Creating GTAP-Money: a GTAP model with financial assets and liabilities; monetary policy; excess capacity; and nominal exchange rates

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Abstract

Standard GTAP provides a medium-term view of the effects of tariffs and other policies. To enhance GTAP for short-run analysis, we create GTAP-Money by adding to standard GTAP: (1) accumulation of financial assets and liabilities; (2) monetary policy; (3) unemployment, excess capacity and inflation; and (4) nominal exchange rates.

Our theoretical specification introduces a financial agent for each country r that solves an optimization problem, taking account of expected rates of return, to determine the allocation of its budget between financing physical capital and buying bonds in all countries. Introduction of financial agents handles features (1) and (2). It provides a framework for keeping track of each country's wealth, saving, current account, and its accumulation of foreign assets and liabilities. Through its optimization problem, financial agent r responds to monetary tightening (e.g. an increase in interest rates on bonds issued by r's central bank) by reallocating part of its budget *away from* acquiring foreign assets and financing r's physical assets and *towards* buying government bonds. This is consistent with the aim of monetary policy, namely, to have a short-run modifying influence on domestic demand mainly through affecting physical investment.

For embracing feature (3), we specify asymmetric responses of factor prices to changes in demand. We assume that nominal wages do not move down significantly in the short run in response to unemployment but move up sharply if unemployment goes below the normal rate of frictional unemployment. We assume similar asymmetric responses for capital rental rates.

We work through the GTAP equations adding a nominal exchange rate for each country r. Endogenous movements in *nominal* exchange rates [feature (4)] provide the extra degrees of freedom required to determine *real* exchange rates when domestic price levels are tied down by sticky nominal factor prices.

The paper contains illustrative simulations.

1. Introduction

In light of recent U.S. trade policies, economists in treasury and trade departments in governments around the world are looking for tools to help them understand the effects of tariffs. GTAP is the obvious choice. However, for economists in these departments with backgrounds in macroeconomic modeling, GTAP assumptions seem problematic. Macro economists are critical of the lack of explicit recognition in the standard GTAP model of:

(1) accumulation of financial assets and liabilities at a national level[1];
(2) monetary policy;
(3) excess capacity and "stranded" physical assets; and
(4) nominal exchange rates.

Standard GTAP provides a medium to long-term view of the effects of tariffs, but the absence of features (1) – (4) limits its value for analyzing first-year effects. Quick access to results on these short-run effects is of central interest to policy makers grappling with the evolving and unpredictable nature of U.S. trade policies.

In this paper, we set out the theory GTAP-Money. This is a version of GTAP that embraces features (1) – (4). The model is still in preliminary form, but we provide illustrative simulations that suggest that GTAP-Money captures the main elements of monetary policy. It is hoped that eventually GTAP-Money will provide insights on the short-run effects of tariffs, while at the same time improve communication between economists working in GTAP's applied general equilibrium paradigm and economists with a macro, monetary viewpoint.

The central ingredient in the extension of GTAP to GTAP-Money is the introduction of a financial agent for each country[2]. At the end of each year, the agent for country r allocates a budget between three classes of financial assets:

bonds issued by governments throughout the world;
equity issued by firms throughout the world; and
liquidity used by residents (firms, households and the government) in country r.

Agent r's budget for the end of year is determined by the end-of-year value of bonds, equity and liquidity that it held at the start of the year, supplemented by saving and the public sector borrowing requirement generated during the year. In allocating the budget, agent r takes account of expected rates of return on financial assets.

The introduction of a financial agent for each country handles feature (1). It provides an accounting framework for keeping track of each country's wealth, saving, current account, and its accumulation of foreign assets and liabilities. The optimization behavior assumed for the financial agents in GTAP-Money handles feature (2). For example, in GTAP-Money, financial agents throughout the world, but especially in country r, respond to an increase in the interest rates on bonds issued by r's government by reallocating part of their budgets away from financing r's physical assets towards buying r's government bonds. This is consistent with the aim of monetary policy, namely to have a short-run modifying influence on domestic demand mainly through affecting physical investment (construction, etc).

[1] Previous versions of GTAP with explicit treatments of foreign assets and liabilities include Ianchovichina and. McDougall (2012), and Dixon *et al.* (2021).
[2] Most regions in our version of GTAP are a single country. We use the expression country even though a few regions in the model are composites of countries, e.g. Rest of Europe.

In standard GTAP, a country's stock of physical capital is mobile between industries and the rental rate paid by industries for using it adjusts to ensure that total demand across industries coincides with the existing stock. Instead, in GTAP-Money, we assume that the stock of capital in each industry is fixed in the short run[3]. While we allow rental increases in response to increases in demand beyond normal levels, we do not allow significant short-run rental reductions when demand for using capital in an industry falls below normal levels. This asymmetric treatment is consistent with feature (3). It gives a version of GTAP that generates short-run unemployment of capital, that is excess capacity. If an industry-specific negative shock persists, then under-use of existing capital in the industry can last into the long run, that is physical assets can become stranded.

It is relatively common in CGE modeling to encapsulate short-run movements in employment through *real* wage rigidities.[4] In GTAP-Money, in common with the practice of monetary economists, we introduce *nominal* wage rigidities. Similar to our treatment of capital rentals, we assume that nominal wage rates respond asymmetrically to variations in demand for labor.

With short-run movements in a country's price level largely tied down by asymmetric movements in nominal rental and wage rates we are in danger of over-determination. Without nominal exchange rates, standard GTAP requires endogenous movements in domestic price levels to determine *real* exchange rate movements. Endogenous real exchange rates are required to reconcile trade flows that are sensitive to relative price movements between countries with trade balances that are given largely as residuals after the determination of:

GDP, via factor markets (that determine inputs of K and L) and total-factor productivity;
private consumption, via consumption functions;
public consumption, via consumption functions or exogenous settings; and
investment, via expected rates of return and monetary policy.

In creating GTAP-Money, we worked through the GTAP equations adding a nominal exchange rate for each country. These exchange rates represent the movement in the number of units of international currency (think $US or SDR) per unit of local currency. Problems concerning revaluations of foreign assets and liabilities are navigated. Endogenous movements in *nominal* exchange rates [feature (4)] provide the extra degrees of freedom required to determine *real* exchange rates when domestic price levels are substantially tied down.

In Section 2, we set out the monetary extensions of GTAP that lead to GTAP-Money. Section 3 provides two illustrative simulations of the effects of monetary tightening by the U.S.: one representing an increase in the cash-rate; and the other representing liquidity operations such as withdrawal of liquidity from commercial banks through compulsory increases in their deposits in the central bank. These applications are proof of concept. They give results that are plausible and will meet qualitative expectations of macro-monetary economists. Concluding remarks are in Section 4, which also contains a list of tasks required to improve GTAP-Money to a point where it produces results that are defensible in policy circles.

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[3] In moving from a model in which capital is mobile between industries to one in which capital is fixed in each industry, we use the method described in Dixon *et al.*(2019).

[4] This assumption underlies much of the early Australian modeling of short-run impacts of tariffs, taxes and other policies, see for example Dixon *et al.* (1977, 1982) and Corden and Dixon (1980).

2. Theory

The standard GTAP model has been described in several publications starting with Hertel (1997). More recent descriptions are in Corong *et.al.* (2017) and Aguiar *et al.* (2019). Here we assume a basic familiarity with GTAP and set out the new theory we have added to form GTAP-Money.

Subsection 2.1 describes the financial agents. This introduces concepts that are developed in later subsections. These include the budget of a financial agent (subsection 2.2), rates of return on different financial assets (subsection 2.3), and valuation factors that translate start-year values to end-year values (subsection 2.5). Subsection 2.4 sets out the relationship between the saving-investment gap for each country and its accumulation of net foreign assets. The specification of income and interest receipts and payments on foreign assets and liabilities is in subsection 2.6. The relationship between the change in liquidity, new bonds and the public-sector borrowing requirement is described in subsection 2.7 and derived in Appendix 1. Our assumptions concerning short-run adjustment in nominal factor prices in response to changes in demand are in subsection 2.8. Subsection 2.9 discusses equilibrating mechanisms in GTAP-Money that connect the real side of the economy with the monetary side.

2.1. Optimization by financial agents

The central addition that we have made to the standard GTAP model is introduction of a financial agent for each country.

At the end of each year the agent for country r

chooses ZB1(s,r) and ZE1(s,r) for all s, and LA1(r)

to maximize

$$\mathrm{U}\left[\mathit{RB(s,r)}*\mathrm{ZB1(s,r)}\ \text{for all s;}\quad \mathit{RE(s,r)}*\mathrm{ZE1(s,r)}\ \text{for all s;}\quad \mathit{RLA(r)}*\mathrm{LA1(r)}\right] \tag{2.1}$$

subject to

$$\mathrm{B1(r)} = \sum_{s}[\mathrm{ZB1(s,r)} + \mathrm{ZE1(s,r)}] + \mathrm{LA1(r)} \quad . \tag{2.2}$$

where

U is a utility function that we assume has a CES form;

ZB1(s,r) is the end-of-year value of r's holdings in r's currency of bonds issued by government in s[5];

ZE1(s,r) is the end-of-year value of r's holdings in r's currency of equity issued by firms in s;

LA1(r) is liquidity in r's currency held by residents, firms and government in r ;

[5] In calculating ZB1(r,r) we exclude bonds issued by r's government and held by r's central bank. We think of r's government as a consolidated entity that includes the central bank. We treat purchases by the central bank of government bonds as creation of new money.

RB(s,r) is the power of the rate of return (1 plus the rate) that agent r expects on holdings of bonds issued by government in s;

RE(s,r) is the power of the rate of return that agent r expects on holdings of equity issued by firms in s;

RLA(r) is the power of the rate of return that agent r expects on holdings of liquidity in r; and

B1(r) is the total value of funds in r's currency that agent r has available at the end of year to allocate between bonds and equity in all countries and liquidity just in r.

In (2.1) and (2.2) we simplify reality by assuming that there are only three financial assets in each country:

Bonds issued by the government. This is public sector debt held outside the government, which includes the central bank. Our definition of bonds includes any financial liability instrument issued by the government. For example, it includes deposits by commercial banks in the central bank. For most countries, public sector debt is a substantial fraction of GDP. As shown in Table 2.1, in our data for the U.S. it is 80.91 per cent of GDP.

Equity issued by firms. This refers to financing physical capital. It includes not only what is normally thought of as equity (that is shares) but also debt instruments issued by firms to finance capital. The total value of equity defined this way is assumed equal to the replacement value of physical capital. We obtained the values shown in Table 2.1 from our GTAP database. For all countries the value of physical capital is 2 to 4 times GDP.

Liquidity held by residents, firms and government. This is narrow money (M1). It consists of notes and coins plus readily accessible bank balances and credit (credit cards). The ratio of narrow money to GDP varies widely across countries, from 10 per cent for Brazil to 120 per cent for Japan.

With a CES utility function we obtain the following percentage change equations for agent r:

$$\mathrm{zb1(s,r)} = \mathrm{b1(r)} + (\sigma-1)*\left(\mathrm{rb(s,r)} - \mathrm{roreave(r)}\right) + \mathrm{fzb(s,r)} - \mathrm{fave(r)} \quad \text{for all s,} \tag{2.3}$$

$$\mathrm{ze1(s,r)} = \mathrm{b1(r)} + (\sigma-1)*\left(\mathrm{re(s,r)} - \mathrm{roreave(r)}\right) + \mathrm{fze(s,r)} - \mathrm{fave(r)} \text{ for all s,} \tag{2.4}$$

$$\mathrm{la1(r)} = \mathrm{b1(r)} + (\sigma-1)*\left(\mathrm{rla(r)} - \mathrm{roreave(r)}\right) + \mathrm{fla(r)} - \mathrm{fave(r)}, \tag{2.5}$$

$$\mathrm{rorave(r)} = \frac{\sum_{s}[\mathrm{ZB1(s,r)}*\mathrm{rb(s,r)} + \mathrm{ZE1(s,r)}*\mathrm{re(s,r)}] + \mathrm{LA1(r)}*\mathrm{rla(r)}}{\sum_{s}[\mathrm{ZB1(s,r)} + \mathrm{ZE1(s,r)}] + \mathrm{LA1(r)}}, \tag{2.6}$$

and

$$\mathrm{fave(r)} = \frac{\sum_{s}[\mathrm{ZB1(s,r)}*\mathrm{fzb(s,r)} + \mathrm{ZE1(s,r)}*\mathrm{fze(s,r)}] + \mathrm{LA1(r)}*\mathrm{fla(r)}}{\sum_{s}[\mathrm{ZB1(s,r)} + \mathrm{ZE1(s,r)}] + \mathrm{LA1(r)}}, \tag{2.7}$$

where
the lowercase symbols in equations (2.3) to (2.7) are percentage changes in the variables represented by the corresponding uppercase symbols; and

Table 2.1. Financial instruments by country as a per cent of GDP, start of 2015

	Bonds issued*	Equity issued**	Liquidity held***
1 USA	80.91	310	30
2 China	35.00	394	55
3 Japan	144.59	386	120
4 Skorea	9.49	385	40
5 France	86.35	384	40
6 Germany	52.17	390	40
7 Brazil	35.64	275	10
8 India	64.05	305	20
9 Russia	15.29	213	20
10 Australia	22.11	342	40
11 RoAmerica	30.74	279	20
12 RoAsia	35.00	283	20
13 RoEurope	66.18	306	40
14 Africa	39.37	242	20
15 RoW	40.00	298	30
16 UK	77.58	355	75
17 Canada	28.62	384	40
18 Mexico	46.49	329	23

* These numbers are estimates of net national debt. This is gross national debt *less* intragovernmental debt such as money owed by the government to the central bank. Our data source is the IMF's World Economic Outlook for 2015 (https://www.imf.org/en/publications/weo/weo-database/2022/october). For India, China and Russia, data were available only for gross national debt. For these countries we obtained the net national debt numbers by making small downward adjustments to the gross numbers. For the amalgamated countries we used average net national debt numbers.

** Based on data for start-of-year values for capital stocks in our version of the GTAP model.

*** This is narrow money. The numbers were derived from Codera Analytics (https://codera.co.za/which-regions-have-the-highest-share-of-narrow-and-broad-money-in-gdp/).

