A brief overview of investment behavior in CGE models

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Abstract: This paper explores several options for implementing investment allocation in a recursive dynamic CGE model. The standard in many models is to form demand for a homogeneous investment good in period $t-1$ equal to total available savings in $t-1$ and to allocate the new capital good across activities in year t, typically assuming perfect capital mobility. This paper explores different investment specifications where the allocation of investment across activities occurs in period $t-1$, in essence making capital sector-specific in period t. This also allows for different investment cost structures by destination activity. These alternative specifications are introduced into the standard GTAP model as implemented in GAMS and illustrates the impacts using a carbon price scenario.

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Chapter 1

Introduction

Most CGE models treat investment in a relatively simple fashion. The main reason is that investment theory is complex and typically involves rates of return, forward-looking expectations, and uncertainty—all topics that are typically excluded from CGE models. Moreover, the use of many CGE models has been comparative static in nature, thus investment is simply treated as a component of final demand with no direct feedback on the supply-side. Despite the complications that arise from the application of CGE model for dynamic analysis, it is increasing in use as some of the challenges of national economies, and the global economy, are very dynamic in nature?most notably climate change, but also changing demographics, and the sustainability of pension systems.

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The most common implementation of investment theory is to assume: (1) savings-led investment for aggregate investment; (2) a single cost structure for the single investment good; and (3) allocation of gross investment across sectors under the assumption of steady-state return to capital, i.e., uniform returns across all sectors. This overview will focus on two of these three issues, investment good heterogeneity and allocation and exclusively implementation for recursive dynamic models. In other words, this survey will not delve into the theory of aggregate investment. For interested readers, the most comprehensive approach to the latter in a CGE framework is the G-Cubed model (McKibbin and Wilcoxen (2013)).

Very few models, and very few databases, incorporate investment good heterogeneity. Most investment cost structures are derived from standard input-output tables (IOT) with a single investment vector. There is no doubt, however, that the structure of investment goods across destination activities vary considerably, e.g., agriculture will mostly be purchasing farm equipment, with low expenditures on buildings, whereas service sector investments will concentrate on buildings and IT infrastructure. One of the seminal sources in the CGE literature (Derviş et al. (1982)), does describe in detail the cost structure of investment demand by destination. A more recent foundation for investment cost by destination activity can be found in the JRC's GEM-E3 model (Capros et al. (2017)), for which they have developed new estimates for the European Union countries (Norman-López et al. (2023)).

One of the issues in using the cost structure by destination information is the time lapse between the decision to invest (in a particular sector, for example) and the actual allocation of investment in recursive dynamic models. In most standard recursive dynamic applications, aggregate investment is decided in period $t-1$, and the allocation of that aggregate investment occurs in period t. The simplest rule is to assume that investment is allocated efficiently across sectors such that the rates of return are equalized. In models with investment by destination, the allocation decision is taken in period $t-1$, which in effect makes capital sector specific in period t, with potential mismatches between supply and demand and thus heterogeneous returns. The rest of the paper will assess a few key model specifications with destination specific investment, their implementation and data requirements, and the required key elasticities.

The paper will assess two different implementations of the allocation of investment by destination activity (in the period of investment expenditures, i.e., prior to the period when the new capital is available for production). The first is the investment theory developed by for the ORANI model (Dixon et al. (1982)), which is a multi-sector national model developed for Australia, and which is also implemented in a modified version of the GTAP model (Dixon et al. (2019)). This specification is similar to the GTAP specification that allocates global savings across regions. It implements a rate-of-return schedule for each activity which is a function of the ratio of K_t/K_{t-1}, which determines an 'expected' rate-of-return and the allocation rule specifies that the expected rate-of-return is equalized across activities. The current rate of return is similar to Tobin?s Q, it is the profitability rate relative to the replacement cost of capital less the depreciation rate. The key parameter in this specification is the elasticity of the expected rate-of-return vis-à-vis the ratio and in principle it could be sector specific. The second specification is derived from the GEM-E3 model (Capros et al. (2017)). In this specification, investment is a share of the current capital stock and Tobin's q, and the key demand elasticity is linked to Tobin's q. Another important factor in the GEM-E3 model is the expected growth rate of the sector. All variables and parameters are activity specific and—given the destination specific heterogeneous cost structure—the replacement cost of capital will also be sector specific.

