A Global Trade Model Calibrated to USDA Baseline Agricultural Projections

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Abstract

The U.S. Department of Agriculture (USDA) publishes, on an annual basis, its 10-year baseline agricultural projections for production, yields, harvested areas, trade and inventories of major agricultural commodities in the U.S. and major markets abroad. Despite the projections' importance in policymaking and other public and private decisions, the USDA lacks the practical ability to run counterfactual analysis exercises to evaluate the projections' response to any observed market change or proposed policy change. This paper develops a theoretically consistent economic model calibrated to the USDA baseline projections to facilitate such exercises. The calibrated model replicates the 10-year projections and allows simulated changes to the projections as a result of shocks to land supply, productivity or to a range of policy variables including tariffs, export taxes and non-tariff measures. Model output is mapped to a user-friendly display that can be used to illustrate the effects of a given shock on the projected outcomes. We conduct four model application exercises to illustrate the model's capabilities and our user-friendly approaches to displaying results on the internet.

Keywords: Global agricultural trade, partial equilibrium model, USDA baseline projections
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1 Introduction

As part of its support for public and private decision-makers in U.S. agriculture, the U.S. Department of Agriculture (USDA) publishes, on an annual basis, its baseline 10-year projections for harvested area, production, consumption, trade and storage of agricultural commodities that are important in global production, consumption and trade.[1] The projections set 10-year expectations for these and other market outcomes in the U.S. and in major agricultural producing and consuming markets abroad. The projections are integral to forecasts of the cost of U.S. farm programs, forecasts that serve as inputs into the U.S. President's annual budget request.[2] Despite their importance to policymaking and to the agricultural sector more broadly, the annual projections are not easily updated in the face of rapidly changing events. Major events that shape world agricultural commodity markets - the outbreak of the Russia-Ukraine war is just one example - can not be easily integrated into the projections until the following February.[3] A fully-consistent economic model calibrated to the baseline projections would allow rapid updates of the projections, updates that are disciplined by economic behavior, cost and trade shares, and plausible values of the relevant elasticities. Open-source code would allow analysts and the outside public to consider their own scenarios, and conduct sensitivity analysis when major shocks hit global commodity markets.

This paper offers an illustration of how a theoretically-consistent economic model can be calibrated to the USDA baseline projections, and shocked to allow theory-consistent updating of the key projection variables that are relevant to agricultural decision-makers. Open-source code that solves the model and produces the visualizations will be available upon publication of the paper. Here, we document the mathematical relationships that we use to represent economic behavior in a model calibrated to the USDA's baseline projections for nine agricultural commodities and 20 countries/regions. The projected variables used in the calibrated model include projected quantities of harvested acres, production, consumption and trade, as well as stocks at both the beginning and the end of each marketing year. Other inputs into the calibrated model include USDA's projected

[1]Dohlman et al. (2025) report the most recent version of the baseline projections. These projections used October 2024 data from the *World Agricultural Supply and Demand Estimates (WASDE)* as inputs to project outcomes for the 2025/2026 to the 2034/2035 marketing years. User friendly visualizations of the baseline projections are available at this USDA website. We describe the projections in more detail in section 2.

[2]USDA (2025) explains that the USDA uses the estimates to project the cost of farm programs, and these estimates are used by the Office of Management and Budget to project long-run budget outlays. The U.S. Office of Management and Budget also uses the information in preparing the President's annual budget request.

[3]The USDA has a mechanism for producing interim forecasts, but the process is slow, relies on proprietary code, and sometimes produces outcomes that are not consistent with micro-economic theory (Ivanic and Nava, 2023).

losses for cotton, and supply-chain relationships that link soybean production and trade to the production of soybean meal, soybean oil and soybeans' use as feed for livestock. The calibrated model replicates these data from the USDA projections. Other market linkages between the commodities are imposed through adding up conditions: an implicit market for land clears in equilibrium and a representative consumer in each country substitutes between commodities as relative prices change.

The role of the calibrated theoretical model is to simulate responses to shocks. The model includes a range of policy variables, including tariffs, export taxes and non-tariff measures (NTMs). The model also allows exogenous shocks to land supplies and to yields (which we formalize as a total factor productivity shock to production of the affected crop). Shocks to the model generate responses in local production, trade, end-of-period stocks, and indirect adjustments that arise via land markets, consumer substitution and foreign responses to endogenous changes in global markets. The user is able to trace, for example, the consequences of a 20 percent reduction in the Ukrainian land supply on all the model's endogenous variables. Key outcomes are the way in which the shock affects production, trade, consumption and storage in the commodity markets included in the model.

The BSAT model is designed to be user-friendly, and we have created a variety of display options that allow the user to easily view the modeled effects of shocks on the projections. In the case of the aforementioned shock to land supply in Ukraine, global wheat markets are most affected. The user can trace the modeled changes in the projections with a variety of charts that allow a focus on either levels of, or changes in, the key variables of the model. Heat maps of changes across countries allow a quick and intuitive understanding of the sectors and countries that are most affected by the shock. We illustrate the visualization in the paper's results section.

As illustrations of the model's capabilities, we conduct four scenario analyses, applying different exogenous shocks to the model's benchmark calibration. As discussed above, we consider a shock to the supply of land in Ukraine. We next consider the imposition of export taxes by the government of India, a policy implemented during the food price spike of 2022-23. We model both illustrations as temporary shocks. In a third example, we consider an increase in Chinese yields that persists over the projections' entire ten-year period, a shock that we motivate by China's recent decision to allow genetically modified corn, soybeans and cotton. A fourth scenario considers a one-year reduction in the productivity of U.S. land used in corn and soybean production, an outcome that could arise as a result of a significant weather shock.

The model recalculates all model outcomes in response to each of the above shocks. A variety of visualization tools allow the user to observe the most affected model outcomes. A key purpose of the paper is to link the model structure and proposed shocks to model results displayed in the online visualization tool. We provide illustrative examples of model outcomes, as reported in the visualization tool constructed to display the effects of each shock on the baseline projections.

The outline of the paper is as follows. Section 2 reviews the USDA baseline agricultural projections. Section 3 describes the theoretical model. Section 4 describes the data. Section 5 describes the computational implementation of the model. Section 6 offers an illustration of model results and our display mechanisms. Section 7 concludes. Appendix A provides a detailed mathematical representation of the model.

2 An overview of the USDA Baseline Agricultural Projections

The USDA produces annual long-term agricultural projections, commonly referred to as "baseline projections," which provide a comprehensive scenario for the U.S. farm sector and global agricultural markets over a 10-year horizon (Economic Research Service, 2025) These projections represent an interagency consensus on the trajectory of the agricultural sector and serve as a fundamental tool for agricultural policy analysis and budget planning.

USDA baseline projections differ fundamentally from monthly World Agricultural Supply and Demand Estimates (WASDE) reports in both their purpose and methodology. While WASDE reports focus on short-term forecasts that aim to predict market outcomes within one or two years, baseline projections examine longer-term underlying trends based on specific conditioning assumptions. These projections are released annually, typically in February (e.g., Dohlman et al., 2025), with early-release tables for U.S. farm sector projections appearing in November following the October WASDE.

The baseline projections cover an extensive range of agricultural topics relevant to the U.S. Conceptually, they comprise two sets of projections, one focused on U.S. indicators and another on international indicators. The U.S.-focused set includes a comprehensive set of projections for major agricultural commodities such as grains, oilseeds, fruits, nuts, vegetables, cotton, sugar, livestock, and livestock products. For each commodity, there are projected indicators such as production, yields, acreage, price, and net returns. Additionally, released reports also include aggregate indicators of the farm sector, most notably farm income projections and specialty crop outlooks. The international baseline projections, on the other hand, cover up to 44 regional markets and only 13 commodities: barley, corn, cotton, rice, soybeans, sorghum, wheat, soybeans, soybean meal, soybean oil, and livestock (beef, pork, and poultry). The set of indicators for international projections is also more limited, focusing mostly on quantities, such as production, yields, acreage, imports, and exports. Notably, the international indicators do not include bilateral trade flows between regions, regional prices, or net returns.

The baseline projections serve multiple critical purposes within the federal government and broader agricultural sector stakeholders. Most importantly, commodity projections are used to forecast farm program costs and prepare the President's annual budget. The baseline projections also provides a neutral policy scenario that serves as a benchmark for analyzing alternative policies and market developments. Researchers and policymakers use the baseline to evaluate potential policy changes, such as modifications to biofuel policies or trade agreements.

The development of USDA's long-term projections involves extensive interagency collaboration. The World Agricultural Outlook Board chairs the Interagency Agricultural Projection Committee, which develops the projections and is responsible for the review and clearance of the final report. The Economic Research Service (ERS) coordinates the inputs to the economic models and drafts the report text. Other agencies involved include the Farm Production and Conservation Business Center, the Foreign Agricultural Service, the Office of the Chief Economist, the Office of Budget and Program Analysis, the Risk Management Agency, the Agricultural Marketing Service, and others.

The projections are built on specific assumptions designed to create a neutral backdrop. Macroeconomic conditions are typically "smoothed," meaning that they exclude recessions or economic booms. Agricultural policies are assumed to remain unchanged from the current law. Weather patterns are assumed to be normal, and no domestic or external shocks to global agricultural markets are incorporated. This neutral framework focuses on fundamental long-term forces affecting agricultural markets rather than short-term volatility.

The USDA's methodological approach combines commodity-specific analytical tools for U.S. projections with an integrated international modeling system. The ERS Country-Commodity Linked System provides the framework for international baseline projections, using country and regional models that employ economic behavioral relationships to project production, consumption, and trade quantities based on world and domestic commodity prices and assumed macroeconomic conditions (Hjort et al., 2018). For livestock and poultry, the USDA Livestock and Poultry Baseline Model provides specialized structural equations capturing the biological and economic dynamics of animal agriculture (Maples et al., 2022). The Excel-based country and regional model equations are converted to Fortran and run together with a linking procedure that iteratively adjusts world prices until global imports and exports balance for each major agricultural commodity market. This solution algorithm uses a numerical version of Newton's method within Gauss-Seidel loops, cycling through price adjustments until all commodity markets simultaneously clear. The system uses historical data primarily from the USDA Foreign Agricultural Service's Production, Supply and Distribution database, supplemented by FAO data where necessary for livestock products and other commodities with incomplete coverage.

The USDA baseline projections provide a consistent framework for long-term agricultural market analysis. Their neutral assumptions make them particularly useful for policy comparison and scenario analysis. The interagency process ensures that diverse expertise and perspectives are incorporated, while the detailed commodity coverage and international scope make them comprehensive analytical tools. However, these projections also have inherent limitations. By design, they exclude shocks and assume smooth macroeconomic trends, which means actual outcomes invariably diverge from projected paths as unforeseen events occur, such as weather extremes, disease outbreaks, policy changes, or economic disruptions. The projections represent one plausible scenario rather than a definitive forecast. Additionally, the complexity of the projection modeling system and the patchwork of data and numerical processing technologies significantly hinder analysts' ability to perform counterfactual simulations using assumptions that depart from the baseline projections. This paper seeks to address these limitations, offering an easy-to-use tool for scenario analyses calibrated to the baseline projections.

3 Theoretical Model

The model produces a multi-product partial equilibrium across multiple regions that is consistent with nested constant elasticity of substitution (CES) preferences in consumption, constant returns to scale in production, and a constant elasticity of transformation (CET) across home, foreign and storage markets. The model is calibrated to data on acres harvested, production, consumption, trade and storage of nine commodities and 20 countries/regions included in the USDA/ERS 10-year baseline projections. Consumer preferences are represented in a nested CES structure with commodities in the top nest and Armington (1969) substitution between domestic and imported products in the secondary nest.[4] Production is modeled with constant returns to scale. Finally, intermediaries' allocate supplies of agricultural goods across markets using a constant elasticity of transformation (CET) structure commonly used in other policy models.[5] Although the BSAT model represents a partial equilibrium, its production and consumption decisions resemble the structure of the GTAP CGE model.[6] Our model includes storage, so we extend the CET structure to include production for end-of-period stocks. The model is static and sequential annual equilibrium solutions are solved independently. The structure abstracts from expectation formation in order to simplify the theory and the model's computational solution.

In order to calibrate the model to the USDA/ERS projections, we use the calibrated share form of Rutherford (1995). The model is initially solved in levels and then converted into *hat algebra*. This approach ensures an accurate calibration of the model to the projections and facilitates its numerical solution. Below, we provide a detailed explanation of the model. We first detail assumptions about the behavior of consumers, producers' and intermediaries in each agricultural market. We then describe market clearance in the model's markets. For simplicity, the subindex r denotes a region in the USDA/ERS projections and i an agricultural good.[7]

3.1 Optimal Economic Behavior in Agricultural Markets

3.1.1 Consumers

Consumers' decisions in the model are determined by Leontief preferences across agricultural and non-agricultural composites, respectively.[8] Within the agricultural composite, consumer preferences are assumed to differ across eight commodity bundles, where each bundle includes com-

[4] Applications of Armington (1969) often use it to explain bilateral trade patterns. The USDA baseline projections do not include bilateral trade flows, so our calibrated model simply differentiates between imports and domestic production for each country-commodity pair.

[5] Powell and Gruen (1968) develop the CET form. De Melo and Robinson (1989) apply it in a single-country computable general equilibrium (CGE) model. Later applications of CET within single-country CGE models include Rutherford et al. (1993) and Konan and Maskus (1997). Our structure is similar to the models used in these papers, but the structure is partial, rather than general equilibrium, and applies to commodity markets rather than to the entire economy. We also model multiple linked economies rather than a single economy trading with the world.

[6] See Hertel (1997) and Van der Mensbrugghe (2018) for further details on the structure of GTAP.

[7] Appendix A provides a detailed mathematical derivation of the model.

[8] This assumption allows partial equilibrium developments in commodity agriculture to be isolated from unmodeled developments in non-agricultural markets for which the projections provide no data. The implicit assumption is that there is perfectly elastic supply of the non-agricultural good at the (fixed) benchmark price. While the lack of economy-wide resource constraints would be problematic if we were calculating welfare, the assumption is not critical to the partial equilibrium outcomes we study. The non-agricultural composite is a perfect complement to the agricultural composite, and the supply of non-agricultural commodities implicitly expand or contract (with no resource costs) in conjunction with the modeled changes in the agricultural composite. Non-agricultural goods are included so that the demand system satisfies homogeneity, without the need to formulate resource costs of changes in the non-agricultural composite.

modities with similar characteristics (e.g. Corn and Sorghum, soy meal and soy oil, etc.). Each commodity bundle is a composite of demands for its individual commodities. Demand for individual commodities in each country are represented as an Armington composite of domestic and imported varieties.

Consumers' preferences are represented with a four-level nested CES utility function (see Figure 1). The upper level nest determines consumers' demand for the agricultural composite, $X_{r,AG}$, and a non-agricultural composite, $X_{r,NA}$. The agricultural composite is another CES nest (hereafter, nest n) that determines demand for commodity bundles $XN_{r,AG}$.[9] The commodity bundles nest demands for individual commodities i in nest n; $XA_{r,i}$.[10] Finally, individual commodity i nests demands for the local and imported varieties, $XD_{r,i}$ and $XM_{r,i}$, respectively. The nested CES structure generates price elasticities of demand for each agricultural product that we calibrate to benchmark elasticities observed in the literature.[11]

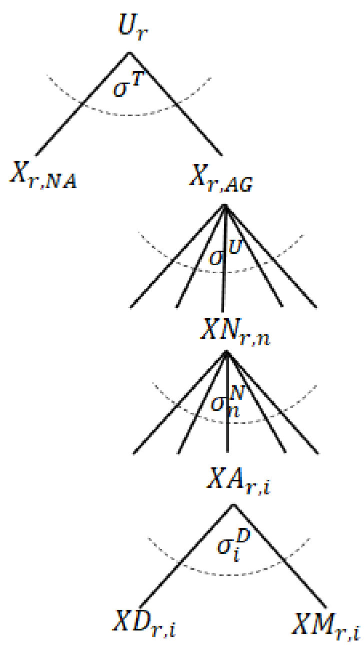

Figure 1: Structure of consumers' nested CES utility function

[9] The model has eight commodity bundles; commodities with similar characteristics are grouped together. In Appendix B, we define the nests as follows: (1) Cereals, including corn and sorghum; (2) Soy Secondary, soybean meal and soybean oil; (3) Livestock, beef and veal, pork and poultry; (4) Barley Cereals, barley; (5) Bread Cereals, wheat, (6) Rice Cereals, rice; (7) Soy Cereals, soybeans; and (8) Other, cotton.

[10] Since every good i belongs to a unique nest n, the subindex n is omitted for the variables and parameters defined at the combination r, n, i.

[11] Appendix B shows the analytical expression for the price elasticity of demand of every agricultural good in the model. Table B5 in this Appendix shows the calibrated price elasticities of demand for each agricultural good in the model and the reference elasticity used in the calibration exercise.

This representation of consumer behavior is admittedly simplistic, especially because consumers do not directly demand the agricultural commodities in the USDA/ERS baseline projections. However, we believe this representation is robust enough for our purposes, since consumers implicitly demand these goods through the final products they demand. In this sense, it is irrelevant whether an agricultural good i was originally used as food or feed for livestock. In the end, consumers demand agricultural goods, and this determines their optimal consumption decisions.

The assumed utility function that consumers maximize in each region r is given by:

$$U_r = \left[\beta_{r,NA}^{\frac{1}{\sigma^T}} X_{r,NA}^{\frac{\sigma^T-1}{\sigma^T}} + \beta_{r,AG}^{\frac{1}{\sigma^T}} X_{r,AG}^{\frac{\sigma^T-1}{\sigma^T}} \right]^{\frac{\sigma^T}{\sigma^T-1}}, \tag{1}$$

where the β terms denote CES distribution parameters, σ^T the elasticity of substitution between agricultural (AG) and non-agricultural (NA) goods, and $P_{r,AG}$ and $P_{r,NA}$ the corresponding prices of these goods.[12]

Within agricultural goods, we assume σ^U denotes the elasticity of substitution among nests, σ_n^N the elasticity of substitution among agricultural goods in every nest n, and σ_i^D the elasticity of substitution between local and imports of every good i.[13] Similarly, $PN_{r,AG}$ denotes the price of all agricultural goods in a nest n in region r, $PA_{r,i}$ the price of each agricultural good i in region r, $PD_{r,i}$ the domestic price of agricultural good i in region r, and $PM_{r,i}$ the import price of each agricultural good i in region r. In addition, because agricultural good i is internationally traded at a reference price $PREF_i$ and countries apply import tariffs (τ^M) and import non-tariff measures (ψ^M), $PM_{r,i}$ is assumed to be equal to $PREF_i \left(1 + \tau_{r,i}^M\right) \left(1 + \psi_{r,i}^M\right)$.

Now, assuming that consumers maximize their utility subject to their budget constraint, and beginning to solve this problem from the bottom level of their utility function, consumers' local $XD_{r,i}$ and imports $XM_{r,i}$ demands for each agricultural good i in region r, written in *hat algebra*, are given by:[14]

[12] Given consumers' Leontief preferences for agricultural and non-agricultural goods, σ^T is *ex ante* assumed equal to 0.

[13] Table B6 shows the value assumed for each elasticity of substitution in the numerical calibration of the model. Notably, it shows that σ^U and most σ_n^N take values ranging between 0 and 1, derived from a *ex ante* calibration of the price elasticities of demand of all products with estimates available in the literature. These estimates then suggest certain degree of complementarity across and within nests in the model. The Table B6 also shows that σ_n^N varies depending on the nest, and is set to zero for nests that bundle only one agricultural good.

[14] All expressions are written in *hat-algebra* by taking the ratio of each variable to its benchmark level (i.e. $\widehat{Z} = \frac{Z}{\bar{Z}}$).

$$\widehat{XD}_{r,i} = \left(\frac{\widehat{PD}_{r,i}}{\widehat{PA}_{r,i}}\right)^{-\sigma_i^D} \cdot \widehat{XA}_{r,i} \tag{2}$$

$$\widehat{XM}_{r,i} = \left(\frac{\widehat{PM}_{r,i}}{\widehat{PA}_{r,i}}\right)^{-\sigma_i^D} \cdot \widehat{XA}_{r,i}, \tag{3}$$

where the price index $\widehat{PA}_{r,i} = \left[\left(1-\phi_{r,i}^{DIMP}\right)\left(\widehat{PD}_{r,i}\right)^{1-\sigma_i^D} + \phi_{r,i}^{DIMP}\left(\widehat{PM}_{r,i}\right)^{1-\sigma_i^D}\right]^{\frac{1}{1-\sigma_i^D}}$, $\widehat{PM}_{r,i} = \widehat{PREF}_i\left(\frac{1+\tau_{r,i}^M}{1+\overline{\tau}_{r,i}^M}\right)\left(\frac{1+\psi_{r,i}^M}{1+\overline{\psi}_{r,i}^M}\right)$, and $\phi_{r,i}^{DIMP}$ denotes the share of agricultural imports of good i in region r on the total value of demand.[15]

Moving up one level in the consumers' utility function, the total demand for each agricultural good i in region r, $XA_{r,i}$, written in *hat algebra*, is given by:

$$\widehat{XA}_{r,i} = \left(\frac{\widehat{PA}_{r,i}}{\widehat{PN}_{r,n}}\right)^{-\sigma_n^N} \widehat{XN}_{r,n}, \tag{4}$$

where the price index $\widehat{PN}_{r,n} = \left[\sum_{i\in n}\phi_{r,i}^N\left(\widehat{PA}_{r,i}\right)^{1-\sigma_n^N}\right]^{\frac{1}{1-\sigma_n^N}}$, and $\phi_{r,i}^N$ denotes the share of the demand for good i in the total value of the demand for goods in nest n.[16]

Turning now to the next upper level, the total demand for agricultural goods in every nest n in region r, $XN_{r,n}$, written in *hat algebra*, is given by:

$$\widehat{XN}_{r,n} = \left(\frac{\widehat{PN}_{r,n}}{\widehat{P}_{r,AG}}\right)^{-\sigma^U} \widehat{X}_{r,AG}, \tag{5}$$

where the price index $\widehat{P}_{r,AG} = \left[\sum_n \phi_{r,n}^U\left(\widehat{PN}_{r,n}\right)^{1-\sigma^U}\right]^{\frac{1}{1-\sigma^U}}$, and $\phi_{r,n}^U$ denotes the share of the total demand for agricultural goods in nest n and region r in the total value of the demand agricultural for goods in r.[17]

[15] $\phi_{r,i}^{DIMP}$ is given by the ratio $\overline{PM}_{r,i}\overline{XM}_{r,i}/\left(\overline{PREF}_i\overline{XD}_{r,i} + \overline{PM}_{r,i}\overline{XM}_{r,i}\right)$, where all variables are evaluated at the benchmark values.

[16] $\phi_{r,i}^N$ is given by the ratio $\overline{PA}_{r,i}\overline{XA}_{r,i}/\sum_{i\in n}\overline{PA}_{r,i}\overline{XA}_{r,i}$, where $\overline{PA}_{r,i}\overline{XA}_{r,i}$ equals to $\left(\overline{PREF}_i\overline{XD}_{r,i} + \overline{PM}_{r,i}\overline{XM}_{r,i}\right)$ and all variables are evaluated at the benchmark values.

[17] $\phi_{r,n}^U$ is given by the ratio $\overline{PN}_{r,n}\overline{XN}_{r,n}/\sum_n\overline{PN}_{r,n}\overline{XN}_{r,n}$, where $\overline{PN}_{r,n}\overline{XN}_{r,n}$ equals to $\sum_{i\in n}\overline{PA}_{r,i}\overline{XA}_{r,i}$ and all variables are evaluated at the benchmark values.

Finally, moving up to the top level of the consumers' utility function, the composite demand for agricultural, $X_{r,AG}$, and non-agricultural goods, X_{NA}, written in *hat algebra*, is given by:

$$\widehat{X}_{r,NA} = \left(\frac{\widehat{P}_{r,NA}}{\widehat{PU}_r}\right)^{-\sigma^T} \frac{\widehat{I}_r}{\widehat{PU}_r} \tag{6}$$

$$\widehat{X}_{r,AG} = \left(\frac{\widehat{P}_{r,AG}}{\widehat{PU}_r}\right)^{-\sigma^T} \frac{\widehat{I}_r}{\widehat{PU}_r} \tag{7}$$

where $\widehat{PU}_r = \left[(1-\alpha_r)\left(\widehat{P}_{rNA}\right)^{1-\sigma^T} + \alpha_r\left(\widehat{P}_{rAG}\right)^{1-\sigma^T}\right]^{\frac{1}{1-\sigma^T}}$, and α_r denotes consumers' expenditure share in agricultural goods in each region r.

3.1.2 Producers

Producers' decisions are modeled assuming that their technology exhibits constant returns to scale. Producers also leverage in Total Factor Productivity (TFP) gains in the production of every agricultural good i in region r, and these gains may differ in the generation of value-added and/or transformation of intermediate inputs. The technical change is land-biased within the value added generation, and conditions are perfectly competitive in the market of all agricultural goods. Additionally, for simplicity, the production of primary agricultural goods (e.g. corn, wheat and Cotton) only generates value added, while the production of secondary goods (e.g. soybean oil) involves exclusively the transformation of other goods (in this case of soybeans).[18]

We formally model producers' behavior, assuming that their production technology can be represented with a nested CES production function (see Figure 2). This function allows us to model that the production of each agricultural good i in region r, $Qprd_{r,i}$, involves the generation of value added, $VA_{r,i}$ for primary goods, and the transformation of intermediate inputs, $IN_{r,i}$ for secondary goods. It also allows us to model that the generation of value added of every primary good i in region r is a function of the demand for land, $LN_{r,i}$, and non-land factors of production, $NL_{r,i}$. Similarly, the transformation of intermediate inputs to produce every secondary good i in region r may involve the demand for local $qLC^{IN}_{r,i,j}$ and imported $qIM^{IN}_{r,i,j}$ intermediate good j, which could itself be another agricultural good (e.g. soybeans for producing soy oil and soy meal).

[18]The model classifies as primary goods: barley, corn, cotton, rice, sorghum soybean, and wheat. Similarly, it denotes as secondary goods: soybean meal and soybean oil.

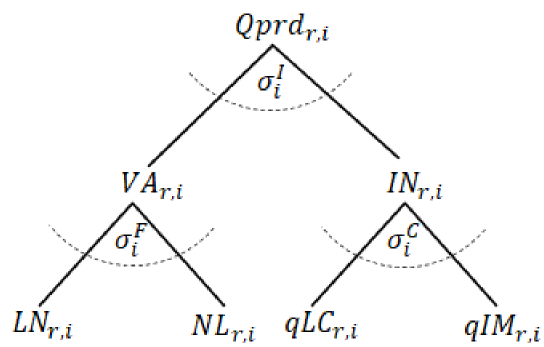

Figure 2: Structure of producers' nested CES production function

Accordingly, the profit function that we assume each producer maximizes is given by:

$$\Pi = (PD_{r,i} - C_{r,i}) \cdot Qprd_{r,i} \tag{8}$$

where $PD_{r,i}$ denotes the price of each agricultural good i in region r on the domestic market, and $C_{r,i}$ the unit cost of production.[19]

The production function for each producer is given as follows, as a function of the generation of value added $VA_{r,i}$ and the transformation if intermediate inputs $IN_{r,i}$,

$$Qprd_{r,i} = \left[{\gamma_{r,i}^{VA}}^{\frac{1}{\sigma_i^I}} \left(\Omega_{r,i}^{VA} VA_{r,i}\right)^{\frac{\sigma_i^I - 1}{\sigma_i^I}} + (1 - \gamma_{r,i}^{VA})^{\frac{1}{\sigma_i^I}} \left(\Omega_{r,i}^{IN} IN_{r,i}\right)^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \right]^{\frac{\sigma_i^I}{\sigma_i^I - 1}}, \tag{9}$$

where σ_i^I denotes the elasticity of substitution between the generation of value added generation and the transformation of intermediate inputs, $\Omega_{r,i}^{VA}$ and $\Omega_{r,i}^{IN}$ TFP shifters to the generation of value added and transformation of intermediates, and both γ's the CES distribution parameters of the production function:[20]

Now, turning to the generation of value added by producers of each primary good i in region r, we model this process using a CES structure that combines the demand for land $LN_{r,i}$ and non-land factors $NL_{r,i}$. Furthermore, we include a parameter $\Theta_{r,i}$ in this function to model producers' land biased technical change. We therefore model the value added of an agricultural primary good i in region r, assuming that it takes the following form:

[19] This means that the model assumes that the total cost of production of every agricultural good i in region r is equal to $C_{r,i} \cdot Qprd_{r,i} = PVA_{r,i} \cdot VA_{r,i} + PIN_{r,i} \cdot IN_{r,i}$.

