Towards consistent and calibratable capital stock accumulation in multi-annual step recursive dynamics

Abstract

Depending on the underlying capital stock accumulation mechanism, recursively dynamic general equilibrium models running in multi-annual steps may have consistency issues in capital stock accumulation, reflected through the consistency with benchmark annual steps, the consistency between capital stock accumulation and wealth accumulation, and the consistency under different model closures. In this paper we introduce novel linearized equations for capital and wealth accumulation that leads to smooth non-linear investment growth within each multi-annual period. It is shown that these new mechanisms can offer overall improved consistency and more flexibility for capital stock calibration. We conduct simplified CGE baseline experiments to shed light on the superiority of these new mechanisms as compared to other accumulation mechanisms and add sensitivity analyses to gain insights into the ballpark of these mechanisms.

Keywords: recursive dynamics, computable general equilibrium, multi-annual step, capital stock, wealth, investment

1. Introduction

Computable general equilibrium (CGE) models with capital-driven recursive dynamics may run in time steps of varying lengths. While running in annual steps is generally considered the benchmark for such CGE models, many large and complex models are specified to run in multi-annual steps (i.e., several years in each projection step/period). These include widely used CGE models such as GTAP-Dyn (Ianchovichina & McDougall, 2000), ENVISAGE (Van der Mensbrugghe, 2008), GEM-E3 (Capros et al., 2013), MAGNET (Woltjer et. al., 2014), MIT EPPA (Chen et al., 2016, 2022) or d-PLACE (Gaska et al., 2020). Apart from these recursively dynamic CGE models, there are also forward-looking CGE models running in multi-annual steps such as Takeda and Arimura (2021).

Compared to running in annual steps, a dynamic model benefits substantially from running in multi-annual steps in several aspects. First, running in multi-annual steps can significantly reduce the time required to solve a model and this especially matters for very large and complex computable general equilibrium models. If running in annual steps, a large CGE model with a detailed coverage of sectors and regions can take hours or even days to solve in order to produce decades-long annual projections. By contrast, if the same model runs, say, in ten-year steps instead of annual steps, then the time required to run the model would be significantly shortened – to about a tenth of the time in an annual setting. Taking also into account the fact that models usually need to run several times for debugging, this type of multi-annual setting would naturally be a time-efficient choice for modellers. Second, the reporting requirements usually prefer model results to be presented for some milestone years such as 2030, 2040, 2050, etc., rather than for all individual years into the far future. Setting models up in multi-annual periods with each period ending at these milestone years enables modellers to make efficient use of research time by skipping those years in between while still meeting the reporting requirements. Third, data availability sometimes may also limit a model's capacity in handling a higher time-resolution. A model baseline constructed upon assumptions across multiple social, economic, and environmental domains may require data inputs from various sources. Some of these sources may not have sufficient data available for all the years exactly matching the baseline setup. In this situation, modellers may have to make certain assumptions and apply the same data to a number of years and run the model in multi-annual steps to wrap up those years in a single time step. Last but not least, for model variables that are updated with quantity changes but initialized from values (e.g. tariff quotas), multi-annual steps would be necessary to minimize potential inconsistencies caused by price-induced value changes coupled with repeated initializations if running in annual steps.

Despite the aforementioned benefits, questions may arise as to whether the multi-annual settings can still generate consistent model solutions. The consistency can be reflected in different aspects. In fact, several criteria can be used to check against the consistency of multi-annual step model solutions resulting from using different capital stock accumulation mechanisms. These criteria include the consistency between capital stock accumulated through multi-annual steps and capital stock accumulated via the benchmark annual steps, the consistency between capital stock accumulation and wealth accumulation, and the consistency of capital stock accumulation between exogenous and endogenous baselines where GDP growth is exogenously given or endogenously determined.

If a CGE model is specified to run in multi-annual steps, a legitimate question may arise as to whether the number of year steps chosen would affect the projected capital stock growth and through which affect other model solutions. In other words, can a multi-annual specification, regardless of how many year steps are applied, yield capital growth projections consistent with the benchmark simulation running in annual (i.e., year on year) steps? This is a fundamental yet legitimate question. Given the annual steps with the default accumulation mechanism being the benchmark, the accumulation mechanism applied in multi-annual steps is expected to yield similar results at the same point of time. Otherwise, the projected model results would be less reliable and less comparable with other models solved with different time steps. However, as this paper shows, once a model is solved multi-annually, the projected capital growth can be sensitive to how capital stock is (assumed to be) accumulated throughout the years in each multi-annual period. Depending on the specified mechanism, the accumulated capital stock may deviate to some extent from the benchmark annual step solutions. Other things being equal, a mechanism with minimal deviations from the benchmark would be preferred.

