The Environmental Impact and Sustainability Applied General Equilibrium (ENVISAGE) Model

Version 9.0

Dominique van der Mensbrugghe[1]
The Center for Global Trade Analysis, Purdue University

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[1] Email: vandermd@purdue.edu

Abstract

This document's main purpose is to provide a full description of the Envisage. Envisage is a global recursive dynamic computable general equilibrium model developed at the World Bank. Envisage has been developed to assess the interactions between economies and the global environment as affected by human-based emissions of greenhouse gases. At its core, Envisage is a relatively standard recursive dynamic multi-sector multi-region CGE model. It has been complemented by an emissions and climate module that links directly economic activities to changes in global mean temperature. And it incorporates a feedback loop that links changes in temperature to impacts on economic variables such as agricultural yields or damages created by sea level rise. One of the overall objectives of the development of Envisage has been to provide a greater focus on the economics of climate change for a more detailed set of developing countries as well as greater attention to the potential economic damages. The model remains a work in progress as there are several key features of the economics of climate change that are planned to be incorporated.

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Chapter 1

Introduction

The purpose of this document is to provide a complete specification of the equations of the World Bank's Environmental Impact and Sustainability Applied General Equilibrium (ENVISAGE) Model. The Envisage Model is designed to analyze a variety of issues related to the economics of climate change:

- Baseline emissions of CO_2 and other greenhouse gases
- Impacts of climate change on the economy
- Adaptation by economic agents to climate change
- Greenhouse gas mitigation policies—taxes, caps and trade
- The role of land use in future emissions and mitigation
- The distributional consequences of climate change impacts, adaptation and mitigation—at both the national and household level.

Envisage is intended to be flexible in terms of its dimensions. The core database—that includes energy volumes and CO_2 emissions—is the GTAP database, currently version 9.0 with a 2011 base year. The latter divides the world into 140 countries and regions, of which 120 are countries and the other region-based aggregations.[1] The database divides global production into 57 sectors—with extensive details for agriculture and food and energy (coal mining, crude oil production, natural gas production, refined oil, electricity, and distributed natural gas). Appendix J provides more detail. Due to numerical and algorithmic constraints, a typical model is limited to some 20-30 sectors and 20-30 regions.

This document describes the current version of Envisage, which is still in a developmental stage. This current version includes the following:

- Capital vintage production technology that permits analysis of the flexibility of economies
- Partially endogenous technical change
- A detailed specification of energy demand in each economy
- Incorporation of a limited set of new energy technologies

[1] The countries defined in GTAP cover well over 90 percent of global GDP and population.

- A resource depletion module for coal, oil and natural gas
- The ability to introduce future alternative energy (or backstop) technologies
- CO_2 emissions that are fuel and demand specific
- Incorporation of the main Kyoto greenhouse gases (methane, nitrous oxide and the fluoridated gases)
- A flexible system for incorporating any combination of carbon taxes, emission caps and tradable permits
- A simplified climate module that links greenhouse gas emissions to atmospheric concentrations combined with a carbon cycle that leads to radiative forcing and temperature changes.

The current work program includes the following tasks:

- Addition of marginal abatement cost curves for the non-CO_2 gases
- Adding a more detailed land-use module

Chapter 2

ENVISAGE Model in a nutshell

The ENVISAGE Model is a descendent of a family of models that originated at the OECD in the late 1980s and early 1990s, WALRAS, RUNS and GREEN (see OECD (1989/1990), Burniaux (1987), Burniaux and van der Mensbrugghe (1994), Burniaux, Nicoletti, and Oliveira-Martins (1992), and van der Mensbrugghe (1994)).[1] ENVISAGE was initially developed at the World Bank in 2007 and was a re-coded version of the World Bank's LINKAGE model (van der Mensbrugghe (2011)), which had a trade focus. It is designed specifically to analyze climate change issues and thus incorporates a more developed energy sector, a climate module (that makes integrated assessment an option), and climate change impact feedbacks. ENVISAGE is coded using the GAMS/MCP package.

ENVISAGE is a recursive dynamic model with flexible step sizes. Each period is solved as a comparative static equilibrium with myopic expectations. Production is modelled using a set of nested production functions. There are three production archetype structures—crops that is intended to capture extensification vs. intensification, livestock that is designed to capture land versus feed substitutability, and all other sectors that rely on the traditional capital versus labor substitutability. In all sectors energy is isolated as a special input that is a complement to capital in the short-term and a substitute for capital in the long-term. A vintage structure is implemented in production that captures putty-semi putty technology with less substitution assumed on installed capital, but more substitution using new capital. This version of ENVISAGE has a single land type that is allowed to expand using a supply function with a region-specific asymptote. Land allocation across sectors relies on a nested CET transformation structure using the LEITAP/MAGNET specification and elasticities.

There are various utility functions embedded in ENVISAGE to model household demand (CDE, LES/ELES, AIDADS). For the AgMIP exercise, the LES utility function has been used—with dynamically re-calibrated parameters (in the baseline) to target FAO's food consumption trends from its most recent report (Alexandratos and Bruinsma (2012)). The Armington specification is used to model trade. In the standard closure, the net government fiscal position is fixed with lump sum taxes endogenous, investment is savings driven, and the trade balance is fixed with endogenous real exchange rates.

Dynamics is captured through three channels:

1. Population and labor force growth are exogenous

2. Capital dynamics relies on the standard motion equation that equates the current capital stock to the sum of the previous period's depreciated stock of capital and volume of investment

[1] The first two of which were actually initially developed at Stanford University and the Université Libre de Bruxelles, respectively.

3. There is an economy-wide (labor) productivity factor that is calibrated in the reference run to meet the overall target for GDP growth, though the model allows for intersectoral differences in productivity. For the AgMIP exercise—agriculture and services have the same labor productivity—it is higher in manufacturing. Exogenous productivity factors are applied to yields in agriculture, energy use in all sectors and final demand, and international trade and transport margins. The food related input-output coefficients linked to household consumption are also adjusted over time to target FAO's food consumption trends-coupled with changes to the LES parameters described above. The latest version also incorporates endogenous productivity growth linked to investments in R&D and knowledge accumulation.

Chapter 3

Model specification

3.1 Model dimensions

The coding of the model is relatively independent of the dimensionality of the underlying GTAP database and other functional dimensions of the data. Table 3.1 provides a listing of the main sets and subsets of the model. One of the key dimensions is aa, which encapsulates all of the Armington agents, including a, which is the subset of production activities. Sectors and commodities have three classifications: a, i, and k, respectively (production) activities, marketed commodities and consumed commodities. In a traditional model the three sets are identical. In the ENVISAGE model, with its multi-input multi-output production structure, output from activities (a) is combined with imports to supply (or 'produce') commodities (i). This allows, for example, to have multiple activities produce a single commodity (for example electricity), and to have single activities produce multiple commodities (e.g. sugar producing sugar, ethanol, rum and even power). In addition, ENVISAGE allows for commodities in final demand (indexed by k) to differ from marketed commodities (i). A consumer-based 'make' or transition matrix maps consumed commodities to supplied commodities. This allows for more realistic demand behavior in the context of an energy model. For example, household demand for transportation services can be a combination of demand for fuel and automobiles. If the price of fuel goes up, the combined demand for fuel/autos would decline. It also allows for specific treatment of the demand for fuel and intra-fuel substitutability.

The next sections of the document describe the different block or modules of the model using the rather traditional circular flow scheme of economics, i.e. starting with production and factor incomes, income distribution, demand, trade, and macro closure. At the end, there is a discussion on the model dynamics.

3.2 Production block

The ENVISAGE production structure relies on a set of nested constant-elasticity-of-substitution (CES) structures.[1] The current version of ENVISAGE includes three production prototypes that are used to specify respectively production in crops, livestock and all other). Figures 3.1 through 3.6 describe the various nested structures. Three of the nests are generic to all three production structures—figures 3.1, 3.5 and 3.6 referring respectively to the top nest (XP), the capital, skilled labor and energy bundle ($KSLE$) and the energy bundle ($XNRG$). The intermediate nests are specific to each of the three prototype production structures. Figure 3.3 illustrates the crop production

[1] Some of the key analytical properties of the CES, and its related constant-elasticity-of-transformation (CET) function, are fully described in Appendix A.

Table 3.1: Sets used in model definition

Set	Description
aa	Armington agents
a	Activities (a subset of aa)
i	Produced (or supplied) goods
$Manu$	Set of manufacturing sectors (subset of i, used in definition of numéraire)
fp	Factors of production
l	Labor categories (subset of fp)
$Captl$	Capital account (subset of fp)
$Landr$	Land account (subset of fp)
$Natrs$	Natural resource account (subset of fp)
k	Consumed commodities
$nrg(k)$	The energy bundle in consumed commodities
in	Institutions (subset of a)
h	Households (subset of in)
Gov	Government account (subset of in)
Inv	Investment account (subset of in)
gy	Government revenue accounts
$ptax$	Production tax account (subset of gy)
$atax$	Sales tax account on Armington goods (subset of gy)
$mtax$	Import tariff account (subset of gy)
$etax$	Export tax account (subset of gy)
$vtax$	Tax on factors of production account (subset of gy)
$ctax$	Carbon tax (subset of gy)
r	Regions
r', d, s	Alias with r, d used for destination region and s for source region
HIC	Set of high-income regions (subset of r, used in definition of numéraire)
$RSAV$	Residual region (subset of r, must be of single dimension)

intermediate nest, Figure 3.4 illustrates the livestock production intermediate nest and Figure 3.2 illustrates the standard production intermediate nest.

The key factor in crops production is intensification vs. extensification and thus the production structure captures substitution between land and other inputs. Livestock is characterized by land and feed substitution. Intermediate inputs for each activity are divided into three bundles—energy ($XNRG$) that is a near complement with capital in the short-run and a substitute in the long-run, sector specific inputs ($ND2$) that are substitutes with factors (such as agricultural chemicals in crops and feed in livestock) and all other intermediate inputs that are typically modeled in fixed proportion to output ($ND1$). Like the Linkage model, production, in the dynamic version of the model is based on a vintage structure of capital, indexed by v. In the standard version, there are two vintages—*Old* and *New*, where *New* is capital equipment that is newly installed at the beginning of the period and *Old* capital is capital greater than a year old. The vintage structure impacts model results through two channels. First, it is typically assumed that *Old* capital has lower substitution elasticities than *New* capital. Thus countries with higher savings rates will have a higher share of *New* capital and thus greater overall flexibility. The second channel is through the allocation of capital across sectors. *New* capital is assumed to be perfectly mobile across sectors. *Old* capital is sluggish and released using an upward sloping supply curve. In sectors where demand is declining, the return to capital will be less than the economy-wide average. This is explained in greater detail in the market equilibrium section.

Most of the equations in the production structure are indexed by v, i.e. the capital vintage. The exceptions are those where it is assumed that the further decomposition of a bundle are no longer vintage specific—such as the demand for non-energy intermediate inputs. Each production activity is indexed by a, and is different from the index of produced commodities, i (allowing for the combination of outputs from different activities into a single produced good, for example electricity).

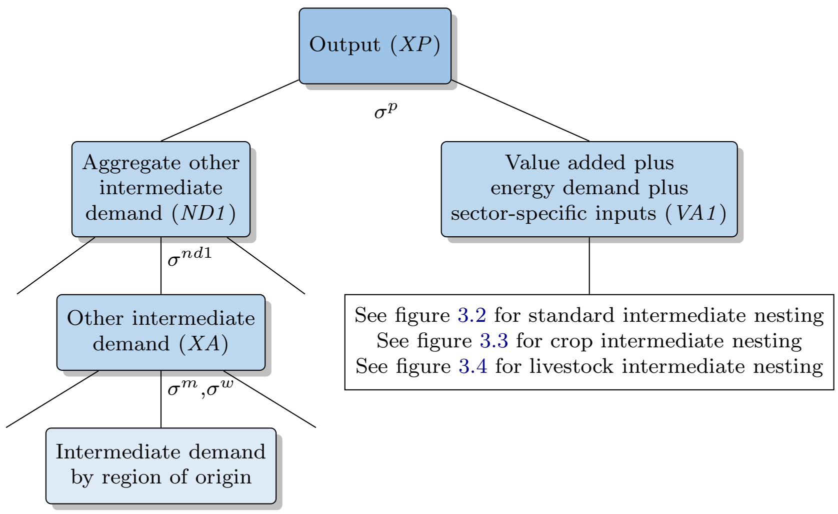


Figure 3.1: **Top nest in production structure**

Equations (P-1) and (P-2) are derived demands for two bundles, one designated as aggregate value added, *VA1*, though it also includes energy demand that is linked to capital and potentially other sector-specific intermediate inputs, and aggregate intermediate demand, *ND1*, a bundle that excludes energy and other sector-specific inputs (that will be part of the *ND2* bundle). Both are shares of output by vintage, XPv, with the shares being price sensitive with respect to the ratio of the vintage-specific unit cost, PXv, and the component prices, respectively *PVA1* and *PND1*. The equations allow for technological change embodied in the λ parameters that are allowed to be node-specific. For uniform technological change, the two parameters can be subject to the same percentage change. Both productivity factors are impacted by the same damage adjustment, δ^{cd}, which is region and sector specific and depends on climate change.[2] Equation (P-3) defines the vintage-specific unit cost, PXv. Almost all CES price equations are based on the dual cost expression instead of the aggregate cost or revenue formulation. The unit cost function includes the effects of productivity improvement and damages. To the extent climate leads to damages, δ^{cd} drops below its initial level of 1, raising unit cost, all else equal. Equation (P-4) determines the aggregate unit cost, PX, the weighted average of the vintage-specific unit costs with the weights given by the vintage-specific output levels. The model allows for a markup, π, to unit cost that is normally exogenous and initialized at 0. The revenue generated by the markup, Π, is defined in equation (P-5). Equation (P-6) determines the final market price for output, PP, that is equal to the unit cost augmented by the output tax (or subsidy), τ^p. The equivalence of the tax-adjusted

[2] Discussed further below.

unit cost to the output price is an implication of assuming constant-returns-to-scale technology and perfect competition (and/or the presence of a fixed markup). The production price can also be adjusted by a volume only tax (or an excise tax), represented by τ^x.

$$VA1_{r,a,v} = \alpha^{va1}_{r,a,v} \left(\delta^{cd}_{r,a}\lambda^{v}_{r,a,v}\right)^{\sigma^p_{r,a,v}-1} \left(\frac{PXv_{r,a,v}}{PVA1_{r,a,v}}\right)^{\sigma^p_{r,a,v}} XPv_{r,a,v} \tag{P-1}$$

$$ND1_{r,a,v} = \sum_v \alpha^{nd1}_{r,a,v} \left(\delta^{cd}_{r,a}\lambda^{n}_{r,a,v}\right)^{\sigma^p_{r,a,v}-1} \left(\frac{PXv_{r,a,v}}{PND1_{r,a}}\right)^{\sigma^p_{r,a,v}} XPv_{r,a,v} \tag{P-2}$$

$$PXv_{r,a,v} = \frac{1}{\delta^{cd}_{r,a}} \left[\alpha^{va}_{r,a,v}\left(\frac{PVA1_{r,a,v}}{\lambda^{v}_{r,a,v}}\right)^{1-\sigma^p_{r,a,v}} + \alpha^{nd}_{r,a,v}\left(\frac{PND1_{r,a}}{\lambda^{n}_{r,a,v}}\right)^{1-\sigma^p_{r,a,v}}\right]^{1/(1-\sigma^p_{r,a,v})} \tag{P-3}$$

$$PX_{r,a} = (1+\pi_{r,a}) \frac{\sum_v PXv_{r,a,v} XPv_{r,a,v}}{XP_{r,a}} \tag{P-4}$$

$$\Pi_{r,a} = \pi_{r,a} \sum_v PXv_{r,a,v} XPv_{r,a,v} \tag{P-5}$$

$$PP_{r,a} = \left(1+\tau^p_{r,a}\right) PX_{r,a} + \tau^x_{r,a} \tag{P-6}$$

The subsequent nests are specific to each of the three production prototypes, i.e. the *VA1* bundle is split into a variety of intermediate bundles—but using a different nesting for each one of the prototypes. The intermediate bundles include a natural resource bundle (*NRB*), two labor bundles (*LAB1* and *LAB2*), an energy bundle (*XNRG*) and a capital/energy/labor bundle (*KP2*). The combination of these intermediate bundles are different for the different prototypes and include other intermediate bundles *VA2* and *KP1* where the same variable name is used for each of the three prototypes but represent different combinations of composite bundles. The following shorthand formulas, in Table 3.2, show the different combinations.

Table 3.2: Production nests for different archetypes

Crops	**Livestock**	**All other**
VA1=CES(*NRB*, *VA2*)	*VA1*=CES(*KP1*, *VA2*)	*VA1*=CES(*NRB*, *VA2*)
VA2=CES(*KP1*, *KP2*)	*VA2*=CES(*LAB1*, *KP2*)	*VA2*=CES(*KP1*, *LAB1*)
KP1=CES(*LAB1*, *ND2*)	*KP1*=(*NRB*, *ND2*)	*KP1*=CES(*KP2*, *ND2*)

The following sub-sections spell-out the full set of equations for each of the production archetypes—though some of the equations are repeated verbatim when they represent the same combination of CES bundles.

3.2.1 Standard production nesting

The first set of equations (P-7) through (P-15) reflect the implementation of the intermediate bundles for the standard production function. The standard production function essentially reflects capital/labor substitution. For most sectors, the *NRB* bundle will not be active, with the exception of the energy sectors where the *NRB* will contain the relevant fossil fuel resource base. The *ND2* bundle will also most likely be empty. The purpose of the *LAB1* and *LAB2* bundles is to allow the differentiation of the use of diffent labor types in production. *LAB1* could be thought of including only unskilled labor types, whereas *LAB2* would include skilled labor types. The latter can then be combined with capital with potentially low elasticities as there is evidence that capital and skilled labor are near complements. The first level nest, therefore, decomposes the top level value added bundle, *VA1*, into two bundles—a natural resource bundle on the one hand (*NRB*) and a subsidiary value added bundle *VA2*. Equations (P-7) and (P-8) represent respectively demand for *VA2* and *NRB* where σ^{v1} is the CES substitution elasticity. Equation (P-9) represents the price formulation for the *VA1* bundle, *PVA1*.

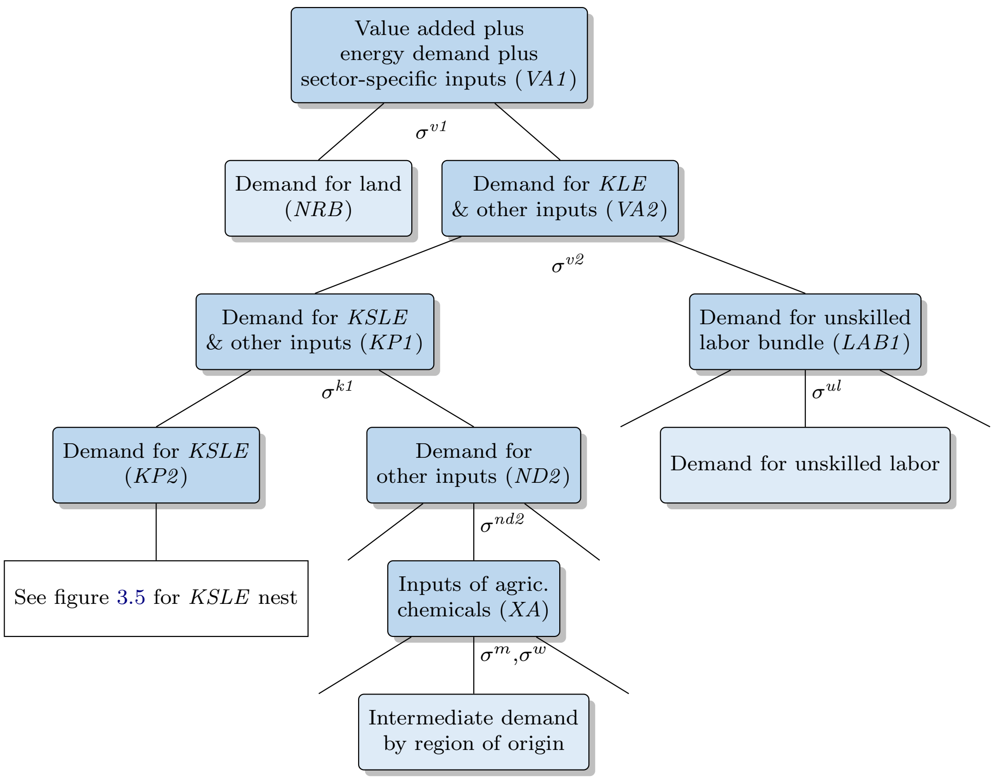


Figure 3.2: **Intermediate nest for standard production structure**

$$VA2_{r,a,v} = \alpha^{va2}_{r,a,v} \left(\frac{PVA1_{r,a,v}}{PVA2_{r,a,v}}\right)^{\sigma^{v1}_{r,a,v}} VA1_{r,a,v} \tag{P-7}$$

$$NRB_{r,a,v} = \alpha^{nrb}_{r,a,v} \left(\frac{PVA1_{r,a,v}}{PNRB_{r,a,v}}\right)^{\sigma^{v1}_{r,a,v}} VA1_{r,a,v} \tag{P-8}$$

$$PVA1_{r,a,v} = \left[\alpha^{va2}_{r,a,v} \left(PVA2_{r,a,v}\right)^{1-\sigma^{v1}_{r,a,v}} + \alpha^{nrb}_{r,a,v} \left(PNRB_{r,a,v}\right)^{1-\sigma^{v1}_{r,a,v}}\right]^{1/(1-\sigma^{v1}_{r,a,v})} \tag{P-9}$$

The next CES nest decomposes the *VA2* bundle into the (unskilled) labor bundle and the capital bundle (with energy and optionally skilled labor), designated *KP1*. Equations (P-10) and (P-11) represent demand for the *KP1* and *LAB1* bundle respectively where σ^{v2} is the relevant substitution elasticity. Equation (P-12) represents the price of the *VA2* bundle, *PVA2*.

$$KP1_{r,a,v} = \alpha^{kp1}_{r,a,v} \left(\frac{PVA2_{r,a,v}}{PKP1_{r,a,v}}\right)^{\sigma^{v2}_{r,a,v}} VA2_{r,a,v} \tag{P-10}$$

$$LAB1_{r,a,v} = \alpha^{lab1}_{r,a,v} \left(\frac{PVA2_{r,a,v}}{PLAB1_{r,a,v}}\right)^{\sigma^{v2}_{r,a,v}} VA2_{r,a,v} \tag{P-11}$$

$$PVA2_{r,a,v} = \left[\alpha^{kp1}_{r,a,v} \left(PKP1_{r,a,v}\right)^{1-\sigma^{v2}_{r,a,v}} + \alpha^{lab1}_{r,a,v} \left(PLAB1_{r,a,v}\right)^{1-\sigma^{v2}_{r,a,v}}\right]^{1/(1-\sigma^{v2}_{r,a,v})} \tag{P-12}$$

The *KP1* bundle is composed of the sector-specific intermediate input bundle, *ND2*, and the augmented capital bundle, *KP2*. Equations (P-13) and (P-14) determine respectively demand for the *KP2* and *ND2* bundles where the substitution elasticity is given by σ^{k1}. Equation (P-15) determines the price of the *KP1* bundle, *PKP1*.

$$KP2_{r,a,v} = \alpha^{kp2}_{r,a,v} \left(\frac{PKP1_{r,a,v}}{PKP2_{r,a,v}}\right)^{\sigma^{k1}_{r,a,v}} KP1_{r,a,v} \tag{P-13}$$

$$ND2_{r,a,v} = \alpha^{nd2}_{r,a,v} \left(\frac{PKP1_{r,a,v}}{PND2_{r,a,v}}\right)^{\sigma^{k1}_{r,a,v}} KP1_{r,a,v} \tag{P-14}$$

$$PKP1_{r,a,v} = \left[\alpha^{kp2}_{r,a,v} \left(PKP2_{r,a,v}\right)^{1-\sigma^{k1}_{r,a,v}} + \alpha^{nd2}_{r,a,v} \left(PND2_{r,a,v}\right)^{1-\sigma^{k1}_{r,a,v}}\right]^{1/(1-\sigma^{k1}_{r,a,v})} \tag{P-15}$$

3.2.2 Crop production nesting

This sections describes the decomposition of the *VA1* bundle for the crops production strucuture. Equations (P-16) through (P-18) are identical to the standard production structure, i.e. *VA1* is decomposed into an *VA2* bundle and the *NRB* bundle.

$$VA2_{r,a,v} = \alpha^{va2}_{r,a,v} \left(\frac{PVA1_{r,a,v}}{PVA2_{r,a,v}}\right)^{\sigma^{v1}_{r,a,v}} VA1_{r,a,v} \tag{P-16}$$

$$NRB_{r,a,v} = \alpha^{nrb}_{r,a,v} \left(\frac{PVA1_{r,a,v}}{PNRB_{r,a,v}}\right)^{\sigma^{v1}_{r,a,v}} VA1_{r,a,v} \tag{P-17}$$

$$PVA1_{r,a,v} = \left[\alpha^{va2}_{r,a,v} \left(PVA2_{r,a,v}\right)^{1-\sigma^{v1}_{r,a,v}} + \alpha^{nrb}_{r,a,v} \left(PNRB_{r,a,v}\right)^{1-\sigma^{v1}_{r,a,v}}\right]^{1/(1-\sigma^{v1}_{r,a,v})} \tag{P-18}$$

The *VA2* bundle is decomposed into two bundles *KP1* and *KP2*. The latter is the same across all production structures and is capital augmented by energy and skilled labor. The *KP1* bundle in the case of crops is the combination of *LAB1* and *ND2*, where the latter is assumed to be agricultural chemicals (fertilizers, pesticides, herbicides, etc.). Equations (P-19) and (P-20) represent respectively demand for the *KP1* and *KP2* bundles, where σ^{v2} is the CES substitution elasticity. Equation (P-21) represents the price of the *VA2* bundle, *PVA2*.

$$KP1_{r,a,v} = \alpha^{kp1}_{r,a,v} \left(\frac{PVA2_{r,a,v}}{PKP1_{r,a,v}}\right)^{\sigma^{v2}_{r,a,v}} VA2_{r,a,v} \tag{P-19}$$

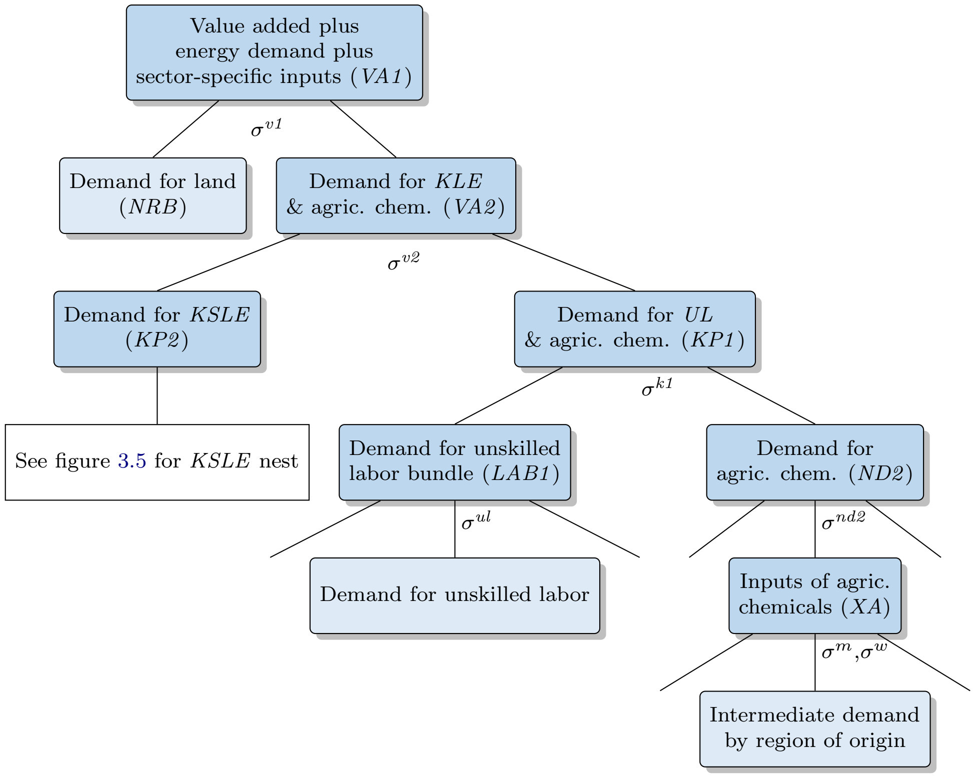


Figure 3.3: **Intermediate nest for crop production structure**

$$KP2_{r,a,v} = \alpha^{kp2}_{r,a,v} \left(\frac{PVA2_{r,a,v}}{PKP2_{r,a,v}}\right)^{\sigma^{v2}_{r,a,v}} VA2_{r,a,v} \tag{P-20}$$

$$PVA2_{r,a,v} = \left[\alpha^{kp1}_{r,a,v} \left(PKP1_{r,a,v}\right)^{1-\sigma^{v2}_{r,a,v}} + \alpha^{kp2}_{r,a,v} \left(PKP2_{r,a,v}\right)^{1-\sigma^{v2}_{r,a,v}}\right]^{1/(1-\sigma^{v2}_{r,a,v})} \tag{P-21}$$

The third nest in the crop production structure decomposes *KP1* into *LAB1* and *ND2*—equations (P-22) and (P-23)—with a CES elasticity of σ^{k1}. Equation (P-24) determines the price of the *KP1* bundle, *PKP1*.

$$LAB1_{r,a,v} = \alpha^{lab1}_{r,a,v} \left(\frac{PKP1_{r,a,v}}{PLAB1_{r,a,v}}\right)^{\sigma^{k1}_{r,a,v}} KP1_{r,a,v} \tag{P-22}$$

$$ND2_{r,a,v} = \alpha^{nd2}_{r,a,v} \left(\frac{PKP1_{r,a,v}}{PND2_{r,a,v}}\right)^{\sigma^{k1}_{r,a,v}} KP1_{r,a,v} \tag{P-23}$$

$$PKP1_{r,a,v} = \left[\alpha^{lab1}_{r,a,v} \left(PLAB1_{r,a,v}\right)^{1-\sigma^{k1}_{r,a,v}} + \alpha^{nd2}_{r,a,v} \left(PND2_{r,a,v}\right)^{1-\sigma^{k1}_{r,a,v}}\right]^{1/(1-\sigma^{k1}_{r,a,v})} \tag{P-24}$$

3.2.3 Livestock production nesting

The key objective in the livestock production nesting is to capture the substitution between land and feed and within feed, the substitution across the different feed components. The *VA1* bundle is first decomposed into a *VA2* and a *KP1* bundle—that both have different components compared with crops and the standard production structure. Equations (P-25) and (P-26) determine respectively the demands for the *VA2* and *KP1* bundles. The price of the *VA1* bundle is given by equation (P-27).

$$VA2_{r,a,v} = \alpha^{va2}_{r,a,v} \left(\frac{PVA1_{r,a,v}}{PVA2_{r,a,v}} \right)^{\sigma^{v1}_{r,a,v}} VA1_{r,a,v} \tag{P-25}$$

$$KP1_{r,a,v} = \alpha^{kp1}_{r,a,v} \left(\frac{PVA1_{r,a,v}}{PKP1_{r,a,v}} \right)^{\sigma^{v1}_{r,a,v}} VA1_{r,a,v} \tag{P-26}$$

$$PVA1_{r,a,v} = \left[\alpha^{va2}_{r,a,v} \left(PVA2_{r,a,v} \right)^{1-\sigma^{v1}_{r,a,v}} + \alpha^{kp1}_{r,a,v} \left(PKP1_{r,a,v} \right)^{1-\sigma^{v1}_{r,a,v}} \right]^{1/(1-\sigma^{v1}_{r,a,v})} \tag{P-27}$$

The next nest decomposes the *VA2* bundle into *LAB1* and *KP2*, equations (P-28) and (P-29) respectively. The price of the *VA2* bundle, *PVA2* is given by equation (P-30).

$$LAB1_{r,a,v} = \alpha^{lab1}_{r,a,v} \left(\frac{PVA2_{r,a,v}}{PLAB1_{r,a,v}} \right)^{\sigma^{v2}_{r,a,v}} VA2_{r,a,v} \tag{P-28}$$

$$KP2_{r,a,v} = \alpha^{kp2}_{r,a,v} \left(\frac{PVA2_{r,a,v}}{PKP2_{r,a,v}} \right)^{\sigma^{v2}_{r,a,v}} VA2_{r,a,v} \tag{P-29}$$

$$PVA2_{r,a,v} = \left[\alpha^{lab1}_{r,a,v} \left(PLAB1_{r,a,v} \right)^{1-\sigma^{v2}_{r,a,v}} + \alpha^{kp2}_{r,a,v} \left(PKP2_{r,a,v} \right)^{1-\sigma^{v2}_{r,a,v}} \right]^{1/(1-\sigma^{v2}_{r,a,v})} \tag{P-30}$$

The key nest decomposes *KP1* into land on the one hand (the *NRB* bundle) and feed on the other hand (the *ND2* bundle). Equations (P-31) and (P-32) determine respectively the demand for the *NRB* and *ND2* bundles. The land/feed substitution elasticity is given by σ^{k1}. The price of the *KP1* bundle, *PKP1*, is given by equation (P-33).

$$NRB_{r,a,v} = \alpha^{nrb}_{r,a,v} \left(\frac{PKP1_{r,a,v}}{PNRB_{r,a,v}} \right)^{\sigma^{k1}_{r,a,v}} KP1_{r,a,v} \tag{P-31}$$

$$ND2_{r,a,v} = \alpha^{nd2}_{r,a,v} \left(\frac{PKP1_{r,a,v}}{PND2_{r,a,v}} \right)^{\sigma^{k1}_{r,a,v}} KP1_{r,a,v} \tag{P-32}$$

$$PKP1_{r,a,v} = \left[\alpha^{nrb}_{r,a,v} \left(PNRB_{r,a,v} \right)^{1-\sigma^{k1}_{r,a,v}} + \alpha^{nd2}_{r,a,v} \left(PND2_{r,a,v} \right)^{1-\sigma^{k1}_{r,a,v}} \right]^{1/(1-\sigma^{k1}_{r,a,v})} \tag{P-33}$$

3.2.4 Decomposition of lower nests

At this stage, the decomposition of the remaining bundles is identical across all sectors. The remaining bundles include two intermediate demand bundles (*ND1* and *ND2*), two labor bundles (*LAB1* and *LAB2*), natural resources (*NRB*) and the *KP2* bundle (that includes *LAB2* and energy).

The next nest is a decomposition of the capital/energy bundle, *KP2*, into demand for a capital plus potentially other factors of production (e.g. skilled labor), *KP3*, and an energy bundle, *XNRG*.

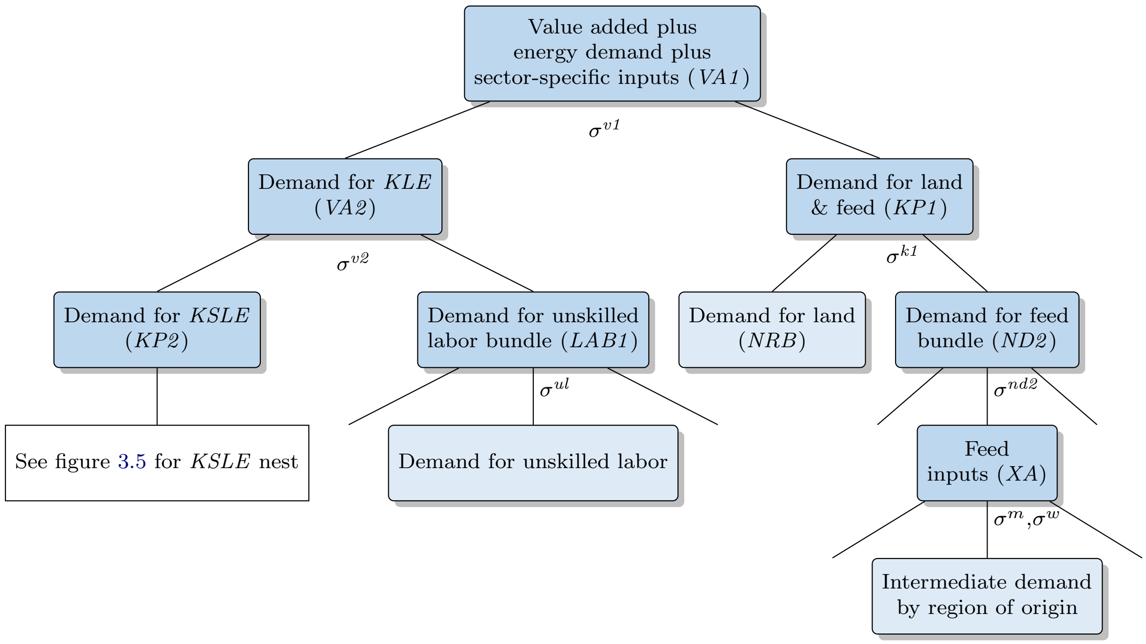


Figure 3.4: **Intermediate nest for livestock production structure**

Equation (P-34) defines the demand for the capital and skilled labor bundle, $KP3$. The substitution elasticity is given by σ^{k2}. Equation (P-35) determines the demand for the energy bundle, $XNRG$. The latter is indexed by eb, a special set that indexes all energy bundles. There is a set mapping that has a one-to-one correspondence between the given activity a and a specific item in eb. The reason for this is to simplify the code for disaggregating the energy bundles across agents in the economy and is described further below. Equation (P-36) defines the price of the $KP2$ bundle, $PKP2$.

$$KP3_{r,a,v} = \alpha^{kp3}_{r,a,v} \left(\frac{PKP2_{r,a,v}}{PKP3_{r,a,v}} \right)^{\sigma^{k2}_{r,a,v}} KP2_{r,a,v} \tag{P-34}$$

$$XNRG_{r,eb,v} = \alpha^{ep}_{r,a,v} \left(\frac{PKP2_{r,a,v}}{PXNRG_{r,eb,v}} \right)^{\sigma^{k2}_{r,a,v}} KP2_{r,a,v} \tag{P-35}$$

$$PKP2_{r,a,v} = \left[\alpha^{kp3}_{r,a,v} \left(PKP3_{r,a,v} \right)^{1-\sigma^{k2}_{r,a,v}} + \alpha^{ep}_{r,a,v} \left(PXNRG_{r,eb,v} \right)^{1-\sigma^{k2}_{r,a,v}} \right]^{1/(1-\sigma^{k2}_{r,a,v})} \tag{P-36}$$

The next nest is a decomposition of the capital plus other labor bundle, $KP3$, into demand for a capital plus potentially some types of labor (e.g. skilled labor). Equation (P-37) determines the demand for the $LAB2$ bundle. Equation (P-38) defines the demand for capital by vintage, KV. The substitution elasticity is given by σ^{k3}. The capital productivity factor is λ^{gf} and potentially includes climate related impacts. The productivity factor also includes an additional shift parameter, φ^w, that is sometimes used to scale base year prices/volumes in the case unitary indices are not used. It is normally constant over time. Equation (P-39) defines the price of the $KP3$ bundle, $PKP3$.

$$LAB2_{r,a,v} = \alpha^{lab2}_{r,a,v} \left(\frac{PKP3_{r,a,v}}{PLAB2_{r,a,v}} \right)^{\sigma^{k3}_{r,a,v}} KP3_{r,a,v} \tag{P-37}$$

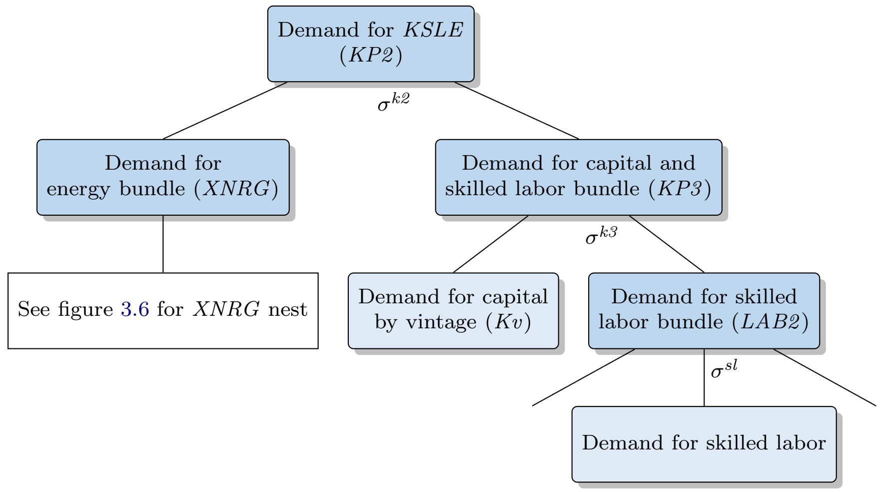


Figure 3.5: **Nesting for *KSLE* bundle**

$$KV_{r,a,v} = \alpha^{f}_{r,Captl,a,v}\left(\lambda^{gf}_{r,Captl,a}\varphi^{w}_{r,Captl,a,v}\right)^{\sigma^{k3}_{r,a,v}-1}\left(\frac{PKP3_{r,a,v}}{PKV_{r,a,v}}\right)^{\sigma^{k3}_{r,a,v}} KP3_{r,a,v} \tag{P-38}$$

$$PKP3_{r,a,v} = \left[\alpha^{lab2}_{r,a,v}PLAB2^{1-\sigma^{k3}_{r,a,v}}_{r,a,v} + \alpha^{f}_{r,Captl,a,v}\left(\frac{PKV_{r,a,v}}{\lambda^{gf}_{r,Captl,a}\varphi^{w}_{r,Captl,a,v}}\right)^{1-\sigma^{k3}_{r,a,v}}\right]^{\frac{1}{1-\sigma^{k3}_{r,a,v}}} \tag{P-39}$$

The next set of steps decompose the *NRB* bundle, the *LAB1* and *LAB2* bundles and the *ND1* and *ND2* bundles. The decomposition of the energy bundle is presented later.