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We will assess the impacts of the specification choice with implementation in the standard GTAP model and an illustrative policy shock and highlight the data requirements and parameterization.[1]

The paper introduces two allocation mechanisms. One is based on the ORANI model (Dixon et al. (1982)) and subsequently introduced into a variant of the GTAP model (Dixon et al. (2019)), and the other is used by the GEM-E3 model. We will also show that the two approaches are quite similar. A third approach is that developed for the MONASH and USAGE models (Dixon and Rimmer (2002) and Dixon and Rimmer (2005)), which is based on activity-specific investment schedules that allows for short-term deviations from long-term conditions.

[1] A third alternative, not discussed herein, is derived from the Monash model (Dixon and Rimmer (2002)), which was also used for the USAGE model (Dixon and Rimmer (2005)). In this case, there is a supply schedule for capital, where capital supply depends on the rate of return, but the supply schedule is based on an S-curve, i.e., a logistic function, which caps the rate of return between lower- and upper-bounds. The implementation allows for short term deviations of the rates of return across activities, but long-term steady-state properties. The specification requires, the curvature parameter (i.e., the variable elasticity) of the supply curve and the two asymptotes, as well as estimates of what is the ?trend? growth in the capital stock.

Chapter 2

Investment allocation

2.1 Aggregate savings and investment

The standard GTAP model (Corong et al. (2017)) and many other CGE models (see for example van der Mensbrugghe (2024)) have a simplified specification for allocating investable funds across activities. Typically investment occurs in year $t-1$ for a single capital good, that is allocated across activities in year t, often assuming perfect mobility.

In standard models, aggregate investable funds is equal to aggregate savings. In GTAP, this translates to:

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$$YI_{r,t} = SAVE_{r,t} + SAVEF_{r,t} + \delta_{r,t} PINV_{r,t} KB_{r,t}$$

where YI is nominal gross investment, $SAVE$ is regional savings that emerges from the top-level utility function that allocate regional income between private and public expenditures and savings, $SAVEF$ is net capital inflows, or foreign savings, and the last term is the value of capital depreciation, where KB is the beginning of period capital stock and $PINV$ is the replacement cost of capital. This equation is not explicit in the GTAP model, nor does the model have the $SAVEF$ variable—both are implicit. Other models, such as Envisage have a different allocation of national income, with more structure provided to government balances. In the case of Envisage the equation becomes:

$$YI_{r,t} = SAVH_{r,t} + SAVG_{r,t} + SAVEF_{r,t} + \delta_{r,t} PINV_{r,t} KB_{r,t} \tag{2.1}$$

where $SAVH$ and $SAVG$ represent private and public savings respectively, modeled separately—the sum of which is equal to $SAVE$. In the GTAP model, $SAVEF$ can be generated by two different mechanisms. We first calculate the aggregate after tax rate of return and the net rate of return which adjusts for the cost of replacement and depreciation (i.e., Tobin's q):

$$Rental_{r,t} = \sum_a PES_{r,cap,a,t} QES_{r,cap,a,t} / KB_{r,t} \tag{2.2}$$

$$RORC_{r,t} = \frac{Rental_{r,t}}{PINV_{r,t}} - \delta_{r,t} \tag{2.3}$$

The next equation defines the expected rate of return, which is a function of the growth of the capital stock, where KE is the end-of-period capital stock given the current level of investment. High growth would tend to dampen the expected rate of return, and vice-versa.

$$RORE_{r,t} = RORC_{r,t} \left(\frac{KE_{r,t}}{KB_{r,t}} \right)^{-RoRFlex_r} \tag{2.4}$$

$$KE_{r,t} = (1 - \delta_r) KB_{r,t} + QINV_{r,t} \tag{2.5}$$

The GTAP model has two closure rules for determining foreign savings. Under the first, the rate of return is equalized across all regions:

$$RORE_{r,t} = RORG_t \tag{2.6}$$

and we include the following equation:

$$\sum_r SAVEF_{r,t} = 0 \tag{2.7}$$

which in this specification determines the global rate of return, i.e., $RORG_t$.

