Representation of fuel usage in general equilibrium modeling

Y.H. Henry Chen[φ] and Jonathan Siegle[§]

Abstract

In computable general equilibrium (CGE) analysis, the values of all commodities including fuels are represented in monetary units. This suggests that for a multi-energy-input sector, under a given output level, inter-fuel substitution under carbon pricing can inadvertently change the energy use levels in physical units and therefore unintentionally contribute to alteration of the underlying energy efficiency. However, a significant change in energy efficiency implied by inter-fuel substitution is unlikely to happen. In this study, we consider an approach to address the issue in a CGE modeling framework.

[φ] Global Change Joint Program, Massachusetts Institute of Technology. Cambridge, MA 02139, USA.
[§] Institute of Energy Economics and Rational Energy Use, University of Stuttgart. Stuttgart, Baden-Württemberg, Germany.

Contents

This paper describes the current state of work at the time of publication. Despite the great care taken in the preparation of this paper and the underlying work, the absence of errors cannot be guaranteed.

1 Introduction

Computable General Equilibrium (CGE) models are usually parameterized in monetary units. Like all commodities, energy carriers in a CGE model are valued in a monetary unit, which serves as the metrics for inter-fuel substitution under model simulations.

Considering two fossil fuels, a fossil fuel with a lower energy-related emissions coefficient can have a higher value-related emissions coefficient – if its energy price is lower. Under carbon pricing, it therefore incurs a higher penalty. Consequently, its usage in a production function declines more. This might be an unexpected, but economically still well-founded result.

There are good reasons for the use of monetary units (Paltsev et al., 2005).[1] Using them allows for the combined view and aggregation of very different kinds of commodities. This practice entails the inclusion of prices, which measure the marginal values of commodity inputs. For example, land parcels can have the same physical sizes, but very different value. Value reflects the contribution of the parcels to the economy. Just as land prices assign different values to the size of different parcels, energy prices assign different values to the energy content of different energy carriers.

Differentiated valuation of the energy content of different energy carriers can be well explained by other attributes such as scarcity, energy density, cleanliness, cost of conversion etc. (Cleveland et al. 2000). Carbon pricing would increasingly align the fuel use pattern with energy-related emissions coefficients only if it also increased the relative importance of energy content among the other attributes relevant for valuation.

But the use of monetary units entails another issue that deserves attention: If multiple fuels with various base year prices (in dollar per joule) are used simultaneously, the variation in joule per dollar between those fuels may unexpectedly result in a change in energy efficiency whenever inter-fuel substitutions are observed.

CGE models often work with monetary reference quantities (e.g. bn. dollar) and relative prices (dimensionless), i.e., the optimization problems in models have no idea about the required energy prescribed by the laws of physics in producing one unit of output, and therefore the simulated input-output structure in a counterfactual scenario may be infeasible in

[1] For more details, see p.10 in (Paltsev et al. 2005).

reality. There are some CGE models that use physical reference quantities for energy carriers. (Fawcett and Sands 2005) describe using a hybrid input-output table for their "Second-Generation-Model", containing physical quantity units with monetary prices for energy commodities and monetary quantity units with relative prices for other commodities. As a successor to the "Second-Generation-Model", the Phoenix Model follows the same approach in terms of units (Sue Wing et al., 2011). However, the change to physical units for the reference quantities and monetary prices does not solve the issues described. Total value of an input is still monetary, and inter-fuel substitution is not affected by this choice.

Although for a given output level, at a small scale of inter-fuel substitution, it is possible that the final energy provided by one dollar of an energy carrier may be replaced by the final energy of one dollar of another via technological adjustment, this is unlikely the case with a much greater extent of inter-fuel substitution, if the underlying energy content per dollar of each fuel is quite different.

CGE models often use multi-level nested CES functions, where not only inter-fuel substitution is possible, but also substitution between the energy aggregate and other inputs, e.g. capital. This is another source of a possible inconsistency with physical constraints besides the inter-fuel substitution.

The issue of the possible inconsistency of top-down model results with physical constraints is well known, and has been mentioned before e.g. by (Hourcade et al. 2006). (Meran 2019) presents a general elaboration of the issue with CES functions and points to solutions - either adding restrictions to the parameters or directly including thermodynamic limits into the design of production functions. The approach presented in this paper does not change function parameters or functions itself, but instead explicitly adds an additional efficiency constraint.

This article proceeds as follows: We demonstrate the issue of inter-fuel substitution numerically in Section 2 using a stylized CGE model. In Section 3, we propose a solution that incorporates the energy efficiency consideration in formulating the producer's optimization problem, and offer an example with our adjustment by revising the simple model in Section 4. With the real-world economic and energy parameterizations in Section 5, we apply the adjustment on a large-scale CGE model to study the energy use implication of carrying out the Nationally Determined Contributions (NDC) on the U.S. economy. In Section 6, we provide conclusions and point out possible directions for future research and improvement.

2 Modeling Challenge

Let us begin from a numerical exercise to demonstrate the modeling challenge. Consider a stylized “1-2-4” Arrow-Debreu model (Arrow and Debreu, 1954) adapted from (Markusen, 2006) with one region, two sectors (X and Y) and four factors (gas G, refined oil O, coal A, and capital K). Sector X represents all activities that generate CO_2 emissions from fuel combustion, and sector Y delineates those without producing emissions. As in (Paltsev et al. 2005), the emissions per joule from the lowest to the highest are: G (50.2 Mt-CO_2/EJ), O (73.0 Mt-CO_2/EJ), and A (90.5 Mt-CO_2/EJ). The input-output structure of this economy is summarized in Table 1 below. For instance, to produce 100 Billion dollars of X, it needs inputs of G, O, A, and K with 25 billion dollars each.

Table 1. The SAM of the 1-2-4 model

	Production sectors			Consumers
Markets	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PG	-25	-25		50
PO	-25	-25		50
PA	-25	-25		50
PK	-25	-25		50

Unit: Billion $

We draw the fuel price data from the GTAP 11 database (Aguiar et al., 2023), and with the assumed input-output structure of Table 1, G provides 6.1 EJ of energy, while O and A provide 2.6 EJ and 10.7 EJ of energy, respectively (Table 2).

Table 2. Input structure of Sector X

		Source	G	O	A	K
[1]	Capital input (bn $)	assumed				25
[2]	Energy input (bn $)	assumed	25	25	25	
[3]	Energy input (EJ)	=[2]/[4]	6.08	2.60	10.69	
[4]	Energy price (bn $/EJ)	GTAP11	4.1	9.6	2.3	
[5]	Emissions per EJ (Mt-CO2/EJ)	EPPA	50.23	72.97	90.53	
[6]	Emissions (Mt-CO2)	= [3]*[5]	305.2	189.7	968.2	
[7]	Emissions /bn$ (Mt-CO2/bn $)	= [6]/[2]	12.2	7.6	38.7	

In the 1-2-4 model, production and consumption functions are constant elasticity of substitution (CES) functions, where the elasticities of substitution between inputs are unity (i.e., Cobb-Douglas). Specifically, the cost function for X features a nested CES function, where the price indices for fuels are combined together by a CES function, and the aggregated price index is then coupled with the price index of K by another layer of CES function. When emissions are priced, the penalties will be proportional to the associated energy use levels.