[20] Since the model assumes that the production of primary agricultural goods only generates value added, while the production of secondary goods involves exclusively the transformation of other goods, σ_i^I is assumed equal to 0 for all agricultural goods.

$$VA_{r,i} = \left[{\gamma_{r,i}^{LN}}^{\frac{1}{\sigma_i^F}} \left(\Theta_{r,i} LN_{r,i}\right)^{\frac{\sigma_i^F - 1}{\sigma_i^F}} + (1 - \gamma_{r,i}^{LN})^{\frac{1}{\sigma_i^F}} NL_{r,i}^{\frac{\sigma_i^F - 1}{\sigma_i^F}} \right]^{\frac{\sigma_i^F}{\sigma_i^F - 1}}, \tag{10}$$

where both γ's denote the CES distribution parameters, and σ_i^F the elasticity of substitution between land and no-land factors in the production of good i in r.[21]

Similarly, we model the transformation of intermediate input j to produce each secondary good i in region r, using a CES structure that combines local $qLC_{r,i,j}^{IN}$ and imports $qIM_{r,i,j}^{IN}$ demand for intermediate input j to produce secondary good i in region r. We therefore model intermediates demand for producing secondary good i in region r, assuming that it takes the following form:

$$IN_{r,i} = \sum_j \left[{\gamma_{r,j}^{IN}}^{\frac{1}{\sigma_i^C}} qLC_{r,i,j}^{IN\,\frac{\sigma_i^C - 1}{\sigma_i^C}} + (1 - \gamma_{r,j}^{IN})^{\frac{1}{\sigma_i^C}} qIM_{r,i,j}^{IN\,\frac{\sigma_i^C - 1}{\sigma_i^C}} \right]^{\frac{\sigma_i^C}{\sigma_i^C - 1}} \tag{11}$$

where both γ's also denote the CES distribution parameters, and σ_i^C the elasticity of substitution between the demand for local $qLC_{r,i,j}^{IN}$ and imported intermediate input j, $qLC_{r,i,j}^{IN}$, to produce good i in region r.[22]

Now, solving producers' profits maximization problem, the optimal generation of value-added of good i in r, $VA_{r,i}$, and its subsequent intermediate inputs demand, $IN_{r,i}$, written in *hat algebra*, are given by:

$$\widehat{VA}_{r,i} = \frac{1}{\widehat{\Omega_{r,i}^{VA}}} \left(\frac{\widehat{PVA}_{r,i}}{\widehat{\Omega_{r,i}^{VA}} \widehat{PD}_{r,i}} \right)^{-\sigma_i^I} \widehat{Qprd}_{r,i} \tag{12}$$

$$\widehat{IN}_{r,i} = \frac{1}{\widehat{\Omega_{r,i}^{IN}}} \left(\frac{\widehat{PIN}_{r,i}}{\widehat{\Omega_{r,i}^{IN}} \widehat{PD}_{r,i}} \right)^{-\sigma_i^I} \widehat{Qprd}_{r,i} \tag{13}$$

[21] σ_i^F is assumed to be equal to 0.9 in the numerical calibration of the model (See Table D1). This value is assigned to this elasticity, given the assumed underlying Cobb-Douglas structure to predict the land cost shares $\phi_{r,i}^f$ (detailed in Appendix C). Ideally, this elasticity must be equal to 1 - that is, the elasticity of substitution in Cobb-Douglas frameworks. However, the numerical solution of the model experiences convergence problems, for which we assign a value close enough, 0.9.

[22] σ_i^C is assumed to be equal to 2 in the numerical calibration of the model (See Table D1). This value relies on the theoretical assumption that producers easily shift their local and import demand for good j to produce good i in region r in response to a price change.

where $\phi^{VA}_{r,i}$ denotes the value added share in the production of each good i in region r, and producers' zero profit condition implies that $\widehat{PD}_{r,i} = \widehat{C}_{r,i} = \left[\left(\phi^{VA}_{r,i}\right) \left(\frac{\widehat{PVA}_{r,i}}{\widehat{\Omega}^{VA}_{r,i}}\right)^{1-\sigma^I_i} + \left(1-\phi^{VA}_{r,i}\right)\left(\frac{\widehat{PIN}_{r,i}}{\widehat{\Omega}^{IN}_{r,i}}\right)^{1-\sigma^I_i} \right]^{\frac{1}{1-\sigma^I_i}}$.

Turning now to producers' demand for land and non-land factors, we find that they are given as follows, as a function of their corresponding prices in each region r, PLN_r and PNL_r:

$$\widehat{LN}_{r,i} = \frac{1}{\widehat{\Theta}_{r,i}} \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i}\widehat{PVA}_{r,i}} \right)^{-\sigma^F_i} \widehat{VA}_{r,i} \tag{14}$$

$$\widehat{NL}_{r,i} = \left(\frac{\widehat{PNL}_r}{\widehat{PVA}_{r,i}} \right)^{-\sigma^F_i} \widehat{VA}_{r,i} \tag{15}$$

where $\widehat{PVA}_{r,i} = \left[\phi^f_{r,i} \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i}}\right)^{1-\sigma^F_i} + \left(1-\phi^f_{r,i}\right)\left(\widehat{PNL}_r\right)^{1-\sigma^F_i} \right]^{\frac{1}{1-\sigma^F_i}}$ and $\phi^f_{r,i}$ denotes the cost share of land in the value of total costs to produce good i in r.[23]

Finally, moving to producers' local and imported demand for agricultural good j to produce i in region r, $qLC^{IN}_{r,i,j}$ and $qIM^{IN}_{r,i,j}$ respectively, written in *hat algebra*, we find are given by:

$$\widehat{qLC}^{IN}_{r,i,j} = \left(\frac{\widehat{PD}_{r,j}}{\widehat{PIN}_{r,i}} \right)^{-\sigma^C_i} \widehat{IN}_{r,i} \tag{16}$$

$$\widehat{qIM}^{IN}_{r,i,j} = \left(\frac{\widehat{PM}_{r,j}}{\widehat{PIN}_{r,i}} \right)^{-\sigma^C_i} \widehat{IN}_{r,i}, \tag{17}$$

where $\widehat{PIN}_{r,i} = \left[\left(1-\phi^{cIM}_{r,j}\right) \widehat{PD}^{1-\sigma^C_i}_{r,j} + \phi^{cIM}_{r,j} \left(\widehat{PM}_{r,j}\right)^{1-\sigma^C_i} \right]^{\frac{1}{1-\sigma^C_i}}$, and $\phi^{cIM}_{r,j}$ the share of imported input j in the total value of the supply of good j used an input in region r.[24]

Hence, the total demand for good j used as intermediate input in the production of all agricultural secondary goods in each region r, $qIN_{r,j}$, is given by:

$$\widehat{qIN}_{r,j} = \sum_i \left(\phi^{LC}_{r,i,j} \cdot \phi^{TTLC}_{r,j}\right) \cdot \widehat{qLC}^{IN}_{r,i,j} + \sum_i \left(\phi^{IM}_{r,i,j} \cdot \phi^{TTIM}_{r,j}\right) \cdot \widehat{qIM}^{IN}_{r,i,j} \tag{18}$$

where $\phi^{LC}_{r,i,j}$ denotes the share of the domestic supply of good i in the total value of domestic supply of goods produced with good j, $\phi^{IM}_{r,i,j}$ the share of imports of good i in the total value of imports of goods produced with good j, $\phi^{TTLC}_{r,j}$ the share of the total domestic supply of goods produced

[23] $\phi^f_{r,i}$ is calculated as detailed in Appendix C.

[24] $\phi^{cIM}_{r,j}$ is given by the ratio $\sum_i \left(\overline{PM}_{r,j} \cdot \overline{qIM}^{IN}_{r,i,j}\right) / \sum_i \left(\overline{PM}_{r,j} \cdot \overline{qIM}^{IN}_{r,i,j} + \overline{PREF}_{r,j} \cdot \overline{qLC}^{IN}_{r,i,j}\right)$, where all variables are evaluated at the benchmark values.

with good j in the sum of the domestic and imported supplies of these goods, and $\phi^{TTIM}_{r,j}$ the corresponding share for imports.[25]

3.1.3 Intermediaries

Finally, intermediaries' supply allocation decisions of agricultural goods across markets are modeled using a CET structure. For simplicity, intermediaries do not incur any cost when allocating the supply, so the model mainly simulates how equilibrium supply allocations respond to changes in fundamentals rather than to frictions in the allocation of the goods. Intermediaries also allocate the supply of each good i in region r, maximizing their revenues $R_{r,i}$. In particular, they do so with the objective of determining (1) how much should they sell and store each good i in region r?; (2) how should they divide the total supply of each good i in region r between local supply and exports?; and (3) how should they allocate the local supply of each good i in region r between the supply for final consumption and as intermediate input to produce other goods?

In order to model intermediaries' behavior, the total supply of each agricultural good i in region r, $XS_{r,i}$, is assumed to be given by an initial level of stocks $BS_{r,i}$, total production $Qprd_{r,i}$ and imports $M_{r,i}$. However, since some activities (such as cotton) experience losses $LS_{r,i}$ in their supply, the model assumes that only the supply of each agricultural good i in region r net of them, $XST_{r,i}$ is allocated. Furthermore, these losses account for a fixed share $\vartheta_{r,i}$ of total supply $XS_{r,i}$, such that the total supply of each agricultural good i in region r, net of losses, is given by:[26]

$$XST_{r,i} = XS_{r,i}\left(1 - \vartheta_{r,i}\right). \tag{19}$$

Additionally, the model uses a three-level nested CET function to simulate intermediaries' allocation of the supply of every good i in region r (see Figure 3). This function allows to model, at the upper level, the supply sold $XCS_{r,i}$ and stored $XSS_{r,i}$ as inventories of each good i in region r. It also allows modeling the local supply $XDS_{r,i}$ and export supply $XExp_{r,i}$. Finally, at the bottom level, this function allows modeling the supply of each good for final consumption $XDF_{r,i}$ and as an intermediate input $XDI_{r,i}$ to produce other goods.

[25] $\phi^{LC}_{r,i,j}$ is given by the ratio $(\overline{PDS}_{r,i,j}\overline{XDS}_{r,i,j}/\sum_i \overline{PDS}_{r,i,j}\overline{XDS}_{r,i,j})$, $\phi^{IM}_{r,i,j}$ to $(\overline{PM}_{r,i,j}\overline{XM}_{r,i,j}/\sum_i \overline{PM}_{r,i,j}\overline{XM}_{r,i,j})$, $\phi^{TTLC}_{r,j}$ to $(\sum_i \overline{PDS}_{r,i,j}\overline{XDS}_{r,i,j}/\left(\sum_i \overline{PDS}_{r,i,j}\overline{XDS}_{r,i,j} + \sum_i \overline{PM}_{r,i,j}\overline{XM}_{r,i,j}\right))$, and $\phi^{TTIM}_{r,j}$ to $(\sum_i \overline{PM}_{r,i,j}\overline{XM}_{r,i,j}/\left(\sum_i \overline{PDS}_{r,i,j}\overline{XDS}_{r,i,j} + \sum_i \overline{PM}_{r,i,j}\overline{XM}_{r,i,j}\right))$; where all variables are evaluated at the benchmark values.

[26] $\vartheta_{r,i}$ is simply equal to the ratio of losses $LS_{r,i}$ to total supply $XS_{r,i}$.

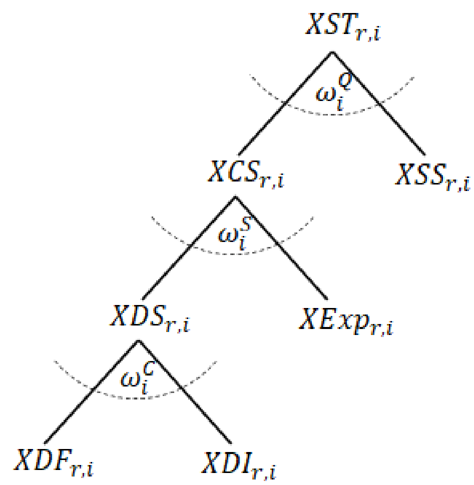

Figure 3: Structure of intermediaries' nested CET function

Hence, the revenue function that we assume every intermediary maximizes is given by:

$$R_{r,i} = PSS_{r,i} \cdot XSS_{r,i} + PCS_{r,i} \cdot XCS_{r,i} \tag{20}$$

which can also be written as:

$$\begin{aligned} &= PSS_{r,i} \cdot PSS_{r,i} + (PE_{r,i} \cdot XExp_{r,i} + PDS_{r,i} \cdot XDS_{r,i}) \\ &= PSS_{r,i} \cdot PSS_{r,i} + (PE_{r,i} \cdot XExp_{r,i} + (PD_{r,i} \cdot XDF_{r,i} + PI_{r,i} XDI_{r,i}))\,, \end{aligned} \tag{21}$$

where $PCS_{r,i}$ denotes the price of agricultural good i sold in region r, $PSS_{r,i}$ the price when stored, $PDS_{r,i}$ the price when sold locally, $PE_{r,i}$ the price when exported, $PDF_{r,i}$ the price when sold locally and as final good, and $PI_{r,i}$ the price when sold locally and as input to produce other good. In addition, because every agricultural good i is internationally traded at a reference price $PREF_i$ and countries sometimes apply export taxes (τ^X) and export non-tariff measures (ψ^M), $PX_{r,i}$ is assumed to be equal to $PREF_i \left(1 - \tau^X_{r,i}\right) \left(1 - \psi^X_{r,i}\right)$.

We also model intermediaries' decision about how much they should optimally sell and store each agricultural good i in region r, using the following CET function:

$$XST_{r,i} = \left({\theta^C_{r,i}}^{-\frac{1}{\omega_i^Q}} XCS_{r,i}^{\frac{\omega_i^Q+1}{\omega_i^Q}} + (1-\theta^C_{r,i})^{-\frac{1}{\omega_i^Q}} XSS_{r,i}^{\frac{\omega_i^Q+1}{\omega_i^Q}} \right)^{\frac{\omega_i^Q}{\omega_i^Q+1}}, \tag{22}$$

where ω_i^Q denotes the elasticity of transformation between both allocations of the supply and all θ's are distribution parameters.[27]

Similarly, we model intermediaries' decision about how they should optimally divide the total supply of each good i in region r between local supply and exports, using the following CET function.

$$XCS_{r,i} = \left({\theta^D_{r,i}}^{-\frac{1}{\omega_i^S}} XDS_{r,i}^{\frac{\omega_i^S+1}{\omega_i^S}} + (1-\theta^D_{r,i})^{-\frac{1}{\omega_i^S}} XExp_{r,i}^{\frac{\omega_i^S+1}{\omega_i^S}} \right)^{\frac{\omega_i^S}{\omega_i^S+1}}, \tag{23}$$

where ω_i^S denotes the elasticity of transformation between both allocations of the supply of each good i in r and all θ's are distribution parameters.[28]

Finally, we represent intermediaries' decision about how they should optimally allocate the local supply of each good i in region r between the supply for final consumption and as intermediate input to produce other goods, assuming a similar CET function,

$$XDS_{r,i} = \left({\theta^F_{r,i}}^{-\frac{1}{\omega_i^C}} XDF_{r,i}^{\frac{\omega_i^C+1}{\omega_i^C}} + (1-\theta^F_{r,i})^{-\frac{1}{\omega_i^C}} XDI_{r,i}^{\frac{\omega_i^C+1}{\omega_i^C}} \right)^{\frac{\omega_i^C}{\omega_i^C+1}}, \tag{24}$$

where ω_i^C denotes the elasticity of transformation between both allocation of the supply and all θ's are distribution parameters.[29]

Now, solving the intermediaries' revenue maximization problem - starting from the bottom level of their CET structure, the supply of good i in region r for final consumption, $XDF_{r,i}$ and the supply as intermediate input to produce other goods $XDI_{r,i}$, written in *hat algebra*, are given by:

[27] ω_i^Q is assumed to be equal to 0 in the numerical calibration of the model (See Table D1). The theoretical reason underlying this assumption is that USDA projection data show that inventories of most agricultural goods exhibit a relatively stable relationship with their market supply.

[28] ω_i^S is assumed to be equal to 5 in the numerical calibration of the model consistently with estimates available for this elasticity in the literature (See Table D1).

[29] ω_i^C is assumed to be equal to 5 in the numerical calibration of the model (See Table D1). This value is relies on the theoretical assumption that intermediaries can easily shift the allocation supply of agricultural good i in region r between the supply for final consumption and as intermediate input to produce other goods in response to a price change.

$$\widehat{XDI}_{r,i} = \left(\frac{\widehat{PI}_{r,i}}{\widehat{PDS}_{r,i}}\right)^{\omega_i^C} \cdot \widehat{XDS}_{r,i} \tag{25}$$

$$\widehat{XDF}_{r,i} = \left(\frac{\widehat{PD}_{r,i}}{\widehat{PDS}_{r,i}}\right)^{\omega_i^C} \cdot \widehat{XDS}_{r,i} \tag{26}$$

where $\widehat{PDS}_{r,i} = \left[\left(1-\phi_{r,i}^{sCRH}\right)\left(\widehat{PD}_{r,i}\right)^{1+\omega_i^C} + \phi_{r,i}^{sCRH}\left(\widehat{PI}_{r,i}\right)^{1+\omega_i^C}\right]^{\frac{1}{1+\omega_i^C}}$ and $\phi_{r,i}^{sCRH}$ denotes the share of good i used in region r as an intermediate input on the total value of its domestic supply.[30]

Moving up one level in the intermediaries' CET structure, the local and export supply of agricultural good i in region r - $XDS_{r,i}$ and $XExp_{r,i}$ respectively, written in *hat algebra*, are given by:

$$\widehat{XDS}_{r,i} = \left(\frac{\widehat{PDS}_{r,i}}{\widehat{PCS}_{r,i}}\right)^{\omega_i^S} \cdot \widehat{XCS}_{r,i} \tag{27}$$

$$\widehat{XExp}_{r,i} = \left(\frac{\widehat{PE}_{r,i}}{\widehat{PCS}_{r,i}}\right)^{\omega_i^S} \cdot \widehat{XCS}_{r,i} \tag{28}$$

where $\widehat{PCS}_{r,i} = \left[\left(1-\phi_{r,i}^{sEXP}\right)\left(\widehat{PDS}_{r,i}\right)^{1+\omega_i^S} + \phi_{r,i}^{sEXP}\left(\widehat{PE}_{r,i}\right)^{1+\omega_i^S}\right]^{\frac{1}{1+\omega_i^S}}$, $\widehat{PE}_{r,i} = \widehat{PREF}_i\left(\frac{1-\tau_{r,i}^X}{1-\overline{\tau}_{r,i}^X}\right)\left(\frac{1-\psi_{r,i}^X}{1-\overline{\psi}_{r,i}^X}\right)$, and $\phi_{r,i}^{sEXP}$ represents the share agricultural exports of good i in region r in the total value of the current supply of this good.[31]

Finally, solving the optimal supply sell and store of each agricultural good i in region r, $XCS_{r,i}$ and $XSS_{r,i}$ respectively, written in *hat algebra*, are given by:

$$\widehat{XSS}_{r,i} = \left(\frac{\widehat{PSS}_{r,i}}{\widehat{PS}_{r,i}}\right)^{\omega_i^Q} \cdot \widehat{XS}_{r,i} \tag{29}$$

$$\widehat{XCS}_{r,i} = \left(\frac{\widehat{PCS}_{r,i}}{\widehat{PS}_{r,i}}\right)^{\omega_i^Q} \cdot \widehat{XS}_{r,i} \tag{30}$$

[30] $\phi_{r,i}^{sCRH}$ is given by the ratio $\overline{PIN}_{r,i}\overline{qIN}_{r,i}/\overline{PDS}_{r,i}\overline{XDS}_{r,i}$, where all variables are evaluated at the benchmark values.

[31] $\phi_{r,i}^{sEXP}$ is given by the ratio $\overline{PREF}_i\overline{XExp}_{r,i}/\overline{PCS}_{r,i}\overline{XCS}_{r,i}$, where all variables are evaluated at the benchmark values.

where $\widehat{PS}_{r,i} = \left[\left(1-\phi^{sSTK}_{r,i}\right)\left(\widehat{PCS}_{r,i}\right)^{1+\omega^S_i} + \phi^{sSTK}_{r,i}\left(\widehat{PSS}_{r,i}\right)^{1+\omega^S_i}\right]^{\frac{1}{1+\omega^S_i}}$ and $\phi^{sSTK}_{r,i}$ denotes the share of the inventories of good i in region r on the total value of the supply of this good.[32]

3.2 Market Clearing Conditions

Given this representation of consumers', producers' and intermediaries' behavior in the model, an essential component to theoretically close and numerically solve the model is the set of market-clearing conditions. We include market-clearing conditions for the following markets (1) the domestic market of each agricultural good i in region r; (2) the international market of each agricultural good i; (3) the factor markets in each region r; (4) the market for inventories of good i in each region r; and (5) the market for each good i in r as intermediate input. The objective is to ensure that consumers', producers' and intermediaries' decisions coherently fit in equilibrium.

3.2.1 Domestic Market of Each Agricultural Good

The market clearing condition imposed in the model on the domestic market of each agricultural good i in region r is given by the equality, written in *hat algebra*, between its total supply $\widehat{XST}_{r,i}$ and its total demand $\widehat{XDT}_{r,i}$. It also establishes that the price that satisfies this condition corresponds to the domestic price in equilibrium, expressed in *hat algebra*, for each good i in country r, $\widehat{PD}_{r,i}$.

$$\widehat{XST}_{r,i} = \widehat{XDT}_{r,i} \tag{31}$$

As explained above, the total supply of each agricultural good i in region r net of losses, $XST_{r,i}$, is defined in the model as a function of initial stocks $BS_{r,i}$, total production $Qprd_{r,i}$, and total imports $M_{r,i}$. Accordingly, we write it as follows using (19):

$$XST_{r,i} = \left(BS_{r,i} + Qprd_{r,i} + M_{r,i}\right)\left(1 - \vartheta_{r,i}\right), \tag{32}$$

where $M_{r,i}$ also corresponds to imports in levels of good i in region r as final good $XM_{r,i}$, as intermediate good to produce good l, $qIM^{IN}_{r,i,l}$, and for any other uses, $OM_{r,i}$.

$$M_{r,i} = XM_{r,i} + \sum_{l} qIM^{IN}_{r,i,l} + OM_{r,i}. \tag{33}$$

[32] $\phi^{sSTK}_{r,i}$ is given by the ratio $\overline{PREF}_i\overline{XSS}_{r,i}/\overline{PS}_{r,i}\overline{XST}_{r,i}$, where all variables are evaluated at the benchmark values.

Turning now to the total demand for each i in r, $XDT_{r,i}$, the model defines it as a function of total consumption $DC_{r,i}$, total exports $XExp_{r,i}$, and final stocks $XSS_{r,i}$. We therefore write it as:

$$XDT_{r,i} = DC_{r,i} + XExp_{r,i} + XSS_{r,i} \tag{34}$$

where $DC_{r,i}$ aggregates the demand for each agricultural good i in region r as final good, $XA_{r,i}$ and as an intermediate good, $qIN_{r,i}$, as follows:

$$DC_{r,i} = XA_{r,i} + qIN_{r,i}. \tag{35}$$

Hence, the total supply of each good i in region r, $XST_{r,i}$, written in *hat algebra*, is given by:

$$\widehat{XST}_{r,i} = \left(\phi^{sBS}_{r,i} \cdot \widehat{BS}_{r,i}\right) + \left(\phi^{sQprd}_{r,i} \cdot \widehat{Qprd}_{r,i}\right) + \left(\phi^{sM}_{r,i} \cdot \widehat{M}_{r,i}\right) \tag{36}$$

where $\phi^{sBS}_{r,i}$, $\phi^{sQpr}_{r,i}$ and $\phi^{sM}_{r,i}$ are the shares of initial stocks, production and imports of good i in region r, respectively, in the value of its total supply.[33] $\widehat{M}_{r,i}$ also corresponds to the total imports of each good i in region r, written in *hat algebra*, as follows:

$$\widehat{M}_{r,i} = \left(\phi^{mFF}_{r,i} \cdot \widehat{XM}_{r,i}\right) + \left(\phi^{mCC}_{r,i} \cdot \sum_{l} \widehat{qIM}^{IN}_{r,i,l}\right) + \left(\phi^{mOO}_{r,i} \cdot \widehat{OM}_{r,i}\right) \tag{37}$$

where $\phi^{mFF}_{r,i}$, $\phi^{mCC}_{r,i}$ and $\phi^{mOO}_{r,i}$ are the share of imports of i in r that are used as final good, as intermediate input and for any other uses, respectively, in the total value of imports of i in r.[34]

Similarly, the total demand for each good i in r, $XDT_{r,i}$, written in *hat algebra*, is given by:

$$\widehat{XDT}_{r,i} = \left(\phi^{dDC}_{r,i} \cdot \widehat{DC}_{r,i}\right) + \left(\phi^{dEX}_{r,i} \cdot \widehat{XExp}_{r,i}\right) + \left(\phi^{dST}_{r,i} \cdot \widehat{XSS}_{r,i}\right) \tag{38}$$

where $\phi^{dDC}_{r,i}$, $\phi^{dEX}_{r,i}$ and $\phi^{dST}_{r,i}$ are the shares of total consumption, exports, final stocks and produc-

[33] $\phi^{sBS}_{r,i}$ is given by the ratio $\left(\overline{PREF}_i \cdot \overline{BS}_{r,i}\right)/\overline{VST}_{r,i}$, $\phi^{sQpr}_{r,i}$ to $\left(\overline{PREF}_i \cdot \overline{Qprd}_{r,i}\right)/\overline{VST}_{r,i}$, and $\phi^{sM}_{r,i}$ to $\left(\overline{PM}_{r,i} \cdot \overline{M}_{r,i}\right)/\overline{VST}_{r,i}$, denoting the total value of supply of agricultural good i in region r, $VTS_{r,i}$, equals to $\left(\overline{PREF}_i \cdot \overline{BS}_{r,i}\right) + \left(\overline{PREF}_i \cdot \overline{Qprd}_{r,i}\right) + \left(\overline{PM}_i \cdot \overline{M}_{r,i}\right)$. Likewise, all variables are evaluated at the benchmark values.

[34] $\phi^{mFF}_{r,i}$ is given by the ratio $\left(\overline{PM}_{r,i} \cdot \overline{XM}_{r,i}\right)/\overline{VTM}_{r,i}$, $\phi^{mCC}_{r,i}$ to $\left(\overline{PM}_{r,i} \cdot \sum_l \overline{qIM}^{IN}_{r,i,l}\right)/\overline{VTM}_{r,i}$, and $\phi^{m,OO}_{r,i}$ to $\left(\overline{PM}_{r,i} \cdot \overline{OM}_{r,i}\right)/\overline{VTM}_{r,i}$, denoting the total value of imports of agricultural good i in region r, $\overline{VTM}_{r,i}$, equals to $\left(\overline{PM}_{r,i} \cdot \overline{M}_{r,i}\right)$. Likewise, all variables are evaluated at the benchmark values.

tion losses, respectively, in the total value of demand of i in r.[35] $\widehat{DC}_{r,i}$ is also defined as:

$$\widehat{DC}_{r,i} = \left(\phi^{cF}_{r,i} \cdot \widehat{XA}_{r,i}\right) + \left(\phi^{cI}_{r,i} \cdot \widehat{qIN}_{r,i}\right) \tag{39}$$

where $\phi^{cF}_{r,i}$ and $\phi^{cI}_{r,i}$ are the shares of total consumption of i in r as final good and as intermediate input, respectively, on the total value of consumption, $VTD_{r,i}$.[36]

3.2.2 International Market of Each Agricultural Good

Turning now to the international market for agricultural goods, the model assumes no differentiation for each good based on their country of origin. Corn from the U.S. is treated simply as corn, and soybeans from China as soybeans. The model therefore assumes that each agricultural good i is traded in a unique international market, where market clearance is given by the equality, written in *hat algebra*, between the world's supply of exports of each good i, $\widehat{WX}_i$, and the world's demand for imports of each good i, $\widehat{WM}_i$. Furthermore, the price that satisfies this condition is the international price of reference in equilibrium, written in *hat algebra*, for each good i, $\widehat{PREF}_i$.

$$\widehat{WX}_i = \widehat{WM}_i \tag{40}$$

We therefore write both the world's supply of exports and the world's demand for imports of each agricultural good i in *hat algebra* as:

$$\widehat{WX}_i = \sum_r \phi^{EX}_{r,i} \cdot \widehat{XExp}_{r,i} \tag{41}$$

$$\widehat{WM}_i = \sum_r \phi^{IM}_{r,i} \cdot \widehat{M}_{r,i}, \tag{42}$$

where $\phi^{EX}_{r,i}$ and $\phi^{IM}_{r,i}$ correspond to the share of agricultural good i exports and imports in country r, respectively, in the total value of exports and imports of i worldwide.[37]

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[35] $\phi^{dDC}_{r,i}$ is given by the ratio to $\overline{VTC}_{r,i}/\overline{VDT}_{r,i}$, $\phi^{dEX}_{r,i}$ to $\left(\overline{PE}_{r,i} \cdot \overline{XExp}_{r,i}\right)/\overline{VDT}_{r,i}$, and $\phi^{dST}_{r,i}$ to $\left(\overline{PREF}_i \cdot \overline{XSS}_{r,i}\right)/\overline{VDT}_{r,i}$; by defining the total value of consumption of agricultural good i in region r, $\overline{VTC}_{r,i}$, equals to $\overline{PREF}_i \cdot \left(\left(\overline{XA}_{r,i} - \overline{XM}_{r,i}\right) + \left(\overline{qIN}_{r,i} - \sum_l \overline{qIM}^{IN}_{r,i,l}\right)\right) + \left(\overline{PM}_{r,i} \cdot \overline{M}_{r,i}\right)$ and the total value of demand of i in r, $\overline{VDT}_{r,i}$, as $\overline{VTC}_{i,j} + \left(\overline{PE}_{r,i} \cdot \overline{XExp}_{r,i}\right) + \left(\overline{PREF}_i \cdot \overline{XSS}_{r,i}\right)$. Likewise, all variables are evaluated at the benchmark values.

[36] $\phi^{cF}_{r,i}$ is given by the ratio $\left(\overline{PA}_{r,i}\overline{XA}_{r,i}\right)/\overline{VTC}_{r,i}$, and $\phi^{cI}_{r,i}$ to $\overline{PIN}_{r,i}\overline{qIN}_{r,i}/\overline{VTC}_{r,i}$; where all variables are evaluated at the benchmark values.

[37] $\phi^{EX}_{r,i}$ is given by the ratio $\overline{PREF}_i\overline{XExp}_{r,i}/\sum_r \overline{PREF}_i\overline{XExp}_{r,i}$, and $\phi^{IM}_{r,i}$ to $\overline{PM}_{r,i}\overline{M}_{r,i}/\sum_r \overline{PM}_{r,i}\overline{M}_{r,i}$; where all variables are evaluated at the benchmark values.