In our own files, bonds and equity are distributed across asset-owning countries in Fin15_170426.xlsx, sheet important data. This file imports data from Table 1_170426.xlsx which gets data from WEO_NET&Gross debt to GDP.xlsx

σ is agent r's elasticity of substitution between earning money on different financial instruments. In our illustrative simulations we assume that σ equals 2.

In (2.3) to (2.5) we include percentage-change shift variables [fzb(s,r), fze(s,r), fla(r)] and their average [fave(r), defined by (2.7)]. These variables can be used to represent changes in the preferences (utility function) of agent r. We use preference shifts in simulations of liquidity operations in which agent r is obliged to shift towards or against buying r-government bonds. Recall that bonds are defined to include deposits by commercial banks in the central bank.

2.2. End-of-year budget for financial agents

We determine the end-of-year budget for agent r, B1(r), by

$$\begin{aligned} B1(r) = & \sum_s ZB0(s,r) * VBScnd(s,r) + \sum_s ZE0(s,r) * VEScnd(s,r) \\ & + LA0(r) + PSBR(r) + SAVE(r) \end{aligned} \tag{2.8}$$

In (2.8), "0" refers to a start-of-year entity. ZB0(s,r) and ZE0(s,r) are the start-of-year values of r's holdings in r's currency of bonds and equity issued in s, and LA0(r) is the start-of-year value of liquidity in r.

PSBR(r) is country r's public-sector borrowing requirement or the public sector deficit. By running a deficit, the government provides money to r's financial agent who can reallocate this money between bonds, equity and liquidity.

SAVE(r) is the value of r's saving (net of depreciation) during the year, defined as net national product less private and public consumption.

VBScnd(s,r) and VEScnd(s,r) are revaluation factors, to be discussed in subsection 2.5. They take account of exchange rate and price changes that affect the values of financial assets in the transition from start of year to end of year. The logic of including "Scnd" in the names of these revaluation factors is that they refer to revaluations in the currency of the second-mentioned country, in this case country r. In subsection 2.5, we will be introducing revaluation factors that include "Frst".

What equation (2.8) says is that the financial resources available to agent r at the end of the year to be allocated between bonds, equity and liquidity to be held in the next year are made up of assets from the beginning of the year, revalued where appropriate, supplemented by the public sector deficit and saving.

2.3. Specification of rates of return

Bonds

We assume that powers of rates of return on bonds are policy instruments:

$$\mathrm{RB(s,r) = FCR(s)*CR(s)} \tag{2.9}$$

where we think of CR(s) as the power of the cash rate in s and FCR(s) as an exogenous markup factor.

Equity

Towards working out a formula for the expected rates of return on business investments (what we refer to as equity), we start by calculating the end-of-year value of a unit of capital obtained during the year for industry j in country r plus its rental earnings, given by:

$$\mathrm{ENDV(j,r) = PCGSDS1(r)*(1-D) + PCAP(j,r)*\frac{XCAPUSE(j,r)}{KB(j,r)}} \tag{2.10}$$

In (2.10),

PCAP(j,r) is the rental applying to units of capital in industry j in country r.

XCAPUSE(j,r) is the quantity of capital in use and KB(j,r) is the quantity of capital in existence in industry (j,r). The ratio XCAPUSE(j,r)/KB(j,r) is the fraction of (j,r)'s available capital that is used. If the ratio is 0.95, then we assume that investors plan as if only 95 per cent of an additional unit of capital will earn the current rental.

PCGDS1(r) is the end-year price of a unit of capital, and PCGDS1(r)*(1-D) is its depreciated value at the end of the year. In the GTAP model, including our version, the input composition and hence the price of units of capital are the same across all industries j in country r. Depreciation rates (D) are the same for all industries globally.

We define the power of the *current* rate of return [RORC(j,r)] on capital in industry (j,r) by dividing ENDV(j,r) by the outlay required to obtain an extra unit of capital during the year.

We assume that this outlay is the average price in the year of units of capital, that is, the average of the start-year and end-year prices. Thus, we obtain

$$\mathrm{RORC}(j,r) \equiv \frac{\mathrm{ENDV}(j,r)}{0.5*\left[\mathrm{PCGDS0}(r)+\mathrm{PCGDS1}(r)\right]}$$

$$= \frac{\mathrm{PCAP}(j,r)*\dfrac{\mathrm{XCAPUSE}(j,r)}{\mathrm{KB}(j,r)}+\mathrm{PCGSDS1}(r)*(1-D)}{0.5*\left[\mathrm{PCGDS0}(r)+\mathrm{PCGDS1}(r)\right]} \qquad (2.11)$$

where PCGDS0(r) is the lagged price, or start-year price of units of capital in country r.

We define the power of the rate of return *expected* by investors from any country s when they invest in any industry j in country r as.

$$\mathrm{RE}(r,s) \ = \mathrm{RORC}(j,r)*\left(\frac{\mathrm{K1}(j,r)}{\mathrm{K0}(j,r)}\right)^{-\mathrm{RORFLEX}} \qquad \text{for all s and j} \qquad (2.12)$$

where K0(j,r) and K1(j,r) are quantities of capital in (j,r) at start- and end-year, and RORFLEX is a positive parameter.[6]

In (2.12), we assume that investors from all countries spread their investments across r's industries to equalize expected rates of return. This determines the end-year allocation of r's capital between industries, and also means that no j argument is required for the expected rates of return on the LHS of (2.12).

Under (2.12), if no increase in (j,r)'s capital stock is planned for the year, then capital creators expect rates of return to stay at their current levels [RORC(j,r)]. If capital is growing through the year, then capital creators discount for additional risk, moving expected rates of return below current rates of return.

As explained in subsection 2.9, expected rates of return determined in (2.12) not only drive optimization behavior by financial agents [ZE1(r,s) in (2.1) – (2.2)] but also connect with the real side of the economy via

$$\sum_{s} \mathrm{ZE1}(r,s)*\frac{\mathrm{E1}(s)}{\mathrm{E1}(r)} = \mathrm{VK1}(r) \qquad (2.13)$$

$$\mathrm{VK1}(r) = \mathrm{VK0}(r)*\frac{\mathrm{PCGDS1}(r)}{\mathrm{PCGDS0}(r)}+\mathrm{I}(r) \qquad (2.14)$$

and

$$\sum_{s} \mathrm{ZE0}(r,s)*\frac{\mathrm{E0}(s)}{\mathrm{E0}(r)} = \mathrm{VK0}(r) \qquad (2.15)$$

where

6 Equation (2.12) is used in standard GTAP. It originates from Dixon *et al*. (1977 and 1982). For RORFLEX we used the value 2. In earlier applications of (2.12), values between 5 and 10 were used. The toned-down value used here reflects our use of *powers* of rates of return rather than rates of return.

ZE1(r,s) and ZE0(r,s) are (as implied by our earlier definition) end-year and start-year values of s's holdings in s's currency of equity issued by firms in r;
E1(s) and E0(s) are s's exchange rate defined as the number of units of international currency (think $US or SDR or gold) per unit of s's currency at end-year and start-year;
VK0(r) and VK1(r) are the start-year and end-year values in r's currency of r's capital stock; and
I(r) is the value of net investment in country r during the year, valued at end-year prices for capital goods [PCGDS1(r)].

On the LHS of (2.13) is the value in r's currency of equity finance provided to r at the end of the year. This is equated to the end-year value of r's capital. In (2.14), the end-year value of r's capital stock is given by the start-year value, revalued by changes through the year in the price of capital goods, *plus* net investment during the year. In (2.15) the start-year value of r's capital stock is equated to the value in r's currency of start-year equity finance. In these three equations, we assume that financial agents value capital stocks at replacement cost, that is we fix Tobin's Q at one. A potential extension of our modeling would be endogenization of Tobin's Q to recognize imperfect competition and barriers to setting up new firms.

Liquidity

We specify the power of the rate of return on liquid assets as

$$\mathrm{RLA(r)} = \left(\frac{\mathrm{GDP(r)} / \mathrm{LA1(r)}}{\mathrm{GDP2LA(r)}} \right)^{\gamma} \tag{2.16}$$

where γ is a positive parameter and GDP2LA (r) is a coefficient representing the normal ratio of nominal GDP to liquidity. We refer to RLA(r) as a shadow rate of return. It reflects the value that residents put on liquidity but doesn't generate actual payments.

If the ratio of nominal GDP to liquidity strays above the normal ratio, then RLA(r) will rise. Via the optimization problem (2.1) - (2.2), this will encourage increased provision of liquidity by financial agent r.

2.4. The saving-investment gap and net foreign assets

The surplus (deficit) of saving relative to investment in country r is balanced by net financial capital outflow from r (inflow to r). This balance is specified by

$$\begin{aligned}
\mathrm{SAVE(r)} - \mathrm{I(r)} = {} & \sum_{s \neq r} \mathrm{ZB1(s,r)} - \sum_{s \neq r} \mathrm{ZB0(s,r)} * \mathrm{VBScnd(s,r)} \\
& + \sum_{s \neq r} \mathrm{ZE1(s,r)} - \sum_{s \neq r} \mathrm{ZE0(s,r)} * \mathrm{VEScnd(s,r)} \\
& - \left\{ \sum_{s \neq r} \mathrm{ZB1(r,s)} * \frac{\mathrm{E1(s)}}{\mathrm{E1(r)}} - \sum_{s \neq r} \mathrm{ZB0(r,s)} * \mathrm{VBFrst(r,s)} \right\} \\
& - \left\{ \sum_{s \neq r} \mathrm{ZE1(r,s)} * \frac{\mathrm{E1(s)}}{\mathrm{E1(r)}} - \sum_{s \neq r} \mathrm{ZE0(r,s)} * \mathrm{VEFrst(r,s)} \right\}
\end{aligned} \tag{2.17}$$

On the LHS of (2.17) is the saving-investment surplus generated in the real side of the economy. On the RHS is the increase in country r's *net* foreign assets from start-year to end-year beyond what can be explained by revaluation factors. This is made up of four terms. The first term is r's end-year holding of foreign bonds *less* start-year holdings revalued. The second term is r's end-year holding of foreign equity *less* start-year holdings revalued. The third and fourth terms deal with of offsetting increases r's foreign liabilities. In these terms, ZB1(r,s) and ZE1(r,s) are end-year values of s's holdings in s's currency of bonds and equity issued by government and firms in r. These values are converted to r's currency via the factor E1(s)/E1(r). ZB0(r,s) and ZE0(r,s) are start-year values of s's holdings in s's currency. They are revalued and converted to r's currency via the factors VBFst(r,s) and VEFrst(r,s). Notice that the use of "Frst" in these valuation factors indicates that the conversion is to the currency of the country identified by the first argument.

2.5. Revaluation factors

In (2.8) and (2.17) we introduced revaluation factors that convert start-year values of financial assets to end-year values.

In setting revaluation factors for equity, we assume that equity in country s held by country r is redeemable in s's currency. This means that its end-year value in s's currency reflects price changes in s during the year. For calculating its end-year value in r's currency we must convert by using r and s's end-year exchange rates.

In setting revaluation factors for bonds, we assume that bonds issued by country s and held by country r are repayable in international currency. This means that their end-year values from the point of view of country r are independent of developments through the year in country s. For calculating their end-year value in r's currency we must convert by using r's end-year exchange rate.

VBScnd(s,r)

The start-year value in r's currency of bonds held by r and issued by s [ZB0(s,r)] transitions to an end-year value in r's currency via the valuation factor:

$$\mathrm{VBScnd}(s,r) = \frac{\mathrm{E0}(r)}{\mathrm{E1}(r)} \quad \text{for } s \neq r \quad \text{and} \quad \mathrm{VBScnd}(r,r) = 1 \tag{2.18}$$

where E0(r) and E1(r) are r's exchange rate at start and end of year. Underlying (2.18) is the assumption that bonds are held for one year and those issued by country s and held by country r ($s \neq r$) and are repayable in international dollars. We assume that bonds issued by r and held by residents of r are repayable in r's currency, giving a valuation factor of 1.

VEScnd(s,r)

The start-year value in r's currency of equity held by r and issued by s [ZE0(s,r)] transitions to an end-year value in r's currency via the valuation factor:

$$\mathrm{VEScnd}(s,r) = \frac{\mathrm{E0}(r)}{\mathrm{E0}(s)} * \frac{\mathrm{PCGDS1}(s)}{\mathrm{PCGDS0}(s)} * \frac{\mathrm{E1}(s)}{\mathrm{E1}(r)} \quad \text{for } s, r \tag{2.19}$$

The first term on the RHS of (2.19) converts r's start-year equity holding into s's currency. In the second term, we assume that equity investments in s appreciate according to movements through the year in capital goods prices. Then at the end of year their value in s's currency is converted to r's currency via the end-year exchange rate ratio.

VBFrst(r,s)

The start-year value in s's currency of bonds held by s and issued by r [ZB0(r,s)] transitions to an end-year value in r's currency via the valuation factor:

$$\mathrm{VBFrst}(r,s) = \frac{\mathrm{E0}(s)}{\mathrm{E1}(r)} \quad \text{for } s \neq r \quad \text{and } \mathrm{VBFrst}(r,r) = 1 \tag{2.20}$$

In (2.20), we continue to assume that bonds are repayable in international currency. The numerator on the RHS translates r's start-year bond liability to s from s's currency to international currency. Then, the denominator translates from international currency to the end-year value in r's currency.

VEFrst(r,s)

The start-year value in s's currency of equity held by s and issued by r [ZE0(r,s)] transitions to an end-year value in r's currency via the valuation factor:

$$\mathrm{VEFrst}(r,s) = \frac{\mathrm{E0}(s)}{\mathrm{E0}(r)} * \frac{\mathrm{PCGDS1}(r)}{\mathrm{PCGDS0}(r)} \quad \text{for } s, r \tag{2.21}$$

The first term on the RHS of (2.21) converts s's start-year equity holding into r's currency. In the second term, we assume that equity investments in r appreciate according to movements through the year in capital goods prices.

