A Computable General Equilibrium Theory of Heterogeneous Firms with Product Quality Differentiation1

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Abstract

In the existing computable general equilibrium (CGE) models, changes in trade flows are based on the assumption that the quality of the products shipped to each export destination and the perception of consumers in those markets toward product qualities remains unchanged. This is contrary to the recent literature emphasizing the supply-side aspects of quality differentiation, that is product quality across firms differ, and/or the demand-side aspects of quality differentiation, that is the willingness of consumers across markets for product qualities offered by firms differs and it may vary. This study provides an extension to the formulation of international trade in the CGE models that rely on Melitz (2003) by allowing firms to produce distinct product qualities and to serve markets in which the willingness of consumers to pay for quality is sufficiently high for their profitability. This extension is based on the theoretical studies that include firm product quality differentiation into the Melitz framework and studies that offer approaches to translate the theoretical frameworks of firms' behavior in international trade into a computable setting accessible to CGE modelers. We, first, bring both supply- and demand-side aspects of product quality into a unified general equilibrium framework defined at the firm level. In the unified framework, the quality of products of firms is determined by their productivity and the decision of firms to operate on each trade link depends on the willingness of consumers to pay for the product quality, which is in turn determined by the per capita income in each destination. Second, we translate the unified framework defined at the firm level to a computable setting that matches the relatively aggregate sectoral data and definitions of available data used by CGE models. Therefore, our proposed CGE framework enhances the accessibility of firm trade models with product quality differentiation to CGE modelers.

1 Introduction

Policymakers are increasingly relying on computable general equilibrium (CGE) models to understand the consequences of their policies and/or external shocks on observed trade patterns (Jafari et al. 2021). The current CGE models simulate the changes in trade flows based on the assumption that the quality of products supplied to each market does not change. This is at odds with the empirical literature that shows product qualities differ across firms, firms charge prices depending on the quality of their products (Kugler and Verhoogen 2012; Feenstra and Romalis 2014; Gervais 2015), and consumers have different tastes for the qualities (Hallak 2010; Hallak and Schott 2011; Feenstra and Romalis 2014). Upon a change in the trade system firms revise their decision whether to export to a given market; this results in changes in the mix of firms exporting to that market, and therefore the quality of product destined to that market. Moreover, incorporating the product quality differentiation into the CGE models will address important features of observed trade patterns; it explains the existence of firms with low productivity in some export markets (Hallak and Sivadasan 2013) and the reasons for some local firms not selling to domestic markets (Balistreri and Tarr 2021).[2] Recent theoretical models of international trade have introduced features to explain such observed trade patterns. However, these theoretical models are not easily accessible in CGE models because of both data and methodological deficiencies. In theoretical models, firms are the unit of the analysis but CGE models must rely on the relatively aggregate sectoral data because of the data deficiency. Moreover, the functional forms that explain the behavior of economic agents in CGE models must be analytically convenient. That is, the parameters of the function could be calibratable and/or estimable based on the available data, and the functional form should not degrade the computability of the model.

The main challenges are to modify the current theoretical frameworks to have analytically convenient functional forms and to translate the equations of the model to a setting that represents the behavior of a representative firm, while explicitly taking into account the heterogeneities firms with product quality differences. For this purpose, we adopt from the literature and introduce both horizontal and vertical quality differentiation into a unified framework, and the willingness of consumers to pay for vertically differentiated products. The conceptual distinction between horizontal and vertical differentiation of products allows distinguishing between subjective attributes of quality reflecting heterogeneous consumer preferences

[2] In case of trade in agri-food products, product differentiation shapes increasingly the trade both at country/regional and firm level, reflecting the growing role of quality differentiation for consumers' purchase decisions (Sexton and Xia 2018).

(horizontal differentiation) and the objective attributes that can be ordered into quality levels (vertical differentiation). Next, we translate the framework into a computable setting that is suitable for a trade simulation analysis by CGE modelers.

Conventional Armington models reflect the quality differentiation of products in the demand function by distribution (share) parameters differentiated by origin.[3] However, these parameters are exogenous, and thereby the simulated changes of trade flows from these models rely on pre-existing trade shares and thus capture the trade adjustments only at the intensive margin. This is at odds with the empirical trade literature that highlights the role of extensive margin in explaining the patterns of trade (Akgul et al. 2016; Jafari and Britz 2018). CGE models that rely on models of monopolistic competition based on increasing returns to scale (i.e., Krugman type models) endogenize a part of the distribution parameter, the taste for variety component that reflects the horizontal differentiation of product quality (Balistreri and Rutherford 2013; Akgul et al. 2016). This is done by linking the number of firms (i.e., the number of available varieties) to the distribution parameter in the Dixit Stiglitz (Dixit and Stiglitz 1977) demand function. The higher the number of (horizontally differentiated) varieties of the product supplied to a given destination market means that consumers in that market give higher preferences to that product. However, these models assume that firms are homogenous and export to every destination, and therefore all the variation in trade flows is still on the intensive margin (i.e., average export per firm).[4] To address the deficiencies, current CGE models incorporate features of Melitz (2003) that identify the differences in firm productivity as an important factor determining the number of firms in each market (i.e., the extensive margin of trade). The number of firms in each market reflects the horizontal differentiation of products in that market and it is representative for the taste component in the demand function. Melitz-type CGE models have greatly improved the simulation of trade patterns. However, these models do not consider the vertical quality differentiation of product across firms, and consumers' taste toward the vertically differentiated products. The vertical quality differentiation reflected in the distribution parameter and the willingness to pay for vertical quality are exogenous in these models, and

[3] In conventional Armington models, the elasticity of substitution parameter in the demand function also partly captures the quality differentiation.

[4] These models assume that firms are homogenous and implicitly give no role to the firms in determining the pattern of trade, while firm differences and their importance in shaping trade pattern has been evident since the mid-1990s due to the availability of more detailed data at the plant and firm level

therefore the simulated changes in trade flows are based on the pre-existing vertical quality component and the level of appreciation for it.[5]

Recently, theoretical literature introduced the concept of vertical product differentiation into the Melitz model and its variants to explain the features of international trade that are not explained by the original Melitz model. The modeling extensions with regard to incorporation of quality differentiation mainly focus on supply-side aspects of quality differentiation and can be classified into three categories (Gervais 2015). First, the Melitz type models in which firms draw product quality instead of productivity from a distribution (Baldwin and Harrigan 2011; Crozet et al. 2012; Dinopoulos and Unel 2013).[6] Second, extensions of Melitz (2003), where quality is endogenous and determined from the random productivity draw (Johnson 2012; Kugler and Verhoogen 2012). Third, models in which firms are heterogeneous in their efficiency, i.e., the ability to increase output from a given input, and in their ability to produce quality (Hallak and Sivadasan 2013).