The *NRB* bundle is decomposed into land and sector-specific factors—the two are indexed by *nr* that is a subset of the set of factors. In the full GTAP database, there are no sectors with both land and a natural resource. Land is only used in the crop and livestock sectors. Natural resources are consumed in six sectors—forestry, fishing, coal, oil, natural gas and other mining. Thus this nest typically will have one inactive node. It could be possible that in an aggregation of the GTAP database a sector could have both land and natural resources. Equation (P-40) determines the sectoral demand for the natural resources. It is summed across vintages. The price of the natural resource bundle, *PNRB*, is given by equation (P-41).

$$XF_{r,nr,a} = \sum_{v}\alpha^{f}_{r,nr,a,v}\left(\lambda^{gf}_{r,nr,a}\varphi^{w}_{r,nr,a,v}\right)^{\sigma^{nr}_{r,a,v}-1}\left(\frac{PNRB_{r,a,v}}{PF_{r,nr,a}}\right)^{\sigma^{nr}_{r,a,v}} NRB_{r,a,v} \tag{P-40}$$

$$PNRB_{r,a,v} = \left[\sum_{nr}\alpha^{f}_{r,nr,a,v}\left(\frac{PF_{r,nr,a}}{\lambda^{gf}_{r,nr,a}\varphi^{w}_{r,nr,a,v}}\right)^{1-\sigma^{nr}_{r,a,v}}\right]^{\frac{1}{1-\sigma^{nr}_{r,a,v}}} \tag{P-41}$$

An additional CES nest decomposes the aggregate land bundle, XF_{LandR}, into land demand by type. One of the satellite accounts of GTAP is the decomposition of land by agro-ecological zones of which there are three broad types—tropical, temperate and boreal—within each of which there are six sub-categories linked to the length of the growing period; thus there is a total of 18 agro-ecological zones or AEZs. The land types are indexed by *lt*. Equations (P-42) and (P-43) define the demand for land by type within each land-using activity, Lnd, and the price of the aggregate land bundle, PF_{LandR}. The coefficient χ^{lnd} is used as a scaling factor for the land demand variable.

$$Lnd_{r,lt,a} = \alpha^{lnd}_{r,lt,a,v} \left(\frac{PF_{r,LandR,a}}{PLnd_{r,lt,a}}\right)^{\sigma^{lt}_{r,a}} \frac{XF_{r,LandR,a}}{\chi^{lnd}_{r,lt,a}} \tag{P-42}$$

$$PF_{r,LandR,a} XF_{r,LandR,a} = \sum_{lt} \chi^{lnd}_{r,lt,a} PLnd_{r,lt,a} Lnd_{r,lt,a} \tag{P-43}$$

The following node decomposes the *LAB1* bundle that is intended to contain unskilled labor types—but could contain all labor types. Equation (P-44) determines the demand for labor indexed by *l1*. It is summed across vintages. Equation (P-45) determines the price of the *LAB1* bundle, *PLAB1*.

$$XF_{r,l1,a} = \sum_{v} \alpha^{f}_{r,l1,a,v} \left(\lambda^{gf}_{r,l1,a} \varphi^{w}_{r,l1,a,v}\right)^{\sigma^{l1}_{r,a,v}-1} \left(\frac{PLAB1_{r,a,v}}{PF_{r,l1,a}}\right)^{\sigma^{l1}_{r,a,v}} LAB1_{r,a,v} \tag{P-44}$$

$$PLAB1_{r,a,v} = \left[\sum_{l1} \alpha^{f}_{r,l1,a,v} \left(\frac{PF_{r,l1,a}}{\lambda^{gf}_{r,l1,a} \varphi^{w}_{r,l1,a,v}}\right)^{1-\sigma^{l1}_{r,a,v}}\right]^{\frac{1}{1-\sigma^{l1}_{r,a,v}}} \tag{P-45}$$

The following node, the decomposition of the *LAB2* bundle, is virtually identical to above, but where labor is in the *LAB2* bundle is indexed by *l2*—intended to represent skilled labor types. Equation (P-46) determines the demand for labor indexed by *l2*. It is summed across vintages. Equation (P-47) determines the price of the *LAB2* bundle, *PLAB2*.

$$XF_{r,l2,a} = \sum_{v} \alpha^{f}_{r,l2,a,v} \left(\lambda^{gf}_{r,l2,a} \varphi^{w}_{r,l2,a,v}\right)^{\sigma^{l2}_{r,a,v}-1} \left(\frac{PLAB2_{r,a,v}}{PF_{r,l2,a}}\right)^{\sigma^{l2}_{r,a,v}} LAB2_{r,a,v} \tag{P-46}$$

$$PLAB2_{r,a,v} = \left[\sum_{l2} \alpha^{f}_{r,l2,a,v} \left(\frac{PF_{r,l2,a}}{\lambda^{gf}_{r,l2,a} \varphi^{w}_{r,l2,a,v}}\right)^{1-\sigma^{l2}_{r,a,v}}\right]^{\frac{1}{1-\sigma^{l2}_{r,a,v}}} \tag{P-47}$$

The next two nodes in the production nest decompose the two *ND* bundles—*ND2* is intended to include all sector-specific intermediate inputs and *ND1* contains the residual intermediate inputs that not only excludes the inputs in *ND2*, but also excludes the energy inputs. Equation (P-48) determines the demand for the (Armington) intermediate inputs, XA, indexed by iox, where iox is the subset of all intermediate inputs that are neither in *ND2* nor in the energy bundle. The set iox is specific to each activity. In general, we assume that the substitution elasticity for *ND1* is zero. The relevant price is the agent (or activity) specific Armington price, PA^a. The latter will be a composite price of domestic and imported goods, augmented by domestic taxes and, in mitigation scenarios, with a tax linked to emissions (described below). The model allows for input-specific efficiency improvements as encapsulated by the λ^{nd} parameter. Equation (P-49) provides the price of the aggregate *ND1* bundle.

$$XA_{r,iox,a} = io_{r,iox,a,Old} \left(\lambda^{nd}_{r,iox,a}\right)^{\sigma^{n1}_{r,a}-1} \left(\frac{PND1_{r,a}}{PA^{a}_{r,iox,a}}\right)^{\sigma^{n1}_{r,a}} ND1_{r,a} \tag{P-48}$$

$$PND1_{r,a} = \left[\sum_{iox} io_{r,iox,a,Old} \left(\frac{PA^a_{r,iox,a}}{\lambda^{nd}_{r,iox,a}} \right)^{1-\sigma^{n1}_{r,a}} \right]^{\frac{1}{1-\sigma^{n1}_{r,a}}} \tag{P-49}$$

Equation (P-50) determines the (Armington) demand for inputs contained in the *ND2* bundle that are intended to be sector specific—for example the feed inputs in the livestock sector. The elasticity in this case is expected to be positive. The index *ion* used in the equation includes only the inputs that are not in *ND1* and not in the energy bundle. The set *ion* is specific to each activity. Equation (P-51) provides the price of the aggregate *ND2* bundle.

$$XA_{r,ion,a} = io_{r,ion,a,Old} \left(\lambda^{nd}_{r,ion,a} \right)^{\sigma^{n2}_{r,a}-1} \left(\frac{PND2_{r,a}}{PA^a_{r,ion,a}} \right)^{\sigma^{n2}_{r,a}} ND2_{r,a} \tag{P-50}$$

$$PND2_{r,a} = \left[\sum_{ion} io_{r,ion,a,Old} \left(\frac{PA^a_{r,ion,a}}{\lambda^{nd}_{r,ion,a}} \right)^{1-\sigma^{n2}_{r,a}} \right]^{\frac{1}{1-\sigma^{n2}_{r,a}}} \tag{P-51}$$

This ends the description of the production structure, though there is a further decomposition of XA_n, i.e. the non-fuels intermediate Armington demand, and the energy bundle. The latter is decomposed across fuels, and then finally decomposed as an Armington good.

3.3 Income block

The model has six indirect tax streams and two direct-tax streams:

1. The output tax, τ^p imposed on the aggregate price of output, PX, with an additional excise tax, τ^x, in some circumstances.
2. A sales tax on sales of domestic Armington goods, τ^{Ap}, which is agent specific and imposed on the economy-wide price of domestic goods, PA.[3]
3. Bilateral import tariff, τ^m, imposed on the landed (or CIF) price of imports, WPM. The model also allows for homogeneous goods, in which case the tariff represents a wedge between the world price and the domestic price.
4. Bilateral export tax (or subsidy), τ^e, imposed on the producer price of exports, PE. In the case of a homogeneous commodity, the export tax represents the wedge between world prices and domestic prices.
5. Taxes on the factors of production, τ^v, imposed on the market-clearing price of factors, NPF.
6. Taxes on emissions, τ^e, imposed on the Armington consumption of goods.
7. Taxes on factor income, κ^f.
8. Direct household taxes, $kappa^h$.

[3] The model allows for agent-specific Armington decompositions, though this increases the size of the model considerably. This is described further in Appendix C on alternative trade specifications.

Equations (Y-1) through (Y-6) correspond to the aggregate revenues generated by each of the six indirect taxes. The notation for the variables not already described will be given below. One important observation concerns the bilateral trade variables. These are indexed either as (r, d, i) where r is the country of origin (the exporter), d the country of destination (the importer) and i is the sector index, or as (s, r, i) where r is the importer, s the source country of imports (i.e. the exporter) and i is the sector index. This explains the switch in the indices equations (Y-3) and (Y-4) where WTF corresponds to the bilateral trade flow from region r to region d. Carbon tax revenues will be a function of the parameter ϕ that allows for full or partial participation—including full exclusion-by agent. Equation (Y-7) describes tax revenues from taxes on factor income. Thus, total factor income is distributed directly to households net of income taxes that accrue to the government. Equation (Y-8) defines the aggregate revenue from taxing household income.[4] Fiscal closure will be discussed below.

$$GREV_{r,ptax} = \sum_a \left[\tau^p_{r,a} PX_{r,a} XP_{r,a} + \tau^x_{r,a} XP_{r,a}\right] \tag{Y-1}$$

$$GREV_{r,atax} = \sum_i \sum_{aa} \tau^{Ap}_{r,i,aa} \left(\gamma^c_{r,i,aa} PAT_{r,i}\right) XA_{r,i,aa} \tag{Y-2}$$

$$GREV_{r,mtax} = \sum_{i \in Arm} \sum_s \tau^m_{s,r,i} WPM_{s,r,i} WTF^d_{s,r,i} + \sum_{i \notin Arm} \tau^m_{r,i} PW_i XMT_{r,i} \tag{Y-3}$$

$$GREV_{r,etax} = \sum_{i \in Arm} \sum_d \tau^e_{r,d,i} PE_{r,d,i} WTF^s_{r,d,i} + \sum_{i \notin Arm} \tau^e_{r,i} PW_i XET_{r,i} \tag{Y-4}$$

$$GREV_{r,vtax} = \sum_{fp} \sum_a \left(\tau^{vtax}_{r,fp,a} + \tau^{vsub}_{r,fp,a}\right) NPF_{r,fp,a} XF_{r,fp,a} \tag{Y-5}$$

$$GREV_{r,ctax} = \sum_{em} \sum_i \sum_{aa} \tau^{emi}_{r,em} \phi_{r,em,i,aa} \chi^e_{em} \rho_{r,em,i,aa} XA_{r,i,aa} \tag{Y-6}$$

$$GREV_{r,ytax} = \sum_{fp} \kappa^f_{r,fp} \sum_a NPF_{r,fp,a} XF_{r,fp,a} + \kappa^f_{r,Captl} \sum_a \Pi_{r,a} \tag{Y-7}$$

$$GREV_{r,htax} = \sum_h \chi^k_r \kappa^h_{r,h} YH_{r,h} \tag{Y-8}$$

Equation (Y-9) defines aggregate fiscal revenues, where the set gy corresponds to the six indirect tax streams ($ptax$, $atax$, $ttax$, $etax$, $vtax$ and $ctax$) and the two direct tax streams ($ytax$ and $htax$). It is also assumed that income from a cap and trade system on emissions accrue to the government. Equation (Y-10) summarizes net household income, YH. It is assumed that all factor income net of income taxes (where NPF represents the market clearing factor price net of taxes) accrues to households as well as profits generated by the markups. Household income is then adjusted for the depreciation allowance, $DeprY$. Equation (Y-11) describes household disposable income, YD, where κ^h represents the base year household-specific direct tax rate. The direct tax rate is adjusted

[4] The GTAP database does not distinguish between public and private savings. Thus net household income tax reflects a balancing item that balances the macro accounts. If additional information is available on the split of domestic savings between private and public, the macro SAM would need to be rebalanced. For example, if the government deficit is equal to x percent of GDP, this amount would have to be added to private savings to balance the investment/savings accounts, and subtracted from the initial level of net current transfers from the government to households to balance both the household and government accounts.

by an economy-wide adjustment factor, χ^k, which can be endogenous to achieve a given target, for example the deficit of the public sector. Macro closure is discussed in more detail below.[5] Disposable income is adjusted by changes in international tourism receipts, IIT, which will be affected by climate change.

$$YG_r = \sum_{gy} GREV_{r,gy} + \sum_{em} QuotaY^E_{r,em} \tag{Y-9}$$

$$YH_r = \sum_{fp} \left(1 - \kappa^f_{r,fp}\right) \sum_a NPF_{r,fp,a} XF_{r,fp,a} + \left(1 - \kappa^f_{r,Captl}\right) \sum_a \Pi_{r,a} - DeprY_r \tag{Y-10}$$

$$YD_r = \left(1 - \chi^k_r \kappa^h_{r,h}\right) YH_r + IIT_{r,h} \tag{Y-11}$$

3.4 Demand block

The demand block is divided into two sections. The first describes the allocation of household disposable income between savings and expenditures on goods and services. The second describes other final demand for goods and services.

Envisage incorporates several different specifications for the household utility function. The default utility function is the constant-difference-in-elasticities (CDE) utility function popularized by the GTAP model (see Hertel (1997)). The other three utility functions are variations of the linear expenditure system (LES), also known as the Stone-Geary utility function. These include the standard LES, the so-called extended linear expenditure system (ELES) that integrates the decision to save with the allocation of income on goods and services, and the third variant, known as an implicitly directly additive demand system (AIDADS), extends the standard LES to include variable marginal budget shares that allow for more plausible Engel behavior. All four utility functions are described in Appendix B.

Households first allocate disposable income between savings on the one hand and total expenditure on goods and services on the other hand.[6] Equation (D-1) determines the household savings rate (relative to disposable income), s^s, as a function of per capita growth (g^{pc}) and the youth and elderly dependency ratios, respectively given by $DRAT^{PLT15}$ and $DRAT^{65UP}$.[7] These variables are typically exogenous in dynamic scenarios. The savings function also captures a persistence factor defined by β^s.[8] Equation (D-2) determines the level of household savings. It should be noted that if the ELES utility function is used to specify household demand for goods and services, equation (D-1) is dropped and equation (D-2) then defines the average propensity to save as the ELES

[5] The GTAP dataset contains only a single representative household per country/region. The model implementation allows for multiple households and hence the need for an economy-wide tax shifter that is uniform across households. This has different distributional consequences than an additive shifter or a more complex direct tax schedule.

[6] The demand block is significantly reformulated compared to the first version of the Envisage model. The latter was largely inspired by the GTAP model. The current version is more similar to the Linkage specification. It drops the top level utility function that allocated national income across savings, and public and private expenditures. In the long-term scenarios this top-level structure was typically over-ruled with other specific assumptions making the theoretical consistency of the top-level formulation less appealing.

[7] The original theory and parameters for this formulation can be found in Loayza, Schmidt-Hebbel, and Servén (2000) and Masson, Bayoumi, and Samiei (1998) and is summarized in van der Mensbrugghe (2011).

[8] Setting all the β parameters to 0 would yield a constant savings rate. It is also possible to use this equation to formulate a different closure—for example to target investment and allow the shift parameter, χ^s, to adjust to achieve the given target.

itself determines the level of savings. Equation (D-3) in essence determines aggregate expenditures on household goods and services, YC. In the case of the ELES, combined with equation (D-4), it defines the level of household savings, which is an outcome of the ELES. Equation (D-4) is only used for the ELES version of the model.

$$s^s_{r,h} = \chi^s_r \alpha^s_{r,h} + \beta^s_r s^s_{r,h,-1} + \beta^g_r g^{pc}_r \beta^y_r DRAT^{PLT15} + \beta^e_r DRAT^{P65UP} \tag{D-1}$$

$$S^h_{r,h} = s^s_{r,h} YD_{r,h} \tag{D-2}$$

$$YD_{r,h} = YC_{r,h} + S^h_{r,h} \tag{D-3}$$

$$YC_{r,h} = \sum_k PHX_{r,k,h} XH_{r,k,h} \tag{D-4}$$

3.4.1 Expenditures

The next block of equations determines the top-level sectoral demands for goods and services for households. In the standard model private expenditures are derived from the CDE specification and are based on consumer-defined goods, XH, indexed by k, not Armington goods, XA, that are indexed by i. The CDE and its properties are described fully in Appendix B. Four equations are needed to implement the top-level CDE utility function. Equation (D-5) defines an auxiliary variable θ^{gh} that simplifies the remaining equations.[9] It is a term that includes the utility level, consumer prices and total per capita expenditure. Equation (D-6) determines the household budget shares and represents the normalized levels of the θ^{gh} variables. Equation (D-7) converts the budget shares to household demand in levels. Utility is defined in per capita terms. However, the variable XH represents aggregate expenditure since the variable YC is also total expenditure on goods and services and not the per capita level. Finally, equation (D-8) defines implicitly the utility level.

$$\theta^{gh}_{r,k,h} = \alpha^h_{r,k,h} b^h_{r,k,h} \left(\frac{U^{e^h_{r,k,h}} PHX_{r,k,h}}{YC_{r,h}/Pop_{r,h}} \right)^{b^h_{r,k,h}} \tag{D-5}$$

$$s^h_{r,k,h} = \frac{\theta^{gh}_{r,k,h}}{\sum_k \theta^{gh}_{r,k,h}} \tag{D-6}$$

$$XH_{r,k,h} = \frac{s^h_{r,k,h} YC_{r,h}}{PHX_{r,k,h}} \tag{D-7}$$

$$\sum_k \frac{\theta^{gh}_{r,k,h}}{b^h_{r,k,h}} \equiv 1 \tag{D-8}$$

Equation (D-9) defines a consumer expenditure price index, PC. The latter can be used to deflate total nominal expenditure in order to derive the aggregate volume of real expenditure, XC, as defined in equation (D-10).

$$PC_{r,h} = \sum_k s^h_{r,k,h} PHX_{r,k,h} \tag{D-9}$$

[9] In the LES variants, the variable θ^{gh} reflects the climate-impacted subsistence minimum.

$$XC_{r,h} = YC_{r,h}/PC_{r,h} \tag{D-10}$$

The LES functional form has been used for special purposes. In some cases the subsistence minima, though calibrated using base year data, are 'climate sensitive'—for example energy demand for cooling and/or heating. This is described further in Appendix B. Another special purpose has been to dynamically re-calibrate the LES parameters to target specific assumptions about budget shares, income elasticities, or in the case of food, specific trends on food consumption. These are also described in Appendix B.

The next set of equations decomposes consumer demand defined as consumer goods into produced (or more accurately, Armington) goods. A transition matrix approach is used where each consumed good is composed of one or more produced goods and combined using a CES aggregator.[10] Each consumer good could also have its own energy bundle—with different demand shares across energy.[11] Equation (D-11) converts consumed goods HX into non-energy Armington goods, XA. Equation (D-12) determines demand for the energy bundle, $XNRG$, for each of the k consumed goods. There is a one-to-one correspondence between each index k and an index in the set eb (that also includes demand for energy bundles in production and other final demand). Equation (D-13) then determines the price of consumer good k.

$$XA_{r,n,h} = \sum_k \alpha^c_{r,n,k,h} \left(\frac{PHX_{r,k,h}}{PA^a_{r,n,h}} \right)^{\sigma^c_{r,k,h}} HX_{r,k,h} \tag{D-11}$$

$$XA_{r,eb,Old} = \sum_h \alpha^{nrgh}_{r,k,h} \left(\frac{PHX_{r,k,h}}{PNRG_{r,eb,Old}} \right)^{\sigma^c_{r,k,h}} HX_{r,k,h} \tag{D-12}$$

$$PHX_{r,k,h} = \left[\sum_{n \in nnrg} \alpha^c_{r,n,k,h} \left(PA^a_{r,n,h} \right)^{1-\sigma^c_{r,k,h}} + \alpha^{nrgh}_{r,k,h} \left(PNRG_{r,eb,Old} \right)^{1-\sigma^c_{r,k,h}} \right]^{\frac{1}{1-\sigma^c_{r,k,h}}} \tag{D-13}$$

The final block of demand equations decomposes aggregate public and investment demands. A CES expenditure function is used that covers all non-energy Armington goods and an energy bundle. Decomposition of the energy bundle is done at a later stage. Equation (D-14) represents the sectoral (Armington) demand for public and investment non-energy expenditures XA, where the index f represents the set spanning (gov and inv). Equation (D-15) determines the demand for the energy bundle (where the index eb is mapped to the respective f index). The expenditure price indices, PC_f, are given by equation (D-16). In the standard model there are no stock-building activities. In some scenarios it is helpful to give éxogenouśdemand shocks. This is most easily done by assuming stock-building activities as defined in equation (D-17), where the level of stock-building is linked to domestic production, XS.

$$XA_{r,n,f} = \alpha^f_{r,n,f} \left(\frac{PC_{r,f}}{PA^a_{r,n,f}} \right)^{\sigma^f_{r,f}} XC_{r,f} \tag{D-14}$$

[10] Using the standard GTAP data, the transition matrix is diagonal—each consumed good corresponds to exactly one produced good. ENVISAGE still uses this approach save for the energy bundle that is combined into one consumed commodity. Work is ongoing to develop a global database of transition matrices. The GREEN model for example (see Burniaux, Nicoletti, and Oliveira-Martins (1992) and van der Mensbrugghe (1994)) had four consumed goods and eight standard produced goods.

[11] For example, a transportation bundle is likely to be dominated by liquid fuel demand, whereas demand for heat is likely to be dominated by electricity and natural gas.

$$XA_{r,eb,Old} = \alpha^{nrgf}_{r,f} \left(\frac{PC_{r,f}}{PNRG_{r,eb,Old}}\right)^{\sigma^f_{r,f}} XC_{r,f} \tag{D-15}$$

$$PC_{r,f} = \left[\sum_{n \in nnrg} PA^a_{r,n,f} XA_{r,n,f} + PNRG_{r,eb,Old} XNRG_{r,eb,Old}\right] / XC_{r,f} \tag{D-16}$$

$$XA_{r,i,stb} = \chi^{stb}_{r,i} XS_{r,i} \tag{D-17}$$

3.5 The fuels block

Each agent in the economy has a specified demand for an aggregate energy bundle. The fuel demanders are indexed by eb that spans all activities (a), each commodity consumed by households (k) and other final demand (f).[12] The equations above provide the bundle $XNRG$ across all eb agents. That bundle is decomposed across all energy sources using a nested CES structure with agent-specific share parameters and substitution elasticities. At the top level, demand is decomposed between electricity and non-electric energy (see figure 3.6). The non-electric bundle is split into coal on the one hand, and gas and oil on the other. The oil and gas bundle is then split into oil on the one hand and gas on the other. Using the standard GTAP classification, the final electric bundle is composed of commodity *ely* alone. The coal bundle is composed of the commodity *coa* alone. The gas bundle is composed of the commodities *gas* and *gdt*. And the oil bundle is composed of the commodities *oil* and *p_c*. In most cases, for these latter two bundles, one component will dominate the other. For example, there may be some residual oil consumption in households, but the bulk of the consumption will be *p_c*. When the new alternative technologies are introduced, they are inserted at the bottom most node for electricity, coal, oil and gas respectively.

The next block of equations is the top of the energy node nest. It decomposes the energy bundle, $XNRG$, into an electric bundle, $XELY$, and a non-electric bundle, $XNELY$. Equations (F-1) and (F-2) define respectively the demands for the electric and non-electric bundles with a substitution elasticity given by σ^e. The equations are defined over all demanders of energy bundles, eb, and are also vintage-specific in production. Equation (F-3) defines the CES price of the energy bundle as a CES aggregation of the respective bundle prices, $PELY$ and $PNELY$.

$$XELY_{r,eb,v} = \alpha^{ely}_{r,eb,v} \left(\frac{PNRG_{r,eb,v}}{PELY_{r,eb,v}}\right)^{\sigma^e_{r,eb,v}} XNRG_{r,eb,v} \tag{F-1}$$

$$XNELY_{r,eb,v} = \alpha^{nely}_{r,eb,v} \left(\frac{PNRG_{r,eb,v}}{PNELY_{r,eb,v}}\right)^{\sigma^e_{r,eb,v}} XNRG_{r,eb,v} \tag{F-2}$$

$$PNRG_{r,eb,v} = \left[\alpha^{ely}_{r,eb,v} \left(PELY_{r,eb,v}\right)^{1-\sigma^e_{r,eb,v}} + \alpha^{nely}_{r,eb,v} \left(PNELY_{r,eb,v}\right)^{1-\sigma^e_{r,eb,v}}\right]^{\frac{1}{1-\sigma^e_{r,eb,v}}} \tag{F-3}$$

The following block decomposes the non-electric bundle into a coal bundle, $XCOA$, and an oil and gas bundle, $XOLG$, given respectively by equations (F-4) and (F-5) with a substitution

12 Note that in the case of energy bundles from the production side, they are also indexed by vintage with potentially different substitution elasticities across vintages. The non-production energy bundles are also indexed by vintage (for simplicity), though only the *Old* vintage is active.

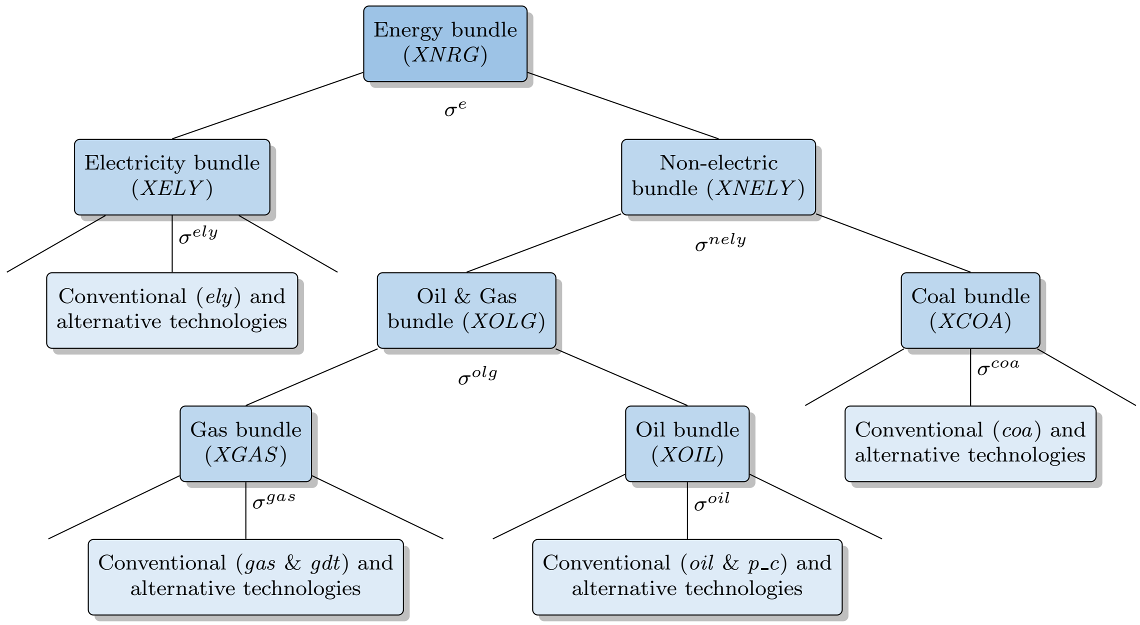


Figure 3.6: **Energy nest**

elasticity of σ^{nely}. Equation (F-6) defines the price of the non-electric bundle, $PNELY$.

$$XCOA_{r,eb,v} = \alpha^{coa}_{r,eb,v} \left(\frac{PNELY_{r,eb,v}}{PCOA_{r,eb,v}}\right)^{\sigma^{nely}_{r,eb,v}} XNELY_{r,eb,v} \tag{F-4}$$

$$XOLG_{r,eb,v} = \alpha^{olg}_{r,eb,v} \left(\frac{PNELY_{r,eb,v}}{POLG_{r,eb,v}}\right)^{\sigma^{nely}_{r,eb,v}} XNELY_{r,eb,v} \tag{F-5}$$

$$PNELY_{r,eb,v} = \left[\alpha^{coa}_{r,eb,v} \left(PCOA_{r,eb,v}\right)^{1-\sigma^{nely}_{r,eb,v}} + \alpha^{olg}_{r,eb,v} \left(POLG_{r,eb,v}\right)^{1-\sigma^{nely}_{r,eb,v}}\right]^{\frac{1}{1-\sigma^{nely}_{r,eb,v}}} \tag{F-6}$$

The third node decomposes the oil and gas bundle into a gas bundle, $XCOA$, and an oil bundle, $XOIL$. Equations (F-7) and (F-8) provide the demand equations for the respective bundles with a substitution elasticity of σ^{log}. Finally, equation (F-9) describes the price of the oil and gas bundle, $POLG$, as a CES aggregation of the gas bundle, $PGAS$, and the oil bundle, $POIL$.

$$XGAS_{r,eb,v} = \alpha^{gas}_{r,eb,v} \left(\frac{POLG_{r,eb,v}}{PGAS_{r,eb,v}}\right)^{\sigma^{olg}_{r,eb,v}} XOLG_{r,eb,v} \tag{F-7}$$

$$XOIL_{r,eb,v} = \alpha^{oil}_{r,eb,v} \left(\frac{POLG_{r,eb,v}}{POIL_{r,eb,v}}\right)^{\sigma^{olg}_{r,eb,v}} XOLG_{r,eb,v} \tag{F-8}$$

$$POLG_{r,eb,v} = \left[\alpha^{gas}_{r,eb,v} \left(PGAS_{r,eb,v}\right)^{1-\sigma^{olg}_{r,eb,v}} + \alpha^{oil}_{r,eb,v} \left(POIL_{r,eb,v}\right)^{1-\sigma^{olg}_{r,eb,v}}\right]^{\frac{1}{1-\sigma^{olg}_{r,eb,v}}} \tag{F-9}$$

At this point, the decomposition of fuels is down to four fundamental energy sources—electricity, coal, gas and oil. In the initial state, with the GTAP data alone, each of the six energies in GTAP is mapped to these four bundles. Four energy sets are defined: *ely*, *coa*, *oil* and *gas* that correspond to a mapping to one of the four types of energy. The GTAP *ely* sector is mapped to *ely*, the GTAP *coa* sector is mapped to *coa*, the GTAP *gas* and *gdt* sectors are mapped to *gas*, and the GTAP *oil* and *p_c* sectors are mapped to *oil*. With the introduction of new technologies, the set mappings will increase. Thus if there is one electric backstop technology, say renewables, and designated by *elybs*, it will be mapped to the *ely* aggregate electric bundle.

Equations (F-10) through (F-17) determine the decomposition of the four basic energy bundles to their respective Armington volumes. For electricity and coal, with the base data, these equations are somewhat redundant since the bundles map to only one Armington commodity. Each demand equation requires a summing over vintages (for only activities), and a summing across *eb* indices. In most cases, the *eb* index maps to one, and only one, agent (aa). In the case of consumption, however, the energy bundle can exist for each consumed commodity (k), and thus there can be as many energy decompositions as there are consumer commodities. Each bundle also allows for energy efficiency improvement, sometimes designated as the autonomous energy efficiency improvement (AEEI) parameter, which is region, agent, fuel and vintage specific (in principle). The price equations need a separate mapping from the *eb* to the *aa* index, though it is assumed that the consumer price for a given fuel is uniform across the k commodities (i.e. natural gas used for heat has the same price as natural gas used for transportation.).

$$XA_{r,ely,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{elybs}_{r,ely,eb,v} \left(\frac{\lambda^e_{r,ely,eb,v} PELY_{r,eb,v}}{PA^a_{r,ely,aa}} \right)^{\sigma^{ely}_{r,eb,v}} \frac{XELY_{r,eb,v}}{\lambda^e_{r,ely,eb,v}} \tag{F-10}$$

$$PELY_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{ely} \alpha^{elybs}_{r,ely,eb,v} \left(\frac{PA^a_{r,ely,aa}}{\lambda^e_{r,ely,eb,v}} \right)^{1-\sigma^{ely}_{r,eb,v}} \right]^{\frac{1}{1-\sigma^{ely}_{r,eb,v}}} \tag{F-11}$$

$$XA_{r,coa,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{coabs}_{r,coa,eb,v} \left(\frac{\lambda^e_{r,coa,eb,v} PCOA_{r,eb,v}}{PA^a_{r,coa,aa}} \right)^{\sigma^{coa}_{r,eb,v}} \frac{XCOA_{r,eb,v}}{\lambda^e_{r,coa,eb,v}} \tag{F-12}$$

$$PCOA_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{coa} \alpha^{coabs}_{r,coa,eb,v} \left(\frac{PA^a_{r,coa,aa}}{\lambda^e_{r,coa,eb,v}} \right)^{1-\sigma^{coa}_{r,eb,v}} \right]^{\frac{1}{1-\sigma^{coa}_{r,eb,v}}} \tag{F-13}$$

$$XA_{r,gas,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{gasbs}_{r,gas,eb,v} \left(\frac{\lambda^e_{r,gas,eb,v} PGAS_{r,eb,v}}{PA^a_{r,gas,aa}} \right)^{\sigma^{gas}_{r,eb,v}} \frac{XGAS_{r,eb,v}}{\lambda^e_{r,gas,eb,v}} \tag{F-14}$$

$$PGAS_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{gas} \alpha^{gasbs}_{r,gas,eb,v} \left(\frac{PA^a_{r,gas,aa}}{\lambda^e_{r,gas,eb,v}} \right)^{1-\sigma^{gas}_{r,eb,v}} \right]^{\frac{1}{1-\sigma^{gas}_{r,eb,v}}} \tag{F-15}$$

$$XA_{r,oil,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{oilbs}_{r,oil,eb,v} \left(\frac{\lambda^e_{r,oil,eb,v} POIL_{r,eb,v}}{PA^a_{r,oil,aa}} \right)^{\sigma^{oil}_{r,eb,v}} \frac{XOIL_{r,eb,v}}{\lambda^e_{r,oil,eb,v}} \tag{F-16}$$

$$POIL_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{oil} \alpha^{oilbs}_{r,oil,eb,v} \left(\frac{PA^a_{r,oil,aa}}{\lambda^e_{r,oil,eb,v}} \right)^{1-\sigma^{oil}_{r,eb,v}} \right]^{\frac{1}{1-\sigma^{oil}_{r,eb,v}}} \tag{F-17}$$

3.6 Trade Block

3.6.1 Top level Armington

The equations above have determined completely the so-called Armington demand for goods across all agents, XA, that include activities (a), private or consumer demand (h), and other final demand (f). The union of these three sets is the set aa. In the standard version of ENVISAGE, all Armington agents are assumed to have the same preference function for domestic and import goods.[13] It is also assumed that the Armington good, for each commodity i, is homogeneous across agents, and can therefore be aggregated in volume terms. However, when using the energy volume data that comes with the GTAP data set, the derived energy prices vary (modestly in most cases) across agents.[14] To maintain the adding up assumption with the price differentials, a shift parameter is associated with each agent. One could think of this intuitively as a quality index, so the gasoline consumed by households has a different quality than that consumed in transportation, where quality differences may simply reflect octane levels.

Equation (T-1) defines aggregate Armington demand, XAT. It is the sum across all agents of their Armington demand—adjusted by the fixed shift (or quality) parameter, γ^c.[15] The agent-specific Armington price is composed of two components. The first, PA, is formed from the nationally determined Armington price, PAT, defined below, adjusted by the quality index, γ^c, and augmented by the user-specific sales tax, τ^{Ap}—see equation (T-2). To this is added the emission tax, τ^{me}, see equation (T-3). The emissions tax is given as a dollar amount per unit of emission, where ρ determines the agent specific level of emissions per unit of demand by agent (aa), per input (i) and per emission type (em). The emissions rate ρ is multiplied by a global emissions factor χ^e that allows for the emissions rate to vary in the baseline scenario to achieve a given global emissions trend.[16] In other words ρ is calibrated to base year data and χ^e represents trend changes in the emissions rate. The model allows for full or partial exemptions using the parameter φ—that can also be agent, input and emission specific. For example it is possible to exempt given sectors or households from paying the emissions tax for specific fuels, say gasoline. By default, the parameter φ is set at 1, i.e. there are no exemptions. The emissions tax can be either set exogenously or be model-derived by imposing an emissions cap at either the country or regional level.[17] Notice that the emission tax is not an *ad valorem* tax, but a Pigouvian per unit tax.

$$XAT_{r,i} = \sum_{aa} \gamma^c_{r,i,aa} XA_{r,i,aa} \tag{T-1}$$

$$PA_{r,i,aa} = \left(1 + \tau^{Ap}_{r,i,aa}\right) \gamma^c_{r,i,aa} PAT_{r,i} \tag{T-2}$$

$$PA^a_{r,i,aa} = PA_{r,i,aa} + \sum_{em} \tau^{emi}_{r,em} \chi^e_{em} \varphi_{r,em,i,aa} \rho_{r,em,i,aa} \tag{T-3}$$

13 The GTAP data decomposes Armington demand into its domestic and import component by agent. Appendix C explains an alternative version of the Armington decomposition that allows for agent-specific behavior. Note that this increases the size of the model considerably.

14 The energy data, derived from the databases of the International Energy Agency (IEA), are expressed in millions of tons of oil equivalent (MTOE) across all energies, and thus prices are in base year dollars per unit of MTOE.

15 The γ^c parameter is initialized at 1 for all non-energy commodities. For energy commodities, it is initialized such that there is uniformity of energy prices in efficiency units.

16 The baseline calibration of the emission rates only affects non-CO_2greenhouse gases.

17 The model does not include equation (T-3) as it has been substituted throughout to minimize the creation of additional variables.

As mentioned above, the decomposition of the Armington aggregate, XAT, is done at the national level. Figure 3.7 illustrates the decomposition of Armington demand into demand for goods by region of origin. A nested structure is assumed. At the top level agents choose the optimal mix of domestic goods and an aggregate import bundle. The second nest decomposes the aggregate import bundle into demand for imports by region of origin. (The Armington equations are all indexed by im. The model allows for homogeneous traded commodities and these are indexed by ih.) Aggregate national demand for domestic goods, XDT^d, is then a fraction of XAT, with the fraction sensitive to the relative price of domestic goods, PD, to the Armington good, PAT—as shown in equation (T-4). The key parameter, known as the Armington substitution elasticity, is σ^m. The model allows for quality differences in the Armington composite goods using the γ^a and γ^t parameters. These in effect allow one to calibrate the CES functions in terms of value shares with the appropriate initialization of the respective γ. Equation (T-5) determines the demand for aggregate imports, XMT, which are further decomposed by trading partner (see below). The price of aggregate imports is tariff-inclusive. Finally, equation (T-6) defines the aggregate (or national) price of the aggregate Armington good, PAT.

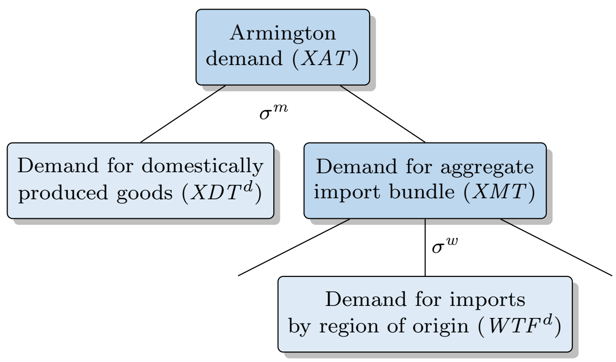


Figure 3.7: **Armington nest**

$$XDT^d_{r,im} = \alpha^d_{r,im} \left(\gamma^a_{r,im}\right)^{\sigma^m_{r,im}-1} \left(\frac{PAT_{r,im}}{PD_{r,im}}\right)^{\sigma^m_{r,im}} XAT_{r,im} \tag{T-4}$$

$$XMT_{r,im} = \alpha^m_{r,im} \left(\gamma^t_{r,im}\right)^{\sigma^m_{r,im}-1} \left(\frac{PAT_{r,im}}{PMT_{r,im}}\right)^{\sigma^m_{r,im}} XAT_{r,im} \tag{T-5}$$

$$PAT_{r,im} = \left[\alpha^d_{r,im} \left(\frac{PD_{r,im}}{\gamma^a_{r,im}}\right)^{1-\sigma^m_{r,im}} + \alpha^m_{r,im} \left(\frac{PMT_{r,im}}{\gamma^t_{r,im}}\right)^{1-\sigma^m_{r,im}}\right]^{\frac{1}{1-\sigma^m_{r,im}}} \tag{T-6}$$

3.6.2 Bilateral trade prices

Each bilateral trade flow is associated with four different prices:

1. PE represents the factory or farm gate price
2. WPE represents the FOB price, an export tax or subsidy induces a wedge between the producer price and the FOB price[18]

[18] The ENVISAGE model specification of export taxes is that they are an *ad valorem* tax on the producer price, thus an export subsidy is negative. An alternative formulation would be to specify the tax as a wedge between the world price and the domestic FOB price in which case the subsidy is measured as a positive wedge.

3. WPM represents the CIF price, international trade and transport margins introduce a wedge between the FOB and CIF price

4. PM represents the agent-price and includes the bilateral tariff

Equations (T-7) through (T-9) describe three of the prices associated with international trade, respectively WPE, WPM and PM (the determination of PE is described below). The first regional index, s, is the source or exporting region. The second regional index, d, is the destination or importing region. The respective wedges are represented by τ^e, the export tax/subsidy, τ^{tm}, the international transport margin, and τ^m the bilateral tariff. The price of a unit of international transport is uniform, irrespective of the transport node and sector.

$$WPE_{s,d,im} = \left(1 + \tau^e_{s,d,im}\right) PE_{s,d,im} \tag{T-7}$$

$$WPM_{s,d,im} = WPE_{s,d,im} + \tau^{tm}_{s,d,im} PWMG \tag{T-8}$$

$$PM_{s,d,im} = \left(1 + \tau^m_{s,d,im}\right) WPM_{s,d,im} \tag{T-9}$$

3.6.3 Second level Armington nest

The second nest in the Armington structure allocates aggregate import demand (across all agents) to specific regions of origin.[19] The bilateral trade flow will reflect preferences, the region of origin-specific export price and the bilateral tariff, τ^m. The price impacts are reflected in the tariff-inclusive bilateral price PM. Equation (T-10) defines import demand, WTF^d, by region r, sourced in region s. Equation (T-11) defines the aggregate import price, PMT. It is an aggregation of the tariff inclusive bilateral import price. All agents are assumed to face the same import price (net of the sales tax), i.e. implicitly we are assuming that the composition of the import bundle by each agent is identical.

$$WTF^d_{s,r,im} = \alpha^w_{s,r,im} \left(\gamma^m_{s,r,im}\right)^{\sigma^w_{r,im}-1} \left(\frac{PMT_{r,im}}{PM_{s,r,im}}\right)^{\sigma^w_{r,im}} XMT_{r,im} \tag{T-10}$$

$$PMT_{r,im} = \left[\sum_s \alpha^w_{s,r,im} \left(\frac{PM_{s,r,im}}{\gamma^m_{s,r,im}}\right)^{1-\sigma^w_{r,im}}\right]^{\frac{1}{1-\sigma^w_{r,im}}} \tag{T-11}$$

3.6.4 Allocation of domestic production to domestic and export markets

Analogous to the two-nested Armington specification described above, the ENVISAGE model allows for imperfect transformation of output across markets of destination—domestic and for export. A two-nested CET structure is implemented, see figure 3.8. At the top level, output is allocated between the domestic market and aggregate exports. At the next level, aggregate exports are allocated across various foreign markets. At either nest, infinite transformation is allowed in which case the CET first order conditions are replaced by the law of one price. The supply of international trade and transport services (XMG) is treated apart and is assumed to be priced at the average producer price, PP.

[19] Note that in either version of the top-level Armington decomposition—national or agent-specific—the decomposition of imports by region of origin is specified at the national level.

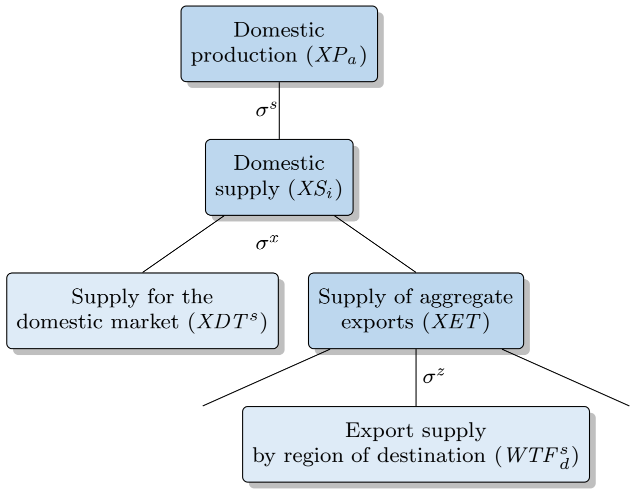


Figure 3.8: **Armington nest**

Equations (T-12) and (T-13) represent the derived supply for domestic, XDT^s, and aggregate export, XET, markets respectively. With finite transformation, these conditions are the standard CET first order conditions based on supply (less supply of international trade and transport services). With perfect transformation, each is replaced with the law of one price whereby the domestic, PD, and export, PET, producer price are set equal to the aggregate supply price, PS. Equation (T-14) represents the market equilibrium for supply. With perfect transformation domestic supply is equal to the sum of supply to the various markets—domestic, XDT, aggregate exports, XET and international trade and transport services, XMG. With finite transformation, the aggregation function is equal to the CET primal function. However, this can be replaced with the CET dual price function as is the case in equation (T-14). All equations allow for a component specific quality or efficiency factor, γ^d and γ^e.

$$\begin{cases} XDT^s_{r,im} = \gamma^{xd}_{r,im} \left(\frac{\gamma^d_{r,im} PD_{r,im}}{PS_{r,im}} \right)^{\sigma^x_{r,im}} \frac{XS_{r,im} - XMG_{r,im}}{\gamma^d_{r,im}} & \text{if} \quad \sigma^x_{r,im} \neq \infty \\ PD_{r,im} = \gamma^d_{r,im} PS_{r,im} & \text{if} \quad \sigma^x_{r,im} = \infty \end{cases} \tag{T-12}$$

$$\begin{cases} XET_{r,im} = \gamma^{xe}_{r,im} \left(\frac{\gamma^e_{r,im} PET_{r,im}}{PS_{r,im}} \right)^{\sigma^x_{r,im}} \frac{XS_{r,im} - XMG_{r,im}}{\gamma^e_{r,im}} & \text{if} \quad \sigma^x_{r,im} \neq \infty \\ PET_{r,im} = \gamma^e_{r,im} PS_{r,im} & \text{if} \quad \sigma^x_{r,im} = \infty \end{cases} \tag{T-13}$$

$$\begin{cases} PS_{r,im} = \left[\gamma^{xd}_{r,im} \left(\frac{PD_{r,im}}{\gamma^d_{r,im}} \right)^{1+\sigma^x_{r,im}} + \gamma^{xe}_{r,im} \left(\frac{PET_{r,im}}{\gamma^e_{r,im}} \right)^{1+\sigma^x_{r,im}} \right]^{\frac{1}{1+\sigma^x_{r,im}}} & \text{if} \quad \sigma^x_{r,im} \neq \infty \\ XS_{r,im} = \gamma^d_{r,im} XDT^s_{r,im} + \gamma^e_{r,im} XET^s_{r,im} + XMG_{r,im} & \text{if} \quad \sigma^x_{r,im} = \infty \end{cases} \tag{T-14}$$

Equations (T-15) and (T-16) reflect the second level CET nest allocating aggregate exports, XET, across various export markets as represented by WTF^s. With perfect transformation, the bilateral export producer price is equal to the aggregate export price, PET, and aggregate export supply is simply the sum across all export markets. With finite transformation, the CET first-order condition determines WTF^s and the aggregate export price is the CET aggregation of the regional export prices. Similar to the other trade equations, a quality or efficiency parameter is introduced that allows for prices to deviate from uniformity even with an infinite transformation elasticity.

$$\begin{cases} WTF^s_{r,d,im} = \gamma^{xw}_{r,d,im} \left(\frac{\gamma^w_{r,d,im} PE_{r,d,im}}{PET_{r,im}} \right)^{\sigma^z_{r,im}} \frac{XET_{r,im}}{\gamma^w_{r,d,im}} & \text{if} \quad \sigma^z_{r,im} \neq \infty \\ PE_{r,d,im} = \gamma^w_{r,d,im} PET_{r,im} & \text{if} \quad \sigma^z_{r,im} = \infty \end{cases} \tag{T-15}$$

$$\begin{cases} PET_{r,im} = \left[\sum_d \gamma^{xw}_{r,d,im} \left(\frac{PE_{r,d,im}}{\gamma^w_{r,d,im}} \right)^{1+\sigma^z_{r,im}} \right]^{\frac{1}{1+\sigma^z_{r,im}}} & \text{if} \quad \sigma^z_{r,im} \neq \infty \\ XET_{r,im} = \sum_d \gamma^w_{r,d,im} WTF^s_{r,d,im} & \text{if} \quad \sigma^z_{r,im} = \infty \end{cases} \tag{T-16}$$

3.6.5 Homogeneous traded goods

The model allows for homogeneous traded goods. In principle, none of the goods in GTAP can be treated immediately as homogeneous goods since there exists bilateral trade for all goods. In principle, some goods are nearly homogeneous since either imports or exports are so small that they could be ignored in an intermediate step that moves from the Armington specification to one based on net trade. It is also possible to introduce new commodities into the model as either Armington or homogeneous goods.

