Alternative Investment Dynamics Schemes:

Implementation Based on A Recursive Dynamic Model

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ABSTRACT

This paper documents a set of alternative investment dynamics schemes implemented in the recursive dynamic MIT's U.S. Regional Energy Policy (USREP) model. Compared to the current investment dynamics that ignores the price-based investment behavior, the alternative investment schemes endogenize investment decisions through various mechanisms that relate investment cost with the rate of return to capital. Specifically, one alternative scheme applies Tobin's q through an endogenous adjustment of investment such that investment cost is equalized to replacement value of capital assets. To enable the representative agent to adjust investment based on past information or future expectation, we introduce hybrid investment dynamic schemes, namely the adaptive expectation and forward-looking rational expectation. Both hybrid schemes follow the Bellman's Principle of Optimality and impose an arbitrage condition across periods on investment cost justified by the rate of return to capital taking into account depreciation. Furthermore, the forward-looking rational expectation investment scheme shows robustness in converging to the optimal path when considering adjustment cost to investment and variation in initial starting values.

Keyword: Investment Dynamics, Bellman's Principle of Optimality, Adaptive and Rational Expectation, General Equilibrium

1. Introduction

In the realm of policy analysis, investment decisions are considered as a linchpin for the effective implementation needed to achieve policy goals. To understand the crucial role of investment, the study assesses the impact of different investment dynamics assumptions in the context of an ambitious emissions mitigation policy. First and foremost, the transition to a low-carbon economy requires substantial capital investment in the development and deployment of cleaner technologies. Moreover, the investment decision affects the cost-effectiveness of emission policies because investments in clean technologies today bear cost implications for future emissions compliance (Willner et al., 2021). Recognizing the growing importance of computable general equilibrium (CGE) application in climate and environmental policy evaluation, this paper uses a recursive dynamic CGE model of the U.S. economy to explore alternative investment dynamics schemes and provide an understanding of the vital role played by investment decisions in optimizing climate and environmental policies.

Leveraging the insights gained from prior efforts in developing MIT Emissions Prediction and Policy Analysis (EPPA) model (Chen et al., 2022) with two distinct investment schemes, we formulate three alternative investment dynamics to illustrate the impact of investment decision within the context of policy considerations. More importantly, we construct a framework facilitating forward-looking rational expectation within a recursive dynamic setting.

In the recursive version of the EPPA model (EPPA-RD), decisions about production, consumption and investment are made only on the basis of prices in the period of the decision, and this is often referred to as “myopic” expectations. Investments (which are converted to capital in the next period) are made as if input costs and output prices will remain unchanged in the future. In EPPA-RD, savings and total consumption are fixed shares of income and so consumers do not alter their savings and consumption on expectations of future returns on investment or on expectations of changes in the price of consumption in the future. In a forward-looking version of EPPA (EPPA-FL) with perfect foresight, optimization over time means that decisions today about production, consumption and investment are based on expectations that are realized in the model simulation. Consumers equate the marginal utility of consumption through time - this feature of an optimizing through time results in a phenomenon sometimes referred to as consumption smoothing because anticipated shocks in consumption are smoothed out by altering savings. This is a "substitution" effect. Agents also look ahead and change savings in anticipation of changes in investment opportunities. This is an "income" effect because, for example, more savings today in anticipation of higher returns creates more future income.

The current version of USREP (Yuan et al., 2019) is a recursive dynamic model based on "myopic" expectation as assumed in EPPA-RD. The investment behavior is driven only by the growth in consumption without considering price signals for optimal investment in each period. Put another way, the rate of return on capital has no direct impact on savings and investment decisions. Households will not invest more when capital return is high or invest less otherwise. This deviates from the core of the Ramsey convention that relates investment decisions to the current and future rates of return, as specified in EPPA-FL.

To endogenize investment decisions by relating investment decisions with the rate of return to capital, we could follow the same procedure of building EPPA-FL from EPPA-RD. However, the current scale of the model dimensionality and the level of details to include make it complex to convert to a forward-looking model with perfect foresight. Moreover, the forward-looking behavior with perfect foresight is not realistic in that uncertainties exist given that the changes to economy, technology, and policy cannot be perfectly foreseen, especially for a long-term evaluation. Therefore, we propose alternative investment behavior schemes that are more realistic compared to perfect foresight and easy to implement into the dynamic recursive USREP.