The paper contributes to the literature in two important ways. First, it extends the second type models that incorporate product quality differentiation into Melitz (2003). It does so by building on Johnson (2012) who allows the endogenous determination of product qualities in each market depending on the supply side aspects, but extending it for the demand side aspects of quality differentiation as well following the insights from Hallak (2010) and Hallak and Schott (2011). In this proposed framework, firms draw their productivity that in turn determines firm product quality. Moreover, the decision of firms to export to each destination market, that is the existence of given product quality in that link, is influenced by both the firm productivity and the perception of consumers in the export destination toward the product quality offered by firms. That is, firms export to each destination if the following two conditions hold: (i) if the consumer's appreciation for each unit of quality in the export destination market is not lower than the marginal costs associated with quality production so that firms will have positive operating profit, and (ii) if the productivity is sufficiently high so that firms' profits exceed the bilateral fixed cost of exporting to a given destination.

[5] While Melitz type models have been implemented in trade simulation models, to date, the implementation of vertical product differentiation in simulation models remains rudimentary, for example in MIRAGE, by distinguishing different quality ranges, according to the country of origin of the product. The main reasons are: (1) the lack of accessible data, (2) and absence of a tractable theoretical general equilibrium framework capturing vertical differentiation but at the same time being computable and applicable into simulation analysis.

[6] In benchmark models -such as Melitz- differences in physical productivity and product quality across firms are observationally equivalent in terms of how they influence participation and trade flows. In contrast, productivity and quality heterogeneity do have distinct implications for export behaviour.

Second, this paper brings the features of product quality differentiation into the recent CGE models. For this purpose, we assume that firms' productivities follow a Pareto distribution and aggregate each of firms' decision variables over firms based on this distribution consistent with the literature that introduced Melitz (2003) into CGE models (Balistreri et al. 2011; Balistreri and Rutherford 2013; Akgul et al. 2016; Jafari and Britz 2018; Dixon et al. 2016). This approach leads to a model in which all its variables and equations explain the behavior of a representative firm without neglecting the productivity differences across firms.

The rest of the paper is organized as follows. We describe a unified model that incorporates both vertical and horizontal product quality differentiation. We then explain how we translate the model, into a commutable setting. Finally, we conclude.

2 Theory

In this section, we introduce a multi-country trade model with a continuum of products that are qualitatively differentiated both horizontally and vertically; and the perception toward vertically differentiated products is asymmetric on the demand side and can change in each market. The model is a Melitz (2003)-type model that builds on Johnson (2012) to reflect the supply-side determinants of quality differentiation. In this respect, we add the features of the general equilibrium framework into the model. We further extend the model to consider the demand side aspects of product quality differentiation following the reasoning of Hallak (2010) and Hallak and Schott (2011).

2.1 Consumption

Assume that a representative consumer in each country (d ∈ r) has an aggregate demand Q_{id} from a continuum of differentiated varieties (ω), among a set of available varieties of product i, Ω_i, sourced from different countries (s ∈ r). That is, consumers in d demand $q_{isd}(\omega)$ of the physical quantity of a variety of ω from s. Considering that the representative consumer has constant elasticity of substitution (CES) preferences over varieties, the aggregate demand of i, Q_{id}, is

$$Q_{id} = \left(\sum_s A_{isd}^{\frac{1}{\sigma_{id}}} \int_{\omega\in\Omega_i} [b_d(\lambda_{isd}(\omega), z_d)\; q_{isd}(\omega)]^{\frac{\sigma_{id}-1}{\sigma_{id}}}\ \mathrm{d}\omega\right)^{\frac{\sigma_{id}}{\sigma_{id}-1}}, \tag{1}$$

where $b_d(\lambda, z)$ is a function that represents the "weight" attributed to the individual characteristics of each physical unit of that variety in country d. This weight is a function of the vertical quality of a variety $\lambda(\omega) \geq 1$ measured in utility per physical unit and some country-specific characteristics summarized in z_d. λ captures all vertical attributes of products, other than its price, for which all consumers are willing to pay more. It captures all the tangible attributes of products such as its durability and functionality and its intangible attributes such as the appeal of design and image that can be ranked by consumers.[7] As individuals in different countries have different perceptions for quality, i.e., consumers have heterogeneous preferences across markets, b_d varies across countries. The country-specific component of heterogeneous consumer preferences has a long tradition in the economy going back to the Linder hypothesis (Linder 1961). Linder argues that countries with higher per capita income demand relatively higher-quality goods. This is also in line with the recent growing literature (Gervais 2009; Hallak 2010; Hallak and Schott 2011; Feenstra and Romalis 2014).[8] z_d reflects also other country-specific characteristics like the culture and religion in determining the taste (Aw et al. 2020). $\sigma_{id} > 1$ is the elasticity of substitution between any two pair of varieties and can vary by product and country. Given $b_d(\lambda_{isd}(\omega), z_d)$ in the CES specification of this paper, the elasticity of substitution does not anymore capture the region-specific product differentiation characteristics that will be summarized in z_d , as opposed to the classical CES (Aw et al. 2020). The demand for each variety is also influenced by the differences between regions, i.e., country pair-specific factors that affect the trade of two nations, that are not linked to diversity in varieties. The Armington preference weight (share parameter), A_{isd}, captures these country pair-specific factors.

We impose a functional form for the quality demand $b_d(\lambda_{isd}(\omega), z_d) = \lambda_{isd}(\omega)^{\gamma(z)_d}$, where $\gamma_d > 1$ is the intensity of preferences for quality in country d (Hallak 2010; Demir 2011; Feenstra and Romalis 2014).[9] Hence, the demand function is

$$Q_{id} = \left(\sum_s A_{isd}^{\frac{1}{\sigma_{id}}} \int_{\omega\in\Omega_i} \left[\lambda_{isd}(\omega)^{\gamma(z)_d}\, q_{isd}(\omega)\right]^{\frac{\sigma_{id}-1}{\sigma_{id}}} \mathrm{d}\omega\right)^{\frac{\sigma_{id}}{\sigma_{id}-1}}. \tag{2}$$

This function shows that the higher the vertical attribute of a product, λ_{isd}, the more likely the consumer is to choose that product conditional on the prices. The power component, γ_d, governs the impact of the relative change in the vertical attributes of product quality on the demand for that product. The component γ_d may reflect the perception of the consumers toward quality and we allow this coefficient to vary with the relative

[7] Consumers demand for each quality-adjusted variety, $\bar{q}_{isd}(\omega)$, is $\left[b_d\left(\lambda_{isd}(\omega)\right) q_{isd}(\omega)\right]$, i.e., consumers only care about $\bar{q}_{isd}(\omega)$.

[8] This is also in line with the earlier work; see e.g. Jaskold Gabszewicz and Thisse 1979; Gabszewicz and Thisse 1980; Shaked and Sutton 1982, 1983 that show the marginal value of quality is higher for the household with greater income.

[9] Where $q_{isd}(\omega)$ represents physical units of the product and $\lambda(\omega)^{\gamma(gdp)_d}$ is a factor that maps physical units of the product into utility units.

income per capita of each country and define it as $\gamma(gdp)_d$ where gdp denotes the per capita income (Hallak 2010; Hallak and Schott 2011; Feenstra and Romalis 2014). For example, low-quality products are more demanded in low-income countries because low product quality firms find it profitable to export to these markets and vice versa (Fajgelbaum et al. 2011), which is consistent with the demand side of the Linder hypothesis. [10] We further assume that the willingness to pay for quality has the following functional form (Baumgärtner et al. 2017) γ_d = k GDP_d^{η}, whereas $\eta > 1$ is the elasticity of willingness to pay for quality with respect to the income.