Relationships between valuation factors

VBFrst(r,s) and VBScnd(r,s) are both applied to the same entity: s's start-year holding of bonds issued by r. They both transition ZB0(r,s) to an end-year value. The difference is that VBFrst(r,s) generates the end-year value in r's currency while VBScnd(r,s) generates the value in s's currency. Consequently, it is reassuring to note from (2.18) and (2.20) that

$$\frac{\mathrm{VBFrst}(r,s)}{\mathrm{VBScnd}(r,s)} = \frac{\mathrm{E1}(s)}{\mathrm{E1}(r)} \tag{2.22}$$

Similarly, (2.19) and (2.22) imply that

$$\frac{\mathrm{VEFrst}(r,s)}{\mathrm{VEScnd}(r,s)} = \frac{\mathrm{E1}(s)}{\mathrm{E1}(r)} \tag{2.23}$$

2.6. Receipts and payments on foreign assets and liabilities

In calculating net national product, we include net foreign income given by

$$\mathrm{NFY}(r) = \mathrm{YFB}(r) + \mathrm{YFE}(r) - \mathrm{PFB}(r) - \mathrm{PFE}(r) \tag{2.24}$$

where

NFY(r) is net foreign income for country r expressed in r's currency;

YFB(r) an PFB(r) are receipts and payments on r's foreign bond assets and liabilities; and

YFE(r) an PFE(r) are receipts and payments on r's foreign equity assets and liabilities.

Receipts by country r and payments by country s are calculated according to

$$\mathrm{YFB(r)} = \sum_{s \neq r} \mathrm{ZB0(s,r)} * \frac{\mathrm{E0(r)}}{\mathrm{E0(s)}} * \left(\mathrm{CR(s)} - 1\right) * \frac{\mathrm{E1(s)}}{\mathrm{E1(r)}} \tag{2.25}$$

$$\mathrm{PFB(s)} = \sum_{r \neq s} \mathrm{ZB0(s,r)} * \frac{\mathrm{E0(r)}}{\mathrm{E0(s)}} * \left(\mathrm{CR(s)} - 1\right) \tag{2.26}$$

$$\mathrm{YFE(r)} = \sum_{s \neq r} \left\{ \frac{\mathrm{ZE0(s,r)} * \mathrm{E0(r)}/\mathrm{E0(s)}}{\mathrm{VKB(s)}} \right\} * \left(\mathrm{VOA("capital",s)} - \mathrm{VDEP(s)}\right) * \frac{\mathrm{E1(s)}}{\mathrm{E1(r)}} \tag{2.27}$$

$$\mathrm{PFE(s)} = \sum_{r \neq s} \left\{ \frac{\mathrm{ZE0(s,r)} * \mathrm{E0(r)}/\mathrm{E0(s)}}{\mathrm{VKB(s)}} \right\} * \left(\mathrm{VOA("capital",s)} - \mathrm{VDEP(s)}\right) \tag{2.28}$$

In (2.25), we calculate r's receipts from its holding of bonds in s (s ≠r) by first looking at the start-year value of these bonds in r's currency, ZB0(s,r). We translate this value into s's currency by multiplying by E0(r)/E0(s). Next, we calculate the interest payable on the bonds in s's currency by multiplying by s's cash rate [CR(s)-1, power less one]. Finally, we translate this value into an end-year payment in r's currency by multiplying by E1(s)/E1(r).

In (2.26), s's payment to r on account of r's holding of bonds in s is the same as r's receipts calculated in (2.25) but expressed in s's currency rather than r's.

In (2.27), we calculate r's receipts from its holding of equity in s (s ≠r) by first looking at the start-year value of this equity in r's currency, ZE0(s,r). We translate this value into s's currency by multiplying by E0(r)/E0(s). By dividing by the start-year value of s's capital [VKB(s)] we obtain r's share in the ownership of s's capital. We apply this share to the total rental earnings of s's capital net of depreciation [VOA("capital",s)-VDEP(s)]. This gives s's payments to r in s's currency. Finally, we translate to an end-year payment in r's currency by multiplying by E1(s)/E1(r).

In (2.28), s's payment to r on account of r's holding of equity in s is the same as r's receipts calculated in (2.27) but expressed in s's currency rather than r's.

Notice in (2.27) and (2.28) that there is no industry dimension. We assume that the fraction of (j,s)'s equity held by r is the same for all j. Under this assumption, we can calculate income and payment flows in terms of s's aggregate earnings, VOA("capital",s), and r's aggregate ownership share of s's capital without mention of industries.

2.7. The relationship between the change in liquidity, the public-sector borrowing requirement and new bonds

In Appendix 1 we use (2.2), (2.8), (2.13) to (2.15) and (2.21) to show that

$$\mathrm{LA1(r)} - \mathrm{LA0(r)} = \mathrm{PSBR(r)} - \left\{ \sum_{s} \mathrm{ZB1(r,s)} * \frac{\mathrm{E1(s)}}{\mathrm{E1(r)}} - \sum_{s} \mathrm{ZB0(r,s)} * \mathrm{VBFrst(r,s)} \right\} \tag{2.29}$$

That is

$$\Delta \mathrm{LA(r)} = \mathrm{PSBR(r)} - \mathrm{nBONDS(r)} \tag{2.30}$$

where

ΔLA(r) is the change in liquidity in country r from start-year to end-year; and nBONDS(r) is the revenue collected by government r from the issue of new bonds during the year, that is the increase in the value of bonds from start-year to end-year, beyond the increase that can be explained by revaluation.

This is given by:

$$\mathrm{nBONDS(r)} = \sum_{s} \mathrm{ZB1(r,s)} * \frac{\mathrm{E1(s)}}{\mathrm{E1(r)}} - \sum_{s} \mathrm{ZB0(r,s)} * \mathrm{VBFrst(r,s)} \qquad (2.31)$$

Equation (2.30) means that the increase during the year in liquid assets held by the residents of country r is what the government borrows from the central bank. This is new money created in r. It is the part of the government deficit that is not financed by new bonds.

2.8. Factor-price-factor-use adjustment curves: sticky nominal wages and rental rates

Managing the macro economy would be relatively easy if prices adjusted quickly and continuously to ensure full employment of labor and capital. However, factor prices move sluggishly and movements in *real* factor prices can be stymied by adjustments in commodity prices. For example, restoration of full employment in the wake of a negative shock such as a tariff imposed by a trade partner may require a reduction in real wages. Even if weakening in the labor market causes nominal wages to fall, reductions in inflation may generate a perverse increase in real wages. Consequently, for looking at macro-policy problems in a CGE model, it is necessary to introduce explicit assumptions about short-run responses in nominal factor prices to changes in factor demands. For both labor and capital, we include in GTAP responses of the form illustrated by the curve in Figure 2.1.

In this figure, the X axis measures factor-use. For labor, this is hours worked relative to labour supply, and for capital it is capital-in-use relative to capital-in-existence. The Y axis measures the nominal wage rate or the nominal rental rate. In the simulations reported in Section 3, the model contains factor-price-factor-use adjustment curves in all countries for skilled and unskilled labor and for capital by industry.

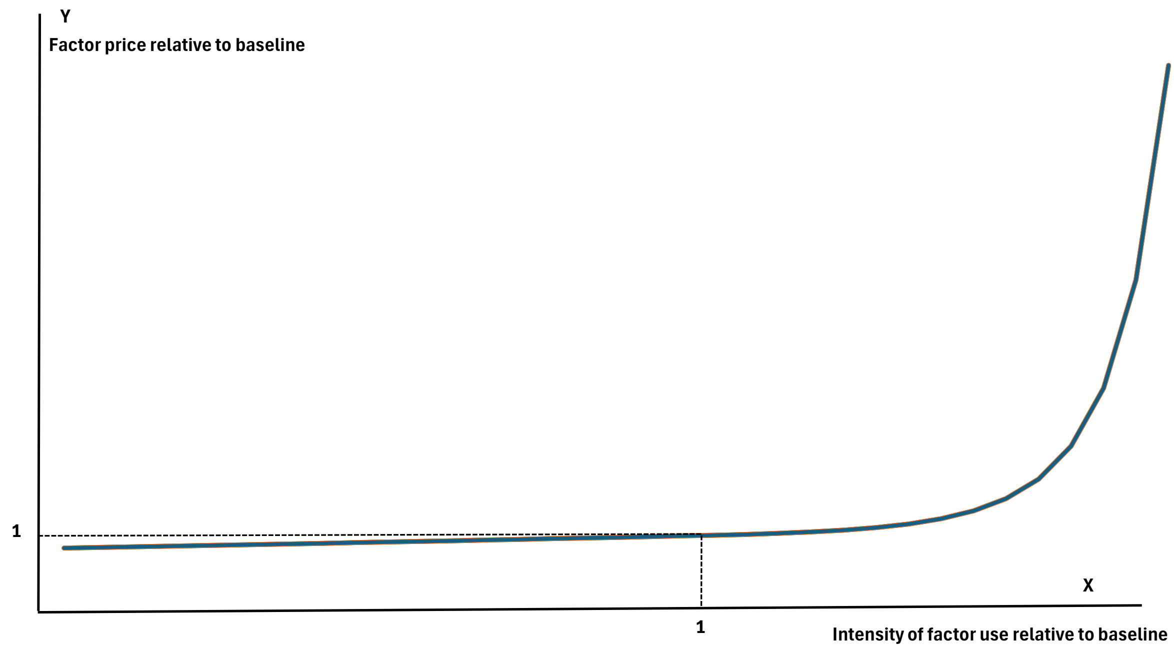
As shown in Figure 2.1, the curves pass through the point (1,1). This is because on the two axes the factor-use and price levels are relative to a business-as-usual baseline. We assume that if factor-use is on baseline, then the factor price will also be on baseline. If in a simulation of the effects of a shock, possibly combined with a policy response, we find that factor-use goes above baseline, then we assume that the factor price goes above baseline. Similarly, if factor-use goes below baseline, then the factor price goes below baseline.

The functional forms we adopt for the factor-price-factor-use curves are asymmetrical around the (1,1) point. As factor-use passes the baseline level (X moves beyond 1) the factor price moves strongly above the baseline level (Y moves strongly above 1). On the other hand, when X moves below 1, the downward movement in Y is muted.

At the upper end, the asymmetric shape recognizes resource constraints. We assume that there are upper limits on labor hours and on the intensity with which capital can be used. As factor-use moves towards its upper limit, demand for the factor is choked off by sharp increases in its price.

At the lower end, the asymmetric shape recognizes resistance, at least in the short run, to reductions in factor prices. This is most obvious for labor, where institutional arrangements

Figure 2.1. Factor-price-factor-use adjustment curve



in many countries mean that cuts in nominal wage rates are unusual. Our labor curves introduce some downward flexibility because wages can go below the level anticipated (baseline) for a year but not require a cut in nominal wages below the level of the previous year.

For capital, the standard CGE assumption in short-run analysis is that the quantity available in each industry is predetermined or exogenous, and that the demand for this capital is brought into equality with the available supply via adjustments in the capital rental rate. We have found that the standard assumption can lead to extreme and unrealistic movements in capital rental rates.[7] For short-run analysis, asymmetric capital-supply curves allow, realistically, for excess capacity (unused capital) in a downturn.

Under the zero-pure-profits assumption in our CGE model, product prices to a large extent reflect factor prices. In a downturn, sluggish downward movements in capital rentals and wages translate into sluggish downward movements in product prices.

As described in Appendix 2, the functional forms for the factor-price-factor-use curves differ between labor and capital. For capital, the Y values go lower at the low end and are higher for the relevant range of X values at the high end. With this configuration and with capital-labor substitution elasticities less than 1 (0.5 in Section 3), the capital share in GDP tends to fall in a downturn and rise in an upturn. This is consistent with a large volume of business-cycle literature, see for example U.S. Bureau of Labor Statistics (2026) and La Cava (2019).

[7] See for example Dixon and Rimmer (2011).

2.9. Equilibrating mechanisms: connecting the real side of the economy with the behavior of the financial agents

The financial agents in our model hold three types of financial assets: equity that finances capital creation; liquidity that facilitates day-to-day transactions; and government bonds that at least partially finance the government deficit. What variables coordinate the decisions by the financial agents to hold these financial assets with demands by capital creators for investment finance, demands by the public for transaction finance and demands by the government for deficit finance?

Equity coordination

For equity, the coordinating variable for each country r is the power of the rate of return expected by the financial agents of all countries s on their equity investments in all industries in r. We denote this expected rate of return by

$$RE\_s(r) = RE(r,s) \quad \text{for all s} \tag{2.32}$$

With higher values for RE_s(r), financial agents through their optimizing decisions supply more equity finance to country r, ZE1(r,s) increases for all s, see (2.4). On the other hand, via (2.12), higher values for RE_s(r) correspond to lower values for the end-year capital stock in country r: as they undertake more projects, risk-averse capital creators in r adopt lower expected rates of return. In the RE_s(r)-VK1(r) space [where VK1(r) is the end-year value of r's capital stock] we have an upward-sloping supply curve encapsulating the behavior of the financial agents and a downward-sloping demand curve encapsulating the behavior of capital creators. The equilibrium or coordinating value for RE_s(r) is determined by the intersection of these two curves.

Favourable shocks on the real side of the r's economy induce increases in current rates of return [the RORC(j,r)s]. This can be thought of as a rightward shift in the demand curve for equity finance by capital creators. A new equilibrium will be established with higher values for RE_s(r) and VK1(r).

Favourable financial shocks for country r mean that financiers are willing to supply more money for capital creation in r at any given expected rate of return. This could reflect increased confidence in r's economy and could be simulated by positive values for the relevant preference variables in (2.4), fze(r,s) for all s. We can think of these favourable financial shocks as rightward shifts in the supply curve for equity finance to country r. At the new equilibrium, RE_s(r) will be lower, and VK1(r) will be higher.

Liquidity coordination

For liquidity, the coordinating variable for each country r is the power of the rate of return on liquid assets, RLA(r). Equation (2.16) provides a downward-sloping demand curve for liquidity in country r and optimizing behavior by r's financial agent provides an upward-sloping supply curve, see (2.5).

An increase in nominal GDP causes a rightward shift in the demand curve (the public requires more liquidity), resulting in a new equilibrium with increased values for RLA(r) and LA1(r).