For CGE models running in multi-annual steps, the consistency between capital stock accumulation and wealth accumulation is vital. CGE models must run in balance in order to give meaningful projections and explanations. In an annual step simulation, the balance between capital stock and wealth at the global level can be warranted if both follow the default accumulation mechanism. This is because they are accumulated respectively from investment and savings, which are already in balance globally due to the way the "Walras Law" is implemented in the model. In a multi-annual setting, however, the balance between global capital stock and wealth may not be guaranteed as this depends further on whether the accumulation mechanisms between capital stock and wealth are consistent throughout each of the multi-annual projection periods. Thus, an additional check is needed to ensure the consistency between the two in specified accumulation mechanisms.

In a typical CGE baseline where GDP growth projections are used as exogenous inputs to the model to generate the region-level total factor productivity change or a similar variable (the exogenous baseline), the projected regional capital growth should be the same as in the endogenous baseline where GDP growth is endogenously determined and swapped with the same region-level total factor productivity change which is then exogenously shocked in this baseline. This theoretical consistency is required not only to GDP and capital stock, but also to all the other variables in the model. In practice, however, model solutions might slightly differ in the two baselines due to known or unknown reasons. Thus, we also need to check the consistency between the exogenous and the endogenous baseline solutions resulting from using different capital stock accumulation mechanisms. Generally speaking, the closer the results between the two baselines, the higher level of consistency the underlying accumulation mechanism achieves, *ceteris paribus*.

Among widely used CGE models, the standard MAGNET model features capital-driven recursive dynamics with an implicit linkage between investment and capital stock. With a multisectoral and multiregional coverage (Woltjer et al., 2014), the MAGNET model core builds upon the standard GTAP modelling framework (Hertel, 1997; Corong et al., 2017). In the default standard version, the MAGNET model runs in multi-annual steps, and capital growth in each region is set exogenously to follow the baseline GDP growth rate. While offering a succinct solution in capital determination and relatively stable capital and investment movements across sectors and regions in usual model outcomes, this approach clearly has its weakness in assessing large-scale investment programmes. A good example of such programmes is the European Green Deal investment plan, which is meant to spend around 25% of the EU budget on environmental objectives until 2030 (European Commission, 2020). Based on exogenously specified capital supply, this approach essentially disconnects the natural linkage between investment and capital endowment. Consequently, the specified capital growth may not be consistent with what is indicated by savings growth in the model. In other words, economic growth under such a specification may be sustained by underestimated or overestimated capital growth rates, which, in turn, may cause unreasonable levels of technological change and unintended repercussions across the model.

Apart from the consistency issue embedded in the standard MAGNET model, the underlying assumption made for capital stock growth, which is simply targeting GDP growth, does not appropriately reflect historical observations especially those over the recent decades. The growth rate targeting essentially leads to a constant ratio between capital and GDP, which might be supported by earlier historical data like a century ago as reflected in the mainstream growth theory around that time (e.g. Kaldor, 1961). Over the recent decades, however, new growth theory (e.g. Romer, 1990; Acemoglu, 2002; and Barro & Sala-i-Martin, 2004) suggests that economic growth has been increasingly driven by capital-augmenting technological progress, and this capital-augmenting trend is especially pronounced in advanced economies (Piketty & Zucman, 2014). The capital-augmenting nature of technological progress implies that capital growth may surpass GDP growth over the long term, which has been supported by historical data for the recent decades. For example, the Penn

World Table (v10), as documented by Martínez & Baeza (2023), shows that the average ratio of annualized capital growth over GDP growth during 1990-2019 was approximately 1.2 for the world and 1.3 for the EU, and thus confirms what was suggested by the new growth theory.As an improvement to the standard MAGNET approach, Smeets-Kristkova et al. (2021) simulate an increase in green investments using recursively dynamic approach on the sectoral level.. In their approach, capital stock at the beginning of each projection period is derived from the accumulated investments over the preceding years. This improvement ensures a consistent transmission between investment growth and capital stock growth. Simulation outcomes show that the resulting dynamic trajectories are largely supported by historical observations as indicated in the Penn World Table (Feenstra et. al., 2015). However, the potential inconsistency between the multi-annual results and the benchmark annual results remains and this inconsistency is caused by the so-called multiplication approach applied to capital stock accumulation. In each multi-annual step, capital stock is accumulated through multiplying the corresponding base year (net) investment by the number of years in that multi-annual step. As shown later, this kind of multiplication mechanism may cause inconsistent capital growth as compared to the benchmark annual (i.e. year-on-year) results. In usual economic growth with a rising trend, this multiplication approach would typically cause an underestimation of the accumulated capital stock. Furthermore, since the wealth flows are not explicitly modelled, a balance check cannot be conducted on the accumulated capital stock. This creates a potential inconsistency between the derived capital stock and the capital stock indicated from the savings side. Therefore, simulation outcomes may still exaggerate the benefits for countries that act as net investors and underestimate the benefits for net lenders (Dixon et al., 2019; Dixon et al., 2021; Lemelin et al., 2013).