All proposed alternative investment schemes address the omission in price-based investment decisions. To keep model tractability, we start with one alternative scheme that applies Tobin's q through an endogenous adjustment of investment such that investment cost is equalized to replacement value of capital assets. To enable the representative agent to adjust investment based on past information or future expectation, we introduce hybrid investment dynamic schemes, namely the adaptive expectation and forward-looking rational expectation. Both hybrid schemes follow the Bellman's Principle of Optimality and impose an arbitrage condition across periods on investment cost justified by the rate of return to capital, adjusted for depreciation.

None of the proposed methods affects the constant marginal propensity to save (MPS) assumption in the model.[1] By regulating investment decisions, consumption adjusts to grow at the same rate as savings and investment. This represents a departure from the rational expectation with perfect foresight that requires the marginal utility of consumption to be equalized across time periods. Therefore, the forward-looking rational expectation scheme presented in the paper only assumes limited foresight given the restriction on how consumption can dynamically adjust in response to future economic variations.

2. Theoretical formulation

A Ramsey dynamic optimization problem maximizes the present value of lifetime utility with a constant elasticity of intertemporal substitution utility function that is increasing, continuous and concave. Aggregate outputs from a constant return production function that combines capital and labor are either consumed or invested in new capital goods. Capital stock evolves according to the law of motion with adjustment for depreciation ($0 < \delta < 1$).

$$Max \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

subject to:

$$C_t = F(L_t, K_t) - I_t \tag{2}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

Given the recursive structure of the dynamic optimization problem, we can represent the problem as a dynamic programming problem. Specifically, we use the Bellman equation to express the infinite horizon optimization into an objective function over two successive periods. The Bellman equation introduces a functional equation V to relate the value of the states in the two periods. There are state variables that describe the current state of the system (K_t), and control variables that are to be chosen in the current period (C_t). The current period consumption determines current and future utilities. Since total output is exhausted by consumption and investment, the choice of consumption affects investment in the current period hence capital

[1] Consumption and savings are set in a fixed proportion, implying that the MPS is constant. The model assumes savings is exhausted by the amount of capital for new investment, suggesting that the dynamics of the model are determined by the constant MPS.

stock available for production in the next period. Applying Bellman's Principle of Optimality, the infinite horizon dynamic programming problem can be written as:

$$V(K_t) = \underset{C_t,\, K_{t+1}}{Max}\, \{U(C_t) + \beta V(K_{t+1})\} \tag{4}$$

Subject to: $C_t \; = \; F(L_t\, , K_t) + (1 - \delta)K_t - K_{t+1}$

We can eliminate C_t and convert the problem into one that chooses an optimal future state K_{t+1}:

$$V(K_t) = \underset{K_{t+1}}{Max}\{U(F(L_t\, , K_t) + (1 - \delta)K_t - K_{t+1}) + \beta V(K_{t+1})\} \tag{5}$$

For an optimal K_{t+1}, we derive first order condition of Equation (5) with respect to K_{t+1}:

$$U'(F(L_t\, , K_t) + (1 - \delta)K_t - K_{t+1}) = \beta V'(K_{t+1}) \tag{6}$$

Equation (6) equates marginal utility of consumption in period t to marginal utility of allocating to investment instead and the augmented consumption in period t+1.

Since V is unknown, we need to replace V with known information. Applying the Envelope Theorem to evaluate the derivative of the value function with respect to K_t yields:

$$V'(K_t) = U'(F(L_t\, , K_t) + (1 - \delta)K_t - K_{t+1}) \cdot [F_K(L_t\, , K_t) + (1 - \delta)] \tag{7}$$

where $F_K(L_t\, , K_t)$ is a partial derivative of production function with respect to K_t, marginal product of capital or the rate of return to capital.