Equation (T-17) defines net trade, NT, for homogeneous goods defined over index ih. It is defined as a value and is the difference between domestic supply, XS, and domestic demand, XAT, evaluated at the world price, PW. Net trade is negative if demand exceeds supply. Equation (T-18) is the market equilibrium condition for homogeneous goods. At equilibrium, the sum of net trade across all countries must equal 0. Equation (T-19) determines the domestic price of homogeneous goods—it is equal to the world price adjusted by the tariff (that is no longer region of origin specific). Both supply and demand prices are equal as provided by equation (T-20).

$$NT_{r,ih} = PW_{ih} \left(XS_{r,ih} - XAT_{r,ih} \right) \tag{T-17}$$

$$\sum_r NT_{r,ih} = 0 \tag{T-18}$$

$$PS_{r,ih} = \left(1 + \tau^m_{r,ih} \right) PW_{ih} \tag{T-19}$$

$$PAT_{r,ih} = PS_{r,ih} \tag{T-20}$$

The next three equations are not strictly necessary for the model, but provide identities that can be useful. The first, equation (T-21), defines the volume of aggregate exports. It is specified as a mixed complementarity formula, or using an orthogonality condition. For the relation to hold, exports must be equal to net trade, or if net trade is negative, exports are set to zero, i.e. they must never fall below zero. The second equation (T-22), almost identical, defines the aggregate volume of imports. If exports are positive, XMT will be zero, else, imports are equal to the negative of net trade and will be positive. The third is a definition of a world price for Armington goods, and is a weighted global average of domestic supply prices, PS.

$$\left(PW_{ih} XET_{r,ih} - NT_{r,ih} \right) XET_{r,ih} = 0 \perp XET_{r,ih} \geq 0 \tag{T-21}$$

$$NT_{r,ih} = PW_{ih} \left(XET_{r,ih} - XMT_{r,ih} \right) \tag{T-22}$$

$$PW_{im} = \sum_r \varphi^w_{r,im} PS_{r,im} \tag{T-23}$$

3.6.6 Domestic supply

The model allows for multi-output production activities (for example producing ethanol and DDGS from ethanol production) and the aggregation of goods produced by multiple activities into a single commodity (for example different streams of electrical production—coal, gas, hydro, nuclear, renewables, etc.—each with its own cost structure, but combined by a distributor into a single commodity).

Multi-output production

Activity a can therefore produce a suite of commodities indexed by i, hence output at this level is indexed by both a and i, $X_{a,i}$.[20] This is implemented using a CET structure with the possibility of infinite transformation. Equation (T-24) defines the supply of $X_{a,i}$ emanating from activity a (or XP_a), where the law of one price holds in the case of a finite transformation. Equation (T-25) represents the zero profit condition, or the revenue balance for the multi-output production function.

$$\begin{cases} X_{r,a,i} = \gamma^p_{r,a,i} \left(\frac{P_{r,a,i}}{PP_{r,a}}\right)^{\omega^s_{r,a}} XP_{r,a} & \text{if} \quad \omega^s_{r,a} \neq \infty \\ P_{r,a,i} = PP_{r,a} & \text{if} \quad \omega^s_{r,a} = \infty \end{cases} \tag{T-24}$$

$$PP_{r,a} XP_{r,a} = \sum_{i \in \{\gamma^p_{r,a,i} \neq 0\}} P_{r,a,i} X_{r,a,i} \tag{T-25}$$

Domestic supply of non-electric commodities

The aggregation of commodities produced by multiple activities uses a single nested CES function for all commodities with the exception of electricity. The aggregate supply of the electric commodity is described in the next section.

For the non-electric commodities, multiple streams of output can be combined into a single supplied commodity, XS_i, with a CES aggregator, or preference function. The specification allows for homogeneous goods in which case the cost of each component must be equal, subject perhaps to an efficiency or quality differential. Equation (T-26) determines the demand for produced commodity X. In the case of a finite elasticity it is a CES formulation. With an infinite substitution elasticity, the law-of-one price must hold, i.e. the producer price of each component must be equalized in efficiency units. Equation (T-27) determines the equilibrium condition in the form of the cost function equality.

$$\begin{cases} X_{r,a,i} = \alpha^s_{r,a,i} \left(\gamma^s_{r,a}\right)^{\sigma^s_{r,i}-1} \left(\frac{PS_{r,i}}{P_{r,a,i}}\right)^{\sigma^s_{r,i}} XS_{r,i} & \text{if} \quad \sigma^s_{r,i} \neq \infty \\ P_{r,a,i} = \gamma^s_{r,a} PS_{r,i} & \text{if} \quad \sigma^s_{r,i} = \infty \end{cases} \tag{T-26}$$

$$PS_{r,i} XS_{r,i} = \sum_{a \in \{\alpha^s_{r,a,i} \neq 0\}} P_{r,a,i} X_{r,a,i} \tag{T-27}$$

[20] In the GTAP database this will be represented by a diagonal matrix where each activity produces one and only one good.

Domestic supply of electricity

The bundling of electricity uses a nested CES structure instead of a single nest, see figure 3.9. The top nest combines aggregate power supply with distribution and transmission services to form aggregate domestic electricity supply. Aggregate power supply is composed of a user-determined number of power bundles to which each power source is mapped. A nested approach is used. The first level nest disaggregates the aggregate power bundle into one or more user-defined sub-power bundles. The second level nest decomposes a subset of the power bundles into additional power bundles. The other power bundles in the first nest will be decomposed directly into specific power activities. The bottom-level power bundles are decomposed into power activities.

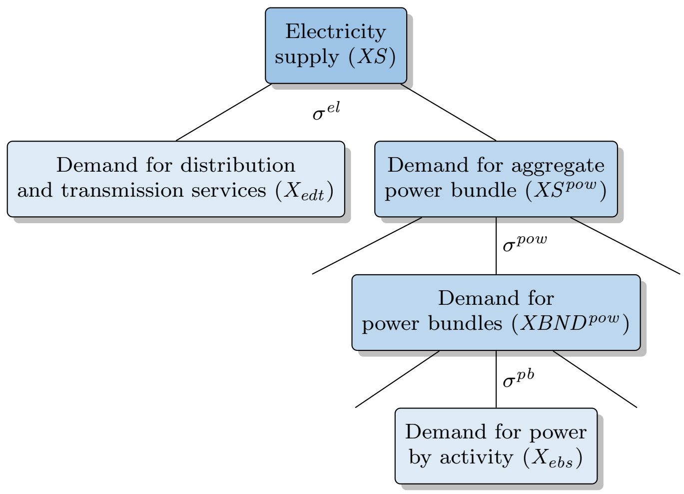


Figure 3.9: **CES nest for power bundle**

Equation (T-28) determines the demand for electricity distribution and transmission services—indexed by activities *edt*—used to produce one or more electric commodities—indexed by *ely*.[21] It is linked to the total supply of power, XS^{pow}, in a CES bundle. The normal specification assumes a Leontief technology, i.e. a substitution elasticity of zero. Equation (T-29) determines the demand for the power bundle, it is a bundle of all electricity generation, and excludes the distribution and transmission services. Equation (T-30) determines the supply price of aggregate electricity.

$$X_{r,edt,ely} = \alpha^s_{r,edt,ely} \left(\frac{PS_{r,ely}}{P_{r,edt,ely}} \right)^{\sigma^{el}_{r,ely}} XS_{r,ely} \tag{T-28}$$

$$XS^{pow}_{r,ely} = \alpha^{pow}_{r,ely} \left(\frac{PS_{r,ely}}{P^{pow}_{r,ely}} \right)^{\sigma^{el}_{r,ely}} XS_{r,ely} \tag{T-29}$$

$$PS_{r,ely} = \left[\alpha^s_{r,edt,ely} \left(P_{r,edt,ely} \right)^{1-\sigma^{el}_{r,ely}} + \alpha^{pow}_{r,ely} \left(P^{pow}_{r,ely} \right)^{1-\sigma^{el}_{r,ely}} \right]^{1/(1-\sigma^{el}_{r,ely})} \tag{T-30}$$

The following stage decomposes aggregate demand for power into a user determined number of power bundles. Many configurations are possible. The one most commonly used in Envisage includes two power bundles at this stage—one called 'CNV' that represents conventional power technologies and is itself a power bundle composed of coal-fired, oil-fired, gas-fired, nuclear, and

[21] Typically there is a single distribution and transmission activity and a single electricity commodity.

hydro-power bundles. The second is a bundle of renewable technologies—wind, solar and other renewables. The latter is a bottom-level power bundle, i.e. it is directly mapped to specific power activities.[22] Equation (T-31) determines the demand for the first level power bundles, these are the power bundles indexed by *pb1*. Notice that the aggregate price used in the expression is PN^{pow} and not P^{pow}. The latter is the aggregate, or average price of the power bundle. The former is a price index that is defined in equation (T-32). In the standard CES, the two price concepts are identical. The power decomposition uses the so-called adjusted CES, which preserves the additivity of the CES components (see Annex A). The demand expressions in both versions of the CES are similar. However, the expression for the aggregate price index differs and the price index is not equal to the average price (as calculated using the zero profit condition). Thus, equation (T-33) is added that evaluates the average price, P^{pow}.

$$XBND^{pow}_{r,pb1,ely} = \alpha^{powbnd}_{r,pb1,ely} \left(\frac{PN^{pow}_{r,ely}}{\lambda^{powbnd}_{r,pb1,ely} PBND^{pow}_{r,pb1,ely}} \right)^{\sigma^{pow1}_{r,ely}} XS^{pow}_{r,ely} \tag{T-31}$$

$$PN^{pow}_{r,ely} = \left[\sum_{pb1} \alpha^{powbnd}_{r,pb1,ely} \left(\lambda^{powbnd}_{r,pb1,ely} PBND^{pow}_{r,pb1,ely} \right)^{-\sigma^{pow1}_{r,ely}} \right]^{-1/\sigma^{pow1}_{r,ely}} \tag{T-32}$$

$$P^{pow}_{r,ely} XS^{pow}_{r,ely} = \sum_{pb1} PBND^{pow}_{r,pb1,ely} XBND^{pow}_{r,pb1,ely} \tag{T-33}$$

The next set of equations deals with the intermediate power bundles that are composed of other power bundles. These intermediate power bundles are indexed by *pb1b*, i.e. they are a subset of *pb1* and are also formed of other power bundles indexed by *pb2*. Equation (T-34) determines the demand for the second level power bundles, these are the power bundles indexed by *pb2*. Each power bundle *pb2* is mapped to one, and only one, first level power bundle indexed by *pb1b*. Equation (T-35) defines the adjusted CES price index for the power bundles indexed by *pb1b*. And, equation (T-36) determines the relative aggregate price, $PBND^{pow}$.

$$XBND^{pow}_{r,pb2,ely} = \alpha^{powbnd}_{r,pb2,ely} \left(\frac{PBNDN^{pow}_{r,pb1b,ely}}{\lambda^{powbnd}_{r,pb2,ely} PBND^{pow}_{r,pb2,ely}} \right)^{\sigma^{pow2}_{r,pb1b,ely}} XBND^{pow}_{r,pb1b,ely} \quad \text{if } pb2 \in \{pb1b\} \tag{T-34}$$

$$PBNDN^{pow}_{r,pb1b,ely} = \left[\sum_{pb2 \in \{pb1b\}} \alpha^{powbnd}_{r,pb2,ely} \left(\lambda^{powbnd}_{r,pb2,ely} PBND^{pow}_{r,pb2,ely} \right)^{-\sigma^{pow2}_{r,pb1b,ely}} \right]^{-1/\sigma^{pow2}_{r,pb1b,ely}} \tag{T-35}$$

$$PBND^{pow}_{r,pb1b,ely} XBND^{pow}_{r,pb1b,ely} = \sum_{pb2 \in \{pb1b\}} PBND^{pow}_{r,pb2,ely} XBND^{pow}_{r,pb2,ely} \tag{T-36}$$

The subsequent nests decompose the various power bundles into their components, which is the output of the various power activities. Each power activity is mapped to one, and only one, power bundle. Equation (T-37) determines the demand for the power generated by activity *ebs*, where activity *ebs* is mapped to power bundle *pba*. Equations (T-38) and (T-39) determine respectively

[22] An earlier version of the power bundle included peak versus base load. This is still possible in the current configuration as it is possible to define two power bundles, base and peak, and map the individual activities to the two power bundles.

the power bundle price index using the adjusted CES price expression and the average price of the power bundle using the zero profit condition.

$$X_{r,ebs,ely} = \alpha^s_{r,ebs,ely} \left(\frac{PBNDN^{pow}_{r,pba,ely}}{P_{r,ebs,ely}} \right)^{\sigma^{pba}_{r,pba,ely}} XBND^{pow}_{r,pba,ely} \quad \text{if } ebs \in pba \tag{T-37}$$

$$PBNDN^{pow}_{r,pba,ely} = \left[\sum_{ebs \in \{pba\}} \alpha^s_{r,ebs,ely} \left(P_{r,ebs,ely} \right)^{-\sigma^{pba}_{r,pba,ely}} \right]^{-1/\sigma^{pba}_{r,pba,ely}} \tag{T-38}$$

$$PBND^{pow}_{r,pba,ely} XBND^{pow}_{r,pba,ely} = \sum_{ebs \in \{pba\}} P_{r,ebs,ely} X_{r,ebs,ely} \tag{T-39}$$

The sample GAMS code provided in listing (3.1) shows an example of a power nesting. This configuration of the GTAP power database uses an aggregation of 8 of the potential power sources (aggregating away the distinction between base and peak load generation), and maintains the electricity distribution and transmission activity. In addition, 3 additional power sources are included—coal- and gas-powered generation with carbon capture and storage (CCS) and an 'advanced nuclear' option. These are introduced with very lower penetration shares initially and with a given cost structure typically higher than their conventional counterparts. Thus there are a total of 11 power generation activities and one transmission and distribution activity. The subset *ebs* refers to the generation activities. The set *pb* defines all possible power bundles. Some may be composed of other power bundles (i.e. they are intermediate power bundles) and the others will be composed directly of power generation activities. The set *pb1* defines the intermediate power bundles—of which there are only two: conventional power and a renewable bundle. The former is composed of other power bundles. The latter is composed of renewable power activities. Hence, there is only one intermediate power bundle, indexed by *pb1b*, referring to the bundle 'CNV'. The set *pb2* defines the power bundles that are mapped to intermediate power bundles. And, the mapping `mappow1` provides the mapping for the 'CNV' bundle, i.e. the mapping for *pb1b* relative to the *pb2* bundles. The set *pba* refers to the bottom-level power bundles. All of these bundles are composed of one or more power activities. The mapping `mappow` provides the mapping of the 11 power generation activities to the bottom-level power bundles. In the configuration described in the listing, the coal power bundle is composed of two power activities—conventional coal and coal with CCS, similarly the gas power bundle is composed of conventional gas power and gas with CCS, and the nuclear power bundle is composed of conventional nuclear and the so-called advanced nuclear power. The renewable power bundle is made up of wind, solar and other renewable power. The other power bundles contain only one activity each.

The purpose of this configuration is three-fold. First, it allows to change the 'preferences' for renewable power independently of the preference for the other technologies. Second, it allows to change the preference for oil-powered electricity within the conventional bundle.[23]. Third it allows for introducing and developing new technologies as close substitutes to existing technologies. To conclude, the implementation of the power nesting provides the user with significant flexibility.

Listing 3.1: Example of a power nesting in GAMS

```
set elya(a) "Electicity activities" /
   clp-a      "Coal-base power"
```

[23] We assume a phase-out of oil-powered electricity.

```
   olp-a      "Oil-base power"
   gsp-a      "Gas-base power"
   nuc-a      "Nuclear power"
   hyp-a      "Hydro power"
   wnd-a      "Wind power"
   sol-a      "Solar power"
   xel-a      "Other power"
   ccs-a      "Coal based CCS"
   gcs-a      "Gas based CCS"
   adv-a      "Advanced nuclear"
   edt-a      "Electricity distribution and transmission"
/ ;

set ebs(elya) "Base power sources" /
   clp-a      "Coal-base power"
   olp-a      "Oil-base power"
   gsp-a      "Gas-base power"
   nuc-a      "Nuclear power"
   hyp-a      "Hydro power"
   wnd-a      "Wind power"
   sol-a      "Solar power"
   xel-a      "Other power"
   ccs-a      "Coal based CCS"
   gcs-a      "Gas based CCS"
   adv-a      "Advanced nuclear"
/ ;

set pb "Power bundles" /
   cnv        "Conventional power"
   coa        "Coal power"
   oil        "Oil power"
   gas        "Gas power"
   nuc        "Nuclear"
   hyd        "Hydro"
   ren        "Renewables"
/ ;

set pb1(pb) "Top level power bundles" /
   cnv         "Conventional power"
   ren         "Renewables"
/ ;

set pb1b(pb)  "Top level power bundle composed of other power bundles" /
   cnv         "Conventional power"
/ ;

set pba(pb) "Power bundles composed of activities" /
   coa        "Coal power"
   oil        "Oil power"
   gas        "Gas power"
   nuc        "Nuclear"
   hyd        "Hydro"
   ren        "Renewables"
/ ;

set pb2(pb) "Second level power bundles" /
   coa        "Coal power"
   oil        "Oil power"
   gas        "Gas power"
   nuc        "Nuclear"
   hyd        "Hydro"
/ ;

set mapPow1(pb1b, pb2) /
   cnv.(coa, oil, gas, nuc, hyd)
/ ;

set mapPow(pb, ebs) "Power bundle mapping" /
```

```
    coa         .clp—a
    coa         .ccs—a
    oil         .olp—a
    gas         .gsp—a
    gas         .gcs—a
    nuc         .nuc—a
    nuc         .adv—a
    hyd         .hyp—a
    ren         .wnd—a
    ren         .xel—a
    ren         .sol—a
/ ;
```

3.6.7 International trade and transport services

The global demand for international trade and transport services will be driven by the overall level of trade. Its allocation across suppliers is specified as a CES function where demand (partially) adjusts to low-cost suppliers. Within each region, production of these services is given by a CES technology.

Equation (T-40) determines the global demand for international trade and transport services, $XWMG$.[24] Regional supply of these services, $XTMG$, is determined in equation (T-41), the CES first order conditions. The global price, $PWMG$, is given in equation (T-42), the CES dual price formula. The regional supply price, $PTMG$, is given in equation (T-43). And the sectoral and regional supply of these services, XMG, is given in equation (T-44).

$$PWMG.XWMG = \sum_s \sum_d \sum_{im} \left(WPM_{s,d,im} - WPE_{s,d,im} \right) WTF^s_{s,d,im} \tag{T-40}$$

$$XTMG_r = \alpha_r^{tmg} \left(\frac{PWMG}{PTMG_r} \right)^{\sigma^t} XWMG \tag{T-41}$$

$$PWMG = \left[\sum_r \alpha_r^{tmg} PTMG_r^{1-\sigma^t} \right]^{1/(1-\sigma^t)} \tag{T-42}$$

$$PTMG_r = \left[\sum_i \alpha_{r,i}^{mg} PP_{r,i}^{1-\sigma^{rt}} \right]^{1/(1-\sigma^{rt})} \tag{T-43}$$

$$XMG_{r,i} = \alpha_{r,i}^{mg} \left(\frac{PTMG_r}{PP_{r,i}} \right)^{\sigma^{rt}} XTMG_r \tag{T-44}$$

3.7 Product market equilibrium

The model has only two 'basic' commodities—domestically produced goods for the domestic market, XDT, and bilateral exports, WTF. All other goods are composite goods. Equations (E-1) and (E-2) determine the equilibrium price for these two sets of goods, respectively PD and PE. With perfect transformation (at both levels), the true goods market equilibrium price is PS and equation (T-14)

[24] Note that the current formulation assumes that homogeneous goods are transported at no cost internationally.

is the market equilibrium condition. In the model implementation, the equilibrium conditions (E-1) and (E-2) are substituted out.

$$XDT^d_{r,i} = XDT^s_{r,i} \tag{E-1}$$

$$WTD^d_{s,d,i} = WTD^s_{s,d,i} \tag{E-2}$$

3.8 Factor market equilibrium

The GTAP database has five factors of production—unskilled and skilled labor[25], capital, land and natural resources (or sector-specific factors: forestry, fishing, coal, oil, natural gas and other mining). The next sections describe factor market equilibrium for these factors. The first describes a resource with a national market—with no, partial or full mobility. In the standard version of Envisage, this covers only the land markets. Labor markets are covered separately. The model allows for labor market segmentation where the rural and urban markets clear separately and with the existence of a Harris-Todaro type rural to urban migration function. Natural resources have a supply curve under various assumptions. Finally, the capital market is handled apart—partially to implement the vintage capital structure.

3.8.1 The market for land

A nested structure is used to allocate land, by type, across the different activities that use land. At the top most level, aggregate land by type, *TLand*, is determined using a price sensitive supply function. Three specifications are implemented. The first is a simple constant elasticity supply function that might be a good local approximation but could pose problems in dynamic simulations where land prices rise significantly. The other two functional forms have an explicit asymptote that does not allow for land supply to increase beyond an exogenous upper supply limit, *LandMax*. The second specification uses a logistic function, and the third uses a hyperbolic (as used in the LEITAP/MAGNET) family of models. Equation (F-1) determines aggregate land supply by type lt according to one of the three specifications. (N.B. The *LandMax* parameter was initially calibrated to the FAO/GAEZ database at the aggregate level, i.e. for a single aggregate land-type. This parameter is assumed to hold for all land-types (as the maximum percent increase), but will be modified in the future to correspond to the 18 individual AEZs.)

$$TLand_{r,lt} = \begin{cases} \alpha^{tl}_{r,lt}\left(\dfrac{PTLand_{r,lt}}{PGDPMP_r}\right)^{\varepsilon^f_{r,lt}} & \text{if } LandMax_{r,lt}{=}\infty \\ \dfrac{LandMax_{r,lt}}{1+\alpha^{tl}_{r,lt}e^{-\gamma^{tl}_{r,lt}(PTLand_{r,lt}/PGDPMP_r)}} & \text{if } LandMax_{r,lt}{\neq}\infty,\ ifLogLand{=}1 \\ LandMax_{r,lt} - \alpha^{tl}_{r,lt}\left(\dfrac{PTLand_{r,lt}}{PGDPMP_r}\right)^{-\gamma^{tl}_{r,lt}} & \text{if } LandMax_{r,lt}{\neq}\infty,\ ifLogLand{=}0 \end{cases} \tag{F-1}$$

We have seen above on the production side that each land-using activity demands land of type lt. The allocation of land supply across activities assumes some friction in land transformability. For example, it may be difficult to convert wheat land into rice land. A nested CET structure is used to determine the allocation of aggregate land, *TLand*, across the various land-using activities

[25] Starting with release 9, there will be five labor categories—2 of which are associated with skilled labor and the other 3 with unskilled labor

indexed by a. Land-using activities are divided into three fundamental land-types, $Land_{ltb}$, indexed by ltb. For a single nested CET, all land-using activities could be allocated to the first land bundle, $Land_{lt1}$, and the other two land bundles would be redundant. In the case of the LEITAP/MAGNET model, the first bundle is composed of fruits and vegetables, other crops and other livestock. The second bundle consists of sugar and pasture land (for ruminant production). And the third bundle consists of cereals. The GTAP AEZ model has a different nesting. The first bundle consists of only forestry.[26] The second bundle consists of pasture (for ruminant production). And the third bundle consists of all crops (not just cereals as is the case for the LEITAP/MAGNET specification.) To add additional flexibility and for more transparency, the implementation of these CET nests includes some additional nests compared to either the LEITAP/MAGNET of GTAP/AEZ specifications. This requires the introduction of an intermediate land bundle, $LndBnd$, and additional CET elasticities, but that can be easily initialized to replicate the specifications of the two aforementioned models. Figure 3.10 provides an illustration of the generic land allocation CET nesting.

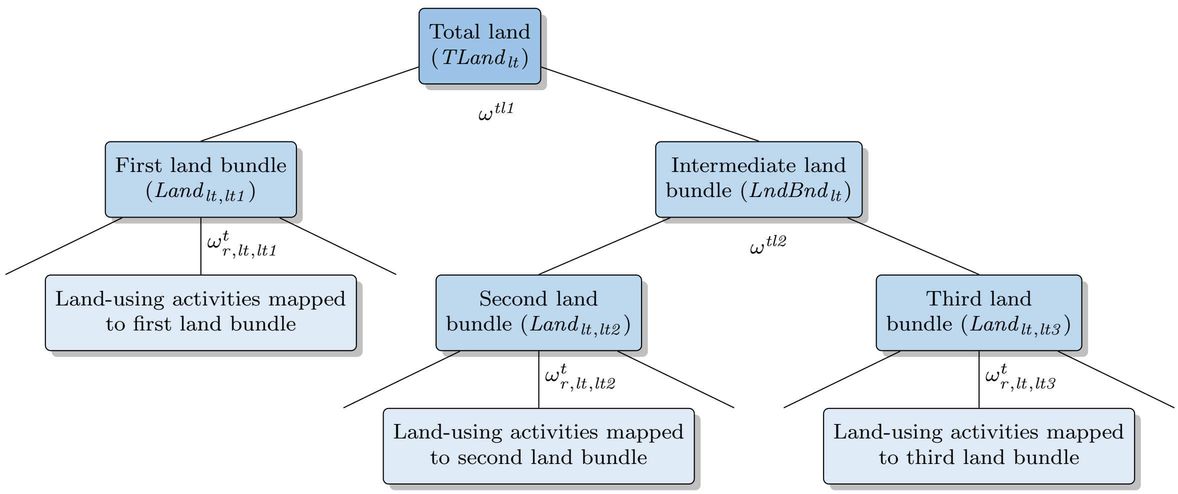


Figure 3.10: **Land allocation nesting**

The aggregate land bundle, $TLand$, is thus first decomposed into the first land bundle, $Land_{lt1}$, and the intermediate land bundle, $LndBnd$, with a CET transformation elasticity of ω^{tl1}. The supply functions are provided in equations (F-2) and (F-3). The revenue consistency equation is given by equation (F-4) and could be replaced by the CET dual price expression (in the case of a finite transformation). The specification allows for perfect transformation in which case the law-of-one-price must hold and the prices of the component bundles must equal the price of the aggregate bundle.

$$\begin{cases} Land_{r,lt,lt1} = \alpha^{Land}_{r,lt,lt1} \left(\frac{PLand_{r,lt,lt1}}{PTLand_{r,lt}}\right)^{\omega^{tl1}_{r,lt}} TLand_{r,lt} & \text{if } \omega^{tl1}_{r,lt} \neq \infty \\ PLand_{r,lt,lt1} = PTLand_{r,lt} & \text{if } \omega^{tl1}_{r,lt} = \infty \end{cases} \tag{F-2}$$

$$\begin{cases} LndBnd_{r,lt} = \alpha^{LndBnd}_{r,lt} \left(\frac{PLndBnd_{r,lt}}{PTLand_{r,lt}}\right)^{\omega^{tl1}_{r,lt}} TLand_{r,lt} & \text{if } \omega^{tl1}_{r,lt} \neq \infty \\ PLndBnd_{r,lt} = PTLand_{r,lt} & \text{if } \omega^{tl1}_{r,lt} = \infty \end{cases} \tag{F-3}$$

[26] Note that in the AEZ database, payments to natural resources in the forestry sector are converted to land payments (after adjustments).

$$PTLand_{r,lt}\,TLand_{r,lt} = PLand_{r,lt,lt1}\,Land_{r,lt,lt1} + PLndBnd_{r,lt}LndBnd_{r,lt} \tag{F-4}$$

In the next node of the CET nest, the intermediate bundle, $LndBnd$, is decomposed into the other two land bundles, with a transformation elasticity given by ω^{tl2}. The supply equations for the bundles are provided in equations (F-5) and (F-6) with the revenue consistency equation given by equation (F-7).

$$\begin{cases} Land_{r,lt,lt2} = \alpha^{Land}_{r,lt,lt12}\left(\frac{PLand_{r,lt,lt2}}{PLndBnd_{r,lt}}\right)^{\omega^{tl2}_{r,lt}} LndBnd_{r,lt} & \text{if } \omega^{tl2}_{r,lt} \neq \infty \\ PLand_{r,lt,lt2} = PLndBnd_{r,lt} & \text{if } \omega^{tl2}_{r,lt} = \infty \end{cases} \tag{F-5}$$

$$\begin{cases} Land_{r,lt,lt3} = \alpha^{Land}_{r,lt,lt13}\left(\frac{PLand_{r,lt,lt3}}{PLndBnd_{r,lt}}\right)^{\omega^{tl2}_{r,lt}} LndBnd_{r,lt} & \text{if } \omega^{tl2}_{r,lt} \neq \infty \\ PLand_{r,lt,lt3} = PLndBnd_{r,lt} & \text{if } \omega^{tl2}_{r,lt} = \infty \end{cases} \tag{F-6}$$

$$PLndBnd_{r,lt}\,LndBnd_{r,lt} = PLand_{r,lt,lt2}\,Land_{r,lt,lt2} + PLand_{r,lt,lt3}\,Land_{r,lt,lt3} \tag{F-7}$$

The final equations in the decomposition of the land bundle allocate land supply to the different land-using activities based on their mappings to one of the three land bundles. The user-cost of land of type lt in activity a is given by $PLnd$, and the supplier cost is given by $NPLnd$. Equation (F-8) determines the supply of land to activity a. (N.B. The land equations—on both the demand and supply side—integrate the market equilibrium condition, hence there is no special notation for the demand and supply variables.) Each of the land-using activities a is mapped to one, and only one of the three land bundles indexed by ltb. Equation (F-9) is the standard revenue consistency equation for the CET function. As mentioned earlier, the LEITAP/MAGNET model maps fruits and vegetables, other crops and other livestock to the first bundle, $lt1$. LEITAP/MAGNET does not actually implement the bundle, instead these activities are mapped directly to the top level land aggregate. The implication of this is that the second level CET elasticity for bundle $lt1$ is equal to the top level CET elasticity:

$$\omega^{t}_{r,lt,lt1} = \omega^{tl1}_{r,lt}$$

Similarly, the second level nest contains the sugar and pasture activities. This implies that the third level elasticity for bundle $lt2$ is equal to the second level CET elasticity of the intermediate land bundle:

$$\omega^{t}_{r,lt,lt2} = \omega^{tl2}_{r,lt}$$

$$\begin{cases} Lnd_{r,lt,a} = \alpha^{s,Lnd}_{r,lt,a}\left(\frac{NPLnd_{r,lt,a}}{PLand_{r,lt,ltb}}\right)^{\omega^{t}_{r,lt,ltb}} Land_{r,lt,ltb} & \text{if } \omega^{t}_{r,lt,ltb} \neq \infty,\ a \in ltb \\ NPLnd_{r,lt,a} = PLand_{r,lt,ltb} & \text{if } \omega^{t}_{r,lt,ltb} = \infty,\ a \in ltb \end{cases} \tag{F-8}$$

$$PLand_{r,lt,ltb}Land_{r,lt,ltb} = \sum_{a\in ltb} NPLnd_{r,lt,a}\chi^{lnd}_{r,lt,a}Lnd_{r,lt,a} \tag{F-9}$$

3.8.2 Labor markets

In the standard Envisage model, labor markets clear nationally with an economy-wide wage rate equating supply and aggregate demand—separately for both skilled and unskilled labor. The model does not allow for international migration. An alternative version of the model allows for national labor market segmentation with a Harris-Todaro type migration function from rural to urban activities.[27] Due to data limitations, rural activities are equated with agricultural sectors and urban activities with all other sectors.

Sectoral labor demand across sectors (indexed by a) is determined by the production function in each sector. Sectors are segmented into two 'zones'—rural and urban, indexed by z. The basic idea behind Harris-Todaro is that migration is a function of the ratio of the urban wage to the rural wage. Equation (F-10) defines the average wage in each zone z, W^a. It is equal to total nominal labor remuneration in each zone divided by total volume demand (in person-years for example). Equation (F-11) then determines the level of migration from rural to urban zones, MG, as a function of the ratio of the nominal average wage in each zone (potentially adjusted for unemployment, i.e. the expected average wage), subject to a migration elasticity (ω^m), where χ^m is a calibrated shift parameter. Equation (F-12) determines the zone-specific labor supply, L^s. It is equated to the previous period's labor supply adjusted by a zone-specific (and exogenous) labor supply growth rate and adjusted for migration. The parameter δ_z is equal to -1 for the rural zone and equal to +1 for the urban zone. In the case of no labor market segmentation, MG is equal to zero.[28] Equation (F-13) represents the equilibrium condition for the two possible specifications. The top equation equates supply by zone to demand by zone (under the assumption of full employment) with segmented markets. The bottom equation holds for the case with a nationally integrated labor market. Finally, equation (F-14) sets the sectoral wage. With segmented markets it is equal to the equilibrium wage in the relevant zone—potentially adjusted by a sector-specific wage premium that allows for inter-sectoral wage differences. With national markets, it is equal to the national equilibrium wage rate with again the possibility of a wage premium.

$$W^a_{r,l,z} = \frac{\sum\limits_{a \in z} NPF_{r,l,a} XF_{r,l,a}}{\sum\limits_{a \in z} XF_{r,l,a}} \tag{F-10}$$

$$MG_{r,l} = \chi^m_{r,l} \left[\frac{\left(1 - UE_{r,l,Urb}\right) W^a_{r,l,Urb}}{\left(1 - UE_{r,l,Rur}\right) W^a_{r,l,Rur}} \right]^{\omega^m_{r,l}} \quad \text{if } \omega^m_{r,l} \neq \infty \tag{F-11}$$

$$L^s_{r,l,z,t} = \left(1 + g^l_{r,l,z,t}\right) L^s_{r,l,z,t-1} + \delta_z MG_{r,l,t} \tag{F-12}$$

$$\begin{cases} L^s_{r,l,z} = \sum\limits_{a \in z} XF_{r,l,a} & \text{if } \omega^m_{r,l} \neq \infty \\ \sum\limits_{z} L^s_{r,l,z} = \sum\limits_{a} XF_{r,l,a} & \text{if } \omega^m_{r,l} = \infty \end{cases} \tag{F-13}$$

$$\begin{cases} NPF_{r,l,a} = \pi^w_{r,l,a} W^{ez}_{r,l,z} & \text{if } \omega^m_{r,l} \neq \infty, a \in z \\ NPF_{r,l,a} = \pi^w_{r,l,a} W^e_{r,l} & \text{if } \omega^m_{r,l} = \infty \end{cases} \tag{F-14}$$

27 See Harris and Todaro (1970).

28 Appendix E describes how model equations are adjusted for inter-period gaps of greater than one year.

3.8.3 Market for sector-specific factors

The sector specific factor—normally the natural resource base in natural resource sectors—is handled using an upward sloping supply curve with the elasticity given by ε^{ff} or by a logistic function with a specified maximum supply, see equation (F-15).[29] If the former is infinite, the return to the sector specific factor is assumed to rise at the same rate as the GDP deflator, see equation (F-16), else it is determined by market equilibrium. The finite supply curve has three shifters. The first, α^{fs}, is calibrated with base year data. The second, α^{rfs}, can be calibrated in a dynamic scenario to target a region specific variable, for example output or the regional producer price. The third, α^{gfs}, can be calibrated in a dynamic scenario to target a global variable, for example global output or the global price. In this case, the shifter moves each country/regional supply curve by the same proportional amount.

$$\begin{cases} XF^s_{r,a} = \alpha^{rfs}_{r,a}\alpha^{gfs}_{a}\alpha^{fs}_{r,NatRs,a}\left(\dfrac{NPF_{r,NatRs,a}}{PGDPMP_r}\right)^{\varepsilon^{ff}_{r,a}} & \text{if } \varepsilon^{ff}_{r,a} \neq \infty, XF^{max}_{r,a} = \infty \\ XF^s_{r,a} = \dfrac{XF^{max}_{r,a}}{1+\alpha^{fs}_{r,NatRs,a}e^{-\gamma^{fs}_{r,a}(NPF_{r,NatRs,a}/PGDPMP_r)}} & \text{if } \varepsilon^{ff}_{r,a} \neq \infty, XF^{max}_{r,a} \neq \infty \end{cases} \tag{F-15}$$

$$\begin{cases} XF^s_{r,a} = XF_{r,NatRs,a} & \text{if } \varepsilon^{ff}_{r,a} \neq \infty \\ NPF_{r,NatRs,a,t} = PGDPMP_r NPF_{r,NatRs,a,t-1} & \text{if } \varepsilon^{ff}_{r,a} = \infty \end{cases} \tag{F-16}$$

3.8.4 Depletion modules for fossil fuel resources

The first part of this section describes the core functioning for the resource depletion modules for the fossil fuel extraction sectors. Reserves are divided into two pools—proven and unproven, or yet-to-discover reserves. Extraction is done at a rate of ρ^x from proven reserves, Res:

$$XF^p = \rho^x Res$$

where XF^p is potential extraction, not necessarily actual extraction. Reserves evolve according to the following motion equation:

$$Res_t = Res_{t-1} - XF_{t-1} + \xi^x YTFR_{t-1}$$

i.e. the level of reserves at the beginning of time period t is equal to the previous period's reserve level, less actual extraction, XF, plus additions to reserves, i.e. the conversion of yet-to-discover reserves into proven reserves. The discovery rate, ξ^x, will be sensitive to the output price of the relevant fossil fuel.If the actual level of extraction is equal to the potential rate of extraction we have:

$$Res_t = (1-\rho^x_{t-1})Res_{t-1} + \xi^x_{t-1} YTFR_{t-1} \iff XF_{t-1} = XF^p_{t-1} = \rho^x_t Res_t$$

Assuming extraction equals potential extraction in all time periods, the equation above can be converted for multi-year time steps to the following:

$$Res_t = (1-\rho^x)^n Res_{t-n} + \frac{(1-\xi^x)^n - (1-\rho^x)^n}{\rho^x - \xi^x}\xi^x YTFR_{t-n}$$

[29] A future version of the model will see development of a resource depletion module for natural gas and crude oil.

though takes the following expression when the two rates are equal:

$$Res_t = (1-\rho^x)^n Res_{t-n} + n\,(1-\rho^x)^{(n-1)}\,\xi^x\,YTFR_{t-n}$$

If actual extraction is less than potential, the motion equation for reserves is derived from the second equation above:

$$Res_t = Res_{t-n} + (1-(1-\xi_t^x)^n)\;YTFR_{t-n} - CXF_t$$

where CFX represents cumulative extraction between period $t-n$ and $t-1$ and is evaluated by the following expression:

$$CXF_t = \frac{XF_t - XF_{t-n}}{(XF_t/XF_{t-n})^{(1/n)} - 1}$$

where the term in the denominator represents the average rate of growth of actual extraction between $t-n$ and t.[30]

The yet-to-find profile derives from the following motion equation:

$$YTFR_t = (1-\xi^x)\;YTFR_{t-1}$$

that in a multi-period time step converts to:

$$YTFR_t = (1-\xi^x)^n\;YTFR_{t-n}$$

For fixed discovery and extraction rates, and if countries are on the depletion profile, cumulative extraction can be described with the following expression:

$$\begin{aligned}\sum_{t=0}^{n-1} Extr_t = \sum_{t=0}^{n-1} \rho_t Res_t \quad &= [1-(1-\rho_t^x)^n]\,Res_{t-n} \\ &+ [\rho_t^x\,(1-(1-\xi_t^x)^n) - \xi_t^x\,(1-(1-\rho_t^x)^n)]\,\frac{YTFR_{t-n}}{\rho_t^x - \xi_t^x}\end{aligned}$$

The formula becomes the following with the additional yet-to-discover reserves shifter:

$$\begin{aligned}\sum_{t=0}^{n-1} Extr_t = \quad &= [1-(1-\rho_t^x)^n]\,Res_{t-n} \\ &+ \left[\rho_t^x\frac{1-(\chi_t^x(1-\xi_t^x))^n}{1-\chi_t^x(1-\xi^x)} - (1-(1-\rho_t^x)^n)\right]\frac{YTFR_{t-n}}{\chi_t^x(1-\xi_t^x) - (1-\rho_t^x)}\end{aligned}$$

Conversion rate

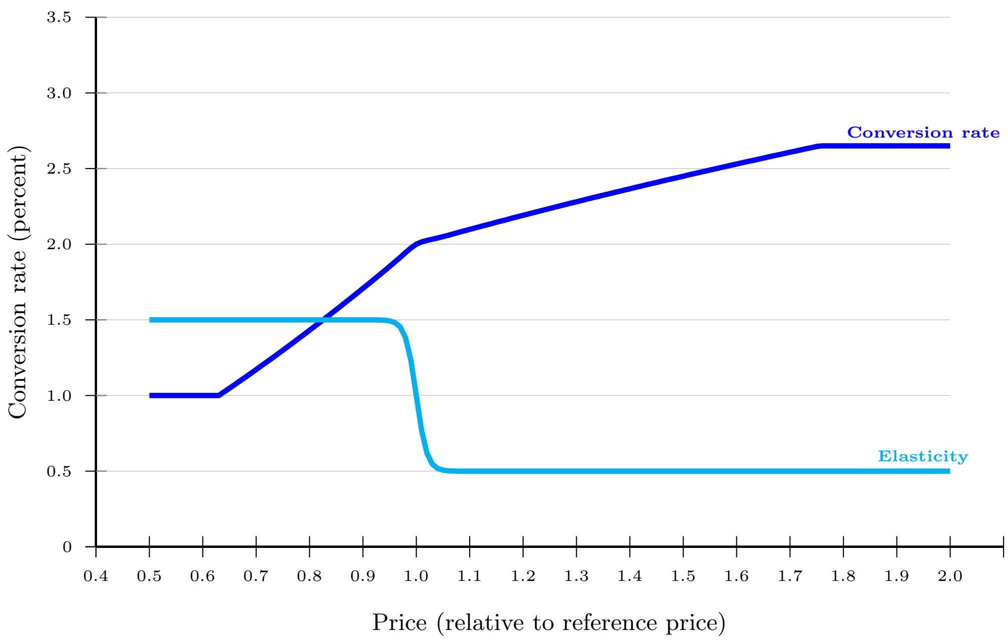
The conversion rate (ξ^x), i.e. the rate at which unproven reserves are converted to proven reserves is assumed to be price sensitive. Within specified ranges, the specification assumes a constant elasticity function. The ranges are identified relative to a reference price growth path. If the actual price growth is greater than the reference growth path, the conversion rate rises with one elasticity. If the actual price growth is below the reference growth path, the conversion rate declines with a different elasticity. Typically it is assumed that the 'low-path' elasticity is greater than the

[30] The denominator becomes 0 in the case of zero growth in the level of actual extraction, in which case the cumulative extraction formula is simply $n.XF_{t-n}$. The current code does not trap this potential issue as the probability of such an occurrence is low.