A willingness by the financial system to provide more liquidity through credit cards or other means causes a rightward shift in the supply curve, with a consequent reduction in RLA(r) and increase in LA1(r). This could be simulated for country r with a positive value for the preference variable fla(r) in (2.5).

Bond coordination

For bonds, identifying the coordinating variable for each country r is less intuitive than for equity and liquidity. It turns out that the answer is the exchange rate, E1(r).

The nature of the problem can be seen by looking at (2.30) which for convenience is repeated here:

$$\Delta LA(r) = PSBR(r) - nBONDS(r) \qquad (\textit{repeated, } 2.30)$$

In (2.30), the LHS is determined by demand for and supply of liquidity. On the RHS, the government deficit (PSBR) is determined by the government's expenditure and taxation policies. Recalling that the rate of return on bonds in country r is tied down by r's cash-rate, and that the cash-rate is exogenous, it is not obvious how (2.30) is put in place.

Rather than asking how (2.30) is equilibrated, we ask an equivalent question: what is the equilibrating variable ensuring decisions of financial agents which determine country r's net foreign assets, are compatible with the saving-investment gap in country r? That is, what puts (2.17) in place?

That this is an equivalent question can be established by noting that given the definitions of r's financial budget in (2.2) and (2.8), (2.17) holds if and only if (2.30) holds. This is proved in Appendix 1. Consequently, if we understand how (2.17) is equilibrated then we also understand how (2.30) is equilibrated.

To aid the discussion, we repeat (2.17) in stripped down notation:

$$NETFA1(r) = SAVE(r) - I(r) + NETFA0\_REVAL(r) \qquad (2.33)$$

where $NETFA1(t)$ is end-year net foreign assets given by

$$NETFA1(r) = \sum_{s \neq r} ZB1(s,r) + \sum_{s \neq r} ZE1(s,r) - \left\{ \sum_{s \neq r} ZB1(r,s) * \frac{E1(s)}{E1(r)} + \sum_{s \neq r} ZE1(r,s) * \frac{E1(s)}{E1(r)} \right\} \qquad (2.34)$$

and $NETFA0\_REVAL(r)$ is the end-year value of the net foreign assets that r had at the start of the year. This is given by the "0" terms in (2.17).

The LHS of (2.33), defined in (2.34), is the outcome of decisions by r's financial agent to hold foreign financial assets and decisions by financial agents in other countries to hold financial liabilities issued by r. The RHS of (2.33) is r's start-year net foreign assets revalued to end-year plus the surplus of saving over investment. The variables on both sides of the equation are affected in different ways by movements in the exchange rate.

In our modeling, we assume that the financial agent for each country is concerned with balancing its portfolio in terms of its own currency. Consequently, a higher exchange rate for country r doesn't make much difference to the amount of foreign dollars that foreigners want to place in country r. However, it reduces the number of *r-dollars*. In this way, a higher exchange rate reduces the dollar-r value of r's foreign liabilities. With regard to r's foreign assets, a higher exchange rate doesn't have much impact on the number of r-dollars that r's financial agent wants to place in other countries. Putting these two ideas together, we conclude that a higher exchange rate increases country r's net foreign assets calculated in r-dollars. That is, a higher exchange rate for r increases the LHS of (2.33).

On the RHS of (2.33), a higher exchange rate for country r doesn't directly affect its saving-investment gap. The dollar-r value of country r's existing foreign assets is reduced by a higher exchange rate, but this is also true for country r's foreign liabilities. Thus, the revaluation term on the RHS of (2.33) normally shows only limited sensitivity to exchange rate movements.

With the LHS of (2.33) responding to exchange rate movements and the RHS being relatively insensitive to these movements, adjustments in the exchange rate can be thought of as the equilibrating variable that brings the two sides together.

For the illustrative simulations in Section 3, we trace out the two curves suggested by (2.33). The first curve is the upward-sloping relationship between r's nominal exchange rate and r's net foreign assets determined by financial agents [the LHS of (2.33)]. The second curve is the comparatively flat relationship between r's nominal exchange rate and r's net foreign liabilities determined by the saving-investment gap and revalued start-year net foreign liabilities [the RHS of (2.33)]. The intersection of these two curves determines r's equilibrium exchange rate and net foreign liabilities. By thinking about shifts in these curves, we can anticipate how r's exchange rate and net foreign assets are likely to respond in our model to both real and monetary shocks.

3. Illustrative applications: tightening of U.S. monetary policy

Tables 3.1 and 3.2 show macroeconomic results for two forms of monetary tightening by the U.S.: an increase in the cash-rate by the Fed that permeates through the economy; and a liquidity operation in which the Fed mops up liquidity by forcing the private sector (mainly the commercial banks) to increase their holding of government bonds.

In Table 3.1, the shock is a 1 per cent increase in the power of the cash-rate. Via (2.9) this generates a 1 per cent increase in RB(US,r) for all r, that is a 1 per cent increase in the power of the rate of return that is earnt by holders of bonds issued by the U.S. government.

In Table 3.2, the shock is a 2.073 per cent increase in the preference of the U.S. financial agent for U.S. bonds [fzb(US, US) =2.073, see (2.3)]. This means that there is a 2.073 per cent increase in the demand for U.S. bonds by the U.S. financial agent at any given levels of the U.S. financial budget [b1(US)] and the expected rates of return appearing in (2.3) to (2.5), [rb(s, US), re(s, US) and rla(US)]. The preference shift could be brought about by an increase in capital adequacy ratios imposed by the Fed on commercial banks requiring them to bias their asset portfolio towards low-risk assets. The 2.073 per cent shock was chosen to aid comparison between the two simulations: with the 2.073 shock, the effect on U.S. real GDP is the same as in the first simulation[-1.56 per cent, row 1, column 1 in Tables 3.1 and 3.2].

The results in Tables 3.1 and 3.2 are short-run percentage deviations from baseline. For example, the GDP result for the U.S. in Table 3.1 means that because of the cash-rate shock, real GDP is 1.56 per cent less in year 1 than it would have been without the shock.

Table 3.1. Macro results from a 1 per cent increase in the power of the cash rate (% deviations from baseline)

	(1) Real GDP	(2) Capital in use	(3) Employment	(4) Real wage	(5) Real investment	(6) 100*ΔBalTrade to GDP ratio	(7) Nom exch rate ($Int/$local)	(8) Price of GDP	(9) Real exchange rate
1 USA	-1.56	-0.84	-1.87	0.66	-2.29	0.32	0.83	-1.50	-0.32
2 China	-0.02	-0.02	-0.03	0.08	0.20	-0.11	-0.22	-0.06	0.10
3 Japan	0.01	0.00	0.01	0.05	0.31	-0.09	-0.20	-0.02	0.16
4 Skorea	0.00	-0.02	-0.01	0.10	0.24	-0.08	-0.20	-0.05	0.13
5 France	0.12	0.05	0.14	-0.01	0.49	-0.10	-0.25	0.06	0.18
6 Germany	0.01	0.00	-0.01	0.06	0.47	-0.13	-0.21	-0.03	0.13
7 Brazil	0.01	0.00	0.01	0.07	0.24	-0.06	-0.23	-0.04	0.11
8 India	0.03	0.01	0.04	0.06	0.20	-0.06	-0.25	-0.02	0.11
9 Russia	-0.05	-0.04	-0.06	0.13	0.16	-0.08	-0.25	-0.19	-0.06
10 Australia	0.01	-0.02	0.01	0.06	0.29	-0.10	-0.20	-0.05	0.12
11 RoAmerica	-0.03	-0.03	-0.04	0.10	0.23	-0.07	-0.22	-0.10	0.05
12 RoAsia	-0.03	-0.03	-0.03	0.10	0.27	-0.13	-0.23	-0.12	0.02
13 RoEurope	0.09	0.04	0.12	0.00	0.57	-0.14	-0.28	0.05	0.15
14 Africa	-0.05	-0.04	-0.08	0.08	0.16	-0.07	-0.23	-0.13	0.01
15 RoW	-0.01	-0.01	-0.03	0.07	0.29	-0.09	-0.23	-0.05	0.10
16 UK	-0.03	-0.02	-0.05	0.07	0.21	-0.05	-0.21	-0.07	0.09
17 Canada	-0.05	-0.06	-0.06	0.12	0.50	-0.19	-0.24	-0.14	0.00
18 Mexico	-0.12	-0.08	-0.18	0.19	0.31	-0.12	-0.25	-0.26	-0.14
Global average							**0**	**-0.29**	

Table 3.2. Macro results from a 2.073 per cent increase in the preference of the U.S. financial agent for U.S. bonds (% deviations from baseline)

	(1) Real GDP	(2) Capital in use	(3) Employment	(4) Real wage	(5) Real investment	(6) 100*ΔBalTrade to GDP ratio	(7) Nom exch rate ($Int/$local)	(8) Price of GDP	(9) Real exchange rate
1 USA	-1.56	-0.82	-1.89	0.65	-3.06	0.23	0.97	-1.50	-0.09
2 China	-0.04	-0.04	-0.07	0.13	0.28	-0.11	-0.25	-0.11	0.09
3 Japan	-0.07	-0.05	-0.10	0.16	0.48	-0.05	-0.23	-0.15	0.07
4 Skorea	-0.03	-0.03	-0.05	0.15	0.35	-0.08	-0.24	-0.09	0.13
5 France	-0.03	-0.03	-0.05	0.12	0.69	-0.06	-0.30	-0.07	0.08
6 Germany	-0.08	-0.06	-0.13	0.14	0.72	-0.09	-0.25	-0.15	0.05
7 Brazil	-0.03	-0.03	-0.06	0.12	0.31	-0.06	-0.26	-0.10	0.09
8 India	0.01	-0.01	0.01	0.12	0.26	-0.07	-0.30	-0.05	0.10
9 Russia	-0.10	-0.08	-0.13	0.20	0.22	-0.08	-0.30	-0.29	-0.13
10 Australia	-0.04	-0.05	-0.06	0.12	0.42	-0.10	-0.24	-0.13	0.08
11 RoAmerica	-0.08	-0.06	-0.11	0.16	0.29	-0.07	-0.26	-0.18	0.02
12 RoAsia	-0.08	-0.07	-0.12	0.18	0.36	-0.10	-0.27	-0.24	-0.05
13 RoEurope	-0.06	-0.04	-0.09	0.12	0.70	-0.04	-0.34	-0.11	0.00
14 Africa	-0.11	-0.07	-0.17	0.14	0.22	-0.08	-0.27	-0.22	-0.04
15 RoW	-0.08	-0.05	-0.11	0.13	0.39	-0.09	-0.26	-0.15	0.05
16 UK	-0.09	-0.06	-0.13	0.14	0.36	-0.05	-0.25	-0.16	0.05
17 Canada	-0.13	-0.11	-0.17	0.16	0.65	-0.17	-0.27	-0.25	-0.07
18 Mexico	-0.17	-0.12	-0.26	0.25	0.35	-0.12	-0.28	-0.35	-0.18
Global average							**0**	**-0.37**	

As set out in Section 2, the main macro assumptions for all countries underlying the two simulations are:

- employment of labor and capital are determined in markets with asymmetric adjustments of nominal wage and capital rentals (subsection 2.8);
- physical capital creation (investment) is determined by the interaction of risk-averse entrepreneurs and financiers who compare expected rates of return on capital with rates of return on other financial assets (subsections 2.3 and 2.9);
- cash-rates are policy instruments and determine the rate of interest applying to government bonds (subsections 2.3 and 2.9);
- nominal exchange rates adjust endogenously to reconcile saving-investment gaps with the outcome of decisions by financial agents (subsection 2.9).

An additional assumption is that private and public consumption move with net national product. This is equivalent to assuming that the savings rate is unaffected by the monetary shocks.

3.1. Overview of results

The results in the two tables are similar. In both tables, tightening U.S. monetary policy reduces U.S. inflation (-1.50 and -1.50 per cent, row 1, column 8 in Tables 3.1 and 3.2). With sticky adjustment in nominal wages, U.S. workers get an increase in real wages (0.66 and 0.65 per cent, row 1, col 4). This reduces employment (-1.87 and -1.89 per cent, row 1, col 3). With less employment, there is a reduction in U.S. real GDP (-1.56 per cent, in both simulations). The reduction in activity in the U.S. leads to a reduction in the intensity with which the existing capital stock is used, that is the emergence of excess capacity. This is indicated in row 1, col 2 of the two tables which show reductions in capital-in-use of 0.84 and 0.82 per cent. As would be expected on the basis of an aggregate production function, the result for U.S. real GDP in both tables lies between the results for capital-in-use and employment.

The increase in the cash-rate in the first simulation and the compulsory preference change towards government bonds in the second simulation cause reallocation of the U.S. financial budget away from financing capital creation. These alternative ways of tightening monetary policy both reduce investment in the U.S. (-2.29 and -3.06 per cent, row 1, col 5). The percentage reductions in investment exceed those in GDP. By assumption private and public consumption track net national product, which moves approximately in line with GDP. Thus, national expenditure (C+I+G) falls relative to GDP, and the trade balance (X – M) for the U.S. moves towards surplus (0.32 and 0.23 per cent of GDP, row 1, col 6).

A move towards balance-of-trade surplus caused by a reduction in national expenditure (in this case triggered by a cut in investment) normally requires an improvement in competitiveness to stimulate exports and inhibit imports. The necessary adjustment in competitiveness is brought about by real devaluation. In our model, deviations in the real exchange rate from the baseline are measured by:

$$\mathrm{re1(r)} = \left[\mathrm{e1(r)} + \mathrm{pgdp(r)}\right] - \left[\mathrm{e1g} + \mathrm{pgdpg}\right] \tag{3.1}$$

where

re1(r) is the percentage deviation from baseline in the end-year real exchange rate for country r (negative for devaluation, positive for appreciation);

e1(r) is the end-year percentage deviation in r's nominal exchange rate;
pgdp(r) is the percentage deviation in the price deflator for GDP in r; and
e1g and pgdpg are global averages of the percentage deviations in the nominal exchange rates and price levels.

In (3.1), positive values for e1(r) and pgdp(r) combine as deteriorating factors in r's international competitiveness. Then in determining the ultimate movement in r's competitiveness, movements in these variables are compared with global averages.

