Towards consistent and calibratable capital stock accumulation in multi-annual step recursive dynamics

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Why multi-annual steps matter

- While many recursively dynamic CGE models run in annual steps, there are large and complex CGE models running in multi-annual steps – to save running time
- Model solutions may differ between annual and multi-annual steps for the same reporting year and this depends on, among other things, how capital stock accumulates in a multi-annual step
- Models with explicit capital stock accumulation mechanisms (i.e. investment-capital linkages) may particularly suffer from this consistency issue, compared to their implicit counterparts
- Some other consistency issues may also arise or become more pronounced in multi-annual steps
- It is key to employ an effective mechanism to minimise these issues, improving the overall consistency

Criteria for comparing capital stock accumulation approaches in multi-annual steps

- Consistency with the benchmark annual step solutions: annual convergence and multi-annual deviations
- Consistency under different model closures: a particular focus on the exogenous and endogenous baselines with the GDP and TFP swap
- Consistency between capital stock accumulation and wealth accumulation: a global balance is required
- Flexibility for calibration: let investment to adjust if capital growth is targeted

Capital stock accumulation approaches in multi-annual steps

- Implicit approaches:
  - Capital targeting GDP (MAGNET)
  - Rate of return equalization (GTAP-Wealth)
- Explicit approaches:
  - Constant investment (GTAP-RD Annual Benchmark): VKB_Q(r) * kb(r) = 100 * $NETINV_0$(r) * *time*
  - Updated investment (GDyn): VKB(r)* kb(r) = 100 * NETINV(r) * time
  - Smooth (gross) investment growth (ENVISAGE, MAGNET):

$$KB_T(r) = (1 - DEP)^T * KB_0(r) + \sum_{s=0}^{T-1} (1 - DEP)^s \left(\frac{INV_T(r)}{INV_0(r)}\right)^{\frac{s}{T}} * INV_0(r)$$

Smooth investment and savings growth

- Smooth investment growth (adapted from van der Mensbrugghe, 2008, 2010):

$$KB_T(r) = (1-DEP)^T * KB_0(r) + \sum_{s=0}^{T-1}(1-DEP)^s \left(\frac{INV_T(r)}{INV_0(r)}\right)^{\frac{s}{T}} * INV_0(r)$$

$$= (1-DEP)^T * KB_0(r) + \frac{\frac{INV_T(r)}{INV_0(r)} - (1-DEP)^T}{\left(\frac{INV_T(r)}{INV_0(r)}\right)^{\frac{1}{T}} - (1-DEP)} * INV_0(r)$$

- Smooth gross savings growth (adapted from Dixon et al. 2019, 2021):

$$WQ_T(r) = (1-DEP)^T * WQ_0(r) + \sum_{s=0}^{T-1}(1-DEP)^s \left(\frac{GSAVE_T(r)}{GSAVE_0(r)}\right)^{\frac{s}{T}} * GSAVE_0(r)$$

$$= (1-DEP)^T * WQ_0(r) + \frac{\frac{GSAVE_T(r)}{GSAVE_0(r)} - (1-DEP)^T}{\left(\frac{GSAVE_T(r)}{GSAVE_0(r)}\right)^{\frac{1}{T}} - (1-DEP)} * GSAVE_0(r)$$

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A simple baseline for comparing accumulation mechanisms

- Model: MAGNET with a GTAP core and a few open-source modules
- Database: based on GTAP database (version 11) 2017
- Regional aggregation: six regions including three major economies (US, EU, and China), two medium-sized economies (Argentina and Ukraine), and Rest of the World, aggregated from the 141 standard GTAP regions
- Commodity aggregation: 81 commodities aggregated from the 131 standard MAGNET commodities
- Baseline: SSP2 for GDP and population; land productivity growth, and natural resource supply

Regional capital stock growth in benchmark annual steps and deviations in alternative annual steps 2017-2050 (%)

	Benchmark Annual Steps	Deviations in Altenative Annual Steps from the Benchmark				
		Capital Targeting GDP	Rate of Return Equalized	Linear Constant Investment	Linear Updated Investment	Smooth Investment Growth
USA	73.6	-5.9	+3.1	+0.0	+0.8	+0.0
EU	63.9	-6.1	+11.5	+0.0	+0.3	+0.0
China	226.4	-59.0	-17.2	+0.0	+2.2	+0.0
Argentina	71.0	+3.0	+7.2	+0.0	+0.7	+0.0
Ukraine	95.5	-2.7	+18.4	+0.0	+1.3	+0.0
ROW	185.7	-16.1	-1.5	+0.0	+2.8	+0.0
World	147.3	-21.8	-1.7	+0.0	+1.7	+0.0

