



POVERTY ANALYSIS USING AN INTERNATIONAL CROSS-COUNTRY DEMAND SYSTEM

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ABSTRACT

This paper proposes a new method for *ex ante* analysis of the poverty impacts arising from policy reforms. Three innovations underlie this approach. The first is the estimation of a global demand system using a combination of micro-data from household surveys, and macro-data from the International Comparisons Project. The second innovation relates to a methodology for post-estimation calibration of the global demand system, giving rise to country specific demand systems and an associated expenditure function which, when aggregated across the expenditure distribution, reproduce observed per capita budget shares exactly. The third innovation is use of the calibrated expenditure function to calculate the change in the head-count of poverty, poverty gap and squared poverty gap arising from policy reforms, where the poverty measures are derived using a unique poverty level of utility, rather than an income or expenditure-based measure. We employ these techniques with a demand system for food, other non-durables and services estimated using a combination of 1996 ICP data set and national expenditure distribution data. To illustrate the usefulness of these calibrated models for policy analysis, we assess the impacts of an assumed five percent food price rise as might be following a multilateral trade agreement.

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INTRODUCTION

Recent research into the evolution of the world distribution of income focuses on using a combination of cross country and within country household survey information to estimate the distribution of income from an *ex post* perspective (Schultz 1998; Bourguignon and Morrisson 2002; Quah 2002; Sala-i-Martin 2002a, 2002b, 2006). These analyses have arrived at mixed results, suggesting either no change, or slower growth, in the extent of poverty worldwide (Bourguignon and Morrisson 2002) or falling poverty (Sala-i-Martin 2006). While useful from a historical perspective, such analyses do not permit us to predict how inequality, and indeed poverty rates, might change in the future. This is particularly important for analyzing the poverty impacts of trade policy changes, where the impact on low income households is likely to be very different from higher income households. Developing such a characterization could be done on a country-by-country basis wherein household expenditure survey data are employed to approximate compensating variation as a budget share weighted average of price changes arising from policy shocks (Chen and Ravallion; Levinsohn, Berry and Friedman).

However, such an approach does suffer from a number of weaknesses. Inconsistencies between national accounts and household survey data (Sala-i-Martin 2006) renders results from household survey based analysis suspect in making predictions about aggregate impacts. Moreover, considerable difficulties exist in obtaining systematically comparably disaggregated expenditure information from household surveys for many countries. Even more substantial difficulties are encountered when attempting to map expenditure patterns on particular goods or services from household survey data to aggregate data, such as those contained in the International Comparisons Project (ICP), and matching these mappings across countries. It is also often difficult to obtain the price data needed to fully characterize preferences in terms of Engel and substitution elasticities from household survey data. Lastly, the share-weighted summation approach to approximating CV lacks theoretical rigor. Indeed the absence of substitution effects can be quite problematic for large price changes (Friedman and Levinsohn).¹ All-in-all, it is often difficult to compare impacts across countries based on country-by-country analyses which utilize survey data. In this regard, a theoretically rigorous, internationally comparable analysis is very attractive.

However, to date most international demand studies have been done using only per capita data which is of limited value to those interested in the distributional consequences of policy reform. One exception is the recent work by Cranfield (1999) and Cranfield *et al.* (2004) using maximum entropy and treating the per capita observation as an explicit aggregation of households. This paper builds on that previous work and contributes a framework which enables estimation of the future impact of poverty arising from exogenous policy shocks. It does so by incorporating more disaggregated household survey data into an entropy-based estimation procedure by which the demand for final goods and services at different points of the expenditure distribution are estimated. This demand system, in turn, provides the basis for analyzing the impact of a global food price increase of the sort anticipated by international trade models in the wake of WTO reforms.

1. More specifically, these authors find that incorporation of substitution effects dampens by 50% the welfare loss from price increases following the Asian financial crisis.

Recovery of disaggregate demands at each point of the national expenditure distribution is achieved using a global demand system (i.e., a demand system with the same set of parameters for each observation) which embodies more flexible income (expenditure) effects compared to alternative demand systems. The value of the demand system is further enhanced via post-estimation calibration of the parameters so that each country-specific demand system reproduces observed per capita demands. The resulting demand system appears to be well-suited to predicting expenditure patterns across the income spectrum in the three focus countries in our study: Indonesia, Thailand and the Philippines. As such it provides a useful vehicle for evaluating the welfare impacts of changes in consumer prices due to international trade reforms.²

One of the great benefits of this approach to poverty analysis derives from the fact that we are able to establish a unique *poverty level of utility* for each country. The poverty level of utility is invariant to the international trade shock and resulting price changes and therefore ideally suited for assessing the impact of a price change on the poverty level. This leads us to define a modified Foster-Greer-Thorbecke (FGT) poverty measure. Recall that the FGT measure depends on the ratio of an individual's expenditure, y_n , to a threshold level of expenditure, z : $P_\delta = N^{-1} \sum_{n=1}^N (1 - y_n z^{-1})^\delta \mathbf{I}(y_n \leq z)$, where δ equals 0, 1 or 2, n indexes individuals (households) in the population of size N and $\mathbf{I}(y_n \leq z)$ is an indicator function equaling one if the individual's (household's) expenditure is less than or equal to the threshold expenditure level.