In the alternative specification, the value of global net investment is allocated across regions using fixed shares:

$$NetInv_{r,t} = \chi^I_{r,t} GBLCGDS_t \tag{2.8}$$

where

$$NetInv_{r,t} = QINV_{r,t} - \delta_{r,t} PINV_{r,t} KB_{r,t} \tag{2.9}$$

and

$$GBLCGDS_t = \sum_r NetInv_{r,t} \tag{2.10}$$

$$RORG_t = \sum_r \chi^I_{r,t} RORE_{r,t} \tag{2.11}$$

For a given level of investment, in the standard model, there is a single expenditure function, which determines the demand for goods and services to produce capital goods:

$$QIA_{r,i,t} = s^i_{r,i,t} QINV_{r,t} \tag{2.12}$$

$$PINV_{r,t} = \sum_i s^i_{r,i,t} PIA_{r,i,t} \tag{2.13}$$

$$QINV_{r,t} = YI_{r,t} / PINV_{r,t} \tag{2.14}$$

under the explicit assumption that the expenditure function takes the Leontief form, i.e., fixed volume shares.

Table 2.1 summarizes macroeconomic closure for aggregate investment. Equations (2.1), (2.7) and (2.14) are not actually part of the GTAP model, nor are variables YI and $SAVF$. In this case, equations (2.6) and (2.8) determine $QINV$, depending on the value of `RORDELTA`.

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Table 2.1: **Aggregate investment closure**

Variable	Equation	Note
Rental	2.2	Average after-tax return to capital
RORC	2.3	Adjusted net return to capital
RORE	2.4	Expected return to capital
KE	2.5	End-of-period capital stock
NetInv	2.9	Net capital stock in region r
GBLCGDS	2.10	Global net capital stock
SAVF	2.6	If `RORDELTA` $= 1$
	2.8	if `RORDELTA` $= 0$
RORG	2.7	`RORDELTA` $= 1$
	2.11	`RORDELTA` $= 0$
PINV	2.1	Aggregate price of investment goods
QINV	2.14	Aggregate volume of investment
YI	2.1	Nominal investment

2.2 A standard formulation for capital allocation

In the standard GTAP model, and many other CGE models, aggregate investment as described above, is used to calculate the end-of-period capital stock, which becomes the beginning-of-period capital stock for the next year, i.e.,

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$$KB_{r,t} = KE_{r,t-1} = (1 - \delta_{r,t-1})KB_{r,t-1} + QINV_{r,t-1}$$

This capital stock is then allocated across all activities in year t, according to a CET transformation specification, typically assuming full mobility. Note that the model carries forth two indicators for the capital stock. The first is the non-normalized level, KB, which is in the same units as investment and typically some multiple of GDP. The other indicator is the level of capital remuneration in the reference year, i.e., profits, which will have a reference price of 1. It is assumed that the ratio of the two indicators is fixed at reference year levels and it represents the pre-tax rate of return. For example, say GDP is 100, and the capital/output ratio is 3 to 1, i.e., the non-normalized level of the capital stock is 300. Say profits are some 40% of GDP, then the pre-tax profit rate is 40/300, or 13.3%, and the constant ratio of the two capital stock indicators will be 4/30. The GTAP variable, QE, which represents the normalized capital stock indicator, is then updated at the beginning of each period as:

$$QE_{r,cap,t} = QE_{r,cap,t-1}\frac{KB_{r,t}}{KB_{r,t-1}}$$

and it is the variable QE that is allocated using the CET specification. Also note that the CET specification is based on the post-tax remuneration of capital, i.e., the variable PES.

2.3 Destination-specific investment allocation

This section describes two alternative specifications for allocating investable funds across activities. The macro closure, as described in the section above is identical, i.e., total savings, or YI, in year $t-1$, is allocated across activities according to different criteria. Both specifications assume that the allocation is done in period $t-1$, and that activity specific capital supply is fixed in period t, i.e., there is no allowance for capital mobility within the year t. This specification also allows for activity-specific cost structures as it is known in advance the destination of the new investment.