Regional capital stock growth in benchmark annual steps and deviations in multi-annual steps

2017-2050 (%)

	Benchmark Annual Steps	Deviations in Multi-Annual Steps from Benchmark Annual Steps				
		Capital Targeting GDP	Rate of Return Equalized	Linear Constant Investment	Linear Updated Investment	Smooth Investment Growth
USA	73.6	-5.9	+3.6	-3.1	+0.5	-0.1
EU	63.9	-6.1	+11.5	-2.0	-0.2	-0.8
China	226.4	-59.0	-17.0	-10.0	+1.4	-1.4
Argentina	71.0	+3.0	+8.0	-3.4	+0.2	-0.2
Ukraine	95.5	-2.7	+18.8	-4.9	+1.2	+0.2
Other	185.7	-16.1	-1.0	-11.8	+2.2	-0.5
World	147.3	-21.8	-1.3	-7.6	+1.2	-0.7

Differences in regional capital stock growth between exogenous and endogenous baselines

2017-2050 (%)

	Capital Targeting GDP	Rate of Return Equalized	Linear Constant Investment	Linear Updated Investment	Smooth Investment Growth
USA	0.00	0.00	0.00	-0.05	0.00
EU	0.00	0.00	0.00	-0.03	0.00
China	0.00	0.00	0.00	-0.27	0.01
Argentina	0.00	0.00	0.00	-0.04	0.00
Ukraine	0.00	-0.01	0.00	-0.07	0.00
Other	0.00	-0.01	0.00	-0.51	0.00
World	0.00	-0.01	0.00	-0.26	0.00

Adjustment required to rebalance global capital stock and wealth (%)

	Capital Targeting GDP	Rate of Return Equalized	Linear Constant Investment	Linear Updated Investment	Smooth Investment Growth
2017-2020	-2.65	0.00	0.00	0.00	-0.01
2020-2025	5.55	0.00	0.00	0.00	-0.02
2025-2030	1.49	0.00	-0.01	0.00	-0.02
2030-2035	-0.75	0.00	-0.01	0.00	-0.02
2035-2040	-2.90	0.00	-0.01	0.00	-0.01
2040-2045	-4.18	0.00	-0.01	0.00	-0.01
2045-2050	-5.36	0.00	-0.01	0.00	-0.01
2017-2050	-8.87	0.00	-0.04	0.00	-0.09

Captial stock and investment growth under the standard and calibrated baselines 2017-20 (%)

	Capital Targeting GDP				Smooth Investment growth			
	Capital stock growth		Investment growth		Capital stock growth		Investment growth	
	Standard Baseline	Calibrated Baseline	Standard Baseline	Calibrated Baseline	Standard Baseline	Calibrated Baseline	Standard Baseline	Calibrated Baseline
USA	3.2	4.6	3.0	4.1	4.8	7.4	4.4	57.5
EU	-1.7	4.6	-1.9	1.2	4.1	7.4	0.9	71.8
China	15.6	4.6	16.6	15.2	9.8	7.4	16.2	-17.8
Argentina	-14.0	4.6	-13.5	-0.8	4.9	7.4	-0.5	50.1
Ukraine	2.8	4.6	2.7	4.2	6.1	7.4	5.5	29.3
Other	1.8	4.6	2.3	3.0	9.1	7.4	4.7	-17.8
World	4.6	4.6	5.3	6.0	7.4	7.4	7.0	10.8

Summary for the capital stock accumulation mechanisms comparison

- Consistency with benchmark annual step solutions: Smooth Investment Growth performs the best while Capital Targeting GDP performs the worst
- Consistency under different market closures (exogenous and endogenous baselines): Linear Updated Investment performs the worst
- Consistency between capital stock accumulation and wealth accumulation: No adjustments are required in Rate of Return Equalization and Updated Net Investment; Largest adjustments are required in Capital Targeting GDP
- Flexibility for capital stock calibration: only Smooth Investment Growth is capable
- Overall, the smooth mechanism performs better compared to the alternative mechanisms under the simplified baseline and only the smooth mechanism is capable of conducting a sensible investment-capital growth calibration.