With the price $p_{isd}(\omega)$ for variety ω in country d sourced from s, the resulting unit expenditure function, i.e., the aggregate of prices of varieties of i available in country d, P_{id}, is

$$P_{id} = \left(\sum_s A_{isd} \int_{\omega \in \Omega_i} \left[\left(\frac{p_{isd}(\omega)}{\lambda_{isd}(\omega)^{\gamma(gdp)_d}}\right]^{1-\sigma_{id}} \mathrm{d}\omega \right)^{\frac{1}{1-\sigma_{id}}}. \quad (3)$$

The unit expenditure function refers to the minimum amount of expenditure to generate one unit of utility ($Q_{id} \equiv U_{id}$). This function is dual to Q_{id}, which is defined in Equation (2).

Applying *Shephard's Lemma* on the expenditure function, the demand for each variety, $q_{isd}(\omega)$, is

$$q_{isd}(\omega) = A_{isd} \text{ N } [\lambda_{isd}(\omega)^{\gamma(gdp)_d}]^{\sigma_{id}-1} \left(\frac{p_{isd}(\omega)}{P_{id}}\right)^{-\sigma_{id}} Q_{id}. \quad (4)$$

This function shows that demand for each variety is negatively related to its price, and conditional on prices, it is positively related to the vertical attributes of the products and the perception of consumers toward the vertical attributes. The demand for each variety of a product i shipped to country d ,q_{isd}, also increase linearly with the aggregate demand for the product i, Q_{id} and nonlinearly with the aggregate price index that reflects the price of substitutes.

2.2 Production

On the production side, there are N_{is} firms in each country s to produce i and each firm maximizes profit by deciding on entering each possible market. Each firm produces a distinct individual differentiated variety ω in a monopolistic competitive market. Firms are heterogeneous with respect to their productivity $\varphi \geq 1$, which

[10] We acknowledge that the substitution elasticity parameter still captures other components of consumers' taste, such as the religions and cultural factors that change the preferences, since we only consider the GDP as factor affecting the willingness to pay for quality. For example, from a religious point of view there is a low preference for alcoholic drinks and pig meat in Islamic countries regardless of the vertical quality of the product. To give another example, from the cultural point of view, the demand for horse meat in the U.S. and U.K. even with very high-quality brands is very low. Allowing γ to vary across products as well as countries will be a natural extension to the framework.

drives each firm to produce distinct product quality $\lambda \geq 1$. We follow the reasoning of Johnson (2012) and assume the functional form $\lambda_{is}(\omega) = \varphi_{is}^{\theta}$ to explain the quality as function of productivity. Firms' productivity refers to the amount of (composite) physical inputs required to produce one unit of the physical output. Each firm chooses whether to enter the specific market d , and therefore to offer a product quality level in that specific market, λ_{isd}, and to charge the price in the destination market, p_{isd}. The decision of firms depends on their productivity that determines their cost advantage as well as the product quality and the perception of consumers toward the quality they offer.

Firms in each region s produce outputs i with a composite unit of inputs c_{is} under constant returns to scale, and the marginal cost of production depends on the firm productivity φ_{is} and the quality of the good produced λ_{is}. Firms become aware of their productivity once they pay a fixed sunk cost f_{is}^{e}. Firms also pay fixed entry cost for every potential market that they will serve, f_{isd}, as well as the overhead fixed production cost $f > 0$. The existence of these fixed costs is the source of increasing returns to scale for firms. Firms' product quality offered to potential markets is endogenous and depends on the consumers' level of quality appreciation that determines taste for quality and firms' productivity parameter that defines the cost to produce quality. Accordingly, firms in each country have a marginal cost $mc(\varphi, \lambda)$ that depends on firms' productivity φ , and product quality λ. The marginal cost of production is given by

$$mc(\varphi, \lambda) = \frac{c_{is}}{\varphi_{is}} k \lambda_{is}^{\beta} \qquad 0 \leq \beta < 1, \tag{5}$$

where k is a constant, c_{is} is the unit input price, and β is the quality elasticity of marginal cost that shows the rate of increase in marginal cost with quality.[11] We assume β is below one to ensure that marginal cost is not increasing excessively fast. [12] We note that the two dimensions can be reduced to a single dimension as firms with the same productivity have the same quality output.

A firm selling to destination d from market s has fixed trade costs f_{isd} in units of composite input so that their fixed cost payment on each bilateral link is $c_{is} f_{isd}$. The shipment of the product to the destination d also involves the variable transport cost. Firms must ship $\tau_{isd} > 1$ units of products such that one unit of

[11] The similar cost schedule can be found in Mandel 2010; Demir 2011; Feenstra and Romalis 2014; Johnson 2012. The difference of these studies is that they use different functions for quality production to arrive to this form.
[12] We note that production of product quality incurs fixed cost; we suppress this term in the overhead fixed production cost.

product will reach destination d. Firms face no fixed and variable transport costs when selling to the domestic market ($f_{isd} = 0, \tau_{isd} = 1$). Accordingly, the firm operating profit is

$$\pi(\varphi, \lambda) = \frac{p_{isd}(\varphi,\lambda)\, q_{isd}(\varphi,\lambda)}{(1+t_{isd})} - \tau_{isd} \left[\frac{c_{is}}{\varphi_{is}} k\lambda_{is}^{\beta}\right] q_{isd}(\varphi, \lambda) - c_{is} f_{isd} \tag{6}$$

$$= q_{isd}(\varphi, \lambda)\left(\frac{p_{isd}(\varphi,\lambda)}{(1+t_{isd})} - \tau_{isd} \frac{c_{is}}{\varphi_{is}} k\lambda_{is}^{\beta}\right) - c_{is} f_{isd},$$

In the first line, the first expression on the right-hand side refers to the income of the firms. Here, upon consideration that $p_{isd}(\varphi, \lambda)$ and $q_{isd}(\varphi, \lambda)$ are delivered product price and product quantity, the income of the firm is corrected for the tariffs t_{isd}. The second and third terms refer to the variable costs and operating fixed bilateral costs, respectively. The second line factors out $q_{isd}(\varphi, \lambda)$ from the first and second expressions.[13]

2.3 Optimal pricing

Optimal pricing results from the derivation of $\pi(\varphi, \lambda)$ with respect to the $p_{isd}(\varphi, \lambda)$,

$$\frac{\delta\pi(\varphi,\lambda)}{\delta p_{isd}} = \frac{\delta q_{isd}}{\delta p_{isd}} \left(\frac{p_{isd}(\varphi,\lambda)}{(1+t_{isd})} - \tau_{isd} \left[\frac{c_{is}}{\varphi_{is}} k\lambda_{is}^{\beta}\right]\right) + \frac{1}{(1+t_{isd})} q_{isd}(\varphi, \lambda) = 0, \tag{7}$$

which after simplification is

$$p_{isd} = \frac{\sigma_{id}}{(\sigma_{id}-1)} \tau_{isd} \left[\frac{c_{is}}{\varphi_{is}} k\lambda_{is}^{\beta}\right] (1 + t_{isd}). \tag{8}$$

The optimal pricing shows that the firms charge a constant markup of $\frac{\sigma_{id}}{(\sigma_{id}-1)}$ over the marginal costs. The price charged by each firm decreases with firm productivity but increases with firm product quality. It also increases with per unit trade costs (transport costs and tariffs). Moreover, as σ_{id} reflects the own-price elasticity of demand for each product, the delivered price, i.e., cif price, increases with the value of the own-price elasticity.