'high-path' elasticity, i.e. it is relatively easier to lower the conversion rate than to raise it. The specification also has lower and upper bounds on the conversion rate. Figure 3.11 provides an example profile for the conversion rate. The reference conversion rate is 2 percent, i.e. of the price path is exactly the reference path, the conversion rate is 2 percent. The elasticity below the reference price path is 1.5, and it is 0.5 for the high-path elasticity. The bounds are set at 1.0 and 2.5 percent respectively for the lower and upper bounds. The figure shows both the conversion rate and the elasticity. In principle there is a discontinuity in the first derivative at the inflection point, i.e. at the reference price path the two elasticities flip.[31] This is converted to a continuous function using the sigmoid (or logistic function), defined as:

$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

Figure 3.11: **Possible conversion rate profile**



The elasticity expression is defined by equation (F-17). The elasticity expression is a linear combination of the 'Lo' elasticity and the 'Hi' elasticity. At low price path, i.e. where $PRAT$ is less than 1, the sigmoid function takes a value close to 0. On a high price path, the sigmoid function takes a value close to 1. The κ parameter determines the 'length' of the transition period between using the 'Lo' and 'Hi' elasticities. The greater is κ the quicker the transition. A value of 30 leads to a relatively quick transition. Equation (F-18) defines the price ratio for determining the elasticity. The first term on the right calculates the annualized growth rate for the output price of the relevant fuel, and this is compared with an exogenous trend reference price. Equation (F-19) defines the price sensitive conversion rate, with the elasticity determined by equation (F-17), where the elasticity depends on the actual price trend relative to the reference price trend. In the implementation of the expression, the lower and upper bounds on the conversion rate are imposed as additional constraints.

$$\omega^{f}_{r,a} = \omega^{f,lo}_{r,a} + \text{sigmoid}\left(\kappa(PRAT_{r,a} - 1)\right)\left(\omega^{f,hi}_{r,a} - \omega^{f,lo}_{r,a}\right) \quad \text{(F-17)}$$

[31] This was the original specification in the GREEN model, van der Mensbrugghe (1994).

$$PRAT_{r,a,t} = \left(\frac{PP_{r,a,t}/PGDPMP_{r,t}}{PP_{r,a,t-n}/PGDPMP_{r,t-n}}\right)^{1/n} \frac{1}{P^{ref}_{r,a,t}} \tag{F-18}$$

$$\xi^x_{r,a} = \xi^0_{r,a} PRAT_{r,a}^{\omega^f_{r,a}} \tag{F-19}$$

Reserve depletion

The next set of equations by and large replicate those from above as they apply to the ENVISAGE model. Equation (F-20) represents the cumulative extraction between period $t-n$ and $t-1$. For $n=1$ it reduces simply to lagged extraction as the fraction drops out.[32] Equation (F-21) describes the profile for actual reserves and equation (F-22) describes the profile for potential reserves, i.e. if production is based on the maximum extraction rate. The former is based on the expression $Res_t = Res_{t-1} - XF_{t-1} + \xi^x YTFR_{t-1}$, whereas the latter is based on the expression $Res_t = (1-\rho^x_{t-1})Res_{t-1} + \xi^x_{t-1} YTFR_{t-1}$. In both cases, the variable $YTFR$ is adjusted by χ^x which can be used to calibrate the reserve profile to a given price scenario or as an exogenous shifter in specific scenarios. Equation (F-23) describes the profile for yet-to-discover reserves. In the absence of any shift in the parameter χ^x, the unproven reserves can only decline over time.

$$CXF_{r,a,t} = XF^{1/n}_{r,a,t-n} \frac{XF_{r,a,t} - XF_{r,a,t-n}}{XF^{1/n}_{r,a,t} - XF^{1/n}_{r,a,t-n}} \tag{F-20}$$

$$Res_{r,a,t} = Res_{r,a,t-n} + \left(1 - \left[\chi^x_{r,a,t}\left(1-\xi^x_{r,a,t}\right)\right]^n\right) YTFR_{r,a,t-n} - CXF_{r,a,t} \tag{F-21}$$

$$Res^p_{r,a,t} = (1-\rho^x_{r,a,t})^n Res^p_{r,a,t-n} + \xi^x_{r,a,t}\chi^x_{r,a,t} YTFR_{r,a,t-n} \frac{\left(\chi^x_{r,a,t}(1-\xi^x_{r,a,t})\right)^n - (1-\rho^x_{r,a,t})^n}{\chi^x_{r,a,t}(1-\xi^x_{r,a,t}) - (1-\rho^x_{r,a,t})} \tag{F-22}$$

$$YTFR_{r,a,t} = \left(\chi^x_{r,a,t}(1-\xi^x_{r,a,t})\right)^n YTFR_{r,a,t-n} \tag{F-23}$$

If a region is off its depletion profile, there is a gap between the depletion profile and actual reserves and this gap can lead to a situation with production potential above production given by the reserve profile. Equation (F-24) defines this gap that is captured in the variable Res^g. The expression is implemented as a mixed complementarity relation as the gap must be positive, i.e. actual production cannot exceed potential production. Equation (F-25) defines potential production. If there is no gap between actual extraction and potential extraction, production is simply equal to the product of the extraction rate and the potential level of reserves. If actual production is below potential, the reserve gap (adjusted for the time-period gap) is added to potential production.

$$Res^g_{r,a} \geq Res_{r,a} - Res^p_{r,a} \perp Res^g_{r,a} \geq 0 \tag{F-24}$$

$$XF^p_{r,a} = \rho^x_{r,a} Res^p_{r,a} + \frac{Res^g_{r,a}}{n} \tag{F-25}$$

[Still to be described]. In GREEN for the oil markets, countries are segmented into two groups—those with price sensitive supply and those on the reserve profile. For those on the reserve profile,

[32] At the moment, the model specification ignores the situation of zero-growth in the level of extraction. The formula would need to be replaced with $n.XF_{r,a,t-n}$.

Res^g is set to zero and $XF = XF^p$. Those with price sensitive supply, their production is described by:

$$XF_t = \min \left[XF_{t-n} \left(\frac{PP_t/PGDP_t}{PP_{t-n}/PGDP_{t-n}} \right)^{\eta_t^f}, XF_t^p \right]$$

For natural gas, the notional supply of the fixed factor is either infinite or with an upward sloping supply curve:

$$XF^{Not} = \begin{cases} XF^d & \text{if } \eta^f = \infty \\ XF_{t-n}^s \left[\frac{PF_t/PGDP_t}{PF_{t-n}/PGDP_{t-n}} \right]^{\eta_t^f} & \text{if } \eta^f \neq \infty \end{cases}$$

$$XF^s = \min \left[XF^{Not}, XF^p \right]$$

3.8.5 Capital markets in comparative statics

This section describes capital allocation across sectors in the comparative static version of the model. It is based on a nested CET structure. The top nest allows for international capital mobility depending on the value of the CET elasticity. The second nest implements inter-sectoral capital mobility. Starting with the top nest, global capital is given by $TKAP$. It is allocated across the modeled regions under a CET specification with a transformation elasticity given by ω^k. Equation (F-26) determines regional supply of aggregate capital and equation (F-27) determines the average global price of capital, $TRENT$.

$$\begin{cases} XFT_{r,Captl}^s = \beta_r^k \left(\frac{PFT_{r,Captl}}{TRENT} \right)^{\omega^k} TKAP & \text{if } \omega^k \neq \infty \\ PFT_{r,Captl} = TRENT & \text{if } \omega^k = \infty \end{cases} \tag{F-26}$$

$$TRENT.TKAP = \sum_r PFT_{r,Captl} XFT_{r,Captl} \tag{F-27}$$

The next nest in the capital allocation module determines the sectoral supply of capital, where the mobility of capital is captured in the CET transformation elasticity given by ω^f. Equation (F-28) determines sectoral capital supply in the case of a finite transformation elasticity. Equation (F-29) determines the aggregate regional return to capital. In the case of an infinite transformation, this is replaced with the adding up condition in volume terms. Note that this equation is also used in the dynamic version of the model which at the margin assumes perfect capital mobility across activities (see below).

$$\begin{cases} XF_{r,Captl,a} = \alpha_{r,Captl,a}^{fs} \left(\frac{NPF_{r,Captl,a}}{PFT_{r,Captl}} \right)^{\omega_{r,Captl}^f} XFT_{r,Captl} & \text{if } \omega_{r,Captl}^f \neq \infty \\ NPF_{r,Captl,a} = PFT_{r,Captl} & \text{if } \omega_{r,Captl}^f = \infty \end{cases} \tag{F-28}$$

$$\begin{cases} PFT_{r,Captl} = \left[\sum_a \alpha_{r,Captl,a}^{fs} \left(NPF_{r,Captl,a} \right)^{1+\omega_{r,Captl}^f} \right]^{1/(1+\omega_{r,Captl}^f)} & \text{if } \omega_{r,Captl}^f \neq \infty \\ XFT_{r,Captl} = \sum_a XF_{r,Captl,a} & \text{if } \omega_{r,Captl}^f = \infty \end{cases} \tag{F-29}$$

3.8.6 Capital markets with the vintage capital specification

This section describes sectoral capital allocation under the assumption of multiple vintage capital. Capital market equilibrium under the vintage capital framework assumes the following:

- *New* capital is perfectly mobile and its allocation across sectors insures a uniform rate of return.
- *Old* capital in expanding sectors is equated to new capital, i.e. the rate of return on *Old* capital in expanding sectors is the same as the economy-wide rate of return on new capital.
- Declining sectors release *Old* capital. The released *Old* capital is added to the stock of *New* capital. The assumption here is that declining sectors will first release the most mobile types of capital, and this capital, being mobile, is comparable to *New* capital (e.g. transportation equipment).
- The rate of return on capital in declining sectors is determined by sector-specific supply and demand conditions.

The result of these assumptions is that if there are no sectors with declining economic activity, there is a single economy-wide rate of return. In the case of declining sectors, there will be an additional sector-specific rate of return on *Old* capital for each sector in decline.

To determine whether a sector is in decline or not, one assesses total sectoral demand (which of course, in equilibrium equals output). Given the capital-output ratio, it is possible to calculate whether the initially installed capital is able to produce the given demand. In a declining sector, the installed capital will exceed the capital necessary to produce existing demand. These sectors will therefore release capital on the secondary capital market in order to match their effective (capital) demand with supply. The supply schedule for released capital is a constant elasticity of supply function where the main argument is the change in the relative return between *Old* and *New* capital. Supply of capital to the declining sector is given by the following formula:

$$K^s_{a,Old} = K^0_a \left[R_{a,Old}/R_{a,New}\right]^{\eta^k_a}$$

where K^s_{Old} is capital supply in the declining sector, K^0 is the initial installed (and depreciated) capital in the sector at the beginning of the period, and η^k is the dis-investment elasticity. (Note that in the model, the variable R is represented by PF.) In other words, as the rate of return on *Old* capital increases towards (decreases from) the rate of return on *New* capital, capital supply in the declining sector will increase (decrease). Released capital is the difference between K^0 and K^s_{Old}. It is added to the stock of *New* capital. In equilibrium, the *Old* supply of capital must equal the sectoral demand for capital:

$$K^s_{a,Old} = KV_{a,Old}$$

Inserting this into the equation above and defining the following variable:

$$RR_a = R_{a,Old}/R_{a,New}$$

yields the following equilibrium condition:

$$KV_{a,Old} = K^0_a \left[RR_a\right]^{\eta^k_a}$$

The supply curve is kinked, i.e. the relative rate of return is bounded above by 1. If demand for capital exceeds installed capital, the sector will demand *New* capital and the rate of return on *Old* capital is equal to the rate of return on *New* capital, i.e. the relative rate of return is 1. The kinked supply curve has been transformed into a mixed complementarity (MCP) relation. The following inequality is inserted in the model:

$$K^s_{a,Old} = K^0_a \left[RR_a\right]^{\eta^k_a} \leq K^{d,Not}_a = \chi^v_a XP_a$$

The right-hand side determines the *notional* demand for capital in sector a, i.e. it assesses aggregate output (equal to demand) and multiplies this by the capital output ratio for *Old* capital. This is then the derived demand for *Old* capital. If the installed capital is insufficient to meet demand for *Old* capital, the sector will demand *New* capital, and the inequality obtains with the relative rates of return capped at 1. If the derived demand for *Old* capital is less than installed capital, the sector will release capital according to the supply schedule. In this case the inequality transforms into an equality, and the relative rate of return is less than 1.

Equation (F-30) determines the capital output ratio, χ^v for *Old* capital. Equation (F-31) specifies the supply schedule of *Old* capital. In effect, this equation determines the variable RR, the relative rate of return between *Old* and *New* capital.

$$\chi^v_{r,a} = \frac{KV_{r,a,Old}}{XPv_{r,a,Old}} \tag{F-30}$$

$$K^0_{r,a} \left(RR_{r,a}\right)^{\eta^k_{r,a}} \leq \chi^v_{r,a} XP_{r,a} \perp RR_{r,a} \leq 1 \tag{F-31}$$

There is a single economy-wide rate of return on *New* capital. The equilibrium rate of return on *New* capital is determined by setting aggregate supply equal to aggregate demand. Aggregate demand for new capital is given by:

$$\sum_{a \in Expanding} \sum_v KV_{a,v}$$

where the set *Expanding* includes all sectors in expansion. Since *Old* capital in expanding sectors is equated with *New* capital, the appropriate sum is over all vintages. The aggregate capital stock of *New* capital is equal to the total capital stock, less capital supply in declining sectors:

$$K^s - \sum_{a \in Declining} K^{s,Old}_a$$

where the set *Declining* covers only those sectors in decline. However, at equilibrium, capital supply in declining sectors must equal capital demand for *Old* capital, and capital demand for *New* capital in these sectors is equal to zero. Hence, the supply of *Old* capital in declining sectors can be shifted to the demand side of the equilibrium condition for *New* capital, and this simplification yields equation (F-32) which determines the economy-wide rate of return on *New* capital. Equation (F-33) adds up capital demand across vintages. Equation (F-34) determines the vintage and sector specific rates of return.[33] For *New* capital, RR is 1 and thus the rate of return on *New* capital is always equal to the economy-wide rate of return (adjusted by the factor tax). For *Old* capital, if the

[33] These are the net rates of return after tax. Thus the relative rate of return variable, RR, is defined in terms of the net rate of return.

sector is in decline, RR is less than 1 and the rate of return on *Old* capital will be less than the economy-wide rate of return (adjusted by the factor tax).

$$\sum_a XF_{r,Captl,a} = XFT_{r,Captl} \tag{F-32}$$

$$XF_{r,Captl,a} = \sum_v KV_{r,a,v} \tag{F-33}$$

$$PKV_{r,a,v} = \left(1 + \tau^{vtax}_{r,Captl,a} + \tau^{vsub}_{r,Captl,a}\right) RR_{r,a} PFT_{r,Captl} \tag{F-34}$$

3.8.7 Allocation of output across vintages

This section describes how output is allocated across vintages. Aggregate sectoral output, XP, is equated to aggregate sectoral demand and is derived from XS, which itself is derived from a CET aggregation of XD and XET. Given the beginning of period installed capital, it is possible to assess the level of potential output produced using the installed capital. If this level of output is greater than the aggregate output (demand) level, the sector appears to be in decline, installed capital will be released, *Old* output will be equated with aggregate output (demand), and *New* output is zero. Equation (F-35) equates aggregate output, XP, to the sum of output across all vintages. Equation (F-36) determines output that can be derived from installed, or *Old* capital, thus equation (F-35) in essence determines output produced with *New* capital by residual. *Old* output is equated to the sectoral supply of *Old* capital, divided by the capital/output ratio. Equation (F-37) sets the aggregate price of capital—in both declining and expanding sectors it is equal to the rate of return on *Old* capital.

$$XP_{r,a} = \sum_v XPv_{r,a,v} \tag{F-35}$$

$$XPv_{r,a,Old} = K^0_{r,a} \left(RR_{r,a}\right)^{\eta^k_{r,a}} / \chi^v_{r,a} \tag{F-36}$$

$$PF_{r,Captl,a} = PKV_{r,a,Old} \tag{F-37}$$

The final set of equations link user prices of factors with the market price (or the returns to the owners of the factors). Equation (F-38) links the user-price of all factors of production to the after-tax sectoral price of each of the factors. With the introduction of the AEZ database, the next set of equations close the land markets by incorporating the various taxes and subsidies on land use. Equation (F-39) incorporates the taxes and subsidies applied at the activity and land-type level, where $LndTax$ and $LndSub$ represent the *ad valorem* wedges. Equations (F-40) and (F-41) determine respectively the average tax and subsidy on land use by activity, where the average is over all land types indexed by lt.

$$PF_{r,fp,a} = \left(1 + \tau^{vtax}_{r,fp,a} + \tau^{vsub}_{r,fp,a}\right) NPF_{r,fp,a} \tag{F-38}$$

$$PLnd_{r,lt,a} = \left(1 + LndTax_{r,lt,a} + LndSub_{r,lt,a}\right) NPLnd_{r,lt,a} \tag{F-39}$$

$$\tau^{vtax}_{r,LandR,a} NPF_{r,LandR,a} XF_{r,LandR,a} = \sum_{lt} LndTax_{r,lt,a} NPLnd_{r,lt,a} Lnd_{r,lt,a} \tag{F-40}$$

$$\tau^{vsub}_{r,LandR,a} NPF_{r,LandR,a} XF_{r,LandR,a} = \sum_{lt} LndSub_{r,lt,a} NPLnd_{r,lt,a} Lnd_{r,lt,a} \tag{F-41}$$

3.9 Macro closure

Equation (M-1) defines the government accounting balance, S^g. It is the difference between revenues and expenditures, the latter including some share of stock-building expenditures. Government expenditures, split between R&D and other expenditures, are typically fixed relative to GDP in the baseline and fixed in level terms in subsequent simulations—see equations (G-7)-(G-9). Equation (M-2) defines real government savings. Real government savings are fixed—to insure at the least debt sustainability. Nominal revenues are endogenous. The direct tax schedule shifts to achieve the given fiscal target (using the χ^k shifter). Equation (M-3) defines foreign savings, S^f. These are fixed in numéraire terms for all regions save a residual region (indexed by $rSav$). Equation (M-4) insures capital flow equilibrium at the global level (and in effect defines foreign savings for the residual region).[34] Equation (M-5) defines the depreciation allowance. Equation (M-6) represents the investment/savings balance, with aggregate gross investment expenditures on the left-side and total available savings on the right, including the depreciation allowance and adjusted for stock-building expenditures. The model's price anchor, or numéraire, $PNUM$, is defined in equation (M-7). It is defined as the unit value of manufactured exports from the high-income countries, where the set defined by $Numer$ spans the manufactured sectors. One equation needs to be dropped from the model specification and typically one equation from equation (M-6) is dropped. This in fact represents a global Walras' Law that has global investment equal to global savings.

$$S_r^g = YG_r - PC_{r,Gov}XC_{r,Gov} - \psi_{r,Gov}^{stb} \sum_i PA_{r,i,stb}XA_{r,i,stb} \tag{M-1}$$

$$RSg_r = S_r^g / PGDPMP_r \tag{M-2}$$

$$S_r^f = PNUM.\overline{S}_r^f \qquad \text{for } r \notin rSav \tag{M-3}$$

$$\sum_r S_r^f \equiv 0 \tag{M-4}$$

$$DeprY_r = \delta_r PC_{r,Inv} K_r \tag{M-5}$$

$$PC_{r,Inv}XC_{r,Inv} = \sum_h S_{r,h}^h + S_r^g + S_r^f + DeprY_r - \psi_{r,Inv}^{stb} \sum_i PA_{r,i,stb}XA_{r,i,stb} \tag{M-6}$$

$$PNUM = \sum_{r \in HIC} \sum_d \sum_{i \in Numer} \phi_{r,d,i}^n WPE_{r,d,i} \tag{M-7}$$

The following block of equations provides the main macroeconomic identities. Nominal and real GDP at market price, $GDPMP$ and $RGDPMP$, are provided in Equations (M-8) and (M-9), respectively. The GDP at market price deflator, $PGDPMP$, is defined in equation (M-10). Per capita real output, $RGDPPC$, is defined in equation (M-11). Equation (M-12) defines real per capita income growth. And the GDP absorption shares, $GDPShr$, are provided in equation (M-13).

[34] Alternative versions of the model allow for partial mobility of global savings. These are described in Appendix D.

Equation (M-14) defines real domestic absorption—it is the sum of household, government and investment real expenditures.

$$\begin{aligned} GDPMP_r &= \sum_i \left[\sum_{in} PA_{r,i,in} XA_{r,i,in} + PA_{r,i,stb} XA_{r,i,stb} \right] \\ &+ \sum_{i \in im} \left[\sum_d WPE_{r,d,i} WTF_{r,d,i} - \sum_s WPM_{s,r,i} WTF_{s,r,i} \right] \\ &+ \sum_{ih} NT_{r,ih} + PTMG_r XTMG_r \end{aligned} \tag{M-8}$$

$$\begin{aligned} RGDPMP_r &= \sum_i \left[\sum_{in} PA_{r,i,in,0} XA_{r,i,in} + PA_{r,i,stb,0} XA_{r,i,stb} \right] \\ &+ \sum_{i \in im} \left[\sum_d WPE_{r,d,i,0} WTF_{r,d,i} - \sum_s WPM_{s,r,i,0} WTF_{s,r,i} \right] \\ &+ \sum_{ih} PW_{ih,0} \left(XET_{r,ih} - XMT_{r,ih} \right) + PTMG_{r,0} XTMG_r \end{aligned} \tag{M-9}$$

$$PGDPMP_r = GDPMP_r / RGDPMP_r \tag{M-10}$$

$$RGDPPC_r = RGDPMP_r / Pop_r \tag{M-11}$$

$$g^{ypc}_{r,t} = \left(\frac{RGDPPC_{r,t}}{RGDPPC_{r,t-n}} \right)^{1/n} - 1 \tag{M-12}$$

$$GDPShr_{r,in} = \frac{PC_{r,in} XC_{r,in}}{GDPMP_r} \tag{M-13}$$

$$RYD_{r,t} = \sum_{in} PA_{r,i,in,0} XA_{r,i,in} \tag{M-14}$$

The default closure rules of the model are as follows:

- Household savings are endogenous and are either driven by the demographic-influenced savings function or as part of the ELES consumer demand system.[35]
- Government revenues are endogenous and government expenditures, as a share of nominal GDP, are fixed, thus total expenditures are endogenous. The government balance is fixed, in part to avoid problems of financing sustainability. The government balance is achieved with a uniform shift in the household direct tax schedule. This implies that new revenues, for example generated by a carbon tax, would lower direct taxes paid by households.
- Investment is savings driven. Household and government savings were discussed above. Foreign savings, in the default closure are fixed. Thus investment is largely influenced through household savings.[36]

[35] The latter may allow for demographics and other factors to influence the ELES parameters between periods in the dynamic setting where ELES parameters may be re-calibrated.

[36] Alternative closures are conceivable, for example targeting investment (as a share of GDP) and allowing the household savings schedule adjust to achieve the target.

- The current account, the mirror entry of the capital account, is exogenous. *Ex ante* changes to trade, for example a rise in the world price of imported oil, is met through emphex post changes in the real exchange rate.

3.10 Model dynamics

3.10.1 Factors and technology

Model dynamics are driven by three factors—similar to most neo-classical growth models. Population and labor force growth rates are exogenous and given essentially by the UN Population Division scenario. The labor force growth rate is equated to the growth rate of the working age population, i.e. the population aged between 15 and 64.[37]

The second factor is capital accumulation. The aggregate capital stock in any given year, *KStock*, is equated to the previous year capital stock, less depreciation at a rate of δ, plus the previous period's volume of investment, XC_{Inv}, see equation (G-1). The latter is influenced by the national savings rate plus foreign savings and, as well, the unit cost of investment. The aggregate capital stock variable takes two forms. The first, *KStock*, is the aggregate capital stock evaluated at base year prices. The second is the 'normalized' aggregate capital stock, XFT. The normalized capital stock is equal to the tax inclusive base year capital remuneration, i.e. the user cost of capital across sectors. It is normalized because its price is set to 1 in the base year. The ratio of the normalized capital stock to the actual capital stock provides a measure of the gross rate of return to capital. It is assumed that both measures of the capital stock grow at the same rate and hence equation (G-2) that equalizes the ratio of the two measures.[38]

$$KStock_{r,t} = (1 - \delta_{r,t})\, KStock_{r,t-1} + XC_{r,inv,t-1} \tag{G-1}$$

$$XFT_{r,Captl,t} = \left(\frac{XFT_{r,Captl,0}}{KStock_{r,0}}\right) KStock_{r,t} \tag{G-2}$$

The third factor is productivity. There are a number of productivity factors peppered throughout the model. The key productivity factor is λ^f that corresponds to converting factor in volume terms to factors in efficiency units. It is typically initialized at 1 in the base year. Equation (G-3) represents labor productivity growth across sectors. The factor γ^l represents an economy-wide labor productivity growth factor—that is either exogenous, or can be used to target a growth rate such as the GDP growth rate. If the factor π is set to 0 and χ^l is set to 1, labor productivity growth is assumed to be uniform across all sectors. The latest version of the model includes a new term, π^n, that represents an endogenous response to expenditures in R&D and knowledge accumulation, described further below.

$$\lambda^f_{r,l,a,t} = \left(1 + \pi^n_{r,a} + \pi_{r,a} + \chi^l_{r,a}\gamma^l_r\right) \lambda^f_{r,l,a,t-1} \tag{G-3}$$

The following assumptions are made regarding productivity:

- Sectors are typically segmented into three groups—agriculture, manufacturing and services.

[37] In future work, these assumptions will be linked to other variables influencing both the decision to work (i.e. the labor force participation rate) and the skill level (via assumptions on education).

[38] It is important to use the actual capital stock in the capital accumulation function since the level of investment must correspond to the actual capital stock, not the normalized level.

- Productivity in agriculture is exogenous and factor neutral. The λ^n and λ^v parameters are set to grow at some exogenous and uniform rate. [To be modified].

- In most sectors, productivity is assumed to be labor augmenting only—and is uniform across both skilled and unskilled labor. This is represented by the variable γ^l.

- There can be sector specific adjustments to the economy-wide labor productivity factor. These adjustments are represented by the parameters π and χ^l—the former is an additive wedge (for example 2 percentage points higher in manufacturing than in services) and the latter is a multiplicative wedge. The default assumptions has π equal to 2 (percentage points) in manufacturing and 0 for all other sectors and χ^l is 1 across all sectors with the exception of agriculture, where it is assumed to have a value of 0. [To be modified]. Various other assumptions are possible and have been used to fine-tune the calibration of the baseline scenario.

- In the calibration, or business-as-usual scenario, the economy-wide productivity factor, γ^l, is calibrated to achieve some target level of per capita growth, at least for some period, including historical validation from the base year to some current year (say from 2007 to 2012), and including some medium term horizon such as 2015. After 2015, the parameter γ^l can be fixed and per capita growth then is an endogenous variable. In most policy scenarios, the γ^l parameter is fixed.

- Energy efficiency is assumed to improve at some exogenous rate that influences the λ^e parameter.

- International trade and transport margins, τ^{rm}, are assumed to improve at some exogenous rate.

3.10.2 R&D and knowledge

The latest version of the model allows for an endogenous component to productivity growth that is linked to expenditures on research and development (R&D) and the accumulation of knowledge. Expenditures on R&D are exogenous (and by default set to some percentage of GDP and financed by the government.) The stock of knowledge is treated in a similar fashion to physical capital:

$$KN_t = (1 - \delta_t^n)KN_{t-1} + R\&D_{t-1}$$

but with some additional twists, where KN is the stock of knowledge, δ^n represents the 'depreciation' of knowledge and $R\&D$ is the level of R&D expenditures. The main twist is the assumption that there is a distributed lag process between the expenditures and their impact on the knowledge stock. The distributed lag process that is implemented is the *Gamma* distribution that has been used in Smeets Křístková, van Dijk, and van Meijl (2016) and CHECK FOR OTHERS (Hertel Baldos, Alston et al Springer) in the case of agriculture. In this case the accumulation of the stock of knowledge becomes:

$$KN_t = (1 - \delta_t^n)KN_{t-1} + \sum_{k=0}^{N} \gamma_k R\&D_{t-k-1}$$

where γ_k represents a set of weights that sum to 1, i.e. eventually each $ spent has a positive and full impact on knowledge, but only over a period of N years and with a different profile depending on the parameters of the *Gamma* distribution. The *Gamma* distribution is given by the following expression:

$$\gamma_k = \chi\,(k+1)^{\delta/(1-\delta)}\,\lambda^k$$

where δ and λ determine the shape of the distribution and χ is a scale factor that guarantees that the weights sum to 1. Figure (3.12) provides a sample of possible shapes for the distribution of the weights.

Equation (G-4) represents the knowledge stock motion equation as implemented in the model.[39] The upper limit on the summation is the minimum of N, the length of the *Gamma* distribution that is region specific, and the term $t - t_0$, which represents the number of years for which lagged R&D expenditures are available. For the first simulation year, for example, there will only be two years for which R&D expenditures are available.[40]

$$KN_{r,t} = (1-\delta^n_t)\,KN_{r,t-1} + \sum_{k=0}^{\min(N_r,t-t_0)} \gamma^n_{r,k} R\&D_{r,t-k} \tag{G-4}$$

The knowledge stock motion equation is paired with an equation that defines the share of the stock of knowledge with respect to GDP, equation (G-5). In the dynamic calibration scenario, this equation can be used to smooth the knowledge stock in the initial periods as typically the model will not have the required lags for R&D expenditures. It is possible to provide a given level for the knowledge stock to GDP ratio and to make the knowledge 'depreciation' rate an endogenous variable—that will most likely take negative values.

$$KNYRat_{r,t} = KN_{r,t}/RGDPMP_{r,t} \tag{G-5}$$

The endogenous part of productivity growth is linked to the growth of the knowledge stock through equation (G-6). The elasticity of endogenous productivity with respect to the growth of knowledge is given by the parameter ϵ^r. It is assumed to be time variant. The parameter γ^r converts the economy-wide impact into a sector-specific impact. By default it is assumed that the sector-specific impact is uniform across sectors. One possible alternative is to set γ^r equal to χ^l which is the sector specific component of the labor productivity multiplicative shifter.

$$\pi^n_{r,a,t} = \gamma^r_{r,a,t}\epsilon^r_{r,t}\left[\frac{KN_{r,t}}{KN_{r,t-1}} - 1\right] \tag{G-6}$$

R&D expenditures are extracted from total government expenditures.[41] Equations (G-7) and (G-8) define the expenditure shares of R&D and other public expenditures as a share of GDP. In the baseline scenario, a path might be set for the ratios that would then determine the expenditure

[39] This equation as currently coded in GAMS requires 1-year time steps.

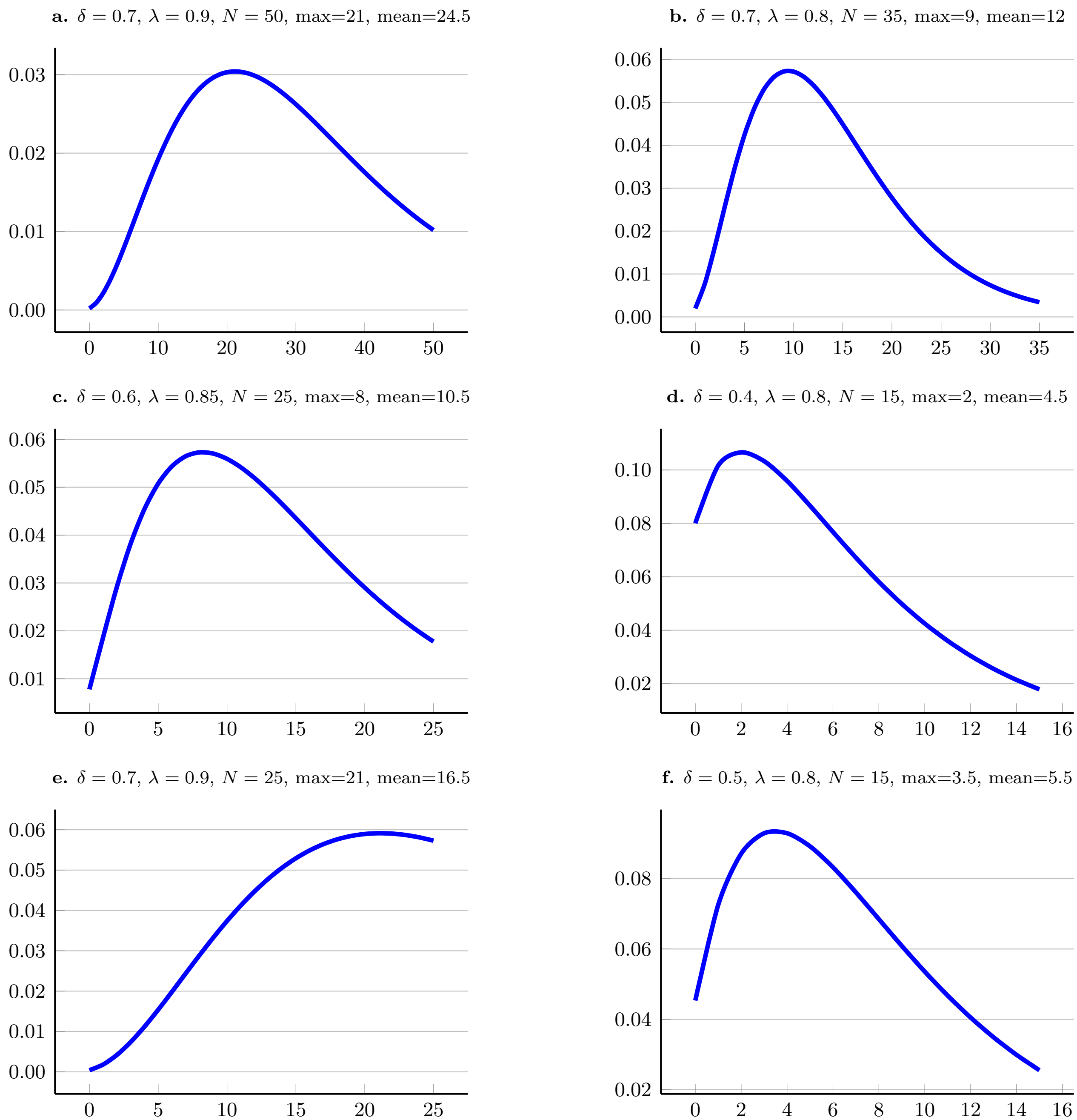
[40] The GAMS code for implementing the stock of knowledge is the following:

```
rstock(r,t) =e= (1 - rdepr(r,t))*rstock(r,t-1)
     + sum(kr$(ord(kr) le min(maxK(r), (years(t) - year0))),
     gammard(r,kr)*sum(tlag$(ord(tlag) eq (ord(t) - ord(kr) + 1)), r_d(r,tlag))) ;
```

The second sum looks to pair the weight k with the appropriate lag. For example, when $k = 0$ (i.e. `ord(kr)` eq 1) we want the contemporaneous level of R&D expenditures (i.e. r_d_t), thus the difference in `ord(tlag)` and `ord(t)` should be zero and so `ord(k)` must be 1.

[41] It is assumed that public R&D expenditures have the same cost structure as other public expenditures on goods and services.

Figure 3.12: Gamma distribution examples



levels. In subsequent scenarios, one would hold fixed the expenditure levels and let the ratios be endogenous. Equation (G-9) links these two expenditures to aggregate public expenditures.

$$R\&DYRat_{r,t} = R\&D_{r,t}/RGDPMP_{r,t} \tag{G-7}$$

$$XCGYRat_{r,t} = XCG_{r,t}/RGDPMP_{r,t} \tag{G-8}$$

$$XC_{r,Gov,t} = R\&D_{r,t} + XCG_{r,t} \tag{G-9}$$

3.10.3 Preferences

The model is designed to handle changes in preferences in two ways. The first way uses a so-called 'twist' specification.[42] As originally formulated, the 'twist' parameter is used to modify the share parameters in CES functions (for example the capital/labor ratio or the ratio of domestic to imported goods in the Armington function) such that the aggregate cost is unchanged.[43]. The twist is given by the following formula:

$$\frac{R_1}{R_0} = 1 + tw$$

The ratio R, in the case of the capital/labor ratio would be K/L and the twist is intended to change that ratio by tw percent. In the case of the adjusted CES, i.e. the volume preserving CES, the twist formula takes the form of:

$$\alpha_1^k = \alpha_0^k \left[r_0^k + \frac{1 - r_0^k}{1 + tw} \right]^{1/\sigma}$$

and

$$\alpha_1^l = \alpha_0^l \left[r_0^k (1 + tw) + (1 - r_0^k) \right]^{1/\sigma}$$

where α^k and α^l are respectively the capital and labor share parameters in the adjusted CES function and r^k is the volume share of capital, i.e. $r^k = K/X$, where X is the aggregate capital/labor bundle. The twist can be generalized to cover multiple-component CES functions, for example a power bundle that is composed of conventional power technologies and renewable technologies.

Say that renewables start out at some share of power, for example r_0. We wish to make them r in year T. For this we implement the 'twist' for the share parameters. The twist is implemented on the power bundles that preserve additivity using the adjusted CES. The annual twist parameter is calculated using the following formula:

$$tw = \left[\frac{r}{r_0} \frac{1 - r_0}{1 - r} \right]^{1/(T - T_0)} - 1$$

where T is the final year (for example 2050) and T_0 is the initial year, for example 2011. If r_0 is 2 percent and r is 10 percent (i.e. a five-fold increase in the share), the relevant annual twist over 39 years is around 4.4 percent.

[42] See Dixon and Rimmer (2002) for its application in the MONASH model and Dixon and Rimmer (2005) for its application in the USAGE model.

[43] This is explained in more details in Appendix A

At the beginning of each time period, the twist formulas can be applied to the relevant bundles. For the renewable bundle, this implies a twist of:

$$\lambda_{ren,t} = \lambda_{ren,t-n}\left[r_{t-n} + \frac{1-r_{t-n}}{(1+tw)^n}\right]^{1/\sigma}$$

For the non-renewable bundle, the twist is expressed as:

$$\lambda_{cnv,t} = \lambda_{cnv,t-n}\left[r_{t-n}\,(1+tw)^n + (1-r_{t-n})\right]^{1/\sigma}$$

where *cnv* covers all non-renewable activities. The variable r is the lagged volume share for renewables where r_t is equal to r_0 for the first year and equal to r for the final year (of implementation of the twist).

If the twists operate in the absence of price shifts, the volume shares can be updated with the following formula:

$$r_t = r_{t-n}\left[\frac{\lambda_{ren,t-n}}{\lambda_{ren,t}}\right]^{\sigma}$$

All of the calculations can occur prior to the simulation using base-year data only.

The second method imposes an exogenous shift in the preference parameters based on 'anticipated' shares if the technologies were available at the same cost. In the case of new technologies, these are typically introduced with very low shares, say 10^{-6}, with the conventional share therefore close to 1. We may expect these shares to evolve to something like 60 percent for the conventional and 40 percent for the new technology were these to have identical costs at some point in the future. These shifts can be either phased in over a time period or simply imposed at some point in the future. The following formulas are used for the phase-in method:

$$\begin{cases} \alpha_t^c = \alpha_{t_0}^c + (\alpha_{tgt}^c - \alpha_{t_0}^c)\dfrac{t-t_0}{T-t_0} & \text{if } t \leq T \\ \alpha_t^c = \alpha_{tgt}^c & \text{if } tgtT \end{cases}$$

and

$$\alpha_t^n = 1 - \alpha_t^c$$

where α^c and α^n represent respectively the share of the conventional technology and the new technology, α_{tgt}^c is the target level of the share for the conventional technology, t_0 is the first year of the phase-in and T is the final year of the phase-in. Note that it could be necessary to 'phase-in' the phase-in for any given time period if the shock proves numerically difficult. In this case, the phase-in can be done iteratively with the following formula:

$$\alpha_{i,t}^c = \alpha_{t-n}^c + n\left(\frac{i-1}{m}\right)\frac{\alpha_{tgt}^c - \alpha_{t_0}^c}{T-t_0}$$

where i iterates between 1 and $m+1$. For i equal to 1, the conventional share is equal to the previous period's share. For i equal to $m+1$, the conventional share is equal to the targeted share for period t. If m is equal to n, i.e. m is equal to the time step, the formula simplifies to:

$$\alpha_{i,t}^c = \alpha_{t-n}^c + (i-1)\frac{\alpha_{tgt}^c - \alpha_{t_0}^c}{T-t_0}$$

3.10.4 Introducing cost curves

Changes in costs in the normal functioning of the model depend on changes in input prices and the standard assumptions regarding technology change. This section explains how to introduce an acceleration in cost reduction coming from some exogenous phenomenon such as learning by doing. The basic idea is to start from an initial price, say P_0, and to reduce costs over time, albeit with a lower limit given by $PMIN$. There also exists a target price for year T, given by P_T, that must be greater than $PMIN$. Let α represent the ratio of the final price, P_T, relative to the base year price, P_0, i.e.:

$$P_T = \alpha P_0$$

and let γ represent the ratio of the minimum price, $PMIN$, relative to the initial price, i.e.:

$$PMIN = \gamma P_0$$

where $\alpha > \gamma$. Two functional forms are introduced to represent the cost curve—the hyperbola and the logistic functions. The hyperbola takes the following form:

$$P_t = PMIN + \chi t^{-\beta}$$

The logistic function takes the form:

$$P_t = \frac{PMIN}{1 + \chi e^{-\beta t}}$$

Calibration of the parameters for the hyperbola function involve the following expressions:

$$\beta = \frac{\ln\left(\dfrac{\alpha - \gamma}{1 - \gamma}\right)}{\ln\left(\dfrac{t_0}{T}\right)} \qquad \chi = P_0\left(1 - \gamma\right)$$

where we define t_0 to start with 1.[44] In the case of the logistic function, the calibration formulas are:

$$\beta = -\frac{1}{T}\ln\left(\frac{\alpha - \gamma}{\alpha\left(1 - \gamma\right)}\right) \qquad \chi = \gamma - 1$$

where we define t_0 to start with 0.[45] Figure 3.13 depicts the shapes of the two cost curves for the same initial and end points and price limit. From the shape of the curves it is clear that costs drop sharply in the initial years using the hyperbola specification. The drop is more gradual (and constant in percentage terms) for the logistic function.

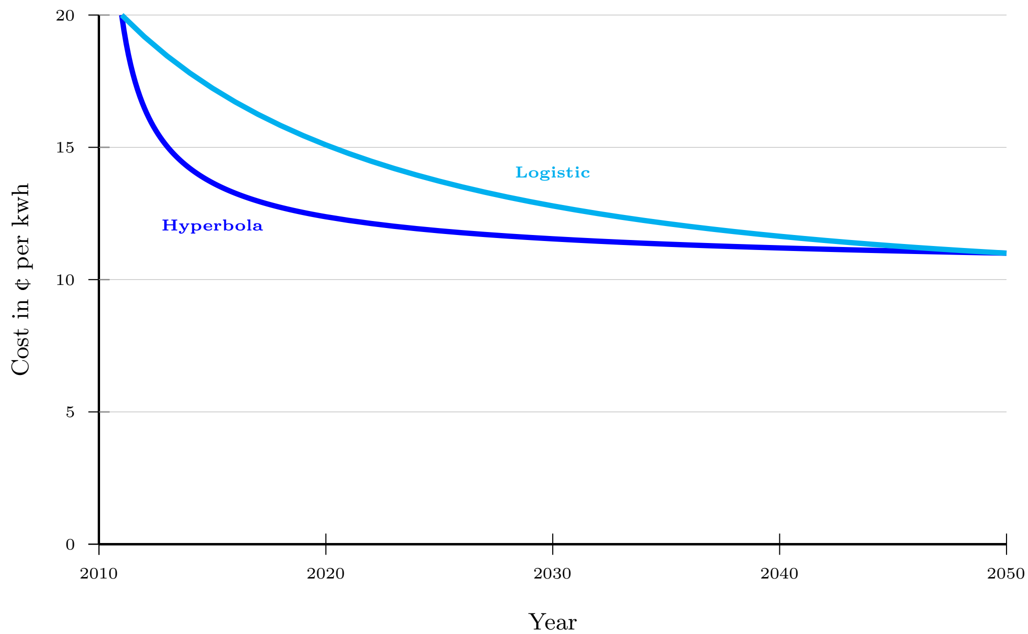
The adjustment to the cost curve is made on total factor productivity (TFP) for the relevant activity. The aggregate cost is given by the following formula:

$$P_t = \frac{1}{\lambda_t} F(P_{i,t})$$

44 If the first year is 2011, t is then defined as $t - 2010$. If the final year is 2050, then T is equal to $2050 - (2011 - 1) = 40$.

45 If the first year is 2011, t is then defined as $t - 2011$. If the final year is 2050, then T is equal to $2050 - 2011 = 39$.

Figure 3.13: **Example of cost curves**



where F is the cost function and depends on the prices of the various inputs given by P_i. In most cases, the cost function will be the dual price expression of the CES function.[46] TFP is assumed to behave according to the following expression:

$$\lambda_t = (1 + \pi_t)^n \lambda_{t-n}$$

Assuming that input prices are invariant, the growth in TFP must be equal to the following expression:

$$\pi_t = \left(\frac{P_{t-n}}{P_t}\right)^{1/n} - 1$$

We can introduce the formulas for the two cost curves to derive expressions for π. For the hyperbola, we have:

$$\pi_t = \left(\frac{PMIN + \chi(t-n)^{-\beta}}{PMIN + \chi t^{-\beta}}\right)^{1/n} - 1 \Longleftrightarrow \left(\frac{\gamma + (1-\gamma)(t-n)^{-\beta}}{\gamma + (1-\gamma)t^{-\beta}}\right)^{1/n} - 1$$

For the logistic function, the expression for π is given by:

$$\pi_t = \left(\frac{e^{\beta t} + \chi}{e^{\beta t} + \chi e^{\beta n}}\right)^{1/n} - 1$$

Both changes in preferences and introduction of the cost curves can be implemented to change the share of any technology in a demand bundle. The latter operates directly on the preference shares of the buyer. The former operates on the perceived cost of the technology. These are formulated independently of the actual cost changes that arise from the endogenous changes in input prices.

46 In most cases λ is initialized at 1 and is held constant. Technology is introduced as labor-augmenting technical change and with an exogenous improvement in energy efficiency.

3.11 Emissions, climate and impact modules

The module's sequence is as follows. First total emissions are derived. The current version of the model includes four greenhouse gases—carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and the fluoridated gases as an aggregate (F-gases). Though most of the emissions are linked to intermediate and final demand, i.e. the consumption of some emitting good or service, in production some may also be linked to capital (e.g. cattle stock in the case of methane), land (in the case of methane and nitrous oxide emissions in agriculture) and/or aggregate output (e.g. municipal waste-base methane emissions). The emissions of greenhouse gases lead to atmospheric concentrations—emissions directly add to the atmosphere, but concentrations in the atmosphere also interact with the ocean and land, creating a dynamic process that would continue even in the absence of emissions. The atmospheric concentration has an impact on radiative forcing, i.e. how much of the sun's energy is reflected back to space. Finally, there is a set of equations that links radiative forcing to temperature global mean temperature change. The final phase of the module links changes in the average mean temperature to economic impacts that feed back into production and demand thereby closing the loop between economic activities, climate, back to economic activities.