Updating Equation (7) forward by one period and substitute for $V'(K_{t+1})$ in the first order condition, we obtain the Euler equation:

$$\frac{U\prime(C_t)}{\beta U\prime(C_{t+1})} = F_K(L_t\, , K_t) + (1 - \delta) \tag{8}$$

The left-hand side of Equation (8) represents the marginal rate of substitution (MRS) or the willingness to substitute present consumption for future consumption. MRS suggests the amount of increase in period *t+1* consumption be required for a one unit of reduction in period *t* consumption. The right-hand side represents the marginal rate of transformation (MRT) or the rate at which present consumption can be traded for future consumption through investment in capital. MRT implies the gain from one unit increase in investment through marginal increase in production and the resale value of capital adjusted for depreciation. The equilibrium between MRS and MRT ensures an intertemporal path of optimal consumption.

If we substitute Equation (6) into Equation (7),

$$V'(K_t) = U'(C_t) \cdot F_K(L_t\,, K_t) \; + \beta V'(K_{t+1})(1-\delta) \tag{9}$$

then replacing marginal value of capital and marginal utility of consumption in terms of prices, Equation (10) indicates that the price of capital in the current period (PK_t) should be sufficient to cover the rental price of capital (RK_t) plus the expected capital price in the next period (PK_{t+1}) adjusted for depreciation.

$$PK_t = \; RK_t + PK_{t+1}(1-\delta) \tag{10}$$

where $RK_t = P_t \cdot F_K(L_t\,, K_t)$ represents the rental price of capital, calculated as the product of the output price and the marginal product of capital. This is an arbitrage condition for endogenous investment decisions by weighing the price of capital against the rental value as well as the expected future value, taking into account capital productivity and the rate of depreciation. Holding everything else equal, an increase (decrease) in the expected future price of capital provides an incentive to invest more (less) in the current period.

3. Overview of the Methodologies

The current version of USREP adopts "myopic" expectation in a recursive dynamic formulation. It assumes that the economy is initially in a steady state where there is no external forcing from growth in population, productivity, energy efficiency, etc., and the investment exactly replaces depreciated capital. Investment growth is driven by trade-off against consumption, and does not follow the price-based investment behavior indicated by Equation (10). The rate of return on capital has no direct impact on savings and investment decisions. Households will not invest more when capital return is high or invest less otherwise, neither do households adjust investment decisions based on past and current information or future expectation of capital prices. To enable the representative agent to make investment decisions based on current state, past information, or future expectation, we introduce three investment dynamic schemes, namely Tobin's q, the adaptive expectation, and forward-looking rational expectation. The alternative investment schemes endogenize investment decisions through various mechanisms that relate investment cost with the rate of return to capital.

3.1. Tobin's q

Tobin (1969) argued that the rate of investment or the growth rate of capital stock should be related to the value of capital relative to its replacement cost. An increase in rate of return would attract more investment until cost of investment reaches an equilibrium with its replacement value. Assume (1) the economy is in a steady state where each period is treated as if the last period so $PK_t = PK_{t+1}$ and (2) capital stock is fully depreciated by the end of the period so $\delta = 1$. These assumptions lead to a stabilized Tobin's q condition that suggests cost of investment equal to replacement value measured by rental price of capital in the model:

$$PINV_t = RK_t \tag{11}$$

3.2. Static with Adaptive Expectation

The adaptive expectations assume that agents form their expectations based on past and current information. The adaptive forward-looking investment scheme we propose regulates the current-period investment by evaluating current investment cost against investment cost and the rate of return in the preceding period with adjustment for depreciation. Assuming capital malleability and reversible investment, we adopt Bellman's Principle of Optimality to establish two-period adaptive dynamics such that investment in the previous period is generated according to a forward-looking behavior with a one-period ahead foresight.

$$(1-\delta)\cdot PINV_t \;=\; PINV_{t-1} - RK_{t-1}\cdot ror \tag{12}$$

where *PINV* is cost of investment, and *RK* is the rental price of capital. δ is the capital depreciation rate, and *ror* the rate of return to capital. Time subscript *t* and *t-1* denote the current and previous period, respectively.

Adaptive expectation is relatively straightforward to implement due to the availability of historical data and trends. It is valuable for pinpointing investment opportunities in the short to medium term, which are primarily influenced by previous trends and price movements. Compared to Tobin's q, adaptive expectation utilizes past information in the decision-making process with limited foresight and imposes fewer restrictive assumptions, making it more suitable for broader applications. However, the adaptive expectation scheme has its limitations because it assumes the future mirrors the past which would not capture the impact of future policies on today's investment decision. This is where the rational expectation scheme comes into play.