Consistent with standard CGE modeling, the trade balance improvements for the U.S. in Tables 3.1 and 3.2 are accompanied by real devaluations (real exchange rate movements of -0.32 and -0.09 per cent, row 1, col 9). These real exchange rate movements are calculated in (3.1) from the results for the nominal exchange rate (col 7), the price of GDP (column 8) and the global averages (at the foot of cols 7 and 8).

Consistent with standard monetary theory, monetary tightening for the U.S. delivers real exchange rate devaluation via nominal exchange rate appreciation (0.83 and 0.97 per cent, row 1, col 7 in Tables 3.1 and 3.2) that is more than offset by reductions in inflation (movements in pgdp for the U.S. of -1.50 per cent in both tables).

The reductions in inflation reflect reductions in factor prices and reductions in the $US prices of exports and imports associated with nominal appreciation. The determination of the nominal exchange rate in the cash-rate simulation is explained in the next subsection. Then, subsection 3.3 focuses on the differences between the cash-rate results and those for liquidity operations.

3.2. Determination of the nominal exchange rate in the cash-rate simulation

Figure 3.1 shows the key role of the nominal exchange rate in tying together the real and monetary sides of the economy.

The two lines in Figure 3.1 show two versions of the change in end-year U.S. net foreign assets (valued in $USb) caused by a 1 per cent increase in the cash-rate. The lines were generated in 5 new simulations with the U.S. nominal exchange rate set exogenously. In these simulations we combined the 1 per cent increase in the U.S. cash-rate with exogenous percentage movements in the U.S. nominal exchange rate of -0.20, 0.00, 0.83, 0.97 and 1.20. To make it possible to exogenize the U.S. exchange rate we introduce an endogenous shift variable on the RHS of (2.33). This can be interpreted as an unidentified revaluation effect.

The upward sloping solid line in Figure 3.1 shows for the 5 simulations the deviations in end-year U.S. net foreign assets determined by the behavior of financial agents [movements in the LHS of (2.33), ΔNETFA1(USA)]. The comparatively flat dashed line shows for the 5 simulations the deviations in end-year U.S. net foreign assets determined by deviations in the U.S. saving-investment gap and the genuine valuation effect [movements in the RHS of (2.33), $\Delta SAVE(r) - \Delta I(r) + \Delta NETFA0\_REVAL(r)$]. The gap between the two lines is the endogenous unidentified revaluation effect. Where the two lines cross, the unidentified revaluation effect is zero. This shows the solution for the nominal exchange rate given in Table 3.1, 0.83 per cent.

Figure 3.1. Exchange rate equilibration of two versions of the movements in end-year U.S. net foreign assets ($b): cash-rate simulations (deviations from baseline)

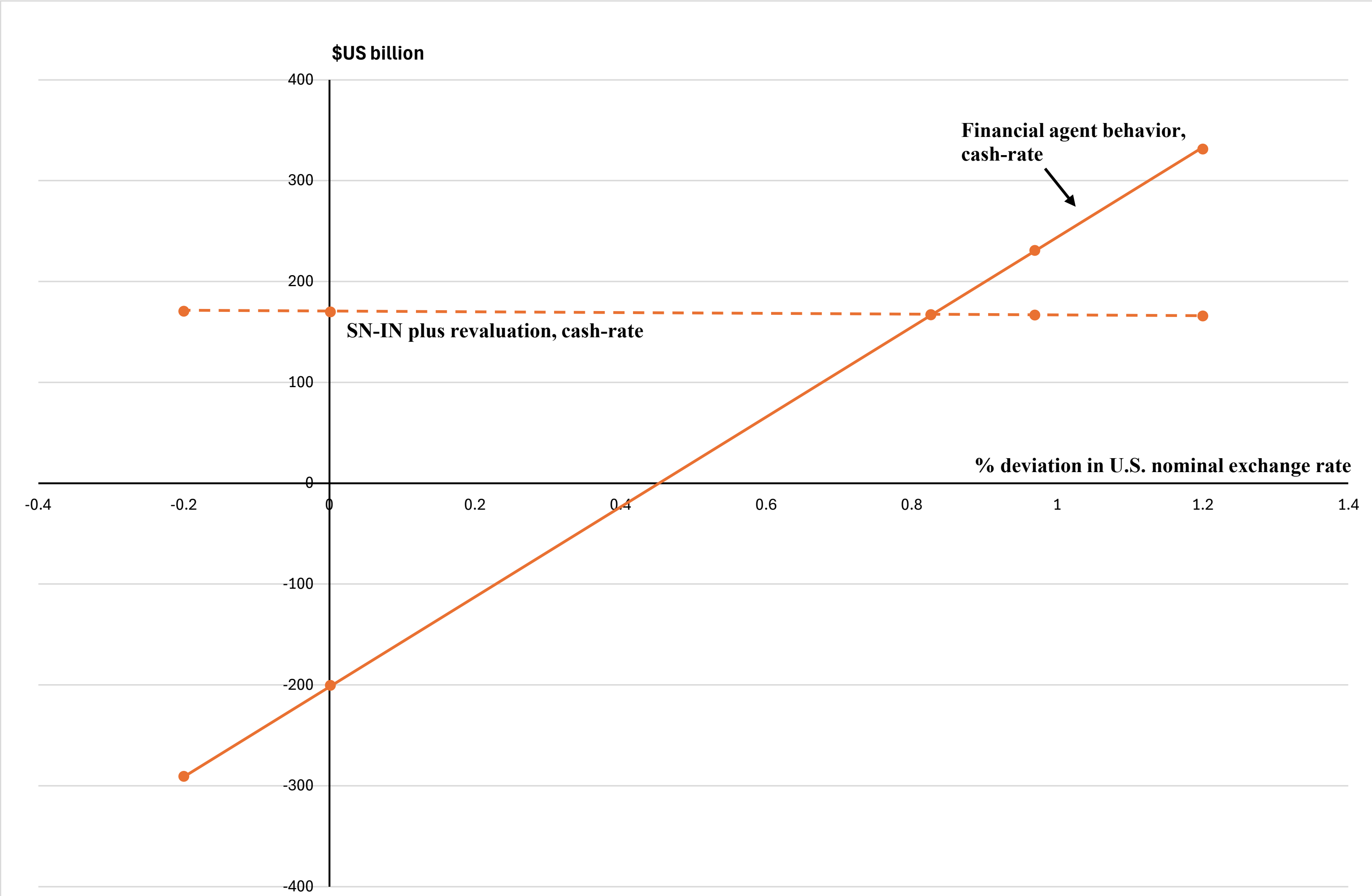
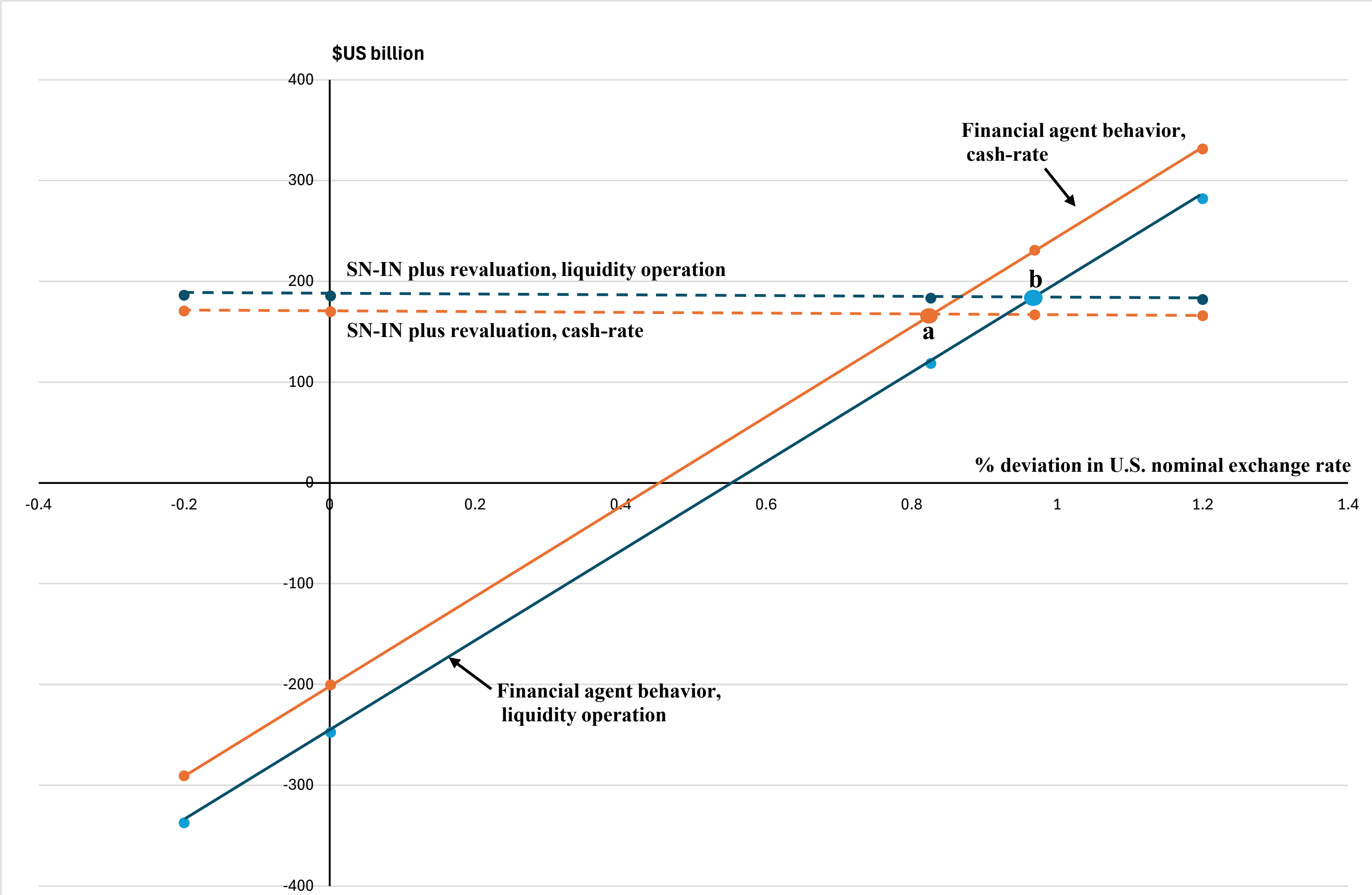


Figure 3.2 Exchange rate equilibration of two versions of the movements in end-year U.S. net foreign assets ($b): cash-rate and liquidity operations simulations (deviations from baseline)



Movements in end-year net foreign assets determined by financial agents [LHS of (2.33)]

In Figure 3.1 we can see that with no change in the exchange rate [e1(USA) = 0.00], the increase in the cash-rate causes financial agents to make decisions that reduce U.S. net foreign assets by $US200b. As confirmed in rows 6 and 7, col 2 of Table 3.3, the increase in the cash-rate causes the U.S. financial agent to bring home money that was previously placed in foreign countries [reductions in holdings of foreign bonds and equity of $US32b and $US179b]. This is a reduction in U.S. foreign assets. U.S. foreign liabilities also fall, but by a relatively small amount. Foreigners are induced by the higher interest rates in the U.S. to place more money in U.S. bonds [$US55b, row 8]. However, this increase in U.S. foreign liabilities is slightly more than offset by a reduction in foreign investments in U.S. equity [$US65b, row 9] reflecting the contraction in U.S. capital creation.

As we move to simulations with higher exchange rates, rows 8 and 9 of Table 3.3 reveal that the main reason for the upward-sloping solid line in Figure 3.1 is reductions in U.S. foreign liabilities.

The deviation in foreign holdings of U.S. bonds moves from $US55b when there is zero change in the exchange rate to -$US25b when the exchange rate increases by 1.2 per cent, and the deviation in foreign holdings of U.S. equities moves from -$US65b to -$US480b. As explained in subsection 2.9, financial agents allocate their budgets in terms of their own currencies. A higher exchange for the U.S. doesn't make much difference to the amount of foreign dollars that foreigners want to place in the U.S. However, it reduces the number of U.S. dollars. In this way, a higher U.S. exchange rate reduces the U.S.-dollar value of U.S. foreign liabilities.

Rows 6 and 7 of Table 3.3 indicate that as the exchange rate moves up, the change in the deviations in U.S. foreign assets also contributes to the increase in U.S. net foreign assets. This is a relatively minor effect. Changes in the exchange rate do not have much impact on the number of U.S. dollars that the U.S. financial agent wants to place in other countries.

Movements in end-year net foreign assets determined in the real economy [RHS of (2.33)]

With no change in the exchange rate [e1(USA) = 0.00], the dashed line in Figure 3.1 indicates that the increase in the cash-rate causes real-side developments that generate an increase in U.S. net foreign assets of about $US170b. As shown in row 4, column 2 of Table 3.3, the main contributor is revaluation effects, $US119b. Via its effects on wage and rental rates, the increase in the cash-rate reduces the U.S. price level, including the price of capital goods. This reduces the U.S.-dollar value of foreign equity liabilities. There is a partially offsetting effect from reductions in price levels in other countries, which reduces the U.S.-dollar value of U.S. foreign equity assets. For bonds, the revaluation effects with the exchange rate fixed on no change are zero, both for U.S. assets and liabilities.

The real-side increase in U.S. net foreign assets in column 2 of Table 3.3 is reinforced by an increase of $US51b in the saving-investment gap (-$US29b for saving relative to -$US80b for investment, rows 2 and 3, col 2). Saving is reduced by a reduction in GDP (and hence net national product) but there is a greater reduction in investment because the cash-rate increase reduces the investment-GDP ratio.

As we move to simulations with higher exchange rates, row 4 of Table 3.3 shows little change in the revaluation contribution to U.S. net foreign assets. This is the outcome of offsetting effects. Because we assume that bonds are repayable in international currency, a higher exchange rate reduces the dollar-U.S. value of U.S. bond assets and liabilities directly, with broadly offsetting effects on the dollar-U.S. value of U.S. net foreign assets. For equity, a higher exchange rate reduces the U.S. price level, including the price of capital goods, thereby reducing the dollar-U.S. value of U.S. equity liabilities. Broadly offsetting this is the reduction in the dollar-U.S. value of U.S. foreign equity assets (redeemable in the currency of the liability country) generated as a direct effect of a higher U.S. exchange rate.