Since our approach yields a single, calibrated expenditure function for the entire population, we replace the individual's expenditure level with an expenditure *function* defined across a price vector, calibrated demand parameters and the individual's level of utility. In a similar vein, the threshold level of expenditure is replaced with the expenditure function defined across the same price vector, the same calibrated demand parameters, and the *poverty level of utility*. To be clear, our modified FGT measure is written as:

$$P_\delta = N^{-1} \sum_{n=1}^N \left(1 - \frac{e(\tilde{\mathbf{p}}, u_n)}{e(\tilde{\mathbf{p}}, \bar{u})}\right)^\delta \mathbf{I}(u_n \leq \bar{u})$$

where $e(\tilde{\mathbf{p}}, u_n)$ is the calibrated expenditure function evaluated at the price vector $\tilde{\mathbf{p}}$ and calibrated utility for the n th household (u_n), while $e(\tilde{\mathbf{p}}, \bar{u})$ is the calibrated expenditure function evaluated using the price vector $\tilde{\mathbf{p}}$ and the poverty level of utility, $\bar{u}$. When $\tilde{\mathbf{p}}$ is set equal to the base (*shocked*) price vector, $e(\tilde{\mathbf{p}}, u_n)$ and $e(\tilde{\mathbf{p}}, \bar{u})$ measure household expenditure and the poverty level of expenditure before (*after*) the price shock, respectively.

Unlike the conventional FGT measure, there is no ambiguity about how the denominator in the modified FGT measure must change in the wake of policy reform. In this framework, the

2. Of course, one important limitation of the work in this paper is that we do not take into account factor earnings. However, the approach to doing so is relatively straightforward, provided estimates of the earnings impacts are available. For an illustration of how this can be done, see the paper by Hertel *et al.* (2004).

threshold level of expenditure is that which is required to attain the initial poverty level of utility, at the new prices. As such, it reflects optimal adjustments in demand, in response to these price changes. Use of an explicit expenditure function in the FGT measure is therefore a valuable innovation and clearly preferable to other income or expenditure-based approaches, which rely on the indexation of a fixed bundle of goods and services to establish a poverty line. In addition to being theoretically more satisfying, the expenditure function approach lends itself to ease of use in the type of partial and general equilibrium modeling often used for trade policy analysis.

This work draws together several recent strands of literature in consumer demand, poverty and trade policy analysis. On the demand side, it represents another step in the long and distinguished literature related to estimation of international consumer demands (e.g. Theil and Clements 1987; Theil *et al.* 1989; Rimmer and Powell 1992; Cranfield *et al.* 2002; Seale *et al.* 2003). By treating per capita national demands as the explicit aggregation of a distribution of demands across the expenditure spectrum, it adds to the literature which merges macro (i.e. per capita) and micro (i.e. individual or household survey based) data for use in the analysis of inequality and poverty (e.g. Schultz 1998; Cranfield 1999; Bourguignon and Morrisson 2002; Quah 2002; Sala-i-Martin 2002a, 2002b, 2006), thus allowing us to look beyond the averages (see, for example, Ravallion 2006). This paper has especially strong ties to Sala-i-Martin (2002a, 2002b, 2006) in that we use per capita expenditure and consumption data coupled with expenditure inequality data to recover an approximation to the expenditure distribution and data sources directly linked to those he has used in his previous work.³ However, we differ from Sala-i-Martin (2002a, 2002b, 2006) in that we are interested in country specific effects arising from policy shocks and we have the means to recover information useful in welfare analysis. In particular, the demand system used here has an explicit expenditure function which we use not only for welfare analysis, but also for poverty analysis. Therefore, our poverty calculations are tied directly back to micro-theory and the behaviour of economic agents; the value of this theoretically grounded approach is to welfare and poverty analysis has been highlighted previously in Ravallion (1998) and Neary (2004).