Substituting (8) into (4) results in

$$q_{isd}(\omega) = [\lambda(\omega)^{\gamma(gdp)_d}]^{\sigma_{id}-1} \left(\frac{\frac{\sigma_{id}}{\sigma_{id}-1} \frac{c_{is}}{\varphi_{is}} \lambda_{isd}^{\beta} \tau_{isd}}{P_{id}}\right)^{-\sigma_{id}} Q_{id} \tag{9}$$

$$= \lambda_{is}^{\sigma_{id}(\gamma_d-\beta)-\gamma_d} \left(\frac{\sigma_{id}}{\sigma_{id}-1} \frac{c_{is}}{\varphi_{is}} \tau_{isd}\right)^{-\sigma_{id}} (P_{id})^{-\sigma_{id}} Q_{id}.$$

Considering that quality-adjusted variety is $\bar{q}_{isd}(\omega) = \lambda(\omega)^{\gamma(gdp)_d} q_{isd}(\omega)$, Equation (9) can be rewritten as

[13] Note that the notations φ_{is} and λ_{is} are identical to φ_{isd} and λ_{isd}.

$$\bar{q}_{isd}(\omega) = \lambda_{is}^{\sigma_{id}(\gamma_d-\beta)} \left(\frac{\sigma_{id}}{\sigma_{id}-1}\frac{c_{is}}{\varphi_{is}}\tau_{isd}\right)^{-\sigma_{id}} (P_{id})^{-\sigma_{id}} Q_{id}. \tag{10}$$

$\sigma_{id}(\gamma_d - \beta)$ shows the elasticity of quantity sold with respect to the changes in its quality (Borsky et al. 2018). As evident, this elasticity is a function of the own-price elasticity and the difference between the intensity for quality γ_d and the elasticity of marginal cost with respect to quality β. If γ_d and β are equal the demand is simplified to the standard demand function from the CES function. Note that here the existence of the term γ_d as a function of income shows that higher per capita income results in higher demand for a product, which means an additional channel for income to affect demand, i.e., the quality channel, compared to the standard case where income increases demand through an increase in Q_{id}.

2.4 Selection to export markets

Firms choose to export to each destination if they earn positive profit from selling to that destination. Export revenue for each firm in s selling to market d, $R(\varphi,\lambda)_{sd}$, is

$$R(\varphi,\lambda)_{sd} = \frac{p_{isd}(\varphi,\lambda)\, q_{isd}(\omega)}{(1+t_{isd})} \tag{21}$$

$$= \frac{1}{(1+t_{isd})} A_{isd} \left[\lambda_{is}(\omega)^{\gamma_d}\right]^{\sigma_{id}-1} \left(p_{isd}(\omega)\right)^{1-\sigma_{id}} P_{id}{}^{-\sigma_{id}} Q_{id}$$

$$= A_{isd}\, \lambda_{is}^{\beta-\sigma_{id}(\beta-\gamma_d)-\gamma_d} \left(\frac{\sigma_{id}}{\sigma_{id}-1}\frac{c_{is}}{\varphi_{is}}\tau_{isd}\right)^{1-\sigma_{id}} P_{id}{}^{-\sigma_{id}} Q_{id}.$$

In the first line, firm revenue is corrected for the tariffs; the second line results from substituting Equation (4) for $q_{isd}(\omega)$; and the third line results from substituting the optimal pricing inclusive of iceberg cost, i.e, Equation (8), into the above equation. The results show that a firm's revenue is a function of its product quality $\lambda(\omega)$, the quality appreciation level in sale destinations γ_d, quality elasticity of marginal cost β, own-price elasticity σ_{id}, and other factors present in $A_{isd}\left(\frac{c_{is}}{\varphi_{is}}\tau_{isd}\right)^{1-\sigma_{id}} P_{id}{}^{-\sigma_{id}} Q_{id}$ reflecting the revenue when the quality of the product is one unit.

Firms choose to export to each destination if they earn positive profit from selling to that destination ($\frac{R(\varphi,\lambda)_{isd}}{\sigma_{id}} \geq f_{isd}$), that is, their operating profit $\frac{R(\varphi,\lambda)_{isd}}{\sigma_{id}}$ at least cover the operating fixed cost f_{isd}. For each trade link, there is a marginal firm with a threshold productivity $\varphi^*{}_{isd}$ such that the profit is zero ($\frac{R(\varphi^*)_{isd}}{\sigma_{id}} = f_{isd}$). Firms with productivity levels above $\varphi^*{}_{isd}$ will export, others will not.

2.5 *Free entry equilibrium condition*

The decision to enter or exit is the same as in the benchmark model of Melitz (2003). As product development and production start-up costs must be disbursed, entry into the industry is costly. All potential entrants must pay the same one-time entry cost f_{is}^{e} irrespective of their decision to operate and the number of markets that they will serve. Considering that fixed costs are paid in terms of the physical units of composite input, the value of fixed cost payment is $c_{is}\, f_{is}^{e}$. This cost will be sunken and firms learn about their productivity, φ, and therefore the value of their investment only after the payment of these costs.

There exists an unlimited number of potential entrants in the industry. Entered firms also face a probability that a shock $\delta \in (0,1)$ will force them out of the market in each of the periods. A firm enters the industry if the expected value from its entry, that is, the expected accumulated operating profit across all markets and periods, is greater than its entry cost. For the marginal firm, i.e., an entrant firm with minimum productivity, the expected profits of a potential entry just equals the sunk costs of entering the market. Accordingly, the free entry condition is defined as

$$E[\boldsymbol{\pi}_{\boldsymbol{is}}] = \sum_{s \in R} \sum_{t=0}^{\infty} (1-\delta)^{t}\, E\left[\boldsymbol{\pi}_{\boldsymbol{isd}}^{t}\right]) = c_{is}\, f_{is}^{e}. \tag{12}$$

This equation shows that once the firm knows its productivity, it can decide to develop and produce a variety in its preferred market segment or to exit the industry immediately. The value of the firm, i.e., its expected profit, is zero if it draws productivity below the profitability threshold and exits. If the firm draws an ability above the minimum productivity value, the cutoff value, the firm produces and its value would equal to the flow of future profits discounted by the probability of exit.

3 Translating the theory into a computing setting

We now turn to translate the theoretical framework into a computable setting compatible with the currently available social accounting matrices and input-output tables that reflect the economic structure at the relatively aggregate levels. We follow the reasoning of Balistreri et al. (2011)[14] and define a representative firm with average productivity whose decision variables are determined by the distribution of the productivity of the operating firms.