3.2.3 Factor Markets

As explained in the producers' section, the model considers only two factors of production: land and non-land. In the land market, the model assumes as market clearance the equality, written in *hat algebra*, between the total land supply, $\widehat{qsLND}_r$, and the total land demand in every region r, $\widehat{qdLND}_r$. Moreover, the price that satisfies this condition is the price in equilibrium, expressed in *hat algebra*, for land in country r, $\widehat{PLN}_r$.

$$\widehat{qsLND}_r = \widehat{qdLND}_r \tag{43}$$

For simplicity, the land supply in each region r is represented using an upward-sloping function with respect to the price of land with price elasticity ϵ^L.[38] Since land supply might also be subject to eventual changes in land availability, this supply function also includes a parameter η_r, ranging between 0 and 1, to control for this effect. Thereby, the total land supply in each region r, written in *hat algebra*, is given by:

$$\widehat{qsLND}_r = \left(\frac{1+\eta_r}{1+\overline{\eta}_r}\right) \times \widehat{PNL}_r^{\epsilon^L}. \tag{44}$$

Now, turning to the total land demand in country r, we know that the land demand to produce each agricultural good i in country r is given by expression (14). Hence, aggregating this demand across all goods in every region r, the total land demand in each region r is given by:

$$\widehat{qdLND}_r = \phi_{ij}^{LN} \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i}\widehat{PVA}_{r,i}}\right)^{-\sigma_i^F} \left(\frac{\widehat{VA}_{r,i}}{\widehat{\Theta}_{r,i}}\right). \tag{45}$$

where ϕ_{ij}^{LN} denotes the share of land to produce good i in region r in the total available land in r.[39]

Turning now to the non-land market, the model assumes that this market is also in equilibrium when supply equals demand. However, the supply in this market is perfectly elastic for simplicity, so that the market is *ex-ante* in equilibrium with a constant price, written in *hat algebra*, $\widehat{PNL}_r$, equal to 1 and an equilibrium quantity set by the demand.

[38] In the numerical calibration of the model ϵ^L is assumed to be equal to 0.11, consistent with estimates available for this elasticity in the literature (See TableD1).

[39] ϕ_{ij}^{LN} is given by the ratio $\overline{LN}_{r,i}/\sum_i \overline{LN}_{r,i}$, where the harvested land is evaluated at the benchmark values.

3.2.4 Market for Inventories of Each Agricultural Good

Modeling the market for inventories of agricultural goods can be challenging. Inventories for agricultural goods vary over time, so dynamic approaches are generally considered as potential options. However, implementing a dynamic method in this type of model introduces serious theoretical, numerical, and computational difficulties. Consequently, standard and widely used models in the literature, such as GTAP, do not model the market for inventories. The perception is that the cost of modeling this market is higher than its potential benefits.

In an effort to include the market for inventories of agricultural goods in this model, as they are part of the USDA/ERS projections, we use a static approach. Specifically, the market for inventories of each agricultural good i in region r is assumed to behave as any other market. There exists a supply $XSS_{r,i}$ and a demand $qdXSS_{r,i}$ for inventories of every good i in region r. Hence, the market clearance in the model, written in *hat algebra*, is given by the equality between demand and supply in every market. Furthermore, the price that satisfies this condition is the price in equilibrium, written in *hat algebra*, for inventories of good i in region r, $\widehat{PSS}_{r,i}$.

$$\widehat{XSS}_{r,i} = \widehat{qdXSS}_{r,i} \tag{46}$$

For simplicity, the model represents the demand for inventories of each agricultural good i in region r using a downward-sloping function with respect to the price with price elasticity ϵ_i^Q.[40] Specifically, we write it, in *hat algebra*, as follows:

$$\widehat{qdXSS}_{r,i} = \widehat{PSS}_{r,i}^{\epsilon_i^Q}. \tag{47}$$

Turning now to the supply for inventories of each agricultural good i in region r, we know it is given by expression (29) as follows:

$$\widehat{XSS}_{r,i} = \left(\frac{\widehat{PSS}_{r,i}}{\widehat{PS}_{r,i}}\right)^{\omega_i^Q} \cdot \widehat{XS}_{r,i}, \tag{48}$$

[40] ϵ_i^Q is assumed to be equal to 0.5 in the numerical calibration of the model (See TableD1). The theoretical assumption behind this value is that we assume that the demand for an agricultural good i increases and its corresponding demand for inventories decreases in response to a decrease in prices, and vice-versa. Hence, assuming a slight response in the inventories, the own price elasticity for inventories for an agricultural good i is positive and inelastic.

3.2.5 Market for Each Agricultural Good as an Intermediate Input

As explained in the producers' section, the model considers that agricultural goods (e.g. soybeans) can be demanded as final but also as intermediate inputs to produce others (e.g. soybean meal or soybean oil). The model therefore specifies a separate market for each agricultural good i in region r, where is traded as an intermediate input. The market clearing condition in this market, written in *hat algebra*, is therefore given by the equality between the supply $\widehat{XDI}_{r,i}$ and demand $\widehat{XDI}_{r,i}$ of good i in country r when used as an intermediate input. Furthermore, the price that satisfies this condition is the price in equilibrium, expressed in *hat algebra*, for that good i in that market in country r, $\widehat{PI}_{r,i}$.

$$\widehat{XDI}_{r,i} = \widehat{qIN}_{r,i} \tag{49}$$

Based on intermediaries' optimal behavior, we know from expression (25) that the supply of each agricultural good i in region r as an intermediate input is given by:

$$\widehat{XDI}_{r,i} = \left(\frac{\widehat{PI}_{r,i}}{\widehat{PDS}_{r,i}} \right)^{\omega_i^C} \cdot \widehat{XDS}_{r,i} \tag{50}$$

We also know from expression (18) that the demand for each agricultural good i in country r as an intermediate good is given by:

$$\widehat{qIN}_{r,j} = \sum_i \left(\phi_{r,i,j}^{LC} \cdot \phi_{r,j}^{TTLC} \right) \cdot \widehat{qLC}_{r,i,j}^{IN} + \sum_i \left(\phi_{r,i,j}^{IM} \cdot \phi_{r,j}^{TTIM} \right) \cdot \widehat{qIM}_{r,i,j}^{IN} \tag{51}$$

4 Data

This section describes the data used to calibrate the model. The data combine publicly available international sources that capture the macroeconomic environment, agricultural production and consumption, and trade policies across regions. Together, they provide a coherent empirical foundation for the benchmark equilibrium and counterfactual analyses.

4.1 USDA Baseline Projections

The baseline projections come from the USDA Economic Research Service's *International Baseline Data* and *Agricultural Baseline Database*.[41] These data provide ten-year international agricultural projections for the supply, demand, and trade of the nine agricultural commodities included in our model. The original data cover 42 regions, which we aggregate into 19 individual regions plus the rest of the world (ROW). The baseline projections serve as the model's benchmark equilibrium, determining observed production, consumption, and trade patterns, as well as the targets used to calibrate structural parameters. We summarize the variables provided in the baseline projections in Table 1.

Table 1: Data from USDA ERS Agricultural Projections

Variable	Description	Source
$LN_{r,i}$	Producers' demand for land	International Baseline Data
$BS_{r,i}$	Initial stocks	International Baseline Data
$XSS_{r,i}$	Final stocks	International Baseline Data
$XA_{r,i}$	Total Consumption	International Baseline Data
$XM_{r,i}$	Import demand	International Baseline Data
$XExp_{r,i}$	Export supply	International Baseline Data
$Qprd_{r,i}$	Production	International Baseline Data
$LS_{r,i}$	Losses (Cotton only)	International Baseline Data
$y_{r,i}$	Yields	International Baseline Data
$VC_{US,i}$	Variable costs for the U.S.	Agricultural Baseline Database

4.2 International Price

In Section 3, the reference international price is used to clear the international market and determine the import price for each agricultural good. Data on international prices come from the IMF's *Primary Commodity Prices* dataset and FAO's *Food Price Monitoring and Analysis (FPMA) Tool*[42]. The raw data provide monthly prices for all agricultural goods in the model, beginning in January 1990. We take prices for non-livestock goods from the IMF dataset and prices for livestock goods from FAO FPMA. To obtain annual prices, we compute the simple average for 2022 based on the *commodity marketing years* listed in Table 2 for each good. The marketing year may differ from the calendar year for some goods. For example, the marketing year for Barley runs from June 1 to May 31, so we average the monthly prices from June 2022 to May 2023 to obtain the

[41] Both databases can be downloaded from USDA Agricultural Projections to 2032.

[42] See IMF Primary Commodity Prices and FAO FPMA.

2022 annual price for Barley. We also perform the necessary unit conversions to ensure consistency between international prices and the quantity data from the baseline projections.

Table 2: Commodity Marketing Years

Marketing Year	Commodities
Jan 1–Dec 31	Beef and veal; Pork; Poultry
Jun 1–May 31	Barley; Wheat
Aug 1–Jul 31	Cotton; Rice
Sep 1–Aug 31	Corn; Sorghum; Soybeans
Oct 1–Sep 30	Soybean Meal; Soybean Oil

Note: Marketing years follow USDA definitions for each good. See Commodity Marketing Years. The marketing year for Poultry is not provided, so we set it to be the same as that for Beef and Pork.

4.3 Income and Expenditure Share on Agricultural Goods

Income in each region I_r is represented by GDP per capita, calculated by dividing GDP (constant 2015 USD) by total population. Data on GDP and population come from the World Bank's *World Development Indicators* and are supplemented by the IMF's *World Economic Outlook*. We do not take GDP per capita directly from the raw data because we need to construct GDP per capita for the rest of the world (ROW).

In Appendix B, we calibrate the price elasticities of demand using parameters estimated in the literature and data on expenditure shares: $\mathcal{S}_r^{AG}$, $\mathcal{S}_{r,n,AG}$, and $\mathcal{S}_{r,i,n}$. To calculate total expenditure on agricultural goods $\mathcal{S}_r^{AG}$, we use data on Gross National Income (GNI) from the World Bank to represent total expenditure and compute the sum of consumption of baseline agricultural goods. Since consumption is already measured in quantities, we use international prices to calculate expenditure values. The expenditure shares $\mathcal{S}_{r,n,AG}$ and $\mathcal{S}_{r,i,n}$ are computed analogously at their corresponding levels.

4.4 Trade Policies

Trade policy variables are constructed using tariff and non-tariff measure (NTM) data from the World Bank's World Integrated Trade Solution (WITS) database and the World Bank's Non-Tariff Measures database[43]. Both tariff and non-tariff measures are expressed as *ad valorem* equivalents and are aggregated to the region–good level.

[43] See World Integrated Trade Solution and Ad Valorem Equivalent of Non-Tariff Measures.

4.4.1 Tariffs

Raw tariff data are reported at the 6-digit Harmonized System (HS) code level for each country in 2022 (or the closest available year) and are downloaded from the WITS database. We first construct a mapping, as listed in Table 3, from each HS6 product to the corresponding good i in our model, and then calculate simple average tariffs for each good i in each country. In addition, we compute weighted average tariffs at the region–good level, using countries' GDP as weights.

Table 3: Mapping Between Commodities and HS Codes

Commodity	HS codes
Beef and veal	0201, 0202, 020610, 020621, 020622, 020629, 021020
Poultry	0207
Pork	0203, 020630, 020641, 020649, 020910, 021011, 021012, 021019
Rice	1006
Wheat	1001
Corn	1005
Barley	1003
Sorghum	1007
Cotton	5201, 5202, 5203
Soybeans	1201
Soybean meal	2304
Soybean oil	1507

Note: HS codes reported at the 4-digit level indicate that all 6-digit HS products under that heading are included.

4.4.2 Non-tariff Measures

There is no direct quantitative measure of NTMs comparable to tariff rates; instead, we only observe the presence of specific types of NTMs, such as sanitary and phytosanitary measures or technical measures. Previous studies typically estimate an econometric model to identify the effects of tariffs and NTMs on import quantities and then compute the ad-valorem equivalents (AVE) of NTMs based on these estimates. The AVE represents the ad-valorem tariff that would generate the same proportionate reduction in quantity as the NTM. Using the estimation approach developed by Kee et al. (2009) and Kee and Nicita (2016), the World Bank provides estimates of the ad-valorem equivalents of NTMs for 40 importing countries plus the European Union at the HS 6-digit code level.

However, the raw NTM data provided by the World Bank contain many missing values, especially for the agricultural goods and regions we focus on. We impute the missing values using observed NTMs together with data on tariffs, GDP per capita, total agricultural imports, and fixed effects. Specifically, we first estimate the following econometric model:

$$ave_{Rci}^{NTB} = \beta_1 ave_{Rci}^{Tariff} + \beta_2 \overline{ave}_{Rc}^{AgTariff} + \beta_3 \log(GDPpc_{Rc}) + \beta_4 \log(AgImport_{Rc}) + \delta_{Ri} + \epsilon_{Rci}, \tag{52}$$

where ave_{Rci}^{NTB} denotes the estimated AVEs of NTMs for good i in country c in region R. Note that R refers to the geographic region rather than the 20 regions in our model defined earlier. ave_{Rci}^{Tariff} and $\overline{ave}Rc^{AgTariff}$ denote the good-level tariff and the average tariff across all agricultural goods, respectively. $\log(GDPpc_{Rc})$ is the log of GDP per capita in country c, and $\log(AgImport_{Rc})$ is the log of total agricultural imports in country c. δ_{Ri} denotes region–good fixed effects, and ϵ_{Rci} is the error term.

Once we estimate Equation (52) and obtain the coefficients and fixed effects, we impute the missing values using these estimates together with the observed data on tariffs, trade flows, and GDP per capita. The raw NTM data were constructed using information from 2012 to 2016. To maintain consistency, we use data from 2014 for all observed variables. Data on trade flows come from the CEPII BACI database. The final step is to compute the region–good (20 regions in our model) level AVEs of NTMs. We calculate weighted averages, using GDP as the weighting factor.

5 Computational Implementation

This section outlines the computational infrastructure of BSAT. The tool can be conceptually regarded as a stack of three solutions: a data specification, a computational representation with its solution algorithm, and a visualization interface. Each layer is described in a subsection below.

5.1 Data Specification

BSAT implements a data standard to ensure all parts of the tool communicate with a consistent format. In this subsection, we present the guiding principles of BSAT's data standard; details can be found in the tool's documentation.

All data is stored in comma-separated values format (CSV) files, and all data files are embedded in a single ZIP package. Files are organized into four main categories: (i) *targets*, which are the baseline projections data that need to be matched; (ii) *exogenous*, such as macroeconomic and tariff data gathered from sources other than the baseline projections; (iii) *parameters* fully describing the model, such as elasticities of substitution and technical transformation; and (iv) *shocks*, which include a list of variables that can be changed from their neutral, non-shocked values for scenario simulation, such as total factor productivity, land supply, and tariffs.

The ZIP package also includes a metadata file describing the contents of the package, such as a brief explanation of the scenario ran, the version of the baseline data, and a timestamp of the scenario run. All data CSV files are organized in long format with five columns identifying dimensions: (i) *region*: a descriptor using ISO 3-letter codes whenever possible; (ii) *year*: if the original data refers to a season across two years, this field reports the earliest year; (iii) *nest*: for data referring to specific commodities within a nest; (iv) *commodity*: a unique identifier for each commodity; and (v) *input*: for when the data refers to an input (such as land or non-land).

When a dimension does not apply, the content of the cell will be blank. For example, if a row refers to a time-varying parameter relative to a single country (such as GDP per capita), only the country and year columns will have information, and the other three will be left blank.

Two columns complete the long format data: *attribute* (i.e., the variable name) and *value* (a numeric data point). In addition to identifiers and numeric values, data files also include metadata columns with descriptors of the variables and units of measurement and long names for regions, nests, and commodities.

5.2 Computational Representation and Solution

The model described in section 3 defines a system of nonlinear equations that determines market equilibrium. Given that the model admits only positive quantities observed in the data, the system is exactly identified, and the functional specifications are well-behaved (monotonic and twice continuously differentiable), the system yields a unique interior solution. Leveraging this characteristic, we adopt a numerical approach based on the Interior Point Optimization (Wächter and Biegler, 2006) solver (IPOpt) implemented in JuMP, a modeling framework embedded in the Julia programming language (Lubin et al., 2023).[44]

Interior point optimization reformulates the constrained equilibrium problem as a sequence of barrier problems which can be solved using Newton-based methods. The original problem consists of finding prices and quantities that satisfy the market equilibrium conditions (equality constraints)

[44]This approach is similar to Ivanic (2025), which translates the GTAP model version 7 (Corong et al., 2017) JuMP and solves it using IPOpt.

while respecting non-negativity restrictions (inequality constraints). Formally, the problem can be stated as:

$$\begin{aligned} \min_{x} \quad & 0 \\ \text{s.t.} \quad & c(x) = 0 \\ & x \geq 0 \end{aligned}$$

where x represents the vector of endogenous variables (prices and quantities), $c(x)$ represents the system of nonlinear equations defining market equilibrium, and the objective function is set to a constant (typically zero) since we seek any feasible solution rather than optimizing a particular criterion. The inequality constraints ensure economically meaningful solutions with non-negative prices and quantities.

The interior point method transforms this constrained problem into an unconstrained barrier problem by incorporating the inequality constraints into a modified objective function through logarithmic barrier terms. The barrier formulation becomes:

$$\begin{aligned} \min_{x} \quad & -\mu \sum_{i} \log(x_i) \\ \text{s.t.} \quad & c(x) = 0 \end{aligned}$$

where $\mu > 0$ is the barrier parameter. The logarithmic barrier function penalizes solutions approaching the boundary (where any $x_i \to 0$), effectively keeping iterates strictly in the interior of the feasible region. As μ is progressively reduced toward zero across iterations, the barrier problem's solution converges to the solution of the original problem. The IPOpt solver employs a primal-dual framework combined with a filter line-search method to solve this sequence of barrier problems efficiently, updating the barrier parameter adaptively based on progress toward optimality and feasibility. This approach proves particularly effective for large-scale problems with sparse Jacobian structures, as is typical in CGE models with many commodities and regions.

A modeling solution based on IPOpt and JuMP has several practical and computational advantages. One of the main advantages of this approach is its accessibility: all the software infrastructure required to run and modify the model is open-source and available at no cost. This contrasts with commercial solvers and modeling interfaces typically used in large-scale trade models, such as GAMS and GEMPACK. JuMP code is also easy to read, since its syntax is modular and supports mathematical abstractions, allowing users to define functions directly from mathematical expressions. Despite its high-level syntax, JuMP delivers performance comparable to low-level languages (such as C++), owing to Julia's language features such as just-in-time compilation and strong typing (Bezanson et al., 2018). Moreover, because JuMP is embedded in a general-purpose programming language, it is easy to integrate sophisticated data processing for model inputs and outputs, including visualization frameworks.

This approach, however, has two main disadvantages. First, it lacks some advanced debugging and warning features available in commercial solutions to prevent errors (e.g., automatic checks of model conditions and fallback parameter settings). Second, although it can easily interface with plotting packages, results from JuMP models lack a default mechanism for visualization and require the user to implement visualization outside the program. BSAT mitigates the drawbacks from this approach by (i) setting default solver parameters and specifying a set of possible shocks to minimize the possibility of errors, and (ii) providing an easy-to-use interface for visualizing model inputs and outputs.

5.3 Visualization Interface

In addition to modeling and simulation capabilities, BSAT also includes a standalone, browser-based dashboard to visualize the results of a model run contained in a zipped data package (described above). This dashboard allows the user to examine the results of a single run and to load a second set of results for scenario comparisons.

The visualization dashboard comprises seven blocks of information: one for metadata, four for graphical visualizations, and two for tabular data display. The metadata section displays user-input information, such as the author, a description of the simulation, and comments; it also shows timestamps for model calibration and model runs. Each of the four graphical blocks allows the user to select different data field combinations. Figures 4–11 reporting the illustrative cases in the next section are directly captured from these blocks. These blocks comprise (i) a time series plot to visualize a selected variable over time for a choice of country and commodity (Figs. 8–10); (ii) a bar plot of percent changes to visualize a selected variable over regions for a choice of commodity and year (Figs. 4 and 11); (iii) a heat map of percent changes to visualize a selected variable over pairs of region and commodity for a choice of year (Fig. 7); and (iv) a horizontal bar plot to visualized the biggest percent changes for a choice of variable, commodity, and year (Figs. 5 and 6). Finally, the two tabular blocks allow the user to inspect the data tables of the baseline projections and the shocks of each scenario; these blocks also include the option to filter and sort rows by column.

This dashboard is implemented as a Python-based web app using open-source frameworks. It can be executed locally on Windows, Linux, or Mac machines or ported to hosting services such as Hugging Face. Implementation details and installation instructions are included in BSAT's documentation.

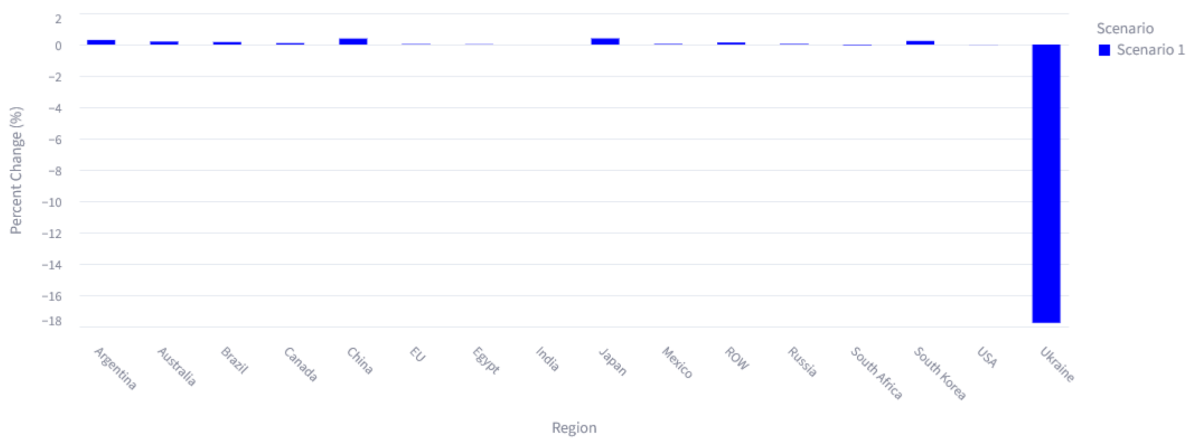
6 Illustration Cases

As explained above, this model is developed to evaluate the response of the USDA Baseline Agricultural Projections to temporal or permanent shocks. Specifically, the model is designed to simulate the effect of changes in trade policy, agricultural productivity, land yields, and land availability either in the US or in any country for which the USDA releases projections. In this section, we present four illustrative cases to demonstrate the model's capabilities. Rather than focusing on the results of each shock, we use these cases to illustrate how the model simulates shocks that differ in nature, magnitude and duration.

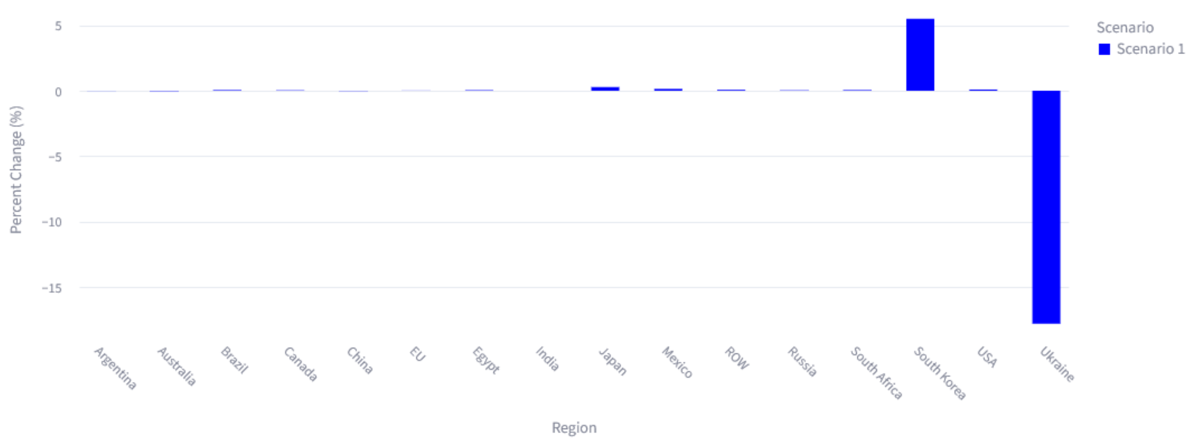
6.1 Case 1: Land Availability

In the first case, we simulate the response of the USDA baseline projections to a 20 percent reduction in Ukraine's land availability during the year 2023/24. With the intensification of the Ukraine-Russia war, the interest in quantifying the response of global agriculture to the reduction of land supply in Ukraine captured the attention of policymakers around the world. Ukraine is among the main suppliers of barley in the world, and barley accounts 9.4 percent of Ukraine's cultivated area. Consequently, many concerns surged about the potential impact not only on the barley market but also on the global market of other grains, such as wheat. We therefore take it as an illustrative case to demonstrate how the model predicts the consequences of a temporal shock affecting the availability of the factors of production.

To simulate the effect of a 20 percent reduction in Ukraine's land availability during the year 2023/24 on the USDA projections, we set the land availability parameter η_r in equation (44) equal to -0.2 for Ukraine in 2023/24. We then execute the model, obtaining the following results.

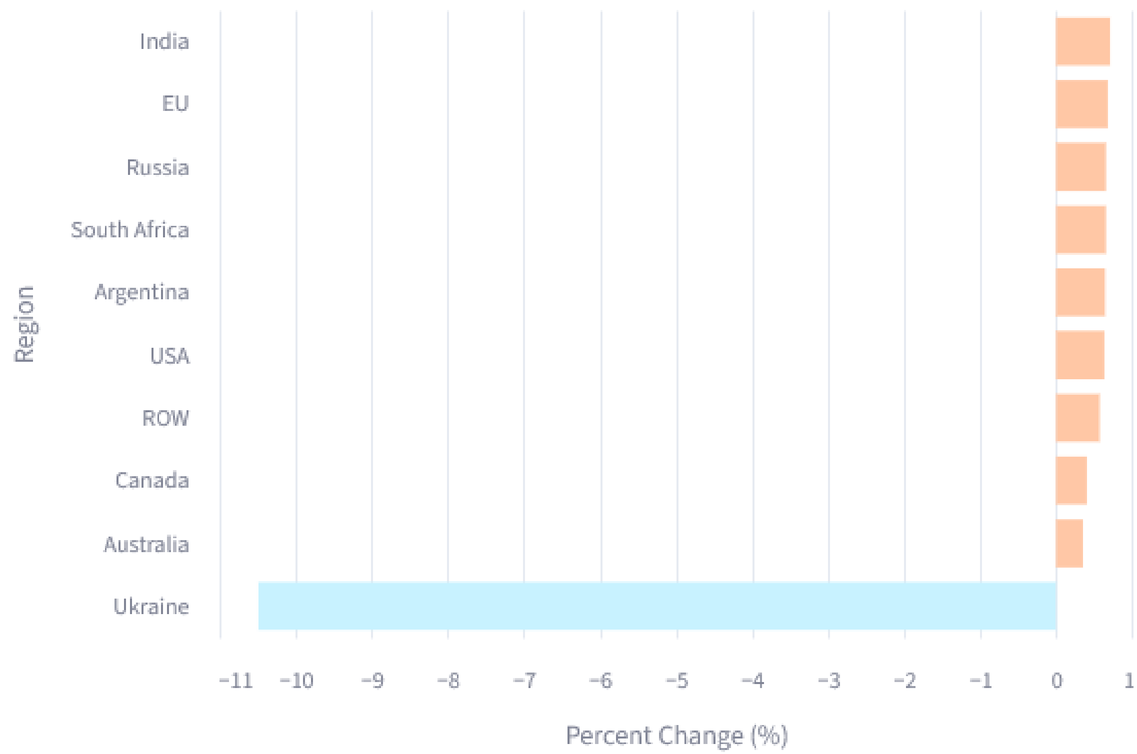


(a) Barley

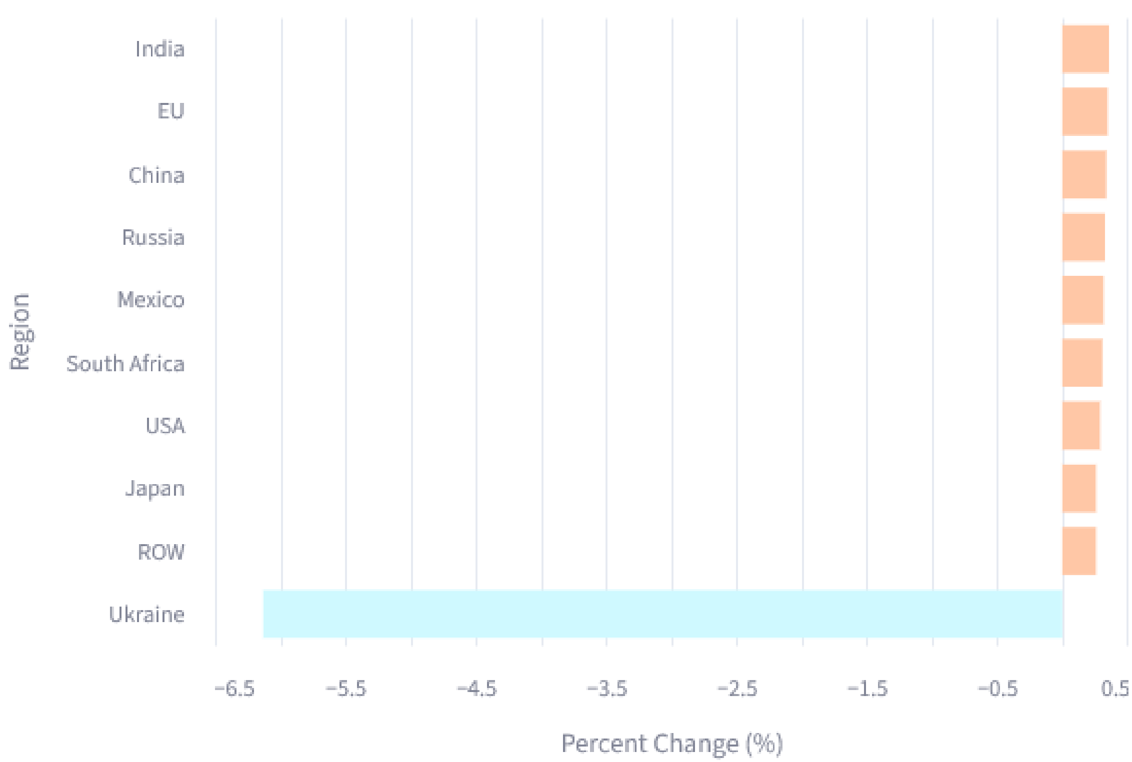


(b) Wheat

Figure 4: % Change in Countries Harvested Area in 2023 in Response to Ukraine's Reduction in Land Availability