Role of the nominal exchange rate

Our analysis of the steep and comparatively flat lines in Figure 3.1 shows that movements in the nominal exchange rate are a robust mechanism for equilibrating the financial and real sides of the determination of a country's net foreign assets. In future applications of GTAP-Money, we expect variants of Figure 3.1 to be a helpful device for explaining results. For any given shock we will ask two questions. First, what is the likely effect on the behavior of financial agents, that is how will the shock affect the position of the solid line in Figure 3.1? Second, what is the likely effect on the saving-investment gap and the value of existing assets, that is how will the shock affect the position of the dashed line in Figure 3.1? Moving the lines will help us to understand the implications for the nominal exchange rate and related price variables. In the presence of sticky nominal factor prices, the determination of the nominal exchange rate may be an important part of understanding the implications of shocks for real factor prices, and consequently for employment and real GDP.

3.3. Comparing the cash-rate simulation with the liquidity-operation simulation

The cash-rate (CR) and the liquidity-operation (LO) simulations reported in Tables 3.1 and 3.2 produce qualitatively similar results but, as shown in those two tables and in Table 3.5, there are significant quantitative differences.

The key to understanding these differences is that with a higher cash-rate, the U.S. makes higher interest payments to foreigners on its bond liabilities. This is not the case in the LO simulation. Consequently, in the transition from CR to LO, there is an increase in U.S. net national product (from -3.29 per cent to -2.98 per cent, row 17, Table 3.5), causing increases in private and public consumption (rows 15 and 16). The consumption increases stimulate activity in the LO simulation relative to the CR simulation, but to make the two simulations comparable we set the LO policy so that it generated the same reduction in real GDP as in the CR simulation (-1.56 in both cases, row 7). To achieve the same GDP results we needed a big enough monetary tightening in the LO simulation to drive real investment below the level in the CR simulation (-3.06 per cent compared with -2.29 per cent, row 3).

More formally, we can explain the differences between the two simulations via the expenditure-side GDP identity:

$$GDPN = CN + GN + IN + XN - MN \tag{3.2}$$

where the LHS is U.S. nominal GDP and the RHS is the sum of nominal private and public consumption, investment and export less imports.

As we go from CR to LO, the movements in real GDP fixed at -1.56 per cent, we would expect the movements in the price deflator for GDP to be very similar, and they are (-1.50 per cent, row 13). If the movements in the price deflator for GDP differed significantly, then

Table 3.3. Exchange rate equilibration of two versions of the movements in end-year U.S. net foreign assets ($b): cash-rate simulations (deviations from baseline)

			(1)	(2)	(3)	(4)	(5)
	Description	***Exch. rate***	***-0.200***	***0.000***	***0.825***	***0.970***	***1.200***
1	**SN - IN plus revaluation**	ΔRHS(2.33)	170.517	169.874	167.202	166.696	165.892
2	Saving	ΔSAVE	-24.280	-28.706	-46.746	-49.854	-54.729
		less					
3	Investment	ΔI	-75.576	-79.536	-95.709	-98.567	-103.123
		plus					
4	Revalued start-year assets	ΔNETFA0_REVAL	119.221	119.044	118.239	117.983	117.499
5	**Financial agent behavior**	ΔLHS(2.33)	-290.707	-200.473	167.002	230.759	331.411
6	Foreign assets, bonds	ΔFA1BONDS	-33.232	-32.047	-27.199	-26.356	-25.025
7	Foreign assets, equity	ΔFA1EQ	-183.983	-178.936	-158.192	-154.498	-148.550
		less					
8	Foreign liabilities, bonds	ΔFL1BONDS	68.089	54.612	-0.381	-9.952	-25.082
9	Foreign liabilities, equity	ΔFL1EQ	5.402	-65.122	-352.013	-401.661	-479.904

Table 3.4. Exchange rate equilibration of two versions of the movements in end-year U.S. net foreign assets ($b): open-market operations simulations (deviations from baseline)

			(1)	(2)	(3)	(4)	(5)
	Description	***Exch. rate***	***-0.200***	***0.000***	***0.825***	***0.970***	***1.200***
1	**SN - IN plus revaluation**	ΔRHS(2.33)	186.139	185.629	183.321	182.819	181.997
2	Saving	ΔSAVE	-16.687	-21.120	-39.163	-42.281	-47.181
		less					
3	Investment	ΔI	-99.779	-103.842	-120.435	-123.335	-127.958
		plus					
4	Revalued start-year assets	ΔNETFA0_REVAL	103.047	102.907	102.049	101.765	101.219
5	**Financial agent behavior**	ΔLHS(2.33)	-337.341	-247.449	118.469	181.941	282.095
6	Foreign assets, bonds	ΔFA1BONDS	-42.777	-41.633	-36.953	-36.138	-34.851
7	Foreign assets, equity	ΔFA1EQ	-248.741	-243.932	-224.278	-220.869	-215.410
		less					
8	Foreign liabilities, bonds	ΔFL1BONDS	9.594	-3.761	-58.252	-67.736	-82.722
9	Foreign liabilities, equity	ΔFL1EQ	36.230	-34.354	-321.447	-371.212	-449.634

with sticky nominal factor prices, real factor prices would differ significantly causing differences in employment of both factors and real GDP. Thus, the movements in nominal GDP, as well as real GDP are the same in the two simulations (-3.04 per cent, row 22).

With similar price movements in the two simulations the boost in nominal consumption, related to the reduction in interest payments to foreigners as we go from CR to LO, translates to increases in real consumption (rows 1 and 2)

Next, we demonstrate that the nominal trade balance, XN – MN, must fall in the transition from CR to LO.

Assume to the contrary that XN – MN rises. With almost no difference in U. S. prices and nominal GDP in the two simulations, improvement in the trade balance in the transition from CR to LO would require an improvement in U.S. competitiveness (real devaluation) generated by a reduction in the U.S. nominal exchange rate.

As explained in subsection 3.2, exchange rate changes do not make much difference to the willingness of foreigners to devote foreign dollars to financing U.S. capital creation, but a lower exchange rate directly increases the number of U.S. dollars that they supply. This would increase U.S. investment. Thus, we would have an increase in absorption (CN + GN + IN) and the trade balance (XN – MN) on the RHS of (3.2). But this is not possible because there is no increase in nominal GDP in the transition from CR to LO.

We can conclude that XN – MN must fall, and the nominal exchange rate must rise. This conclusion is reflected in in rows 12 and 11.

What about the saving-investment gap (SN – IN)? With a higher exchange rate, we know that U.S. net foreign assets rise. As explained in subsection 3.2, this is caused mainly by a reduction in the dollar-U.S. value of U.S. foreign liabilities. We also found that a higher exchange rate has only a small revaluation effect. Consequently, via (2.33), with a higher exchange rate there must be an increase in the saving-investment gap. This can be seen in rows 23 and 24 of Table 3.5.

In the transition from CR to LO, the increase in the saving-investment gap is caused mainly by a reduction in investment. The saving propensity in the U.S. is about 9 per cent. Consequently, the boost in NNP generates only a modest increase in saving (about $US 4b, row 23). A relatively large reduction in investment ($US27b, row 24) is required to generate the increase in the saving-investment gap consistent with the increase in U.S. net foreign assets.

Figure 3.2 reproduces the material from Figure 3.1 for the CR simulation and adds equivalent material for the LO simulation. At any exchange rate, the monetary tightening policy in the LO simulation reduces investment by more than the policy in the CR simulation. As can be checked by comparing the upper panels in Tables 3.3 and 3.4, the weaker investment in the LO simulations is the main reason that the dashed LO line is above the dashed CR line. The solid LO line, representing behavior by financial agents, lies below the corresponding CR line. Relative to the CR policy, the LO policy, with a greater requirement to reduce U.S. investment, induces a stronger reallocation of the U.S. financial agent's budget towards U.S. bonds. This requires strong reductions in U.S. foreign assets (rows 6 and 7, Table 3.4) as the U.S. agent complies with the compulsory preference shift in favor of U.S. bonds and against all other financial assets.

Table 3.5. Macro effects: comparison of the cash-rate (CR) and liquidity operation (LO) simulations (percentage deviations from baseline)

	simulation	CR	LO
1	Real private consumption (C)	-1.75	-1.44
2	Real public consumption (G)	-2.01	-1.70
3	Real investment (I)	-2.29	-3.06
4	Real absorption (C + I + G)	-1.89	-1.78
5	Real exports (X)	0.87	0.14
6	Real imports (M)	-1.95	-1.74
7	Real GDP	-1.56	-1.56
8	Employment	-1.87	-1.89
9	Capital in use	-0.84	-0.82
10	Exchange rate (real)	-0.32	-0.09
11	Exchange rate (nominal)	0.83	0.97
12	Trade balance to GDP ratio (%)	0.32	0.23
13	Price deflator for GDP	-1.50	-1.50
14	Price deflator for absorption	-1.48	-1.50
15	Nominal private consumption (CN)	-3.29	-2.98
16	Nominal public consumption (GN)	-3.29	-2.98
17	Nominal net national product (NNP)	-3.29	-2.98
18	Nominal investment (IN)	-3.56	-4.42
19	Nominal absorption (CN + IN + GN)	-3.34	-3.25
20	Nominal exports (XN)	-0.54	-1.36
21	Nominal imports (MN)	-3.17	-3.22
22	Nominal GDP	-3.04	-3.04
23	Nominal saving, change $USb (SN)	-46753	-42322
24	Nominal investment, change $USb (SN)	-95717	-123371

As illustrated in Figure 3.2, the transition from the CR to the LO simulation moves the equilibrium from point a to point b. The percentage increase in the nominal exchange rate goes from 0.83 to 0.97 (row 11 Table 3.5, and the increase in net foreign assets goes from $U.S.167b to $US183b (row 1, col 3, Table 3.3 and row 1, col 4, Table 3.4).

4. Concluding remarks

GTAP-Money extends the range of potential GTAP applications to include analysis of monetary policy. Consistent with prior expectations, GTAP-Money simulations of monetary tightening by the U.S. showed:

- reductions in U.S. real GDP;
- reductions in U.S. real investment relative to GDP with consequent improvements in the U.S. trade balance;
- reductions in U.S. employment;
- emergence in the U.S. of excess capacity;
- reductions in the U.S. price level and consequent increases in real wage rates;

- nominal appreciation of the U.S. exchange rate;
- U.S. real devaluation; and
- relatively minor effects for other countries.

We also expect that GTAP-Money will generate new insights on simulations in traditional GTAP areas such as tariffs, taxes, labor markets, environmental policies and technical change.

The main theoretical addition to GTAP in GTAP-Money is the introduction of an optimizing financial agent for each country. These agents hold three types of financial assets: equity that finances capital creation; liquidity that facilitates day-to-day transactions; and government bonds that at least partially finance the government deficit. In linking the monetary and real sides of the economy, we needed to identify variables that coordinate the decisions by the financial agents to hold these financial assets with demands by capital creators for investment finance, demands by the public for transaction finance and demands by the government for deficit finance.

We showed that the coordinating variables are: expected rates of return on investments in physical capital; the shadow rate of return to holding liquidity; and the nominal exchange rate.

In our earlier financial version of GTAP [see Dixon *et al.* (2021)] the financial agents held only one type of financial asset, used to finance capital creation. In that model we needed only one coordinating variable, expected rates of return on physical capital. Neither shadow rates of return on liquidity nor nominal exchange rates were explicitly recognised. Implicitly, there were exogenous on zero change. The inclusion in GTAP-Money of endogenous nominal exchange rates is a significant advance.

For the model described in this paper to reach its full potential as a contribution in policy and academic circles, a considerable amount of extra work will be required. The tasks that we have currently identified are as follows.

(a) The additional data for GTAP-Money (beyond that for standard GTAP) must be updated and documented. The additional data are for: end-year and start-year bond and equity asset holdings by country r of liabilities issued by country s [ZB1(s,r), ZB0(s,r), ZE1(s,r), ZE0(s,r)]; end-year and start-year holdings of liquidity by residents of country r [LA1(r), LA0(r)]; and the public-sector borrowing requirement by country r [PSBR(r)]. In the current implementation of GTAP-Money, the data were put together in a stylized form and refer to 2015. The task of replacing these data with authoritative data for 2023 is well advanced. The next version of this paper will report simulations using the new data.

(b) The GEMPACK code for GTAP-Money must be edited. The current code is a working document. It includes blocks of equations that were introduced in the research process but eventually proved irrelevant. These should be cleansed out. User-friendly explanations of variables and equations should be included in the code. Editing of the code will facilitate applications of GTAP-Money and make it a potentially valuable policy tool.

(c) More complete explanations of results such as those in Tables 3.1 and 3.2 should be included in the next draft of this paper. For example, the explanations in Section 3 should be extended to cover not only the U.S. but other countries.

(d) Deeper insights into how the model works should be developed. This means understanding whether the plausible qualitative results reported in Section 3 are an inevitable consequence of the theoretical structure of the model or whether they could be overturned with different but acceptable datasets and parameter values.

(e) We should conduct a calibration exercise to help determine appropriate values for parameters such as the financial agents' elasticity of substitution (σ) between different asset classes, and the sensitivity (γ) of the shadow rate of return on liquidity to variations in the GDP-to-liquidity ratio. The calibration exercise would involve bringing GTAP-Money results for a standard simulation (e.g. an increase in the cash-rate) into line with those generated by well-regarded macro models.

(f) Multi-period simulations could be conducted with GTAP-Money. These might narrow the range of acceptable parameter values by showing that some values, while acceptable for the short run, can produce unacceptable long-run results.

(g) Policy-relevant simulations should be undertaken. An example is simulations of the effects of oil-price increases with different monetary-policy reactions. Another example is the effect of tariff increases by the U.S. with different monetary reactions in the U.S. and by U.S. trade partners such as Canada. These studies would show how GTAP-Money can cope with a broader range of shocks than macro models and can produce results for a broader range of variables such as employment by industry.

(h) For academic publication, we will need to compare our model with other models and cite relevant theoretical literature.