Representative firm's productivity and quality

[14] Other related papers that introduced the Melitz model in a computing setting are Zhai 2008; Balistreri et al. 2011; Balistreri and Rutherford 2013; Akgul et al. 2016; Dixon et al. 2016; Jafari and Britz 2018; Oyamada 2017; Bekkers and Francois 2018.

We define the equilibrium by introducing a representative firm (variety) on each bilateral trade link. We follow Melitz (2003) and define the representative firm as a firm with representative productivity which is a CES aggregation of productivities of all firms operating on a given bilateral trade link

$$\tilde{\varphi}_{isd} = \left[\int_{\varphi^*_{isd}}^{\infty} \varphi_{isd}^{\sigma_{id}-1} \frac{g(\varphi_{is})}{1-G(\varphi^*_{isd})} \, d\varphi_{is}\right]^{\frac{1}{1-\sigma_{id}}}, \tag{13}$$

where $g(\varphi_{is})$ is the probability distribution function (pdf) of the firm productivities and $G\left(\varphi_{isd}\right)$ refers to the cumulative distribution function (cdf). Given that only those firms having a productivity above a certain level $\varphi_{is} > \varphi^*_{isd}$ will operate in a market, $1 - G(\varphi^*_{isd})$ is the probability that the firm is active in the market. Thus, the pdf, conditional on the firm being active on the market d, is $\frac{g(\varphi_{is})}{1-G(\varphi^*_{isd})}$.

Given the imposed relation of quality as a function of productivity $\lambda = \varphi^{\psi}$ and considering the productivity of the representative firm $\tilde{\varphi}$, the product quality of the representative firm $\tilde{\lambda}_{isd}$ is

$$\tilde{\lambda}_{isd} = \tilde{\varphi}_{isd}^{\psi}. \tag{14}$$

Following the reasoning of the literature, we rely here on a Pareto productivity distribution, which has analytical tractability, and empirical evidence supports that the distribution of firms' productivity follows this distribution (Cabral and Mata 2003). The Pareto pdf is

$$g(\varphi_{is}) = \frac{a}{\varphi_{is}} (\frac{b}{\varphi_{is}})^{a}, \tag{15}$$

and thus, its cdf is

$$G(\varphi_{is}) = 1 - (\frac{b}{\varphi_{is}})^{a}, \tag{16}$$

where $b > 0$ is the minimum productivity and $a > 0$ is a shape parameter. Lower values of the shape parameter imply higher productivity dispersion among firms. Taking φ_{is}= φ^*_{isd} in the cdf, the conditional pdf is

$$\frac{g(\varphi_{is})}{1-G(\varphi^*_{isd})} = \frac{\frac{a}{\varphi_{is}}(\frac{b}{\varphi_{is}})^{a}}{(\frac{b}{\varphi^*_{isd}})^{a}} = \frac{a(\varphi^*_{isd})^{a}}{\varphi_{is}{}^{a+1}} \text{ if } \varphi_{is} \geq \varphi^*_{isd} \tag{37}$$

$$= 0 \text{ if } \quad \varphi_{is} < \varphi^*_{isd}.$$

Given the conditional Pareto pdf, we may derive the ratio of the productivity of the representative firm to the marginal firm, i.e., zero-profit productivity cutoff. Substituting (16) into (13) results in the productivity of the representative firm

$$(\frac{\tilde{\varphi}_{isd}}{\tilde{\varphi}_{isd}})^{1-\sigma_{id}} = \frac{a+1-\sigma_{id}}{a}, \quad (48)$$

whereas we assume $a > \sigma_{id} - 1$ for the convergence of the integral to ensure the finite average productivity level in the industry (see Melitz 2003).

Moreover, taking $\varphi_{is} = \varphi^*_{isd}$ in the cdf the probability that firm productivity is greater than φ^*_{isd} , is $1 - G(\varphi^*_{isd}) = (\frac{b}{\varphi^*_{isd}})^a = \frac{N_{isd}}{M_{is}}$, which after solving for φ^*_{isd} results in

$$\varphi^*_{isd} = \mathrm{b}\ (\frac{N_{isd}}{M_{is}})^a. \quad (59)$$

Substituting (19) in (18) and writing the function in terms of $\tilde{\varphi}_{isd}$ results in the representative firm productivity

$$\tilde{\varphi}_{isd} = b(\frac{a}{a+1-\sigma_{id}})^{1/\sigma_{id-1}}\ (\frac{N_{isd}}{M_{is}})^{-1/a}. \quad (20)$$

Representative firm's optimal price

We substitute the particular productivity level $\tilde{\varphi}$ for φ in optimal pricing equation (8), thus the optimal price of the representative firm is

$$\tilde{p}_{isd} = \frac{\sigma_{isd}}{(\sigma_{isd}-1)}\tau_{isd}\ \left[\frac{c_{is}}{\tilde{\varphi}_{is}}\ k\tilde{\lambda}^{\beta}_{is}\right](1+t_{isd}) \quad (21)$$

$$= \mathscr{b}_{isd}\ c_{is}k\tilde{\lambda}^{\beta}_{is}\left[\frac{1}{\tilde{\varphi}_{is}}\right],$$

whereas the second line defines $\mathscr{b}_{isd} = \frac{\sigma_{id}}{(\sigma_{id}-1)}\tau_{isd}\ (1+t_{isd})$. Here, we use $p_{isd}(\tilde{\varphi}) \equiv \tilde{p}_{isd}$ and $\lambda_{isd}(\tilde{\lambda}) \equiv \tilde{\lambda}_{isd}$.

Aggregate price and bilateral firm demand

Dividing both sides of Equation (21) by $\tilde{\lambda}_{isd}(\omega)^{\gamma_d}$ results in the adjusted consumer price

$$\frac{\tilde{p}_{isd}}{\tilde{\lambda}_{isd}(\omega)^{\gamma_d}} = \frac{\mathscr{b}_{isd}\ c_{is}k\tilde{\lambda}^{\beta}_{is}}{\tilde{\lambda}_{isd}(\omega)^{\gamma_d}}\left[\frac{1}{\tilde{\varphi}_{is}}\right], \quad (22)$$

Considering the conditional pdf, we write the average Dixit-Stieglitz price index as

$$P_{id} = \left(\sum_{s\in \mathrm{R}} A_{jsd}\int_{\varphi^*_{isd}} N_{isd}\left[\ (\frac{p_{isd}(\varphi)}{\lambda_{isd}(\varphi)^{\gamma_d}}\right]^{1-\sigma_{id}}\frac{g(\varphi_{is})}{1-G(\varphi^*_{isd})}\mathrm{d}\varphi_{is}\ \right)^{\frac{1}{1-\sigma_{id}}}, \quad (23)$$

where N_{isd} is the mass of firms in sector i of region s that sell in region d. We note that ω for each product variety is identical to φ because each firm with unique productivity produces a unique variety. Therefore, the notations ω are replaced by φ in (23).