(a) Barley



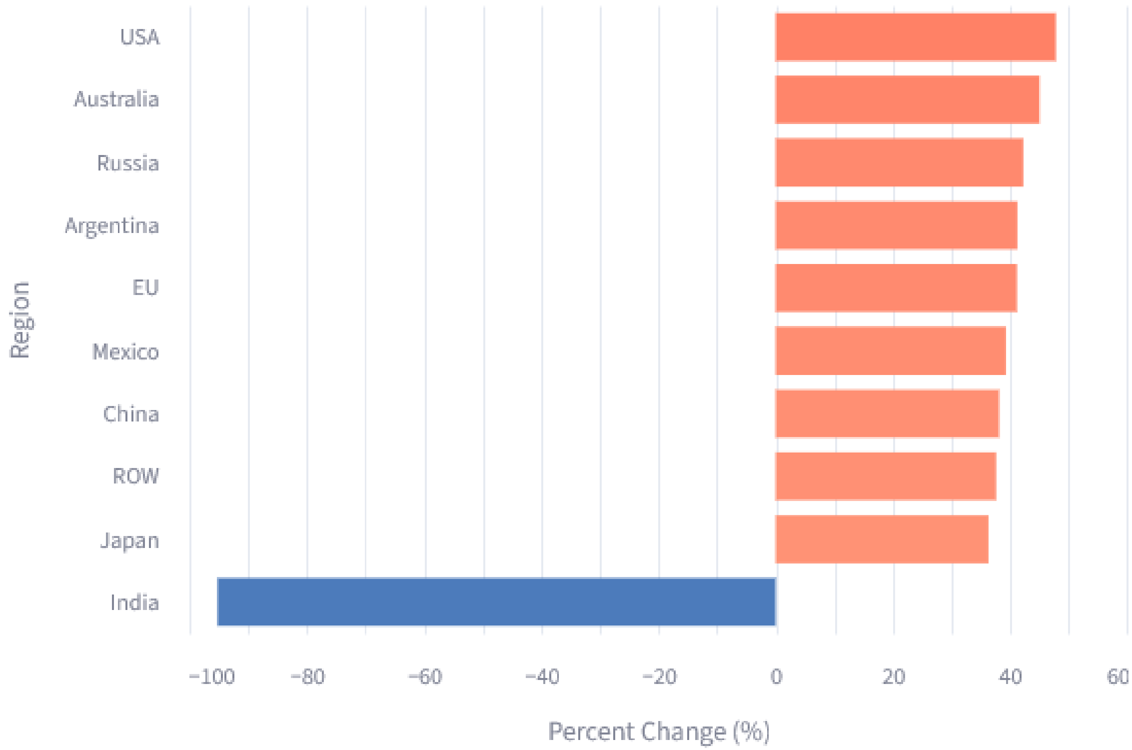
(b) Wheat

Figure 5: % Change in Exports in 2023: Top 10 Countries with the Largest Response to Ukraine's Reduction in Land Availability

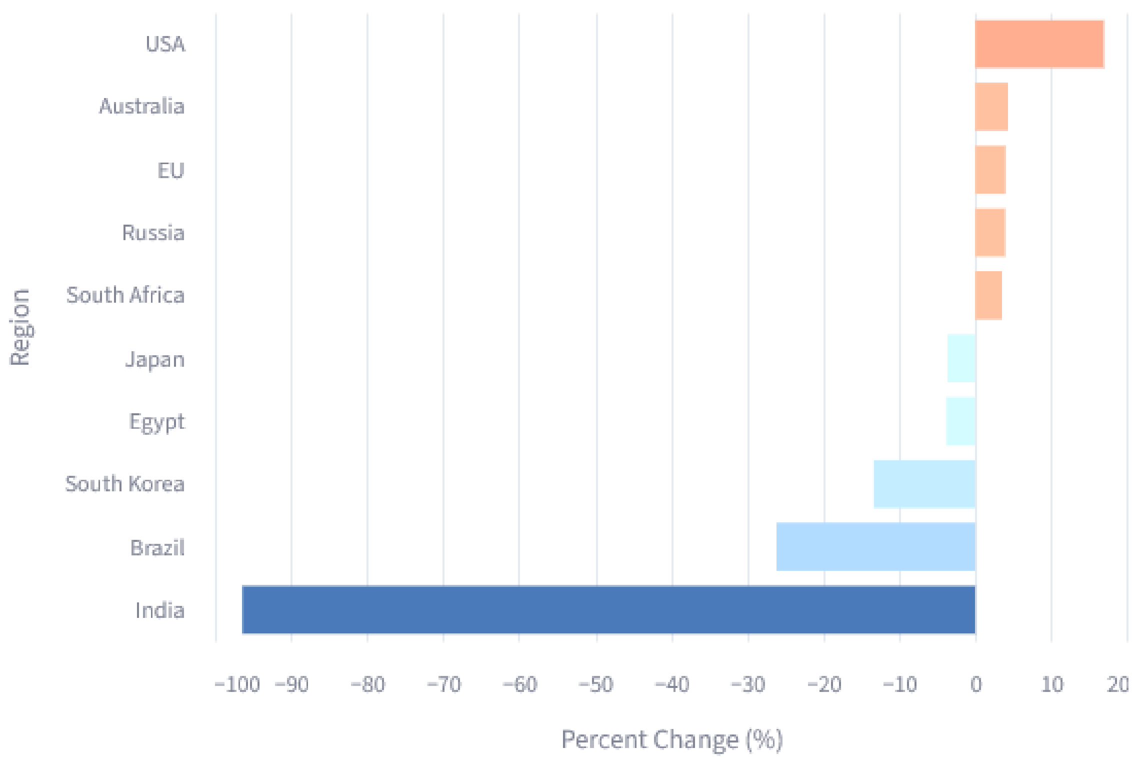
6.2 Case 2: Export Tax

In this second case, we simulate the response of the USDA baseline projections to a 50 percentage point increase in export taxes for rice and wheat in India during the year 2023/2024. For decades, India maintained high trade barriers on agricultural products as a strategy to safeguard their domestic food security. Although most of these restrictions have been released in recent decades, export taxes are frequently used tool to limit price increases in domestic markets. For example, India implemented taxes on rice exports in 2023 and 2025 as a strategy to control domestic prices. Hence, as another illustrative case, we assess how the model simulates the effect of these export taxes on global markets. This is a good case for visualizing how the model predicts the impact of a policy implemented to pursue national security interests on international markets.

To simulate the effect of a 50 percent-points increase in export taxes for rice and wheat in India during the year 2023/2024, we set the export tax parameter $\tau^X_{r,i}$ equal to $\overline{\tau}^X_{r,i} + 0.5$ for rice and wheat in India during 2023/24. We then execute the model, obtaining the following results



(a) Rice



(b) Wheat

Figure 6: % Change in Exports in 2023: Top 10 Countries with the Largest Response to an Export Tax in India

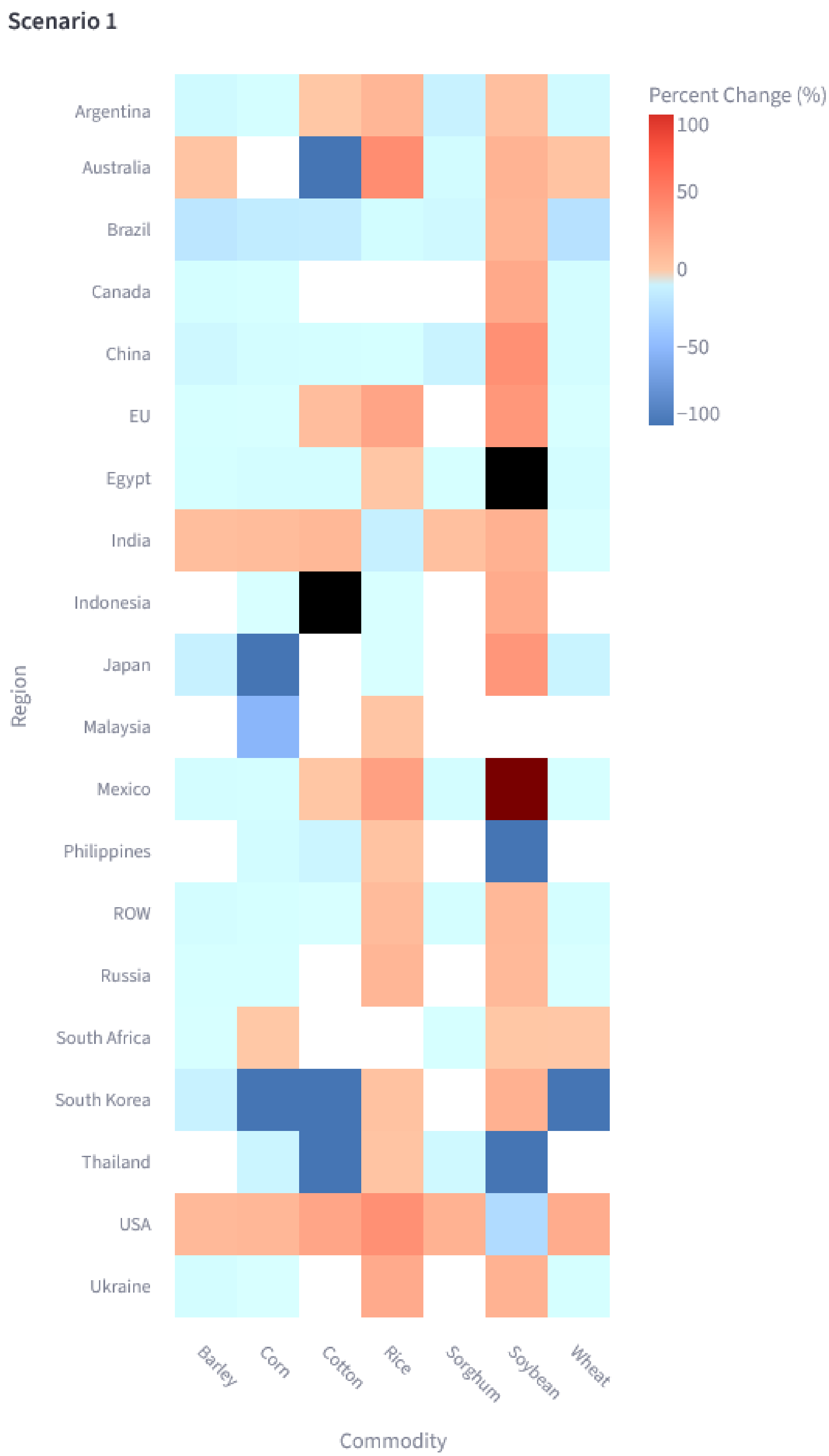


Figure 7: % Change in Countries' Harvested Area in 2023 in Response to an Export Tariff in India

6.3 Case 3: TFP improvement

As a third case, we simulate the response of the USDA baseline projections to two simultaneous shocks to the TFP in China's soybean sector. The first shock (denoted by Shock 1 in the figure) is a steady annual increase of one percentage point in TFP, and the second (denoted by Shock 2) is a steady annual increase of 0.5 percentage points. The development and subsequent adaptation of Genetically Modified (GM) soybean varieties have led to higher yields per hectare and lower costs of production. Although the debate has focused on the consumption of GM agricultural products, a significant enhancement in TFP to produce soybeans is evident. We therefore use this case to illustrate how the model predicts the effects of a permanent technological shock in production, in this case, in the production of soybeans in China. Additionally, we illustrate how the predictions change when the magnitude of the shock is different. In this case, the magnitude of Shock 1 is twice the magnitude of Shock 2.

To simulate the impact of a steady annual increase to the TFP in China's soybean sector, Shock 1 required setting the TFP parameter $\Omega_{r,i}^{VA}$ equal to 1.01 in 2023, 1.02 in 2023, until 1.10 in 2032 for soybeans in China and executing the model. Shock 2 required setting $\Omega_{r,i}^{VA}$ equal to 1.005 in 2023, 1.001 in 2023, until 1.05 in 2032, and executing the model. The results we obtain, plotted in the same figures, are given as follows:

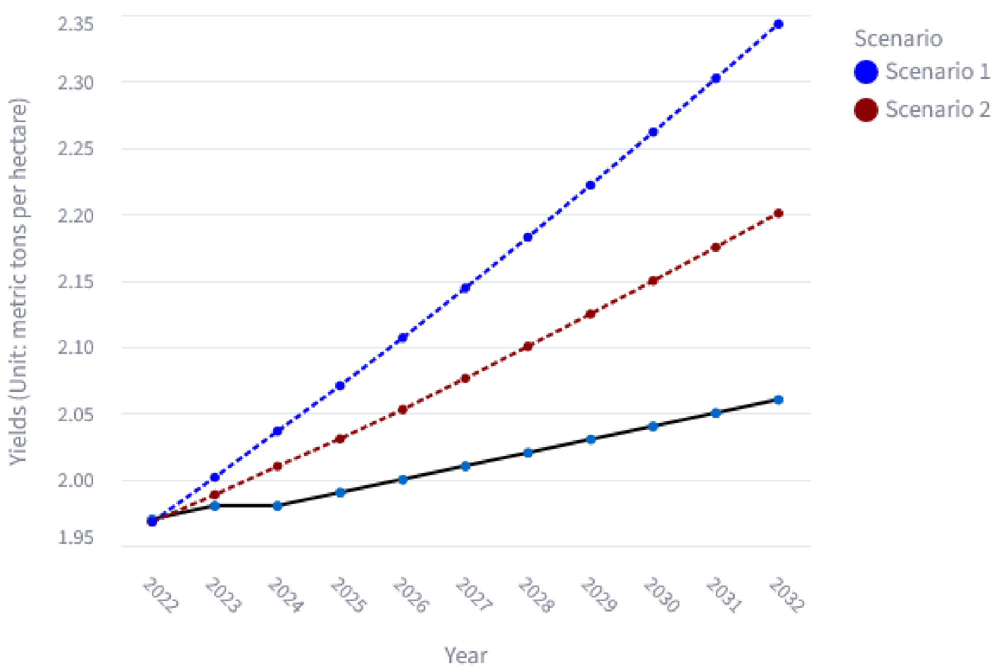
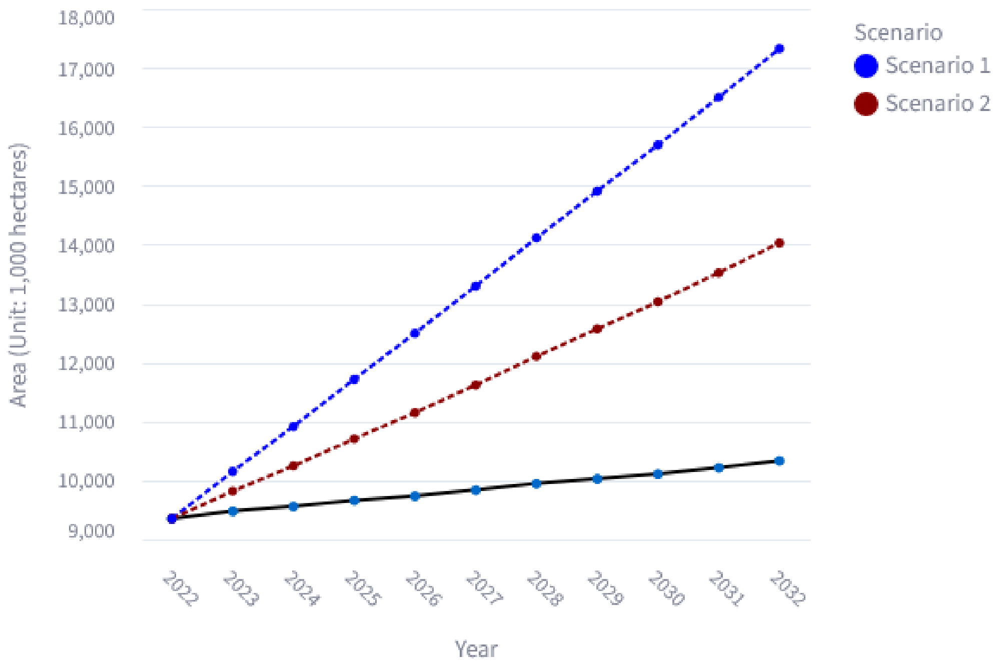
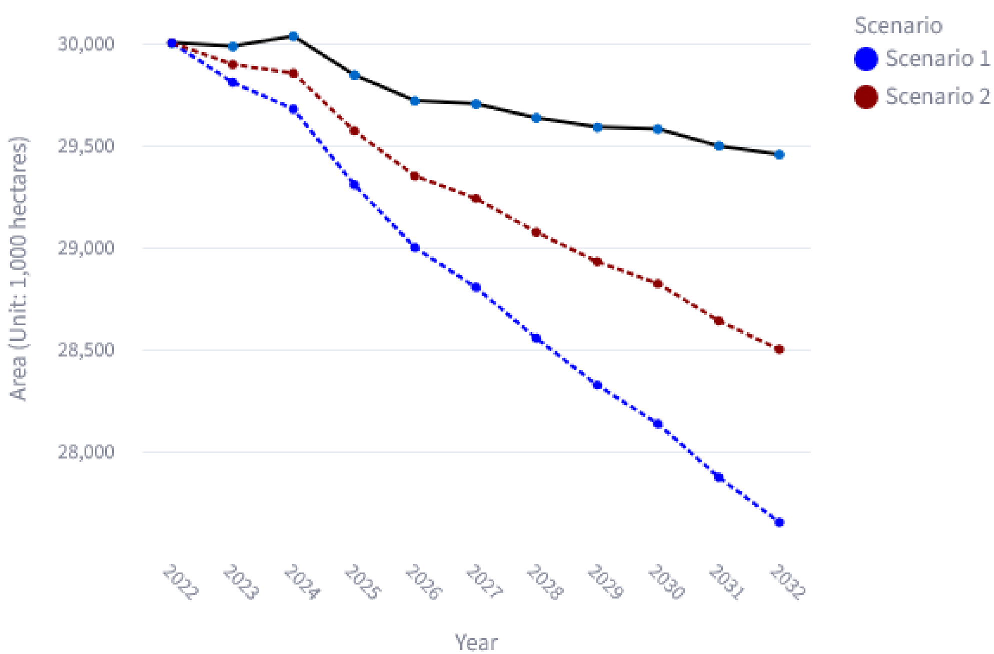


Figure 8: Soybean Yield Projections for China, 2023–2032: Baseline vs. TFP Shocks

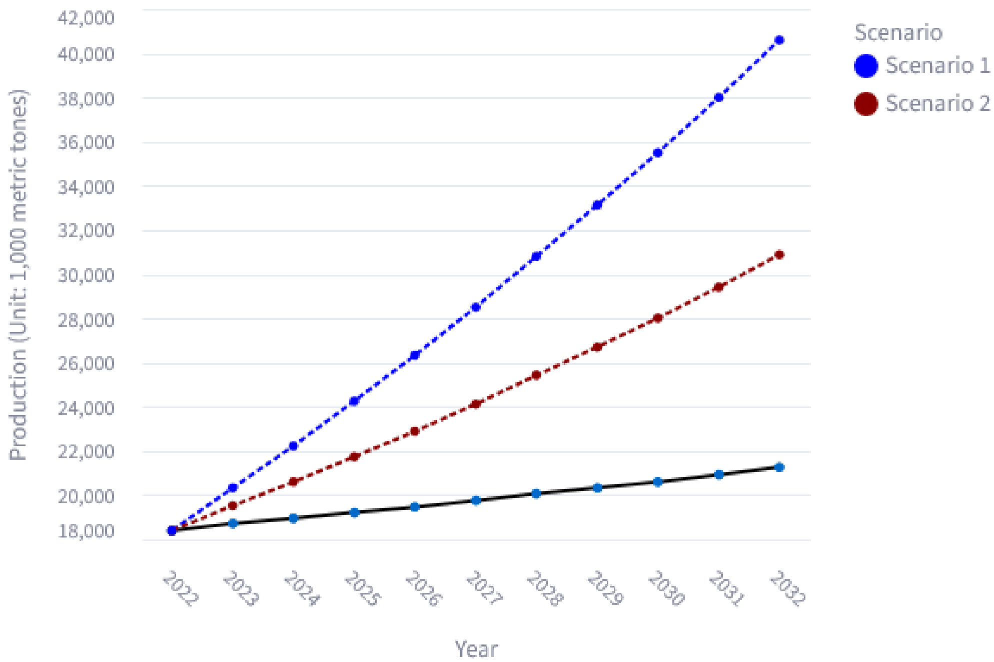


(a) Soybeans

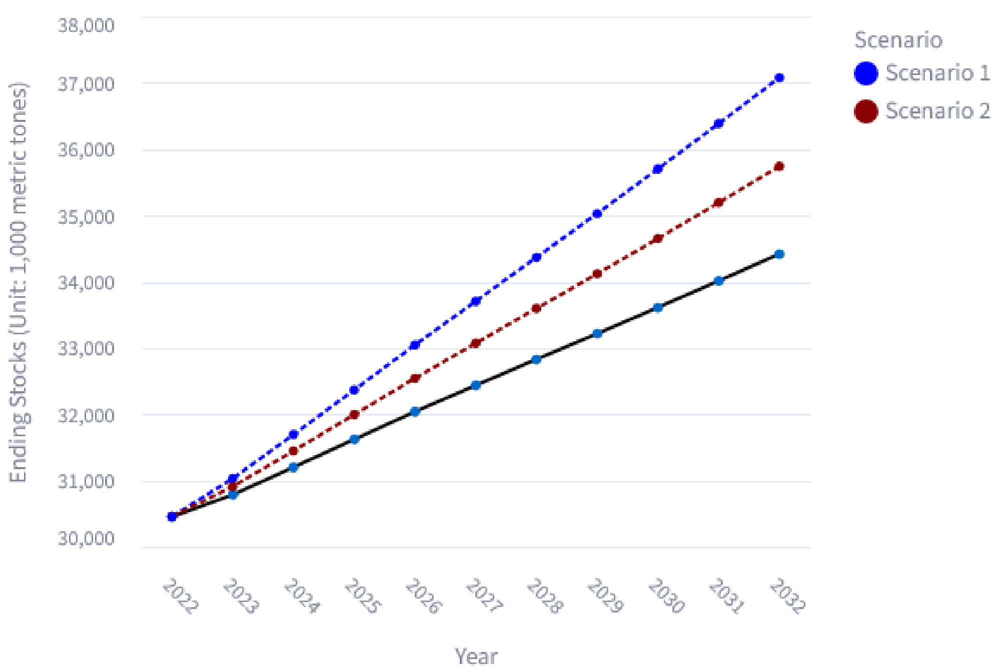


(b) Rice

Figure 9: Harvested Area (1,000 Hectares) Projections for China, 2023–2032: Baseline vs. TFP Shocks



(a) Production



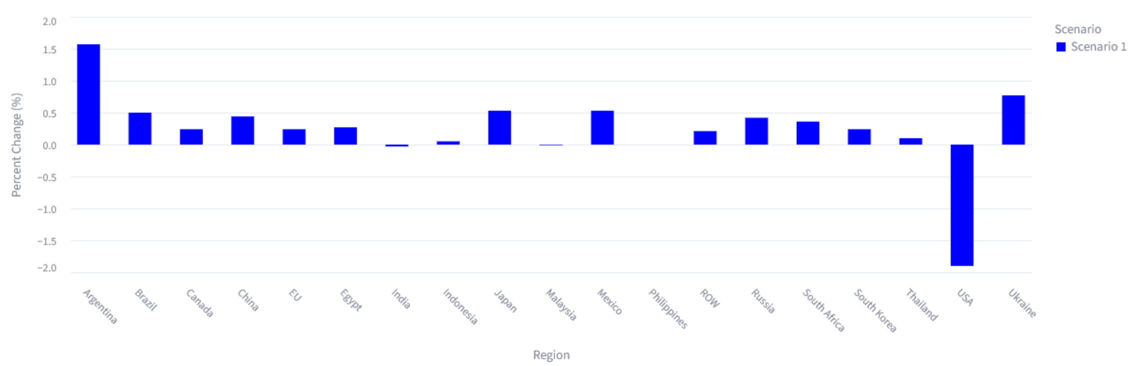
(b) Ending Stocks

Figure 10: Soybeans Projections for China, 2023–2032: Baseline vs. TFP Shocks

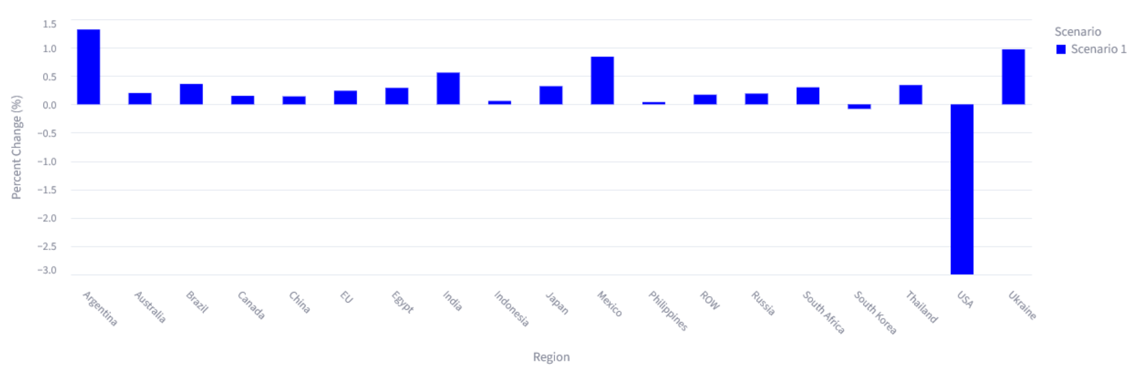
6.4 Case 4: Land Productivity Decline

As a final case, we simulate the response of the USDA baseline projections to a 10 percent decline in land productivity for corn and soybean production in the U.S during the year 2023/24. In recent decades, the substantial increase in greenhouse gas concentrations in the atmosphere has led to an intensification of extreme weather conditions worldwide. Consequently, land productivity for agricultural production is believed to have declined. We therefore examine, in this last illustrative case, how a decline in land productivity for corn and soybeans in the U.S. affects the yields per harvested hectare worldwide.

To simulate the effect of a 10 percent decline in land productivity for corn and soybean production in the U.S. during the year 2023/24 on the USDA projections, we set the land productivity parameter $\Theta_{r,i}$ in equation (10) equal to 0.9 for corn and soybeans in the U.S. during 2023/24. We then execute the model and examine the change in yields per hectare of corn and soybeans worldwide, obtaining the following results.



(a) Corn



(b) Soybeans

Figure 11: % Change in Countries Yields per Harvested Hectare in 2023 in Response to a Decline in Land Productivity for Producing Corn and Soybeans in the U.S.

7 Conclusion

The USDA's baseline agricultural projections integrate a wide body of expertise within the U.S. Department of Agriculture. The projections set expectations for production, trade flows, storage and other relevant commodity market variables for a large set of agricultural commodities and national markets. The projections play an influential role in domestic U.S. policymaking and provide market and other participants with a user-friendly guide to expert views on the medium-term direction of agricultural markets. While important for these purposes, the USDA has difficulty updating the projections in real time, and it is nearly impossible for outside users to explore alternative scenarios that use the baseline projections as a benchmark.

In this paper we develop an integrated partial equilibrium model that represents the baseline projection data as a single (sequential) partial equilibrium. Using response parameters from the literature, we calibrate the model to the baseline predictions data, incorporating policy and other levels that can be used to shock the calibrated model. The model is coded in publicly available software, and our code will be publicly available to allow USDA and other users to conduct analyses of scenarios that consider shocks to a benchmark defined by baseline projections. Such scenarios can be used to update the projections quickly, or to consider the sensitivity of outcomes to particular policy chains. We create a number of data visualization tools that illustrate percentage changes and the new equilibrium levels of variables that are associated with each shock provided to the model.

Our main contribution is to provide a fully-consistent partial equilibrium model of global production and trade in agricultural commodities. We also provide open-source code and visualizations that allow users to consider model outcomes for alternative scenarios quickly. Important limitations of the model are tied to the characteristics of the data in the projections. The model only considers agricultural commodities; it lacks links to both upstream and downstream sectors (since the USDA provides no comparable baseline for these sectors). The projections offer limited data on the consumption of feed by some forms of livestock, so it is difficult to completely represent feed markets without comparable data on livestock for which the data are not included. Our model attributes the demand for commodities to an optimizing consumer, an approach that generates a useful homogeneity property, but considerably understates the complexity of the demand for agricultural commodities. Finally, the projections do not include data on bilateral trade flows, and so our model is not able to represent discriminatory trade policies or other shocks that change bilateral trade patterns.