3.3. Forward-Looking with Limited Foresight

A rational expectation scheme is formed based on all information including the expectation of future policies. The scheme is formulated to regulate the current-period investment by evaluating current investment cost against the rate of return in the current period and the resale value of the investment capital in the next period adjusted for depreciation. We assume investment is reversible in a sense that capital is malleable. The investment is justified by its cost weighed against its rental value plus the resale value in the next period:

$$PINV_t = (1 - \delta) \cdot PINV_{t+1} + RK_t \cdot ror \tag{13}$$

where time subscript *t* and *t+1* denote the current and next period, respectively.

According to Bellman's Principle of Optimality, the rational expectation scheme results in an optimal path for investment given that all parts of the optimal path are optimal. It eliminates biases inherent in the adaptive expectation scheme by integrating forward-looking information over a more extended period. This scheme not only accounts for historical data but also adjusts proactively for expected future conditions and policy changes, providing a sophisticated framework for designing investment strategies. Such foresight enables investors to preemptively adapt to economic changes, optimize asset allocation, and mitigate risks more effectively than the models lacking this foresight capability.

Alternative investment schemes proposed in this paper, particularly the rational expectation scheme, raise implications for the determination of consumption paths through endogenous adjustment in capital investment. The timing and scale of the adjustments heavily depend on the effective utilization of available information and the capability to anticipate the future conditions and policy changes. By integrating these investment decisions within the framework of capital market equilibrium, these approaches create a more proactive strategy that ensures the investment decisions are closely aligned with evolving economic dynamics.

4. Results

In this analysis, we employ a recursive dynamic general equilibrium model of the U.S. economy based on an open-source WiNDC database (WiNDC 3.1, 2022) with 2017 as the benchmark year. To illustrate the crucial role of investment, we assess the impact of different investment dynamics assumptions in the context of an ambitious emissions mitigation policy for the following reasons. First, the transition to a low-carbon economy requires substantial capital investment in developing and deploying cleaner technologies; Second, the investment decision affects the cost-effectiveness of carbon policies because investment in clean technologies today has cost implications for future emissions compliance.

We compare alternative investment dynamic schemes with the original scheme, labeling the reference and carbon scenarios accordingly to reflect the applied investment schemes. For instance, ‘Static’ denotes the original scheme, ‘Tobinq’ represents Tobin’s q scheme, ‘Adaptive’ indicates the static adaptive expectation scheme with a one-period ahead foresight, and ‘Rational’ refers to the forward-looking rational expectation scheme with limited foresight. For all four Reference scenarios, economic growth is calibrated to the GDP growth provided by U.S. Energy Information Administration Annual Energy Outlook in 2023.

In the Reference scenario, the ‘Static’ scheme maintains stable investment prices throughout the horizon. In sharp contrast, the ‘Tobinq’ scheme results in a rapid price increase over time due to its assumption that capital fully depreciates at the end of each period, requiring higher investment prices to attract investment for production in the next period. The schemes with expectation, both ‘Adaptive’ and ‘Rational’, exhibit decreasing investment price trends. The decline is more pronounced under the ‘Rational’ scheme. Consistent with the investment price trajectories, total investment increases most rapidly with the ‘Tobinq’ scheme while grows at the slowest rate under the ‘Rational’ scheme.