2.3.1 ORANI

The first specification was implemented in the ORANI model (Dixon et al. (1982)) and subsequently implemented in the GTAP model (Dixon et al. (2019)). It is quite similar to the foreign saving closure described in the section above, and in fact was its inspiration. So most of the equations from the earlier section are reprised here, with the addition of an activity index.

$$Rental^a_{r,a,t} = PES_{r,cap,a,t} QES_{r,cap,a,t} / KB^a_{r,t} \tag{2.15}$$

$$RORC^a_{r,a,t} = \frac{Rental^a_{r,a,t}}{PINV^a_{r,a,t}} - \delta^a_{r,a,t} \tag{2.16}$$

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$$RORE^a_{r,a,t} = RORC^a_{r,a,t} \left(\frac{KE^a_{r,a,t}}{KB^a_{r,a,t}} \right)^{-RoRFlex^a_{r,a,t}} \tag{2.17}$$

$$KE^a_{r,a,t} = (1 - \delta^a_{r,a,t}) KB^a_{r,a,t} + QINV^a_{r,a,t} \tag{2.18}$$

Closure of this specification holds that the expected rate-of-return is equal to a common rate-of-return, $RORG^a$ and that the sum of investment expenditures across activities sums to the level of investable funds, i.e., YI.

$$RORE^a_{r,a,t} = RORG^a_{r,t} \tag{2.19}$$

$$YI_{r,t} = \sum_a PINV^a_{r,a,t} QINV^a_{r,a,t} \tag{2.20}$$

2.3.2 GEM-E3

In the case of the GEM-E3 model (Capros et al. (2017)), the allocation mechanism relies on the so-called accelerator model of investment and Tobin's q. The former assumes that the optimal level of capital in a sector is simply a multiple of its output level, irrespective of other variables. This implies that investment is directly linked to anticipated changes in output, with or without an adjustment cost. Tobin's q, already present in the ORANI specification, assumes that the optimal level of investment depends on its market rate of return relative to its replacement cost. Accordingly, the authors of GEM-E3 specify the simple model of investment as:

$$QINV^a_{r,a,t} = KB^a_{r,a,t} \left[\frac{Rental^a_{r,a,t}}{PINV^a_{r,a,t} \left(RORG^a_{r,t} + \delta^a_{r,a,t} \right)} - (1 - \delta^a_{r,a,t}) \right]$$

where it is also assumed that firms always replace their depreciated capital (i.e., $\delta^a KB^a$). This is further generalized to yield:

$$\begin{aligned} QINV^a_{r,a,t} \;\; = \;\; & \chi^{inv\_a}_{r,a,t} KB^a_{r,a,t} \Bigg[(1 + egr_{r,a,t}) \\ & \left(\frac{Rental^a_{r,a,t}}{PINV^a_{r,a,t} \left(RORG^a_{r,t} + \delta^a_{r,a,t} \right)} \right)^{\chi^{ror}_{r,a,t} RoRFlex^a_{r,a,t}} - (1 - \delta^a_{r,a,t}) \Bigg] \end{aligned} \tag{2.21}$$

According the GEM-E3 documentation χ^{inv_a} represents a capital adjustment parameter and $RoRFlex^a$ represents the price elasticity, i.e., when capital adjustment does not occur immediately according to the accelerator model. Also entering the specification is the anticipated growth of output in sector a, represented by the variable egr.

Note, that the ORANI and GEM-E3 specifications are not so dissimilar. One could re-arrange equation (2.17):

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$$KE^a_{r,a,t} = KB^a_{r,a,t} \left(\frac{RORC^a_{r,a,t}}{RORE^a_{r,a,t}} \right)^{1/RoRFlex^a_{r,a,t}}$$

and insert equation (2.16):

$$KE^a_{r,a,t} = KB^a_{r,a,t} \left(\frac{\dfrac{Rental^a_{r,a,t}}{PINV^a_{r,a,t}} - \delta^a_{r,a,t}}{RORE^a_{r,a,t}} \right)^{1/RoRFlex^a_{r,a,t}}$$

and replace $KE^a_{r,a,t}$ and $RORE^a$ with $RORG^a$:

$$QINV^a_{r,a,t} = KB^a_{r,a,t} \left[\left(\frac{Rental^a_{r,a,t}}{PINV^a_{r,a,t} RORG^a_{r,t}} - \frac{\delta^a_{r,a,t}}{RORG^a_{r,t}} \right)^{1/RoRFlex^a_{r,a,t}} - (1 - \delta^a_{r,a,t}) \right]$$

The exponent used in the GEM-E3 specification should be equal (more or less) to the reciprocal of the exponent used for the ORANI specification.