When an emissions control aiming at slashing 50% baseline emissions by carbon pricing is in place, the consumption levels of G, O, and A are cut by 29.5%, 19.3%, 62.5%, respectively (Figure 1). Note that while G has the lowest energy-related emissions factor (50.2 Mt-CO_2/EJ), the percentage reduction is higher than that of O, due to the fact that the base year refined oil price (9.6 bn $/EJ) is more than twice as high as the gas price (4.1 bn $/EJ; see Table 2).Therefore, even though refined oil has a higher penalty per joule, the percentage increase in refined oil price (14.7%) is less than half of the gas price increase (31.2%).

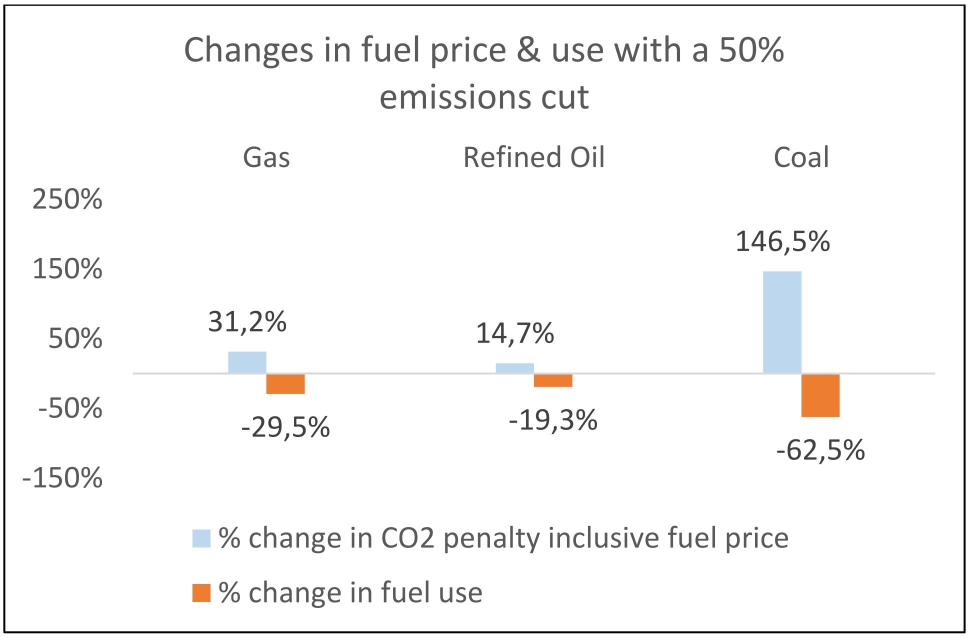

Figure 1. Change in fuel use under the emissions control scenario

The real issue, however, could be in the implied energy efficiency (Figure 2). Under the mitigation, the total energy input in EJ declines by 46.3%, while total output only reduces by 32.0%. This suggests a 26.7% gain in energy efficiency[2]. In reality, an efficiency

[2] energy efficiency gain is calculated according to the definition of the energy efficiency improvement index γ in Section 3.

improvement at this level is likely too high. Having this result is because the optimizing behavior of the producer does not take into account the physical constraint on the production technologies.

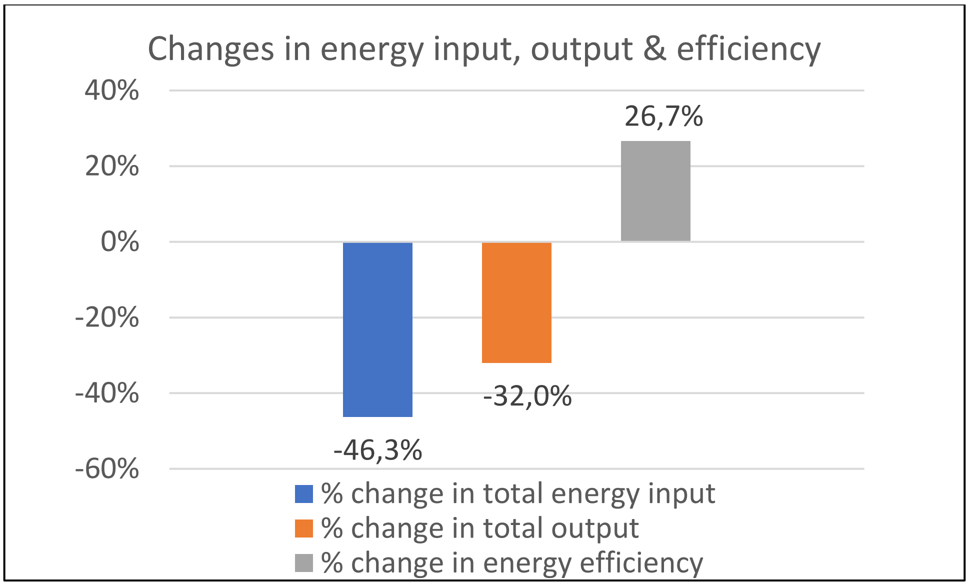

Figure 2. Change in energy input, output and efficiency under the emissions control scenario

3 Proposed Solution

Our strategy to overcome the issue is to incorporate this constraint into the producer's cost minimization problem, so that the problem is subject to the physical constraint as well. The idea is to allocate part of the revenue to fuel expenditure to incentivize a revised production plan with an input-output structure that conforms with the physical constraint.

To proceed, let us define an energy efficiency improvement index γ as follows:

$$\gamma = \frac{X/(gxp \cdot GX + oxp \cdot OX + axp \cdot AX)}{1/(gxp + oxp + axp)} \quad (1)$$

where

- X is the output index for sector X
- GX, OX, and AX are the consumption indices of G, O, and A inputs to X
- gxp, oxp, and axp are the energy contents of GX, OX and AX in physical units

All input and output indices are normalized to unity in the baseline, and therefore $\gamma = 1$ in the baseline. Suppose that the physical constraint requires that the energy efficiency index γ is subject to an upper limit of γ_h. When the constraint is binding, it suggests that the producer of

X needs to spend more on energy input, as for a given output, the cost minimization problem without considering the physical constraint leads to a total energy input that is too low.