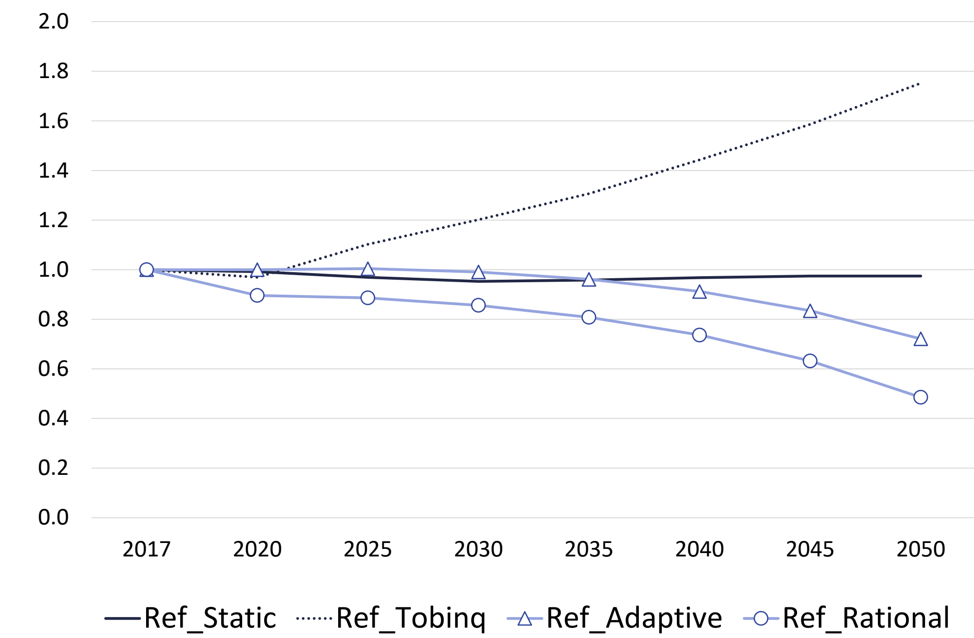

Figure 1: Baseline investment price (2017=1)

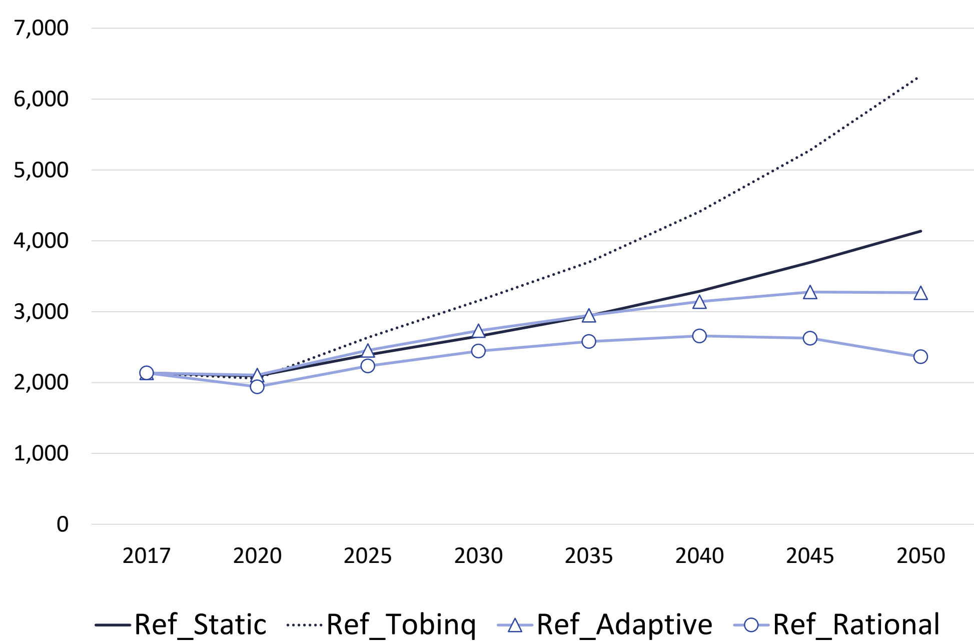

Figure 2: Baseline investment (Billion$)

In the Reference scenarios, the rate of return to capital varies across different investment schemes. The 'Rational' scheme exhibits the most rapid growth in the rate of return, suggesting high efficiency in capital utilization for output production. This scheme's forward-looking capacity optimizes the use of capital by aligning production with future market conditions, therefore generating higher economic return. The capital efficiency is reflected in the falling investment prices (see Figure 1), suggesting there is less demand to attract investment through higher cost when capital is used more efficiently. Conversely, the 'Tobinq' scheme shows the slowest growth in rate of return. Despite high investment price (see Figure 1) that is supposed to attract sufficient capital for future production, the complete depreciation assumption and the lack of foresight about future production dynamics contribute to the inefficiency in aligning investment decision with future market conditions.