The capital stock accumulation equation applied in Smeets-Kristkova et al. (2021) is similar to the accumulation equation applied in GTAP-Dyn (Ianchovichina & McDougall, 2000) as both equations employ a simple linear approach, that is, multiplying annual investment by the number of years in a multi-annual period. The difference though is that GTAP-Dyn would keep investments updated in a multi-step solution, thereby effectively reducing the inconsistency with the benchmark annual solutions. However, as the paper shows, there exist certain inconsistencies between exogenous and endogenous baselines associated with the GTAP-Dyn approach.

To better address the consistency issues in multi-annual recursive dynamics, we introduce in MAGNET two alternative capital stock accumulation mechanisms based on smooth in-period investment growth that, as compared to the benchmark, would lead to generally more consistent multi-annual solutions. The new capital stock accumulation mechanisms are coupled with a corresponding wealth accumulation mechanism adapted from Dixon et al. (2019), leading also to consistent wealth accumulations in multi-annual steps. In addition, the new capital stock and wealth accumulation mechanisms offer more flexibility to calibrate investment and savings growth for desired capital stock and wealth accumulation trajectories, a feature not available in earlier CGE models. We then compare the baseline projections between these different capital stock accumulation mechanisms to shed light on the superiority of the proposed mechanisms.

The structure of this paper is organized as follows. In Section 2 we discuss the existing representative capital stock accumulation mechanisms in annual and multi-annual settings and introduce two new accumulation mechanisms. In Section 3, we discuss the consistency between capital stock accumulation and wealth accumulation in different mechanisms. In Section 4, we discuss the consistency in capital stock accumulation between exogenous and endogenous baselines. Section 5 explains the calibration capability in the newly introduced mechanisms. Section 6 describes the model and data to be used for running test simulations. Section 7 presents and compares the simulation results from these different accumulation mechanisms. In Section 8, a sensitivity analysis is conducted to gain further insights into the strengths and limitations in these mechanisms in comparison. Section 9 concludes with remarks.

2. Consistency with benchmark annual steps

When establishing a linkage between investment and capital stock, a standard approach is to add current period investment to the net capital stock accumulated at the start of the current period. This will generate capital stock at the end of this period or the start of the next period. Mathematically, this can be expressed as

$$KB_{T+1} = (1 - DEP) * KB_T + INV_T \quad (1)$$

where KB_{T+1} denotes capital (K) stock at the beginning (B) of the period T+1, which is the sum of the capital stock at the start of the current period KB_T, depreciated at the rate DEP, and the gross investment in the last period INV_T. If DEP and INV refer respectively to annual depreciation rate and (gross) investment, equation (1) would generate the annual accumulation of capital stock. Here, we ignore the denotation for regions. Expressing equation (1) in the linearized form, as implemented in GEMPACK, the above equation becomes:

$$VKB\_Q(r) * kb(r) = 100 * NETINV_0(r) * d\_unity \quad (1')$$

where VKB_Q(r) represents capital stock at the start of the current period in region r and the lower-case *kb* denotes the percentage change in capital stock from the start of this period to the start of the next period. The real change in capital stock, VKB_Q * kb, where VKB_Q is updated by kb, is derived from the investment net of capital stock depreciation in the current period, $NETINV_0$. *d_unity* is a homotopy variable always shocked by 1. Note that we ignore all the subscript T in this equation as this would be the default format to indicate that all the coefficient values are for the current period unless indicated otherwise.