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8 Appendixes

A Detailed Mathematical Solution of the Model

A.1 Consumers

$$\max_{\substack{X_{r,AG},\ X_{r,NA},\ XN_{r,n},\\ X_{r,n,i},\ XD_{r,n,i},\ XM_{r,n,i}}} U_r = \left[\beta_{r,NA}^{\frac{1}{\sigma^T}} X_{r,NA}^{\frac{\sigma^T-1}{\sigma^T}} + \beta_{r,AG}^{\frac{1}{\sigma^T}} X_{r,AG}^{\frac{\sigma^T-1}{\sigma^T}}\right]^{\frac{\sigma^T}{\sigma^T-1}} \tag{A.1.1}$$

subject to

$$I_r = P_{r,NA} \cdot X_{r,NA} + P_{r,AG} \cdot X_{r,AG}$$

which could also be written as

$$\begin{aligned} I_r &= P_{r,NA} \cdot X_{r,NA} + \sum_n PN_{r,n} \cdot XN_{r,n} \\ &= P_{r,NA} \cdot X_{r,NA} + \sum_n \sum_{i \in n} PA_{r,n,i} \cdot XA_{r,n,i} \\ &= P_{r,NA} \cdot X_{r,NA} + \sum_n \sum_{i \in n} (XD_{r,n,i} \cdot PD_{r,n,i} + XM_{r,n,i} \cdot PM_{r,n,i}) \end{aligned}$$

where

$$\begin{aligned} X_{r,AG} &= \left[\sum_n \beta_{r,n}^{\frac{1}{\sigma^U}} \cdot XN_{r,n}^{\frac{\sigma^U-1}{\sigma^U}}\right]^{\frac{\sigma^U}{\sigma^U-1}} \\ XN_{r,n} &= \left[\sum_{i \in n} \beta_{n,i}^{\frac{1}{\sigma_n^N}} \cdot XA_{r,n,i}^{\frac{\sigma_n^N-1}{\sigma_n^N}}\right]^{\frac{\sigma_n^N}{\sigma_n^N-1}} \\ XA_{r,n,i} &= \left[(\beta_{r,n,i}^D)^{\frac{1}{\sigma_i^D}} \cdot XD_{r,n,i}^{\frac{\sigma_i^D-1}{\sigma_i^D}} + (\beta_{r,n,i}^M)^{\frac{1}{\sigma_i^D}} \cdot XM_{r,n,i}^{\frac{\sigma_i^D-1}{\sigma_i^D}}\right]^{\frac{\sigma_i^D}{\sigma_i^D-1}}. \end{aligned}$$

Now, using the Lagrangian multiplier method, the consumers' utility maximization problem can be written as:

$$\mathcal{L} = U_r - \lambda\left[(P_{r,NA} \cdot X_{r,NA} + P_{r,AG} \cdot X_{r,AG}) - I_r\right] \tag{A.1.2}$$

such that the First-Order conditions (F.O.C.) with respect to every choice variable are given by:

$$\frac{\partial \mathcal{L}}{\partial X_{r,NA}} = A \cdot \beta_{r,NA}^{\frac{1}{\sigma^T}} X_{r,NA}^{-\frac{1}{\sigma^T}} - \lambda P_{r,NA} = 0 \tag{A.1.3a}$$

$$\frac{\partial \mathcal{L}}{\partial X_{r,AG}} = A \cdot \beta_{r,AG}^{\frac{1}{\sigma^T}} X_{r,AG}^{-\frac{1}{\sigma^T}} - \lambda P_{r,AG} = 0 \tag{A.1.3b}$$

$$\frac{\partial \mathcal{L}}{\partial XN_{r,n'}} = B \cdot \beta_{r,n'}^{\frac{1}{\sigma^U}} XN_{r,n'}^{-\frac{1}{\sigma^U}} - \lambda PN_{r,n'} = 0 \tag{A.1.3c}$$

$$\frac{\partial \mathcal{L}}{\partial XN_{r,n''}} = B \cdot \beta_{r,n''}^{\frac{1}{\sigma^U}} XN_{r,n''}^{-\frac{1}{\sigma^U}} - \lambda PN_{r,n''} = 0 \tag{A.1.3d}$$

$$\frac{\partial \mathcal{L}}{\partial XA_{r,n,i'}} = C \cdot \beta_{n,i'}^{\frac{1}{\sigma^N_n}} XA_{r,n,i'}^{-\frac{1}{\sigma^N_n}} - \lambda PA_{r,n,i'} = 0 \tag{A.1.3e}$$

$$\frac{\partial \mathcal{L}}{\partial XA_{r,n,i''}} = C \cdot \beta_{n,i''}^{\frac{1}{\sigma^N_n}} XA_{r,n,i''}^{-\frac{1}{\sigma^N_n}} - \lambda PA_{r,n,i''} = 0 \tag{A.1.3f}$$

$$\frac{\partial \mathcal{L}}{\partial XD_{r,n,i}} = D \cdot (\beta_{r,n,i}^{D})^{\frac{1}{\sigma^D_i}} XD_{r,n,i}^{-\frac{1}{\sigma^D_i}} - \lambda PD_{r,n,i} = 0 \tag{A.1.3g}$$

$$\frac{\partial \mathcal{L}}{\partial XM_{r,n,i}} = D \cdot (\beta_{r,n,i}^{M})^{\frac{1}{\sigma^D_i}} XM_{r,n,i}^{-\frac{1}{\sigma^D_i}} - \lambda PM_{r,n,i} = 0 \tag{A.1.3h}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = (P_{r,NA} \cdot X_{r,NA} + P_{r,AG} \cdot X_{r,AG}) - I_r = 0 \tag{A.1.3i}$$

where A equals to $[U]^{\frac{1}{\sigma^T - 1}}$, B to $A \cdot \beta_{r,AG}^{\frac{1}{\sigma^T}} [X_{r,AG}]^{\left(\frac{\sigma^N_n - 1}{\sigma^N_n}\right) - \left(\frac{\sigma^U - 1}{\sigma^U}\right)}$, C to $B \cdot \beta_{r,n}^{\frac{1}{\sigma^U}} [XN_{r,n}]^{\left(\frac{\sigma^U - 1}{\sigma^U}\right) - \left(\frac{\sigma^N_n - 1}{\sigma^N_n}\right)}$, and D to $C \cdot \beta_{n,i}^{\frac{1}{\sigma^N_n}} [X_{r,n,i}]^{\left(\frac{\sigma^N_n - 1}{\sigma^N_n}\right) - \left(\frac{\sigma^D_i - 1}{\sigma^D_i}\right)}$.

Then, in order to solve this consumers' maximization problem, we begin by solving their optimal choice in the lower-level of their nested CES utility function: the optimal local and imported demand from each agricultural good i in region r. To do it, we take the ratio of expression (A.1.3g) to (A.1.3h). We also omit the subindex n for all variables and parameters defined at the combination r, n, i. This is redundant because we assume that every good i belongs to a unique nest n.

$$\left(\frac{\beta^D_{r,i}}{\beta^M_{r,i}}\right)^{\frac{1}{\sigma^D_i}} \left(\frac{XD_{r,i}}{XM_{r,i}}\right)^{-\frac{1}{\sigma^D_i}} = \frac{PD_{r,i}}{PM_{r,i}}$$

$$\frac{PD_{r,i}}{PM_{r,i}} \frac{XD_{r,i}}{XM_{r,i}} = \left(\frac{PD_{r,i}}{PM_{r,i}}\right)^{1-\sigma^D_i} \left(\frac{\beta^D_{r,i}}{\beta^M_{r,i}}\right)$$

$\Rightarrow$

$$\beta^M_{r,i} (PM_{r,i})^{1-\sigma^D_i} = \frac{PM_{r,i}}{PD_{r,i}} \frac{XM_{r,i}}{XD_{r,i}} \beta^D_{r,i} (PD_{r,i})^{1-\sigma^D_i}$$

$$\beta^D_{r,i} (PD_{r,i})^{1-\sigma^D_i} + \beta^M_{r,i} (PM_{r,i})^{1-\sigma^D_i} = \beta^D_{r,i} (PD_{r,i})^{1-\sigma^D_i} \left(\frac{PM_{r,i}}{PD_{r,i}} \frac{XM_{r,i}}{XD_{r,i}} + 1\right), \tag{A.1.4}$$

where denoting price index $PA_{r,i} = \left[\beta^D_{r,i} (PD_{r,i})^{1-\sigma^D_i} + \beta^M_{r,i} (PM_{r,i})^{1-\sigma^D_i}\right]^{\frac{1}{1-\sigma^D_i}}$ and rearranging terms yields:

$$
\begin{aligned}
XD_{r,i} &= \beta^D_{r,i}\left(\frac{PD_{r,i}}{PA_{r,i}}\right)^{1-\sigma_i^D}\cdot\left(\frac{PD_{r,i}XD_{r,i}+PM_{r,i}XM_{r,i}}{PD_{r,i}}\right)\\
&= \beta^D_{r,i}\left(\frac{PD_{r,i}}{PA_{r,i}}\right)^{1-\sigma_i^D}\cdot\frac{PA_{r,i}XA_{r,i}}{PD_{r,i}}\\
&= \beta^D_{r,i}\left(\frac{PD_{r,i}}{PA_{r,i}}\right)^{-\sigma_i^D}\cdot XA_{r,i}, \qquad \text{(A.1.5)}
\end{aligned}
$$

and by symmetry

$$
XM_{r,i} = \beta^M_{r,i}\left(\frac{PM_{r,i}}{PA_{r,i}}\right)^{-\sigma_i^D}\cdot XA_{r,i}. \qquad \text{(A.1.6)}
$$

Expressions (A.1.5) and (A.1.6) then denote the optimal solution of $XD_{r,i}$ and $XM_{r,i}$ in levels. We write these expressions in *hat algebra*, by taking the ratio of each optimal solution to its benchmark level. Specifically, we express any variable Z in *hat algebra* as: $\frac{Z}{\overline{Z}}$. Therefore, we write $\widehat{XD}_{r,i}$ and $\widehat{XM}_{r,i}$ as follows

$$
\widehat{XD}_{r,i} = \left(\frac{\widehat{PD}_{r,i}}{\widehat{PA}_{r,i}}\right)^{-\sigma_i^D}\cdot\widehat{XA}_{r,i} \qquad \text{(A.1.7)}
$$

$$
\widehat{XM}_{r,i} = \left(\frac{\widehat{PM}_{r,i}}{\widehat{PA}_{r,i}}\right)^{-\sigma_i^D}\cdot\widehat{XA}_{r,i}, \qquad \text{(A.1.8)}
$$

where we solve for the price index $\widehat{PA}_{r,i}$ as follows:

$$
\begin{aligned}
\widehat{PA}_{r,i} &= \frac{PA_{r,i}}{\overline{PA}_{r,i}}\\
&= \frac{\left(\beta^D_{r,i}\left(PD_{r,i}\right)^{1-\sigma_i^D}+\beta^M_{r,i}\left(PM_{r,i}\right)^{1-\sigma_i^D}\right)^{\frac{1}{1-\sigma_i^D}}}{\left(\beta^D_{r,i}\left(\overline{PD}_{r,i}\right)^{1-\sigma_i^D}+\beta^M_{r,i}\left(\overline{PM}_{r,i}\right)^{1-\sigma_i^D}\right)^{\frac{1}{1-\sigma_i^D}}}.
\end{aligned}
$$

Now, rearranging terms yields

$$
\begin{aligned}
\widehat{PA}_{r,i}^{1-\sigma_i^D} &= \frac{\beta^D_{r,i}\left(PD_{r,i}\right)^{1-\sigma_i^D}}{\beta^D_{r,i}\left(\overline{PD}_{r,i}\right)^{1-\sigma_i^D}+\beta^M_{r,i}\left(\overline{PM}_{r,i}\right)^{1-\sigma_i^D}}+\frac{\beta^M_{r,i}\left(PM_{r,i}\right)^{1-\sigma_i^D}}{\beta^D_{r,i}\left(\overline{PD}_{r,i}\right)^{1-\sigma_i^D}+\beta^M_{r,i}\left(\overline{PM}_{r,i}\right)^{1-\sigma_i^D}}\\
&= \frac{1}{\left(\frac{1}{\widehat{PD}_{r,i}}\right)^{1-\sigma_i^D}\left(1+\frac{\beta^M_{r,i}}{\beta^D_{r,i}}\left(\frac{\overline{PM}_{r,i}}{\overline{PD}_{r,i}}\right)^{1-\sigma_i^D}\right)}+\frac{1}{\left(\frac{1}{\widehat{PM}_{r,i}}\right)^{1-\sigma_i^D}\left(1+\frac{\beta^D_{r,i}}{\beta^M_{r,i}}\left(\frac{\overline{PD}_{r,i}}{\overline{PM}_{r,i}}\right)^{1-\sigma_i^D}\right)},
\end{aligned}
$$

and using (A.1.4) which shows that:

$$\frac{PD_{r,i}}{PM_{r,i}}\frac{XD_{r,i}}{XM_{r,i}} = \left(\frac{\beta^D_{r,i}}{\beta^M_{r,i}}\right)\left(\frac{PD_{r,i}}{PM_{r,i}}\right)^{1-\sigma^D_i} \quad \text{and} \quad \frac{PM_{r,i}}{PD_{r,i}}\frac{XM_{r,i}}{XD_{r,i}} = \left(\frac{\beta^M_{r,i}}{\beta^D_{r,i}}\right)\left(\frac{PM_{r,i}}{PD_{r,i}}\right)^{1-\sigma^D_i},$$

we can write this expression as:

$$\begin{aligned}
\widehat{PA}_{r,i} &= \left(\frac{1}{\left(\frac{1}{\widehat{PD}_{r,i}}\right)^{1-\sigma^D_i}\left(1+\frac{\overline{PM}_{r,i}}{\overline{PD}_{r,i}}\frac{\overline{XM}_{r,i}}{\overline{XD}_{r,i}}\right)} + \frac{1}{\left(\frac{1}{\widehat{PM}_{r,i}}\right)^{1-\sigma^D_i}\left(1+\frac{\overline{PD}_{r,i}}{\overline{PM}_{r,i}}\frac{\overline{XD}_{r,i}}{\overline{XM}_{r,i}}\right)}\right)^{\frac{1}{1-\sigma^D_i}} \\
&= \left[\left(1-\phi^{DIMP}_{r,i}\right)\left(\widehat{PD}_{r,i}\right)^{1-\sigma^D_i} + \phi^{DIMP}_{r,i}\left(\widehat{PM}_{r,i}\right)^{1-\sigma^D_i}\right]^{\frac{1}{1-\sigma^D_i}},
\end{aligned}$$

where $\phi^{DIMP}_{r,i}$ denotes the share of agricultural imports of good i in region r on the total value of its demand. We calculate it as $\overline{PM}_{r,i}\overline{XM}_{r,i}/\left(\overline{PREF}_i\overline{XD}_{r,i}+\overline{PM}_{r,i}\overline{XM}_{r,i}\right)$. $\widehat{PD}_{r,i}$ and $\widehat{PM}_{r,i}$ represent the *hat algebra* prices of agricultural commodity i when locally produced and imported, respectively. The import price $\widehat{PM}_{r,i}$ is also calculated as $\widehat{PREF}_i\left(\frac{1+\tau^M_{r,i}}{1+\overline{\tau}^M_{r,i}}\right)\left(\frac{1+\psi^M_{r,i}}{1+\overline{\psi}^M_{r,i}}\right)$.

Now, moving up one level in the consumers' nested CES utility function, we solve the optimal total demand for every commodity i in region r, $XA_{r,i}$. To do it, we follow the same approach we use to derive for optimal local and import demand, but we use expressions (A.1.3e) and (A.1.3f).

$$\begin{aligned}
\left(\frac{\beta_{n,i'}}{\beta_{n,i''}}\right)^{\frac{1}{\sigma^N_n}}\left(\frac{XA_{r,i'}}{XA_{r,i''}}\right)^{-\frac{1}{\sigma^N_n}} &= \frac{PA_{r,i'}}{PA_{r,i''}} \\
\frac{PA_{r,i'}}{PA_{r,i''}}\frac{XA_{r,i'}}{XA_{r,i''}} &= \left(\frac{PA_{r,i'}}{PA_{r,i''}}\right)^{1-\sigma^N_n}\left(\frac{\beta_{n,i'}}{\beta_{n,i''}}\right)
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
\beta_{n,i''}\left(PA_{r,i''}\right)^{1-\sigma^N_n} &= \frac{PA_{r,i''}}{PA_{r,i'}}\frac{XA_{r,i''}}{XA_{r,i'}}\beta_{n,i'}\left(PA_{r,i'}\right)^{1-\sigma^N_n} \\
\sum_{i''\in n}\beta_{n,i''}\left(PA_{r,i''}\right)^{1-\sigma^N_n} &= \frac{\beta_{n,i'}\left(PA_{r,i'}\right)^{1-\sigma^N_n}}{PA_{r,i'}XA_{r,i'}}\sum_{i''\in n}PA_{r,i''}XA_{r,i''}, \qquad \text{(A.1.9)}
\end{aligned}$$

where denoting $PN_{r,n} = \left[\sum_{i''\in n}\beta_{n,i''}\left(PA_{r,i''}\right)^{1-\sigma^N_n}\right]^{\frac{1}{1-\sigma^N_n}}$ and rearranging terms yields:

$$\begin{aligned}
XA_{r,i'} &= \beta_{n,i'}\left(\frac{PA_{r,i'}}{PN_{r,n}}\right)^{1-\sigma^N_n}\frac{\sum_{i''\in n}PA_{r,i''}XA_{r,i''}}{PA_{r,i'}} \\
&= \beta_{n,i'}\left(\frac{PA_{r,i'}}{PN_{r,n}}\right)^{1-\sigma^N_n}\frac{PN_{r,n}\cdot XN_{r,n}}{PA_{r,i''}} \\
&= \beta_{n,i'}\left(\frac{PA_{r,i'}}{PN_{r,n}}\right)^{-\sigma^N_n}XN_{r,n}. \qquad \text{(A.1.10)}
\end{aligned}$$

Now, writing these expressions in *hat algebra* yields:

$$\widehat{XA}_{r,i'} = \left(\frac{\widehat{PA}_{r,i'}}{\widehat{PN}_{r,n}}\right)^{-\sigma_n^N} \widehat{XN}_{r,n} \tag{A.1.11}$$

and

$$\widehat{PN}_{r,n} = \left[\sum_{i''\in n} \phi_{r,i''}^N \left(\widehat{PA}_{r,i''}\right)^{1-\sigma_n^N}\right]^{\frac{1}{1-\sigma_n^N}}, \tag{A.1.12}$$

where $\phi_{r,i''}^N$ denotes the share of the demand for good i'' in the total value of the demand for goods in nest n. We calculate it as $PA_{r,i''}XA_{r,i''}/\sum_{i''\in n} PA_{r,i''}XA_{r,i''}$, where $PA_{r,i''}XA_{r,i''}$ equals $\left(\overline{PREF}_{i''}\overline{XD}_{r,i''} + \overline{PM}_{r,i''}\overline{XM}_{r,i''}\right)$.

Turning now to the next level in consumers' nested CES utility function, we solve for the optimal total demand for agricultural products in nest n in region r, $XN_{r,n}$. To do it, we use expressions (A.1.3c) and (A.1.3d).

$$\begin{aligned}
\left(\frac{\beta_{r,n'}}{\beta_{r,n''}}\right)^{\frac{1}{\sigma^U}} \left(\frac{XN_{r,n'}}{XN_{r,n''}}\right)^{-\frac{1}{\sigma^U}} &= \frac{PN_{r,n'}}{PN_{r,n''}} \\
\frac{PN_{r,n'}}{PN_{r,n''}}\frac{XN_{r,n'}}{XN_{r,n''}} &= \left(\frac{PN_{r,n'}}{PN_{r,n''}}\right)^{1-\sigma^U} \left(\frac{\beta_{r,n'}}{\beta_{r,n''}}\right)
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
\beta_{r,n''}\left(PN_{r,n''}\right)^{1-\sigma^U} &= \frac{PN_{r,n''}}{PN_{r,n'}}\frac{XN_{r,n''}}{XN_{r,n'}}\beta_{r,n'}\left(PN_{r,n'}\right)^{1-\sigma^U} \\
\sum_{n''}\beta_{r,n''}\left(PN_{r,n''}\right)^{1-\sigma^U} &= \frac{\beta_{r,n'}\left(PN_{r,n'}\right)^{1-\sigma^U}}{PN_{r,n'}XN_{r,n'}}\sum_{n''}PN_{r,n''}XN_{r,n''},
\end{aligned} \tag{A.1.13}$$

where denoting $P_{r,AG} = \left[\sum_{n''}\beta_{r,n''}\left(PN_{r,n''}\right)^{1-\sigma^U}\right]^{\frac{1}{1-\sigma^U}}$ and rearranging terms yields:

$$\begin{aligned}
XN_{r,n'} &= \beta_{r,n'}\left(\frac{PN_{r,n'}}{P_{r,AG}}\right)^{1-\sigma^U} \frac{\sum_{n''}PN_{r,n''}XN_{r,n''}}{PN_{r,n'}} \\
&= \beta_{r,n'}\left(\frac{PN_{r,n,'}}{P_{r,AG}}\right)^{1-\sigma^U} \frac{P_{r,AG}\cdot X_{r,AG}}{PN_{r,n''}} \\
&= \beta_{r,n'}\left(\frac{PN_{r,n'}}{P_{r,AG}}\right)^{-\sigma^U} X_{r,AG},
\end{aligned} \tag{A.1.14}$$

such that writing this expression in *hat algebra* yields:

$$\widehat{XN}_{r,n'} = \left(\frac{\widehat{PN}_{r,n'}}{\widehat{P}_{r,AG}}\right)^{-\sigma^U} \widehat{X}_{r,AG} \tag{A.1.15}$$

and the price index $P_{r,AG}$ as follows:

$$\widehat{P}_{r,AG} = \left[\sum_{n''} \phi^U_{r,n''} \left(\widehat{PN}_{r,n''}\right)^{1-\sigma^U}\right]^{\frac{1}{1-\sigma^U}}, \tag{A.1.16}$$

where $\phi^U_{r,n''}$ denotes the share of the demand for goods in nest n'' and region r in the total value of the demand for agricultural goods in r. We calculate it as $PN_{r,n''}XN_{r,n''}/\sum_{n''} PN_{r,n''}XN_{r,n''}$, where $PN_{r,n''}XN_{r,n''}$ also equals to $\sum_{i\in n''} PA_{r,i}XA_{r,i}$.

Finally, moving up to the top level of the consumers' nested CES utility function, we solve for the optimal demand for agricultural and non-agricultural goods in region r, $X_{r,AG}$ and X_{NA} respectively. To do it, we follow the same approach as above, but we use expressions (A.1.3a) and (A.1.3b).

$$\begin{aligned}
\left(\frac{\beta_{r,NA}}{\beta_{r,AG}}\right)^{\frac{1}{\sigma^T}} \left(\frac{X_{r,NA}}{X_{r,AG}}\right)^{-\frac{1}{\sigma^T}} &= \frac{P_{r,NA}}{P_{r,AG}} \\
\frac{P_{r,NA}}{P_{r,AG}}\frac{X_{r,NA}}{X_{r,AG}} &= \left(\frac{P_{r,NA}}{P_{r,AG}}\right)^{1-\sigma^T} \left(\frac{\beta_{r,NA}}{\beta_{r,AG}}\right)
\end{aligned}$$

$$\beta_{r,AG}\left(P_{r,AG}\right)^{1-\sigma^T} = \frac{P_{r,AG}}{P_{r,NA}}\frac{X_{r,AG}}{X_{r,NA}}\beta_{r,NA}\left(P_{r,NA}\right)^{1-\sigma^T}$$

$$\beta_{r,NA}\left(P_{r,NA}\right)^{1-\sigma^T} + \beta_{r,AG}\left(P_{r,AG}\right)^{1-\sigma^T} = \beta_{r,NA}\left(P_{r,NA}\right)^{1-\sigma^T}\left(\frac{P_{r,AG}}{P_{r,NA}}\frac{X_{r,AG}}{X_{r,NA}} + 1\right) \tag{A.1.17}$$

where defining $PU_r = \left[\beta_{r,NA}\left(P_{r,NA}\right)^{1-\sigma^T} + \beta_{r,AG}\left(P_{r,AG}\right)^{1-\sigma^T}\right]^{\frac{1}{1-\sigma^T}}$ and rearranging terms yields:

$$\begin{aligned}
X_{r,NA} &= \beta_{r,NA}\left(\frac{P_{r,NA}}{PU_r}\right)^{1-\sigma^T}\left(\frac{P_{r,NA}X_{r,NA} + P_{r,AG}X_{r,AG}}{P_{r,NA}}\right) \\
&= \beta_{r,NA}\left(\frac{P_{r,NA}}{PU_r}\right)^{-\sigma^T}\frac{I_r}{PU_r}
\end{aligned} \tag{A.1.18}$$

and by symmetry

$$X_{r,AG} = \beta_{r,AG}\left(\frac{P_{r,AG}}{PU_r}\right)^{-\sigma^T}\frac{I_r}{PU_r}. \tag{A.1.19}$$

Lastly, writing these expressions in *hat algebra*, including the expression of the price index PU_r

as a function of the expenditure shared allocated to agricultural goods α_r, yields that:

$$\widehat{X}_{r,NA} = \left(\frac{\widehat{P}_{r,NA}}{\widehat{PU}_r}\right)^{-\sigma^T} \frac{\widehat{I}_r}{\widehat{PU}_r} \tag{A.1.20}$$

$$\widehat{X}_{r,AG} = \left(\frac{\widehat{P}_{r,AG}}{\widehat{PU}_r}\right)^{-\sigma^T} \frac{\widehat{I}_r}{\widehat{PU}_r} \tag{A.1.21}$$

$$\widehat{PU}_r = \left[(1-\alpha_r)\left(\widehat{P}_{rNA}\right)^{1-\sigma^T} + \alpha_r\left(\widehat{P}_{rAG}\right)^{1-\sigma^T}\right]^{\frac{1}{1-\sigma^T}}. \tag{A.1.22}$$

A.2 Producers

$$\max_{\substack{Qprd_{r,i},\ VA_{r,i},\ IN_{r,i},\\ LN_{r,i},\ NL_{r,i},\\ qLC^{IN}_{r,i,j},\ qIM^{IN}_{r,i,j}}} \Pi = (PD_{r,i} - C_{r,i}) \cdot Qprd_{r,i} \tag{A.2.1}$$

where

$$\begin{aligned}
Qprd_{r,i} &= \left[\gamma^{VA\ \frac{1}{\sigma^I_i}}_{r,i} \left(\Omega^{VA}_{r,i} VA_{r,i}\right)^{\frac{\sigma^I_i - 1}{\sigma^I_i}} + (1 - \gamma^{VA}_{r,i})^{\frac{1}{\sigma^I_i}} \left(\Omega^{IN}_{r,i} IN_{r,i}\right)^{\frac{\sigma^I_i - 1}{\sigma^I_i}} \right]^{\frac{\sigma^I_i}{\sigma^I_i - 1}} \\
VA_{r,i} &= \left[\gamma^{LN\ \frac{1}{\sigma^F_i}}_{r,i} \left(\Theta_{r,i} LN_{r,i}\right)^{\frac{\sigma^F_i - 1}{\sigma^F_i}} + (1 - \gamma^{LN}_{r,i})^{\frac{1}{\sigma^F_i}} NL_{r,i}^{\frac{\sigma^F_i - 1}{\sigma^F_i}} \right]^{\frac{\sigma^F_i}{\sigma^F_i - 1}} \\
IN_{r,i} &= \sum_j \left[\gamma^{IN\ \frac{1}{\sigma^C_i}}_{r,j}\, qLC^{IN\ \frac{\sigma^C_i - 1}{\sigma^C_i}}_{r,i,j} + (1 - \gamma^{IN}_{r,j})^{\frac{1}{\sigma^C_i}}\, qIM^{IN\ \frac{\sigma^C_i - 1}{\sigma^C_i}}_{r,i,j} \right]^{\frac{\sigma^C_i}{\sigma^C_i - 1}} \\
C_{r,i} \cdot Qprd_{r,i} &= PVA_{r,i} \cdot VA_{r,i} + PIN_{r,i} \cdot IN_{r,i} \\
PVA_{r,i} \cdot VA_{r,i} &= PLN_r \cdot LN_{r,i} + PNL_r \cdot NL_{r,i} \\
PIN_{r,i} \cdot IN_{r,i} &= \sum_j PD_{r,j} \cdot qLC^{IN}_{r,i,j} + PM_{r,j} \cdot qIM^{IN}_{r,i,j}
\end{aligned}$$

Now, solving this producer maximization problem, we know that F.O.C. are given as follows:

$$\frac{\partial \Pi}{\partial Qprd_{r,i}} = PD_{r,i} - C_{r,i} = 0 \tag{A.2.2a}$$

$$\frac{\partial \Pi}{\partial VA_{r,i}} = E \cdot \left(\Omega^{VA}_{r,i}\right)^{\frac{\sigma^I_i - 1}{\sigma^I_i}} \gamma^{VA\ \frac{1}{\sigma^I_i}}_{r,i} VA_{r,i}^{-\frac{1}{\sigma^I_i}} - PVA_{r,i} = 0 \tag{A.2.2b}$$

$$\frac{\partial \Pi}{\partial IN_{r,i}} = E \cdot \left(\Omega^{IN}_{r,i}\right)^{\frac{\sigma^I_i - 1}{\sigma^I_i}} \left(1 - \gamma^{VA}_{r,i}\right)^{\frac{1}{\sigma^I_i}} IN_{r,i}^{-\frac{1}{\sigma^I_i}} - PIN_{r,i} = 0 \tag{A.2.2c}$$

$$\frac{\partial \Pi}{\partial LN_{r,i}} = F \cdot (\Theta_{r,i})^{\frac{\sigma^F_i - 1}{\sigma^F_i}} \gamma^{LN\ \frac{1}{\sigma^F_i}}_{r,i} LN_{r,i}^{-\frac{1}{\sigma^F_i}} - PLN_r = 0 \tag{A.2.2d}$$

$$\frac{\partial \Pi}{\partial NL_{r,i}} = F \cdot \left(1 - \gamma^{LN}_{r,i}\right)^{\frac{1}{\sigma^F_i}} NL_{r,i}^{-\frac{1}{\sigma^F_i}} - PNL_r = 0 \tag{A.2.2e}$$

$$\frac{\partial \Pi}{\partial qLC^{IN}_{r,i,j}} = G \cdot \gamma^{IN\ \frac{1}{\sigma^C_i}}_{r,j}\, qLC^{IN\ -\frac{1}{\sigma^C_i}}_{r,i,j} - PD_{r,j} = 0 \tag{A.2.2f}$$

$$\frac{\partial \Pi}{\partial qIM^{IN}_{r,i,j}} = G \cdot \left(1 - \gamma^{IN}_{r,j}\right)^{\frac{1}{\sigma^C_i}} qIM^{IN\ -\frac{1}{\sigma^C_i}}_{r,i,j} - PM_{r,j} = 0 \tag{A.2.2g}$$

where E equals to $PD_{r,i}\left(Qprd_{r,i}\right)^{\frac{1}{\sigma_i^I}}$, F to $E\cdot\left(\Omega_{r,i}^{VA}\right)^{\frac{\sigma_i^I-1}{\sigma_i^I}}\gamma_{r,i}^{VA\,\frac{1}{\sigma_i^I}}VA_{r,i}^{\left(\frac{\sigma_i^I-1}{\sigma_i^I}\right)-\left(\frac{\sigma_i^F-1}{\sigma_i^F}\right)}$, and G to $E\cdot\left(\Omega_{r,i}^{IN}\right)^{\frac{\sigma_i^I-1}{\sigma_i^I}}\left(1-\gamma_{r,i}^{VA}\right)^{\frac{1}{\sigma_i^I}}IN_{r,i}^{\left(\frac{\sigma_i^I-1}{\sigma_i^I}\right)-\left(\frac{\sigma_i^C-1}{\sigma_i^C}\right)}$.