Since the producer needs to allocate more funds for fuel expenditure, let us use TX in denoting the proportion of revenue that will be spend on funding additional fuel expenditure to ensure the physical constraint is met, and use SF to denote the percentage of additional expenditure on each fuel. The optimization problem can be formulated as a set of mixed-complementarity problems (MCP) (Problems (2) and (5)) and the equation representing the funding source (Equation (6)):

$$MC - MB \geq 0; X \geq 0\,; [MC - MB\,] \cdot X = 0 \tag{2}$$

Where MC is the marginal cost for the production of X, which is equal to the unit cost function of X, denoted by C, since the CES technology is constant return to scale (CRTS). Note that C is a function of indices of input prices and SF. On the other hand, MB is marginal revenue, which is a function of output price index and TX. In Equations (3) and (4), P_G, P_O, P_A, P_K, and P_X are price indices of G, O, A, K, and X, respectively:

$$MC = C(P_K, P_G \cdot (1 + SF), P_O \cdot (1 + SF), P_A \cdot (1 + SF)) \tag{3}$$

$$MB = P_X \cdot (1 - TX) \tag{4}$$

In the above formulation, SF is determined by solving the following MCP problem:

$$\gamma_h - \gamma \geq 0; -SF \geq 0\,; [\gamma_h - \gamma\,] \cdot (-SF) = 0 \tag{5}$$

Problem (5) is interpreted as: if the constraint on energy efficiency index is nonbinding, the producer will not provide an additional resource as fuel expense. On the other hand, if the efficiency constraint is binding, the producer will “subsidize” the fuel expenditure using part of its revenue:

$$TX \cdot V_X \cdot P_X \cdot X = -SF \cdot (V_{GX} \cdot P_{GX} \cdot GX + V_{OX} \cdot P_{OX} \cdot OX + V_{AX} \cdot P_{AX} \cdot AX) \tag{6}$$

In Equation (6), V_x, V_{GX}, V_{OX}, and V_{AX} are baseline values of X output, inputs of G, O, and A used by X, respectively.

Back to our 1-2-4 model example, let us consider an example where for engineering reasons, the gain in energy efficiency cannot exceed 10%. When the energy efficiency constraint is imposed on the 1-2-4 model under the same scenario, the extent of transitioning from A to G and O is increased (Figure 3), as the efficiency constraint requires more energy is needed in producing the same level of output, and this makes decarbonization trickier (the carbon price goes up from \$49/t-CO_2 without the efficiency constraint to around \$64/t-CO_2 when the constraint is in place). Specifically, A has the highest emissions per dollar (Table 2) and as a result, when the emissions penalty is increased, the use of A is discouraged more than the use of other fuels. With that, the exercise reveals that the change in total energy input is similar with or without the efficiency constraint, as the mitigation target remains the same. Nevertheless, the output level is cut more with the efficiency constraint (Figure 4).

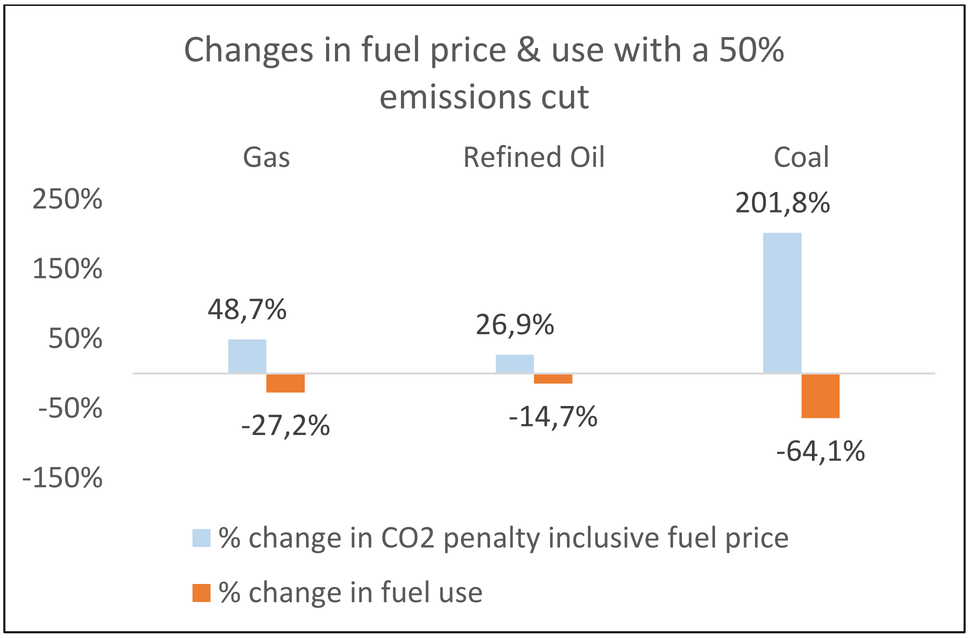

Figure 3. Change in fuel use under the emissions control scenario (with efficiency constraint)

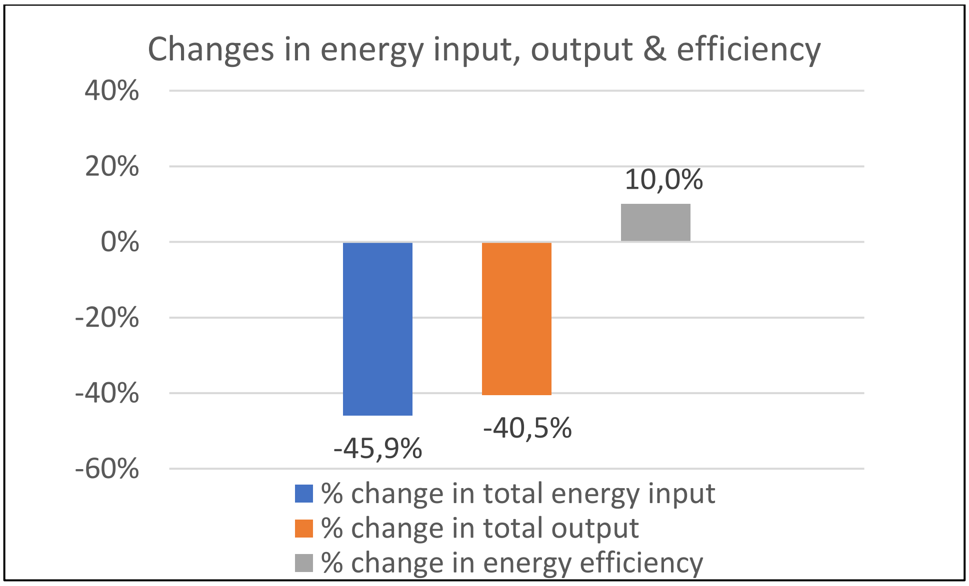

Figure 4. Change in energy input, output and efficiency under the emissions control scenario (with efficiency constraint)

4 A large-scale CGE application

To apply our adjustment on a model calibrated to real-world data, we build a static CGE for the US economy based on GTAPinGAMS (Lanz and Rutherford, 2016). The model has two regions: The United States (USA) and the rest of the world (ROW), and it is used here to study the fossil fuel use implications when the NDCs are carried out effectively.

In the model, the production and consumption functions are CES, and they are embedded in a series of MCP using the modeling languages of GAMS (GAMS, 2023) and MPSGE (Rutherford, 1999), a subsystem of GAMS.[3] The economic, energy, and emissions data are from the Global Trade Analysis Project (GTAP) version 11 (Aguiar et al., 2023) database with a base year of 2017.

For demonstration purposes, primary factors are classified into labor, capital, land, and natural resources. While labor and capital are allowed to move between sectors, land and natural resources are sector specific. Industries are classified into 13 sectors where those producing fossil fuels are separated into coal, gas, and refined oil (Appendix A1). In addition, the power sector is broken down into coal-fired, gas-fired, oil-fired, and other generation. At the same time, to avoid the "bang-bang solution," for each type of generation, a fixed factor (1% of output quantity) with a substitution elasticity of 0.3 is added into the cost function (Morris et al., 2019). The fixed factor supply is assumed to remain unchanged in counterfactual simulations relative to the reference run for this static model exercise.