2.4 Demand for investment goods

Investment by destination sector is represented by the variable $QINV^a$ and is determined in period $t-1$ and total capital in sector a at the beginning of period t is:

$$KB^a_{r,a,t} = KE^a_{r,a,t-1} = (1-\delta^a_{r,a,t-1})KB^a_{r,a,t-1} + QINV^a_{r,a,t-1}$$

Each investment demand by activity a is assumed to have a separate cost structure. Equation () represents the demand for commodity i to produce investment goods for activity a, QIA^a, using a generic CES specification. The unit cost of investment good, $PINV^a$, is provided by equation (2.23).

$$QIA^a_{r,i,a,t} = s^a_{r,i,a,t} QINV^a_{r,a,t-1} \left(\frac{PINV^a_{r,a,t}}{PA^a_{r,i,a,t}}\right)^{\sigma^{i,a}_{r,a}} \tag{2.22}$$

$$PINV^a_{r,a,t} = \left[\sum_i s^a_{r,i,a,t} \left(PA^a_{r,i,a,t}\right)^{1-\sigma^{i,a}_{r,a}}\right]^{1/(1-\sigma^{i,a}_{r,a})} \tag{2.23}$$

Equation (2.12) is replaced with equation (2.12'), which represents the sum across all activities for good i used to produce an investment good in activity a. The aggregate investment price index will be calculated using a Fisher ideal price index. We are assuming that the Armington decomposition of investment is uniform across all investment activities, i.e., $PA^a_{r,i,a,t} = PA_{r,i,inv,t}$.

$$QIA_{r,i,t} = \sum_a QIA^a_{r,i,a,t} \tag{2.12'}$$

$$PINV_{r,t} = PINV_{r,t-1} \sqrt{\frac{MQINV_{r,t,t-1}}{MQINV_{r,t-1,t-1}} \cdot \frac{MQINV_{r,t,t}}{MQINV_{r,t-1,t}}} \tag{2.13'}$$

where

$$MQINV_{r,tp,tq} = \sum_i PA_{r,i,inv,tp} QIA_{r,i,inv,tq}$$

and equation (2.14) continues to determine the aggregate volume of investment, $QINV$.

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Chapter 3

An illustrative simulation

We are going to test the three different specifications using a version of the standard GTAP model in GAMS. The model assumes the nested structure of the GTAP-E model. It is calibrated to an aggregation of the GTAP 9 database, which is freely available. It has a reference year of 2011. The analysis requires two scenarios. The first is to calculate a baseline scenario, which we will refer to as BaU, or business-as-usual. To keep this simple, we will run the simulation from 2011 through 2020 in annual time steps, with very little exogenous inputs. The baseline changes include:

- Population in each region is set to observed changes (as derived from the SSP database).
- Employment in each region is set to observed changes in the 15-64 population. Consistent with this version of the GTAP model, the labor supply curve shifter is the calibrating variable as the supply elasticity is assumed to be zero.
- Real GDP growth in per capita terms is exogenous and is set to observed changes (as derived from the SSP ddatabase). The productivity shifter, `afereg`, is made endogenous and impacts the level of labor productivity (uniformly across skills).
- At the beginning of each period, the beginning-of-period capital stock is set equal to the lagged end-of-period capital stock.

The baseline simulation contains no other changes, such as specific structural changes.

The second scenario puts a price on carbon starting in year 2012. The tax starts at \$5 per ton of CO_2 in 2012 rising to \$45 in 2020. The tax is uniform across the globe. One would anticipate sharp drops in coal output and the production of coal-based electricity. Figure ?? shows global output of the various energy carriers. We see a sharp decline in coal output and coal thermal power plants, with more modest reductions for gas output and gas-based electricity. On the other hand, there is a sharp rise in non-fossil fuel based electricity output.

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Chapter 4

Concluding remarks

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Appendix A

Figures

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Figure 1: **Global energy output with standard model**