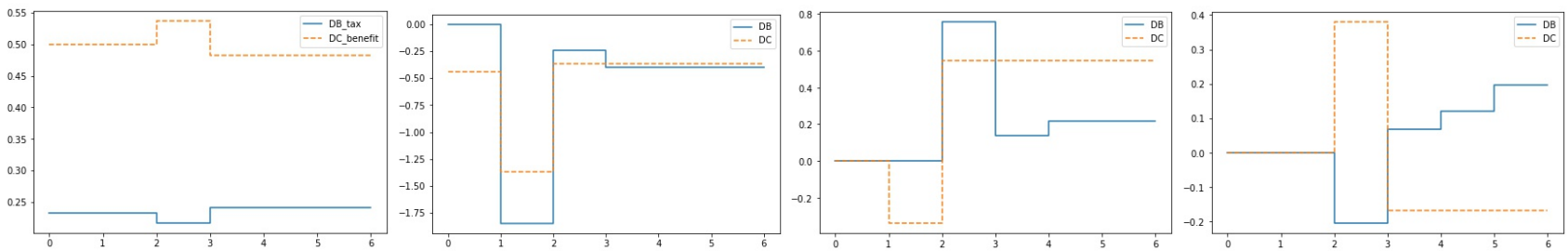
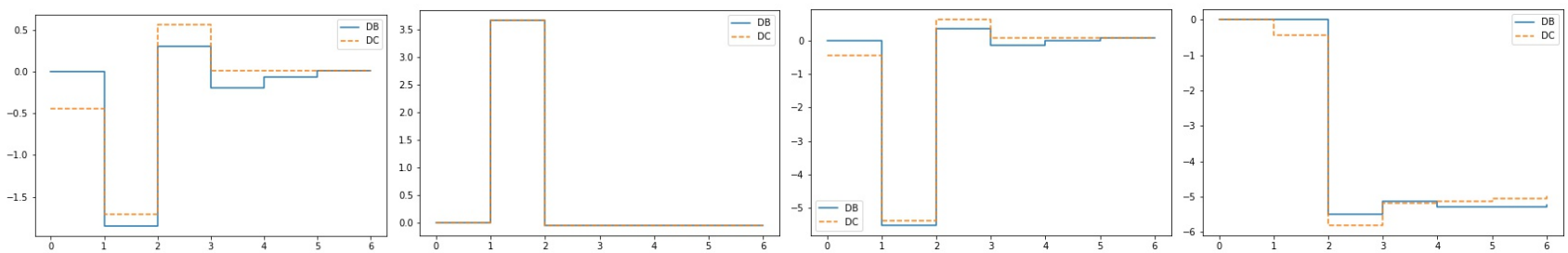


Figure 9: Permanent fertility decrease and persistent productivity increase with pensions

(a) Tax and benefit (b) Young saving (c) Old saving (d) Retiree saving



(e) National saving (f) Investment (g) Current account (h) Foreign assets

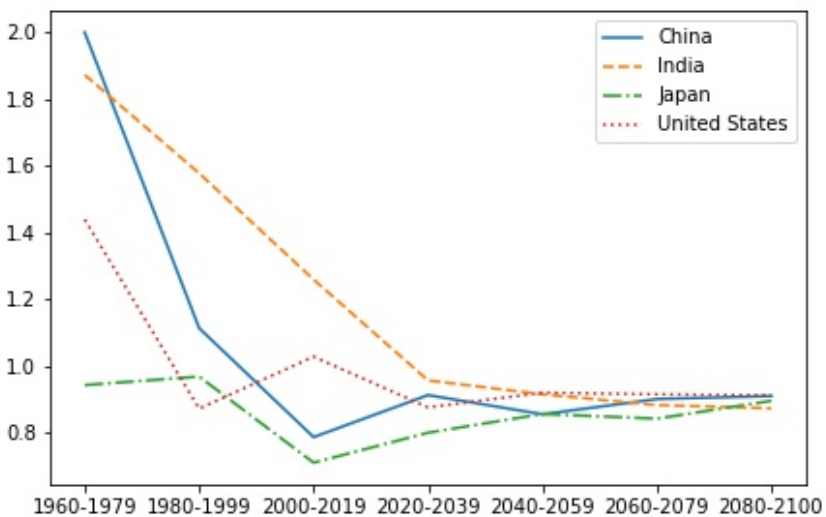


4. Quantitative results

- Applies the model to US, Japan, China and India.
- Child, young worker, old worker, retiree: 20 years each.
- Demographic transitions (UN World Population Prospects).