We forward calibrate the reference (no NDC) scenario to the year of 2030, with the growth rates for GDP, capital, and autonomous energy efficiency improvement (AEEI) borrowing from (Chen et al. 2022), and the population growth projection coming from (United Nations 2020). For simplicity, only CO_2 emissions coming from fossil fuel combustion included in the GTAP database are considered, and carbon pricing is used to achieve the NDC targets. For USA, the target is to cut emissions by 36% below the 2005 level in 2030 (UNFCCC, 2023),[4] which is translated into a 25.2% reduction over the reference (no NDC) level of 2030. For ROW, the target (cutting 20.8% of reference emissions) is derived from the simulated emissions reduction of Chen et al. under the same scenario and regional coverage.

---

[3] The core model is included in the Appendix, and the complete model is available upon request.
[4] The NDC includes all GHGs. But since we only consider CO_2 emissions from burning fossil fuels, combusted CO_2 emissions in 2005 (5992 Mt-CO_2) sourced from (EIA 2019) are used in calculating the 2030 target.

The simulations show that before implementing our adjustment, under the NDC scenario, the refined oil usage in the U.S. was cut by around 10.9% relative to the reference level, while the gas and coal usage was reduced by about 26.8% and 56.6%, respectively (Figure 5).

To provide an example with our adjustment, let us consider the case where the upper bound of the energy efficiency improvement is 20% relative to the baseline case for the year of 2017. With the constraint, gas and refined oil consumption will be reduced less (by 24.8% and 5.2%). However, coal consumption will be cut further up to 69.9% (Figure 5). In fact, we find that for most sectors, the efficiency constraint is binding (Figure 6). Similar to our finding in Section 3, this is because a binding energy efficiency constraint suggests that more energy input per unit of output is needed, and this implies that decarbonization will become harder, as also reflected in higher simulated CO_2 prices when the efficiency constraint is in place.

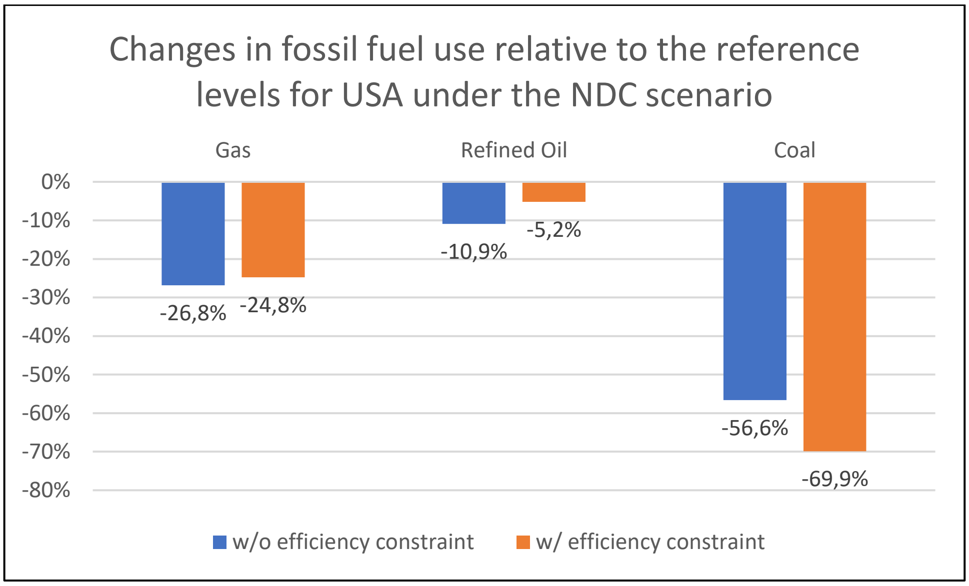

Figure 5. Changes in fuel use for USA under the NDC scenario

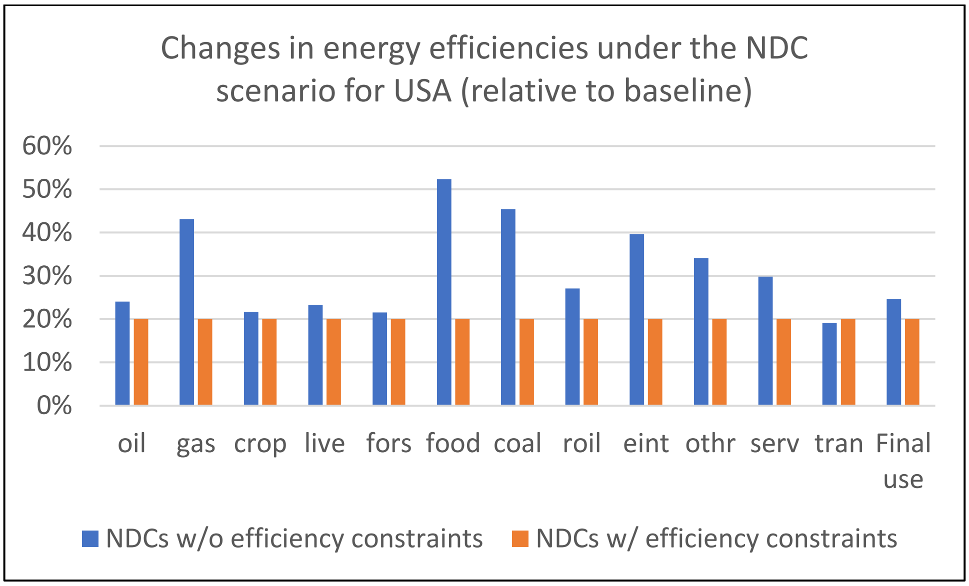

Figure 6. Changes in energy efficiencies under the NDC scenario.

5 Conclusions

The main datasets in CGE modeling are input-output tables, and sectors with multi-energy inputs are ubiquitous in the datasets, either due to the nature of production technologies or data aggregation resulting from the sectoral definition. As energy-economic CGE models are widely used in studying the implications of energy or climate policies, we demonstrate the unintended consequence of inter-fuel substitution in monetary units: Inter-fuel substitution may significantly contribute to alteration of the sectoral energy efficiencies to an unrealistic extent, if the base year fuel prices vary.

In this study, we offer a solution that resolves the challenges. From a technical perspective, introducing the energy efficiency constraint and fund-allocating mechanism require only minimal revisions, and therefore the strategy has the potential to be applied on a variety of large-scale CGE models without the penalties of significantly increasing the overall model complexity.

Nevertheless, it should be pointed out that the energy efficiency constraint deals with the issue in an aggregate manner. In reality, substitution between fossil fuels and electricity likely includes greater efficiency potentials than substitution among fossil fuels themselves, because electric technologies are in many cases much more efficient than fossil ones. In its current form, such fuel-specific differentiations are not considered in the efficiency constraint.