The first emissions equation, equation (C-1), determines the level of emissions, EMI, of type em for each unit of consumption of commodity i by agent aa, where aa covers all production activities and final demand accounts. It is simply a fixed coefficient with respect to the demand level. The emissions rate, ρ, can be adjusted in the baseline by the factor χ to allow for autonomous improvements in the emission rates.[47] Equation (C-2) captures emissions linked to the use of factors of production such as capital and/or land. Equation (C-3) are emissions linked to generic production activities and not to a specific technology, i.e. they are simply output based emissions. The aggregate emission by region (or country r), $EMITot$, is defined in equation (C-4) and is the double sum over all agents and sources (consumption, factor use and production level), with the possibility of an additional exogenous level of emissions, $EMIOth$. The level of global emissions, $EMIGbl$, is the summation across all countries and regions, with an additional exogenous component not accounted for in the regional models—see equation (C-5).

$$EMI_{r,em,i,aa} = \chi^e_{em} \rho_{r,em,i,aa} XA_{r,i,aa} \tag{C-1}$$

$$EMI_{r,em,fp,a} = \chi^e_{em} \rho_{r,em,fp,a} XF_{r,fp,a} \tag{C-2}$$

$$EMI_{r,em,Total,a} = \chi^e_{em} \rho_{r,em,Total,a} XP_{r,a} \tag{C-3}$$

$$EMITot_{r,em} = \sum_{aa} \sum_{is} EMI_{r,em,is,aa} + EMIOth_{r,em} \tag{C-4}$$

$$EMIGbl_{em} = \sum_{r} EMITot_{r,em} + EMIOthGbl_{em} \tag{C-5}$$

3.11.1 Emission taxes, caps and trade

There are a number of different potential regimes to limit carbon emissions. The simplest is simply to impose a carbon tax, i.e. set the variable τ^{emi} to some value (measured as dollars in base year

[47] This has only been used to calibrate the emissions rate of non-CO_2 greenhouse gases.

prices per unit of emitted C). Emission caps can be set on either a single region (or country) basis, with a differentiated carbon tax across regions/countries, or on a region-wide basis with a uniform carbon tax. Quota regions are indexed by *rq* and can be assigned one or more countries. Examples of cap and cap and trade scenarios are provided in Appendix G. Equation (C-6) implements emissions caps for each coalition of regions subject to a cap (potentially just a single country). The sum of emissions across all regions belong to region *rq* is capped to *EMICap* (the shifter is explained below). Equation (C-6) determines the regional emissions tax, τ^{emiR}. Equation (C-7) is an accounting identity that equates the country/region tax, τ^{emi}, to the region-wide emissions tax. The regional carbon tax is normally equated to the coalition carbon tax, i.e. by default, χ^{ct} is equal to 1 and β^{ct} is equal to 0. It is possible to provide either multiplicative and/or additive weights to the equilibrium coalition carbon tax for specific distributional purposes. One such example is to target global emissions, i.e. a coalition of all regions, but where specific regions are not part of the formal coalition, in which case χ^{ct} would be set to zero for regions not part of the coalition. The weights could also constitute a 'desired' carbon tax across different coalitions (in terms of dollars per ton of carbon) and then the variable τ^{emiR} would have an initial value of 1 and either adjust proportionately or additively subject to the overall cap.

$$\sum_{r \in rq} EMITot_{r,em} = \chi^{Cap}_{em} EMICap_{rq,em} \tag{C-6}$$

$$\tau^{emi}_{r,em} = \chi^{ct}_{r,em} \tau^{emiR}_{rq,em} + \beta^{ct}_{r,em} \qquad \text{for } r \in rq \tag{C-7}$$

$$QuotaY^{emi}_{r,em} = \tau^{emi}_{r,em} \left[EMIQuota_{r,em} - EMITot_{r,em}\right] \qquad \text{if Cap \& Trade is active} \tag{C-8}$$

The shifter in equation (C-6) allows for additional targeting, for example a cap on global emissions. Say for example one wants to cap global emissions by 20 percent but only impose a cap on Annex I emissions. There is some potential leakage from the cap on Annex I countries—with non-Annex I countries increasing their emissions—because the world price of fossil fuels may decline and because they increase their production of carbon intensive goods for export to the now less competitive Annex I markets. The cap on Annex I countries can then be thought of as setting the burden shares across Annex I countries and the shifter, χ^{Cap}, in equation (C-6) is then endogenous to meet the overall objective, for example capping global emissions.

Equation (C-8) determines the value of the trade in emissions quota when country/region specific quotas, *EMIQuota*, are allocated. The value of the quota is the difference between the quota and actual emissions, *EMITot*, valued at the emissions tax level. Currently, it is assumed that the quota rents are recycled back to the government.

3.11.2 Concentration, forcing and temperature

The current version of ENVISAGE includes a highly simplified climate module that is largely inspired by the climate module in the MERGE model.[48] It replaces the original climate module that was based on Nordhaus' DICE 2007 model[49] because the latter has a CO_2-only focus and ENVISAGE requires a module that can handle other greenhouse gases. We may eventually also assess the implementation of the PAGE09 climate module that has the added advantage of providing spatially distinct temperature change in part linked to regional differences in latitude.[50]

48 As described in Manne, Mendelsohn, and Richels (1995) and implemented in the GAMS version of MERGE.

49 Nordhaus (2008), also described in greater detail in Appendix F.

50 Hope (2010).

Greenhouse gases are treated differently in their impacts on temperature. Carbon emissions are released into the atmosphere that is divided into five boxes, *Box*, indexed by *b*. New emissions are released into the five boxes, equation (C-9), where the fraction parameter, φ^b, sums to 1.[51] The level of carbon in each of the boxes decays over time at the rate δ^d (that is box-specific). Equation (C-10) then determines the total concentration of atmospheric carbon (or the stock) in gtC, *Conc*, summing over all of the boxes and added to the pre-industrial concentration. For the other greenhouse gases, indexed by *xghg*, the atmospheric concentration is equal to the previous period's concentration with a decay parameter δ^x, to which is added new emissions, equation (C-11). The total concentration is the sum of the transient concentration, $Conc^t$, to which is added some equilibrium stock, equation (C-12).

$$Box_{b,t} = \delta^d_{\mathrm{CO_2},b} Box_{b,t-1} + \varphi^b_b EMIGbl_{\mathrm{CO_2},t} \tag{C-9}$$

$$Conc_{\mathrm{CO_2},t} = \sum_b Box_{b,t} + PIC_{\mathrm{CO_2}} \tag{C-10}$$

$$Conc^t_{xghg,t} = \delta^x_{xghg,t} Conc^t_{xghg,t-1} + EMIGbl_{xghg,t} \tag{C-11}$$

$$Conc_{xghg,t} = Conc^t_{xghg,t} + EqConc_{xghg,t} \tag{C-12}$$

The radiative forcing impact, *RF*, of carbon concentration is a logarithmic function of the concentration level where ρ^f is a critical parameter that determines the climate sensitivity—typically measured as the radiative forcing impact of a doubling of CO_2 concentration relative to the pre-industrial level, equation (C-13). The radiative forcing impacts of the other greenhouse gases is normally captured by a power equation in the difference in concentration (from base levels) where the power is either the square root, or linear, equation (C-14). The concentrations measured in ENVISAGE are in gtCeq and are converted back to millions of tons using the global warming potential conversion factor, *GWP*. The ζ parameter captures atmospheric chemical interaction effects across the different greenhouse gases.

$$RF_{\mathrm{CO_2},t} = \rho^f_{\mathrm{CO_2}} \ln\left(\frac{Conc_{\mathrm{CO_2},t}}{Conc_{\mathrm{CO_2},0}}\right) \tag{C-13}$$

$$RF_{xghg,t} = \zeta^f_{xghg}\rho^f_{xghg}\left[\left(\frac{\chi_{xghg} Conc_{xghg,t}}{GWP_{xghg}}\right)^{\omega^f_{xghg}} - \left(\frac{\chi_{xghg} Conc_{xghg,0}}{GWP_{xghg}}\right)^{\omega^f_{xghg}}\right] \tag{C-14}$$

The actualized mean global surface temperature lags behind the potential temperature change as it takes time for atmosphere and ocean temperature transfer. Equation (C-15) captures the potential temperature impact, $Temp^{eq}$, of the changes in radiative forcing which is a linear function of the aggregate change in radiative forcing. The actual change in temperature, *Temp*, is a weighted average of the previous temperature change and the potential temperature—with potential adjustments due to exogenous cooling and radiative forcing (e.g. sulfates), equation (C-16).

$$Temp^{eq}_t = \rho^t \left[RF_0 + \sum_{ghg} RF_{ghg,t}\right] \tag{C-15}$$

$$Temp_t = \left(1-\lambda^t\right) Temp_{t-1} + \lambda^t Temp^{eq}_{t-1} - \rho^t\left(Cool_t - Cool_0 + RF^X_t\right) \tag{C-16}$$

51 More details on the underlying theory, parameterization, and handling of the multi-step time periods is provided in Appendix F.

3.11.3 Climate change economic impacts

The incorporation of climate-related impacts in models of climate change has largely been relegated to highly aggregate economic models (Nordhaus (1994), Nordhaus and Boyer (2000) and Nordhaus (2008), Hope (2006)) using a macro damage function that link changes in temperature to a percentage impact on productivity—normally with an assumption of non-linearity. Hope's damages were initially split into three distinct impacts—macroeconomic, non economic (such as eco-systems), and a third damage linked to a sudden discontinuity that could happen after a given temperature threshold. Hope 2010 has added a fourth channel that splits the impact of sea level rise from the macroeconomic damage function. Nordhaus (2010) has similarly split his macroeconomic damage function into two components with sea level rise split from the rest of the impacts.[52] The FUND model (Anthoff and Tol (2008)) is also a macro model, but they have vastly extended the impact side to include agriculture, forestry, water resources, energy consumption, sea level rise, eco-systems, human health and extreme weather. The initial version of Envisage only incorporated agricultural damages—calibrated to estimates in Cline (2007), but this limited impact has been superseded by a new and more complete set of impact estimates and described below.[53]

Impacts are based on a 2-dimensional table of impact *sources* and impact *destinations*. The impact sources are listed in table (3.3).[54]

Table 3.3: Sources of climate change impacts

Set	Description
sea	Sea level rise
agr	Agricultural productivity
wat	Water availability
onj	On the job (labor) productivity
tou	Tourism
hhe	Human health
end	Energy demand

Table 3.4 describes the destinations of the climate change impacts.

The bulk of the impacts are assumed to be linear with respect to temperature change and are summarized by equation (C-17), where *ddam* is the specific damage function by source and destination. It is implemented as a deviation from the no-damage situation, where *ddam* takes the value of 1 in the absence of climate change. A quadratic damage function is used in the case of agriculture—affecting multi-factor productivity, as depicted in equation (C-18). The following set of equations implements the damages directly on the relevant model variables. Equation (C-19) implements the (quadratic) damage on the top-level productivity parameter in the crop sectors, i.e. it is a uniform shift in the production possibilities frontier across all inputs. It enters in equations (P-1) through (P-3).[55] The next set of four equations determines the impacts on the factors

[52] The critical part of sea level rise is that the lagged structure of temperature exchange between the atmosphere and the sea is unusually long so that even if atmospheric temperature rise is reduced relatively rapidly, the impact on sea level rise would take centuries to dissipate.

[53] Much of the remainder of this section is based on Roson (2009).

[54] At the moment, the human health component only reflects the direct effect on labor productivity, and not the increased demand for health services.

[55] The parameters of the damage function are region specific and reflect to some extent the base year structure of

Table 3.4: Destinations of climate change impacts

Set	Description
lp	Labor productivity (or stock)
kp	Capital productivity (or stock)
tp	Land productivity (or stock)
mp	Multi-factor productivity
hc	Household consumption of energy
hcser	Household consumption of market services
incab	Income from abroad

of production in efficiency units. Equations (C-20) through (C-22) determine the cumulative impact on respectively labor, land and capital. The standard productivity factors, λ^f, are determined in the dynamics module, and the climate-impact adjusted parameters, λ^{gf}, enter the production functions. Equation (C-23) is a simple identity as, for the moment, it is assumed that climate change does not have an impact on the availability of natural resources (i.e. fossil fuels).

$$ddam_{r,src,dst,t} = 1 + \chi^{ccd}_{r,src,dst,t}\left(T_t - T_0\right) \qquad \text{for } src \in Linear \tag{C-17}$$

$$ddam_{r,agr,mp,t} = 1 + \alpha^{a1}_r \min\left(1, T_t - T_0\right) + \alpha^{a2}_r\left(T_t - T_0\right) + \alpha^{a3}_r\left(T_t - T_0\right)^2 \tag{C-18}$$

$$\delta^{cd}_{r,cr} = ddam_{r,agr,mp} \tag{C-19}$$

$$\lambda^{gf}_{r,l,a} = \lambda^f_{r,l,a}\prod_{src} ddam_{r,src,lp} \tag{C-20}$$

$$\lambda^{gf}_{r,LandR,a} = \lambda^f_{r,LandR,a}\prod_{src} ddam_{r,src,tp} \tag{C-21}$$

$$\lambda^{gf}_{r,Captl,a} = \lambda^f_{r,Captl,a}\prod_{src} ddam_{r,src,kp} \tag{C-22}$$

$$\lambda^{gf}_{r,natrs,a} = \lambda^f_{r,natrs,a} \tag{C-23}$$

Equation (C-24) represents the impact on household demand. The impact is assumed to affect the minimal subsistence bundle as represented by the θ parameter. The impact parameter is calibrated to the impact on overall consumption, hence the impact on the subsistence level is scaled for the share of the subsistence level in the overall level of consumption (per capita). Finally, equation (C-25) represents the impact on tourism revenues, iit. This is a linear function of the temperature change, where iit_0 represents base year tourism revenues. Tourism revenues accrue

agricultural production, however, the damage as currently formulated applies uniformly across all crop sectors.

to households and this requires a change to equation (Y-11).[56] Parametrization of the damage functions is described in Appendix H.

$$\theta^{gh}_{r,k,h} = \theta^{h}_{r,k,h}\left[1 + \frac{\left(ddam_{r,end,hc}\text{sign}\left(\theta^{h}_{r,k,h}\right) - 1\right)}{\theta^{h}_{r,k,h}/\left(HX_{r,k,h,0}/Pop_{r,h,0}\right)}\right] \tag{C-24}$$

$$iit_{r,h} = \chi^{iit}_{r,h} iit_{r,0}\left(T_t - T_0\right) \tag{C-25}$$

[56] Similar to all variables that deal with households, the income is allocated across households using the χ^{iit} allocation vector—but for the moment, ENVISAGE has only a single representative household and thus the parameter has unit value.

Appendix A

The CES and CET functions

This appendix describes in full detail the two functional forms most widely used in CGE models- the constant-elasticity-of-substitution (CES) and constant-elasticity-of-transformation (CET) functions. CES functions are widely used in demand functions where substitutability across different products and/or factors is needed and where the main objective is to minimize cost. CET functions are broadly used to determine supply functions across different markets where the main objective is to maximize revenues. The two are very similar in many ways and the algebraic derivations below will be more detailed for the CES function.

A.1 The CES function

A.1.1 Basic formulas

In production, the CES function is used to select an optimal combination of inputs (either goods and/or factors) subject to a CES production function. In consumer demand, the CES is used as a utility (or sub-utility) or preference function. In either case, the purpose is to minimize the cost of purchasing the 'inputs' subject to the production or utility function. In generic terms the system takes the following form:

$$\min_{X_i} \sum_i P_i X_i$$

subject to the constraint:

$$V = A\left[\sum_i a_i(\lambda_i X_i)^\rho\right]^{1/\rho}$$

The objective function represents aggregate expenditure. The constraint expression will be referred to as the CES primal function. The parameter A is an aggregate shifter that can be used to shift the overall production function (or utility function). Each input, X_i, is multiplied by an input-specific shifter, λ_i, that can be used to implement input-specific productivity increases (for example biased technological change), or specific changes in consumer preferences. The (primal) share coefficients, a_i, are typically calibrated to some base year data and held fixed. The CES exponent, ρ, is linked to the curvature of the CES function (and will be explained further below). For given component prices, P_i, and a given level of production or utility V, solving the optimization program above will yield optimal demand functions for the inputs, X_i.

The Lagrangian can be set up as:

$$\mathcal{L} = \sum_i P_i X_i + \Lambda \left(V - A \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho} \right)$$

Taking the partial derivative with respect to X_i and the Lagrange multiplier Λ yields the following system of equations:

$$P_i = \Lambda a_i \lambda_i^\rho X_i^{\rho-1} A \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{(1-\rho)/\rho} = \Lambda a_i A^\rho \lambda_i^\rho X_i^{\rho-1} V^{1-\rho}$$

$$V = A \left[\sum_i a_i (\Lambda_i X_i)^\rho \right]^{1/\rho}$$

Taking the first expression, it can be multiplied by X_i, and then summed. This of course is equal to the value of the bundle, i.e. $P.V$, where P is the aggregate price:

$$P.V = \sum_i P_i X_i = \Lambda V^{1-\rho} A^\rho \sum_i a_i \lambda_i^\rho X_i^\rho = \Lambda V^{1-\rho} V^\rho = \Lambda V$$

This shows that Λ, the Lagrange multiplier is the same as the aggregate price, P. We can re-arrange expression above to get an expression for optimal input demand, where Λ is replaced by P:

$$X_i = a_i^{1/(1-\rho)} A^{\rho/(1-\rho)} \left(\frac{P}{P_i} \right)^{1/(1-\rho)} \lambda_i^{\rho/(1-\rho)} V$$

We finally end up with the following expression, where the CES primal exponent, ρ, is replaced by the so-called CES elasticity of substitution, σ:

$$X_i = \alpha_i (A \lambda_i)^{\sigma-1} \left(\frac{P}{P_i} \right)^\sigma V \tag{A.1-1}$$

where we made the following substitutions:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma - 1}{\sigma} \Leftrightarrow \frac{\rho}{1-\rho} = \sigma - 1 \Leftrightarrow \rho.\sigma = \sigma - 1$$

and

$$\alpha_i = a_i^{1/(1-\rho)} = a_i^\sigma \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

Abstracting from the technology parameters, the demand equation implies that demand for 'input' X_i is a (volume) share of total demand V. The share, with equal prices is simply equal to α_i. With a positive elasticity of substitution, the share is sensitive to the ratio of prices relative to the aggregate price index. Since the component price is in the denominator, the demand for that component declines if its price rises relative to the average and vice versa if its price declines vis-à-vis the average price. The α parameters will be referred to as the CES dual share parameters (for reasons described below), and the a parameters are the primal CES share parameters. Notice that expression (A.1-1) simplifies if it is expressed in terms of efficiency inputs, X^e and efficiency prices, P^e:

$$X_i^e = \alpha_i A^{\sigma-1} \left(\frac{P}{P_i^e} \right)^\sigma V$$

where

$$X_i^e = \lambda_i X_i$$

and

$$P_i^e = \frac{P_i}{\lambda_i}$$

The aggregate price P can be determined using two expressions. The first is the zero profit condition:

$$P = \frac{\sum_i P_i X_i}{V}$$

The other is by inserting the optimal demand relation X_i (equation A.1-1) in the zero profit condition :

$$P.V = \sum_i P_i X_i = A^{\sigma-1} \sum_i P_i \alpha_i \left(\frac{P}{P_i}\right)^{\sigma} \lambda_i^{\sigma-1} V = P^{\sigma} A^{\sigma-1} V \sum_i \alpha_i \left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}$$

The V's cancel out, and the aggregate price can then be expressed by the following formula:

$$P = \frac{1}{A}\left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)} = \frac{1}{A}\left[\sum_i \alpha_i \left(P_i^e\right)^{1-\sigma}\right]^{1/(1-\sigma)} \tag{A.1-2}$$

This is sometimes referred to as the dual price expression. It has virtually the same functional form as the CES primal, which is a CES aggregation of the input volumes using the primal share parameters as weights. The CES dual price formula is a CES aggregation of the input prices using the CES dual share parameters as weights and a different exponent. In a CGE model, the zero-profit condition or the dual price formula can be used interchangeably (with the proviso that the substitution elasticity differs from 1).[1] There is a simple formula for the budget shares given by:

$$s_i = \frac{P_i X_i}{P.V} = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{P}{P_i}\right)^{\sigma} V \left(\frac{P_i}{P}\right) \frac{1}{V} = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{P}{P_i}\right)^{\sigma-1} \tag{A.1-3}$$

Notice that this expression for the budget shares is only a function of prices. With the technology parameters set to 1, this simplifies further to:

$$s_i = \alpha_i \left(\frac{P}{P_i}\right)^{\sigma-1}$$

It turns out that the parameter σ measures the elasticity of substitution for the CES function and is constant over the entire domain. The elasticity of substitution is an indication of the curvature of an isoquant, see Varian (1992), i.e. it measures the rate of change of the ratio of inputs (in a 2-input case), relative to the change in their relative prices. For example, if the CES combines capital and labor to form output, a large substitution elasticity suggests that the factor proportions will change rapidly as one of the inputs becomes cheaper relative to the other. There are two limiting cases of interest. If the substitution elasticity is zero, then there is no substitution across inputs and the optimal choice is to use them in fixed proportion. At the other extreme, if

[1] We shall see below that when the substitution elasticity is 1, both primal and dual expressions take a different functional form.

the substitution elasticity is infinite, this is equivalent to saying the inputs are identical, and in this case, in equilibrium, the two inputs would have the same price. This could potentially be the case for electricity production. If there is a regional or national buyer of electricity, the buyer is most likely indifferent about how the electricity is produced and thus will purchase from the lowest cost producer (a perhaps somewhat simplified view of electricity markets.) This implies that the cost of the electricity inputs, from all sources (e.g. thermal, nuclear, etc.) would be (nearly) identical.

The elasticity of substitution across inputs is defined by the following formula:

$$\sigma = \frac{\partial\left(\frac{X_i}{X_j}\right)}{\partial\left(\frac{P_i}{P_j}\right)} \frac{\left(\frac{P_i}{P_j}\right)}{\left(\frac{X_i}{X_j}\right)}$$

The ratio of the optimal inputs using expression (A.1-1) is:

$$\frac{\alpha_i}{\alpha_j}\left(\frac{P_i}{P_j}\right)^{-\sigma}\left(\frac{\lambda_i}{\lambda_j}\right)^{\sigma-1}$$

Taking the partial derivative of the expression with respect to the ratio P_i/P_j and multiplying it by the second term of the elasticity of substitution yields the conclusion that the substitution elasticity is $-\sigma$. It is logical that it is negative. If the price of one input increases, say labor, relative to the other, say capital, producers would substitute away from labor towards capital, i.e. the ratio of labor to capital would drop as the price of labor increases relative to capital. Varian (1992) in fact defines the elasticity of substitution in terms of the absolute value of the technical rate of substitution, that measures the slope of the budget line. Numerically what it represents is the relative change in the ratios. If σ is 1, for example, and the price of labor increases by 10 percent relative to capital, the labor to capital ratio would decrease by (around) 10 percent.[2] The higher is σ, the more the proportion changes.

A.1.2 Special cases

There are three special cases that require additional derivations due to numerical restrictions on the primal and dual exponents. A substitution elasticity of 0 is clearly a special case and is referred to as a Leontief technology. From the dual price formula, it is clear that σ equal to 1 is a special case and is known as a Cobb-Douglas technology (or utility function). Finally, a value of ρ equal to 1 corresponds to infinite substitution elasticity and a linear primal aggregation function. This is also referred to as a case of perfect substitution.

The Leontief case

The first special case is for the so-called Leontief functional form.[3] In this case the substitution elasticity is 0 and corresponds to a value for ρ that is $-\infty$. In this case the optimization program takes the following form:[4]

$$\min_{X_i} \sum_i P_i X_i$$

subject to the constraint:

2 The elasticity is a marginal concept that holds only approximately for large changes.

3 Leontief, winner of the 1973 Nobel prize in Economics, is renowned for his work on input-output tables, much of which focused on fixed input technologies (!!!! reference).

4 !!!! need a reference

$$V = \min\left(\frac{a_i}{\lambda_i X_i}\right)$$

The visual implementation has L-shaped isoquants. The Leontief technology constraint or production/utility function is discontinuous. Fortunately, the optimal demand functions are easy to implement and are just special cases of expression (A.1-1):

$$X_i = \frac{\alpha_i}{\lambda_i}\frac{V}{A}$$

$$P = \frac{1}{A}\sum_i \alpha_i\left(\frac{P_i}{\lambda_i}\right)$$

Thus the Leontief specification implies that inputs are always in fixed proportion relative to output and the aggregate price is simply the linear weighted aggregation of the input prices, where the weights are given by the input-output coefficients, adjusted by changes in efficiency. The efficiency parameter has a nice intuitive interpretation in this case. Say λ increases by 10 percent, then demand for the input declines by 10 percent.

The Cobb-Douglas function

Another special case is the so-called Cobb-Douglas function, very frequently used in introductory text books in microeconomics. The Cobb-Douglas function has a substitution elasticity of 1 implying that ρ is equal to 0. Clearly, this creates a problem for specifying the CES primal function as well as the CES dual price function. As with the Leontief, the optimal demand conditions are given by expression (A.1-1), with σ set to 1:

$$X_i = \alpha_i\left(\frac{P}{P_i}\right)V \Leftrightarrow s_i = \frac{P_i X_i}{P.V} = \alpha_i$$

The Cobb-Douglas specification has constant budget shares irrespective of relative prices (and changes in technology). Another implication of the Cobb-Douglas specification is that the dual shares must add up to 1 as they are equivalent to the budget shares. By definition, as well, the primal and dual shares are the same. The Cobb-Douglas primal and dual price functions have the following expressions:

$$V = A\prod_i (\lambda_i X_i)^{\alpha_i}$$

$$P = \frac{1}{A}\prod_i\left(\frac{P_i}{\alpha_i\lambda_i}\right)^{\alpha_i}$$

Rather than code the Cobb-Douglas function as a special case, many modelers choose to replace the elasticity of 1 with a value close to 1 such as 1.01. This would have only marginal repercussions on the results.

Perfect substitution

The third special case is for a substitution elasticity of infinity. In this case ρ takes the value of 1 and the primal function is a straight linear aggregation of the inputs. The optimal demand conditions cannot be used in the case of an infinite substitution elasticity. In its stead, the optimal demand condition is replaced with the law-of-one-price, adjusted by efficiency differentials, and the zero profit condition is replaced with the CES primal function, i.e. the linear weighted aggregation of the inputs:

$$\frac{P_i}{\alpha_i \lambda_i} = P$$

$$V = \sum_i \alpha_i \lambda_i X_i$$

The aggregation function can be replaced by the zero profit condition:[5]

$$P.V = \sum_i P_i X_i$$

A.1.3 Calibration of the CES function

Calibration typically involves inverting functional forms to evaluate the value of a parameter given initial values for variables. Prices and volumes, P_i, X_i, V and P, are normally initialized to a given database or SAM. This may or may not include actual price/volume splits. If not, prices will typically be initialized at unit value—potentially adjusted for a price wedge such as a tax or a margin. The substitution elasticities are also normally inputs—either derived from econometric estimation, other data bases or models, or from a literature review. This leaves the parameters λ_i, α_i and A to calibrate. The technology parameters are normally associated with dynamics, so there is little reason not to initialize them to unit value as they can be incorporated in the initial share parameter value without any loss in generality. Thus, the only parameters left to calibrate are the α_i from which it is possible to derive the primal share parameters, a_i, if needed. The calibration formula is derived from the inversion of equation (A.1-1):

$$\alpha_i = \left(\frac{X_i}{V}\right)\left(\frac{P_i}{P}\right)^{\sigma}(A.\lambda_i)^{1-\sigma} = \left(\frac{X_i}{V}\right)\left(\frac{P_i}{P}\right)^{\sigma}$$

The right-most term is the most used formula where the technology parameters are explicitly set to 1.[6]

A.1.4 Comparative statics

Elasticities

This section will derive some of the key elasticities of the CES function. The first relationship is the elasticity of the aggregate price with respect to a component price:

$$\frac{\partial P}{\partial P_i}\frac{P_i}{P} = s_i = \frac{P_i X_i}{P.V}$$

[5] Modelers have the choice of using the primal aggregation function or the revenue function. The latter holds in all three special cases for the substitution elasticity.

[6] In many introductions to CGE models, the calibration formulas explicitly exclude the price term. This is a dangerous practice that can lead to model bugs that can be hard to detect. It is best to explicitly initialize prices to 1 and use the correct calibration formula. In fact, one way to test model calibration and specification is to initialize prices to an arbitrary value and initialize volumes subject to these prices. Simulating a counterfactual with no shocks should replicate the initial data solution. If not, there is an error in initialization, calibration and/or specification.

The elasticity of the aggregate price relative to an input price is equal to the budget share, irrespective of the substitution elasticity. The matrix of own- and cross-price elasticities, holding the aggregate volume constant is given by the following formula:

$$\varepsilon_{ij} = \frac{\partial X_i}{\partial P_j}\frac{P_j}{X_i} = \sigma(s_j - \delta_{ij})$$

where δ_{ij} is the so-called Kronecker's delta that takes the value 1 for i equal to j, else it takes the value 0. Since σ is positive, all components are gross substitutes in the CES.

Formulas in percent differences

It is useful in terms of comparative static analyses to convert the basic equations into percent differences. It is easy to trace out the impacts of a change in one of the 'exogenous' variables on demand and the overall price index. This is also the form of the equations used for models implemented in GEMPACK such as MONASH-style models.

The following expressions convey expressions (A.1-1) and (A.1-2) into their percent difference form:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \sigma\left(\dot{P} - \dot{P}_i\right) - (\sigma - 1)\left(\dot{A} + \dot{\lambda}_i\right)$$

$$\frac{\partial P}{P} = \dot{P} = -\dot{A} + \sum_i s_i \dot{P}_i - \sum_i s_i \dot{\lambda}_i = -\dot{A} + \sum_i s_i\left(\dot{P}_i - \dot{\lambda}_i\right)$$

Thus the percent change in the unit cost, P, for a change in the input price, P_i, all else equal, is (approximately) the value share of component i—as already noted above.

A.1.5 Growth Accounting

Use can be made of the linearization above to derive the linearized growth accounting formula:

$$\frac{\Delta V}{V} = \frac{\Delta A}{A} + \sum_i s_i \frac{\Delta x_i}{x_i} + \sum_i s_i \frac{\Delta \lambda_i}{\lambda_i}$$

A.1.6 Parameter twists

The basic analytics

This final section on the CES describes how to adjust the share parameters in a dynamic scenario under a specific assumption—this is called the twist adjustment and is a core feature of the dynamic MONASH model, see Dixon and Rimmer (2002). The basic idea is to alter the share parameter, in a two-component CES, to target a given change in the ratio of the two components, however, with neutral impacts on the aggregate cost. For example, the target may be a cost-neutral increase in the capital/labor ratio by x%, or an increase in the import to domestic ratio of y%.

The ratio of the two components is given by the following expression using equation (A.1-1) as the starting point:

$$R = \frac{\alpha_1 \lambda_1^{\sigma-1} {P_2}^{\sigma}}{\alpha_2 \lambda_2^{\sigma-1} {P_1}^{\sigma}}$$

The idea is to move the initial ratio, R_{t-1} to R_t by tw percent.

$$\frac{R_t}{R_{t-1}} = (1 + tw_t)$$

Using the formulas above, we have:

$$\frac{R_t}{R_{t-1}} = (1 + tw_t) = \frac{\left(\frac{\lambda_{1,t}}{\lambda_{1,t-1}}\right)^{\sigma-1}}{\left(\frac{\lambda_{2,t}}{\lambda_{2,t-1}}\right)^{\sigma-1}} = \frac{(1 + \pi_{1,t})^{\sigma-1}}{(1 + \pi_{2,t})^{\sigma-1}}$$

The π variables represent the growth (either positive or negative) that will be applied to the technology parameters under the assumption of cost-neutral technological change. We can start with the dual cost function for year t, but with year $t-1$ prices:

$$\begin{aligned} P_{t-1}^{1-\sigma} &= \alpha_1 \left(\frac{P_{1,t-1}}{\lambda_{1,t}}\right)^{1-\sigma} + \alpha_2 \left(\frac{P_{2,t-1}}{\lambda_{2,t}}\right)^{1-\sigma} \\ &= \alpha_1 (1 + \pi_{1,t})^{\sigma-1} \left(\frac{P_{1,t-1}}{\lambda_{1,t}}\right)^{1-\sigma} + \alpha_2 (1 + \pi_{2,t})^{\sigma-1} \left(\frac{P_{2,t-1}}{\lambda_{2,t}}\right)^{1-\sigma} \end{aligned}$$

Recall that the share equation is given by:

$$s_{i,t-1} = \alpha_i \lambda_{i,t-1}^{\sigma-1} \left(\frac{P_t}{P_{i,t-1}}\right)^{\sigma-1}$$

Dividing through the expression above by $P_t^{1-\sigma}$ and inserting the share expressions for year $t-1$, we end up with:

$$1 = s_{1,t-1}(1 + \pi_{1,t})^{\sigma-1} + s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}$$

Solving in terms of π_1, we have:

$$(1 + \pi_{1,t})^{\sigma-1} = \frac{1 - s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}}{s_{1,t-1}}$$

and this can be inserted into the twist target formula to get:

$$1 + tw_t = \frac{1 - s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}}{s_{1,t-1}(1 + \pi_{2,t})^{\sigma-1}} = \frac{(1 + \pi_{2,t})^{1-\sigma} - s_{2,t-1}}{s_{1,t-1}}$$

Finally, π_2 can be isolated to yield:

$$1 + \pi_{2,t} = [s_{1,t-1}(1 + tw_t) + s_{2,t-1}]^{1/(1-\sigma)} = [1 + s_{1,t-1} tw_t]^{1/(1-\sigma)}$$

We can re-insert this into the expression above to derive an expression for π_1:

$$1 + \pi_{1,t} = \left[\frac{1 + s_{1,t-1} tw_t}{1 + tw_t}\right]^{1/(1-\sigma)}$$

Finally, the productivity update formulas that incorporate the twist adjustment take the form:

$$\lambda_{1,t} = (1 + \pi_{1,t})\lambda_{1,t-1} = \left[\frac{1 + s_{1,t-1} tw_t}{1 + tw_t}\right]^{1/(1-\sigma)} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t})\lambda_{2,t-1} = [1 + s_{1,t-1} tw_t]^{1/(1-\sigma)} \lambda_{2,t-1}$$

It is possible to generalize these formulas by partitioning the set of CES components into two sets—a set indexed by 1 that is the target set, and a set indexed by 2 that is the complement. For example, think of a set of electricity technologies that includes conventional and advanced. It is possible then to provide the same twist to all of the new technologies relative to the conventional technologies. The only change in the formulas above is that the share variable for the single component is replaced by the sum of the shares for the bundle of components:

$$\lambda_{1,t} = (1+\pi_{1,t})\lambda_{1,t-1} = \left[\frac{1+tw_t \sum_{i\in 1} s_{i,t-1}}{1+tw_t}\right]^{1/(1-\sigma)} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1+\pi_{2,t})\lambda_{2,t-1} = \left[1+tw_t \sum_{i\in 1} s_{i,t-1}\right]^{1/(1-\sigma)} \lambda_{2,t-1}$$

Converting to percent differences

The π factors reflect a percentage change in the relevant productivity factors for each of the components. Using a Taylor series approximation, the formulas above can be converted to a linear equation that is used by the Monash-style models. For the first component, we have:

$$\pi_1 = F(tw) = \left[\frac{1+s_1 tw}{1+tw}\right]^{1/(1-\sigma)} - 1 \approx F(0) + tw.F'(0) = -tw\frac{1-s_1}{1-\sigma}$$

For the second component we have:

$$\pi_2 = F(tw) = [1+s_1 tw]^{1/(1-\sigma)} - 1 \approx F(0) + tw.F'(0) = tw\frac{s_1}{1-\sigma}$$

Note that in the Monash models, the signs are reversed because the productivity factors divide the volume components whereas in the formulation above the productivity factors are multiplicative.

Examples of twisting the share parameters

We demonstrate these concepts with two examples. The first is a CES production function of capital and labor, where the labor share is 60% and the capital/labor substitution elasticity (i.e. σ) is set to 0.9. Prices are initialized at 1, therefore the original capital/labor ratio is 2/3. The target is to raise the capital/labor ratio 10% assuming cost neutrality. Table A.1 shows the key results. Labor efficiency would increase by 48% and capital efficiency would decline by 43%.

Table A.1: Example of capital/labor twist

	Labor	Capital	Capital/labor ratio
Initial	60.0	40.0	0.6667
After twist	57.7	42.3	0.7333
Percent change	-3.8	5.8	10.0
Growth factor	0.48	-0.43	

The second example comes from trade and the Armington assumption. Assume an 80/20 split between domestic goods and imports in value and volume implying a ratio of imports to demand of domestic goods of 0.25. Table A.2 shows the twist parameters needed to achieve an increase in this ratio of 10 percent with an Armington elasticity of 2. The preference parameter for imports increases by nearly 8 percent, while that for domestic goods decreases by 2 percent.

Table A.2: Example of Armington import/domestic twist

	Domestic	Import	Import/domestic ratio
Initial	80.0	20.0	0.250
After twist	78.4	21.6	0.275
Percent change	-2.0	7.8	10.0
Growth factor	-0.02	0.08	

A.1.7 Summary

In summary, the CES functional form is often used as a production (or sub-production) function that combines two or more inputs to form output (or an intermediate composite bundle), under the assumption of cost minimization. It is also frequently used to maximize utility (or sub-utility) over a set of two or more goods, again with the assumption of cost minimization. Table A.3 highlights the two main expressions to emerge from the optimization—the derived demand functions, X_i, and the CES dual price expression, P. The top row shows the expression with all technology parameters initialized at 1, and the bottom row the most generic version.

Table A.3: Key equations for CES implementation

	Demand	Aggregate price
Basic	$X_i = \alpha_i V \left(\frac{P}{P_i}\right)^{\sigma}$	$P = \left[\sum_i \alpha_i P_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$
with full technology	$X_i = \alpha_i (A\lambda_i)^{\sigma-1} V \left(\frac{P}{P_i}\right)^{\sigma}$	$P = \frac{1}{A}\left[\sum_i \alpha_i \left(\frac{P}{\lambda_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$

A.2 The CET Function

A.2.1 The basic formulation

This section describes the constant-elasticity-of-transformation (CET) function. The CET function is often used to describe a transformation frontier between two or more outputs. For example, a producer may produce two or more products and decides how much of each to produce based on market conditions, i.e. relative prices. The CET is often used to represent a producer's decision on the allocation of output between domestic and foreign markets. Another example is land supply, where land will be allocated across different uses according to the relative returns. The transformation elasticity is assumed to be uniform between any pair of outputs and therefore is analogous to the demand-based CES function described in detail above. The exposition of the CET will be much more succinct than that of the CES because most of the derivations can be derived in a similar fashion.

The CET can be setup as a revenue maximization problem, subject to a transformation frontier:

$$\max_{X_i} \sum_i P_i X_i$$

subject to

$$V = A\left[\sum_i g_i(\lambda_i X_i)^v\right]^{1/v}$$

where V is the aggregate volume (e.g. aggregate supply), X_i are the relevant components (sector-specific supply), P_i are the corresponding prices, g_i are the CET (primal) share parameters, and ν is the CET exponent. The CET exponent is related to the CET transformation elasticity, ω via the following relation:

$$\nu = \frac{\omega+1}{\omega} \Leftrightarrow \omega = \frac{1}{\nu-1}$$

The transformation elasticity is assumed to be positive. Solution of this maximization problem leads to the following first order conditions:

$$X_i = \gamma_i (A\lambda_i)^{-1-\omega}\left(\frac{P_i}{P}\right)^{\omega} V \tag{A.1-4}$$

and

$$P = \frac{1}{A}\left[\sum_i \gamma_i \left(\frac{P_i}{\lambda_i}\right)^{1+\omega}\right]^{1/(1+\omega)} \tag{A.1-5}$$

where the γ_i parameters are related to the primal share parameters, g_i, by the following formula:

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \left(\frac{1}{\gamma_i}\right)^{1/\omega}$$

From expression A.1-4, and ignoring the technology parameters for the moment, the clear difference with the CES expression for optimal demand (equation A.1-1) is that the component price is in the numerator and the aggregate price in the denominator. This is intuitively logical. If the supply price to a market goes up relative to the average market price, one would anticipate that supply would increase to that market. The greater the transformation elasticity the greater are the market shifts.

Calibration is similar to the CES case. Prices and volumes are initialized using base year data. Equation (A.1-4) can then be inverted to calculate the share parameters, γ_i, with typically the technology parameters initialized to the value 1. In most implementations, there is no need to carry around the primal share parameters, nor the primal exponent.

The main interesting case for the CET is the case of perfect transformation, i.e. the transformation elasticity is infinity. In this case the CET exponent is 0 and the aggregation function is a linear weighted aggregation of the components. The standard CET equations are then replaced by the law-of-one price and the linear aggregation function (or alternatively, the zero profit condition).

$$\frac{P_i}{A\lambda_i} = P\forall i$$

$$A\sum_i \lambda_i X_i = X$$

A.2.2 Converting to percent differences

It is easier to interpret or decompose the results of a simulation by looking at the CET equations in percent differences form—that is the standard form for MONASH-style models and implementation in GEMPACK. The following equations show the equations in percent difference form:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \omega\left(\dot{P}_i - \dot{P}\right) - (\omega+1)\left(\dot{\lambda}_i + \dot{A}\right)$$

$$\frac{\partial P}{P} = \dot{P} = -\dot{A} + \sum_i s_i\dot{P}_i - \sum_i s_i\dot{\lambda}_i = -\dot{A} + \sum_i s_i\left(\dot{P}_i - \dot{\lambda}_i\right)$$

where the variable s_i is the value share of component i in total revenue:

$$s_i = \frac{P_i X_i}{P \cdot V} = \gamma_i\left(\frac{P_i}{A \cdot \lambda_i \cdot P}\right)^{\omega+1}$$

A.3 Modified CES and CET that incorporate additivity

The standard CET supply allocation specification does not preserve physical additivity, i.e. the sum of the volume components is not necessarily equal to the total volume. There are a number of alternative specifications that do preserve volume homogeneity, for example the multinomial logit. One alternative, described below, uses a modified form of the CET preference function. This specification has been used for labor and land supply allocations (see respectively Dixon and Rimmer (2006) and Giesecke et al. (2013)).

A.3.1 The CET implementation

The CET alternative involves solving the following optimization:

$$\max_{X_i} U = \left[\sum_i g_i\left(\lambda_i P_i X_i\right)^{\nu}\right]^{1/\nu}$$

subject to the constraint:

$$V = \sum_i X_i$$

The variable definitions are similar to above, X_i are the volume components, P_i are the relevant component prices and V is aggregate volume. The λ_i parameters are preference parameters. The CET utility function is not simply a function of the volumes, but explicitly a function of the preference-adjusted revenues of the individual components. The closed-form solution to the above system is the following set of equations:

$$X_i = \gamma_i V\left(\frac{\lambda_i P_i}{P^c}\right)^{\omega} \quad \text{(A.1-6)}$$

$$P^c = \left[\sum_i \gamma_i\left(\lambda_i P_i\right)^{\omega}\right]^{1/\omega} \quad \text{(A.1-7)}$$

Both equations are similar to their standard CET counterparts, but with some differences. First, P^c is a price index, but it is not the average price of the components, i.e. $P^c X \neq \sum_i P_i X_i$. Second, this price index is based on ω not $1+\omega$ as in the standard CET dual price expression. The revenue correct price index is defined by the following formula:

$$P = \frac{\sum\limits_i \gamma_i \lambda_i^\omega P_i^{\omega+1}}{\sum\limits_i \gamma_i \lambda_i^\omega P_i^\omega} = \frac{\sum\limits_i \gamma_i \lambda_i^\omega P_i^{\omega+1}}{(P^c)^\omega} = \sum_i \gamma_i P_i \left(\frac{\lambda_i P_i}{P^c}\right)^\omega = \sum_i \frac{X_i}{V} P_i \qquad \text{(A.1-8)}$$

The other transformations include:

$$\gamma_i = {g_i}^{1+\omega}$$

$$\omega = \frac{\nu}{1-\nu} \iff \nu = \frac{\omega}{1+\omega}$$

It is worth noting that the relation between ω and ν differs from the standard CET relation as the respective formula is inverted. The implication of this is that ν is bounded below by 0 instead of ∞, but is otherwise positive over the entire (positive) range of ω. And, in both the standard and adjusted CET ν converges to 1 as ω converges to ∞. As regards calibration, there is an extra degree of freedom as the value for utility is not specified. It is easiest to simply set P^c to 1 as for given P_i and λ_i the calibration of the γ parameters is straightforward:

$$\gamma_i = \frac{X_i}{V}\left(\frac{\lambda_i P_i}{P^c}\right)^{-\omega}$$

If prices and technology or preference parameters are initialized at 1, the calibrated γ parameters are equal to the initial volume shares.

Converting this to a Monash-style equation in percent differences, the derived supply function is:

$$\dot{X}_i = \dot{V} + \omega \left[\dot{P}_i + \dot{\lambda}_i - \sum_{j=1}^n \frac{X_j}{V}\left(\dot{P}_j + \dot{\lambda}_j\right)\right]$$

This equation uses volume shares as weights for cross-price (and cross-preference) effects. In the standard CET formulation, value shares are used as weights.

The standard specification needs some modifications for two special cases—perfect transformation and perfect immobility. The case of perfect transformation, i.e. a transformation elasticity of ∞, leads to all prices moving in unison with the aggregate price index. Thus equation (A.1-6) is replaced with the following expression:

$$\lambda_i P_i = P^c$$

where λ_i is calibrated to the initial price ratios. The price index expression, equation (A.1-7) is replaced with the volume constraint:

$$V = \sum_i X_i$$

In the model implementation of the adjusted CET, this latter expression can be used in all cases and can replace equation (A.1-7).