Then, in order to solve for the optimal generation of value-added $VA_{r,i}$ and the demand for inputs $IN_{r,i}$, we take the ratio of expression (A.2.2b) to (A.2.2c) and rearrange terms as follows:

$$\begin{aligned}\left(\frac{\Omega_{r,i}^{VA}}{\Omega_{r,i}^{IN}}\right)^{\frac{\sigma_i^I-1}{\sigma_i^I}}\left(\frac{\gamma_{r,i}^{VA}}{1-\gamma_{r,i}^{VA}}\right)^{\frac{1}{\sigma_i^I}}\left(\frac{VA_{r,i}}{IN_{r,i}}\right)^{-\frac{1}{\sigma_i^I}}&=\frac{PVA_{r,i}}{PIN_{r,i}}\\ \frac{PVA_{r,i}}{PIN_{r,i}}\frac{VA_{r,i}}{IN_{r,i}}&=\left(\frac{PVA_{r,i}}{PIN_{r,i}}\right)^{1-\sigma_i^I}\left(\frac{\gamma_{r,i}^{VA}}{1-\gamma_{r,i}^{VA}}\right)\left(\frac{\Omega_{r,i}^{IN}}{\Omega_{r,i}^{VA}}\right)^{1-\sigma_i^I}\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}(1-\gamma_{r,i}^{VA})\left(\frac{PIN_{r,i}}{\Omega_{r,i}^{IN}}\right)^{1-\sigma_i^I}&=\frac{PIN_{r,i}}{PVA_{r,i}}\frac{IN_{r,i}}{VA_{r,i}}\gamma_{r,i}^{VA}\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}}\right)^{1-\sigma_i^I}\\ (\gamma_{r,i}^{VA})\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}}\right)^{1-\sigma_i^I}+(1-\gamma_{r,i}^{VA})\left(\frac{PIN_{r,i}}{\Omega_{r,i}^{IN}}\right)^{1-\sigma_i^I}&=(\gamma_{r,i}^{VA})\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}}\right)^{1-\sigma_i^I}\left(\frac{PIN_{r,i}}{PVA_{r,i}}\frac{IN_{r,i}}{VA_{r,i}}+1\right)\end{aligned}\tag{A.2.3}$$

where denoting $PD_{r,i}=\left[\left(\gamma_{r,i}^{VA}\right)\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}}\right)^{1-\sigma_i^I}+\left(1-\gamma_{r,i}^{VA}\right)\left(\frac{PIN_{r,i}}{\Omega_{r,i}^{IN}}\right)^{1-\sigma_i^I}\right]^{\frac{1}{1-\sigma_i^I}}$ and rearranging terms yields

$$\begin{aligned}VA_{r,i}&=\gamma_{r,i}^{VA}\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}PD_{r,i}}\right)^{1-\sigma_i^I}\left(\frac{PIN_{r,i}IN_{r,i}+PVA_{r,i}VA_{r,i}}{PVA_{r,i}}\right)\\ &=\gamma_{r,i}^{VA}\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}PD_{r,i}}\right)^{1-\sigma_i^I}\frac{PD_{r,i}Qprd_{r,i}}{PVA_{r,i}}\\ &=\frac{\gamma_{r,i}^{VA}}{\Omega_{r,i}^{VA}}\left(\frac{PVA_{r,i}}{\Omega_{r,i}^{VA}PD_{r,i}}\right)^{-\sigma_i^I}Qprd_{r,i}\end{aligned}\tag{A.2.4}$$

and by symmetry

$$IN_{r,i}=\frac{1-\gamma_{r,i}^{VA}}{\Omega_{r,i}^{IN}}\left(\frac{PIN_{r,i}}{\Omega_{r,i}^{IN}PD_{r,i}}\right)^{-\sigma_i^I}Qprd_{r,i}.\tag{A.2.5}$$

Now, writing both expressions in *hat algebra* yields:

$$\widehat{VA}_{r,i} = \frac{1}{\widehat{\Omega^{VA}_{r,i}}} \left(\frac{\widehat{PVA}_{r,i}}{\widehat{\Omega^{VA}_{r,i}}\widehat{PD}_{r,i}} \right)^{-\sigma_i^I} \widehat{Qprd}_{r,i} \tag{A.2.6}$$

$$\widehat{IN}_{r,i} = \frac{1}{\widehat{\Omega^{IN}_{r,i}}} \left(\frac{\widehat{PIN}_{r,i}}{\widehat{\Omega^{IN}_{r,i}}\widehat{PD}_{r,i}} \right)^{-\sigma_i^I} \widehat{Qprd}_{r,i} \tag{A.2.7}$$

Turning now to producer's optimal choice of land $LN_{r,i}$ and non-land sources $NL_{r,i}$, we follow the same approach but using expressions (A.2.2d) and (A.2.2e).

$$(\Theta_{r,i})^{\frac{\sigma_i^F - 1}{\sigma_i^F}} \left(\frac{\gamma_{r,i}^{LN}}{1-\gamma_{r,i}^{LN}} \right)^{\frac{1}{\sigma_i^F}} \left(\frac{LN_{r,i}}{NL_{r,i}} \right)^{-\frac{1}{\sigma_i^F}} = \frac{PLN_r}{PNL_r}$$

$$\frac{PLN_r}{PNL_r}\frac{LN_{r,i}}{NL_{r,i}} = \left(\frac{PLN_r}{PNL_r} \right)^{1-\sigma_i^F} \left(\frac{\gamma_{r,i}^{LN}}{1-\gamma_{r,i}^{LN}} \right) (\Theta_{r,i})^{\sigma_i^F - 1}$$

$\Rightarrow$

$$\left(1-\gamma_{r,i}^{LN}\right)(PNL_r)^{1-\sigma_i^F} = \frac{PNL_r}{PLN_r}\frac{NL_{r,i}}{LN_{r,i}} \left(\gamma_{r,i}^{LN}\right) \left(\frac{PLN_r}{\Theta_{r,i}} \right)^{1-\sigma_i^F}$$

$$\left(\gamma_{r,i}^{LN}\right) \left(\frac{PLN_r}{\Theta_{r,i}} \right)^{1-\sigma_i^F} + \left(1-\gamma_{r,i}^{LN}\right)(PNL_r)^{1-\sigma_i^F} = \left(\gamma_{r,i}^{LN}\right) \left(\frac{PLN_r}{\Theta_{r,i}} \right)^{1-\sigma_i^F} \left(\frac{PNL_r}{PLN_r}\frac{NL_{r,i}}{LN_{r,i}} + 1 \right) \tag{A.2.8}$$

where defining $PVA_{r,i} = \left[\left(\gamma_{r,i}^{LN}\right) \left(\frac{PLN_r}{\Theta_{r,i}} \right)^{1-\sigma_i^F} + \left(1-\gamma_{r,i}^{LN}\right)(PNL_r)^{1-\sigma_i^F} \right]^{\frac{1}{1-\sigma_i^F}}$ and rearranging terms yields:

$$\begin{aligned} LN_{r,i} &= \gamma_{r,i}^{LN} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}} \right)^{1-\sigma_i^F} \left(\frac{PNL_r NL_{r,i} + PLN_r LN_{r,i}}{PLN_r} \right) \\ &= \gamma_{r,i}^{LN} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}} \right)^{1-\sigma_i^F} \frac{PVA_{r,i} VA_{r,i}}{PLN_r} \\ &= \frac{\gamma_{r,i}^{LN}}{\Theta_{r,i}} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}} \right)^{-\sigma_i^F} VA_{r,i} \end{aligned} \tag{A.2.9}$$

and by symmetry

$$NL_{r,i} = \left(1-\gamma_{r,i}^{LN}\right) \left(\frac{PNL_r}{PVA_{r,i}} \right)^{-\sigma_i^F} VA_{r,i} \tag{A.2.10}$$

Now, writing this expression in *hat algebra*, including the price index of the value added, yields:

$$\widehat{LN}_{r,i} = \frac{1}{\widehat{\Theta}_{r,i}} \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i} \widehat{PVA}_{r,i}} \right)^{-\sigma_i^F} \widehat{VA}_{r,i} \tag{A.2.11}$$

$$\widehat{NL}_{r,i} = \left(\frac{\widehat{PNL}_r}{\widehat{PVA}_{r,i}} \right)^{-\sigma_i^F} \widehat{VA}_{r,i} \tag{A.2.12}$$

$$\widehat{PVA}_{r,i} = \left[\phi_{r,i}^f \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i}} \right)^{1-\sigma_i^F} + \left(1 - \phi_{r,i}^f\right) \left(\widehat{PNL}_r\right)^{1-\sigma_i^F} \right]^{\frac{1}{1-\sigma_i^F}} \tag{A.2.13}$$

denoting $\phi_{r,i}^f$ as the cost share of land in the value of total costs to produce good i in r. We calculate it as $PLN_r \cdot LN_{r,i} / \left(PLN_r \cdot LN_{r,i} + PNL_r \cdot NL_{r,i}\right)$.

Finally, in order to determine the optimal choice of local and imported inputs, we follow the same approach as above, but using expressions (A.2.2f) and (A.2.2g).

$$\begin{aligned} \left(\frac{\gamma_{r,j}^{IN}}{1-\gamma_{r,j}^{IN}} \right)^{\frac{1}{\sigma_i^C}} \left(\frac{qLC_{r,i,j}^{IN}}{qIM_{r,i,j}^{IN}} \right)^{-\frac{1}{\sigma_i^C}} &= \frac{PD_{r,j}}{PM_{r,j}} \\ \frac{PD_{r,j}}{PM_{r,j}} \frac{qLC_{r,i,j}^{IN}}{qIM_{r,i,j}^{IN}} &= \left(\frac{PD_{r,j}}{PM_{r,j}} \right)^{1-\sigma_i^C} \left(\frac{\gamma_{r,j}^{IN}}{1-\gamma_{r,j}^{IN}} \right) \end{aligned}$$

$\Rightarrow$

$$\left(1-\gamma_{r,j}^{IN}\right) \left(PM_{r,j}\right)^{1-\sigma_i^C} = \frac{PM_{r,j}}{PD_{r,j}} \frac{qIM_{r,i,j}^{IN}}{qLC_{r,i,j}^{IN}} \left(\gamma_{r,j}^{IN}\right) PD_{r,j}^{1-\sigma_i^C}$$

$$\left(\gamma_{r,j}^{IN}\right) PD_{r,j}^{1-\sigma_i^C} + \left(1-\gamma_{r,j}^{IN}\right) \left(PM_{r,j}\right)^{1-\sigma_i^C} = \left(\gamma_{r,j}^{IN}\right) PD_{r,j}^{1-\sigma_i^C} \left(\frac{PM_{r,j}}{PD_{r,j}} \frac{qIM_{r,i,j}^{IN}}{qLC_{r,i,j}^{IN}} + 1 \right) \tag{A.2.14}$$

where assuming that each good i uses at most one good j as input such that $PIN_{r,i,j} = PIN_{r,i}$, and $PIN_{r,i} = \left[\left(\gamma_{r,j}^{IN}\right) PD_{r,j}^{1-\sigma_i^C} + \left(1-\gamma_{r,j}^{IN}\right) PM_{r,j}{}^{1-\sigma_i^C} \right]^{\frac{1}{1-\sigma_i^C}}$, we can rearrange terms to obtain

$$\begin{aligned} qLC_{r,i,j}^{IN} &= \gamma_{r,j}^{IN} \left(\frac{PD_{r,j}}{PIN_{r,i}} \right)^{1-\sigma_i^C} \left(\frac{PM_{r,j} \cdot qIM_{r,i,j}^{IN} + PD_{r,j} \cdot qLC_{r,i,j}^{IN}}{PD_{r,j}} \right) \\ &= \gamma_{r,j}^{IN} \left(\frac{PD_{r,j}}{PIN_{r,i}} \right)^{1-\sigma_i^C} \left(\frac{PIN_{r,i} IN_{r,i}}{PD_{r,j}} \right) \\ &= \gamma_{r,j}^{IN} \left(\frac{PD_{r,j}}{PIN_{r,i}} \right)^{-\sigma_i^C} IN_{r,i}, \end{aligned} \tag{A.2.15}$$

and by symmetry

$$qIM_{r,i,j}^{IN} = \left(1-\gamma_{r,j}^{IN}\right) \left(\frac{PM_{r,j}}{PIN_{r,i}} \right)^{-\sigma_i^C} IN_{r,i}. \tag{A.2.16}$$

Then, writing these expressions in *hat algebra* yields:

$$\widehat{qLC}^{IN}_{r,i,j} = \left(\frac{\widehat{PD}_{r,j}}{\widehat{PIN}_{r,i}}\right)^{-\sigma_i^C} \widehat{IN}_{r,i} \tag{A.2.17}$$

$$\widehat{qIM}^{IN}_{r,i,j} = \left(\frac{\widehat{PM}_{r,j}}{\widehat{PIN}_{r,i}}\right)^{-\sigma_i^C} \widehat{IN}_{r,i} \tag{A.2.18}$$

$$\widehat{PIN}_{r,i} = \left[\left(1-\phi^{cIM}_{r,j}\right)\widehat{PD}^{1-\sigma_i^C}_{r,j} + \phi^{cIM}_{r,j}\left(\widehat{PM}_{r,j}\right)^{1-\sigma_i^C}\right]^{\frac{1}{1-\sigma_i^C}} \tag{A.2.19}$$

where $\phi^{cIM}_{r,j}$ denotes the share imported of input j in the total value of the supply of good j used an input in region r. We calculate it as $\sum_i \left(PM_{r,j} \cdot qIM^{IN}_{r,i,j}\right) / \left(PM_{r,j} \cdot qIM^{IN}_{r,i,j} + PREF_{r,j} \cdot qLC^{IN}_{r,i,j}\right)$.

Hence, the total demand for good j as an intermediate input to produce others in region r, $qIN_{r,j}$, is given by:

$$qIN_{r,j} = \sum_i \left(qLC^{IN}_{r,i,j} + qIM^{IN}_{r,i,j}\right), \tag{A.2.20}$$

where this expression written in *hat algebra* is given by:

$$\begin{aligned}
&= \widehat{qIN}_{r,j} \\
&= \frac{\sum_i \left(qLC^{IN}_{r,i,j} + qIM^{IN}_{r,i,j}\right)}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)} \\
&= \frac{\sum_i qLC^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)} + \frac{\sum_i qIM^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)} \\
&= \frac{\sum_i qLC^{IN}_{r,i,j}}{\sum_i \overline{qLC^{IN}_{r,i,j}}}\left(\frac{\sum_i \overline{qLC}^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)}\right) + \frac{\sum_i qIM^{IN}_{r,i,j}}{\sum_i \overline{qIM^{IN}_{r,i,j}}}\left(\frac{\sum_i \overline{qIM}^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)}\right) \\
&= \left(\frac{qLC^{IN}_{r,i,j}}{\overline{qLC^{IN}_{r,i,j}}}\frac{\overline{qLC^{IN}_{r,i,j}}}{\sum_i \overline{qLC^{IN}_{r,i,j}}} + \ldots\right)\left(\frac{\sum_i \overline{qLC}^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)}\right) + \\
&\quad + \left(\frac{qIM^{IN}_{r,i,j}}{\overline{qIM^{IN}_{r,i,j}}}\frac{\overline{qIM^{IN}_{r,i,j}}}{\sum_i \overline{qIM^{IN}_{r,i,j}}} + \ldots\right)\frac{\sum_i qIM^{IN}_{r,i,j}}{\sum_i \overline{qIM^{IN}_{r,i,j}}}\left(\frac{\sum_i \overline{qIM}^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)}\right) \\
&= \left(\sum_i \widehat{qLC}^{IN}_{r,i,j}\frac{\overline{qLC^{IN}_{r,i,j}}}{\sum_i \overline{qLC^{IN}_{r,i,j}}}\right)\left(\frac{\sum_i \overline{qLC}^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)}\right) + \\
&\quad + \left(\sum_i \widehat{qIM}^{IN}_{r,i,j}\frac{\overline{qIM^{IN}_{r,i,j}}}{\sum_i \overline{qIM^{IN}_{r,i,j}}}\right)\left(\frac{\sum_i \overline{qIM}^{IN}_{r,i,j}}{\sum_i \left(\overline{qLC^{IN}_{r,i,j}} + \overline{qIM^{IN}_{r,i,j}}\right)}\right) \\
&= \sum_i \left(\phi^{LC}_{r,i,j} \cdot \phi^{TTLC}_{r,j}\right)\widehat{qLC}^{IN}_{r,i,j} + \sum_i \left(\phi^{IM}_{r,i,j} \cdot \phi^{TTIM}_{r,j}\right)\widehat{qIM}^{IN}_{r,i,j}
\end{aligned} \tag{A.2.21}$$

denoting $\phi^{LC}_{r,i,j}$ equal to the share of the domestic supply of good i in the total value of domestic supply of goods produced with good j ($XDS_{r,i,j}/\sum_i XDS_{r,i,j}$), $\phi^{IM}_{r,i,j}$ to the share of imports of good i in the total value of imports of goods produced with good j ($PM_{r,i,j}XM_{r,i,j}/\sum_i PM_{r,i,j}XM_{r,i,j}$), $\phi^{TTLC}_{r,j}$ to the share of the total domestic supply of goods produced with good j in the sum of the domestic and imported supplies of these goods ($\sum_i XDS_{r,i,j}/(\sum_i XDS_{r,i,j} + \sum_i PM_{r,i,j}XM_{r,i,j})$), and $\phi^{TTIM}_{r,j}$ as the corresponding share for imports ($\sum_i PM_{r,i,j}XM_{r,i,j}/(\sum_i XDS_{r,i,j} + \sum_i PM_{r,i,j}XM_{r,i,j})$).

A.3 Intermediaries

$$\max_{\substack{XCS_{r,i},\ XSS_{r,i},\ XDS_{r,i},\\ XExp_{r,i},\ XDF_{r,i},\ XDI_{r,i}}} \quad R_{r,i} = PSS_{r,i} \cdot XSS_{r,i} + PCS_{r,i} \cdot XCS_{r,i} \tag{A.3.1}$$

which can be written as:

$$\begin{aligned} &= PSS_{r,i} \cdot PSS_{r,i} + (PE_{r,i} \cdot XExp_{r,i} + PDS_{r,i} \cdot XDS_{r,i}) \\ &= PSS_{r,i} \cdot PSS_{r,i} + (PE_{r,i} \cdot XExp_{r,i} + (PD_{r,i} \cdot XDF_{r,i} + PI_{r,i} XDI_{r,i})) \end{aligned} \tag{A.3.2}$$

subject to

$$XST_{r,i} = \left({\theta^C_{r,i}}^{-\frac{1}{\omega_i^Q}} XCS_{r,i}^{\frac{\omega_i^Q+1}{\omega_i^Q}} + (1-\theta^C_{r,i})^{-\frac{1}{\omega_i^Q}} XSS_{r,i}^{\frac{\omega_i^Q+1}{\omega_i^Q}} \right)^{\frac{\omega_i^Q}{\omega_i^Q+1}}$$

$$XCS_{r,i} = \left({\theta^D_{r,i}}^{-\frac{1}{\omega_i^S}} XDS_{r,i}^{\frac{\omega_i^S+1}{\omega_i^S}} + (1-\theta^D_{r,i})^{-\frac{1}{\omega_i^S}} XExp_{r,i}^{\frac{\omega_i^S+1}{\omega_i^S}} \right)^{\frac{\omega_i^S}{\omega_i^S+1}}$$

$$XDS_{r,i} = \left({\theta^F_{r,i}}^{-\frac{1}{\omega_i^C}} XDF_{r,i}^{\frac{\omega_i^C+1}{\omega_i^C}} + (1-\theta^F_{r,i})^{-\frac{1}{\omega_i^C}} XDI_{r,i}^{\frac{\omega_i^C+1}{\omega_i^C}} \right)^{\frac{\omega_i^C}{\omega_i^C+1}}$$

Using the Lagrangian multiplier method to solve this maximization problem yields:

$$\frac{\partial \mathcal{L}}{\partial XCS_{r,i}} = PCS_{r,i} - \lambda \cdot H \cdot {\theta^C_{r,i}}^{-\frac{1}{\omega_i^Q}} XCS_{r,i}^{\frac{1}{\omega_i^Q}} \tag{A.3.3a}$$

$$\frac{\partial \mathcal{L}}{\partial XSS_{r,i}} = PSS_{r,i} - \lambda \cdot H \cdot \left(1-\theta^C_{r,i}\right)^{-\frac{1}{\omega_i^Q}} XSS_{r,i}^{\frac{1}{\omega_i^Q}} \tag{A.3.3b}$$

$$\frac{\partial \mathcal{L}}{\partial XDS_{r,i}} = PDS_{r,i} - \lambda J \cdot {\theta^D_{r,i}}^{-\frac{1}{\omega_i^S}} XDS_{r,i}^{\frac{1}{\omega_i^S}} \tag{A.3.3c}$$

$$\frac{\partial \mathcal{L}}{\partial XExp_{r,i}} = PE_{r,i} - \lambda J \cdot \left(1-\theta^D_{r,i}\right)^{-\frac{1}{\omega_i^S}} XExp_{r,i}^{\frac{1}{\omega_i^S}} \tag{A.3.3d}$$

$$\frac{\partial \mathcal{L}}{\partial XDF_{r,i}} = PD_{r,i} - \lambda K \cdot {\theta^F_{r,i}}^{-\frac{1}{\omega_i^C}} XDF_{r,i}^{\frac{1}{\omega_i^C}} \tag{A.3.3e}$$

$$\frac{\partial \mathcal{L}}{\partial XDI_{r,i}} = PI_{r,i} - \lambda K \cdot \left(1-\theta^F_{r,i}\right)^{-\frac{1}{\omega_i^C}} XDI_{r,i}^{\frac{1}{\omega_i^C}} \tag{A.3.3f}$$

where H equals $(XST_{r,i})^{-\frac{1}{\omega_i^Q}}$, J to $H \cdot {\theta^C_{r,i}}^{-\frac{1}{\omega_i^Q}} XCS_{r,i}^{\left(\frac{\omega_i^Q+1}{\omega_i^Q}\right)-\left(\frac{\omega_i^S+1}{\omega_i^S}\right)}$, and K to $J \cdot {\theta^D_{r,i}}^{-\frac{1}{\omega_i^S}} XDS_{r,i}^{\left(\frac{\omega_i^S+1}{\omega_i^S}\right)-\left(\frac{\omega_i^C+1}{\omega_i^C}\right)}$.

Then, to solve this intermediaries' problem, we begin by solving the optimal supply of each good i in region r for final demand, $XDF_{r,i}$, and the optimal supply for intermediate consumption, $XDI_{r,i}$. To do it, we take the ratio of expression (A.3.3e) to (A.3.3f) and rearrange terms as follows:

$$\frac{PD_{r,i}}{PI_{r,i}} = \left(\frac{\theta^F_{r,i}}{1-\theta^F_{r,i}}\right)^{-\frac{1}{\omega^C_i}} \left(\frac{XDF_{r,i}}{XDI_{r,i}}\right)^{\frac{1}{\omega^C_i}}$$

$$\left(\frac{\theta^F_{r,i}}{1-\theta^F_{r,i}}\right)\left(\frac{PD_{r,i}}{PI_{r,i}}\right)^{1+\omega^C_i} = \frac{PD_{r,i}}{PI_{r,i}}\frac{XDF_{r,i}}{XDI_{r,i}}$$

$\Rightarrow$

$$\theta^F_{r,i}\left(PD_{r,i}\right)^{1+\omega^C_i} = \frac{PD_{r,i}}{PI_{r,i}}\frac{XDF_{r,i}}{XDI_{r,i}}\left(1-\theta^F_{r,i}\right)\left(PI_{r,i}\right)^{1+\omega^C_i}$$

$$\theta^F_{r,i}\left(PD_{r,i}\right)^{1+\omega^C_i} + \left(1-\theta^F_{r,i}\right)\left(PI_{r,i}\right)^{1+\omega^C_i} = \left(1-\theta^F_{r,i}\right)\left(PI_{r,i}\right)^{1+\omega^C_i}\left(\frac{PD_{r,i}}{PI_{r,i}}\frac{XDF_{r,i}}{XDI_{r,i}}+1\right) \quad \text{(A.3.4)}$$

where denoting $PDS_{r,i} = \left[\theta^F_{r,i}\left(PD_{r,i}\right)^{1+\omega^C_i} + \left(1-\theta^F_{r,i}\right)\left(PI_{r,i}\right)^{1+\omega^C_i}\right]^{\frac{1}{1+\omega^C_i}}$ and rearranging terms yields:

$$\begin{aligned} XDI_{r,i} &= \left(1-\theta^F_{r,i}\right)\left(\frac{PI_{r,i}}{PDS_{r,i}}\right)^{1+\omega^C_i} \cdot \left(\frac{PD_{r,i}XDF_{r,i}+PI_{r,i}XDI_{r,i}}{PI_{r,i}}\right) \\ &= \left(1-\theta^F_{r,i}\right)\left(\frac{PI_{r,i}}{PDS_{r,i}}\right)^{1+\omega^C_i} \cdot \frac{PDS_{r,i}XDS_{r,i}}{PI_{r,i}} \\ &= \left(1-\theta^F_{r,i}\right)\left(\frac{PI_{r,i}}{PDS_{r,i}}\right)^{\omega^C_i} \cdot XDS_{r,i}, \end{aligned} \quad \text{(A.3.5)}$$

and by symmetry

$$XDF_{r,i} = \theta^F_{r,i}\left(\frac{PD_{r,i}}{PDS_{r,i}}\right)^{\omega^C_i} \cdot XDS_{r,i}. \quad \text{(A.3.6)}$$

Then, writing both expressions in *hat algebra* yields:

$$\widehat{XDI}_{r,i} = \left(\frac{\widehat{PI}_{r,i}}{\widehat{PDS}_{r,i}}\right)^{\omega^C_i} \cdot \widehat{XDS}_{r,i} \quad \text{(A.3.7)}$$

$$\widehat{XDF}_{r,i} = \left(\frac{\widehat{PD}_{r,i}}{\widehat{PDS}_{r,i}}\right)^{\omega^C_i} \cdot \widehat{XDS}_{r,i} \quad \text{(A.3.8)}$$

where $\widehat{PDS}_{r,i}$ is given as follows:

$$\widehat{PDS}_{r,i} = \frac{PDS_{r,i}}{\overline{PDS}_{r,i}}$$
$$= \frac{\left(\theta^F_{r,i}\left(PD_{r,i}\right)^{1+\omega^C_i} + \left(1-\theta^F_{r,i}\right)\left(PI_{r,i}\right)^{1+\omega^C_i}\right)^{\frac{1}{1+\omega^C_i}}}{\left(\theta^F_{r,i}\left(\overline{PD}_{r,i}\right)^{1+\omega^C_i} + \left(1-\theta^F_{r,i}\right)\left(\overline{PI}_{r,i}\right)^{1+\omega^C_i}\right)^{\frac{1}{1-\omega^C_i}}}.$$

Rearranging terms yields

$$\widehat{PDS}^{1+\omega^C_i}_{r,i} = \frac{\theta^F_{r,i}\left(PD_{r,}\right)^{1+\omega^C_i}}{\left(\theta^F_{r,i}\left(\overline{PD}_{r,i}\right)^{1+\omega^C_i} + \left(1-\theta^F_{r,i}\right)\left(\overline{PI}_{r,i}\right)^{1+\omega^C_i}\right)^{\frac{1}{1-\omega^C_i}}} + \frac{\left(1-\theta^F_{r,i}\right)\left(PI_{r,i}\right)^{1+\omega^C_i}}{\left(\theta^F_{r,i}\left(\overline{PD}_{r,i}\right)^{1+\omega^C_i} + \left(1-\theta^F_{r,i}\right)\left(\overline{PI}_{r,i}\right)^{1+\omega^C_i}\right)^{\frac{1}{1-\omega^C_i}}}$$
$$= \frac{1}{\left(\frac{1}{\widehat{PD}_{r,i}}\right)^{1+\omega^C_i}\left(1+\frac{1-\theta^F_{r,i}}{\theta^F_{r,i}}\left(\frac{\overline{PI}_{r,i}}{\overline{PD}_{r,i}}\right)^{1+\omega^C_i}\right)} + \frac{1}{\left(\frac{1}{\widehat{PI}_{r,i}}\right)^{1+\omega^C_i}\left(1+\frac{\theta^F_{r,i}}{1-\theta^F_{r,i}}\left(\frac{\overline{PD}_{r,i}}{\overline{PI}_{r,i}}\right)^{1+\omega^C_i}\right)},$$

such that using (A.3.4), which shows that

$$\frac{PI_{r,i}}{PD_{r,i}}\frac{XDI_{r,i}}{XDF_{r,i}} = \left(\frac{1-\theta^F_{r,i}}{\theta^F_{r,i}}\right)\left(\frac{PI_{r,i}}{PD_{r,i}}\right)^{1+\omega^C_i} \text{ and } \frac{PD_{r,i}}{PI_{r,i}}\frac{XDF_{r,i}}{XDI_{r,i}} = \left(\frac{\theta^F_{r,i}}{1-\theta^F_{r,i}}\right)\left(\frac{PD_{r,i}}{PI_{r,i}}\right)^{1+\omega^C_i},$$

we write this expression as:

$$\widehat{PDS}_{r,i} = \left[\frac{1}{\left(\frac{1}{\widehat{PD}_{ri}}\right)^{1+\omega^C_i}\left(1+\frac{\overline{PI}_{r,i}}{\overline{PD}_{r,i}}\frac{\overline{XDI}_{r,i}}{\overline{XDF}_{r,i}}\right)} + \frac{1}{\left(\frac{1}{\widehat{PI}_{r,i}}\right)^{1+\omega^C_i}\left(1+\frac{\overline{PD}_{r,i}}{\overline{PI}_{r,i}}\frac{\overline{XDF}_{r,i}}{\overline{XDI}_{r,i}}\right)}\right]^{\frac{1}{1+\omega^C_i}}$$
$$= \left[\left(1-\phi^{sCRH}_{r,i}\right)\left(\widehat{PD}_{r,i}\right)^{1+\omega^C_i} + \phi^{sCRH}_{r,i}\left(\widehat{PI}_{r,i}\right)^{1+\omega^C_i}\right]^{\frac{1}{1+\omega^C_i}},$$

where $\phi^{sCRH}_{r,i}$ denotes the share of the value of agricultural good i used in region r as an intermediate input in the total domestic supply of this good. We calculate it as $PIN_{r,i}qIN_{r,i}/PDS_{r,i}XDS_{r,i}$.