Equation (1') represents an annual change and thus carries the default assumption that the depreciation reflects the written-off value of capital stock on an annual basis. Dynamically, this equation is similar to the capital stock accumulation equation used in Dixon et al. (2021) since both equations capture the annual change in capital stock. The only difference is that equation (1') characterizes the real change in capital stock (i.e. capital stock changes due to investment) rather than the nominal change in capital stock which also includes the price effect (i.e. capital stock changes due to changes in capital goods prices). Despite this difference, both equations would produce the same change in capital stock, denoted by the lowercase variable, *kb*, in equation (1'). As such, this equation can be used as the benchmark for the accumulation of capital stock.

The limitation of equation (1') is that it runs only in annual steps but not in multi-annual steps since investment is counted annually. A capital stock accumulation mechanism that runs in multi-annual steps needs to properly account for the change in investment throughout a multi-annual period such that it matches the annual change in investment when the model runs annually with equation (1'). This is the way to ensure consistency between the capital stock accumulated in multi-annual steps and its benchmark, i.e., the capital stock accumulated annually with equation (1'). Put simply, capital stock growth via multi-annual steps should in principle match the growth via annual steps over the same period. However, depending on the specification of the underlying accumulation mechanism, capital stock accumulated in multi-annual steps may deviate from the benchmark result. Thus, to what extent the multi-annual accumulation may deviate from the benchmark serves as a basis upon which the consistency of the applied mechanism can be checked.

There are varying approaches to specifying how capital stock can accumulate over a multi-annual period. Broadly, these approaches can be categorized into two groups. One group is called the implicit approach which does not use an explicit functional form to specify how capital stock accumulates from investment. Instead, capital stock is determined in alternative ways. For instance, capital stock growth may target GDP growth or is determined via the equalization of current rates of return. As will be shown later, capital stock determined using these implicit approaches can deviate greatly from the benchmark result. The other group is the explicit approach which applies specific functional forms to accumulate capital stock from investment. Depending on the underlying assumptions on investment growth over a multi-annual period, these accumulation functions range from simple forms such as linear investment growth to more complex ones which may assume certain types of smooth investment growth. In this paper, we will discuss two types of linear growth mechanisms: linear constant and linear updated, with the difference between the two depending mainly on whether the underlying investment remains unchanged or updated throughout a multi-annual period. We will also introduce two types of smooth growth mechanisms: smooth gross and smooth net, which rely on the smooth/exponential investment growth assumptions made respectively on gross investment or net investment.

When comparing these multi-annual capital stock accumulation mechanisms, an important criterion we use to judge their relative superiority is consistency with the benchmark equation (1'). This consistency can be reflected in two aspects: One is to what extent these multi-annual mechanisms can generate capital stock growth consistent with the benchmark annual steps, and the other is whether these multi-annual mechanisms converge to the benchmark equation (1') if running in annual steps. The first aspect needs to be checked with model solutions which will be conducted in the results comparison section, while the second aspect mainly concerns the equational forms. Since no equational form exists with the implicit mechanisms, we know for sure that these mechanisms are unlikely to converge to the benchmark equation. However, for those explicit mechanisms with capital stock accumulation equations, we can check this equational consistency when introducing them below.

A simple approach to explicitly accumulating capital stock in a multi-annual setting is to multiply net investment by period length - the number of years in a multi-annual period, which would give the change in capital stock by assuming that investment does not change within this period. This approach, as adopted by Smeets-Kristkova et al. (2021) is expressed as

$$VKB(r) * kb(r) = 100 * NETINV_0(r) * \textit{time} \quad (2)$$

where the real change in capital stock, VKB * kb, is derived from the base net investment, $NETINV_0$ (i.e. net investment in the first year of the current period), multiplied by the number of years, denoted as *T*, in each multi-annual period. With net investment being evaluated at the base price, this equation captures the real change in capital stock throughout the multi-annual period. However, since net investment is assumed to be constant over the multi-annual period, this equation tends to underestimate capital stock growth in normal scenarios where investment has a positive growth trend over time.

One way to minimize this potential underestimation is to allow for updating of the net investment, such as the one adopted in Ianchovichina and McDougall (2012), where the capital stock accumulation equation is given by

$$VKB(r) * kb(r) = 100 * NETINV(r) * time \quad (3)$$

where the net investment coefficient, NETINV, is allowed to be updated continuously throughout the multi-annual period, with the update frequency depending on the solution steps applied. These continuous updates allow capital stock accumulation to keep up with the changing investment in the model in a multi-step solution. It should be noted though that, while this equation may help minimize an underestimation of multi-annual capital stock accumulation as in equation (2), equation (3) may be inconsistent in an annual setting. This is because in the benchmark annual setting, the real change in capital stock is given by the base net investment, rather than the updated net investment, as shown in equation (1').

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