Furthermore, the efficiency constraint not only restricts efficiency improvement resulting from the inter-fuel-substitution itself, but affects the overall efficiency improvement, which can also be induced by substitution between fuels and other inputs like capital. The upper efficiency bound should be carefully set such that it will not rule out higher efficiency potential. For example, in a dynamic model, it can be increased over time.

Future research may find ways to more precisely constrain efficiency gains resulting purely from the substitution among fossil fuels. However, our attempts with more specific formulations so far caused difficulties in solution finding.

Appendix

Appendix A1. Sectors in the large-scale CGE

Table A1. Sectors in the CGE

Sector	*Notation*
Agriculture - Crops	*CROP*
Agriculture - Livestock	*LIVE*
Agriculture - Forestry	*FORS*
Food Products	*FOOD*
Coal	*COAL*
Crude Oil	*OIL*
Refined Oil	*ROIL*
Gas	*GAS*
Electricity	*ELEC*
Coal-fired	*cele*
Gas-fired	*gele*
Oil-fired	*oele*
other	*rele*
Energy-Intensive Industries	*EINT*
Other Industries	*OTHR*
Services	*SERV*
Transport	*TRAN*

Appendix A2. Adapted "1-2-4" model

```
$TITLE simple.gms: 1-2-4 model


table   sam(*,*)    Social Accounting MAXrix

     X0     Y0     GX     OX     AX     W0     CONS0
PX0  100                                -100
PY0         100                         -100
PW0                                     200    -200
PG0         -25    -25                         50
PO0         -25           -25                  50
PA0         -25                  -25           50
PGX0 -25           25
POX0 -25                  25
PAX0 -25                         25
PK0  -25    -25                                50

;

parameters
        cpg             carbon penalty for using gas,
        cpo             carbon penalty for using oil,
        cpa             carbon penalty for using coal,
        gxp             physical unit of gas used by X,
        oxp             physical unit of oil used by X,
        axp             physical unit of coal used by X,
        ctax            flag for CO2 pricing,
        eff             flag for energy efficiency adjustment,
        xfp(*)          percentage change of fuel use,
        df(*)           EJ change in fuel,
        pf(*)           percentage change in CO2 penalty excluded fuel price,
        pfx(*)          percentage change in CO2 penalty included fuel price,
        vxfp(*)         percentage change of the value of fuel input to X,
        xfpay           CO2 payment of X from burning fuel,
        ecr             emissions cut ratio,
        emis            emissions,
        eta0            energy efficiency index - baseline;

* Emissions coefficients 50.23, 72.97, and 90.53 (Mt-CO2) are from EPPA;
* Energy use 6.08, 2.60, and 10.69 (EJ) are derived from the assumed monetary inputs and the average
fuel prices from GTAP11


gxp     = 6.08;
oxp     = 2.60;
axp     = 10.69;
cpg     = gxp*50.23;
cpo     = oxp*72.97;
cpa     = axp*90.53;
eta0    = 1;

ctax    = 0;
eff     = 0;
ecr     = 0.5;

$ONTEXT

$MODEL:simple

$SECTORS:
        X       ! Activity level for sector X
        Y       ! Activity level for sector Y
        W       ! Activity level for sector W (Hicksian welfare index)
        GX      ! Gas input to X
        OX      ! Oil input to X
        AX      ! Coal input to X

$COMMODITIES:
        PX      ! Price index for commodity X
        PY      ! Price index for commodity Y
        PG      ! Price index for primary factor gas
        PO      ! Price index for primary factor oil
```

```
        PA      ! Price index for primary factor coal
        PK      ! Price index for primary factor K
        PW      ! Price index for welfare (expenditure function)
        PC$ctax ! CO2 Price (thousand USD per tCO2)

        PGX     ! Price index for gas - CO2 penalty included
        POX     ! Price index for oil - CO2 penalty included
        PAX     ! Price index for coal - CO2 penalty included

$CONSUMERS:
        CONS    ! Income level for consumer CONS

$AUXILIARY:
        SF$eff          ! Subsidy rate on fuel inputs used by X
        TX$eff          ! Tax rate on producing X
        ETA             ! energy efficiency index

$PROD:X s:1     s1(s):1
          O:PX          Q:sam("PX0","X0")           a:CONS      n:TX$eff
          I:PGX         Q:(-sam("PGX0","X0"))   a:CONS  n:SF$eff
          I:POX         Q:(-sam("POX0","X0"))   a:CONS  n:SF$eff
          I:PAX         Q:(-sam("PAX0","X0"))   a:CONS  n:SF$eff
          I:PK          Q:(-sam("PK0","X0"))

$PROD:GX        s:0
          O:PGX         Q:(-sam("PGX0","X0"))
          I:PG          Q:(-sam("PGX0","X0"))
          I:PC$ctax     Q:cpg

$PROD:OX        s:0
          O:POX         Q:(-sam("POX0","X0"))
          I:PO          Q:(-sam("POX0","X0"))
          I:PC$ctax     Q:cpo

$PROD:AX        s:0
          O:PAX         Q:(-sam("PAX0","X0"))
          I:PA          Q:(-sam("PAX0","X0"))
          I:PC$ctax     Q:cpa

$PROD:Y s:1
          O:PY          Q:sam("PY0","Y0")
          I:PG          Q:(-sam("PG0","Y0"))
          I:PO          Q:(-sam("PO0","Y0"))
          I:PA          Q:(-sam("PA0","Y0"))
          I:PK          Q:(-sam("PK0","Y0"))

$PROD:W s:1
          O:PW          Q:sam("PW0","W0")
          I:PX          Q:(-sam("PX0","W0"))
          I:PY          Q:(-sam("PY0","W0"))

$DEMAND:CONS
          D:PW          Q:(-sam("PW0","CONS0"))
          E:PG          Q:sam("PG0","CONS0")
          E:PO          Q:sam("PO0","CONS0")
          E:PA          Q:sam("PA0","CONS0")
          E:PK          Q:sam("PK0","CONS0")
          E:PC$ctax     Q:((cpg+cpo+cpa)*ecr)

$CONSTRAINT:ETA
        ETA =e= ((X*sam("PX0","X0"))/(GX*gxp+OX*oxp+AX*axp))/(sam("PX0","X0")/(gxp+oxp+axp));

$CONSTRAINT:SF$eff
        ETA =l= 1.1;

$CONSTRAINT:TX$eff
    TX*sam("PX0","X0")*PX*X =e= (-sam("PGX0","X0"))*PGX*GX*(-SF)
                               +(-sam("POX0","X0"))*POX*OX*(-SF)
                               +(-sam("PAX0","X0"))*PAX*AX*(-SF);

$OFFTEXT
;
$SYSINCLUDE mpsgeset simple

ETA.L           = eta0;
```

```
SF.LO           = -inf;
PW.FX           = 1;
simple.iterlim  = 0;

$INCLUDE simple.GEN
SOLVE simple USING MCP;

execute_unload ".\results\simple_calibration.gdx";

ctax            = 1;
simple.iterlim  = 2000;

$INCLUDE simple.GEN
SOLVE simple USING MCP;

xfp("G")= (GX.L-1);
xfp("O")= (OX.L-1);
xfp("A")= (AX.L-1);
df("G") = (GX.L-1)*gxp;
df("O") = (OX.L-1)*oxp;
df("A") = (AX.L-1)*axp;
pf("G") = (PG.L-1);
pf("O") = (PO.L-1);
pf("A") = (PA.L-1);
pfx("G")= (PGX.L-1);
pfx("O")= (POX.L-1);
pfx("A")= (PAX.L-1);

vxfp("G")       = (PG.L*GX.L-1);
vxfp("O")       = (PO.L*OX.L-1);
vxfp("A")       = (PA.L*AX.L-1);
xfpay("G")      = GX.L*(cpg);
xfpay("O")      = OX.L*(cpo);
xfpay("A")      = AX.L*(cpa);
emis            = cpg*GX.L+cpo*OX.L+cpa*AX.L;

execute_unload ".\results\simple.gdx";

ctax            = 1;
eff             = 1;
simple.iterlim  = 2000;

$INCLUDE simple.GEN
SOLVE simple USING MCP;

xfp("G")= (GX.L-1);
xfp("O")= (OX.L-1);
xfp("A")= (AX.L-1);
df("G") = (GX.L-1)*gxp;
df("O") = (OX.L-1)*oxp;
df("A") = (AX.L-1)*axp;
pf("G") = (PG.L-1);
pf("O") = (PO.L-1);
pf("A") = (PA.L-1);
pfx("G")= (PGX.L-1);
pfx("O")= (POX.L-1);
pfx("A")= (PAX.L-1);
vxfp("G")       = (PG.L*GX.L-1);
vxfp("O")       = (PO.L*OX.L-1);
vxfp("A")       = (PA.L*AX.L-1);
xfpay("G")      = GX.L*(cpg);
xfpay("O")      = OX.L*(cpo);
xfpay("A")      = AX.L*(cpa);
emis            = cpg*GX.L+cpo*OX.L+cpa*AX.L;

execute_unload ".\results\simple_eff.gdx";
```