Now, moving up one level in intermediaries' CET supply structure, we solve for the optimal domestic supply of agricultural good i in region r, $XDS_{r,i}$ and the optimal supply of exports, $XExp_{r,i}$. To do it, we follow the same approach as above, but using expressions (A.3.3c) and (A.3.3d).

$$\frac{PDS_{r,i}}{PE_{r,i}} = \left(\frac{\theta^D_{r,i}}{1-\theta^D_{r,i}}\right)^{-\frac{1}{\omega^S_i}}\left(\frac{XDS_{r,i}}{XExp_{r,i}}\right)^{\frac{1}{\omega^S_i}}$$
$$\left(\frac{\theta^D_{r,i}}{1-\theta^D_{r,i}}\right)\left(\frac{PDS_{r,i}}{PE_{r,i}}\right)^{1+\omega^S_i} = \frac{PDS_{r,i}}{PE_{r,i}}\frac{XDS_{r,i}}{XExp_{r,i}}$$

$\Rightarrow$

$$\theta^D_{r,i}\left(PDS_{r,i}\right)^{1+\omega^S_i} = \frac{PDS_{r,i}}{PE_{r,i}}\frac{XDS_{r,i}}{XExp_{r,i}}\left(1-\theta^D_{r,i}\right)\left(PE_{r,i}\right)^{1+\omega^S_i}$$

$$\theta^D_{r,i}\left(PDS_{r,i}\right)^{1+\omega^S_i} + \left(1-\theta^D_{r,i}\right)\left(PE_{r,i}\right)^{1+\omega^S_i} = \left(1-\theta^D_{r,i}\right)\left(PE_{r,i}\right)^{1+\omega^S_i}\left(\frac{PDS_{r,i}}{PE_{r,i}}\frac{XDS_{r,i}}{XExp_{r,i}}+1\right) \tag{A.3.9}$$

where denoting $PCS_{r,i} = \left[\theta^D_{r,i}\left(PDS_{r,i}\right)^{1+\omega^S_i} + \left(1-\theta^D_{r,i}\right)\left(PE_{r,i}\right)^{1+\omega^S_i}\right]^{\frac{1}{1+\omega^S_i}}$ and rearranging terms yields:

$$\begin{aligned} XExp_{r,i} &= \left(1-\theta^D_{r,i}\right)\left(\frac{PE_{r,i}}{PCS_{r,i}}\right)^{1+\omega^S_i}\cdot\left(\frac{PDS_{r,i}XDS_{r,i}+PE_{r,i}XExp_{r,i}}{PE_{r,i}}\right) \\ &= \left(1-\theta^D_{r,i}\right)\left(\frac{PE_{r,i}}{PCS_{r,i}}\right)^{1+\omega^S_i}\cdot\frac{PCS_{r,i}XCS_{r,i}}{PE_{r,i}} \\ &= \left(1-\theta^D_{r,i}\right)\left(\frac{PE_{r,i}}{PCS_{r,i}}\right)^{\omega^S_i}\cdot XCS_{r,i}, \end{aligned} \tag{A.3.10}$$

and by symmetry

$$XDS_{r,i} = \theta^D_{r,i}\left(\frac{PDS_{r,i}}{PCS_{r,i}}\right)^{\omega^S_i}\cdot XCS_{r,i}. \tag{A.3.11}$$

Hence, writing these expressions in *hat algebra* yields:

$$\widehat{XExp}_{r,i} = \left(\frac{\widehat{PE}_{r,i}}{\widehat{PCS}_{r,i}}\right)^{\omega^S_i}\cdot\widehat{XCS}_{r,i} \tag{A.3.12}$$

$$\widehat{XDS}_{r,i} = \left(\frac{\widehat{PDS}_{r,i}}{\widehat{PCS}_{r,i}}\right)^{\omega^S_i}\cdot\widehat{XCS}_{r,i} \tag{A.3.13}$$

$$\widehat{PCS}_{r,i} = \left[\left(1-\phi^{sEXP}_{r,i}\right)\left(\widehat{PDS}_{r,i}\right)^{1+\omega^S_i} + \phi^{sEXP}_{r,i}\left(\widehat{PE}_{r,i}\right)^{1+\omega^S_i}\right]^{\frac{1}{1+\omega^S_i}}, \tag{A.3.14}$$

where $\phi^{sCRH}_{r,i}$ represents the share of agricultural exports of good i in region r in the total value of its current supply. We calculate it as $PREF_i XExp_{r,i}/PCS_{r,i}XCS_{r,i}$.

Finally, in order to solve for the optimal supply of good i in region r currently sold on the market, $XCS_{r,i}$, and optimal supply stored as inventories, $XSS_{r,i}$, we take the ratio of expressions (A.3.3a) and (A.3.3b).

$$\frac{PCS_{r,i}}{PSS_{r,i}} = \left(\frac{\theta^C_{r,i}}{1-\theta^C_{r,i}}\right)^{-\frac{1}{\omega^Q_i}}\left(\frac{XCS_{r,i}}{XSS_{r,i}}\right)^{\frac{1}{\omega^Q_i}}$$

$$\left(\frac{\theta^C_{r,i}}{1-\theta^C_{r,i}}\right)\left(\frac{PCS_{r,i}}{PSS_{r,i}}\right)^{1+\omega^Q_i} = \frac{PCS_{r,i}}{PSS_{r,i}}\frac{XCS_{r,i}}{XSS_{r,i}}$$

$\Rightarrow$

$$\theta_{r,i}^{C}\left(PCS_{r,i}\right)^{1+\omega_i^Q} = \frac{PCS_{r,i}}{PSS_{r,i}}\frac{XCS_{r,i}}{XSS_{r,i}}\left(1-\theta_{r,i}^{C}\right)\left(PSS_{r,i}\right)^{1+\omega_i^Q}$$

$$\theta_{r,i}^{C}\left(PCS_{r,i}\right)^{1+\omega_i^Q} + \left(1-\theta_{r,i}^{C}\right)\left(PSS_{r,i}\right)^{1+\omega_i^Q} = \left(1-\theta_{r,i}^{C}\right)\left(PSS_{r,i}\right)^{1+\omega_i^Q}\left(\frac{PCS_{r,i}}{PSS_{r,i}}\frac{XCS_{r,i}}{XSS_{r,i}}+1\right) \tag{A.3.15}$$

where denoting $PS_{r,i} = \left[\theta_{r,i}^{C}\left(PCS_{r,i}\right)^{1+\omega_i^Q} + \left(1-\theta_{r,i}^{C}\right)\left(PSS_{r,i}\right)^{1+\omega_i^Q}\right]^{\frac{1}{1+\omega_i^Q}}$ and rearranging terms yields:

$$\begin{aligned}
XSS_{r,i} &= \left(1-\theta_{r,i}^{C}\right)\left(\frac{PSS_{r,i}}{PS_{r,i}}\right)^{1+\omega_i^Q}\cdot\left(\frac{PCS_{r,i}XCS_{r,i}+PSS_{r,i}XSS_{r,i}}{PSS_{r,i}}\right)\\
&= \left(1-\theta_{r,i}^{C}\right)\left(\frac{PSS_{r,i}}{PS_{r,i}}\right)^{1+\omega_i^Q}\cdot\frac{PS_{r,i}XST_{r,i}}{PSS_{r,i}}\\
&= \left(1-\theta_{r,i}^{C}\right)\left(\frac{PSS_{r,i}}{PS_{r,i}}\right)^{\omega_i^Q}\cdot XST_{r,i}, \qquad (A.3.16)
\end{aligned}$$

and by symmetry

$$XCS_{r,i} = \theta_{r,i}^{C}\left(\frac{PCS_{r,i}}{PS_{r,i}}\right)^{\omega_i^Q}\cdot XST_{r,i}. \tag{A.3.17}$$

Then, writing these expressions in *hat algebra* yields:

$$\widehat{XSS}_{r,i} = \left(\frac{\widehat{PSS}_{r,i}}{\widehat{PS}_{r,i}}\right)^{\omega_i^Q}\cdot\widehat{XS}_{r,i} \tag{A.3.18}$$

$$\widehat{XCS}_{r,i} = \left(\frac{\widehat{PCS}_{r,i}}{\widehat{PS}_{r,i}}\right)^{\omega_i^Q}\cdot\widehat{XS}_{r,i} \tag{A.3.19}$$

$$\widehat{PS}_{r,i} = \left[\left(1-\phi_{r,i}^{sSTK}\right)\left(\widehat{PCS}_{r,i}\right)^{1+\omega_i^Q} + \phi_{r,i}^{sSTK}\left(\widehat{PSS}_{r,i}\right)^{1+\omega_i^Q}\right]^{\frac{1}{1+\omega_i^Q}}, \tag{A.3.20}$$

where $\phi_{r,i}^{sSTK}$ denotes the share of the value of the stocks of agricultural good i in region r on the total value of the supply of this good. We calculate it as $PSS_{r,i}XSS_{r,i}/PS_{r,i}XST_{r,i}$.

A.4 Market Clearing Conditions

A.4.1 Domestic Market Each Agricultural Good

The market clearance condition included in the model to simulate this market implies that:

$$XST_{r,i} = XDT_{r,i} \tag{A.4.1}$$

The total supply of each agricultural commodity i in region r is defined as

$$XST_{r,i} = BS_{r,i} + Qprd_{r,i} + M_{r,i}, \tag{A.4.2}$$

where $BS_{r,i}$ are the initial stocks, $Qprd_{r,i}$ the total production and $M_{r,i}$ aggregates the imports of i in r as final good, $XM_{r,i}$, as intermediate good to produce good l, $qIM^{IN}_{r,i,l}$, and for any other uses, $OM_{r,i}$. We write $M_{r,i}$ as:

$$M_{r,i} = XM_{r,i} + \sum_{l} qIM^{IN}_{r,i,l} + OM_{r,i} \tag{A.4.3}$$

The total demand for agricultural good i in region r is also defined as

$$XDT_{r,i} = DC_{r,i} + XExp_{r,i} + XSS_{r,i} + LS_{r,i} \tag{A.4.4}$$

where $DC_{r,i}$ aggregates the demand for each good i in r both as final good, $XA_{r,i}$ and as an intermediate good, $qIN_{r,i}$, $XExp_{r,i}$ denotes the optimal exports of i in r, $XSS_{r,i}$ the optimal level of final stocks, and $LS_{r,i}$ the residual loss of production. We write $DC_{r,i}$ as:

$$DC_{r,i} = XA_{r,i} + qIN_{r,i}, \tag{A.4.5}$$

and $LS_{r,i}$ is fixed share $\vartheta_{r,i}$ in the total supply of agricultural good i in country r, $XST_{r,i}$,

$$LS_{r,i} = \vartheta_{r,i} \cdot XST_{r,i}. \tag{A.4.6}$$

Finally, we write the corresponding expressions in *hat algebra* as follows:

$$\begin{aligned}
\widehat{XST}_{r,i} &= \frac{XST_{r,i}}{\overline{XST}_{r,i}} \\
&= \frac{BS_{r,i} + Qprd_{r,i} + M_{r,i}}{\overline{BS}_{r,i} + \overline{Qprd}_{r,i} + \overline{M}_{r,i}} \\
&= \left(\frac{\overline{BS}_{r,i}}{\overline{BS}_{r,i} + \overline{Qprd}_{r,i} + \overline{M}_{r,i}}\right)\widehat{BS}_{r,i} + \left(\frac{\overline{Qprd}_{r,i}}{\overline{BS}_{r,i} + \overline{Qprd}_{r,i} + \overline{M}_{r,i}}\right)\widehat{Qprd}_{r,i} + \\
&\quad + \left(\frac{\overline{M}_{r,i}}{\overline{BS}_{r,i} + \overline{Qprd}_{r,i} + \overline{M}_{r,i}}\right)\widehat{M}_{r,i} && \text{(A.4.7)} \\
&= \left(\phi^{sBS}_{r,i} \cdot \widehat{BS}_{r,i}\right) + \left(\phi^{sQprd}_{r,i} \cdot \widehat{Qprd}_{r,i}\right) + \left(\phi^{sM}_{r,i} \cdot \widehat{M}_{r,i}\right) && \text{(A.4.8)}
\end{aligned}$$

where $\phi^{sBS}_{r,i}$, $\phi^{sQpr}_{r,i}$ and $\phi^{sM}_{r,i}$ are the shares of initial stocks, production and imports of good i in region r, respectively, in the value of the total supply of i in r. Then, defining that the value of the total supply of i in r, $VTS_{r,i}$, equals $(PREF_i \cdot BS_{r,i}) + (PREF_i \cdot Qprd_{r,i}) + (PM_i \cdot M_{r,i})$, we calculate $\phi^{sBS}_{r,i}$ equals $(PREF_i \cdot BS_{r,i})/VST_{r,i}$, $\phi^{sQpr}_{r,i}$ to $(PREF_i \cdot Qprd_{r,i})/VST_{r,i}$, and $\phi^{sM}_{r,i}$ to

$\left(PM_{r,i} \cdot M_{r,i}\right) / VST_{r,i}$.

$\widehat{M}_{r,i}$ is also calculated as:

$$\begin{aligned}
\widehat{M}_{r,i} &= \frac{M_{r,i}}{\overline{M}_{r,i}} \\
&= \frac{XM_{r,i} + \sum_l qIM^{IN}_{r,i,l} + OM_{r,i}}{\overline{XM}_{r,i} + \sum_l \overline{qIM}^{IN}_{r,i,l} + \overline{OM}_{r,i}} \\
&= \left(\frac{\overline{XM}_{r,i}}{\overline{XM}_{r,i} + \sum_l \overline{qIM}^{IN}_{r,i,l} + \overline{OM}_{r,i}}\right) \widehat{XM}_{r,i} + \left(\frac{\sum_l \overline{qIM}^{IN}_{r,i,l}}{\overline{XM}_{r,i} + \sum_l \overline{qIM}^{IN}_{r,i,l} + \overline{OM}_{r,i}}\right) \sum_l \widehat{qIM}^{IN}_{r,i,l} + \\
&\quad + \left(\frac{\overline{OM}_{r,i}}{\overline{XM}_{r,i} + \sum_l \overline{qIM}^{IN}_{r,i,l} + \overline{OM}_{r,i}}\right) \widehat{OM}_{r,i} \\
&= \left(\phi^{mFF}_{r,i} \cdot \widehat{XM}_{r,i}\right) + \left(\phi^{mCC}_{r,i} \cdot \sum_l \widehat{qIM}^{IN}_{r,i,l}\right) + \left(\phi^{mOO}_{r,i} \cdot \widehat{OM}_{r,i}\right) \qquad \text{(A.4.9)}
\end{aligned}$$

where $\phi^{mFF}_{r,i}$, $\phi^{mCC}_{r,i}$ and $\phi^{mOO}_{r,i}$ are the share of imports of i in r as final good, as intermediate input and for any other use, respectively, in the total value of imports of i in r. Then, defining the value of the total imports of i in r, $VTM_{r,i}$, equals to $\left(PM_{r,i} \cdot M_{r,i}\right)$, we calculate $\phi^{mFF}_{r,i}$ equals to $\left(PM_{r,i} \cdot XM_{r,i}\right) / VTM_{r,i}$, $\phi^{mCC}_{r,i}$ to $\left(PM_{r,i} \cdot \sum_l qIM^{IN}_{r,i,l}\right) / VTM_{r,i}$, and $\phi^{m,OO}_{r,i}$ to $\left(PM_{r,i} \cdot OM_{r,i}\right) / VTM_{r,i}$.

Turning now to the total demand, we write it in *hat algebra* as

$$\begin{aligned}
\widehat{XDT}_{r,i} &= \frac{XDT_{r,i}}{\overline{XDT}_{r,i}} \\
&= \frac{DC_{r,i} + XExp_{r,i} + XSS_{r,i} + LS_{r,i}}{\overline{DC}_{r,i} + \overline{XExp}_{r,i} + \overline{XSS}_{r,i} + \overline{LS}_{r,i}} \\
&= \left(\frac{\overline{DC}_{r,i}}{\overline{DC}_{r,i} + \overline{XExp}_{r,i} + \overline{XSS}_{r,i} + \overline{LS}_{r,i}}\right) \widehat{DC}_{r,i} + \left(\frac{\overline{XExp}_{r,i}}{\overline{DC}_{r,i} + \overline{XExp}_{r,i} + \overline{XSS}_{r,i} + \overline{LS}_{r,i}}\right) \widehat{XExp}_{r,i} + \\
&\quad + \left(\frac{\overline{XSS}_{r,i}}{\overline{DC}_{r,i} + \overline{XExp}_{r,i} + \overline{XSS}_{r,i} + \overline{LS}_{r,i}}\right) \widehat{XSS}_{r,i} + \left(\frac{\overline{LS}_{r,i}}{\overline{DC}_{r,i} + \overline{XExp}_{r,i} + \overline{XSS}_{r,i} + \overline{LS}_{r,i}}\right) \widehat{LS}_{r,i} \\
&= \left(\phi^{dDC}_{r,i} \cdot \widehat{DC}_{r,i}\right) + \left(\phi^{dEX}_{r,i} \cdot \widehat{XExp}_{r,i}\right) + \left(\phi^{dST}_{r,i} \cdot \widehat{XSS}_{r,i}\right) + \left(\phi^{dLS}_{r,i} \cdot \widehat{LS}_{r,i}\right) \qquad \text{(A.4.10)}
\end{aligned}$$

where $\phi^{dDC}_{r,i}$, $\phi^{dEX}_{r,i}$, $\phi^{dST}_{r,i}$ and $\phi^{dLS}_{r,i}$ are the shares of total consumption, exports, final stocks and production losses, respectively, in the total value of demand of i in r. Then, defining the total value of consumption of i in r, $VTC_{r,i}$, equals to $PREF_i \cdot \left(\left(XA_{r,i} - XM_{r,i}\right) + \left(qIN_{r,i} - \sum_l qIM^{IN}_{r,i,l}\right)\right) + \left(PM_{r,i} \cdot M_{r,i}\right)$ and the total value of demand of i in r, $VDT_{r,i}$, as $VTC_{i,j} + \left(PE_{r,i} \cdot XExp_{r,i}\right) + \left(PREF_i \cdot XSS_{r,i}\right) + \left(PREF_i \cdot LS_{r,i}\right)$, $\phi^{dDC}_{r,i}$ equals $VTC_{r,i}/VDT_{r,i}$, $\phi^{dEX}_{r,i}$ to $\left(PE_{r,i} \cdot XExp_{r,i}\right) / VDT_{r,i}$, $\phi^{dST}_{r,i}$ to $\left(PREF_i \cdot XSS_{r,i}\right) / VDT_{r,i}$, and $\phi^{dLS}_{r,i}$ to $\left(PREF_i \cdot LS_{r,i}\right) / VDT_{r,i}$.

$\widehat{DC}_{r,i}$ is also calculated as:

$$
\begin{aligned}
\widehat{DC}_{r,i} &= \frac{DC_{r,i}}{\overline{DC}_{r,i}} \\
&= \frac{XA_{r,i} + qIN_{r,i}}{\overline{XA}_{r,i} + \overline{qIN}_{r,i}} \\
&= \left(\frac{\overline{XA}_{r,i}}{\overline{XA}_{r,i} + \overline{qIN}_{r,i}} \right) \widehat{XA}_{r,i} + \left(\frac{\overline{qIN}_{r,i}}{\overline{XA}_{r,i} + \overline{qIN}_{r,i}} \right) \widehat{qIN}_{r,i} \\
&= \left(\phi^{cF}_{r,i} \cdot \widehat{XA}_{r,i} \right) + \left(\phi^{cI}_{r,i} \widehat{qIN}_{r,i} \right) \qquad \text{(A.4.11)}
\end{aligned}
$$

where $\phi^{cF}_{r,i}$ and $\phi^{cI}_{r,i}$ are the shares of consumption of i in r as final consumption and as intermediate input, respectively, on the total value of consumption. $\phi^{cF}_{r,i}$ equals $\left(PREF_i \left(XA_{r,i} - XM_{r,i}\right) + \left(PM_{r,i} \cdot XM_{r,i}\right)\right)/VT$ and $\phi^{cI}_{r,i}$ to $\left(PREF_i \left(qIN_{r,i} - \sum_l qIM^{IN}_{r,i,l}\right) + \left(PM_{r,i} \cdot \sum_l qIM^{IN}_{r,i,l}\right)\right)/VTC_{r,i}$.

Finally, $LS_{r,i}$ is calculated as:

$$
\begin{aligned}
\widehat{LS}_{r,i} &= \frac{LS_{r,i}}{\overline{LS}_{r,i}} \\
&= \frac{\vartheta_{r,i} \cdot XST_{r,i}}{\vartheta_{r,i} \cdot \overline{XST}_{r,i}} \\
&= \widehat{XST}_{r,i} \qquad \text{(A.4.12)}
\end{aligned}
$$

A.4.2 International Market of Each Agricultural Good

The market clearance condition included in the model to simulate the international market of each agricultural good i is given by:

$$WX_i = WM_i. \qquad \text{(A.4.13)}$$

where the global supply and demand of agricultural good i is defined as:

$$WX_i = \sum_r XExp_{r,i} \qquad \text{(A.4.14)}$$

$$WM_i = \sum_r M_{r,i}. \qquad \text{(A.4.15)}$$

Now, writing these expressions in *hat algebra* yields:

$$
\begin{aligned}
\widehat{WX}_i &= \frac{WX_i}{\overline{WX}_i} \\
&= \frac{\sum_r XExp_{r,i}}{\sum_r \overline{XExp}_{r,i}} \\
&= \sum_r \left(\frac{\overline{XExp}_{r,i}}{\sum_r \overline{XExp}_{r,i}} \right) \widehat{XExp}_{r,i} \\
&= \sum_r \phi^{EX}_{r,i} \cdot \widehat{XExp}_{r,i} \qquad \text{(A.4.16)}
\end{aligned}
$$

and by symmetry

$$\begin{aligned}\widehat{WM}_i &= \frac{WM_i}{\overline{WM}_i} \\ &= \sum_r \phi^{IM}_{r,i} \cdot \widehat{M}_{r,i},\end{aligned} \tag{A.4.17}$$

where $\phi^{EX}_{r,i}$ and $\phi^{IM}_{r,i}$ is the share of the total exports and imports of i in r, respectively, on the global value of exports and imports of i. Therefore, $\phi^{EX}_{r,i}$ equals $\frac{PREF_i \overline{XExp}_{r,i}}{\sum_r PREF_i \overline{XExp}_{r,i}}$, and $\phi^{IM}_{r,i}$ to $\frac{PM_{r,i}\overline{M}_{r,i}}{\sum_r PM_{r,i}\overline{M}_{r,i}}$

A.4.3 Factor Markets

The market clearance condition included in the model to simulate the land market is given by:

$$qLND^S_r = \sum_i LN_{r,i}. \tag{A.4.18}$$

The model simulates the supply of land in each region r assuming an upward-sloping function with respect to the price of land with price elasticity $\epsilon^L > 0$.

$$qsLND_r = (1 + \eta_{r,i}) \times PNL_r^{\epsilon^L}, \tag{A.4.19}$$

such that written in *hat algebra* is given by:

$$\widehat{qsLND}_r = \left(\frac{1 + \eta_{r,i}}{1 + \overline{\eta}_{r,i}}\right) \times \widehat{PNL}_r^{\epsilon^L}. \tag{A.4.20}$$

Now, using the land demand to produce good k in each region r given by:

$$LN_{r,i} = \frac{\gamma^{LN}_{r,i}}{\Theta_{r,i}} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}}\right)^{-\sigma^F_i} VA_{r,i},$$

and denoting ϕ^{LN}_{ij} as the share of land to produce good i in region r in the total available land in r, calculated as $\frac{LN_{r,i}}{\sum_i LN_{r,i}}$, the clearance condition in the land market of every region r can be written as:

$$
\begin{aligned}
\widehat{qsLND}_r &= \sum_i \frac{\frac{\gamma_{r,i}^{LN}}{\Theta_{r,i}} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}} \right)^{-\sigma_i^F} VA_{r,i}}{\sum_i \frac{\gamma_{r,i}^{LN}}{\overline{\Theta}_{r,i}} \left(\frac{\overline{PLN}_r}{\overline{\Theta}_{r,i} \overline{PVA}_{r,i}} \right)^{-\sigma_i^F} \overline{VA}_{r,i}} \\
&= \sum_i \frac{\frac{\gamma_{r,i}^{LN}}{\Theta_{r,i}} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}} \right)^{-\sigma_i^F} VA_{r,i}}{\sum_i \frac{\gamma_{r,i}^{LN}}{\overline{\Theta}_{r,i}} \left(\frac{\overline{PLN}_r}{\overline{\Theta}_{r,i} \overline{PVA}_{r,i}} \right)^{-\sigma_i^F} \overline{VA}_{r,i}} \\
&= \sum_i \frac{\frac{\gamma_{r,i}^{LN}}{\Theta_{r,i}} \left(\frac{PLN_r}{\Theta_{r,i} PVA_{r,i}} \right)^{-\sigma_i^F} VA_{r,i}}{\sum_i \frac{\gamma_{r,i}^{LN}}{\overline{\Theta}_{r,i}} \left(\frac{\overline{PLN}_r}{\overline{\Theta}_{r,i} \overline{PVA}_{r,i}} \right)^{-\sigma_i^F} \overline{VA}_{r,i}} \\
&= \left(\frac{\frac{\gamma_{r,i}^{LN}}{\overline{\Theta}_{r,i}} \left(\frac{\overline{PLN}_r}{\overline{\Theta}_{r,i} \overline{PVA}_{r,i}} \right)^{-\sigma_i^F} \overline{VA}_{r,i}}{\sum_i \frac{\gamma_{r,i}^{LN}}{\overline{\Theta}_{r,i}} \left(\frac{\overline{PLN}_r}{\overline{\Theta}_{r,i} \overline{PVA}_{r,i}} \right)^{-\sigma_i^F} \overline{VA}_{r,i}} \right) \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i} \widehat{PVA}_{r,i}} \right)^{-\sigma_i^F} \left(\frac{\widehat{VA}_{r,i}}{\widehat{\Theta}_{r,i}} \right) \\
&= \phi_{ij}^{LN} \left(\frac{\widehat{PLN}_r}{\widehat{\Theta}_{r,i} \widehat{PVA}_{r,i}} \right)^{-\sigma_i^F} \left(\frac{\widehat{VA}_{r,i}}{\widehat{\Theta}_{r,i}} \right) \qquad \text{(A.4.21)}
\end{aligned}
$$

Turning now to the non-land market, we assume that this market is also in equilibrium when supply equals demand. However, for simplicity, we assume that the supply in this market is perfectly elastic, so that the market is *ex-ante* in equilibrium with a constant price normalized, PLN_r, equal to 1 and an equilibrium quantity set by the demand.

A.4.4 The Market for Inventories of Each Agricultural Good

The market clearance condition included in the model to simulate the market for inventories of each agricultural good i in region r is given by:

$$XSS_{r,i} = qdXSS_{r,i} \qquad \text{(A.4.22)}$$

Now, assuming that the demand for inventories of each agricultural good i can be represented with the following downward-sloping function with respect to the price with price elasticity $\epsilon^Q < 0$

$$qdXSS_{r,i} = PSS_{r,i}^{\epsilon^Q}, \qquad \text{(A.4.23)}$$

and using the expression for the supply of inventories of each i in r - derived above in (29),

$$\widehat{XSS}_{r,i} = \left(\frac{\widehat{PSS}_{r,i}}{\widehat{PS}_{r,i}} \right)^{\omega_i^Q} \cdot \widehat{XS}_{r,i}, \qquad \text{(A.4.24)}$$

the clearance condition on the market for inventories of good i in region r is given by

$$\widehat{XSS}_{r,i} = \widehat{PSS}_{r,i}^{\epsilon^Q} \qquad \text{(A.4.25)}$$

A.4.5 Market for Each Agricultural Good as an Intermediate Input

The market clearance condition included in the model to simulate the market for each agricultural good i in region r when used as intermediate input is given by

$$XDI_{r,i} = qIN_{r,i} \tag{A.4.26}$$

Now, using the expression for the supply of each agricultural good i in region r used as intermediate input defined above in (25)

$$\widehat{XDI}_{r,i} = \left(\frac{\widehat{PI}_{r,i}}{\widehat{PDS}_{r,i}}\right)^{\omega_i^C} \cdot \widehat{XDS}_{r,i} \tag{A.4.27}$$

and the expression for the demand derived above in (18)

$$\widehat{qIN}_{r,j} = \sum_i \left(\phi_{r,i,j}^{LC} \cdot \phi_{r,j}^{TTLC}\right) \cdot \widehat{qLC}_{r,i,j}^{IN} + \sum_i \left(\phi_{r,i,j}^{IM} \cdot \phi_{r,j}^{TTIM}\right) \cdot \widehat{qIM}_{r,i,j}^{IN} \tag{A.4.28}$$

the clearance condition on the market for good i in region r is given by:

$$\widehat{XDS}_{r,i} = \sum_i \left(\phi_{r,i,j}^{LC} \cdot \phi_{r,j}^{TTLC}\right) \cdot \widehat{qLC}_{r,i,j}^{IN} + \sum_i \left(\phi_{r,i,j}^{IM} \cdot \phi_{r,j}^{TTIM}\right) \cdot \widehat{qIM}_{r,i,j}^{IN} \tag{A.4.29}$$

B Price Elasticities of Demand

B.1 Theoretical Derivation

In this section, we show the theoretical derivation of the price elasticities of demand that we calibrate to execute the model. As detailed in the following section, these theoretical expressions of the price demand elasticities are essential to calibrate them with other estimates in the literature, using the data from the USDA/ERS projections.

B.1.1 Total Demand for Agricultural Goods

In expression A.1.19, we show that the total demand for agricultural goods in each region r is given by:

$$X_{r,AG} = \beta_{r,AG} \left(\frac{P_{r,AG}}{PU_r} \right)^{-\sigma^T} \frac{I_r}{PU_r} \tag{B.1.1}$$

where $PU_r = \left[\beta_{r,NA} \left(P_{r,NA} \right)^{1-\sigma^T} + \beta_{r,AG} \left(P_{r,AG} \right)^{1-\sigma^T} \right]^{\frac{1}{1-\sigma^T}}$.