We are currently looking for research funding to make it possible to undertake these tasks.

Appendix 1. The demand for and supply of bond finance equilibrated via the nominal exchange rate: supporting algebra

In GTAP-Money, we imposed the condition that the saving-investment gap in country r equals the increase in r's net foreign assets beyond what can be explained by revaluation effects, see (2.17). Then we asserted that this condition, together with the definitions, (2.2) and (2.8), of the budgets of the financial agents implies that the increase in liquidity in country r is the unfunded public-sector deficit, (2.30). The first part of this appendix provides the proof.

In the second part, we assume that it is the liquidity constraint, (2.30), that is imposed, and show that this constraint implies that (2.17) must hold. Thus, we demonstrate an equivalence between (2.30) and (2.17).

This equivalence is useful for identifying the equilibrating variable for reconciling government r's demand for finance through the sale of bonds with the willingness of financial agents to purchase bonds. At first glance, the obvious equation on which to base the identification is (2.30) which says that financial agents must be induced to finance the gap (determined mainly on the real side of the economy) between the public-sector deficit and the demand for liquidity. However, we have found that it is preferable (and equivalent) to work with (2.17). Via (2.17), we can think intuitively of movements in r's exchange rate as the variable that equilibrates movements in r's net foreign liabilities determined by the behavior of financial agents with r's saving-investment gap determined primarily on the real side of the economy. Thus, through the equivalence we can see that exchange rate movements can be identified as the variable that equilibrates the demand for and the supply of bonds.

A1.1. Deriving (2.30) using (2.17)

From (2.2) and (2.8) we have

$$\begin{aligned}&\sum_{s}\left[ZB1(s,r)+ZE1(s,r)\right]+LA1(r)\\&=\sum_{s}\left[ZB0(s,r)*VBScnd(s,r)+ZE0(s,r)*VEScnd(s,r)\right]\\&\quad+LA0(r)+PSBR(r)+SAVE(r)\end{aligned} \tag{A1.1}$$

Separating out diagonal terms and rearranging, we obtain

$$\begin{aligned}&\sum_{s\neq r}\left[ZB1(s,r)+ZE1(s,r)\right]-\sum_{s\neq r}[ZB0(s,r)*VBScnd(s,r)+ZE0(s,r)*VEScnd(s,r)]\\&\quad+ZB1(r,r)+ZE1(r,r)+LA1(r)\\&\quad=ZB0(r,r)*VBScnd(r,r)\\&\quad+ZE0(r,r)*VEScnd(r,r)\\&\quad+LA0(r)+PSBR(r)+SAVE(r)\end{aligned} \tag{A1.2}$$

Using (2.17) in (A1.2) gives

$$\begin{aligned}&SAVE(r)-I(r)\\&+\left\{\sum_{s\neq r}ZB1(r,s)*\frac{E1(s)}{E1(r)}-\sum_{s\neq r}ZB0(r,s)*VBFrst(r,s)\right\}\\&+\left\{\sum_{s\neq r}ZE1(r,s)*\frac{E1(s)}{E1(r)}-\sum_{s\neq r}ZE0(r,s)*VEFrst(r,s)\right\}\\&\quad+ZB1(r,r)+ZE1(r,r)+LA1(r)\\&\quad=ZB0(r,r)*VBScnd(r,r)\\&\quad+ZE0(r,r)*VEScnd(r,r)\\&\quad+LA0(r)+PSBR(r)+SAVE(r)\end{aligned} \tag{A1.3}$$

We cancel SAVE(r) from the left and right sides of (A1.3). Then we group the blue terms and the pink terms in (A1.3). Via (2.13) to (2.15) and (2.21), and recognizing that VEScnd(r,r) equals VEFrst(r,r), we see that the blue and pink groupings give offsetting values of VK1(r). This leads to:

$$\begin{aligned}&\left\{\sum_{s}ZB1(r,s)*\frac{E1(s)}{E1(r)}-\sum_{s}ZB0(r,s)*VBFrst(r,s)\right\}\\&+LA1(r)-LA0(r)=PSBR(r)\end{aligned} \tag{A1.4}$$

which can be rewritten as (2.30).

A1.2. Deriving (2.17) using (2.30)

Again, we use (2.2) and (2.8) to obtain (A1.1). Now introducing (2.30) and separating out diagonal terms we obtain:

$$
\begin{aligned}
&\sum_{s\neq r} ZB1(s,r)+\sum_{s\neq r} ZE1(s,r)+ZB1(r,r)+ZE1(r,r)= \\
&=\sum_{s\neq r} ZB0(s,r)*VBScnd(s,r)+\sum_{s\neq r} ZE0(s,r)*VEScnd(s,r) \\
&+ZB0(r,r)*VBScnd(r,r)+ZE0(r,r)*VEScnd(r,r) \\
&\quad +SAVE(r) \\
&+\sum_{s\neq r} ZB1(r,s)*\frac{E1(s)}{E1(r)}+ZB1(r,r) \\
&-\sum_{s\neq r} ZB0(r,s)*VBFrst(r,s)-ZB0(r,r)*VBFrst(r,r)
\end{aligned}
\tag{A1.5}
$$

We cancel out the coloured terms from (A1.5), rearrange and add a new coloured term on the RHS. Via (2.14) to (2.16) and (2.21), this new coloured term is zero. With its inclusion we have:

$$
\begin{aligned}
&\left\{\sum_{s\neq r} ZB1(s,r)-\sum_{s\neq r} ZB0(s,r)*VBScnd(s,r)\right\} \\
&+\left\{\sum_{s\neq r} ZE1(s,r)-\sum_{s\neq r} ZE0(s,r)*VEScnd(s,r)\right\} \\
&+ZE1(r,r) \\
&=ZE0(r,r)*VEScnd(r,r)\quad +SAVE(r) \\
&+\left\{\sum_{s\neq r} ZB1(r,s)*\frac{E1(s)}{E1(r)}-\sum_{s\neq r} ZB0(r,s)*VBFrst(r,s)\right\} \\
&-\left\{I(r)+\sum_{s} ZE0(r,s)*VEFrst(r,s)-\sum_{s} ZE1(r,s)*\frac{E1(s)}{E1(r)}\right\}
\end{aligned}
\tag{A1.6}
$$

Recognizing that VEFrst(r,r) is the same as VEScnd(r,r), we can quickly rearrange (A1.6) to obtain (2.17).

Appendix 2. Specifying asymmetric functional forms for the factor-price-employment adjustment curves

Nominal wage rates

As set out in subsection 2.8, we require an equation to be used in policy simulations to connect nominal wages and employment. The properties that we want for this equation are:

(a) if the employment to labour-supply ratio in a policy simulation is the same as in baseline, then the nominal wage in the policy simulation is the same as in baseline;
(b) if employment to labour supply in policy falls below baseline, then there must be a muted decline in the nominal wage rate below baseline;

(c) if employment to labour supply in policy moves above baseline, then there must be a sharp increase in the nominal wage rate above baseline; and
(d) the elasticity of the nominal wage in the policy simulation with respect to the employment/labour-supply ratio must be consistent with judgements based on macro-economic modeling.

The function we used in the simulations reported in this paper is illustrated in Figure A2.1. To achieve a rapid change in slope of the wage relationship in policy simulations at the (1,1) point, we used a spliced functional form given by:

$$Y = DUM(X) * M_0(X) + \left[1 - DUM(X)\right] * M_1(X) \tag{A2.1}$$

where

$$DUM(X) = \begin{cases} 1 & \text{if } 0 \le X \le 1 \\ 0 & \text{if } X > 1 \end{cases} \tag{A2.2}$$

$$M_0(X) = \frac{-S}{(X-2)} + 1 - S \tag{A2.3}$$

$$M_1(X) = A \exp\left(X^{\alpha}\right) + B \tag{A2.4}$$

In these equations,

X is the employment/labour-supply ratio in the policy simulation, expressed as a fraction of the ratio in baseline. If the ratio is same in policy as in baseline, X = 1. If the employment/labour-supply ratio in policy rises relative to baseline, then X moves above 1. If the ratio falls in policy relative to baseline, then X moves below 1.

Y is the nominal wage rate in policy relative to baseline. If the nominal wage rate in policy is the same as in baseline, Y = 1. Consistent with property (a) above, if X = 1, then via (A2.1) – (A2.3), we see that Y = 1.

S, A, B and α are positive parameters.

The parameters are set so that the relationship between Y and X is continuous and has a continuous first derivative. These properties facilitate the application of the relationship in GEMPACK. The two continuity requirements were met by setting the parameters so that

$$M_0(1) = M_1(1) = 1 \tag{A2.5}$$

$$\frac{\partial M_0}{\partial X}\Big|_{X=1} = \frac{\partial M_1}{\partial X}\Big|_{X=1} \tag{A2.6}$$

From (A2.5) and (A2.6) we obtain

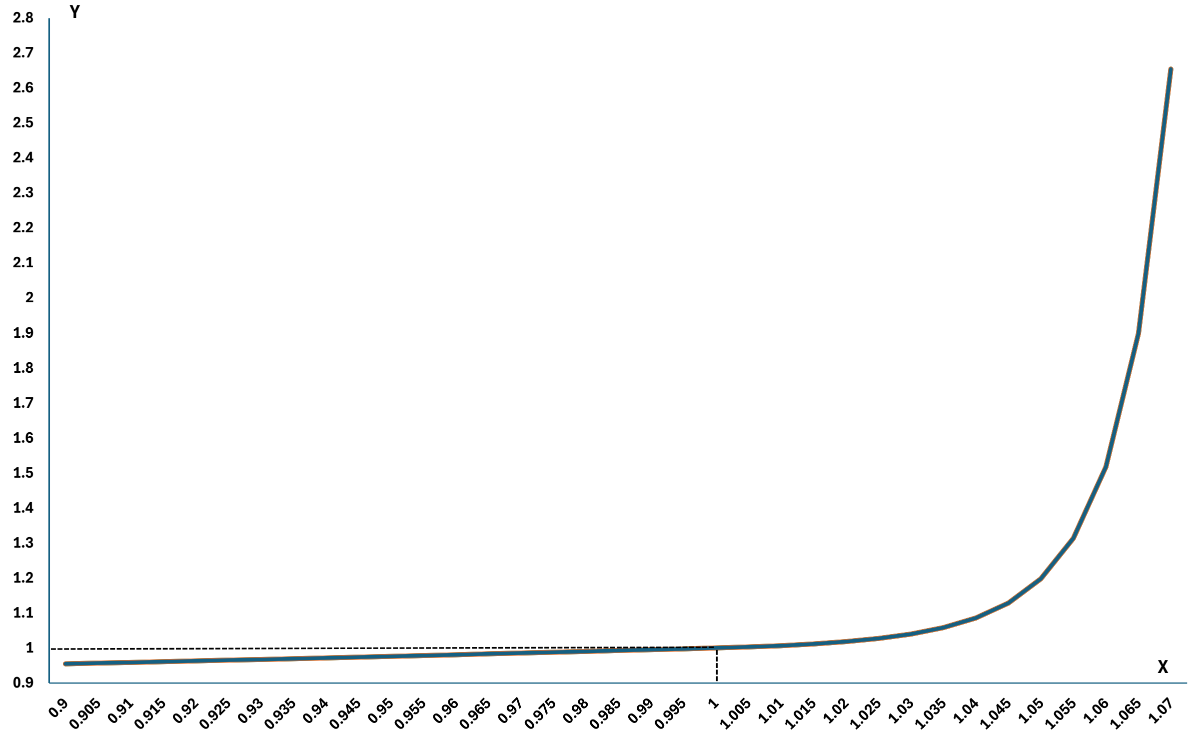
$$1 = A * \exp(1) + B \tag{A2.7}$$

$$S = A * \alpha * \exp(1) \tag{A2.8}$$

Solving for A and B in terms S and α gives

$$A = \frac{S}{\alpha * \exp(1)} \tag{A2.9}$$

Figure A2.1. Relationship between nominal wage relative to baseline (Y) and employment relative to baseline (X) in policy simulations



and

$$B = 1 - \frac{S}{\alpha} \qquad \text{(A2.10)}$$

We set S at 0.5. This is the value we have used in previous projects for the derivative of *real* wages to employment in the vicinity of the baseline solution [(X,Y) = (1,1)], see for example Dixon *et al* (2021). We now use 0.5 for the derivative (which is also the elasticity) of *nominal* wages to employment at the (1,1) point. Further research will be necessary to establish the compatibility of 0.5 with macro-economic modeling, that is property (d).

For α we use the value 25. With S = 0.5 and α = 25, A =0.007358 and B = 0.98.

With these parameter values, (A2.1) meets requirements (b) and (c) above. If the employment/labour-supply ratio falls 5 per cent below its baseline value (X = 0.95), then the effect on the nominal wage rate is muted. It falls by only 2.38 per cent (Y moves from 1 to 0.9762). If the employment/labour-supply ratio rises 5 per cent above its baseline value (X = 1.05), then the nominal wage rate rises sharply, by 19.75 per cent (Y moves from 1 to 1.1975).

Nominal rental rates on capital

In policy simulations we specify a relationship that connects the rental rate per unit of capital [RENTAL(j,r)] in each industry and country to the ratio of capital-in-use to capital-in-existence [KU(j,r)/KE(j.r)]. We follow a broadly similar approach to that used for wages and employment. We require that

(a) if the KU(j,r)/KE(j.r) ratio in the policy simulation is the same as in baseline, then the nominal rental [RENTAL(j,r)] in the policy simulation will be the same as in baseline;
(b) if the KU(j,r)/KE(j.r) ratio in policy falls below baseline, then there is a muted decline in the nominal rental rate below baseline;
(c) if the KU(j,r)/KE(j.r) ratio in policy moves above baseline, then there is a sharp increase in the nominal rental rate above baseline.

We judged that profit margins (reflected in rental rates) can exhibit more short-run flexibility than nominal wage rates. As shown in Figure A2.2, we allow a 22.5 per cent reduction in the rental rate on (j,r)'s capital in response to a 10 per cent reduction in capital-in-use relative to capital-in-existence (10 per cent excess capacity). By contrast, the reduction in the nominal wage for a 10 per cent reduction in employment relative to available labour is only 4.5 per cent. At the other extreme, we assume that the rise in rental rates in response to more intensive use of capital stocks is slower than the rise in nominal wage rates in response to reductions in unemployment below normal levels. Equivalently, we assume in the baseline that there is considerable scope for getting more product out of a given stock of capital, whereas the economy-wide labour constraint binds once employment is more than 2 or 3 per cent above baseline (assumed to be approximately full employment).