Appendix A3. Core of the adapted GTAPinGAMS model

```
$title   Adapted GTAPinGAMS model

$if not set yr $set yr 17
$if not set ds $set ds gvcus
$include ..\build\gtap9data

$if not set sens $set sens 1
$if not set csnm $set csnm fparis
$include ..\scenario\%csnm%.cas

$ontext
$model:gtap11

$sectors:
        y(g,r)$vom(g,r)                   ! Supply
        m(i,r)$vim(i,r)                   ! Imports
        vxmdr(i,s,r)$vxmd(i,s,r)          ! Trade flow from s to r
        yt(j)$vtw(j)                      ! Transportation services
        vstr(j,r)$vst(j,r)                ! Realized vst
        ft(f,r)$(sf(f) and evom(f,r))     ! Specific factor transformation
        a(i,g,r)$vafm(i,g,r)              ! Armington good i used by g
        ac(i,g,r)$(e(i)$vafm(i,g,r))      ! Activity level of carbon inclusive energy transformation
        ye(j,se,r)$(elec(j)$so(se,r))     ! Electricity subsector activity level
        ya(j,se,r)$(elec(j)$so(se,r))     ! Electricity subsector aggregation

$commodities:
        p(g,r)$vom(g,r)                                 ! Domestic output price
        p_(i,r)$(sum(s, vxmd(i,r,s)) or vst(i,r))       ! Domestic output price
        pm(j,r)$vim(j,r)                                ! Import price
        pt(j)$vtw(j)                                    ! Transportation services
        pf(f,r)$evom(f,r)                               ! Primary factors rent
        ps(f,g,r)$(sf(f) and vfm(f,g,r))                ! Sector-specific primary factors
        pa(i,r)$(sum(g, vafm(i,g,r)))                   ! Armington price for i
        pc(r)$co2c(r)                                   ! Carbon penalty
        pac(i,g,r)$(e(i)$vafm(i,g,r))                   ! Carbon inclusive energy price
        pe(se,r)$so(se,r)                               ! Electricity subsector producer price index
        pe_(j,r)$elec(j)                                ! Electricity subsector producer price index
        pr(se,r)                                        ! Fixed factor price index

$consumers:
        ra(r)                                           ! Representative agent

$auxiliary:

        rgdp(r)                                 ! Real GDP
        gprod(r)$(forwc ne 0)                   ! TFP coefficient for matching target rgdp
        emis(r)                                 ! Realized emissions in region r
        emisadj(r)$(aadj(r) ne 0)               ! Quantity adjustment for emissions quota
        v_vafmg(i,r)$fe(i)                      ! Energy use by all sectors (10 bn US$)
        TY(g,r)$((not elec(g))$eff(g,r))        ! Tax rate on producing Y
        SB(g,r)$((not elec(g))$eff(g,r))        ! Subsidy rate on energy inputs used by Y
        ET(g,r)$((not elec(g))$(not gig(g)))    ! Energy efficiency index

$report:
        v:v_vafm(i,g,r)                         i:pa(i,r)       prod:y(g,r)
        v:v_etoe(fe,g,r)                        i:pac(fe,g,r)   prod:y(g,r)
        v:v_etoe_(fe,g,r)                       o:pac(fe,g,r)   prod:ac(fe,g,r)

$prod:a(i,g,r)$vafm(i,g,r)      s:esubd(i)
        o:pa(i,r)               q:vafm(i,g,r)
        i:p(i,r)                q:vdfm(i,g,r)   p:(1+rtfd0(i,g,r))      a:ra(r) t:rtfd(i,g,r)
        i:pm(i,r)               q:vifm(i,g,r)   p:(1+rtfi0(i,g,r))      a:ra(r) t:rtfi(i,g,r)

$prod:ac(i,g,r)$(e(i)$vafm(i,g,r))      s:0
        o:pac(i,g,r)            q:vafm(i,g,r)
        i:pa(i,r)               q:vafm(i,g,r)
        i:pc(r)$co2c(r)         q:eco2(i,g,r)

$prod:y(j,r)$(vom(j,r)$(not elec(j)))   s:0.5   s1(s):0 ee(s1):1va(ee):1en(ee):2ne(en):2
        o:p(j,r)        q:vom(j,r)                              a:ra(r) t:rto(j,r)
        n:TY(j,r)$eff(j,r)
        i:ps(sf,j,r)    q:vfm(sf,j,r)   p:(1+rtf0(sf,j,r))      a:ra(r) t:rtf(sf,j,r)
```

```
        i:pf(mf,r)      q:vfm(mf,j,r)   p:(1+rtf0(mf,j,r))      a:ra(r) t:rtf(mf,j,r)   va:
        i:pa(note,r)    q:vafm(note,j,r)                                                s1:
        i:pac(elec,j,r) q:vafm(elec,j,r)                        a:ra(r)
        n:SB(j,r)$eff(j,r)      en:
        i:pac(fe,j,r)   q:vafm(fe,j,r)  p:(1+itax(fe,j,r))      a:ra(r) t:itax(fe,j,r)
        n:SB(j,r)$eff(j,r)      ne:

$prod:y(j,r)$(vom(j,r)$elec(j))
        o:p(j,r)        q:vom(j,r)
        i:pe_(j,r)      q:vom(j,r)

$prod:ya(j,se,r)$(elec(j)$so(se,r))
        o:pe_(j,r)      q:(so(se,r)*vom(j,r))
        i:pe(se,r)      q:(so(se,r)*vom(j,r))

$prod:ye(j,se,r)$(elec(j)$so(se,r))     s:0.3 s0(s):0.5 s1(s0):0 ee(s1):1 va(ee):1 en(ee):2 ne(en):2
        o:pe(se,r)                      q:(so(se,r)*vom(j,r))
                                                a:ra(r) t:rto(j,r)
        i:ps(sf,j,r)                    q:(si(se,r)*vfm(sf,j,r))
                p:(1+rtf0(sf,j,r))              a:ra(r) t:rtf(sf,j,r)           s0:
        i:pf("lab",r)                   q:(si(se,r)*vfm("lab",j,r))
                p:(1+rtf0("lab",j,r))           a:ra(r) t:rtf("lab",j,r)        va:
        i:pf("cap",r)                   q:(si(se,r)*vfm("cap","elec",r)-evomfd(se,r))
                p:(1+rtf0("cap",j,r))           a:ra(r) t:rtf("cap",j,r)        va:
        i:pa(note,r)                    q:(si(se,r)*vafm(note,j,r))
                                                                                s1:
        i:pac(elec,j,r)                 q:(si(se,r)*vafm(elec,j,r))
                                                                                en:
        i:pac("coal",j,r)$cele(se)      q:vafm("coal",j,r)
                p:(1+itax("coal",j,r))          a:ra(r) t:itax("coal",j,r)      ne:
        i:pac("gas",j,r)$gele(se)       q:vafm("gas",j,r)
                p:(1+itax("gas",j,r))           a:ra(r) t:itax("gas",j,r)       ne:
        i:pac("roil",j,r)$oele(se)      q:vafm("roil",j,r)
                p:(1+itax("roil",j,r))          a:ra(r) t:itax("roil",j,r)      ne:
        i:pr(se,r)                      q:evomfd(se,r)
                p:(1+rtf0("cap","elec",r))      a:ra(r) t:rtf("cap","elec",r)

$prod:y("c",r)$vom("c",r)   s:esub("c")
        o:p("c",r)              q:vom("c",r)                                    a:ra(r)
                t:rto("c",r)            n:TY("c",r)$eff("c",r)
        i:pa(note,r)            q:vafm(note,"c",r)
        i:pac(elec,"c",r)       q:vafm(elec,"c",r)                              a:ra(r)
                                        n:SB("c",r)$eff("c",r)
        i:pac(fe,"c",r)         q:vafm(fe,"c",r)        p:(1+itax(fe,"c",r))    a:ra(r)
                t:itax(fe,"c",r)        n:SB("c",r)$eff("c",r)


$prod:y("g",r)$vom("g",r)   s:esub("g")
        o:p("g",r)      q:vom("g",r)    a:ra(r)  t:rto("g",r)   n:TY("g",r)$eff("g",r)
        i:pa(i,r)       q:vafm(i,"g",r) a:ra(r)                 n:SB("g",r)$eff("g",r)

$prod:y("i",r)$vom("i",r)   s:esub("i")
        o:p("i",r)      q:vom("i",r)    a:ra(r)  t:rto("i",r)   n:TY("i",r)$eff("i",r)
        i:pa(i,r)       q:vafm(i,"i",r) a:ra(r)                 n:SB("i",r)$eff("i",r)

$prod:yt(j)$vtw(j)  s:1
        o:pt(j)                 q:vtw(j)
        i:p_(j,r)               q:vst(j,r)

$prod:vstr(j,r)$vst(j,r)
        o:p_(j,r)               q:vst(j,r)
        i:p(j,r)                q:vst(j,r)

$prod:m(i,r)$vim(i,r)   s:esubm(i)  s.tl:0
        o:pm(i,r)               q:vim(i,r)
        i:p_(i,s)               q:vxmd(i,s,r)   p:pvxmd(i,s,r) s.tl: a:ra(s) t:(-rtxs(i,s,r))
                a:ra(r) t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
        i:pt(j)#(s)             q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl: a:ra(r) t:rtms(i,s,r)

$prod:vxmdr(i,s,r)$vxmd(i,s,r)
        o:p_(i,s)               q:vxmd(i,s,r)
        i:p(i,s)                q:vxmd(i,s,r)

$prod:ft(sf,r)$evom(sf,r)  t:etrae(sf)
        o:ps(sf,j,r)            q:vfm(sf,j,r)
```

```
        i:pf(sf,r)              q:evom(sf,r)

$demand:ra(r)
        d:p("c",r)              q:vom("c",r)
        e:p("c",rnum)           q:vb(r)
        e:p("g",r)              q:(-vom("g",r))
        e:p("i",r)              q:(-vom("i",r))
        e:pf(f,r)               q:evom(f,r)     r:gprod(r)$(forwc ne 0)
        e:pr(se,r)              q:evomfs(se,r)  r:gprod(r)$(forwc ne 0)
        e:pc(r)$co2c(r)         q:eco2tot(r)    r:emisadj(r)$(aadj(r) ne 0)

* p("c",r)*100 = GDP deflator of region r
$constraint:rgdp(r)
        p("c",r)*rgdp(r) =e= sum(k, p(k,r)*y(k,r)*vom(k,r))
                        + sum((i,s), rtxs(i,r,s)*p(i,r)*vxmdr(i,r,s))
                        - sum((i,s), rtms(i,s,r)*(p(i,s)*vxmdr(i,s,r)*(1-
rtxs(i,s,r))+sum(j,pt(j)*vtwr(j,i,s,r)/vxmd(i,s,r)*vxmdr(i,s,r))))
                        + sum((i,s)$vxmd(i,r,s), p(i,r)*vxmdr(i,r,s))
                        + sum(i$vst(i,r), p(i,r)*vstr(i,r))
                        - sum((i,s)$vxmd(i,s,r), (p(i,s)*vxmdr(i,s,r)))
                        - sum((j,i,s)$vxmd(i,s,r), pt(j)*vtwr(j,i,s,r)/vxmd(i,s,r)*vxmdr(i,s,r))
                        ;

$constraint:gprod(r)$(forwc ne 0 and endrgdp eq 0)
        rgdp(r) =g= rgdp0(r);

$constraint:gprod(r)$(forwc ne 0 and endrgdp ne 0)
        gprod(r) =e= gprod_(r);

$constraint:emis(r)
        emis(r) =e= sum((fe,g),ac(fe,g,r)*eco20(fe,g,r));

$constraint:emisadj(r)$(aadj(r) ne 0)
        emis(r) =l= eco2tot(r);

$constraint:v_vafmg(i,r)$fe(i)
        v_vafmg(i,r) =e= sum(g, vafm(i,g,r)*ac(i,g,r));

$CONSTRAINT:ET(g,r)$((not elec(g))$(not gig(g)))
        ET(g,r) =e= (((Y(g,r)*vom(g,r))/(sum(e, AC(e,g,r)*eva(e,g,r))))/(vom(g,r)/sum(e, eva(e,g,r))));

$CONSTRAINT:SB(i,r)$((not elec(i))$eff(i,r))
        ET(i,r) =l= 1.2;

$CONSTRAINT:SB(cg,r)$eff(cg,r)
        ET(cg,r) =l= 1.2;

$CONSTRAINT:SB(gig,r)$eff(gig,r)
        SB(gig,r) =e= 0;

$CONSTRAINT:TY(i,r)$((not elec(i))$eff(i,r))
        TY(i,r)*vom(i,r)*P(i,r)*Y(i,r) =e= -SB(i,r)*sum(e, vafm(e,i,r)*AC(e,i,r)*PAC(e,i,r));

$CONSTRAINT:TY(cg,r)$eff(cg,r)
        TY(cg,r)*vom(cg,r)*P(cg,r)*Y(cg,r) =e= -SB(cg,r)*sum(e, vafm(e,cg,r)*AC(e,cg,r)*PAC(e,cg,r));

$CONSTRAINT:TY(gig,r)$eff(gig,r)
        TY(gig,r) =e= 0;

$offtext

$sysinclude mpsgeset gtap11

* Variable initialization
rgdp.l(r) =     (
                sum(k, p.l(k,r)*y.l(k,r)*vom(k,r))
                + sum((i,s), rtxs(i,r,s)*p.l(i,r)*vxmdr.l(i,r,s))
                - sum((i,s), rtms(i,s,r)*(p.l(i,s)*vxmdr.L(i,s,r)*(1-
rtxs(i,s,r))+sum(j,pt.l(j)*vtwr(j,i,s,r)/vxmd(i,s,r)*vxmdr.l(i,s,r))))

                + sum((i,s)$vxmd(i,r,s), p.l(i,r)*vxmdr.l(i,r,s))
                + sum(i$vst(i,r), p.l(i,r)*vstr.l(i,r))
                - sum((i,s)$vxmd(i,s,r), (p.l(i,s)*vxmdr.l(i,s,r)))
                - sum((j,i,s)$vxmd(i,s,r), pt.l(j)*vtwr(j,i,s,r)/vxmd(i,s,r)*vxmdr.l(i,s,r))
                )/p.l("c",r)
```

```
                ;

emisadj.l(r)    = 1;

* Set the price numeraire
p.fx("c",rnum)  = 1;

SB.L(g,r)$((not elec(g))$eff(g,r))      = 0;
TY.L(g,r)$((not elec(g))$eff(g,r))      = 0;

SB.LO(g,r)$((not elec(g))$eff(g,r))     = -inf;
SB.UP(g,r)$((not elec(g))$eff(g,r))     = 0;
TY.LO(g,r)$((not elec(g))$eff(g,r))     = 0;

v_vafmg.l(i,r)  = sum(g, vafm(i,g,r));
emis.l(r)       = sum((fe,g), eco20(fe,g,r));
ET.L(g,r)       = 1;

gtap11.workspace= 256;
gtap11.iterlim  = 0;
gtap11.iterlim  = 20000;
gtap11.optfile  = 1;

$include gtap11.gen
solve gtap11 using mcp;

if (gtap11.modelstat ne 1, gtap11.optfile = 2;
$include gtap11.GEN
SOLVE gtap11 USING MCP;
);


Parameters
modelstat_      model solve status,
enertot(e,r)    total energy use level,
xfp(e,r)        percentage change of aggregate fuel use;

modelstat_      = gtap11.modelstat;
enertot(e,r)    = sum(g, eva(e,g,r)*ac.l(e,g,r));
xfp(e,r)        = sum(g, eva(e,g,r)*ac.l(e,g,r))/sum(g, eva(e,g,r)) - 1;

$if %sens%==1 execute_unload "..\results\all_%ds%_%csnm%.gdx";
$if not %sens%==1 execute_unload "..\results\all_%ds%_%csnm%_%sens%.gdx";
```