Substituting (22) into (23), the integral phrase reads as

$$\int_{\varphi_{isd}^*} N_{isd} \left[\left(\frac{p_{isd}(\varphi)}{\lambda_{isd}(\varphi)^{\gamma_d}} \right]^{1-\sigma_{id}} \frac{g(\varphi_{is})}{1-G(\varphi_{isd}^*)} \mathrm{d}\varphi_{is} \tag{24}$$

$$= N_{isd} \int_{\varphi_{isd}^*} \left[\frac{\mathscr{b}_{isd}\, c_{is} k \lambda_{is}^{\beta}}{\lambda_{isd}(\omega)^{\gamma_d}} \left[\frac{1}{\varphi_{is}}\right] \right]^{1-\sigma_{id}} \frac{g(\varphi_{is})}{1-G(\varphi_{isd}^*)} \mathrm{d}\varphi_{is}$$

$$= N_{isd} \mathscr{b}_{isd}\, c_{is} k \int_{\varphi_{isd}^*} \left[\frac{\lambda_{is}^{\beta}}{\lambda_{isd}(\omega)^{\gamma_d}} \frac{1}{\varphi_{is}} \right]^{1-\sigma_{id}} \frac{g(\varphi_{is})}{1-G(\varphi_{isd}^*)} \mathrm{d}\varphi_{is}$$

$$= N_{isd} \mathscr{b}_{isd}\, c_{is} k \left[\frac{\tilde{\lambda}_{is}^{\beta}}{\tilde{\lambda}_{isd}(\omega)^{\gamma_d}} \right]^{1-\sigma_{id}} \tilde{\varphi}_{is}{}^{1-\sigma_{id}}$$

$$= N_{isd} \frac{\tilde{p}_{isd}}{\tilde{\lambda}_{isd}(\omega)^{\gamma_d}}$$

Lines two and three follow from the substitution and simplification, respectively. Line 4 comes from combining the definition of the productivity of the representative firm (13), and the product quality of the representative firm (14). Line 5 comes from combing line 3 and the adjusted prices in equation (20). Substituting the last term in (23) results in the unit expenditure function, where products are differentiated vertically by $\tilde{\lambda}_{isd}$, and horizontally by N_{isd} , and γ_d is the willingness to pay for $\tilde{\lambda}_{isd}$

$$P_{id} = \left(\sum_{\omega} A_{isd}\, N_{isd}\, \tilde{\lambda}_{isd}{}^{\gamma_d(\sigma_{id}-1)}\, \tilde{p}_{isd}{}^{1-\sigma_{id}} \right)^{\frac{1}{1-\sigma_{id}}}. \tag{25}$$

The number of varieties in this equation N_{isd} captures the horizontal differentiation of the products. The higher the number of varieties, the lower will be the amount of expenditure to obtain one unit of utility. The higher the quality of the product conditional on $\gamma > 1$ leads to the similar. Moreover, the higher the consumers' perception of quality, the higher is the utility per unit of expenditure.

In Equation (4), if we choose $\varphi = \tilde{\varphi}$, the demand for the representative variety is[15]

$$\tilde{q}_{isd}(\omega) = A_{isd}\, [\tilde{\lambda}_{isd}(\omega)^{\gamma(gdp)_d}]^{\sigma_{id}-1} \left(\frac{\tilde{p}_{isd}(\omega)}{P_{id}}\right)^{-\sigma_{id}} Q_{id}. \tag{26}$$

Zero profit condition and selection to market condition

Considering the demand for average variety (26), the expected revenue of the average firm is obtained by multiplying $\tilde{q}_{isd}(\omega)$ by $\tilde{p}_{isd}(\omega)$ and correcting it for tariffs

[15] Alternatively, the demand for the representative variety can be obtained by Applying *Shephard's Lemma* on the expenditure function defined in (18).

$$\tilde{r}_{isd} = \frac{\tilde{p}_{isd}\tilde{q}_{isd}}{(1+t_{isd})} \tag{27}$$

$$= \frac{\boldsymbol{A_{isd}}\,[\tilde{\lambda}(\omega)^{\gamma_d}]^{\sigma_{id}-1}\left(\frac{\tilde{p}_{isd}(\omega)}{P_{id}}\right)^{-\sigma_{id}} Q_{id}}{(1+t_{isd})}$$

$$= \frac{\boldsymbol{A_{isd}}\,[\tilde{\lambda}(\omega)^{\gamma_d}]^{\sigma_{id}-1}\left(\tilde{p}_{isd}(\omega)\right)^{1-\sigma_{id}} (P_{id})^{\sigma_{id}} Q_{id}}{(1+t_{isd})}.$$

The ratio of the revenue of any two firms in sector i of region s on their sales in region d may be expressed as a function of the ratio of productivities and qualities

$$\frac{\tilde{r}_{isd}}{\hat{r}_{isd}} = \frac{A_{isd}\,[\tilde{\lambda}(\omega)^{\gamma_d}]^{\sigma_{id}-1}\left(\tilde{p}_{isd}(\omega)\right)^{1-\sigma_{id}} (P_{id})^{\sigma_{id}} Q_{id}}{A_{isd}\,[\hat{\lambda}(\omega)^{\gamma_d}]^{\sigma_{id}-1}\left(\hat{p}_{isd}(\omega)\right)^{1-\sigma_{id}} (P_{id})^{\sigma_{id}} Q_{id}} \tag{28}$$

$$= \frac{\tilde{\lambda}^{\gamma_d \, \sigma_{id}-1}\left(\frac{\tilde{\lambda}_{is}^{\beta}}{\tilde{\varphi}_{isd}}\right)^{1-\sigma_{id}}}{\hat{\lambda}^{\gamma_d \, \sigma_{id}-1}\left(\frac{\hat{\lambda}_{is}^{\beta}}{\hat{\varphi}_{isd}}\right)^{1-\sigma_{id}}}$$

$$= \left(\frac{\tilde{\varphi}_{isd}}{\hat{\varphi}_{isd}}\right)^{\sigma_{id}-1} \left(\frac{\tilde{\lambda}_{isd}^{\gamma_d-\beta}}{\hat{\lambda}_{isd}^{\gamma_d-\beta}}\right)^{\sigma_{id}-1},$$

whereas line 2 follows from the substitution of optimal pricing (21) and the productivity of the representative firm $\tilde{\varphi}_{isd}$ and the marginal firm $\hat{\varphi}_{isd}$ (20) into the first line. The last line simplifies. Thus, the revenue of the average firm is

$$\tilde{r}_{isd} = \hat{r}_{isd}\left(\frac{\tilde{\varphi}_{isd}}{\hat{\varphi}_{isd}}\right)^{\sigma_{id}-1} \left(\frac{\tilde{\lambda}_{isd}^{\gamma_d-\beta}}{\hat{\lambda}_{isd}^{\gamma_d-\beta}}\right)^{\sigma_{id}-1}. \tag{29}$$

If the marginal cost elasticity of quality is equal to the intensity rate of quality acceptance in destination countries, this equation reduces to the standard prediction stating that firm revenues are only different in terms of the productivity.