With a flatter curve for the rental relationship, we don't need the spliced functional form used for the wage relationship. Instead, for the rental relationship the specification is:

$$Y = L - A * \exp(B / U) + A * \exp\left\{B / \left[U - X\right]\right\} \qquad \text{(A2.11)}$$

where

X is the capital-in-use/ capital-in-existence ratio in a policy simulation, expressed as a fraction of the ratio in baseline. If the ratio is the same in policy as in baseline, X = 1;

Y is the nominal rental rate in policy relative to baseline; and

A, B, L and U are positive parameters.

Looking at (A2.11), we see that as X approaches U from below, Y goes to infinity. This tells us that U is an upper bound on the ratio of capital-in-use to capital-in-existence relative to the normal (baseline) ratio. We assume that U = 1.2, that is, the use-to-availability ratio can rise no higher than 20 per cent above the normal ratio.

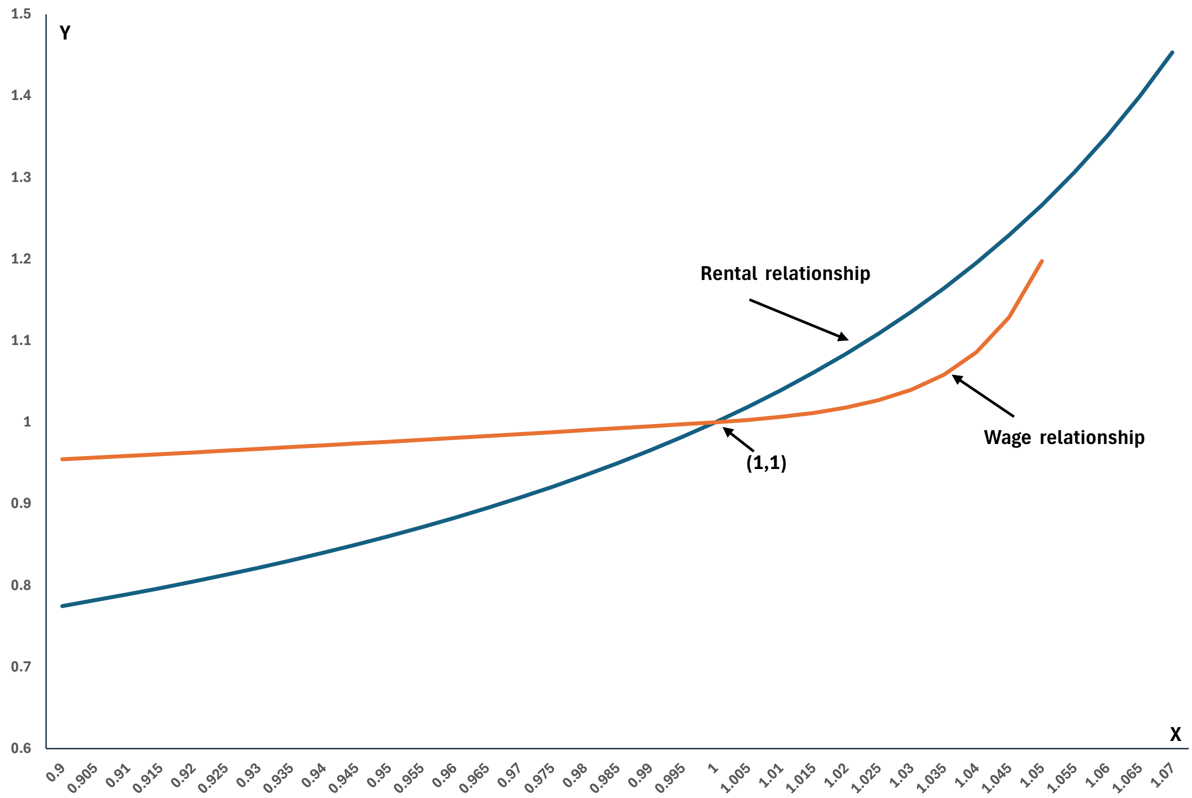
Next, we see that if capital in use falls to zero then Y = L. This tells us that L is a lower bound on the rental rate for the capital in any industry j in country r relative to its initial value. We set L at 0.5. This limits the reductions in capital rentals to 50 per cent of their baseline values.

The steepness of the rental curve is dictated by the value of B. After some trial and error, we set B at 0.1. Now, in accordance with property (a) above, we can determine the value of the last parameter, A, by requiring that when capital use is at baseline (X = 1), then the rental rate is at baseline (Y = 1):

$$1 = 0.5 - A * \exp(0.1/1.2) + A * \exp(0.1/\left(1.2 - 1\right)) \qquad \text{(A2.12)}$$

giving A = 0.889969.

Figure A2.2. Comparison of wage and rental relationships



With all the parameters in place, our choice of B implies that the rental rate (Y) increases sharply relative to baseline when the use-to-availability ratio for capital moves above baseline [property (b)], and falls slowly when the use-to-availability ratio moves below baseline [property (c)]. For example, if the use-to-availability ratio for capital moves 10 per cent above baseline then the capital rental rate increases by about 95 per cent relative to baseline:

$$1.951876 = 0.5 - 0.889969 * \exp(0.1/1.2) + 0.889969 * \exp(0.1/\left(1.2 - 1.1\right)) \qquad \text{(A2.10)}$$

As already mentioned, if the use-to-availability ratio moves 10 per cent below baseline, then the capital rental rate decreases by only 22.5 per cent relative to baseline:

$$0.775 = 0.5 - 0.889969 * \exp(0.1/1.2) + 0.889969 * \exp(0.1/\left(1.2 - 0.9\right)) \qquad \text{(A2.10)}$$

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Creating GTAP-Money: a GTAP model with financial assets and liabilities; monetary policy; excess capacity; and nominal exchange rates

CoPS

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presentation
by
Peter Dixon

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1

Motivation

CoPS

To improve the credibility of short-run GTAP simulations in the eyes of macro-trained economists in government policy departments by explicit recognition in the standard GTAP model of:

(1) accumulation of financial assets and liabilities at a national level;

(2) monetary policy;

(3) excess capacity and "stranded" physical assets; and

(4) endogenous nominal exchange rates.

2

Financial agents: main theoretical construction

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The financial agent for country r choses:

- end-year holdings of government bonds, ZB1(s,r)
- end-year holdings of business liabilities (“equities”), ZE1(s,r)
- end-year liquidity (narrow money, M1), LA1(r)

to maximize

$$U\left[RB(s,r)*ZB1(s,r) \text{ for all s}; \quad RE(s,r)*ZE1(s,r) \text{ for all s}; \quad RLA(r)*LA1(r)\right] \tag{2.1}$$

subject to

$$\text{Budget1}(r)=\sum_s[ZB1(s,r)+ZE1(s,r)]+LA1(r) \quad . \tag{2.2}$$

where

$$\text{Budget1}(r)=\text{Budget0\_reval}(r)+\text{SAVE}(r)+\text{PSBR}(r) \tag{2.8}$$

3

Connecting real and financial sides: business coordination

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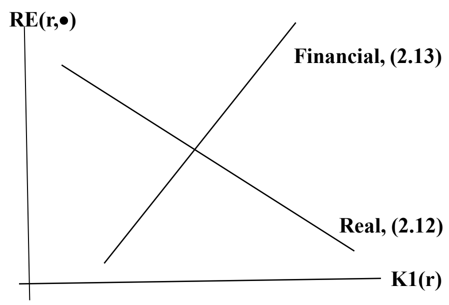
The *expected* rate of return offered by capital creators in country r:

$$RE(r,s)=RE(r,\bullet)=RORC(j,r)*\left(\frac{K1(j,r)}{K0(j,r)}\right)^{-RORFLEX} \quad \text{for all s and j} \tag{2.12}$$

$RE(r,\bullet)$ is determined so that the total amount of end-year finance equals the end of year replacement value of r’s capital stock:

$$\sum_s ZE1(r,s)*\frac{E1(s)}{E1(r)}=VK1(r) \tag{2.13}$$



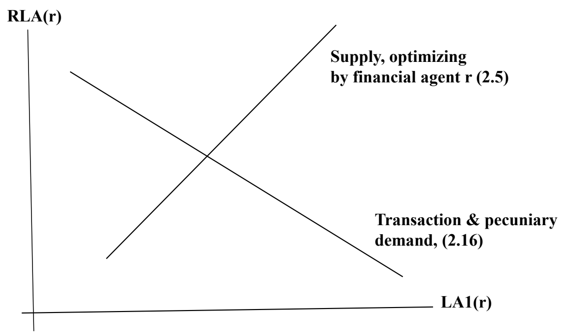
4

Connecting real and financial sides: liquidity coordination

Demand for liquidity is a declining function of shadow rate of return on holding liquidity

$$LA1(r) = \frac{GDP(r)}{\beta(r)} * RLA(r)^{-1/\gamma} \qquad (2.16)$$

Supply of liquidity is an increasing function of shadow rate of return derived from optimization of financial agent r



5

Connecting real and financial sides: bond coordination

$$nBONDS(r) = PSBR(r) - \Delta LA(r) \qquad (2.30)$$

Problem: RB(s,r) cannot be the bond-equilibrating variable. It is a policy variable.

Towards a solution:

We show that (2.30) holds if and only if $NETFA1\_R(r) = NETFA1\_F(r)$

where

$$NETFA1\_R(r) = SAVE(r) - I(r) + NETFA0\_reval(r) \qquad (2.33)$$

and

$$NETFA1\_F(r) = \sum_{s \neq r} ZB1(s,r) + \sum_{s \neq r} ZE1(s,r) - \left\{ \sum_{s \neq r} ZB1(r,s) * \frac{E1(s)}{E1(r)} + \sum_{s \neq r} ZE1(r,s) * \frac{E1(s)}{E1(r)} \right\} \qquad (2.34)$$

Fgn assets(r) – *Fgn liabilities(r)*

6

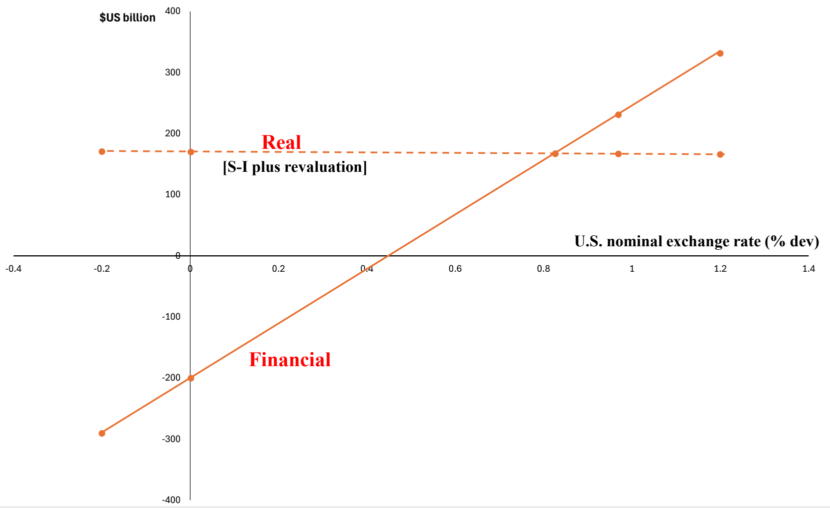
Equating NETFA1_R and NETFA1_F

CoPS

Real $$NETFA1\_R(r) = SAVE(r) - I(r) + NETFA0\_reval(r)$$

Financial $$NETFA1\_F(r) = \sum_{s\neq r} ZBI(s,r) + \sum_{s\neq r} ZEI(s,r) - \left\{ \sum_{s\neq r} ZBI(r,s) * \frac{EI(s)}{EI(r)} + \sum_{s\neq r} ZEI(r,s) * \frac{EI(s)}{EI(r)} \right\}$$

Effect of 1 percentage point increase in the U.S. cash rate on U.S. net fgn assets



7

Macro results from a 1 per cent increase in the power of the cash rate (% deviations from baseline)

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	(1) Real GDP	(2) Capital in use	(3) Employ	(4) Real wage	(5) Real investment	(6) 100*ΔBT /GDP	(7) Nom exch rate ($Int/$local)	(8) Price of GDP	(9) Real exchange rate
1 USA	-1.56	-0.84	-1.87	0.66	-2.29	0.32	0.83	-1.50	-0.32
2 China	-0.02	-0.02	-0.03	0.08	0.20	-0.11	-0.22	-0.06	0.10
…	…	…	…	…	…	…	…	…	…
17 Canada	-0.05	-0.06	-0.06	0.12	0.50	-0.19	-0.24	-0.14	0.00
18 Mexico	-0.12	-0.08	-0.18	0.19	0.31	-0.12	-0.25	-0.26	-0.14

- reduction in U.S. real GDP;
- reduction in U.S. real investment relative to GDP
- improvement in the U.S. trade balance;
- reduction in U.S. employment;
- emergence in the U.S. of excess capacity;
- reduction in the U.S. price level and consequent increase in real wage rates;
- nominal appreciation of the U.S. exchange rate;
- U.S. real devaluation; and
- relatively minor effects for other countries.

8

Concluding remarks

CoPS

GTAP-Money extends the range of GTAP applications to include monetary policy.

We expect GTAP-Money to generate new insights in traditional GTAP areas.

GTAP-Money introduces an optimizing financial agent for each country which holds:
- equity that finances capital creation;
- liquidity that facilitates day-to-day transactions; and
- government bonds that partially finance the government deficit.

In linking the monetary and real sides of the economy, we identified variables that coordinate decisions by financial agents with:
- demands by capital creators for investment finance;
- demands by the public for transaction finance; and
- demands by the government for deficit finance.

The coordinating variables are: expected rates of return on investments in physical capital; the shadow rate of return to holding liquidity; and the nominal exchange rate.

9

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The End

10