The case of zero mobility is readily implemented by dropping completely equation (A.1-7) (or its equivalent, i.e. the volume adding up constraint). With a transformation elasticity of 0, the price composite index in equation (A.1-6) simply drops out and the volume components are in strict proportion to the aggregate volume.

A.3.2 The CES implementation

The adjusted CET (and CRETH) functions replace their counterparts for the allocation problem that preserves additivity. Analogous specifications exist for the CES and CRESH functions that emulate the implementation of their standard counterparts but also allow for additivity.

The CES alternative involves solving the following optimization:

$$\min_{X_i} U = \left[\sum_i a_i \left(\lambda_i P_i X_i\right)^{\rho}\right]^{1/\rho}$$

subject to the constraint:

$$V = \sum_i X_i$$

As in the case of the adjusted CET, the adjusted CES utility function is a function of the preference adjusted cost components. The closed-form solution to the above system is the following set of equations:

$$X_i = \alpha_i V \left(\frac{P^c}{\lambda_i P_i}\right)^{\sigma} \tag{A.1-9}$$

$$P^c = \left[\sum_i \alpha_i \left(\lambda_i P_i\right)^{-\sigma}\right]^{-1/\sigma} \tag{A.1-10}$$

Both equations are similar to their standard CES counterparts, but with some differences. First, P^c is a price index, but it is not the average price of the components, i.e. $P^c X \neq \sum_i P_i X_i$. Second, this price index is based on $-\sigma$ not $1-\sigma$ as in the standard CES dual price expression. The revenue correct price index is defined by the following formula:

$$P = \frac{\sum\limits_i \alpha_i \lambda_i^{\sigma} P_i^{1-\sigma}}{\sum\limits_i \alpha_i \lambda_i^{\sigma} P_i^{\sigma}} = \frac{\sum\limits_i \alpha_i \lambda_i^{\sigma} P_i^{1-\sigma}}{\left(P^c\right)^{-\sigma}} = \sum_i \alpha_i P_i \left(\frac{\lambda_i P_i}{P^c}\right)^{-\sigma} = \sum_i \frac{X_i}{V} P_i \tag{A.1-11}$$

The other transformations include:

$$\alpha_i = {a_i}^{1-\sigma}$$

$$\sigma = \frac{\rho}{\rho - 1} \iff \rho = \frac{\sigma}{\sigma - 1}$$

It is worth noting that the relation between σ and ρ differs from the standard CES relation as the respective formula is inverted. The implication of this is that ρ is bounded below by 0 instead of $-\infty$. It decreases towards $-\infty$, as σ increases towards 1, which is a discontinuity point. It decreases from ∞ towards 1 as σ increases from 1 to ∞.

It is relatively easy to show that the following simple expression holds for the utility function:

$$U = P^c V \tag{A.1-12}$$

As regards calibration, there is an extra degree of freedom as the value for utility is not specified. It is easiest to simply set P^c to 1 as for given P_i and λ_i the calibration of the α parameters is straightforward:

$$\alpha_i = \frac{X_i}{V}\left(\frac{\lambda_i P_i}{P^c}\right)^{\sigma}$$

If prices and technology or preference parameters are initialized at 1, the calibrated α parameters are equal to the initial volume shares.

Converting this to a Monash-style equation in percent differences, the derived demand function is:

$$\dot{X}_i = \dot{V} - \sigma\left[\dot{P}_i + \dot{\lambda}_i - \sum_{j=1}^{n}\frac{X_j}{V}\left(\dot{P}_j + \dot{\lambda}_j\right)\right]$$

This equation uses volume shares as weights for cross-price (and cross-preference) effects. In the standard CES formulation, value shares are used as weights.

A.3.3 Using twists with the adjusted CES

The 'twist' idea described for the normal CES can be applied to the adjusted CES. The concept is somewhat different given the type of optimization problem posed. Rather than change the share parameters in a given direction with cost neutrality, the idea is to change the share parameters with utility neutrality. The problem posed, therefore, is to change the ratio of demand for two goods by a specified amount, while maintaining the same level of utility.

The ratio of the two components is given by the following expression using equation (A.1-9) as the starting point:

$$R = \frac{\alpha_1 \lambda_2 {P_2}^{\sigma}}{\alpha_2 \lambda_1 {P_1}^{\sigma}}$$

The idea is to move the initial ratio, R_{t-1} to R_t by tw percent.

$$\frac{R_t}{R_{t-1}} = (1 + tw_t)$$

while holding U constant. The two expressions above imply that the preference shifters, given by the π parameters, are linked via the following expression:

$$1 + \pi_2 = (1 + \pi_1)\,(1 + tw)^{1/\sigma} \tag{A.1-13}$$

Given equation (A.1-12), holding U constant is equivalent to holding the price index, P^c, constant as well (for a fixed aggregate volume). Thus we can solve the following equation for the parameter π_1:

$$
\begin{aligned}
\left(P_{t-1}^{c}\right)^{-\sigma} &= \alpha_1\left(P_{1,t-1}\lambda_{1,t-1}\right)^{-\sigma}+\alpha_2\left(P_{2,t-1}\lambda_{2,t-1}\right)^{-\sigma}\\
&= \alpha_1\left(P_{1,t-1}\lambda_{1,t-1}\left(1+\pi_{1,t}\right)\right)^{-\sigma}+\alpha_2\left(P_{2,t-1}\lambda_{2,t-1}\left(1+\pi_{2,t}\right)\right)^{-\sigma}\\
&= \alpha_1\left(P_{1,t-1}\lambda_{1,t-1}\left(1+\pi_{1,t}\right)\right)^{-\sigma}+\alpha_2\left(P_{2,t-1}\lambda_{2,t-1}\left(1+\pi_{1,t}\right)\left(1+tw\right)^{1/\sigma}\right)^{-\sigma}\\
&= \left(P_{t}^{c}\right)^{-\sigma}
\end{aligned}
$$

The π variables represent the growth (either positive or negative) that will be applied to the preference parameters under the assumption of utility-preserving preference shifts. This formula can be written in terms of the initial volume shares, $r_i = X_i/V$, simplified and re-arranged to yield:

$$(1+\pi_1)^{\sigma} = r_1 + \frac{r_2}{1+tw}$$

and when re-inserted in equation (A.1-13) we get:

$$(1+\pi_2)^{\sigma} = \left(r_1 + \frac{r_2}{1+tw}\right)(1+tw)$$

The final formulas for the two twist parameters only depend on the initial volume shares, the substitution elasticity and the level of the 'twist':

$$\pi_1 = \left(r_1 + \frac{r_2}{1+tw}\right)^{1/\sigma} - 1 \tag{A.1-14}$$

$$\pi_2 = \left(r_1\left(1+tw\right)+r_2\right)^{1/\sigma} - 1 \tag{A.1-15}$$

It is possible to generalize these formulas by partitioning the set of CES components into two sets—a set indexed by 1 that is the target set, and a set indexed by 2 that is the complement. For example, think of a set of electricity technologies that includes conventional and advanced. It is possible then to provide the same twist to all of the new technologies relative to the conventional technologies. The only change in the formulas above is that the volume share variable for the single component is replaced by the sum of the volume shares for the bundle of components:

$$\lambda_{1,t} = (1+\pi_{1,t})\lambda_{1,t-1} = \left[\sum_{i\in 1} r_{i,t-1} + \frac{\sum_{i\in 2} r_{i,t-1}}{1+tw_t}\right]^{1/\sigma}\lambda_{1,t-1}$$

$$\lambda_{2,t} = (1+\pi_{2,t})\lambda_{2,t-1} = \left[\sum_{i\in 1}\left(1+tw_t\right)r_{i,t-1} + \sum_{i\in 2} r_{i,t-1}\right]^{1/\sigma}\lambda_{2,t-1}$$

Converting to percent differences

The π factors reflect a percentage change in the relevant productivity factors for each of the components. Using a Taylor series approximation, the formulas above can be converted to a linear equation that is used by the Monash-style models. For the first component, we have:

$$\pi_1 = F(tw) = \left[r_1 + \frac{r_2}{1+tw}\right]^{1/\sigma} - 1 \approx F(0) + tw.F'(0) = -tw\frac{r_2}{\sigma}$$

For the second component we have:

$$\pi_2 = F(tw) = \left[r_1\left(1+tw\right)+r_2\right]^{1/\sigma} - 1 \approx F(0) + tw.F'(0) = tw\frac{r_1}{\sigma}$$

Appendix B

The demand systems

The model contains four different possible demand systems for determining household demand for goods and services:

- CDE or constant differences in elasticities—largely derived from the GTAP model
- ELES or extended linear expenditure system
- LES or linear expenditure system
- AIDADS of an implicitly directly additive demand system, an extension of the LES that allows for more plausible Engel behavior

The default demand system is the CDE demand system. It is a relatively flexible funtional form allowing for non-homotheticity. It is not ideal for dynamic scenarios, in the absence of re-calibration of parameters, as the income elasticities are relatively invariant to changes in per capita income. The AIDADS has more plausible Engel behavior, but is more difficult to calibrate and has less than ideal flexibility in terms of cross-price elasticities.

Three of the demand systems (CDE, LES, AIDADS) use a two-tiered structure to first allocate income between savings and expenditures on goods and services. The ELES integrates the savings allocation within its specification. All four systems determine the demand for consumer goods that are different from produced goods. A transition matrix approach is subsequently used to convert consumer goods into produced goods.

B.1 The CDE demand system

The *Constant Difference of Elasticities* (CDE) function is a generalization of the CES function, but it allows for more flexibility in terms of substitution effects across goods and for non-homotheticity.[1] The starting point is an implicitly additive indirect utility function (see Hanoch (1975)) from which we can derive demand using Roy's identity (and the implicit function theorem).

[1] More detailed descriptions of the CDE can be found in Hertel et al. (1991), Surry (1993) and Hertel (1997).

B.1.1 General form

A dual approach is used to determine the properties of the CDE function. The indirect utility function is defined implicitly via the following expression:

$$V(p,u,Y) = \sum_{i=1}^{n} \alpha_i u^{e_i b_i} \left(\frac{p_i}{y}\right)^{b_i} \equiv 1 \tag{B.2-1}$$

where p is the vector of commodity prices, u is (per capita) utility and y is per capita income. Using Roy's identity and the implicit function theorem[2] we can derive demand, x, where v is the indirect utility function (defined implicitly):

$$x_i = -\frac{\partial v}{\partial p_i} / \frac{\partial v}{\partial Y} = -\left(\frac{\partial V}{\partial p_i} / \frac{\partial V}{\partial u}\right) / \left(\frac{\partial V}{\partial Y} / \frac{\partial V}{\partial u}\right) = -\left(\frac{\partial V}{\partial p_i} / \frac{\partial V}{\partial Y}\right) \tag{B.2-2}$$

This then leads to the following demand function:

$$x_i = \frac{\alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{y}\right)^{b_i - 1}}{\sum\limits_j \alpha_j b_j u^{e_j b_j} \left(\frac{p_j}{y}\right)^{b_j}} \tag{B.2-3}$$

Implementation is easier if we define the following variable:

$$\theta_i = \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{y}\right)^{b_i} \tag{B.2-4}$$

Then the budget shares can be expressed as:

$$s_i = \frac{\theta_i}{\sum_j \theta_j} \tag{B.2-5}$$

and the demand expression is:

$$x_i = \frac{s_i}{p_i} y \tag{B.2-6}$$

Implementation also requires evaluating u. This can be done by implementing equation (B.2-1) and inserting the expression for θ:

$$\sum_{i=1}^{n} \frac{\theta_i}{b_i} \equiv 1 \tag{B.2-7}$$

B.1.2 Elasticities

In order to calibrate the CDE system, it is necessary to derive the demand and income elasticities of the CDE. The algebra is tedious, but straightforward.

[2] See Varian (1992), p. 109.

The own-price elasticity is given by the following:

$$\varepsilon_i = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = \frac{s_i\left[\sum_j s_j e_j b_j - e_i b_i\right]}{\sum_j s_j b_j} + b_i\left(1 - s_i\right) - 1 \tag{B.2-8}$$

In deriving the elasticity, we make use of the following formula that defines the elasticity of utility with respect to price (and again makes use of the implicit function theorem):

$$\frac{\partial u}{\partial p_i}\frac{p_i}{u} = -\frac{p_i}{u}\left(\frac{\partial V}{\partial p_i}\right)/\left(\frac{\partial V}{\partial u}\right) = -\frac{s_i}{\sum_j s_j e_j} \tag{B.2-9}$$

The price elasticity of utility is approximately the value share of the respective demand component as long as the weighted sum of the expansion parameters, e, is close to unity. The value (or budget) share is defined in the next equation:

$$s_i = \frac{p_i x_i}{y} \tag{B.2-10}$$

Letting $\sigma_i = 1 - b_i$ (or $b_i = 1 - \sigma_i$), we can also write:

$$\varepsilon_i = s_i\left[\sigma_i - \frac{e_i(1-\sigma_i)}{\sum_j s_j e_j} - \frac{\sum_j s_j e_j \sigma_j}{\sum_j s_j e_j}\right] - \sigma_i \tag{B.2-11}$$

With σ uniform, we also have:

$$\varepsilon_i = -\frac{s_i e_i(1-\sigma)}{\sum_j s_j e_j} - \sigma \tag{B.2-12}$$

With both e and σ uniform, the formula simplifies to:

$$\varepsilon_i = -s_i(1-\sigma) - \sigma = \sigma(s_i - 1) - s_i \tag{B.2-13}$$

Equation (B.2-13) reflects the own-price elasticity for the standard CES utility function. Finally, with e uniform but not σ, we have:

$$\varepsilon_i = s_i\left[2\sigma_i - 1 - \sum_j s_j \sigma_j\right] - \sigma_i \tag{B.2-14}$$

The derivation of the cross elasticities is almost identical and will not be carried out here. Combining both the own-and cross price elasticities, the matrix of substitution elasticities takes the following form where we use the Kronecker product, δ:[3]

$$\varepsilon_{ij} = s_j\left[-b_j - \frac{e_i b_i}{\sum_k s_k e_k} + \frac{\sum_k s_k e_k b_k}{\sum_k s_k e_k}\right] + \delta_{ij}(b_i - 1) \tag{B.2-15}$$

[3] δ takes the value of 1 along the diagonal (i.e. when $i = j$) and the value 0 off-diagonal (i.e. when $i \neq j$).

Again, we replace b by $1-\sigma$, to get:

$$\varepsilon_{ij} = s_j \left[\sigma_j - \frac{e_i(1-\sigma_i)}{\sum\limits_k s_k e_k} - \frac{\sum\limits_k s_k e_k \sigma_k}{\sum\limits_k s_k e_k} \right] - \delta_{ij}\sigma_i \tag{B.2-16}$$

For uniform σ, equation (B.2-16) takes the form:

$$\varepsilon_{ij} = -\frac{e_i s_j (1-\sigma)}{\sum\limits_k s_k e_k} - \delta_{ij}\sigma \tag{B.2-17}$$

And with a uniform σ and e, i.e. the CES assumption, we have:

$$\varepsilon_{ij} = -s_j(1-\sigma) - \delta_{ij}\sigma = \sigma(s_j - \delta_{ij}) - s_j \tag{B.2-18}$$

Finally, for a uniform e only, the matrix of elasticities is:

$$\varepsilon_{ij} = s_j \left[\sigma_j - (1-\sigma_i) - \sum_k s_k \sigma_k \right] - \delta_{ij}\sigma_i \tag{B.2-19}$$

The income elasticities are derived in a similar fashion:

$$\eta_i = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i} = \frac{1}{\sum\limits_k s_k e_k} \left[e_i b_i - \sum_k s_k e_k b_k \right] - (b_i - 1) + \sum_k b_k s_k \tag{B.2-20}$$

For this, we need the elasticity of utility with respect to income:

$$\frac{\partial u}{\partial Y}\frac{Y}{u} = -\frac{Y}{u}\left(\frac{\partial V}{\partial Y}\right) / \left(\frac{\partial V}{\partial u}\right) = \frac{1}{\sum\limits_k s_k e_k} \tag{B.2-21}$$

Note that for a uniform and unitary e, the income elasticity of utility is 1.
Replacing b with $1-\sigma$, equation (B.2-20) can be re-written to be:

$$\eta_i = \frac{1}{\sum\limits_k s_k e_k} \left[e_i(1-\sigma_i) + \sum_k s_k e_k \sigma_k \right] + \sigma_i - \sum_k s_k \sigma_k \tag{B.2-22}$$

With a uniform σ, the income elasticity becomes:

$$\eta_i = \frac{1}{\sum\limits_k s_k e_k} \left[e_i(1-\sigma) + \sigma \sum_k s_k e_k \right] = \frac{e_i(1-\sigma)}{\sum\limits_k s_k e_k} + \sigma \tag{B.2-23}$$

With e uniform, the income elasticity is unitary, irrespective of the values of the σ parameters.

From the Slutsky equation, we can calculate the compensated demand elasticities:

$$\xi_{ij} = \varepsilon_{ij} + s_j \eta_i = -\delta_{ij}\sigma_i + s_j \left[\sigma_j + \sigma_i - \sum_k s_k \sigma_k \right] \tag{B.2-24}$$

The cross-Allen partial elasticities are equal to the compensated demand elasticities divided by the share:

$$\sigma_{ij}^a = \sigma_j + \sigma_i - \sum_k s_k \sigma_k - \delta_{ij}\sigma_i / s_j \tag{B.2-25}$$

It can be readily seen that the difference of the partial elasticities is constant, hence the name of *constant difference in elasticities.*

$$\sigma_{ij}^a - \sigma_{il}^a = \sigma_j - \sigma_l \tag{B.2-26}$$

With a uniform σ, we revert back to the standard CES where there is equivalence between the CES substitution elasticity and the cross-Allen partial elasticity:

$$\sigma_{ij}^a = \sigma \tag{B.2-27}$$

B.1.3 Calibration of the CDE

Calibration assumes that we know the budget shares, the own uncompensated demand elasticities and the income elasticities. The weighted sum of the income elasticities must equal 1, so the first step in the calibration procedure is to make sure Engel's law holds. One alternative is to fix some (or none) of the income elasticities and re-scale the others using least squares. The problem is to minimize the following objective function:

$$\sum_{i \in \Omega} \left(\eta_i - \eta_i^0\right)^2$$

subject to

$$\sum_{i \in \Omega} s_i \eta_i = 1 - \sum_{i \notin \Omega} s_i \eta_i$$

where the set Ω contains all sectors where the income elasticity is not fixed, i.e. its complement contains those sectors with fixed income elasticities. The solution is:

$$\eta_i = \eta_i^0 + s_i \frac{1 - \sum_{j \notin \Omega} s_j \eta_j - \sum_{j \in \Omega} s_j \eta_j^0}{\sum_{j \in \Omega} s_j^2} \quad \forall i \in \Omega$$

Calibration of the σ parameters is straightforward given the own elasticities and the input value shares. The first step is to calculate the Allen partial elasticities, these are simply the income elasticity adjusted by the own elasticities divided by the budget shares:

$$\sigma_{ii}^a = \eta_i + \frac{\varepsilon_{ii}}{s_i} \tag{B.2-28}$$

Next, equation (B.2-25) is setup in matrix form:

$$\sigma_{ii}^a = A\sigma_i \tag{B.2-29}$$

where the matrix A has the form:

$$A = \begin{bmatrix} 2 - \dfrac{1}{s_1} - s_1 & -s_2 & \dots & -s_n \\ -s_1 & 2 - \dfrac{1}{s_2} - s_2 & \dots & -s_n \\ \vdots & \vdots & \ddots & \vdots \\ -s_1 & -s_2 & \dots & 2 - \dfrac{1}{s_n} - s_n \end{bmatrix} \tag{B.2-30}$$

or each element of A has the following formula:

$$a_{ij} = \delta_{ij}(2 - 1/s_i) - s_j$$

We can then solve for σ (and back-out the b parameters):

$$\sigma_i = A^{-1}\sigma_{ii}^a \tag{B.2-31}$$

There is nothing which guarantees the consistency of the calibrated σ parameters, which are meant to be positive. The calculation of the σ parameters depends only on the budget shares and the own-price uncompensated elasticities. If the calibrated σ parameters are not all positive, one could modify the elasticities until consistency is achieved. In practice, problems have occurred when a sector's budget share dominates total expenditure.

The e parameters are derived from Equation (B.2-22) and normalizing them so that their share weighted sum is equal to 1. Equation (B.2-22) can then be converted to matrix form and inverted:

$$B = \begin{bmatrix} s_1\sigma_1 + (1-\sigma_1) & s_2\sigma_2 & \dots & s_n\sigma_n \\ s_1\sigma_1 & s_2\sigma_2 + (1-\sigma_2) & \dots & s_n\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ s_1\sigma_1 & s_2\sigma_2 & \dots & s_n\sigma_n + (1-\sigma_n) \end{bmatrix} \tag{B.2-32}$$

or

$$b_{ij} = s_j\sigma_j + \delta_{ij}(1 - \sigma_i)$$

Then the e parameters are derived from matrix inversion:

$$e_i = B^{-1}C_i = B^{-1}\left(\eta_i - \sigma_i + \sum_k s_k\sigma_k\right) \tag{B.2-33}$$

Calibration of the α parameters is based on equations (B.2-1) and (B.2-3). Start first with equation (B.2-3) and write it in terms relative to consumption of good 1, i.e.:

$$\frac{x_i}{x_1} = \frac{\alpha_i b_i u^{e_i b_i}\left(\dfrac{p_i}{Y}\right)^{b_i - 1}}{\alpha_1 b_1 u^{e_1 b_1}\left(\dfrac{p_1}{Y}\right)^{b_1 - 1}} \tag{B.2-34}$$

This equation can be used to isolate α_i:

$$\alpha_i = \frac{x_i}{x_1}\frac{\alpha_1 b_1 u^{e_1 b_1}\left(\dfrac{p_1}{Y}\right)^{b_1 - 1}}{b_i u^{e_i b_i}\left(\dfrac{p_i}{Y}\right)^{b_i - 1}} \tag{B.2-35}$$

and then inserted into equation (B.2-3):

$$\sum_{i=1}^{n} \alpha_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i} = \alpha_1 u^{e_1 b_1}\frac{b_1}{s_1}\left(\frac{p_1}{Y}\right)^{b_1}\left[\sum_{i=1}^{n}\frac{s_i}{b_i}\right] \equiv 1 \tag{B.2-36}$$

The final expression in equation (B.2-36) can be used to solve for α_1 since the formula must equal 1 by definition:

$$\alpha_1 = u^{-e_1 b_1} \frac{s_1}{b_1} \left(\frac{Y}{p_1}\right)^{b_1} \left[\sum_{i=1}^{n} \frac{s_i}{b_i}\right]^{-1} \tag{B.2-37}$$

Substituting back into equation (B.2-36) we get:

$$\alpha_i = \frac{x_i}{b_i} u^{-e_i b_i} \left(\frac{Y}{p_i}\right)^{b_i - 1} \left[\sum_{j=1}^{n} \frac{s_j}{b_j}\right]^{-1} \tag{B.2-38}$$

The final calibration expression is then the following:

$$\alpha_i = \frac{s_i}{b_i} \left(\frac{Y}{p_i}\right)^{b_i} \frac{u^{-e_i b_i}}{\sum\limits_{j=1}^{n} \frac{s_j}{b_j}} \tag{B.2-39}$$

Utility is undefined in the base data and it is easiest to simply set it to 1.

In conclusion, for calibration we need the budget shares, initial prices, total expenditure, income elasticities and the own-price uncompensated elasticities. From this, we can derive base year consumption volumes, the Allen partial substitution elasticities through equation (B.2-28), σ (and therefore b) through equation (B.2-31) and the inversion of the A-matrix, e through equation (B.2-33) and inversion of the B-matrix, and finally α through equation (B.2-39).

It is possible that the initial shares and elasticities lead to inconsistent calibrated values for the b or e parameters. One solution, modified from Hertel (1997), is to implement some sort of maximum entropy method—explicitly imposing the constraints on the parameters. Step 1 is to calibrate the b-parameters using the following minimization problem:

$$\min L = \sum_i s_i (\varepsilon_{ii} - \varepsilon_{ii}^0)^2$$

subject to

$$\varepsilon_{ii} = (1 - b_i)(s_i - 1) - s_i \left[b_i + \eta_i - \sum_j s_j b_j\right]$$

$$0 < b_i < 1$$

The loss function is a weighted some of square errors where ε^0 represents the initial or target own-price elasticity and ε will be the estimated elasticity with the constraints holding. The first constraint is a transformation of equation (B.2-8) where the income elasticity is substituted into the definition of the own-price elasticity (swapping out for the yet unknown e-coefficients). One critical issue is to ascertain what income elasticities to use in the formula above. One could use the target income elasticities, or an initial transformation of the target elasticities such as described above.

The next step calibrates the e-parameters with some target income elasticities as given as well as the now calibrated b-parameters. The minimization problem is formulated as the following:

$$\min L = \sum_i s_i (\eta_i - \eta_i^0)^2$$

subject to

$$\eta_i = \frac{1}{\sum\limits_k s_k e_k}\left[e_i b_i - \sum_k s_k e_k b_k\right] - (b_i - 1) + \sum_k b_k s_k$$

$$\sum_i s_i \eta_i \equiv 1$$

$$(\eta_i - 1)\ \left(\eta_i^0 - 1\right) > 0$$

The final constraint insures that the estimated income elasticities preserve their relationship relative to 1, i.e. target elasticities lower than 1 remain lower than 1 in the estimation procedure.

B.1.4 CDE in first differences

It is useful to decompose changes in demand using a linearized version of the demand function, and that which is used in the standard GEMPACK version of the CDE function. The CDE implicit utility function can be used to derive a relation between changes in income, utility and prices (all in per capita terms). The first step in the differentiation of the utility function, equation (B.2-1), leads to the following expression:

$$\begin{aligned} 0 &= \sum_i \alpha_i e_i b_i u^{e_i b_i - 1}\left(\frac{p_i}{Y}\right)^{b_i} du \\ &- \sum_i \alpha_i b_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i - 1}\frac{p_i}{Y^2} dY \\ &+ \sum_i \alpha_i b_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i - 1}\frac{1}{Y} dp_i \end{aligned}$$

This can be simplified by inserting the expression for the demand equation, equation (B.2-3), and replacing demand with the budget shares (s_i):

$$0 = \frac{du}{u}\sum_i e_i s_i - \frac{dY}{Y}\sum_i s_i + \sum_i s_i \frac{dp_i}{p_i}$$

And the final expression can be written as:

$$\dot{Y} = \sum_i e_i s_i \dot{u} + \sum_i s_i \dot{p}_i \qquad \text{(B.2-40)}$$

where the dotted variables represent the percent change (and noting that the sum of the budget shares is equal to 1).

The differentiation of the demand function, equation (B.2-3) is somewhat more tedious. The first step leads to the following expression:

$$
\begin{aligned}
dx_i &= \alpha_i b_i e_i b_i u^{e_i b_i - 1}\left(\frac{p_i}{Y}\right)^{b_i-1}\frac{du}{D} \\
&+ \alpha_i b_i u^{e_i b_i}\left(b_i-1\right)\left(\frac{p_i}{Y}\right)^{b_i-2}\frac{1}{Y}\frac{dp_i}{D} \\
&- \alpha_i b_i u^{e_i b_i}\left(b_i-1\right)\left(\frac{p_i}{Y}\right)^{b_i-2}\frac{p_i}{Y^2}\frac{dY}{D} \\
&- \alpha_i b_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i-1}D^{-2}\sum_j \alpha_j b_j e_j b_j u^{e_j b_j - 1}\left(\frac{p_j}{Y}\right)^{b_j} du \\
&- \alpha_i b_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i-1}D^{-2}\sum_j \alpha_j b_j b_j u^{e_j b_j}\left(\frac{p_j}{Y}\right)^{b_j-1}\frac{1}{Y}dp_j \\
&+ \alpha_i b_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i-1}D^{-2}\sum_j \alpha_j b_j b_j u^{e_j b_j}\left(\frac{p_j}{Y}\right)^{b_j-1}\frac{p_j}{Y^2}dY
\end{aligned}
$$

where D is the denominator in the demand equation. This can be simplified to the following expression in terms of the percent changes:

$$
\begin{aligned}
\dot{x}_i &= e_i b_i \dot{u} + \left(b_i-1\right)\dot{p}_i - \left(b_i-1\right)\dot{Y} \\
&- \sum_j e_j b_j s_j \dot{u} - \sum_j b_j s_j \dot{p}_j + \sum_j b_j s_j \dot{Y}
\end{aligned}
$$

Re-grouping terms, the expression becomes:

$$
\begin{aligned}
\dot{x}_i &= (b_i-1)\dot{p}_i - \sum_j b_j s_j \dot{p}_j \\
&+ \dot{u}\left[e_i b_i - \sum_j e_j b_j s_j\right] \\
&+ \dot{Y}\left[\sum_j b_j s_j - (b_i-1)\right]
\end{aligned}
$$

The percent change in u can be replaced with the expression above, equation (B.2-40), to yield the following after re-arrangement:

$$
\begin{aligned}
\dot{x}_i &= \left(b_i-1\right)\dot{p}_i - \sum_j b_j s_j \dot{p}_j - \frac{1}{\sum_k e_k s_k}\sum_j s_j \dot{p}_j\left[e_i b_i - \sum_k e_k b_k s_k\right] \\
&+ \dot{Y}\left[\sum_k b_k s_k - \left(b_i-1\right) + \frac{1}{\sum_k e_k s_k}\left(e_i b_i - \sum_k e_k b_k s_k\right)\right]
\end{aligned}
$$

The final formula inserts the formulas for the income and price elasticities from above to simplify further to the following expression:

$$
\dot{x}_i = \sum_j \varepsilon_{ij}\dot{p}_j + \eta_i \dot{Y} \qquad \text{(B.2-41)}
$$

B.2 The ELES demand system

Many models assume separability in household decision making between saving and current consumption. Lluch and Howe[4] introduced a relatively straightforward extension of the LES consumer

[4] See Lluch (1973) and Howe (1975).

demand function to include the saving decision simultaneously with the allocation of income on goods and services, this has become known as the extended linear expenditure system or the ELES. The ELES is based on consumers maximizing their intertemporal utility between a bundle of current consumption and an expected future consumption bundle represented in the form of savings.

B.2.1 Basic formulation

The utility function of the ELES has the following form:

$$u = \prod_i (x_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s} \tag{B.2-42}$$

with

$$\sum_i \mu_i + \mu_s = 1 \tag{B.2-43}$$

where u is utility, x is the vector of consumption goods, S is household saving (in value), P^s is the price of saving, and μ and θ are ELES parameters.

The consumer solves the following problem:

$$\max \prod_i (x_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s}$$

subject to

$$\sum_{i=1}^{n} p_i x_i + S = Y$$

where p is the vector of consumer prices, and Y is disposable income. The demand functions are:

$$x_i = \theta_i + \frac{\mu_i}{p_i}\left(Y - \sum_{j=1}^{n} p_j \theta_j\right) \tag{B.2-44}$$

$$S = \mu_s \left(Y - \sum_{j=1}^{n} p_j \theta_j\right) = Y - \sum_{j=1}^{n} p_j x_j \tag{B.2-45}$$

The term in parentheses is sometimes called supernumerary income, i.e. it is the income that remains after subtracting total expenditures on the so-called subsistence (or floor) expenditures as represented by the θ parameter. The parameter μ then represents the marginal budget share out of supernumerary income.

B.2.2 ELES elasticities

From the demand equation we can derive the income and price elasticities:

$$\eta_i = \frac{\mu_i Y}{p_i x_i} = \frac{\mu_i}{s_i} \quad \eta_s = \frac{\mu_s Y}{S} = \frac{\mu_s}{s} \tag{B.2-46}$$

$$\varepsilon_i = \frac{\theta_i (1 - \mu_i)}{x_i} - 1 \quad \varepsilon_s = -1 \tag{B.2-47}$$

$$\varepsilon_{ij} = -\frac{\mu_i p_j \theta_j}{p_i x_i} = -\frac{\mu_i p_j \theta_j}{s_i Y} \quad \varepsilon_{sj} = -\frac{\mu_s p_j \theta_j}{sY} = -\frac{p_j \theta_j}{Y^*} \tag{B.2-48}$$

where s is the average propensity to save. Note that the matrix of elasticities can be collapsed to a single formula using the Kronecker factor:

$$\varepsilon_{ij} = -\frac{\mu_i p_j \theta_j}{p_i x_i} - \delta_{ij} \frac{p_i x_i - p_i \theta_i}{p_i x_i} = -\frac{\mu_i}{s_i Y} \left[\delta_{ij} Y^* + p_j \theta_j \right] = -\eta_i \left[\delta_{ij} \frac{Y^*}{Y} + \frac{p_j \theta_j}{Y} \right] \tag{B.2-49}$$

The last expression shows that there is clear linkage between the income and price elasticities. At the limit, when income is much larger than supernumerary income, the two are virtually identical in levels (with opposite signs).

B.2.3 Welfare

With the addition of saving, the indirect utility function is given by:

$$v(p, Y) = \prod_i \left(\frac{\mu_i}{p_i} Y^* \right)^{\mu_i} \left(\frac{\mu_s}{P^s} Y^* \right)^{\mu_s} \tag{B.2-50}$$

or

$$v(p, Y) = \frac{Y^*}{P} \tag{B.2-51}$$

where

$$P = \prod_i \left(\frac{p_i}{\mu_i} \right)^{\mu_i} \left(\frac{P^s}{\mu_s} \right)^{\mu_s}$$

The expenditure function is derived by minimizing the cost of achieving a given level of utility, u. It is set-up as:

$$\min \sum_{i=1}^{n} p_i x_i + S$$

subject to

$$\prod_i (x_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s} \right)^{\mu_s} = u$$

The final expression for the expenditure function is:

$$E(p, u) = \sum_{i=1}^{n} p_i \theta_i + uP \tag{B.2-52}$$

where P, the aggregate price index (including the price of savings) is defined as above.

B.2.4 Calibration

Calibration of the ELES uses the budget share information from the base SAM, including the household saving share. Typically, calibration uses income elasticities for all of the n commodities represented in the demand system and uses equation (B.2-46) to derive the marginal budget shares, μ_i. This procedure leads to a residual income elasticity, which in this case is the income elasticity of saving. The derived savings income elasticity may be implausible, in which case adjustments need to be made to individual income elasticities for the goods, or adjustments can be made on the group of goods, assuming some target for the savings income elasticity.

The first step is therefore to calculate the marginal budget shares using the average budget shares and the initial income elasticity estimates.

$$\mu_i = \frac{\eta_i p_i x_i}{Y} = \eta_i s_i$$

The savings marginal budget share is derived from the consistency requirement that the marginal budget shares sum to 1:

$$\mu_s = 1 - \sum_{i=1}^{n} \mu_i$$

Assuming this procedure leads to a plausible estimate for the savings income elasticity, the next step is to calibrate the subsistence minima, θ. This can be done by seeing that the demand equations, (B.2-44), are linear in the θ parameters. Note that in the case of the ELES the system of equation are of full rank because the μ_i parameters do not sum to 1 (over the n commodities)—they only sum to 1 including the marginal saving share.[5] This may lead to calibration problems if the propensity to save is 0, which may be the case in some SAMs with poor households. The linear system can be written as:

$$C = I\theta + MY - M\Pi\theta$$

where I is an $n \times n$ identity matrix, M is a diagonal matrix with μ_i/P_i on the diagonal, and Π is a matrix, where each row is identical, each row being the transpose of the price vector. The above system of linear equations can be solved via matrix inversion for the parameter θ:

$$\theta = A^{-1}C^*$$

where

$$A = I - M\ \Pi$$

$$C^* = C - MY$$

The matrices A and C^* are defined by:

$$A = [a_{ij}] = \left[\delta_{ij} - \mu_i \frac{p_j}{p_i}\right] = \begin{cases} 1 - \mu_i & \text{if} \quad i = j \\ -\mu_i \dfrac{p_j}{p_i} & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = x_i - \frac{\mu_i Y}{p_i}$$

[5] Note that the calibration and the setup of the ELES assume explicitly that the minimal expenditure on savings is zero.

The A and C^* matrices are simplified if the price vector is initialized at 1:

$$A = [a_{ij}] = \begin{cases} 1 - \mu_i & \text{if} \quad i = j \\ -\mu_i & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = x_i - \mu_i Y$$

In GAMS one could invert the system of equations embodied in equation (B.2-44) directly by solving for the endogenous θ while holding all of the other variables and parameters fixed.

B.3 The AIDADS demand system

Many commonly used utility functions typically exhibit poor Engel behavior—particularly in a dynamic framework. The CDE utility function, popularized by the GTAP model (see Hertel (1997)), has relatively constant income elasticities. The LES utility function has even worse behavior, as in the absence of any shifts in the underlying parameters, the LES converges relatively quickly to a Cobb-Douglas utility function as rapidly rising consumption tends to dominate the floor consumption parameters, even when adjusting the latter to take into account population growth. Rimmer and Powell (see Rimmer and Powell (1992b), Rimmer and Powell (1992a) and Rimmer and Powell (1996)) examine an extension to the standard LES demand system that in effect allows the marginal propensity to consumer parameter to be driven by changes in utility. Their utility function has been called An Implicitly Direct Additive Demand System, or AIDADS. The LES function is a special case of the AIDADS system where the marginal propensity variable is constant. This extension allows for more complex demand behavior, as well as providing better validation for observed changes in consumption patterns.[6]

B.3.1 Basic formulation

AIDADS starts with the implicitly additive utility function given by:

$$\sum_i U_i\left(x_i, u\right) \equiv 1 \tag{B.2-53}$$

Assume the following functional form for the utility function:

$$U_i = \mu_i \ln\left(\frac{x_i - \theta_i}{Ae^u}\right) \tag{B.2-54}$$

where

$$\mu_i = \frac{\alpha_i + \beta_i G\left(u\right)}{1 + G\left(u\right)} \tag{B.2-55}$$

with the restrictions

$$\sum_i \alpha_i = \sum_i \beta_i = 1$$
$$0 \leq \alpha_i \leq 1$$
$$0 \leq \beta_i \leq 1$$

[6] AIDADS has also been explored in the context of the GTAP model, see for example Yu et al. (2003).

$$\theta_i < x_i$$

Cost minimization implies the following:

$$\min \sum_i p_i x_i$$

subject to

$$\sum_i \mu_i \ln\left(\frac{x_i - \theta_i}{Ae^u}\right) \equiv 1 \tag{B.2-56}$$

The first order conditions lead to:

$$\lambda \frac{\partial U_i}{\partial x_i} = p_i = \lambda \frac{\mu_i}{x_i - \theta_i} \Rightarrow \lambda \mu_i = p_i x_i - p_i \theta_i \tag{B.2-57}$$

Taking the sum over i and using the fact that the μ_i sum to unity implies:

$$\lambda = \sum_i p_i x_i - \sum_i p_i \theta_i = Y - \sum_i p_i \theta_i = Y^* \tag{B.2-58}$$

where Y is aggregate expenditure, and Y^*, sometimes referred to as supernumerary income, is residual expenditure after subtracting total expenditure on the so-called subsistence minima, θ.

Re-inserting equation (B.2-58) into (B.2-57) yields the consumer demand equations:

$$x_i = \theta_i + \frac{\mu_i}{p_i} Y^* = \theta_i + \frac{\mu_i}{p_i}\left[Y - \sum_j p_j \theta_j\right] \tag{B.2-59}$$

Equation (B.2-59) is virtually identical to the LES demand equation. Similar to the linear expenditure system (LES), demand is the sum of two components—a subsistence minimum, θ, and a share, μ, of supernumerary income. Unlike the LES, the share parameter, μ, is not constant, but depends on the level of utility itself. AIDADS collapses to the LES if each α parameter is equal to the corresponding β parameter, with the ensuing function of utility, $G(u)$, dropping from equation (B.2-55).

B.3.2 Elasticities

This section develops the main expressions for the income and price elasticities. These formulas will be needed to calibrate the initial parameters of the AIDADS function.

Income elasticities

To derive further properties of AIDADS requires specifying a functional form for $G(u)$. Rimmer and Powell (1996) propose the following:

$$G(u) = e^u \tag{B.2-60}$$

The first step is to calculate the marginal budget share, ρ , defined as:

$$\rho_i = p_i \frac{\partial x_i}{\partial Y}$$

The following expression can be derived from equation (B.2-59):

$$\frac{\partial x_i}{\partial Y} = \frac{Y^*}{p_i}\frac{\partial \mu_i}{\partial Y} + \frac{\mu_i}{p_i}\frac{\partial Y^*}{\partial Y} = \frac{Y^*}{p_i}\frac{\partial \mu_i}{\partial u}\frac{\partial u}{\partial Y} + \frac{\mu_i}{p_i}$$

Thus:

$$\rho_i = \mu_i + Y^* \frac{\partial \mu_i}{\partial u}\frac{\partial u}{\partial Y} \tag{B.2-61}$$

Expression (B.2-61) can be expanded in two steps—first evaluating the partial derivative of the share variable, μ, with respect to utility, and then the more difficult calculation of the partial derivative of u with respect to income. The marginal share formula is:

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u}$$

Its derivative is:

$$\frac{\partial \mu_i}{\partial u} = \frac{(1 + e^u)(\beta_i e^u) - (\alpha_i + \beta_i e^u) e^u}{(1 + e^u)^2} = \frac{e^u (\beta_i - \alpha_i)}{(1 + e^u)^2} \tag{B.2-62}$$

Utility and income are combined in implicit form and thus we will invoke the implicit function theorem to calculate the partial derivative of u with respect to Y. First, insert equation (B.2-59) into equation (B.2-56):

$$\sum_i \mu_i \ln\left(\frac{x_i - \theta_i}{Ae^u}\right) = \sum_i \mu_i \ln\left(\frac{\mu_i Y^*}{Ae^u p_i}\right) = 1$$

Expanding the latter expression yields:

$$f(u, Y) = \sum_i \mu_i \ln\left(\frac{\mu_i}{p_i}\right) + \ln(Y^*) - \ln(A) - u = 1 \tag{B.2-63}$$

which provides the implicit relation between Y and u. The implicit function theorem states the following:

$$\frac{\partial u}{\partial Y} = -\frac{\partial f}{\partial Y}\left[\frac{\partial f}{\partial u}\right]^{-1} \tag{B.2-64}$$

The partial derivative of f with respect to Y is simply:

$$\frac{\partial f}{\partial Y} = \frac{1}{Y^*} \tag{B.2-65}$$

The partial derivative of f with respect to u is:

$$\begin{aligned}
\frac{\partial f}{\partial u} &= -1 + \sum_i \left[\frac{\partial \mu_i}{\partial u} \ln\left(\frac{\mu_i}{p_i}\right) + \mu_i \frac{p_i}{\mu_i} p_i \frac{\partial \mu_i}{\partial u}\right] \\
&= -1 + \frac{e^u}{(1 + e^u)^2} \sum_i \left[\left(\ln\left(\frac{\mu_i}{p_i}\right) + 1\right)(\beta_i - \alpha_i)\right] \\
&= \frac{e^u}{(1 + e^u)^2}\left[\sum_i (\beta_i - \alpha_i) \ln(x_i - \theta_i) - \frac{(1 + e^u)^2}{e^u}\right] \\
&= \frac{e^u}{(1 + e^u)^2}\Omega^{-1}
\end{aligned} \tag{B.2-66}$$

where

$$\Omega = \left[\sum_i (\beta_i - \alpha_i) \ln (x_i - \theta_i) - \frac{(1+e^u)^2}{e^u}\right]^{-1} \tag{B.2-67}$$

The second line uses equation (B.2-62). In the third line, equation (B.2-59) substitutes for the expression in the logarithm, and the adding up constraint allows for the deletion of non-indexed variables. Substituting equations (B.2-65) and (B.2-66) into equation (B.2-64) yields:

$$\frac{\partial u}{\partial Y} = -\frac{\Omega}{Y^*}\frac{(1+e^u)^2}{e^u} \tag{B.2-68}$$

Substituting equations (B.2-62) and (B.2-68) into equation (B.2-61) yields the following expression for ρ:

$$\rho_i = \mu_i - (\beta_i - \alpha_i)\ \Omega$$

The income elasticities are derived from the following expression:

$$\eta_i = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i} = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i}\frac{p_i}{p_i} = \frac{\rho_i}{s_i}$$

where s_i is the average budget share:

$$s_i = \frac{p_i x_i}{Y} = \frac{p_i\theta_i}{Y} + \mu_i\frac{Y^*}{Y} = \mu_i + \left(\frac{p_i\theta_i - \mu_i \sum_j p_j\theta_j}{Y}\right)$$

Thus the income elasticity, η, is equal to the ratio of the marginal budget share, ρ, and the average budget share, s. Finally, equation (B.2-69) describes one formulation of the income elasticity:

$$\eta_i = \frac{\mu_i - (\beta_i - \alpha_i)\,\Omega}{s_i} \tag{B.2-69}$$

Price elasticity

The matrix of substitution elasticities is identical to the expression for the LES and has the form:

$$\sigma_{ij} = [\mu_j - \delta_{ij}]\,\frac{\mu_i Y^*}{s_i s_j Y} \tag{B.2-70}$$

where δ is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

It is clear that the matrix is symmetric. The matrix of substitution elasticities is also equal to:

$$\sigma_{ij} = [\mu_j - \delta_{ij}]\,\frac{\mu_i Y^*}{s_i s_j Y} = \frac{(x_i - \theta_i)}{x_i}\frac{(x_j - \theta_j)}{x_j}\frac{Y}{Y^*} - \frac{\delta_{ij}}{s_j}\frac{(x_i - \theta_i)}{x_i}$$

The compensated demand elasticities derive from the following:

$$\xi_{ij} = s_j\sigma_{ij} = [\mu_j - \delta_{ij}]\,\frac{\mu_i Y^*}{s_i Y} \tag{B.2-71}$$

Finally, the matrix of uncompensated demand elasticities is given by:

$$\varepsilon_{ij} = \xi_{ij} - s_j\eta_i = [\mu_j - \delta_{ij}] \; \frac{\mu_i Y^*}{s_i Y} - s_j\eta_i \tag{B.2-72}$$

The uncompensated demand elasticities can also be written as:

$$\varepsilon_{ij} = -\frac{\mu_i}{s_i Y}\left[p_j\theta_j + \delta_{ij}Y^*\right] + \frac{s_j}{s_i}\left(\beta_i - \alpha_i\right)\Omega \tag{B.2-73}$$

The first term on the right-hand side is always negative. The second term differs from the LES expression for the uncompensated demand elasticities.[7] We can see from expression (B.2-73) that the AIDADS specification allows for both gross complementarity and substitution. As well, it allows for luxury goods, i.e. positive own-price demand elasticities should the second term be positive and greater than the first term.