References

Aguiar, A., Chepeliev, M., Corong, E., and van der Mensbrugghe, D. (2023). The Global Trade Analysis Project (GTAP) Data Base: Version 11. Journal of Global Economic Analysis, 7(2). (https://doi.org/10.21642/JGEA.070201AF)

Arrow, K.J. and Debreu, G. (1954). Existence of an equilibrium for a competitive economy. Econometrica 22, 265–90. (https://www.jstor.org/stable/1907353).

Cleveland, C., Kaufmann, K. and Stern, D. (2000) Aggregation and the role of energy in the economy. (https://doi.org/10.1016/S0921-8009(99)00113-5)

Energy Information Administration (EIA) (2019). Energy-Related Carbon Dioxide Emissions by State, 2005-2016. U.S. Energy Information Administration. 1000 Independence Ave., SW

Washington, DC 20585 (https://www.eia.gov/environment/emissions/state/analysis/excel/table1.xlsx). Downloaded on April 4, 2024.

Fawcett, A. A. and Sands, R. D. (2005). The Second Generation Model: Data, Parameters, and Implementation. (https://www.researchgate.net/publication/298212037_The_Second_Generation_Model_Data_Parameters_and_Implementation)

GAMS (2023). GAMS Documentation Center. GAMS Development Corp. Fairfax, VA 22031 (https://www.gams.com/latest/docs/).

Hourcade, J. C., Jaccard, M., Bataille, C. and Ghersi, F. (2006). Hybrid Modeling: New Answers to Old Challenges (https://shs.hal.science/halshs-00471234)Lanz, B., and Rutherford, T. F. (2016). GTAPinGAMS: Multiregional and Small Open Economy Models. Journal of Global Economic Analysis, 1(2), 1–77. (https://doi.org/10.21642/JGEA.010201AF).

Markusen, J. (2006). Extension of the Simple Model. University of Colorado, Boulder (http://spot.colorado.edu/~markusen/teaching_files/applied_general_equilibrium/GAMS/ch2.pdf)

Meran, G. (2019). Thermodynamic constraints and the use of energy-dependent CES-production functions: A cautionary comment (https://doi.org/10.1016/j.eneco.2019.03.009)

Morris, J., Reilly, J. M., and Chen, Y.-H. H. (2019). Advanced technologies in energy-economy models for climate change assessment. Energy Economics, 80, 476–490. (https://doi.org/10.1016/j.eneco.2019.01.034)

Paltsev, S., Reilly, J. M., Jacoby, H. D., Eckaus, R.S., McFarland, J., Sarofim, M., Asadoorian M. and Babiker M. (2005). The MIT Emissions Prediction and Policy Analysis (EPPA) Model: Version 4 (https://globalchange.mit.edu/publication/14578)

Rutherford, T. (1999). Applied General Equilibrium Modeling with MPSGE as a GAMS Subsystem: An Overview of the Modeling Framework and Syntax. Computational Economics 14: 1–46.

Sue Wing, I., Daenzer, K., Fisher-Vanden, K., Calvin, K. (2011). Phoenix Model Documentation. (https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=a808d631aa87bd3387d8603a038bbb442218ff8c)

United Nations (2020). World Population Prospects 2019. Population Division of the United Nations. New York, NY.

United Nations Framework Convention on Climate Change (UNFCCC) (2023). NDC Registry. UNFCCC. Bonn, Germany (https://unfccc.int/NDCREG).