Firm profit for the representative firm is

$$\tilde{\pi}_{isd} = \frac{\tilde{r}_{isd}}{\sigma_{id}} - c_{is} f_{isd}, \tag{30}$$

and for the marginal firm, the profit would be zero; that is the firm operating profit $\frac{\hat{r}_{isd}}{\sigma_{id}}$ would be equal to its operating fixed cost and this results in

$$\hat{r}_{isd} = \sigma_{id}\, c_{is} f_{isd}. \tag{31}$$

Substituting (31) into (29) results in the revenue of the representative firm

$$\tilde{r}_{isd} = (\frac{\tilde{\varphi}_{isd}}{\hat{\varphi}_{isd}})^{\sigma_{id}-1} (\frac{\tilde{\lambda}_{isd}^{\gamma_d-\beta}}{\hat{\lambda}_{isd}^{\gamma_d-\beta}})^{\sigma_{id}-1} f_{isd}\, c_{is}\, \sigma_{id}. \tag{32}$$

Furthermore, substituting (20) for $(\frac{\tilde{\varphi}_{isd}}{\hat{\varphi}_{isd}})^{\sigma_{id}-1}$ into (32) and combine it with (31) results in

$$f_{isd}\, c_{is} = \tilde{r}_{isd} \frac{(a+1-\sigma_{id})}{a\, \sigma_{id}} (\frac{\tilde{\lambda}_{isd}^{\gamma_d-\beta}}{\hat{\lambda}_{isd}^{\gamma_d-\beta}})^{1-\sigma_{id}} \tag{33}$$

$$= \frac{\tilde{p}_{isd}\tilde{q}_{isd}}{(1+t_{isd})} \frac{(a+1-\sigma_{id})}{a\, \sigma_{id}} (\frac{\tilde{\lambda}_{isd}}{\hat{\lambda}_{isd}})^{(\sigma_{id}-1)(\gamma_d-\beta)}$$

$$= \frac{\tilde{p}_{isd}\tilde{q}_{isd}}{\sigma_{id}(1+t_{isd})} (\frac{\sigma_{id}-1}{a})^{1+\psi(\gamma_d-\beta)} ,$$

whereas the second line follows from the definition of the average revenue by substituting the first line in this equation with the first line in (27). The third line follows from the consideration of $\lambda = \varphi^{\psi}$ that determines $\frac{\tilde{\lambda}_{isd}}{\hat{\lambda}_{isd}} = (\frac{\tilde{\varphi}_{isd}}{\hat{\varphi}_{isd}})^{\psi} = \left[\frac{a+1-\sigma_{id}}{a}\right]^{\frac{\psi}{1-\sigma_{id}}}$. Equation (33) determines the selection to each bilateral market.

The free entry equilibrium condition

The conditionally expected profits for an individual firm in sector i of region s of sales in region d, i.e., $E\left[\pi_{isd}^{t}\right]$ in (14) are

$$E[\pi_{isd}] = \frac{E[r_{isd}]}{\sigma_{id}} - f_{isd}\, c_{is} = \frac{\tilde{p}_{isd}\tilde{q}_{isd}}{(1+t_{isd})} - f_{isd}\, c_{is} \text{ if } \varphi_{is} \geq \varphi_{isd}^{*} \tag{34}$$

$$= 0 \text{ if } \varphi_{is} < \varphi_{isd}^{*} \;.$$

The firms' aggregate variables including profits are on the steady-state and there is an annual probability of death to each firm $0 < \delta < 1$, that is, the probability of survival is 1- δ. Then, the conditional expected profit of a new entrant firm on the steady-state is

$$\sum_{t=0}^{\infty}(1-\delta)^{t} E\left[\pi_{isd}^{t}\right] = \frac{1}{\delta}\left[E[\pi_{isd}]\right] = \frac{1}{\delta}\left[\frac{E[\boldsymbol{r}_{isd}]}{\sigma_{id}} - f_{isd}\, c_{is}\right] \tag{35}$$

$$= \frac{1}{\delta}\left[\frac{\tilde{p}_{isd}\tilde{q}_{isd}}{\sigma_{id}(1+t_{isd})} - f_{isd}\, c_{is}\right] \text{ if } \varphi_{is} \geq \varphi_{isd}^{*}$$

$$= 0 \text{ if } \varphi_{is} < \varphi_{isd}^{*}.$$

A potential entrant will realize these profits with a probability of $1 - G(\varphi_{isd}^{*})$. Considering this probability and substituting (35) in (14) results in the free entry equilibrium condition; that is, the expected profits of a potential entrant summed over all markets just equals the sunk costs of entering the markets

$$\sum_{s\in R} 1 - G(\varphi_{isd}^{*})\ [\ \frac{\tilde{p}_{isd}\tilde{q}_{isd}}{(1+t_{isd})\sigma_{id}} - f_{isd}\ c_{is}\ ] = \delta\ c_{is}\ f_{is}^{e}\ . \tag{36}$$

In the steady-state equilibrium, the probability that a portion of M_{is} firms will operate in the s market is $\frac{N_{isd}}{M_{is}}$, which means that $1 - G(\varphi_{isd}^{*}) = \frac{N_{isd}}{M_{is}}$. Considering this in (36) results in

$$\sum_{s\in R} \frac{N_{isd}}{M_{is}}\ [\ \frac{\tilde{p}_{hrs}\tilde{q}_{hrs}}{(1+t_{isd})\sigma_{id}} - f_{isd}\ c_{is}\ ] = \delta\ c_{is}\ f_{is}^{e}. \tag{37}$$

Substitute f_{isd} with c_{is} from (36) results in the free entry equilibrium condition

$$\sum_{s\in R} \frac{N_{isd}}{M_{is}}\ [\frac{\tilde{p}_{isd}\tilde{q}_{isd}}{\sigma_{id}(1+t_{isd})}\ (\frac{\sigma_{id}-1}{a})^{1+\psi(\gamma_d-\beta)})] = \delta\ c_{is}\ f_{is}^{e}. \tag{38}$$

Market clearance for composite goods

We must ensure that the supply and demand for the composite input are equal. That is the amount of the composite input required for variable costs and fixed costs is equal to its total supply Y_{is}. The total demand for the composite input is the sum of the demand for variable costs, bilateral fixed costs, and fixed entry costs. The amount of the composite input required for variable costs by the average productivity firm in sector i of region s for sale in region d is $\frac{\tau_{isd}\tilde{q}_{isd}}{\tilde{\varphi}_{is}}$. Aggregating over all firms results in $N_{isd}\,\frac{\tau_{isd}\tilde{q}_{isd}}{\tilde{\varphi}_{is}}$. In addition, we must account for the use of the composite input to cover the fixed entry costs that are sunk and the bilateral fixed costs to operate on different trade links. Summing over all markets gives us the market clearance condition

$$Y_{is} = \delta\ f_{is}^{e} M_{is} +\ N_{isd}\ (f_{isd} +\ \frac{\tau_{isd}\ q_{isd}}{\tilde{\varphi}_{isd}}). \tag{39}$$