Turning now to calculating the price elasticity of demand, we first write $X_{r,AG}$ as follows, using $\mathcal{K}$ to represent all constant terms in this demand expression.

$$X_{r,AG} = \mathcal{K} \left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}} \right). \tag{B.1.2}$$

We also calculate $\frac{\partial PU_r}{\partial P_{r,AG}}$ in an effort to facilitate the derivation of the elasticity of demand for agricultural goods in region r.

$$\begin{aligned} \frac{\partial PU_r}{\partial P_{r,AG}} &= \beta_{r,AG} \left(\frac{P_{r,AG}}{PU_r} \right)^{-\sigma^T} \\ &= \left[\frac{P_{r,AG} X_{r,AG}}{I_r} \right] \frac{PU_r}{P_{r,AG}} \\ &= \alpha_r \frac{PU_r}{P_{r,AG}} \end{aligned} \tag{B.1.3}$$

where α_r represents the expenditure share in region r devoted to agricultural goods.

We then simply calculate the price elasticity of demand for agricultural goods in region r, ε_r, as follows:

$$\begin{aligned}
\varepsilon_r &= \frac{\partial X_{r,AG}}{\partial P_{r,AG}} \frac{P_{r,AG}}{X_{r,AG}} \\
&= \mathcal{K}\left[\frac{\partial}{\partial P_{r,AG}}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\right]\frac{P_{r,AG}}{X_{r,AG}} \\
&= \mathcal{K}\left[\left(\frac{1}{\left(PU_r^{1-\sigma^T}\right)^2}\right)\left(-\sigma^T P_{r,AG}^{-\sigma^T-1} PU_r^{1-\sigma^T} - P_{r,AG}^{-\sigma^T}\left(1-\sigma^T\right) PU_r^{-\sigma^T}\frac{\partial PU_r}{\partial P_{r,AG}}\right)\right]\frac{P_{r,AG}}{X_{r,AG}} \\
&= \mathcal{K}\left[\left(\frac{1}{\left(PU_r^{1-\sigma^T}\right)^2}\right)\left(-\sigma^T P_{r,AG}^{-\sigma^T-1} PU_r^{1-\sigma^T} - P_{r,AG}^{-\sigma^T}\left(1-\sigma^T\right) PU_r^{-\sigma^T}\alpha_r\frac{PU_r}{P_{r,AG}}\right)\right]\frac{P_{r,AG}}{X_{r,AG}} \\
&= \mathcal{K}\left[\left(\frac{1}{PU_r^{1-\sigma^T}}\right)\left(-\sigma^T P_{r,AG}^{-\sigma^T-1} - P_{r,AG}^{-\sigma^T-1}\left(1-\sigma^T\right)\alpha_r\right)\right]\frac{P_{r,AG}}{X_{r,AG}} \\
&= \mathcal{K}\left[\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\left(-\sigma^T - \left(1-\sigma^T\right)\alpha_r\right)\right]\frac{1}{X_{r,AG}} \\
&= -\sigma^T - \left(1-\sigma^T\right)\alpha_r
\end{aligned} \tag{B.1.4}$$

B.1.2 Demand for an Agricultural Good

In expression A.1.10, we show that the demand for an agricultural good i in each region r can be written as:

$$XA_{r,i} = \beta_{n,i}\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N} XN_{r,n}. \tag{B.1.5}$$

Similarly, expression A.1.14 shows that the total demand for agricultural good in nest n and region r is given by:

$$XN_{r,n} = \beta_{r,n}\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U} X_{r,AG}, \tag{B.1.6}$$

and expression A.1.19 indicates the total demand for agricultural good is given by:

$$X_{r,AG} = \beta_{r,AG}\left(\frac{P_{r,AG}}{PU_r}\right)^{-\sigma^T}\frac{I_r}{PU_r} \tag{B.1.7}$$

Hence, the demand for and agricultural good i in region r can also be written as:

$$XA_{r,i} = \beta_{n,i}\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N}\beta_{r,n}\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\beta_{r,AG}\left(\frac{P_{r,AG}}{PU_r}\right)^{-\sigma^T}\frac{I_r}{PU_r} \tag{B.1.8}$$

where $PU_r = \left[\beta_{r,NA}\left(P_{r,NA}\right)^{1-\sigma^T} + \beta_{r,AG}\left(P_{r,AG}\right)^{1-\sigma^T}\right]^{\frac{1}{1-\sigma^T}}$, $P_{r,AG} = \left[\sum_n \beta_{r,n}\left(PN_{r,n}\right)^{1-\sigma^U}\right]^{\frac{1}{1-\sigma^U}}$, and $PN_{r,n} = \left[\sum_{i\in n}\beta_{n,i}\left(PA_{r,i}\right)^{1-\sigma_n^N}\right]^{\frac{1}{1-\sigma_n^N}}$.

Turning now to calculate the price elasticity of demand, we also begin by writing $XA_{r,i}$ as follows, using $\mathcal{K}$ to represent all constant terms in this demand expression.

$$XA_{r,i} = \mathcal{K} \left(\frac{PA_{r,i}}{PN_{r,n}} \right)^{-\sigma_n^N} \left(\frac{PN_{r,n}}{P_{r,AG}} \right)^{-\sigma^U} \left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}} \right) \tag{B.1.9}$$

We also solve the following derivatives in an effort to facilitate the calculation of this elasticity of demand of each agricultural good i in region r.

$$\begin{aligned} \frac{\partial PN_{r,n}}{\partial PA_{r,i}} &= \beta_{n,i} \left(\frac{PA_{r,i}}{PN_{r,n}} \right)^{-\sigma_n^N} \\ &= \left[\frac{PA_{r,i} XA_{r,i}}{PN_{r,n} XN_{r,n}} \right] \frac{PN_{r,n}}{PA_{r,i}} \\ &= \mathcal{S}_{r,i,n} \frac{PN_{r,n}}{PA_{r,i}} \end{aligned} \tag{B.1.10}$$

$$\begin{aligned} \frac{\partial P_{r,AG}}{\partial PA_{r,i}} &= \beta_{r,n} \left(\frac{PN_{r,n}}{P_{r,AG}} \right)^{-\sigma^U} \frac{\partial PN_{r,n}}{\partial PA_{r,i}} \\ &= \left[\frac{PN_{r,n} XN_{r,n}}{P_{r,AG} X_{r,AG}} \right] \frac{P_{r,AG}}{PN_{r,n}} \mathcal{S}_{r,i,n} \frac{PN_{r,n}}{PA_{r,i}} \\ &= \mathcal{S}_{r,n,AG} \cdot \mathcal{S}_{r,i,n} \frac{P_{r,AG}}{PA_{r,i}} \end{aligned} \tag{B.1.11}$$

$$\begin{aligned} \frac{\partial PU_r}{\partial PA_{r,i}} &= \beta_{r,AG} \left(\frac{P_{r,AG}}{PU_r} \right)^{-\sigma^T} \frac{\partial P_{r,AG}}{\partial PA_{r,i}} \\ &= \left[\frac{P_{r,AG} X_{r,AG}}{I_r} \right] \frac{PU_r}{P_{r,AG}} \mathcal{S}_{r,n,AG} \cdot \mathcal{S}_{r,i,n} \frac{P_{r,AG}}{PA_{r,i}} \\ &= \alpha_r \cdot \mathcal{S}_{r,n,AG} \cdot \mathcal{S}_{r,i,n} \frac{PU_r}{PA_{r,i}} \end{aligned} \tag{B.1.12}$$

where $\mathcal{S}_{r,i,n}$ represents the expenditure share in agricultural good i in region r within the total expenditure on goods in nest n, $\mathcal{S}_{r,n,AG}$ the expenditure share in good in nest n in region r within all agricultural products, and α_r the expenditure share in region r devoted to agricultural goods.

We then turn to calculate the price elasticity of demand for agricultural good i in region r, $\varepsilon_{r,i}$, as follows:

$$
\begin{aligned}
\varepsilon_{r,i} &= \frac{\partial XA_{r,i}}{\partial PA_{r,i}} \frac{PA_{r,i}}{XA_{r,i}} \\
&= \mathcal{K}\left(\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N}\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\right]\right)\frac{PA_{r,i}}{XA_{r,i}} \\
&= \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N}\underbrace{\left(\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\right]\right)}_{\mathcal{A}}\right]+ \\
&\quad + \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\underbrace{\left(\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N}\right]\right)}_{\mathcal{B}}\right].
\end{aligned}
\tag{B.1.13}
$$

Due to the length of this derivation, we solve by separated terms $\mathcal{A}$ and $\mathcal{B}$, and then substitute them back. In this process, we also use the result of expressions B.1.10, B.1.11, and B.1.12.

$$
\begin{aligned}
&= \mathcal{A} \\
&= \frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\right] \\
&= \left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\right]+\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\right] \\
&= \left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\left[\frac{1}{\left(PU_r^{1-\sigma^T}\right)^2}\right]\left[-\sigma^T P_{r,AG}^{-\sigma^T-1}\left(\frac{\partial P_{r,AG}}{\partial PA_{r,i}}\right)PU_r^{1-\sigma^T}-P_{r,AG}^{-\sigma^T}\left(1-\sigma^T\right)PU_r^{-\sigma^T}\left(\frac{\partial PU_r}{\partial PA_{r,i}}\right)\right]\right)+ \\
&\quad + \left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\left(\left[\frac{1}{\left(P_{r,AG}^{-\sigma^U}\right)^2}\right]\left[-\sigma^U PN_{r,n}^{-\sigma^U-1}\left(\frac{\partial PN_{r,n}}{\partial PA_{r,i}}\right)P_{r,AG}^{-\sigma^U}-PN_{r,n}^{-\sigma^U}\left(-\sigma^U\right)P_{r,AG}^{-\sigma^U-1}\left(\frac{\partial P_{r,AG}}{\partial PA_{r,i}}\right)\right]\right) \\
&= \left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\left[\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}PA_{r,i}}\right]\left[-\mathcal{S}_{r,n,AG}\cdot\mathcal{S}_{r,i,n}\right]\left[\sigma^T+\left(1-\sigma^T\right)\alpha_r\right]\right)+ \\
&\quad + \left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\left(\left[\frac{PN_{r,n}^{-\sigma^U}}{P_{r,AG}^{-\sigma^U}PA_{r,i}}\right]\left[-\sigma^U\mathcal{S}_{r,i,n}\right]\left[1-\mathcal{S}_{r,n,AG}\right]\right) \\
&= -\left[\frac{P_{r,AG}^{-\sigma^T}PN_{r,n}^{-\sigma^U}\mathcal{S}_{r,i,n}}{PU_r^{1-\sigma^T}PA_{r,i}P_{r,AG}^{-\sigma^U}}\right]\left[\mathcal{S}_{r,n,AG}\left[\sigma^T+\left(1-\sigma^T\right)\alpha_r\right]+\sigma^U\left[1-\mathcal{S}_{r,n,AG}\right]\right]
\end{aligned}
\tag{B.1.14}
$$

$$
\begin{aligned}
&= \mathcal{B} \\
&= \frac{\partial}{\partial PA_{r,i}} \left[\left(\frac{PA_{r,i}}{PN_{r,n}} \right)^{-\sigma_n^N} \right] \\
&= \left[\frac{1}{\left(PN_{r,n}^{-\sigma_n^N} \right)^2} \right] \left[-\sigma_n^N PA_{r,i}^{-\sigma_n^N - 1} PN_{r,n}^{-\sigma_n^N} - PA_{r,i}^{-\sigma_n^N} \left(-\sigma_n^N \right) PN_{r,n}^{-\sigma_n^N - 1} \left(\frac{\partial PN_{r,n}}{\partial PA_{r,i}} \right) \right] \\
&= - \left[\frac{\sigma_n^N}{\left(PN_{r,n}^{-\sigma_n^N} \right)^2} \right] \left[PA_{r,i}^{-\sigma_n^N - 1} PN_{r,n}^{-\sigma_n^N} - PA_{r,i}^{-\sigma_n^N} PN_{r,n}^{-\sigma_n^N - 1} \left(\mathcal{S}_{r,i,n} \frac{PN_{r,n}}{PA_{r,i}} \right) \right] \\
&= - \left[\frac{\sigma_n^N}{\left(PN_{r,n}^{-\sigma_n^N} \right)^2} \right] \left[PA_{r,i}^{-\sigma_n^N - 1} PN_{r,n}^{-\sigma_n^N} - PA_{r,i}^{-\sigma_n^N - 1} PN_{r,n}^{-\sigma_n^N} \mathcal{S}_{r,i,n} \right] \\
&= - \left[\frac{\sigma_n^N PA_{r,i}^{-\sigma_n^N - 1} PN_{r,n}^{-\sigma_n^N}}{\left(PN_{r,n}^{-\sigma_n^N} \right)^2} \right] \left[1 - \mathcal{S}_{r,i,n} \right] \\
&= - \left[\frac{\sigma_n^N PA_{r,i}^{-\sigma_n^N - 1}}{PN_{r,n}^{-\sigma_n^N}} \right] \left[1 - \mathcal{S}_{r,i,n} \right]. \qquad \text{(B.1.15)}
\end{aligned}
$$

Now, substituting B.1.14 and B.1.15 in B.1.13, the price elasticity of demand for agricultural good i in region r is given by:

$$
\begin{aligned}
\varepsilon_{r,i} = \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}} & \left[\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N} \underbrace{\left(\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\right]\right)}_{\mathcal{A}}\right] + \\
& + \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\underbrace{\left(\frac{\partial}{\partial PA_{r,i}}\left[\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N}\right]\right)}_{\mathcal{B}}\right] \\
& = \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\left(\frac{PA_{r,i}}{PN_{r,n}}\right)^{-\sigma_n^N}\left(-\left[\frac{P_{r,AG}^{-\sigma^T}PN_{r,n}^{-\sigma^U}\mathcal{S}_{r,i,n}}{PU_r^{1-\sigma^T}PA_{r,i}P_{r,AG}^{-\sigma^U}}\right]\left[\mathcal{S}_{r,n,AG}\left[\sigma^T+\left(1-\sigma^T\right)\alpha_r\right]+\sigma^U\left[1-\mathcal{S}_{r,n,AG}\right]\right]\right)\right] + \\
& + \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\left(\frac{PN_{r,n}}{P_{r,AG}}\right)^{-\sigma^U}\left(\frac{P_{r,AG}^{-\sigma^T}}{PU_r^{1-\sigma^T}}\right)\left(-\left[\frac{\sigma_n^N PA_{r,i}^{-\sigma_n^N-1}}{PN_{r,n}^{-\sigma_n^N}}\right]\left[1-\mathcal{S}_{r,i,n}\right]\right)\right] \\
& = -\mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\frac{XA_{r,i}}{\mathcal{K}}\left(\left[\frac{\mathcal{S}_{r,i,n}}{PA_{r,i}}\right]\left[\mathcal{S}_{r,n,AG}\left[\sigma^T+\left(1-\sigma^T\right)\alpha_r\right]+\sigma^U\left[1-\mathcal{S}_{r,n,AG}\right]\right]\right)\right] - \\
& - \mathcal{K}\frac{PA_{r,i}}{XA_{r,i}}\left[\frac{XA_{r,i}}{\mathcal{K}}\left(\left[\frac{\sigma_n^N}{PA_{r,i}}\right]\left[1-\mathcal{S}_{r,i,n}\right]\right)\right] \\
& = -\mathcal{S}_{r,i,n}\left[\mathcal{S}_{r,n,AG}\left[\sigma^T+\left(1-\sigma^T\right)\alpha_r\right]+\sigma^U\left[1-\mathcal{S}_{r,n,AG}\right]\right]-\sigma_n^N\left[1-\mathcal{S}_{r,i,n}\right] \\
& = -\left(1-\sigma^T\right)\alpha_r\cdot\mathcal{S}_{r,n,AG}\cdot\mathcal{S}_{r,i,n}-\left(\sigma^T-\sigma^U\right)\mathcal{S}_{r,n,AG}\cdot\mathcal{S}_{r,i,n}-\left(\sigma^U-\sigma_n^N\right)\mathcal{S}_{r,i,n}-\sigma_n^N
\end{aligned}
\tag{B.1.16}
$$

so that when a nest n pools a unique agricultural good k, case in which $\mathcal{S}_{r,i,n} = 1$ and $\sigma_n^N = 0$, the demand elasticity of agricultural good i in r is given by:

$$
\varepsilon_{r,i} = -\left(1-\sigma^T\right)\alpha_r\cdot\mathcal{S}_{r,n,AG}-\left(\sigma^T-\sigma^U\right)\mathcal{S}_{r,n,AG}-\sigma^U \tag{B.1.17}
$$

B.2 Calibration

In order to calibrate the price demand elasticities of each good k in region r with data from USDA/ERS projections, we use elasticity estimates available in the literature. Since most of these estimates are reported at the agricultural good level-i rather than at the more granular good-region level-r,i, we treat them as global elasticity estimates for each agricultural good i (See Table B4). Then, to ensure consistency, we calibrate these own price elasticities of demand using the data reported under the "World" region in the USDA/ERS projections.

Table B4: Reference Own Price Demand Elasticities for Agricultural Goods

Agricultural Good	$\overline{\varepsilon_i}$	Source
Barley	-0.273	Fally and Sayre (2018)
Beef and veal	-0.559	Maples et al. (2022)
Corn	-0.245	Fally and Sayre (2018)
Cotton	-0.950	Russo et al. (2008)
Pork	-0.790	Maples et al. (2022)
Poultry	-0.584	Maples et al. (2022)
Rice	-0.272	Fally and Sayre (2018)
Sorghum	-0.275	Fally and Sayre (2018)
Soybeans	-0.190	Fally and Sayre (2018)
Soybean meal	-2.490	Susanto (2006)
Soybean oil	-0.867	USDA/ERS (2011)
Wheat	-0.845	Fally and Sayre (2018)

Note: this table reports reference own price elasticity estimates available in the literature for agricultural goods in USDA/ERS projections. All elasticities with source Fally and Sayre (2018) correspond to the average point estimate calculated using the reported maximum and minimum value.

Mathematically, the calibration exercise consists in minimizing the squared normalized distance between the elasticity estimates available in the literature $\overline{\varepsilon}_i$, shown in Table B4, and the model own price elasticities of demand for each agricultural good ε_r, i, solved above in expressions B.1.16 and B.1.17. The objective is to estimate the elasticity of substitution among nests of agricultural goods σ^U and the elasticity of substitution among agricultural goods within every nest n, σ_n^N resulting from this minimization problem, given that α_r, $\mathcal{S}_{r,n,AG}$, and $\mathcal{S}_{r,i,n}$ are observed expenditure shares. The only constraints we add are that both elasticities of substitution must be non-negative (i.e. $\sigma^U \geq 0$, and $\sigma_n^N \geq 0$). In addition we impose that $\sigma^U \geq \sigma_n^N$ as a *Henning Conundrum Constraint Condition* to ensure that all nests of agricultural goods in the model are gross and net substitutes.[45] Therefore, the direct effect of a shock will always dominate the composite effect.

$$\min_{\sigma^U, \sigma_n^N} \sqrt{\sum_i \left(\frac{\epsilon_{r,i} - \overline{\epsilon}_i}{\overline{\epsilon}_i} \right)^2}. \tag{B.2.1}$$

subject to

$$\begin{aligned} \sigma^U &\geq 0 \\ \sigma_n^N &\geq 0 \\ \sigma^U &\geq \sigma_n^N \end{aligned}$$

This exercise yields the following calibrated elasticities when solved assuming eight distinct nests of agricultural goods in the model. These nests are: (1) Livestock, including beef and veal, poultry, and pork; (2) Cereals, including corn and sorghum; (3) Soy Secondary, including soybean meal and soybean oil; (4) Bread Cereals, including wheat; (5) Rice Cereals; including rice; (6) Soy Cereals; including soybeans; (7) Barley Cereals, including barley; and (8) Other, including cotton.

[45] See Francois and Hall (2007) for a detailed explanation about the importance of imposing the *Henning Conundrum Constraint Condition*.

Table B5: Reference vs. Calibrated Own Price Elasticities

Agricultural Good	Reference Values $\bar{\epsilon}_i$	Calibrated $\bar{\epsilon}_i$	World Consumption Share in USDA/ERS AG goods
Barley	-0.273	-0.276	0.012
Beef and veal	-0.559	-0.499	0.215
Corn	-0.245	-0.243	0.149
Cotton	-0.950	-0.273	0.024
Pork	-0.790	-0.650	0.118
Poultry	-0.584	-0.694	0.089
Rice	-0.272	-0.256	0.098
Sorghum	-0.275	-0.277	0.009
Soybeans	-0.190	-0.258	0.086
Soybean meal	-2.490	-0.749	0.054
Soybean oil	-0.867	-1.006	0.035
Wheat	-0.845	-0.253	0.110

Note: this table reports both the reference and calibrated own price elasticity estimates for each agricultural good included in the model. The world consumption share per good is included as reference, and is calculated using data from world consumption in year 2022/23 according to USDA/ERS projections and the average international prices of each agricultural good in 2022.

This exercise also indicates that the optimal values of the elasticities of substitution σ^U, and σ_n^N are:

Table B6: Calibrated Elasticities of Substitution

Nest	σ^U	σ_n^N
	0.279	
Livestock		0.833
Cereals		0.279
Soy Secondary		1.497
Bread Cereals		0
Rice Cereals		0
Soy Cereals		0
Barley Cereals		0
Other		0

Note: this table reports the calibrated elasticity of substitutions σ^U and σ_n^N resulting from calibrating the own prices of elasticities of the agricultural commodities included in the model.

C Cost Share of Land

In order to calculate the demand for land and for non-land factors, expressions (14) and (15) show that both are functions of the cost share of land, $\phi^f_{r,i}$. We therefore describe in this section the two-step process we follow to calculate it for each agricultural good i in region r.

For transparency, the available data used in the calculation is the following.

- $s_{r,i}$: share of land for agricultural good i in region r.
- $y_{r,i}$: yields for agricultural good i and region r.
- $\bar{\phi}^f_r$: cost share of land for region r.
- $VC_{US,i}$: variable costs in agricultural good i, only for the U.S.

The source of all data are the USDA's baseline projections, as described in Section 4. $s_{r,i} = \frac{LN_{r,i}}{\sum_i LN_{r,i}}$, where $LN_{r,i}$ is producers' demand for land. In addition, we take the regional-level land share $\bar{\phi}^f_r$ from Fuglie (2015). We download these data from USDA's *International Agricultural Productivity*.

C.1 Land Rents

The first step to calculate the cost share of land for every agricultural good i in region r consists in estimating the land rents w paid at that same level of disaggregation. These data are only available at the region level for which we estimate them, making two simplifying assumptions. First, all agricultural goods face the same land rents in a region r, $w_{r,i} = w_r$. This assumption allows us to write an expression for the cost share of land for each agricultural good i in region r as the ratio of land rents in region r to the total variable costs of producing agricultural good i in r.

$$\phi^f_{r,i} = \frac{w_r}{VC_{r,i}}. \tag{C.1.1}$$

Second, we assume that the observed cost share of land in every region r, denoted hereafter by $\bar{\phi}^f_r$, is simply a weighted average of such shares at the region and agricultural good i level as follows:

$$\bar{\phi}^f_r = \sum_i s_{r,i}\phi^f_{r,i}. \tag{C.1.2}$$

Then, substituting (C.1.1) in (C.1.2) yields

$$\begin{aligned}\bar{\phi}^f_r &= \sum_i s_{r,i}\frac{w_r}{VC_{r,i}} \\ &= w_r \sum_i s_{r,i}\frac{1}{VC_{r,i}}\end{aligned} \tag{C.1.3}$$

such that rearranging terms, the land rents in region r are given as follows:

$$w_r = \frac{\bar{\phi}^f_r}{\sum_i s_{r,i}\frac{1}{VC_{r,i}}}. \tag{C.1.4}$$

Finally, substituting (C.1.4) in (C.1.1), the cost share of land for an agricultural good i in region r is simply given by:

$$\phi^f_{r,i} = \bar{\phi}^f_r \frac{1}{\sum_{i'} s_{r,i'} \frac{VC_{r,i}}{VC_{r,i'}}}. \tag{C.1.5}$$

so that these shares are simply a function of observed data on $\bar{\phi}^f_r$ and $VC_{r,i}$. However, data on variable cost $VC_{r,i}$ are only available for the U.S. We therefore estimate those variable costs for other countries in the second step of our approach to calculate the cost share of land for every agricultural good i in region r. We detail specifically how we do it in the next section.

C.2 Variable Costs in Other Regions

In order to calculate the variable cost for producing each agricultural good i in region r, $VC_{r,i}$, we assume that the cross-sectional variation in the yields per harvested hectare of each agricultural good i in region r, $y_{r,i}$, with respect to such yields in the U.S., $y_{US,i}$, is only explained by cross-sectional variation in the variable cost of non-land factors for producing every good k in r. We therefore assume that yield ratios between other regions and the US can be written in terms of non-land costs as follows :

$$\frac{y_{r,i}}{y_{US,i}} = \frac{(1-\phi^f_{r,i})VC_{r,i}}{(1-\phi^f_{US,i})VC_{US,i}}. \tag{C.2.1}$$

Now, using (C.1.1) to determine the variable costs of producing agricultural good i in region r and rearranging the terms, we write (C.2.1) as follows:

$$\begin{aligned}
(1-\phi^f_{r,i})VC_{r,i} &= \frac{y_{r,i}}{y_{US,i}}(1-\phi^f_{US,i})VC_{US,k} \\
(1-\phi^f_{r,i})\frac{w_r}{\phi^f_{r,i}} &= \frac{y_{r,i}}{y_{US,i}}(1-\phi^f_{US,i})VC_{US,i} \\
&= H_{r,i}
\end{aligned} \tag{C.2.2}$$

such that the variable cost of producing each agricultural good i in region r, $VC_{r,i}$ is written in terms of the central unknown of this exercise: the cost share of land to produce i in r, $\phi^f_{r,i}$. $H_{r,i}$ also gathers all the observed non-land variable cost to produce agricultural good i in region r.

Hence, this result leads us to the same initial question: what is the value of $\phi^f_{r,i}$? However, the difference is that it shows us that we can write an expression for the land rents in region r as a function of the cost share of land for producing agricultural good i in region r by rearranging the terms as follows:

$$\phi^f_{r,i} = \frac{w_r}{H_{r,i} + w_r}. \tag{C.2.3}$$

Then, substituting this resulting expression in (C.1.2) yields:

$$\begin{aligned}
\bar{\phi}^f_{r,i} &= \sum_i s_{r,i} \frac{w_r}{H_{r,i} + w_r} \\
&= w_r \sum_i \frac{s_{r,i}}{H_{r,i} + w_r},
\end{aligned} \tag{C.2.4}$$

that is a non-linear equation with respect to w_r. More importantly, we can use it to solve the value of w_r by finding the root of the following function:

$$f(w_r) = w_r \sum_i \frac{s_{r,i}}{H_{r,i} + w_r} - \bar{\phi}^f_{r,i} \tag{C.2.5}$$

We therefore use this expression to solve for w_r. We then use the solution to find the cost share of land to produce agricultural good k in region r, $\phi^f_{r,i}$, substituting it into (C.2.3). Finally, we filter extreme values from the solution, by winsorizing the results at the 5% and 95% quantiles. All values that are smaller than the 5% quantile or greater than 95% quantile are replaced by the corresponding quantile.

D Parameters of the Model

The following Table shows the value of all parameters used in the calibration of model and its corresponding sources. They represent our preferred values for each parameter, but users can change them as needed. The model is designed to adjust to any specific value in the parameters, except for the strict zeros assumed for σ^T, σ_i^I, and ω_i^Q. These zeros are fundamental theoretical assumptions of the model. Any change in them may cause the model to crash.

Table D1: Values of Parameters in Calibration of Model

Parameter	Description	Value	Source
σ^T	ES between AG and non-AG goods	0^*	Theoretical assumption
σ^U	ES among AG nests	$\star$	Calibrated value
σ_n^N	ES among AG goods within a nest	$\star$	Calibrated value
σ_i^D	ES between local and imported demand	3.8	Bernard et al. (2007)
σ_i^I	ES between value added generation and intermediates transformation	0^*	Theoretical assumption
σ_i^F	ES between land and no-land factors	$0.9^\dagger$	Theoretical assumption
σ_i^C	ES between local and imported good used as input to produce another	$2^\bullet$	Theoretical assumption
ω_i^Q	ET between current and stored supply	0^*	Theoretical assumption
ω_i^S	ET between the local and exported supply	$5^\ddagger$	Hillberry and Hummels (2013)
ω_i^C	ET between supply of a good sold as final good and as input to produce another	$5^\bullet$	Theoretical assumption
ϵ_i^Q	OPE of demand of inventories	0.5°	Theoretical assumption
ϵ^L	OPE of supply of land	0.11	Hertel and Baldos (2016)

Note: ES stands for elasticity of substitution, ET for elasticity of transformation, OPE for own price elasticity, and AG for agricultural. The subindex n refers to a nest of agricultural goods and the subindex i to each agricultural good. Any parameter without a subindex indicates that the parameter takes a unique value in the calibration of the model.
* This value is assigned based on the theoretical assumption that optimal decisions explained by this ES or ET are modeled assuming a Leontief structure.
* This is a calibrated value shown in Table B6.
† This value is assigned based on the theoretical assumptions made to predict the land cost shares in see Appendix C.
‡ This value corresponds to the average point estimate of the interval in which ω_i^S ranges according to Hillberry and Hummels (2013).
$^\bullet$ Both values are assigned relying on the theoretical assumption that producers of AG goods used as input to produce others can easily shift local and exports in response to a price change. Similarly, consumers of these AG goods can easily make it between local and imports. Both parameters are set to our preferred values, but users can adjust them as needed.
$^\circ$ This value is assigned based on the theoretical assumption that the demand for an AG good increases and its corresponding demand for inventories decreases in response to a decrease in prices, and vice-versa. Hence, assuming a slight response in the inventories, the own price elasticity for inventories of every AG good in the model is assumed positive and inelastic. This parameters is set to our preferred value, but users can adjust it as needed.