B.3.3 Implementation

Implementation of AIDADS is somewhat more complicated than the LES since the marginal propensity to consume out of supernumerary income is endogenous, and utility is defined implicitly. The following four equations are needed for model implementation:

$$Y^* = Y - \sum_i p_i\theta_i \tag{B.2-74}$$

$$x_i = \theta_i + \frac{\mu_i}{p_i}Y^* \tag{B.2-75}$$

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u} \tag{B.2-76}$$

$$u = \sum_i \mu_i \ln\left(x_i - \theta_i\right) - 1 - \ln\left(A\right) \tag{B.2-77}$$

Equations (B.2-74) and (B.2-75) are identical to their LES and ELES counterparts.[8] Equation (B.2-76) determines the level of the marginal propensity to consume out of supernumerary income, μ, which is a constant in the case of the LES (ELES). It requires however the calculation of the utility level, u, which is defined in equation (B.2-77).

B.3.4 Calibration

[To be updated] Calibration requires more information than the LES. Where the LES has $2n$ parameters to calibrate (subject to consistency constraints), AIDADS has $3n$ parameters (less the consistency requirements)—α, β and θ. The calibration system includes equations (B.2-74) through (B.2-77) which have $2+2n$ endogenous variables (Y^*, θ, μ, and A). There are no equations for calibrating the α and β parameters. If we have knowledge of the income elasticities, we can add the following equations:

$$\Psi = \frac{1}{\Omega} = \left[\sum_i \left(\beta_i - \alpha_i\right)\ln\left(x_i - \theta_i\right) - \frac{\left(1+e^u\right)^2}{e^u}\right] \tag{B.2-78}$$

7 Recall that for the LES, the α and β terms are equal and thus the second term drops.

8 Though the definition of Y includes savings in the case of the ELES.

$$\eta_i = \frac{\rho_i}{s_i} = \frac{\mu_i - (\beta_i - \alpha_i)\,\Omega}{s_i} = \frac{\mu_i}{s_i} - \frac{(\beta_i - \alpha_i)}{s_i \Psi} \tag{B.2-79}$$

There are an additional $1 + n$ equations, solving for Ψ and α. There is need for an additional n equations. Assuming we have knowledge of at least n price elasticities, for example the own-price elasticities, we can add the following equation:

$$\varepsilon_{ii} = -\frac{\mu_i}{s_i Y}\left[p_i \theta_i + Y^*\right] + (\beta_i - \alpha_i)\,\Omega \tag{B.2-80}$$

The α and β parameters are not independent, the following restrictions must hold:

$$\sum_i \alpha_i = 1 \tag{B.2-81}$$

$$\sum_i \beta_i = 1 \tag{B.2-82}$$

The system is under-determined, there are $5+4n$ equations and $3+4n$ variables. One solution, is to make the own-price elasticities endogenous. In this case, we are adding n variables, but then the system is over-determined. We can minimize a loss function with respect to the price elasticities:

$$L = \sum_i \left(\varepsilon_i - \varepsilon_i^0\right)^2$$

where ε^0 represents an initial guess of the own-price elasticities and the calibration algorithm will calculate the endogenous ε in order to minimize the loss function, subject to constraints (B.2-78) through (B.2-82) and the model equations (B.2-74) through (B.2-77). The exogenous parameters in the calibration procedure include p, x, s, Y, η, ε^0 and u.

Appendix C

Alternative trade specification

The GTAP database decomposes aggregate demand for goods and services by agent aa into a domestic component and an (aggregate) import component. It is thus possible to implement the Armington specification at the agent level—though the default model uses a national Armington specification, in part to (significantly) reduce the size of the model. This short appendex describes how the model needs to be modified to allow for agent-specific behavior. Equations (T-1) through (T-6) in the model would be replaced by the equations in the block below. Equations (C.3-1) and (C.3-2) define the Armington domestic and import components at the agent level, where all share parameters and substitution elasticities are agent-specific. Equation (C.3-3) defines the agent-specific Armington price. And equations (C.3-4) and (C.3-5) determine the aggregate domestic demand for domestic production and imports respectively. Under this specification, the variables XAT and PAT are dropped. (N.B. This specification has not been reviewed for use with the 'energy aware' version of ENVISAGE. Among other potential issues, there are no γ parameters that allow for the adding up of energy volumes in efficiency units.)

$$XD_{r,i,aa} = \alpha^d_{r,i,aa} \left(\frac{PA_{r,i,aa}}{\left(1 + \tau^{Ad}_{r,i,aa}\right) PD_{r,i}} \right)^{\sigma^m_{r,i,aa}} XA_{r,i,aa} \tag{C.3-1}$$

$$XM_{r,i,aa} = \alpha^m_{r,i,aa} \left(\frac{PA_{r,i,aa}}{\left(1 + \tau^{Am}_{r,i,aa}\right) PMT_{r,i}} \right)^{\sigma^m_{r,i,aa}} XA_{r,i,aa} \tag{C.3-2}$$

$$\begin{aligned} PA_{r,i,aa} &= \Bigg[\alpha^d_{r,i,aa} \left(\left(1 + \tau^{Am}_{r,i,aa}\right) PD_{r,i}\right)^{1-\sigma^m_{r,i,aa}} \\ &+ \alpha^m_{r,i,aa} \left(\left(1 + \tau^{Am}_{r,i,aa}\right) PMT_{r,i}\right)^{1-\sigma^m_{r,i,aa}} \Bigg]^{1/(1-\sigma^m_{r,i,aa})} \end{aligned} \tag{C.3-3}$$

$$XDT^d_{r,i} = \sum_{aa} XD_{r,i,aa} \tag{C.3-4}$$

$$XMT_{r,i} = \sum_{aa} XM_{r,i,aa} \tag{C.3-5}$$

Appendix D

Alternative capital account closures

[To be completed]

... ENVISAGE has three different closures for the capital account. The simplest is simply to fix the capital account at base year levels. The second option, as described in Hertel (1997), is to allow the capital account equilibrate changes in the expected rate of return to capital across regions, i.e. the percentage change of regional rates of return are equal. If returns are equal initially, this is equivalent to assuming perfect international capital mobility. The third option, also described in Hertel (1997), assumes that the 'global' investor has an optimal portfolio initially, and adjusts capital flows to maintain the same portfolio ex post.

Equation (58) defines the average rate of return to capital in each region, *AvgRoR*. It is the weighted average of the sectoral rates of return. [? Should the weights be fixed, i.e. indexed by t0 ?]. The current net rate of return, *RoRC*, is then defined as the average gross regional rate of return, adjusted by changes to the unit cost of capital, and less depreciation-equation (59). Equation (60) defines the motion equation for aggregate capital. The end-of-period capital stock, K_{t+1}, is equal to the beginning period capital stock, K_t, adjusted for depreciation, and augmented by the current period's volume of investment, XC_{Inv}. The expected rate of return, *RoRE*, is assumed to decline with positive additions to the capital stock. This is the motivation behind equation (61). [See Hertel (1997) for a more detailed description.] Equation (62) defines the value of net investment, *NInv*. Equation (63) defines the average global rate of return, *RoRG*.

The three foreign capital closure rules are encapsulated in equation (64) and are driven by a model flag labeled *KFlowFlag*. The first rule is simply to fix the capital account. To preserve model homogeneity, the initial volume is multiplied by the model numéraire to provide a nominal foreign saving. The second rule equates the percentage change in the expected rate of return in each region. The third rule assumes that global investment is allocated across regions such that the regional composition of investment is invariant. This implies that the percent change in net investment is equal across regions [Shouldn't we be using as a rule that the capital stock in value terms is proportionately the same across regions]. Equation (64) is defined for all regions except for one. The left out region is indexed by *rSAV* that is a subset of the set of regions, r. Closure of the model is guaranteed by equation (65) that forces the global sum of the capital flows to be identically equal to zero.

Appendix E

Dynamic model equations with multi-step time periods

The step size in the model scenarios are allowed to vary across time—in order to save compute time and storage. Particularly in the long-run scenarios, annual increments are not particularly useful. Some of the equations in the model—essentially almost any equation that relies on a lagged variable need to take into account the variable step size, for example equation (G-1), the capital accumulation equation.

$$KStock_{r,t} = (1 - \delta_{r,t})\, KStock_{r,t-1} + XC_{r,Inv,t-1}$$

In fact, this equation is not even necessary in the model for a step size of 1 since both variables on the right-hand side of the equation are lags. However, let n be the step-size, possibly even 1. Then through recursion, the capital accumulation function becomes:

$$KStock_t = (1 - \delta)^n\, KStock_{t-n} + \sum_{j=1}^{n} (1 - \delta)^{j-1}\, XC_{Inv,t-j}$$

If the model is run in step sizes greater than 1, the intermediate values of real investment are not calculated. They can be replaced by assuming a compound growth model for investment:

$$XC_{Inv,t} = \left(1 + \gamma^I\right)^n XC_{Inv,t-n} \tag{E.5-1}$$

Replacing this in the accumulation function yields:

$$KStock_t = (1 - \delta)^n\, KStock_{t-n} + \sum_{j=1}^{n} (1 - \delta)^{j-1} \left(1 + \gamma^I\right)^{n-j} XC_{Inv,t-n}$$

With some algebraic manipulation (that is done for a number of similar expressions below), this formula can be reduced to the following:

$$KStock_t = (1 - \delta)^n\, KStock_{t-n} + \frac{\left(1 + \gamma^I\right)^n - \left(1 - \delta^I\right)^n}{\gamma^I + \delta} XC_{Inv,t-n} \tag{E.5-2}$$

where we incorporate equation (E.5-1) in the model to evaluate the growth rate of investment.

The growth rate is itself a function of contemporaneous investment. If n is equal to 1, it is clear that this equation simplifies to the simple 1-step accumulation function. The capital accumulation function is no longer exogenous since it depends on the investment growth rate, which itself is endogenous. To avoid scale problems, equations (E.5-3) and (E.5-4) are used in place of (E.5-1) and (E.5-2) to provide the n-step capital stock accumulation function. Equation (E.5-3) is likely to evaluate to somewhere between 10 and 20 since the first term is 1 plus the average annual growth of investment, to which is added the depreciation rate less 1. If investment growth is 5 percent and depreciation is likewise 5 percent, then the value is 10. The first term on the right-hand side of equation (E.5-4) is likely to be relatively small since it takes the previous capital stock and subtracts a multiple of the previous period's investment (lagged n years), and then multiplies by the depreciation factor, so that the largest term is the second term, which is a multiple of the current volume of investment.

$$IGFACT_{r,t} = \left[\left(\frac{XC_{r,Inv,t}}{XC_{r,Inv,t-n}} \right)^{1/n} - 1 + \delta_{r,t} \right]^{-1} \tag{E.5-3}$$

$$KStock_t = \left[KStock_{r,t-n} - IGFACT_{r,t} XC_{r,Inv,t-n} \right] \left(1 - \delta_{r,t} \right)^n + IGFACT_{r,t} XC_{r,Inv,t} \tag{E.5-4}$$

The savings function, equation (D-1) also needs modification in a dynamic scenario with multiple years between solution periods. The new equation (E.5-5) below shows the modification of the savings function. It is readily seen that equation (E.5-5) collapses to equation (D-1) when the inter-period gap is equal to 1.

$$\begin{aligned} s^s_{r,t} &= \chi^s_r \alpha^s_r \frac{1 - (\beta^s_r)^n}{1 - \beta^s_r} + (\beta^s_r)^n s^s_{r,t-n} \\ &+ \frac{\beta^g_r g^{pc}_{r,t} - \beta^g_r g^{pc}_{r,t-n} (\beta^s_r)^n}{1 - \beta^s_r \left(g^{pc}_{r,t-n} / g^{pc}_{r,t} \right)^{1/n}} \\ &+ \frac{\beta^y_r DRAT^{PLT15}_{r,t} - \beta^y_r DRAT^{PLT15}_{r,t-n} (\beta^s_r)^n}{1 - \beta^s_r \left(DRAT^{PLT15}_{r,t-n} / DRAT^{PLT15}_{r,t} \right)^{1/n}} \\ &+ \frac{\beta^e_r DRAT^{P65UP}_{r,t} - \beta^e_r DRAT^{P65UP}_{r,t-n} (\beta^s_r)^n}{1 - \beta^s_r \left(DRAT^{P65UP}_{r,t-n} / DRAT^{P65UP}_{r,t} \right)^{1/n}} \end{aligned} \tag{E.5-5}$$

Appendix F

Climate Modules

F.1 Introduction

This annex describes some of the features of the climate modules for the MERGE and DICE models. The current version of ENVISAGE uses the MERGE climate module as it accounts for more of the greenhouse gases.

F.2 Multi-period functional forms and the MERGE climate module

ENVISAGE is designed to allow for variable time steps. This section describes the conversion of the single-period time step equations to the multi-step time equations. The multi-period time steps rely on a common stock-flow motion equation:

$$S_{t+1} = \delta S_t + \beta F_t$$

where δ is a decay rate of the stock, S, and β is an adjustment factor to the flow, F.[1] Using induction, the motion equation becomes the following over multi-periods:

$$S_{t+n} = \delta^n S_t + \beta \sum_{i=0}^{n-1} \delta^{n-i-1} F_{t+i}$$

If we assume that flows grow at a constant rate between t and $t+n$, the equation above becomes:

$$S_{t+n} = \delta^n S_t + \beta F_t \sum_{i=0}^{n-1} \delta^{n-i-1} (1+g)^i = \delta^n S_t + \beta \delta^n F_t \sum_{i=0}^{n-1} \left(\frac{1+g}{\delta} \right)^i$$

The summation term can be simplified and the final formulation is:

$$S_{t+n} = \delta^n S_t + \beta F_t \frac{(1+g)^n - \delta^n}{1+g-\delta} = \delta^n S_t + \beta \frac{F_{t+n} - \delta^n F_t}{1+g-\delta}$$

where

$$g = \left(\frac{F_{t+n}}{F_t} \right)^{1/n} - 1$$

[1] In the standard capital accumulation function, δ is 1 minus the depreciation rate and β is 1.

If the flow is contemporaneous, instead of lagged by one period, it is easy to show that the multi-period stock/flow motion equation is:

$$S_{t+n} = \delta^n S_t + \beta\left(1+g\right)F_t\frac{\left(1+g\right)^n - \delta^n}{1+g-\delta} = \delta^n S_t + \beta\left(1+g\right)\frac{F_{t+n} - \delta^n F_t}{1+g-\delta}$$

i.e. the initial flow is multiplied by the factor $(1+g)$.

Note that MERGE uses a different formulation to account for the inter-period accumulation (and decay). It uses the mid-point of the flows and the formula is then:

$$S_{t+n} = \delta^n S_t + \beta 0.5\left(E_t + E_{t+n}\right)\frac{1-\delta^n}{1-\delta}$$

Given these preliminaries, the first derivation refers to equation (C-9) that determines the level of CO_2 concentration in the different atmospheric boxes. In simplified form, the one-step equation is:

$$B_{t+1} = \delta B_t + \varphi E_{t+1}$$

where δ is the decay rate of concentration in the box and φ is the share of contemporaneous emissions accruing to the box. From the formulas above, the multi-period version of the equation (with contemporaneous emissions) is:

$$B_{t+n} = \delta^n B_t + \varphi(1+g)E_t\frac{\left(1+g\right)^n - \delta^n}{1+g-\delta} = \delta^n B_t + \varphi(1+g)\frac{E_{t+n} - \delta^n E_t}{1+g-\delta} \quad \text{(F.6-1)}$$

where

$$g = \left(\frac{E_{t+n}}{E_t}\right)^{1/n} - 1$$

The code in ENVISAGE follows the MERGE formulation:

$$B_{t+n} = \delta^n B_t + \varphi 0.5\left(E_t + E_{t+n}\right)\frac{1-\delta^n}{1-\delta} \quad \text{(F.6-2)}$$

where the fraction term is replaced by n in the case of $\delta = 1$. In practical terms, both expressions F.6-1 and F.6-2 provide similar results with the former needing in addition an equation for the intra-period emissions growth rate.

The equation for the other greenhouse gases, equation (C-11), takes roughly the same form:

$$C_{t+n} = \delta^n C_t + (1+g)E_t\frac{\left(1+g\right)^n - \delta^n}{1+g-\delta} = \delta^n C_t + (1+g)\frac{E_{t+n} - \delta^n E_t}{1+g-\delta}$$

where there is a single box and thus the parameter φ drops out. Similar to carbon, the final concentration equation uses a modified version of the MERGE equation:

$$C_{t+n} = \delta^n C_t + 0.5\left(E_t + E_{t+n}\right)\frac{1-\delta^n}{1-\delta} \quad \text{(F.6-3)}$$

The temperature equation is similar. In its simplest form, the temperature equation has the change in temperature, ΔT, adjusting over a long-period to the potential temperature change, ΔPT:[2]

[2] The period of adjustment is over 26 years, with λ equal to 0.038.

$$\Delta T_{t+1} = \Delta T_t + \lambda\left(\Delta PT_t - \Delta T_t\right) = \left(1-\lambda\right)\Delta T_t + \lambda \Delta PT_t$$

Using the generic formulas from above, the multi-period version of the temperature change equation is:

$$\Delta T_{t+n} = \left(1-\lambda\right)^n \Delta T_t + \lambda \Delta PT_t \frac{\left(1+g\right)^n - \left(1-\lambda\right)^n}{g+\lambda}$$

where g is the growth of ΔPT between periods t and $t+n$. The MERGE formulation is:

$$\Delta T_{t+n} = \left(1-\lambda\right)^n \Delta T_t + 0.5 * \left(\Delta PT_t + \Delta PT_{t+n}\right)\left(1-\left(1-\lambda\right)^n\right)$$

F.3 DICE 2007 climate module

This section describes the climate module used in the DICE 2007 model.[3] It was the climate module used in the original version of ENVISAGE. It was replaced by the MERGE climate module in order to handle the non-CO_2 greenhouse gases. While not in current use, its description may be of use to interested readers and the appendix also shows how the DICE module has been extended to cover any time definition—not the fixed 10-year time steps of the DICE model.

The model contains three sinks for CO_2 emissions—the atmosphere and the upper and deep oceans. These three sinks are indexed by z. In each period, there is a flow of carbon across the three sinks using a 3×3 transition matrix, K. Each column of the transition matrix represents the share of the stock in the sink that flows to a different sink. Thus the diagonal element represents the share of the stock that stays in its own sink. The current values of the concentration transition matrix are provided in more detail below.

$$Conc_z = \mathrm{K}.Conc_{z,-1} + EMIGbl_{z,CO2,-1} \tag{F.6-4}$$

$$Forc_{atmos} = fCO2x \frac{\log_{10}\left(Conc_{atmos}/ConcPI\right)}{\log_{10}(2)} + ForcOth \tag{F.6-5}$$

$$Temp_{zt} = \mathrm{T}.Temp_{zt,-1} + \Theta.Forc_{zt} \tag{F.6-6}$$

Equation (F.6-4) determines the concentration level in each sink. The concentration level is equal to its lagged value, multiplied by the transition matrix. In the absence of new emissions, one can determine the long-term equilibrium by multiplying the matrix K n-times, where n is large enough that the transition matrix converges towards a constant matrix. Carbon emissions are entirely added to atmospheric concentration.[4] Note that emissions in the model are in terms of carbon. To convert to CO_2, multiply the carbon emissions by the factor (44/12).

Equation (F.6-5) converts atmospheric concentrations to its impact on radiative forcing. Forcing is a logarithmic function (based 10) of concentration with two key parameters. The first is the pre-industrial concentration level, $ConcPI$. The second is the amount of forcing induced by a doubling of concentration from its pre-industrial level, $fCO2x$. The relation allows for an exogenous amount of forcing, that could eventually be negative, as is the current case, due to SO_2 emissions.

Temperature, measured as the increment to temperature in °C since 1900, like concentration, has interactions between the atmosphere and the oceans. In this case the ocean is treated as a

[3] See Nordhaus (2008).

[4] The variable *EMIGbl* is a vector defined overall all sinks, but emissions to the two ocean sinks are always 0.

single sink and the subset *zt* of *z* covers only *atmos* and *dpocn*. Equation (F.6-6) provides the link between temperature in the two sinks with their previous respective temperatures, through a transition matrix T, and the incremental impact from forcing through the matrix Θ.[5] The temperature transition and forcing matrices are further developed below.

The transition matrices in the DICE model are based on a fixed 10-year time step between years. In the ENVISAGE model, the time gap is variable. The model therefore requires two modifications to the DICE version of the climate module. First, it is necessary to convert the 10-year transition matrices to a single-year transition matrix, and then to code the dynamic equations to allow for variable gap dynamic expression.

F.3.1 Emissions and concentration

In the DICE model, the 10-year concentration transition matrix, B, has the following form and values:

$$\mathrm{B} = \left\{ \begin{array}{cccc} & atmos & upocn & dpocn \\ atmos & b_{11} & b_{12} & b_{13} \\ upocn & b_{21} & b_{22} & b_{23} \\ dpocn & b_{31} & b_{32} & b_{33} \end{array} \right\} = \left\{ \begin{array}{cccc} & atmos & upocn & dpocn \\ atmos & 0.810712 & 0.097213 & 0 \\ upocn & 0.189288 & 0.852787 & 0.003119 \\ dpocn & 0 & 0.050000 & 0.996881 \end{array} \right\}$$

Nearly 19 percent of atmospheric carbon is absorbed by the upper sea (over a decade), and the upper sea releases about 10 percent of its carbon to the atmosphere (over a decade).

If emissions end at some point T, then the equilibrium concentration of carbon can be given by the following equation:

$$Conc_{\infty} = \mathrm{B}^{\infty} Conc_T$$

The equilibrium B matrix, B^{∞}, is given by:

$$\mathrm{B}^{\infty} = \left\{ \begin{array}{cccc} & atmos & upocn & dpocn \\ atmos & 0.029269 & 0.029269 & 0.029269 \\ upocn & 0.056991 & 0.056991 & 0.056991 \\ dpocn & 0.913739 & 0.913739 & 0.913739 \end{array} \right\}$$

This implies that in the long run the atmosphere will contain just under 3 percent of total carbon in all three physical zones (or sinks) as of the terminal year of emissions, with about 6 percent in the upper ocean and the remaining 91 percent absorbed in the deep ocean. At today's level of carbon concentrations we would get the following equilibrium concentration levels (assuming all emissions stop today):[6]

$$Conc_{\infty} = \left[\begin{array}{c} 598 \\ 1,164 \\ 18,667 \end{array} \right] = \mathrm{B}^{\infty} \left[\begin{array}{c} 809 \\ 1,255 \\ 18,365 \end{array} \right]$$

This translates into a reduction of 26 percent in atmospheric concentration and a rise of 1.6 percent in deep ocean concentration. If the entire estimated amount of fossil fuels is spewed out into the atmosphere, over the very long run, the atmospheric concentration would stabilize at 713 GTC, lower than today's level, but in the intermediate years, concentration levels could rise dramatically.

5 The variable *Forc* is a vector defined over both sinks but is only non-zero for the atmosphere.

6 The concentration is expressed at the stock of carbon (C) in gigatons.

In the DICE baseline with no mitigation efforts, concentration levels in the atmosphere max out at around 3,000 GTC in around 2250.

The matrix B is valid for a time horizon spanning 10 years. In other words, equation (F.6-4) in terms of the DICE model is:

$$Conc_{z,t+10} = \mathrm{B}C_{z,t} + E_t$$

where E_t represents the cumulated emissions over 10 years through year t. It is possible to convert B into an annual transition matrix with some matrix algebra and numerical evaluation. If the matrix B is a positive definite matrix, than all of its eigenvalues are positive and it is possible to take the n^{th} root of the matrix B. The eigenvalues and eigenvectors of a real matrix B solve the following matrix equation:

$$\mathrm{B}x = \lambda x$$

In other words the projection of the vector x, by the matrix B is equal to that same vector multiplied by a scalar, λ. The eigenvalues, λ, can be calculated by solving an n-degree polynomial derived from the determinant of the above system:

$$\mathrm{B}x = \lambda x \Leftrightarrow (\mathrm{B} - \lambda.I)\, x = 0 \Rightarrow |\mathrm{B} - \lambda.I| = 0$$

Let V be the matrix of (right) eigenvectors of B (in columns), and Λ the diagonal matrix composed of the eigenvalues (in the same order as the respective eigenvectors), then B is diagonalized by:

$$\mathrm{B} = V\Lambda V^{-1}$$

It can be shown that if Λ has only positive eigenvalues[7], than the n^{th} root of B can be derived from:[8]:

$$\mathrm{B}^{1/n} = V\Lambda^{1/n}V^{-1}$$

In the case of the B matrix above, a numerical package has been used to numerically calculate the eigenvalues and eigenvectors:[9]

$$\Lambda = \begin{bmatrix} 0.694258 & 0 & 0 \\ 0 & 0.966122 & 0 \\ 0 & 0 & 1.000000 \end{bmatrix} \quad V = \begin{bmatrix} -0.635745 & -0.311457 & 0.031954 \\ 0.761574 & -0.497912 & 0.062219 \\ -0.125829 & 0.809369 & 0.997551 \end{bmatrix}$$

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7 If the diagonal elements of a square matrix B are all positive, and if B and B' are both diagonally dominant, then B is positive definite. The definition of diagonally dominant is that the absolute value of each diagonal element is greater than the sum of absolute values of the non-diagonal elements in its row. That is if for all i we have $|a_{ii}| > \sum_{\{j|j\neq i\}} |a_{ij}|$.

8 It is pretty easy to see this if n=2:

$$C = V\Lambda^{1/2}V^{-1} = \mathrm{B}^{1/2} \Leftrightarrow C.C = \mathrm{B} \Leftrightarrow V\Lambda^{1/2}V^{-1}.V\Lambda^{1/2}V^{-1} = V\Lambda^{1/2}.\Lambda^{1/2}V^{-1} = \mathrm{B}$$

In the next to the last step the square root of the diagonal matrix is simply the square root of each diagonal element and the multiplication of the two diagonal matrices is simply the original diagonal matrix. This is easy to generalize for any integer root.

9 The eigenvectors are determined up to a scalar multiple. In the case above, they have been normalized to be on the unit circle.

Thus the annual transition matrix is given by:

$$\mathrm{K} = \mathrm{B}^{1/10} = V\Lambda^{1/10}V^{-1} = \begin{bmatrix} 0.978025 & 0.011566 & -0.000017 \\ 0.022520 & 0.983021 & 0.000338 \\ -0.000545 & 0.005413 & 0.999680 \end{bmatrix}$$

Intuitively, one can see that the diagonal elements of K are roughly equal to the diagonal elements of B raised to the power 0.1 and that the off-diagonal elements are roughly 10 percent of the off-diagonal elements of B.

It is worth noting that the third eigenvector reflects the same distribution as the long-run equilibrium distribution described above, corresponding to the eigenvalue 1. The equilibrium matrix can also be derived from the following formula:

$$\mathrm{B}^{\infty} = \underset{n\to\infty}{Lim}\ V\mathrm{K}^{n}V^{-1} = V\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}V^{-1} = \begin{Bmatrix} 0.029269 & 0.029269 & 0.029269 \\ 0.056991 & 0.056991 & 0.056991 \\ 0.913739 & 0.913739 & 0.913739 \end{Bmatrix}$$

Equation (F.6-4) can be written in cumulative form as:

$$Conc_t = \mathrm{K}^n\, Conc_{t-n} + \sum_{j=0}^{n-1} \mathrm{K}^{n-1-j} E_{t+j-n}$$

Assuming that emissions grow at a compound growth rate of g^e between $t-n$ and t, we have the following:

$$\begin{aligned} Conc_t &= \mathrm{K}^n\, Conc_{t-n} + \sum_{j=0}^{n-1} \mathrm{K}^{n-1-j}(1+g)^j E_{t-j-1} \\ &= \mathrm{K}^n\, Conc_{t-n} + (1+g)^{n-1}\sum_{j=0}^{n-1} V\Lambda^{n-1-j}(1+g)^{-n+1+j}V^{-1}E_{t-n} \\ &= \mathrm{K}^n\, Conc_{t-n} + (1+g)^{n-1}V\left[\sum_{j=0}^{n-1}\Lambda^{n-1-j}(1+g)^{-n+1+j}\right]V^{-1}E_{t-n} \end{aligned}$$

The expression within brackets is a diagonal matrix, so it is possible to use standard formulas for a geometric progression to give the following:

$$Conc_t = V\Lambda^n V^{-1}\, Conc_{t-n} + V\Phi V^{-1} E_{t-n}$$

where Λ is defined as above, and the diagonal matrix Φ is given by:

$$\Phi_{ij} = \begin{cases} \dfrac{\lambda_i^n - (1+g^e)^n}{\lambda_i - (1+g^e)} & \text{if} \quad i=j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

Based on these expressions, equation (F.6-4) that defines concentration for a one period gap is replaced by equations (F.6-7), (F.6-8) and (F.6-9). Equation (F.6-7) determines the growth of emissions between period $t-n$ and t assuming a constant annual growth rate, g^{emi}. Equation (F.6-8), similar to the expression above, defines a 3×3 matrix, *EMIGFact*, which is used to determine the growth factor in the cumulative concentration expression. The parameter λ^c in the expression represents the eigenvalues of the transition matrix K. And equation (F.6-9), replacing equation (F.6-4) determines the cumulative concentration, *Conc*, in period t. The first component on the right is the evolution of the existing stock of carbon concentration where V^c is the matrix of eigenvectors of the K matrix and Λ^c is the diagonal matrix of eigenvalues. The second component represents cumulative emissions over the period n, with adjustments for the transition of the lagged emissions across the sinks.

$$g^{emi}_{em,t} = \left(\frac{EMIGbl_{atmos,em,t}}{EMIGbl_{atmos,em,t-n}} \right)^{1/n} - 1 \tag{F.6-7}$$

$$EMIGFact_{z,t} = \frac{(\lambda^c_z))^n - \left(1 + g^{emi}_{em,t}\right)^n}{\lambda^c_z - \left(1 + g^{em}_{em,t}\right)} \tag{F.6-8}$$

$$Conc_t = V^c \left(\Lambda^c\right)^n \left(V^c\right)^{-1} Conc_{t-n} + (12/44).V^c EMIGFact_{z,t} \left(V^c\right)^{-1} EMIGbl_{z,CO2,t-n} \tag{F.6-9}$$

F.3.2 Temperature

The temperature transition, similar to concentration, has to be modified to allow for multiple year gaps. This section describes how the DICE formulation has been adapted for variable time steps.

The temperature module in DICE can be collapsed into matrix form:

$$T = M.T_{-1} + \mathrm{B}.F = \begin{bmatrix} 1 - \beta_1(\lambda + \beta_2) & \beta_1\beta_2 \\ \beta_3 & 1 - \beta_3 \end{bmatrix} .T_{-1} + \begin{bmatrix} \beta_1 \\ 0 \end{bmatrix} .F$$

where the transition and impact matrices, M and B are defined for a 10-year transition period. In the steady-state, this can be written as:

$$T^e = [I - M]^{-1}\mathrm{B}.F$$

where F is a constant level of radiative forcing. The inverse matrix has a rather simple expression:

$$[I - M]^{-1} = \begin{bmatrix} 1/(\lambda\beta_1) & \beta_2/(\lambda\beta_1) \\ 1/(\lambda\beta_1) & (\lambda + \beta_2)/(\lambda\beta_3) \end{bmatrix}$$

This implies that the equilibrium temperature for both the atmosphere and the deep ocean is given simply by:

$$T^e = F/\lambda$$

With the default value for λ, the equilibrium temperature is about 0.8 times the equilibrium forcing level.

Similar to the concentration equation above, the temperature equation is recursive and can be collapsed into multi-period form by the following formula:

$$T_{t+n} = V\Lambda^n V^{-1} T_t + V\Phi V^{-1}\mathrm{B}.F_{t+n}$$

where Λ is the diagonal matrix of eigenvalues of the one-period transition matrix, and the diagonal matrix Φ is given by:

$$\Phi_{ij} = \begin{cases} \dfrac{\lambda_i^n - \left(1 + g^f\right)^n}{\lambda_i \left(1 + g^f\right)^{n--1} - \left(1 + g^f\right)^n} & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

where g^f is the annual compound growth rate of forcing (F) over the n-period range.

There are two differences with the concentration accumulation equation. The first is that the forcing variable, F, is pre-multiplied by the matrix B. The second is that in the DICE code the forcing variable is contemporaneous and not lagged—this changes the accumulation expression compared to the one for concentration. Both nevertheless collapse to 1 when n is equal to 1.

Similar to the concentration matrix, but with additional complications, one must convert the DICE-based 10-period M and B matrices into a 1-period matrix—hopefully preserving as well the particular relations across the different cells of the matrices. The following steps provide one way to do this:

1. Calculate the 10-period eigenvalues and eigenvectors of the 10-period M matrix so that the following holds:

$$M = V\Lambda V^{-1}$$

2. Calculate the $1/10^{th}$ roots of the eigenvalues and then evaluate the 1-period M matrix, Γ:

$$\Gamma = V\Lambda^{0.1}V^{-1}$$

3. Calculate the β coefficients consistent with the values of the cells in Γ. There are too few degrees of freedom, so some choices must be made. For example:

$$\beta_2 = \frac{\lambda\Gamma_{12}}{1-\Gamma_{11}-\Gamma_{12}} \quad \beta_1 = \frac{\Gamma_{12}}{\beta_2} \quad \beta_3 = \Gamma_{21} \quad \Gamma_{22} = 1-\beta_3$$

   Thus, the bottom right cell of Γ is adjusted so that the sum along the bottom row is 1. The one-period B matrix, B_1 then becomes:

$$B_1 = \beta_1$$

4. Since the Γ matrix has been modified, it is necessary to re-calculate the eigenvalues and eigenvectors consistent with the adjusted 1-period Γ matrix. The new one-period β coefficients and the one-period eigenvalues and eigenvectors can be used for models that use one- or multi-period steps.

Equations (F.6-5) and (F.6-6) are then replaced with (F.6-10) and (F.6-11), (F.6-12) and (F.6-13). In equation (F.6-10) the only difference with Equation (F.6-5) is that the expression uses the average concentration between years $t-n$ and t, rather than the concentration of a single year. Equation (F.6-11) defines the average annual growth rate in forcing, g^f, between years $t-n$ and t. Equation (F.6-12) defines a 2×2 diagonal matrix that is used to provide the forcing growth factor, *ForcGFact*, for the cumulative temperature transition equation. It is similar to expression (F.6-8) save that the denominator is adjusted to account for the fact that forcing is assumed to impact current temperatures, and not future temperatures, i.e. forcing and temperature are contemporaneous variables. The λ^t parameters are the eigenvalues of the temperature transition matrix. Equation (F.6-13) represents the temperature/forcing relation for a multi-year transition period, where V^t is the matrix of eigenvectors of the temperature transition matrix, Λ^t is the matrix of eigenvalues, and Θ is the direct impact of forcing on temperature.

$$Forc_{atmos} = fCO2x\frac{\log_{10}\left(0.5\left[Conc_{atmos,t-n}+Conc_{atmos,t}\right]/ConcPI\right)}{\log_{10}(2)} + ForcOth \qquad \text{(F.6-10)}$$

$$g_t^f = \left(\frac{Forc_t}{Forc_{t-n}}\right)^n - 1 \tag{F.6-11}$$

$$ForcGFact_{zt,t} = \frac{\left(\lambda_{zt}^t\right)^n - \left(1 + g_t^f\right)^n}{\lambda_{zt}^t \left(1 + g_t^f\right)^{n-1} - \left(1 + g_t^f\right)} \tag{F.6-12}$$

$$Temp_{zt,t} = V^t \left(\Lambda^t\right)^n \left(V^t\right)^{-1} Temp_{zt,t-n} + V^t ForcGFact_{zt,t} \left(V^t\right)^{-1} \Theta.Forc_{zt} \tag{F.6-13}$$

Appendix G

Examples of carbon taxes, emission caps and tradable permits

G.1 Border tariff adjustment simulations

Border tariff adjustments (BTAs) have been a hotly contested subject since at least the early negotiations over the Kyoto Protocol. The issue arises when a small coalition of countries unilaterally implement emission reduction policies that raise domestic energy prices but have no direct impact on energy prices outside the coalition. This has two consequences. The first is that the coalition countries witness a loss in competitiveness and therefore market share—both at home and on export markets. And, their emission reduction efforts could be partially—or even entirely—offset by rising emission in non-coalition countries, the so-called carbon leakage effect. One response to either or both of these effects is to raise a carbon tax on competing imports thereby raising the relative price of imports with the aim of neutralizing the competiveness effect without of course affecting directly market shares in other countries. The coalition countries are also likely to rebate the carbon tax to exporters to maintain competitiveness outside the coalition. If the main concern is competitiveness, coalition countries could use the domestic carbon content to calculate the needed border tax adjustment. On the other hand, if the main concern is leakage, there could be a case for using the carbon content of the import competing countries under an assumption that all producers would therefore be paying the same price for their carbon emissions. This section describes how the Envisage model has been modified to handle BTAs under these different assumptions.

G.1.1 Adjustment based on carbon content of domestic producers (or importers)

The main idea is to level the playing field for domestic producers. The model calculates how much the costs of production increase due to both the direct and indirect effects of the carbon tax. It is an *ex ante* calculation in the sense that the cost structure of a reference year is chosen before implementation of the BTA and assumes a given price and production structure. In a GE framework prices and quantities will adjust so that the ex post increase in cost is likely to be lower than the *ex ante* increase thus the BTA will over-compensate domestic producers.

Equation (G.7-1) defines the *ex ante* increase in unit cost, PX^1, generated by the carbon tax, where the year tr, refers to a reference year prior to the implementation of the BTA and the superscripted price variables are the prices that reflect the direct and indirect costs of the carbon tax. It assumes therefore a fixed technology and the same prices for factors—but augments the cost of intermediate goods by the carbon tax. The new unit cost will include both the direct and indirect costs of the carbon tax. Equation (G.7-2) determines the *ex ante* price of intermediate goods, PA^1. It is assumed that the increase in intermediate goods is equal to the increase in the unit cost. Equations (G.7-1) and (G.7-2) represent a recursive system in prices that will generate in the end the relative increase in the unit cost of production induced by carbon taxation. Equation (G.7-3) calculates the *ex ante* wedge in the unit cost, ω^d, i.e. the wedge that is induced by the carbon tax given the cost structure of the reference year. Equation (G.7-4) calculates the equivalent tariff that is applied to imported goods that offsets the *ex ante* increase in the cost of domestic production. A complicating factor is the multi-output structure of production. This is dealt with by taking the weighted average of the different production streams (indexed by a) to produce commodity, i. The weights are represented by the α^p parameter and are calculated using the reference year shares:

$$\alpha^p_{r,a} = \frac{\displaystyle\sum_{a \in \{\gamma^p_{r,a,i} \neq 0\}} PP_{r,a,tr} XP_{r,a,tr}}{\displaystyle\sum_{i \in \{\gamma^p_{r,a,i} \neq 0\}} PS_{r,i,tr} XP_{r,i,tr}}$$

For most commodity/activity combinations there is a one-to-one correspondence and the α^p parameter takes the value 1. Note that equations (G.7-1) and (G.7-2) hold for all activities and commodities, whereas equation (G.7-4) is only applied to a subset of commodities indexed by it. The index r represents countries that are self-imposing a carbon tax, whereas the index s in equation (G.7-4) is for all countries that are not limiting emissions. Thus, even if there is uneven effort (or uneven carbon taxes) across countries with GHG emission limits, there is no assumption that a compensating mechanism will be in effect among these countries.

$$\begin{aligned} PX^1_{r,a,t} &= \left[\sum_i PA^1_{r,i,a,t} XA_{r,i,a,tr} + \sum_{fp} PF_{r,fp,a,tr} XF_{r,fp,a,tr} \right. \\ &+ \left. \sum_i \sum_{em} \tau^{emi}_{r,em,t} \rho_{r,em,i,a} XA_{r,i,a,tr} \right] / XP_{r,a,tr} \end{aligned} \quad \text{(G.7-1)}$$

$$PA^1_{r,i,a,t} = PA_{r,i,tr} \frac{\displaystyle\sum_{a' \in \{\gamma^p_{r,a,i} \neq 0\}} \alpha^p_{r,a'} PX^1_{r,a',tr}}{\displaystyle\sum_{a' \in \{\gamma^p_{r,a,i} \neq 0\}} \alpha^p_{r,a'} PX_{r,a',tr}} \quad \text{(G.7-2)}$$

$$\omega^d_{r,a,t} = \left(PX^1_{r,a,t} - PX_{r,a,tr}\right) / PX_{r,a,tr} \quad \text{(G.7-3)}$$

$$\tau^a_{s,r,it,t} = \sum_{a \in \{\gamma^p_{r,a,i} \neq 0\}} \alpha^p_{r,a} \omega^d_{r,a,t} \quad \text{(G.7-4)}$$

Most border adjustment regimes would also include a cost-compensation for exports. A symmetric additive adjustment factor can be included to existing export taxes subsidies. Equation (G.7-5) defines the export subsidy adjustment, where r is the exporting country and has imposed a carbon tax, and d is a destination country with no carbon tax.

$$\tau^{ae}_{r,d,it,t} = -\sum_{a \in \{\gamma^p_{r,a,i} \neq 0\}} \alpha^p_{r,a} \omega^d_{r,a,t} \quad \text{(G.7-5)}$$

These new instruments require changes to equations that contain bilateral tariffs and/or export tax subsidies. This reduces to four equations—equations (T-7) and (T-9) that define bilateral trade prices, FOB and end-user respectively; and equations (Y-3) and (Y-4) that determine fiscal revenues associated with import tariffs and export subsidies respectively.

$$WPE_{r,d,im} = \left(1 + \tau^{e}_{r,d,im} + \tau^{ea}_{r,d,im}\right) PE_{r,d,im} \tag{G.7-6}$$

$$PM_{s,r,im} = \left(1 + \tau^{m}_{s,r,im} + \tau^{a}_{s,r,im}\right) WPM_{s,r,im} \tag{G.7-7}$$

$$GREV_{r,mtax} = \sum_{i \in Arm} \sum_{s} \left(\tau^{m}_{s,r,i} + \tau^{a}_{s,r,i}\right) WPM_{s,r,i} WTF^{d}_{s,r,i} + \sum_{i \notin Arm} \tau^{m}_{r,i} PW_{i} XMT_{r,i} \tag{G.7-8}$$

$$GREV_{r,etax} = \sum_{i \in Arm} \sum_{d} \left(\tau^{e}_{r,d,i} + \tau^{ae}_{r,d,i}\right) PE_{r,d,i} WTF^{s}_{r,d,i} + \sum_{i \notin Arm} \tau^{e}_{r,i} PW_{i} XET_{r,i} \tag{G.7-9}$$

G.1.2 Adjustment based on carbon content of exporters

A tax on the carbon content of imports may help to attenuate the leakage effect of unilateral carbon taxes. The implementation involves assessing the *ex ante* cost of production in the exporting countries (i.e. those not imposing a carbon tax) applying the carbon tax of the destination country. This implies that the cost wedge and the tariff adjustment will vary across exporting countries for the same destination economy, unlike the adjustment above where the tariff adjustment is uniform across importing countries.