Unit expenditure function, welfare, and sectoral demand

The demand for the aggregate commodity i, Q_{id}, is determined in the general equilibrium. We assume that consumers have CES preferences over composite goods $i \in$ I and therefore the unit expenditure (price index) for each region P_d is

$$P_d =\ \left(\sum_{i\in \mathrm{I}} A_{id}\ \ \mathrm{P}_{id}{}^{1-\sigma_s}\ \right)^{\frac{1}{1-\sigma_d}}, \tag{40}$$

whereas P_d is the minimum expenditure required to obtain one unit of utility and it reflects the true cost of the living price index. σ_d denotes the elasticity of substitution between every two products in I, and A_{id} indicates the relative preference across composite goods in I. In such setting, the welfare, i.e., total demand quantity, in region d, $U_d \equiv Q_d$ is simply the value of nominal income GDP over the price index

$$U_d = \frac{GDP_d}{P_d}, \tag{41}$$

and the application of Shephard's Lemma on the aggregate expenditure function results in the demand for each composite good

$$Q_{id} = U_d \left(\frac{A_{id}\, P_d}{P_{id}}\right)^{\sigma_s}. \tag{42}$$

Unit cost function, primary factor markets

Considering the input markets, we assume that the unit input Y_{is} is produced by a Cobb-Douglas technology such that the price of composite input, i.e., the unit cost function c_{is} as a function of the prices of primary factor inputs w_{fr} is

$$c_{is} = \prod_f (w_{fr})^{\mu_{fir}}, \tag{43}$$

whereas μ_{fir} denotes the share of each factor input from total input use.

Considering the demand across all sectors, the total supply of L_{fr} must be equal to the sum of its demand use. Under the assumption of the fixed factor endowment, the total supply of each primary factor $\bar{L}_{fr}$ is equal to the demands

$$\bar{L}_{fr} = \frac{\mu_{fir}\, Y_{id}\, C_{is}}{w_{fr}}, \tag{44}$$

whereas applying Shephard's lemma to the cost function delivers the demand for primary factors.

Income and willingness to pay for quality

The income in each region GDP_d is the value of factor endowments in that region,

$$GDP_d = \sum \quad w_{fr}\, \bar{L}_{fr}. \tag{45}$$

Given the GDP level and assuming that the population is exogenous, the willingness to pay for the quality of the consumers in each market is

$$\gamma_d = \text{k}\, GDP_d^{\eta}. \tag{46}$$

We summarize the equations of this section that will be directly used in the model in Table 1. The system of equations is a square system with a dimension of 5 (I × R× R) + 6 (R× R) + 4R.

Table 1: Melitz model with product quality differentiation

Equations definition	Equations	Associated variable	Dimensions
Product quality of the representative firm (14)	$\tilde{\lambda}_{isd} = \tilde{\varphi}_{isd}^{\psi}$	$\tilde{\lambda}_{is}$: Quality	I × R
Productivity of representative firm (20)	$\tilde{\varphi}_{isd} = b(\frac{a}{a+1-\sigma_{id}})^{1/\sigma_{id-1}} (\frac{N_{isd}}{M_{is}})^{-1/a}$	$\tilde{\varphi}_{isd}$: Productivity	I × R× R
Markup condition (21)	$\tilde{p}_{isd} = \frac{\sigma_{isd}}{(\sigma_{isd}-1)} \tau_{isd} \left[\frac{c_{is}}{\tilde{\varphi}_{isd}} k\tilde{\lambda}_{isd}^{\beta}\right] (1+t_{isd})$	$\tilde{q}_{isd}$: Firm output	I × R× R
Price index on composite commodity (25)	$P_{id} = \left(\sum_{\omega} A_{isd}\, N_{isd}\, \tilde{\lambda}_{isd}{}^{\gamma_d(\sigma_{id}-1)}\, \tilde{p}_{isd}{}^{1-\sigma_{id}}\right)^{\frac{1}{1-\sigma_{id}}}$	Q_{id}: Aggregate quantity	I × R
Bilateral firm-level demand (26)	$\tilde{q}_{isd}(\omega) = A_{isd}\, \mathrm{N}\, [\tilde{\lambda}_{isd}(\omega)^{\gamma(gdp)_d}]^{\sigma_{id}-1} \left(\frac{\tilde{p}_{isd}(\omega)}{P_{id}}\right)^{-\sigma_{id}} Q_{id}$	$\tilde{p}_{isd}$: Firm-level price	I × R× R
Zero Profits (33)	$f_{isd}\, c_{is} = = \frac{\tilde{p}_{isd}\tilde{q}_{isd}}{\sigma_{id}(1+t_{isd})} (\frac{\sigma_{id}-1}{a})^{1+\psi(\gamma_d-\beta)}$	N_{isd}: Operating firms	I × R× R
Free Entry (38)	$\delta\, c_{is}\, f_{is}^{e} = \sum_{s\in R} \frac{N_{isd}}{M_{is}} [\frac{\tilde{p}_{isd}\tilde{q}_{isd}}{\sigma_{id}(1+t_{isd})} (\frac{\sigma_{id}-1}{a})^{1+\psi(\gamma_d-\beta)})]$	M_{id}: Entrant firms	I × R× R
Composite input market-clearing (39)	$Y_{is} = \delta\, f_{is}^{e} M_{is} + N_{isd}\, (f_{isd} + \frac{\tau_{isd}\, q_{isd}}{\tilde{\varphi}_{isd}})$	C_{is}: Unit price for composite inputs	I × R
Unit expenditure function (40)	$P_d = \left(\sum_i A_{id}\ \mathrm{P}_{id}{}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_d}}$	U_d: Welfare	R
Total demand (41)	$U_d = \frac{GDP_d}{P_d}$	P_d: Consumer price index	R
Demand by sector (42)	$Q_{id} = U_d \left(\frac{A_{id}\, P_d}{P_{id}}\right)^{\sigma_s}$	P_{id}: Product price	I × R
Unit cost function (43)	$C_{is} = \prod_f (w_{fs})^{\mu_{fis}}$	Y_{is}: Composite input	I × R

4 Conclusion

Trade literature shows that product quality differentiation explains important features of international trade. The quality differentiation explains the reasons for the increase in unit value of products in the countries with high GDP per capita, for the presence (absence) of firms with low (high) productivity on international trade markets, and for the higher average prices charged for similar products by exporters compared to non-exporters.

To date, the trade CGE models, as a main ex-ante tool for impact assessment of policies, simulate trade changes based on the assumption that product qualities are available on each market and the perception of consumers toward the available qualities do not change. This is contrary to the empirical evidence. Against this background, this study developed a unified trade model based on the existing trade literature to consider quality differences of products offered by each firm and to allow heterogeneous perceptions of consumers across countries for the product quality. Within this framework, the latter may vary with countries' income per capita.

The model is transformed to a computable setting in a CGE framework to broaden its accessibility to the modelers. The model shows that active firms export to a given destination if two conditions are held. First, if the marginal cost of quality products in the domestic market is not bigger than the willingness of consumers in the export markets to pay for that unit of quality. Second, if the operating profit of firms on each trade link is higher than the fixed bilateral cost of trading on that link.

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