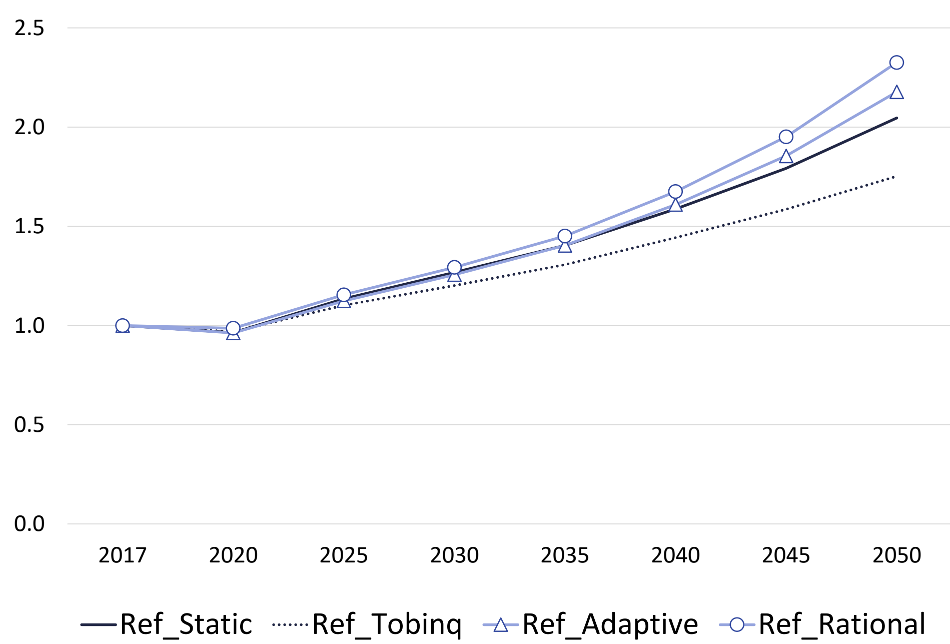

Figure 3: Baseline rate of return to capital (2017=1)

Figure 4 shows that welfare trajectories in all four baseline scenarios maintain the same order as the rate of return to capital. This alignment has implications for the impact of these investment schemes on overall economic welfare. Efficient allocation of investment leads to efficient use of capital for output, which contributes to a gain in economic welfare. Therefore, the 'Rational' scheme that finds an optimal path for investment decision is associated with the highest welfare gains.

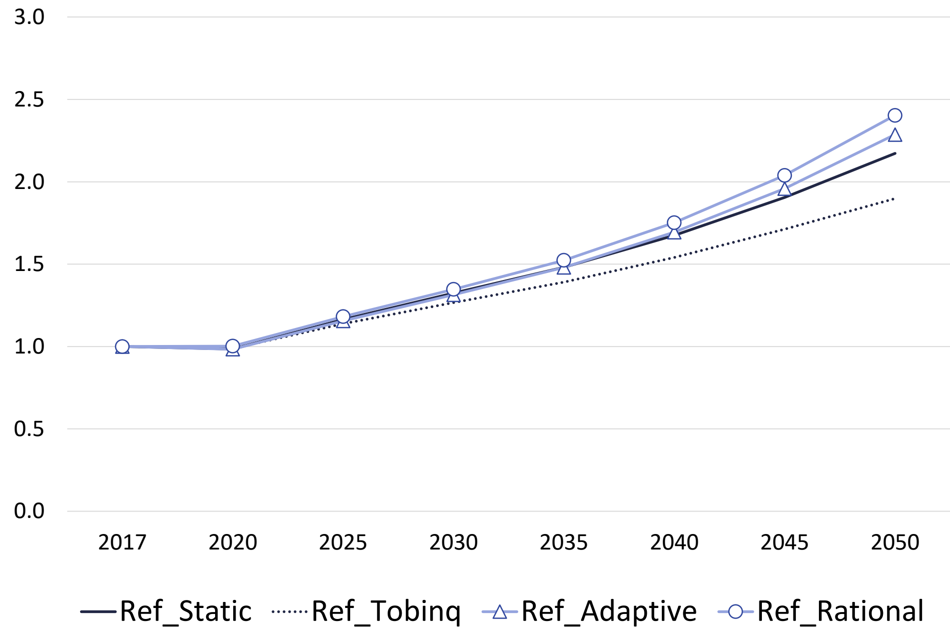

Figure 4: Baseline welfare (2017=1)

Policymakers should therefore promote investment strategies with expectations, such as the ‘Adaptive’ and ‘Rational’ schemes, as these follow the arbitrage condition that regulates investment decisions in an optimal manner. Efficiency in capital use should be an important consideration for these policies, as more efficient schemes not only yield higher returns but also improve economic welfare. Hence, policies should be designed to encourage investors and firms to adopt practices that enhance long-term economic outcomes by optimizing investment efficiency across various industries.