Equation (G.7-10) defines the *fictitious* carbon tax that is imposed on the cost of production in the exporting country. It is bilateral as each of the importing countries has a different carbon tax. Equation (G.7-11) defines the ex ante increase in unit cost, PX^2, generated by the carbon tax, where the year tr, refers to a reference year prior to the implementation of the BTA and the super-scripted price variables are the prices that reflect the direct and indirect costs of the carbon tax. It assumes therefore a fixed technology and the same prices for factors—but augments the cost of intermediate goods by the carbon tax. The new unit cost will include both the direct and indirect costs of the carbon tax. It is a bilateral price as the carbon tax differs across countries of destination. Equation (G.7-12) determines the *ex ante* price of intermediate goods, PA^2. It is assumed that the increase in intermediate goods is equal to the increase in the unit cost. Equation (G.7-13) calculates the *ex ante* wedge in the unit cost, ω^m, i.e. the wedge that is induced by the carbon tax given the cost structure of the reference year. Equation (G.7-14) calculates the equivalent tariff that is applied to imported goods that offsets the *ex ante* increase in the cost of domestic production. Note that equations (G.7-11) through (G.7-13) hold for all activities and commodities, whereas equation (G.7-14) is only applied to a subset of commodities indexed by it. The index r represents countries that are self-imposing a carbon tax, whereas the index s is for all countries that are not limiting emissions.

$$\tau^{emi,2}_{r,s,em,t} = \tau^{emi}_{r,em,t} \tag{G.7-10}$$

$$\begin{aligned} PX^{2}_{r,s,a,t} &= \Bigg[\sum_{i} PA^{2}_{r,s,i,a,t} XA_{s,i,a,tr} + \sum_{fp} PF_{s,fp,a,tr} XF_{s,fp,a,tr} \\ &+ \sum_{i} \sum_{em} \tau^{emi,2}_{r,s,em,t} \rho_{s,em,i,a} XA_{s,i,a,tr}\Bigg] / XP_{s,a,tr} \end{aligned} \tag{G.7-11}$$

$$PA^2_{r,s,i,a,t} = PA_{s,i,tr} \frac{\sum\limits_{a' \in \{\gamma^p_{s,a,i} \neq 0\}} \alpha^p_{s,a'} PX^2_{r,s,a',tr}}{\sum\limits_{a' \in \{\gamma^p_{s,a,i} \neq 0\}} \alpha^p_{s,a'} PX_{s,a',tr}} \tag{G.7-12}$$

$$\omega^d_{r,s,a,t} = \left(PX^2_{r,s,a,t} - PX_{s,a,tr}\right) / PX_{s,a,tr} \tag{G.7-13}$$

$$\tau^a_{s,r,it,t} = \sum_{a \in \{\gamma^p_{s,a,i} \neq 0\}} \alpha^p_{s,a} \omega^m_{r,s,a,t} \tag{G.7-14}$$

Some scenarios allow for compensating a subset of regions for the imposition of a carbon tax. Thus while high-income countries benefit from a uniform global price on carbon, developing countries may not desire to impose a tax that could be harmful to their economy, especially since high-income countries are as at present mostly responsible for the current stock of GHG atmospheric concentration. The compensation mechanism is implemented as a government to government transfer, GTR. Given the fiscal closure rule, this implies that direct taxes on households will shift—lower taxes for receiving countries and higher taxes for donor countries. Equation (G.7-15) guarantees that the sum of the transfers adds up to zero globally where countries are divided into donor and recipient countries. (N.B. The transfers are not bilateral.) Equation (G.7-16) is an allocation mechanism across donor countries that insures that the transfers are equalized on a per capita basis.

$$\sum_{r \in Donors} GTR_r + \sum_{r \in Recipients} GTR_r \equiv 0 \tag{G.7-15}$$

$$GTRpc_r = GTR_r / Pop_r \qquad \text{for } r \in Recipients \tag{G.7-16}$$

The implementation requires exogenizing some objective for the recipient countries. In the standard implementation, real domestic absorption is fixed to baseline values, where the superscript BaU refers to the baseline level:

$$RYD_r = RYD^{BaU}_r \qquad \text{for } r \in Recipients$$

The addition of the new GTR variable requires a change to the government revenue equation:

$$YG_r = \sum_{gy} GREV_{r,gy} + \sum_{em} QuotaY^E_{r,em} + GTR_r \tag{G.7-17}$$

Appendix H

Base ENVISAGE parameters

Like most CGE models, Envisage contains a mix of calibrated and key parameters—the latter sourced from a variety of studies. The basic framework of a comparative static CGE model is that a base year data set is given—the GTAP world social accounting matrix (SAM), for example. This represents value flows. Base prices are typically initialized at unit value, with some exceptions if volume flows and/or stocks are available—for example energy in physical units or the stock of labor. Parameters are then divided into two sets: key parameters—typically substitution, supply, price and income elasticities—, and calibrated parameters. The model can be represented compactly by the following formula:

$$F\left(Y, X; \theta_1, \theta_2\right)$$

where Y represents endogenous variables and X is the set of exogenous variables including policy instruments. Both F and Y are typically of the same size—say n endogenous variables and n equations—and X is of dimension m. θ_1 is the set of key parameters and θ_2 is the set of calibrated parameters. The key parameters are given and typically estimated using outside sources and data. In the calibration phase, both Y and X are given (by the base year data), and the function F is inverted to calibrate the θ_2 parameters such that the model can replicate the base year data. Alternative scenarios then involve perturbing one or more elements in X and inverting the function F to calculate a new Y, holding θ_1 and θ_2 constant.[1] Note that sensitivity analysis on θ_1, the key parameters, typically requires re-calibrating θ_2 for each new set of θ_1. The rest of this appendix presents the key parameter values used for Envisage.

H.1 Production elasticities

The basic production substitution elasticities are provided in Tables H.1 and H.2. They replicate those used for the OECD GREEN model (see Burniaux, Nicoletti, and Oliveira-Martins (1992)) and likewise underlie the World Bank's LINKAGE model. The original OECD GREEN model had a single energy nest rather than the nested structure of Envisage. The uniformity of the energy substitution elasticities therefore replicates the structure of GREEN.

1 Given the recursive nature of the model specification, calibration is typically done block by block with explicit formulas rather than inverting the full model to calibrate the θ_2 parameters.

Table H.1: Production elasticities

Name	Symbol	*Old*	*New*	Note
sigmap	σ^p	0.00	0.00	
sigmav	σ^v	0.12	1.00	Leontief technology in fossil fuel production
sigmake	σ^{ke}	0.00	0.80	Leontief technology in fossil fuel production
sigman	σ^n	0.00	0.00	Not differentiated by vintage

Table H.2: Energy substitution elasticities in production

Name	Symbol	*Old*	*New*	Note
sigmae	σ^e	0.25	2.00	
sigmanely	σ^{nely}	0.25	2.00	
sigmaolg	σ^{olg}	0.25	2.00	
sigmaely	σ^{ely}	0.25	2.00	
sigmacoa	σ^{coa}	0.25	2.00	
sigmaoil	σ^{oil}	0.25	2.00	
sigmagas	σ^{gas}	0.25	2.00	

H.2 Final demand elasticities

Consumer final demand elasticities in the Envisage model are derived in large part from estimates produced by the Economic Research Service (ERS) of the U.S. Department of Agriculture (USDA).[2] The available estimates are for a reduced set of goods and these estimates are allocated over the 57 sectoring scheme of GTAP. They are aggregated using GTAP consumption shares for specific aggregations of GTAP. Table H.3 provides the income elasticities for all 57 GTAP goods for a selected aggregation across GTAP regions.[3] The calibration procedure of Envisage may make some adjustments to these elasticities to insure that the consumption weighted sum of the income elasticities adds up to unity. Table H.4 provides the initial price elasticities used in the calibration procedure for the same aggregate regions as in Table H.3—again using simple averages within regions rather than consumption weighted.[4]

Equations (D-11) and (D-12) convert consumed goods to produced goods using a transition matrix approach with a CES preference structure. The transition matrix is currently diagonal.

2 Regmi (2001), Seale, Jr., Regmi, and Bernstein (2003) and Regmi and Seale, Jr. (2010).

3 Regional aggregations in Table H.3 are simple—not consumption weighted.

4 The price elasticities are treated as negative values upon input.

Table H.3: Consumer income elasticities for select regions

	CHN	XEA	IND	XSA	RUS	XEC	MNA	SSA	LAC	WEU	JPN	USA	RHY
PDR	0.19	0.46	0.49	0.49	0.40	0.41	0.43	0.52	0.38	0.22	0.16	0.05	0.18
WHT	0.19	0.46	0.48	0.49	0.40	0.40	0.42	0.52	0.38	0.22	0.16	0.05	0.18
GRO	0.19	0.46	0.49	0.49	0.40	0.40	0.42	0.43	0.38	0.22	0.16	0.05	0.18
V_F	0.38	0.55	0.58	0.55	0.50	0.49	0.51	0.50	0.48	0.33	0.24	0.08	0.29
OSD	0.23	0.48	0.50	0.52	0.43	0.43	0.45	0.54	0.41	0.25	0.18	0.06	0.21
C_B	0.48	0.74	0.76	0.77	0.65	0.66	0.68	0.76	0.64	0.43	0.31	0.11	0.39
PFB	0.91	0.92	0.92	0.92	0.91	0.91	0.91	0.92	0.91	0.91	0.90	0.90	0.91
OCR	0.48	0.73	0.74	0.75	0.65	0.66	0.68	0.68	0.63	0.43	0.31	0.11	0.38
CTL	0.48	0.74	0.77	0.74	0.66	0.65	0.68	0.74	0.64	0.43	0.31	0.11	0.39
OAP	0.48	0.70	0.75	0.74	0.65	0.65	0.68	0.74	0.64	0.43	0.31	0.11	0.39
RMK	0.51	0.81	0.84	0.76	0.68	0.71	0.74	0.84	0.69	0.46	0.33	0.12	0.41
WOL	0.91	0.92	0.92	0.92	0.91	0.91	0.92	0.92	0.91	0.91	0.90	0.90	0.91
FRS	1.26	1.47	1.49	1.47	1.33	1.37	1.36	1.70	1.34	1.26	1.24	1.22	1.25
FSH	0.53	0.79	0.87	0.84	0.74	0.75	0.77	0.87	0.72	0.48	0.34	0.12	0.42
COA	1.17	1.23	1.24	1.24	1.19	1.21	1.20	1.27	1.20	1.17	1.16	1.15	1.16
OIL	1.18	1.25	1.26	1.26	1.21	1.22	1.22	1.29	1.21	1.18	1.17	1.16	1.17
GAS	1.17	1.23	1.24	1.24	1.19	1.20	1.20	1.27	1.20	1.17	1.16	1.15	1.16
OMN	1.26	1.48	1.50	1.49	1.33	1.37	1.36	1.74	1.35	1.26	1.24	1.22	1.25
CMT	0.48	0.73	0.77	0.77	0.61	0.65	0.67	0.74	0.60	0.42	0.31	0.10	0.38
OMT	0.48	0.70	0.77	0.77	0.64	0.65	0.64	0.72	0.61	0.41	0.31	0.10	0.38
VOL	0.24	0.46	0.51	0.48	0.42	0.43	0.45	0.51	0.39	0.25	0.18	0.06	0.20
MIL	0.51	0.79	0.82	0.82	0.69	0.69	0.71	0.83	0.66	0.43	0.33	0.11	0.41
PCR	0.19	0.39	0.46	0.38	0.40	0.41	0.43	0.49	0.36	0.22	0.15	0.05	0.17
SGR	0.48	0.73	0.76	0.73	0.64	0.66	0.67	0.74	0.63	0.43	0.31	0.11	0.38
OFD	0.48	0.61	0.72	0.73	0.61	0.62	0.64	0.58	0.57	0.39	0.27	0.08	0.36
B_T	0.58	0.90	1.07	1.03	0.85	0.87	0.90	1.18	0.79	0.51	0.36	0.12	0.45
TEX	0.91	0.87	0.88	0.86	0.91	0.90	0.89	0.88	0.89	0.89	0.90	0.90	0.90
WAP	0.91	0.84	0.92	0.90	0.89	0.90	0.87	0.88	0.86	0.88	0.89	0.88	0.88
LEA	0.91	0.89	0.92	0.92	0.91	0.91	0.91	0.91	0.89	0.90	0.90	0.90	0.90
LUM	1.26	1.47	1.50	1.49	1.32	1.36	1.35	1.73	1.33	1.26	1.24	1.22	1.25
PPP	1.26	1.46	1.49	1.49	1.33	1.36	1.35	1.73	1.32	1.24	1.23	1.21	1.24
P_C	1.18	1.15	1.20	1.24	1.17	1.19	1.17	1.25	1.14	1.14	1.15	1.14	1.15
CRP	1.27	1.41	1.48	1.45	1.30	1.34	1.33	1.67	1.26	1.23	1.23	1.20	1.23
NMM	1.26	1.47	1.50	1.48	1.33	1.37	1.35	1.73	1.34	1.26	1.24	1.22	1.25
I_S	1.26	1.48	1.50	1.49	1.33	1.37	1.36	1.74	1.34	1.26	1.24	1.22	1.25
NFM	1.26	1.47	1.50	1.49	1.33	1.37	1.36	1.74	1.34	1.26	1.24	1.22	1.25
FMP	1.26	1.47	1.49	1.49	1.33	1.37	1.36	1.73	1.34	1.26	1.24	1.22	1.25
MVH	1.18	1.17	1.26	1.25	1.18	1.20	1.18	1.27	1.18	1.13	1.15	1.13	1.14
OTN	1.18	1.23	1.26	1.26	1.21	1.22	1.21	1.29	1.19	1.17	1.17	1.15	1.17
ELE	1.27	1.41	1.49	1.48	1.33	1.36	1.35	1.73	1.31	1.25	1.22	1.21	1.22
OME	1.27	1.43	1.49	1.49	1.31	1.34	1.35	1.73	1.32	1.24	1.23	1.20	1.23
OMF	1.26	1.41	1.49	1.48	1.32	1.36	1.34	1.71	1.30	1.24	1.23	1.21	1.23
ELY	1.17	1.17	1.22	1.21	1.12	1.13	1.16	1.24	1.16	1.13	1.15	1.14	1.15
GDT	1.17	1.23	1.24	1.24	1.18	1.17	1.20	1.27	1.20	1.17	1.16	1.15	1.16
WTR	1.17	1.23	1.24	1.24	1.18	1.20	1.20	1.25	1.19	1.16	1.15	1.14	1.16
CNS	1.17	1.23	1.24	1.24	1.19	1.20	1.20	1.26	1.19	1.16	1.16	1.15	1.16
TRD	1.19	0.92	1.17	1.19	0.97	1.11	1.08	1.08	1.03	1.01	0.99	0.98	0.94
OTP	1.18	1.19	1.22	1.15	1.15	1.18	1.18	1.23	1.12	1.14	1.13	1.14	1.14
WTP	1.18	1.25	1.25	1.23	1.20	1.22	1.22	1.29	1.21	1.17	1.17	1.16	1.17
ATP	1.18	1.22	1.26	1.23	1.20	1.21	1.21	1.28	1.19	1.17	1.16	1.16	1.15
CMN	1.18	1.22	1.26	1.25	1.20	1.20	1.20	1.27	1.17	1.15	1.15	1.13	1.15
OFI	1.27	1.44	1.49	1.49	1.33	1.36	1.34	1.70	1.30	1.23	1.23	1.15	1.22
ISR	1.26	1.45	1.50	1.48	1.33	1.37	1.35	1.72	1.32	1.24	1.21	1.18	1.22
OBS	1.27	1.41	1.48	1.46	1.28	1.33	1.27	1.67	1.31	1.09	1.22	1.21	1.22
ROS	1.33	1.71	1.79	1.71	1.42	1.52	1.47	2.47	1.41	1.28	1.23	1.16	1.25
OSG	1.28	1.42	1.49	1.46	1.30	1.36	1.29	1.77	1.25	1.19	1.18	0.99	1.16
DWE	1.18	1.06	1.18	1.18	1.20	1.19	1.17	1.21	1.02	1.16	1.02	0.99	1.03

Note: The regional abbreviations are China (CHN), Rest of East Asia & Pacific (XEA), India (IND), Rest of South Asia (XSA), Russia (RUS), Rest of Europe & Central Asia (XEC), Middle East & North Africa (MNA), Sub-Saharan Africa (SSA), Latin America & Caribbean (LAC), Western Europe (WEU), Japan (JPN), the United States (USA), and Rest of High-income (RHY).

Table H.4: Consumer price elasticities for select regions

	CHN	XEA	IND	XSA	RUS	XEC	MNA	SSA	LAC	WEU	JPN	USA	RHY
PDR	0.15	0.37	0.41	0.40	0.33	0.33	0.35	0.43	0.31	0.18	0.13	0.04	0.14
WHT	0.15	0.37	0.41	0.40	0.33	0.33	0.35	0.43	0.31	0.18	0.13	0.04	0.14
GRO	0.15	0.37	0.41	0.40	0.33	0.33	0.35	0.43	0.31	0.18	0.13	0.04	0.14
V_F	0.30	0.48	0.51	0.50	0.43	0.43	0.44	0.51	0.42	0.28	0.20	0.07	0.25
OSD	0.19	0.39	0.43	0.42	0.35	0.35	0.37	0.44	0.33	0.20	0.15	0.05	0.17
C_B	0.39	0.60	0.62	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
PFB	0.69	0.69	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
OCR	0.39	0.60	0.62	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
CTL	0.39	0.60	0.63	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
OAP	0.39	0.60	0.63	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
RMK	0.41	0.65	0.68	0.68	0.58	0.58	0.60	0.69	0.56	0.38	0.27	0.10	0.34
WOL	0.69	0.69	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
FRS	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
FSH	0.42	0.68	0.71	0.71	0.60	0.61	0.63	0.73	0.59	0.39	0.28	0.10	0.35
COA	0.77	0.85	0.87	0.87	0.81	0.82	0.82	0.89	0.81	0.77	0.75	0.74	0.76
OIL	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
GAS	0.77	0.85	0.87	0.87	0.81	0.82	0.82	0.89	0.81	0.77	0.75	0.74	0.76
OMN	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
CMT	0.39	0.60	0.63	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
OMT	0.39	0.60	0.63	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
VOL	0.19	0.39	0.43	0.42	0.35	0.35	0.37	0.44	0.33	0.20	0.15	0.05	0.17
MIL	0.41	0.65	0.68	0.68	0.58	0.58	0.60	0.69	0.56	0.38	0.27	0.10	0.34
PCR	0.15	0.37	0.41	0.40	0.33	0.33	0.35	0.43	0.31	0.18	0.13	0.04	0.14
SGR	0.39	0.60	0.62	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
OFD	0.39	0.60	0.62	0.62	0.53	0.54	0.55	0.62	0.52	0.35	0.25	0.09	0.31
B_T	0.47	0.82	0.88	0.86	0.71	0.73	0.76	1.07	0.69	0.44	0.31	0.11	0.38
TEX	0.69	0.69	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
WAP	0.69	0.69	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
LEA	0.69	0.69	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
LUM	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
PPP	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
P_C	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
CRP	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
NMM	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
I_S	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
NFM	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
FMP	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
MVH	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
OTN	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
ELE	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
OME	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
OMF	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
ELY	0.77	0.85	0.87	0.87	0.81	0.82	0.82	0.89	0.81	0.77	0.75	0.74	0.76
GDT	0.77	0.85	0.87	0.87	0.81	0.82	0.82	0.89	0.81	0.77	0.75	0.74	0.76
WTR	0.87	0.93	0.94	0.94	0.90	0.91	0.90	0.96	0.90	0.87	0.86	0.85	0.86
CNS	0.77	0.85	0.87	0.87	0.81	0.82	0.82	0.89	0.81	0.77	0.75	0.74	0.76
TRD	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
OTP	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
WTP	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
ATP	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
CMN	0.82	0.91	0.92	0.92	0.86	0.87	0.87	0.95	0.86	0.82	0.80	0.79	0.81
OFI	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
ISR	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
OBS	0.86	1.07	1.09	1.09	0.94	0.97	0.96	1.29	0.95	0.86	0.83	0.81	0.85
ROS	0.99	1.41	1.40	1.39	1.10	1.17	1.14	1.98	1.12	0.99	0.95	0.93	0.97
OSG	0.91	1.13	1.15	1.14	0.98	1.02	1.01	1.39	1.00	0.91	0.88	0.86	0.90
DWE	0.77	0.85	0.87	0.87	0.81	0.82	0.82	0.89	0.81	0.77	0.75	0.74	0.76

Note: The regional abbreviations are China (CHN), Rest of East Asia & Pacific (XEA), India (IND), Rest of South Asia (XSA), Russia (RUS), Rest of Europe & Central Asia (XEC), Middle East & North Africa (MNA), Sub-Saharan Africa (SSA), Latin America & Caribbean (LAC), Western Europe (WEU), Japan (JPN), the United States (USA), and Rest of High-income (RHY).

Appendix I

The accounting framework

This appendix provides a visual representation of the accounting framework in the GTAP dataset and linked to the model specified in this document. We use the Social Accounting Matrix (or SAM) framework, which is somewhat space consuming, but has the advantage of providing a consistent picture in a single snapshot.[1] There is no unique representation of a SAM, the one depicted in Figure A9-1 has the advantage of reflecting the accounts in basic prices and with all individual price wedges.

I.1 SAM accounts

The figure itself does not need much description. There are 18 accounts in total, some of which are purely pass-through accounts (to stick to the tradition that a SAM should be a square and balanced matrix). Table (I.1) summarizes the accounts.

I.2 Correspondence with GTAP variables

Envisage variables are initialized using data available from the GTAP database. Tables (I.2) and (I.3) describes this correspondence.

[1] For an introduction to Social Accounting Matrices, see Pyatt and Round (1985) and Reinert and Roland-Holst (1997).

Table I.1: Description of the SAM accounts

Account	Description
ACT	Represents the production activities. The total is domestic output at producer price, the latter includes the producer tax. Revenues are exhausted by payments to intermediate goods (including sales tax) and to factors of production.
COMM	Represents total supply—domestic production and imports. The latter enter at CIF prices, to which are added import taxes. The disposition of total supply includes domestic sales of domestic goods, XD, aggregate imports, exports and supply of international trade and transport services.
DAP	This is the disposition of domestic sales of domestic production at producer price (PD).
MAP	This is the disposition of import sales at tariff inclusive import prices (that are uniform across all agents).
DIT	Revenues generated by the agent-specific sales tax on domestic products.
MIT	Revenues generated by the agent-specific sales tax on imported products.
VA	Value added accounts. In the activity column, it reflects the net of tax cost of the factors of production. All factor remuneration is attributed to the single representative household.
VA_TAX	Revenues from taxes on the factors of production. All tax revenues are attributed to the government account.
PTAX	Output tax revenues.
EXP_TAX	Revenues (or cost) from export taxes (or subsidies).
IMP_TAX	Revenues from import tariffs.
HH	Represents the accounts of the private sector. From a national account perspective, this is a consolidated private sector that includes enterprises and non-governmental organizations. In this SAM, the sole source of income for households is net factor remuneration. Expenditures include demand for goods and services and savings net of depreciation. Households save and pay income taxes to the government. Note that in the GTAP database the fiscal accounts are not closed. The model is initialized to assume a zero government deficit and direct taxes represent a residual to balance the household (and government accounts).
GOV	The government collects all indirect taxes and purchases goods and services. Its account is closed by assuming a lump-sum tax on households.
INV	The investment account purchases goods and services. Its income comes from domestic private savings gross of depreciation and foreign saving. Public saving is implicitly assumed to be zero.
DEPR	The depreciation account is a pass-through account.
TRADE	The trade account measures the flow of exports (by region of destination) at FOB prices and the flow of imports (by region of origin) at CIF prices. Aggregate exports and imports (across sectors) are recorded in the balance of payment accounts (BoP) by region. The total for these columns/rows is therefore the sum of exports and imports. The difference between exports and imports provides the net trade with each region (though using different prices since exports are evaluated FOB and imports are evaluated CIF). Aggregate exports (by region) show up in the BoP column since they represent foreign income. Aggregate imports (by region) show up in the BoP row.
ITT_MARG	This account shows the regional supply of international trade and transport services. Its aggregate sum will show up in the BoP column since it is foreign revenue.
BoP	This account has the consolidated balance of payments. Exports and supply of international trade and transport services will show up as revenues in the column. Imports will show up in the row as a payment to the rest of the world. The balancing item is the capital account that appears in the column as a payment to the investment sector. If it is positive, the region is a net capital importer. If it is negative, the region is a net capital exporter. In the aggregation of all regional SAMs, this item should show up as a zero. Also, the sum of exports across all regions and the sum of international trade and transport services should equal the sum of imports.

Table I.2: Correspondence of Envisage variables with GTAP database—domestic agents

Envisage	**GTAP**	**Notes**
$PD_{r,i}XD_{r,i,j}$	$VDFM_{i,j,r}$	Firms' intermediate purchases of domestic goods at market prices
$PMT_{r,i}XM_{r,i,j}$	$VIFM_{i,j,r}$	Firms' intermediate purchases of imported goods at market prices
$\tau^{Ad}_{r,i,j}PD_{r,i}XD_{r,i,j}$	$VDFA_{i,j,r} - VDFM_{i,j,r}$	Tax revenues generated by sales tax on firms' intermediate purchases of domestic goods (difference between purchases at agents' price and market price)
$\tau^{Am}_{r,i,j}PMT_{r,i}XM_{r,i,j}$	$VIFA_{i,j,r} - VIFM_{i,j,r}$	Tax revenues generated by sales tax on firms' intermediate purchases of imported goods (difference between purchases at agents' price and market price)
$PD_{r,i}XD_{r,i,h}$	$VDPM_{i,r}$	Household (or private) purchases of domestic goods at market prices.
$PMT_{r,i}XM_{r,i,h}$	$VIPM_{i,r}$	Household (or private) purchases of imported goods at market prices.
$\tau^{Ad}_{r,i,h}PD_{r,i}XD_{r,i,h}$	$VDPA_{i,r} - VDPM_{i,r}$	Tax revenues generated by sales tax on household purchases of domestic goods (difference between purchases at agents' price and market price)
$\tau^{Am}_{r,i,h}PMT_{r,i}XM_{r,i,h}$	$VIPA_{i,r} - VIPM_{i,r}$	Tax revenues generated by sales tax on household purchases of imported goods (difference between purchases at agents' price and market price)
$PD_{r,i}XD_{r,i,Gov}$	$VDGM_{i,r}$	Government (or public) purchases of domestic goods at market prices.
$PMT_{r,i}XM_{r,i,Gov}$	$VIGM_{i,r}$	Government (or public) purchases of imported goods at market prices.
$\tau^{Ad}_{r,i,Gov}PD_{r,i}XD_{r,i,Gov}$	$VDGA_{i,r} - VDGM_{i,r}$	Tax revenues generated by sales tax on government purchases of domestic goods (difference between purchases at agents' price and market price)
$\tau^{Am}_{r,i,Gov}PMT_{r,i}XM_{r,i,Gov}$	$VIGA_{i,r} - VIGM_{i,r}$	Tax revenues generated by sales tax on government purchases of imported goods (difference between purchases at agents' price and market price)
$PD_{r,i}\ XD_{r,i,Inv}$	$VDFM_{i,cgds,r}$	Investment purchases of domestic goods at market prices. N.B. Investment expenditure in the GTAP database is part of the matrix of firms' expenditures on intermediate goods. It has the label 'CGDS', i.e. expenditures on capital goods. The 'CGDS' sector only purchases goods and services (domestic and imported), but unlike other firm expenditures, the 'CGDS' sector does not purchase factors of production.
$PMT_{r,i}XM_{r,i,Inv}$	$VIFM_{i,cgds,r}$	Investment purchases of imported goods at market prices.
$\tau^{Ad}_{r,i,Inv}PD_{r,i}XD_{r,i,Inv}$	$VDFA_{i,cgds,r} - VDFM_{i,cgds,r}$	Tax revenues generated by sales tax on investment purchases of domestic goods (difference between purchases at agents' price and market price)
$\tau^{Am}_{r,i,Inv}PMT_{r,i}XM_{r,i,Inv}$	$VIFA_{i,cgds,r} - VIFM_{i,cgds,r}$	Tax revenues generated by sales tax on investment purchases of imported goods (difference between purchases at agents' price and market price)

Table I.3: Correspondence of Envisage variables with GTAP database—other accounts

Envisage	**GTAP**	**Notes**
$NPF_{r,fp,j}XF_{r,fp,j}$	$EVFA_{fp,j,r}$	Firms' payments to factors of production net of tax/subsidies. Standard factors include unskilled and skilled labor, capital, land (in agriculture and forestry) and natural resources (in fisheries and mineral extraction sectors). Land can also be broken out by agro-ecological zone (AEZ), of which there are 18 categories.
$\tau^v_{r,fp,j}NPF_{r,fp,j}XF_{r,fp,j}$	$FBEP_{fp,j,r}+FTRV_{fp,j,r}$	Revenues generated by taxes/subsidies on the factors of production
$\tau^p_{r,j}PX_{r,j}XP_{r,j}$	$-OSEP_{j,r}$	Revenues generated by taxes/subsidies on output (change of sign in Envisage relative to GTAP, i.e. tax rates are positive and subsidies are negative, wedge is applied to cost of production, not the market price)
$PP_{r,j}XP_{r,j}$	$TVOM_{j,r}$	Value of domestic output at market prices
$PE_{r,d,i}WTF_{r,d,i}$	$VXMD_{i,r,d}$	Exports from region r into importing region d at producer (or market) price.
$WPE_{r,d,i}WTF_{r,d,i}$	$VXWD_{i,r,d}$	Exports from region r into importing region d at border (or FOB) price.
$WPM_{s,r,j}WTF_{s,r,j}$	$VIWS_{i,s,r}$	Imports into region r from exporter s at border (or CIF) price.
$PM_{s,r,j}WTF_{s,r,j}$	$VIMS_{i,s,r}$	Imports into region r from exporter s tariff inclusive.
$\tau^e_{r,d,i}PE_{r,d,i}WTF_{r,d,i}$	$VXWD_{i,r,d}-VXMD_{i,r,d}$	Export tax/subsidy revenues (for exporter r exporting to region d). N.B. Export tax rates are positive and subsidies negative—unlike in the standard GTAP definition, wedge is applied to the producer (or domestic market) price and not to the world (or FOB price).
$\tau^m_{s,r,j}WPM_{s,r,j}WTF_{s,r,j}$	$VIMS_{i,s,r}-VIWS_{i,s,r}$	Tariff revenues (for importer r for imports sourced in s)
$TMARG_{r,d,i}$	$VIWS_{i,r,d}-VXWD_{i,r,d}$	Value of transport margin between exporter r and importer d, i.e. wedge between CIF and FOB prices.
$S^h_{r,h}$	$SAVE_r$	Household savings. N.B. The GTAP database does not distinguish private and public savings, the value of $SAVE$ represents aggregate domestic savings.
$DeprY^h_{r,h}$	$VDEP_r$	Value of depreciation allowance
$PP_{r,i}XMG_{r,i}$	$VST_{i,r}$	The supply of international trade and transport services from region r.

Table I.4: The schematic social accounting matrix for the ENVISAGE model

	ACT	COMM	DAP
Activities		$P_{a,j}X_{a,j}$	
Commodities			$\text{Diag}(PD_{r,j}XDT_{r,j})$
Dom. Comm.	$PD_{r,i}XD_{r,i,a}$		
Imp. Comm.	$PMT_{r,i}XM_{r,i,a}$		
Tax Dom. Comm.	$\tau^{Ad}_{r,i,a}PD_{r,i}XD_{r,i,a}$		
Tax Imp. Comm.	$\tau^{Am}_{r,i,a}PMT_{r,i}XM_{r,i,a}$		
Value Added	$NPF_{r,fp,a}XF_{r,fp,a}$		
VA tax	$\tau^{f}_{r,fp,a}NPF_{r,fp,a}XF_{r,fp,a}$		
Prod. tax	$\tau^{p}_{r,a}PX_{r,a}XP_{r,a}$		
Import tax		$\tau^{m}_{s,r,j}WPM_{s,r,j}WTF_{s,r,j}$	
Export tax		$\tau^{e}_{r,d,j}PE_{r,d,j}WTF_{r,d,j}$	
Households			
Government			
Investment			
Depreciation			
Trade		$WPM_{s,r,j}WTF_{s,r,j}$	
Intl. Margins			
Rest of the world			

Table I.4: The schematic social accounting matrix for the ENVISAGE model (cont.)

	MAP	DIT	MIT
Activities			
Commodities	$\text{Diag}(PMT_{r,j}XMT_{r,j})$		
Dom. Comm.			
Imp. Comm.			
Tax Dom. Comm.			
Tax Imp. Comm.			
Value Added			
VA tax			
Prod. tax			
Import tax			
Export tax			
Households			
Government		$\sum_{aa}\tau^{Ad}_{r,i,aa}PD_{r,i}XD_{r,i,aa}$	$\sum_{aa}\tau^{Am}_{r,i,aa}PMT_{r,i}XM_{r,i,aa}$
Investment			
Depreciation			
Trade			
Intl. Margins			
Rest of the world			

Table I.4: The schematic social accounting matrix for the ENVISAGE model (cont.)

	VA	VA_TAX
Activities		
Commodities		
Dom. Comm.		
Imp. Comm.		
Tax Dom. Comm.		
Tax Imp. Comm.		
Value Added		
VA tax		
Prod. tax		
Import tax		
Export tax		
Households	$\sum_a NPF_{r,fp,a} XF_{r,fp,a}$	
Government		$\sum_a \tau^f_{r,fp,a} NPF_{r,fp,a} XF_{r,fp,a}$
Investment		
Depreciation		
Trade		
Intl. Margins		
Rest of the world		

Table I.4: The schematic social accounting matrix for the ENVISAGE model (cont.)

	PTAX	EXP_TAX	IMP_TAX
Activities			
Commodities			
Dom. Comm.			
Imp. Comm.			
Tax Dom. Comm.			
Tax Imp. Comm.			
Value Added			
VA tax			
Prod. tax			
Import tax			
Export tax			
Households			
Government	$\sum_a \tau^p_{r,a} PX_{r,a} XP_{r,a}$	$\sum_j \tau^e_{r,d,j} PE_{r,d,j} WTF_{r,d,j}$	$\sum_j \tau^m_{s,r,j} WPM_{s,r,j} WTF_{s,r,j}$
Investment			
Depreciation			
Trade			
Intl. Margins			
Rest of the world			

Table I.4: The schematic social accounting matrix for the ENVISAGE model (cont.)

	HH	**GOV**	**INV**
Activities			
Commodities			
Dom. Comm.	$PD_{r,i}XD_{r,i,h}$	$PD_{r,i}XD_{r,i,gov}$	$PD_{r,i}XD_{r,i,inv}$
Imp. Comm.	$PMT_{r,i}XM_{r,i,h}$	$PMT_{r,i}XM_{r,i,gov}$	$PMT_{r,i}XM_{r,i,inv}$
Tax Dom. Comm.	$\tau^{Ad}_{r,i,h}PD_{r,i}XD_{r,i,h}$	$\tau^{Ad}_{r,i,gov}PD_{r,i}XD_{r,i,gov}$	$\tau^{Ad}_{r,i,inv}PD_{r,i}XD_{r,i,inv}$
Tax Imp. Comm.	$\tau^{Am}_{r,i,h}PMT_{r,i}XM_{r,i,h}$	$\tau^{Am}_{r,i,gov}PMT_{r,i}XM_{r,i,gov}$	$\tau^{Am}_{r,i,inv}PMT_{r,i}XM_{r,i,inv}$
Value Added			
VA tax			
Prod. tax			
Import tax			
Export tax			
Households			
Government	$\chi^c_r\kappa^h_r YH_r$		
Investment	S^h_r	S^g_r	
Depreciation	$DeprY_r$		
Trade			
Intl. Margins			
Rest of the world			

Table I.4: The schematic social accounting matrix for the ENVISAGE model (cont.)

	DEPR	**TRADE**	**ITT_MARG**	**RoW**
Activities				
Commodities		$WPE_{r,d,i}WTF_{r,d,i}$	$PP_{r,i}XMG_{r,i}$	
Dom. Comm.				
Imp. Comm.				
Tax Dom. Comm.				
Tax Imp. Comm.				
Value Added				
VA tax				
Prod. tax				
Import tax				
Export tax				
Households				
Government				
Investment	$DeprY_r$			S^f_r
Depreciation				
Trade				$\sum_j WPE_{r,d,j}WTF_{r,d,j}$
Intl. Margins				$\sum_i PP_{r,i}XMG_{r,i}$
Rest of the world		$\sum_j WPM_{s,r,j}WTF_{s,r,j}$		

Appendix J

Dimensions of the GTAP database, release 9.0

Table J.1: Regional dimension of GTAP database

1	AUS	**Australia**
2	NZL	**New Zealand**
3	XOC	**Rest of Oceania** American Samoa (asm), Cook Islands (cok), Fiji (fji), French Polynesia (pyf), Guam (gum), Kiribati (kir), Marshall Islands (mhl), Federated States of Micronesia (fsm), Nauru (nau), New Caledonia (ncl), Norfolk Island (nfk), Northern Mariana Islands (mnp), Niue (niu), Palau (plw), Papua New Guinea (png), Samoa (wsm), Solomon Islands (slb), Tokelau (tkl), Tonga (ton), Tuvalu (tuv), Vanuatu (vut), Wallis and Futura Islands (wlf)
4	CHN	**China**
5	HKG	**Hong Kong (China)**
6	JPN	**Japan**
7	KOR	**Republic of Korea**
8	MNG	**Mongolia**
9	TWN	**Taiwan (China)**
10	XEA	**Rest of East Asia** Macao (mac), North Korea (prk)
12	KHM	**Cambodia**
13	IDN	**Indonesia**
13	IDN	**Indonesia**
14	LAO	**Lao, PDR**
15	MYS	**Malaysia**
16	PHL	**Philippines**
17	SGP	**Singapore**
18	THA	**Thailand**
19	VNM	**Vietnam**
20	XSE	**Rest of Southeast Asia** Myanmar (mmr), Timor-Leste (tmp)

Table J.1: Regional dimension of GTAP database (cont.)

21	BGD	**Bangladesh**
22	IND	**India**
23	LKA	**Sri Lanka**
24	NPL	**Nepal**
25	PAK	**Pakistan**
26	XSA	**Rest of South Asia** Afghanistan (afg), Bhutan (btn), Maldives (mdv)
27	CAN	**Canada**
28	USA	**United States**
29	MEX	**Mexico**
30	XNA	**Rest of North America** Bermuda (bmu), Greenland (grl), Saint Pierre & Miquelon (spm)
31	ARG	**Argentina**
32	BOL	**Bolivia**
33	BRA	**Brazil**
34	CHL	**Chile**
35	COL	**Colombia**
36	ECU	**Ecuador**
37	PRY	**Paraguay**
38	PER	**Peru**
39	URY	**Uruguay**
40	VEN	**Venezuela, Republica Bolivariana de**
41	XSM	**Rest of South America** Falkland Islands (flk), French Guiana (guf), Guyana (guy), Suriname (sur)
42	CRI	**Costa Rica**
43	GTM	**Guatemala**
44	HND	**Honduras**
45	NIC	**Nicaragua**
46	PAN	**Panama**
47	SLV	**El Salvador**
48	XCA	**Rest of Central America** Belize (blz)
49	DOM	**Dominican Republic**
50	JAM	**Jamaica**
51	PRI	**Puerto Rico**
52	TTO	**Trinidad & Tobago**
53	XCB	**Caribbean** Anguilla (aia), Antigua & Barbuda (atg), Aruba (abw), Bahamas (bhs), Barbados (brb), Cayman Islands (cym), Cuba (cub), Dominica (dma), Grenada (grd), Guadeloupe (glp), Haiti (hti), Martinique (mtq), Montserrat (msr), Netherlands Antilles (ant), Saint Kitts & Nevis (kna), Saint Lucia (lca), Saint Vincent & the Grenadines (vct), Turks and Caicos Islands (tca), British Virgin Islands (vgb), United States Virgin Islands (vir)

Table J.1: Regional dimension of GTAP database (cont.)

54	AUT	**Austria**
55	BEL	**Belgium**
56	BGR	**Bulgaria**
57	CYP	**Cyprus**
58	CZE	**Czech Republic**
59	DNK	**Denmark**
60	EST	**Estonia**
61	FIN	**Finland**
62	FRA	**France**
63	DEU	**Germany**
64	GRC	**Greece**
65	HUN	**Hungary**
66	IRL	**Ireland**
67	ITA	**Italy**
68	LVA	**Latvia**
69	LTU	**Lithuania**
70	LUX	**Luxembourg**
71	MLT	**Malta**
72	NLD	**Netherlands**
73	POL	**Poland**
74	PRT	**Portugal**
75	ROU	**Romania**
76	SVK	**Slovakia**
77	SVN	**Slovenia**
78	ESP	**Spain**
79	SWE	**Sweden**
80	GBR	**United Kingdom**
81	NOR	**Norway**
82	CHE	**Switzerland**
83	XEF	**Rest of European Free Trade Area (EFTA)** Iceland (isl), Liechtenstein (lie)
84	ALB	**Albania**
85	BLR	**Belarus**
86	HRV	**Croatia**
87	RUS	**Russian Federation**
88	UKR	**Ukraine**
89	XEE	**Rest of Eastern Europe** Moldova (mda)
90	XER	**Rest of Europe** Andorra (and), Bosnia and Herzegovina (bih), Faroe Islands (fro), Gibraltar (gib), Macedonia (mkd, former Yugoslav Republic of), Monaco (mco), San Marino (smr), Serbia and Montenegro (scg)

Table J.1: Regional dimension of GTAP database (cont.)

91	KAZ	**Kazakhstan**
92	KGZ	**Kyrgyz Republic**
93	XSU	**Rest of Former Soviet Union** Tajikistan (tjk), Turkmenistan (tkm), Uzbekistan (uzb)
94	ARM	**Armenia**
95	AZE	**Azerbaijan**
96	GEO	**Georgia**
97	BHR	**Bahrain**
98	IRN	**Iran**
99	ISR	**Israel**
100	JOR	**Jordan**
101	KWT	**Kuwait**
102	OMN	**Oman**
103	QAT	**Qatar**
104	SAU	**Saudi Arabia**
105	TUR	**Turkey**
106	ARE	**United Arab Emirates**
107	XWS	**Rest of Western Asia** Iraq (irq), Lebanon (lbn), West Bank and Gaza (pse), Syrian Arab Republic (syr), Republic of Yemen (yem)
108	EGY	**Egypt**
109	MAR	**Morocco**
110	TUN	**Tunisia**
111	XNF	**Rest of North Africa** Algeria (dza), Libya (lby)

Table J.1: Regional dimension of GTAP database (cont.)

112	BEN	**Benin**
113	BFA	**Burkina Faso**
114	CMR	**Cameroon**
115	CIV	**Côte d'Ivoire**
116	GHA	**Ghana**
117	GIN	**Guinea**
118	NGA	**Nigeria**
119	SEN	**Senegal**
120	TGO	**Togo**
121	XWF	**Rest of Western Africa**
		Cape Verde (cpv), Gambia, The (gmb), Guinea-Bissau (gnb), Liberia (lbr), Mali (mli), Mauritania (mrt), Niger (ner), Saint Helena (shn), Sierra Leone (sle)
122	XCF	**Central Africa** Central African Republic (caf), Chad (tcd), Congo (cog), Equatorial Guinea (gnq), Gabon (gab), Sao Tome & Principe (stp)
123	XAC	**South-Central Africa** Angola (ago), Democratic Republic of the Congo (cod)
124	ETH	**Ethiopia**
125	KEN	**Kenya**
126	MDG	**Madagascar**
127	MWI	**Malawi**
128	MUS	**Mauritius**
129	MOZ	**Mozambique**
130	RWA	**Rwanda**
131	TZA	**Tanzania**
132	UGA	**Uganda**
133	ZMB	**Zambia**
134	ZWE	**Zimbabwe**
135	XEC	**Rest of Eastern Africa** Burundi (bdi), Comoros (com), Djibouti (dji), Eritrea (eri), Mayotte (myt), Réunion (reu), Seychelles Islands (syc), Somalia (som), Sudan (sdn)
136	BWA	**Botswana**
137	NAM	**Namibia**
138	ZAF	**South Africa**
139	XSS	**Rest of South African Customs Union** Lesotho (lso), Swaziland (swz)
140	XTW	**Rest of the World** Antarctica (ata), Bouvet Island (bvt), British Indian Ocean Territory (iot), French Southern Territories (atf)

Table J.2: Commodity dimension of GTAP database

1	PDR	**Paddy rice**
2	WHT	**Wheat**
3	GRO	**Cereal grains, n.e.s.**
4	V_F	**Vegetables and fruits**
5	OSD	**Oil seeds**
6	C_B	**Sugar cane and sugar beet**
7	PFB	**Plant-based fibers**
8	OCR	**Crops, n.e.s.**
9	CTL	**Bovine cattle, sheep and goats, horses**
10	OAP	**Animal products n.e.s.**
11	RMK	**Raw milk**
12	WOL	**Wool, silk-worm cocoons**
13	FRS	**Forestry**
14	FSH	**Fishing**
15	COA	**Coal**
16	OIL	**Oil**
17	GAS	**Gas**
18	OMN	**Minerals n.e.s.**
19	CMT	**Bovine cattle, sheep and goat, horse meat products**
20	OMT	**Meat products n.e.s.**
21	VOL	**Vegetable oils and fats**
22	MIL	**Dairy products**
23	PCR	**Processed rice**
24	SGR	**Sugar**
25	OFD	**Food products n.e.s.**
26	B_T	**Beverages and tobacco products**
27	TEX	**Textiles**
28	WAP	**Wearing apparel**
29	LEA	**Leather products**
30	LUM	**Wood products**
31	PPP	**Paper products, publishing**
32	P_C	**Petroleum, coal products**
33	CRP	**Chemical, rubber, plastic products**
34	NMM	**Mineral products n.e.s.**
35	I_S	**Ferrous metals**
36	NFM	**Metals n.e.s.**
37	FMP	**Metal products**
38	MVH	**Motor vehicles and parts**
39	OTN	**Transport equipment n.e.s.**
40	ELE	**Electronic equipment**

Table J.2: Commodity dimension of GTAP database (cont.)

41	OME	**Machinery and equipment n.e.s.**
42	OMF	**Manufactures n.e.s.**
43	ELY	**Electricity**
44	GDT	**Gas manufacture, distribution**
45	WTR	**Water**
46	CNS	**Construction**
47	TRD	**Trade**
48	OTP	**Transport n.e.s.**
49	WTP	**Sea transport**
50	ATP	**Air transport**
51	CMN	**Communication**
52	OFI	**Financial services n.e.s.**
53	ISR	**Insurance**
54	OBS	**Business services n.e.s.**
55	ROS	**Recreation and other services**
56	OSG	**Public administration and defense, education, health services**
57	DWE	**Dwellings**

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