Figure 10: Future demographic change

(a) Fertility



(b) Mortality

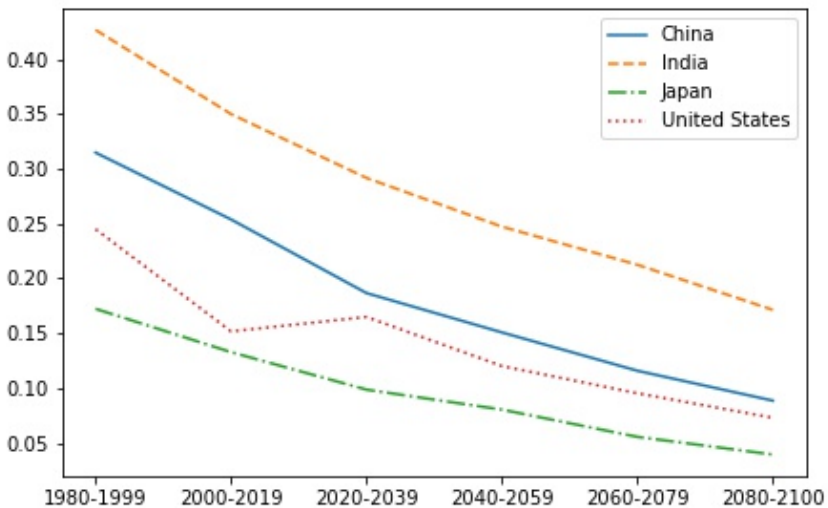
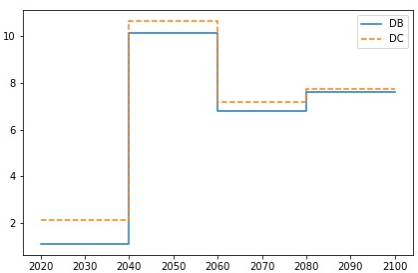


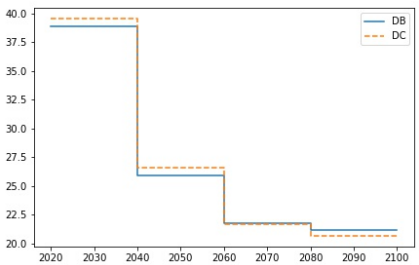
Table 2: Parameters

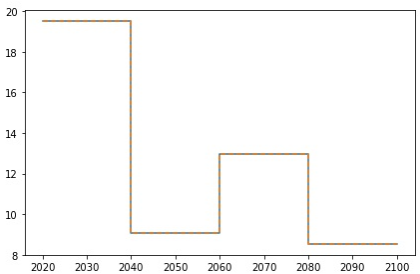
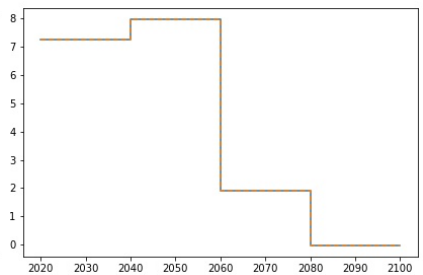
Parameters	US	Japan	China	India
r	0.79	0.49	0.55	1.79
α	0.65	0.65	0.5	0.35
e	2	2	2	2
σ	2	2	2	2
β	0.6	0.6	0.6	0.6
λ	1	1	1	1
ρ	0.5	0.5	0	0

Figure 11: Demographic impacts in four economies over 2020-2100

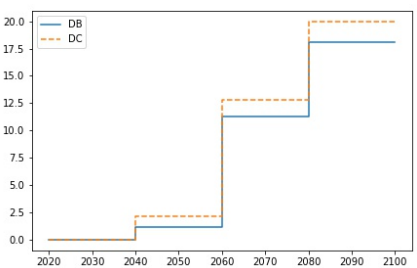
(a) US CA (b) Japanese CA (c) Chinese CA (d) Indian CA

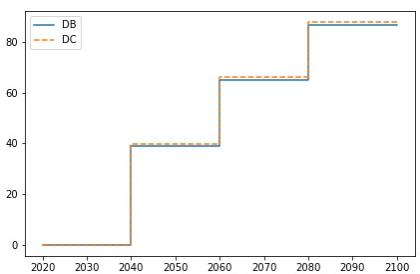


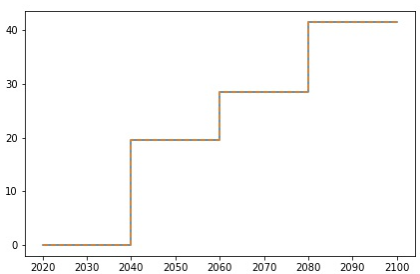
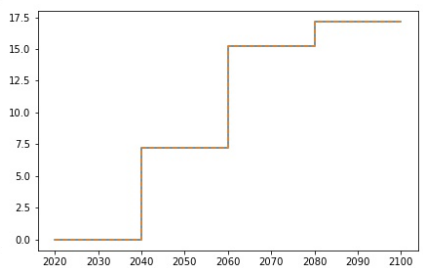


(e) US NFA (f) Japanese NFA (g) Chinese NFA (h) Indian NFA





5. Conclusion

- Demographic change has important implications on national saving and international capital flows.
- The patterns of capital flows depend on the nature of demographic shocks (permanent or transitory; fertility or mortality) and also on the stage of demographic shocks.
- Pension reform towards more private funding in advanced economies tends to increase national saving and drive capital outflows; pension expansion in developing economies tends to decrease national saving and reduce capital outflows.
- Defined-contribution systems tend to increase national saving more than defined-benefit systems in response to population aging.
- Japan and China would experience strong capital outflows driven by population aging in the next several decades.