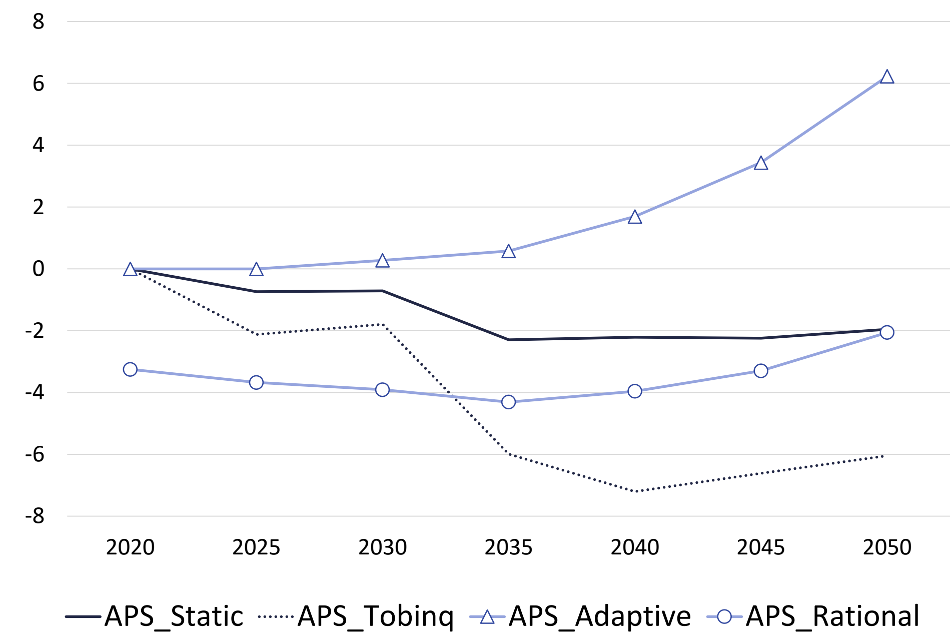

Figure 5: Investment price in the carbon scenario (percent change from the baseline)

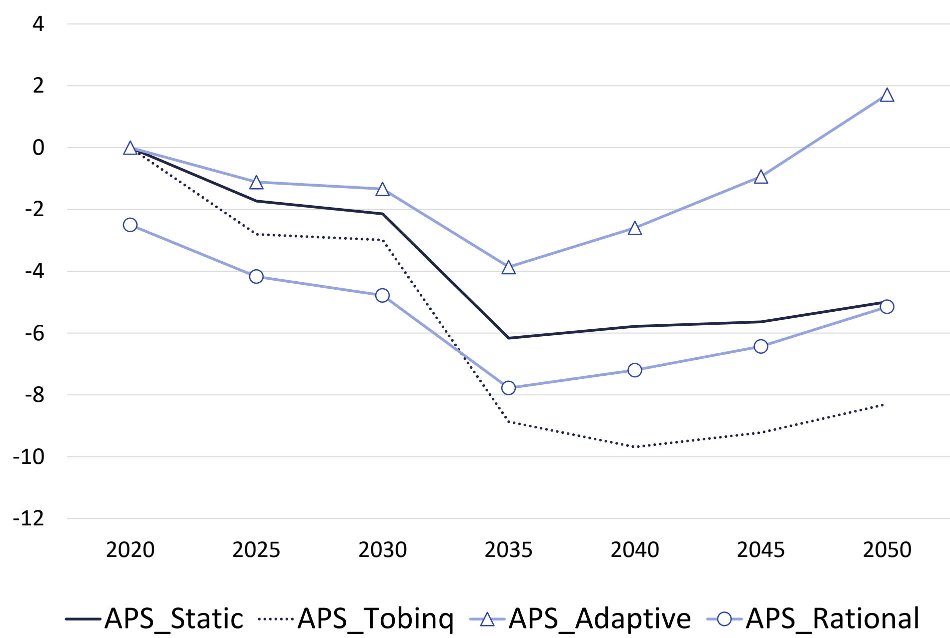

Figure 6: Investment in the carbon scenario (percent change from the baseline)

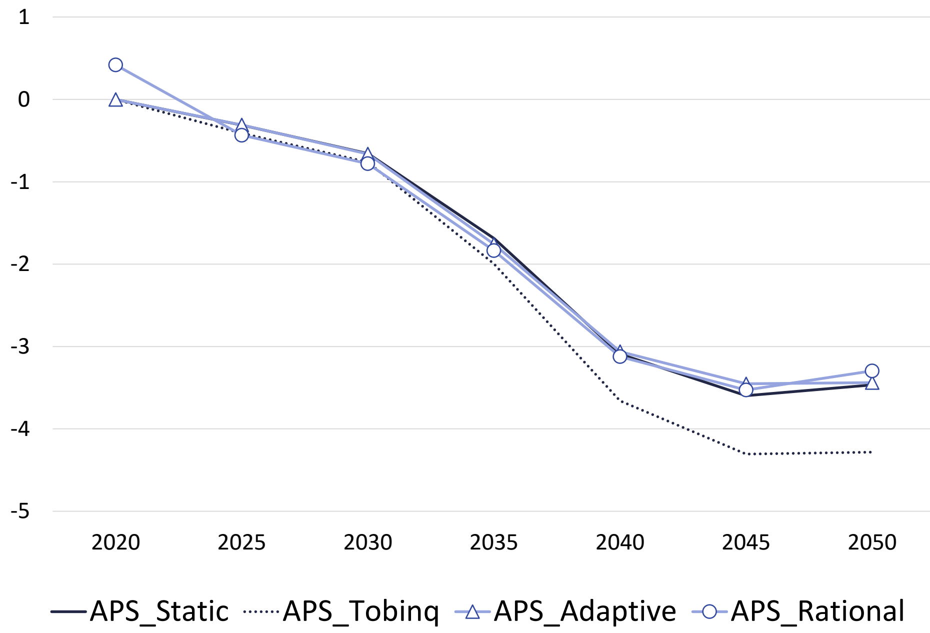

Figure 7: Welfare in the carbon scenario (percent change from the baseline)

Reference

Babiker, M., A. Gurgel, S. Paltsev, J. Reilly. (2009). Forward-Looking Versus Recursive-Dynamic Modeling in Climate Policy Analysis: A Comparison. Economic Modelling, 26(6): 1341-1354 (http://dx.doi.org/10.1016/j.econmod.2009.06.009)

Chen, Y.-H. Henry, S. Paltsev, A. Gurgel, J. Reilly and J. Morris (2022): A multisectoral dynamic model for energy, economic and climate scenario analysis. Low Carbon Economy, 13(2) (doi: 10.4236/lce.2022.132005)

Dixon, Peter & Pearson, K.R. & Picton, Mark & Rimmer, Maureen. (2005). Rational expectations for large CGE models: A practical algorithm and a policy application. Economic Modelling. 22. 1001-1019. 10.1016/j.econmod.2005.06.007.

HM Revenue & Customs (HMRC). (2013). Analysis of the dynamic effects of Corporation Tax reductions. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/263560/4069_CT_Dynamic_effects_paper_20130312_IW_v2.pdf

Tobin, J. (1969). "A General Equilibrium Approach to Monetary Theory". Journal of Money, Credit and Banking, Volume 1, No. 1, 15-29.

Willner, S.N., Glanemann, N. & Levermann. (2021). A. Investment incentive reduced by climate damages can be restored by optimal policy. Nat Commun 12, 3245. https://doi.org/10.1038/s41467-021-23547-5

WiNDC 3.1. (2022). WiNDC: Open-Source Datasets for Economic Analysis. Wisconsin National Data Consortium. https://windc.wisc.edu/index.html

Yuan, M., S. Rausch, J. Caron, S. Paltsev, J. Reilly (2019): The MIT U.S. Regional Energy Policy (USREP) Model: The Base Model and Revisions. Joint Program Technical Note TN #18, August, 24 p. (http://globalchange.mit.edu/publication/17331)