The next section presents a brief discussion of the demand system we estimate. The empirical methods and data are then discussed. Results of the econometric estimation are then presented, followed by development of the calibration scheme and subsequent results. The calibrated demand systems are then used to evaluate the consumption-based poverty impacts of international trade reforms. While this paper only performs these calculations for three countries in Southeast Asia, Indonesia, Philippines and Thailand, the approach could be extended to evaluating the poverty impacts of consumer price changes for all countries in the international data set. This kind of comprehensive, econometrically-based analysis of the international poverty impacts of trade reform has hitherto been missing from the literature.

3. Granted, Sala-i-Martin (2006) used per capita GDP and income inequality measures rather than expenditure measures. Note, however, that use of expenditure data is consistent with the World Bank's approach to modeling poverty issues.

AN IMPLICIT, DIRECTLY ADDITIVE DEMAND SYSTEM

The demand system used to characterize consumer preferences is an implicit, directly additive demand system (nicknamed AIDADS). AIDADS is best characterized as a generalization of the Linear Expenditure System (LES) which allows for non-linear Engel curves while maintaining a parsimonious parameterization of consumer preferences. Rimmer and Powell (1996) developed AIDADS⁴ based on Hanoch's (1975) seminal work on implicit additivity. Written in budget share form AIDADS appears as:

$$w_i = \frac{p_i \gamma_i}{y} + \frac{\alpha_i + \beta_i \exp(u)}{1 + \exp(u)} \left(1 - \frac{\mathbf{p}' \boldsymbol{\gamma}}{y} \right) \quad \forall i \quad (1)$$

where w_i is the i th good's budget share, $\mathbf{p}$ is a n vector of prices with typical element $p_i \in \Re_{++}$, $\alpha_i, \beta_i, \gamma_i$ are unknown parameters, $\boldsymbol{\gamma}$ is a n vector with typical element γ_i , u is utility and $q_i > \gamma_i \geq 0$. In AIDADS, the following parametric restrictions are used to ensure well-behaved demands: $0 \leq \alpha_i, \beta_i \leq 1$ for all i , and $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \beta_i = 1$.

Further details on AIDADS can be found in Cranfield *et al.* (2002, 2004); however, a few points are worthy of mention here. Firstly, as with the LES, AIDADS characterizes consumption at the subsistence level using the parameters γ_i which represent the quantity of good i required for survival, and therefore not subject to discretionary adjustment. Estimation of the subsistence quantities permits us to say something meaningful about consumers' behavioral response (or rather the lack of it) at extremely low income levels.

While AIDADS and the LES share the subsistence parameters, AIDADS generalizes the LES with a re-parameterization of the marginal expenditure share, such that the marginal expenditure shares change with the level of expenditure. When $\alpha_i = \beta_i$, AIDADS collapses to the LES and the marginal expenditure share on good i is constant. The parameter α_i characterizes the marginal expenditure shares on good i in the neighborhood of subsistence income, whereas β_i describes the marginal budget share at extremely high levels of expenditure. If $\alpha_i > \beta_i$, then the marginal (and average) budget share falls with rising income. The opposite is true when $\alpha_i < \beta_i$. From the point of view of characterizing consumption behavior at very low income levels, this additional flexibility is very important, as the marginal expenditure shares of the very poor are generally quite different from their counterparts evaluated at national, per capita income levels.

4. AIDADS is in the family of demand systems satisfying Cooper and McLaren's (1992) conditions for effective global regularity (see Rimmer and Powell 1996 for details). AIDADS also has rank three (see Gorman 1980 and Lewbel 1991 for further discussion on demand system rank).

EMPIRICAL METHODS & DATA

As the main purpose of this paper is to utilize an international, cross-country demand system for poverty analysis, we do not focus on estimation methods *per se*. Indeed, the entropy-based empirical methods used to recover the approximation to the distribution of expenditure, estimate parameters of AIDADS and recover unobservable levels of consumption have been published previously in Cranfield *et al.* (2004). However, since the present analysis incorporates a more refined approach to the estimation problem, a technical appendix containing the empirical model accompanies this paper.

Nevertheless, to contextualize the results, note that the empirical framework is developed in a mathematical programming environment, wherein the underlying demand system parameters and approximation to the distribution of expenditure are calculated. The numerical optimization program minimizes an objective function composed of a concentrated log-likelihood function and entropy function; the former allows for estimation of the demand system parameters while the latter enables recovery of the approximation to the distribution to expenditure. Constraints are used to define the AIDADS demand system, associated parametric restrictions and regression error terms, as well as the level of utility in the AIDADS model. Additional constraints are included to ensure the recovered approximation to the expenditure distribution matches the known moment conditions for expenditure and to ensure that the recovered disaggregate demands aggregate back to the observed level of per capita demand.

Our analysis uses price, per capita expenditure, and budget share data from a cross section set of countries in the most recent (1996) International Comparisons Project (ICP). These data are useful in analyzing international demand patterns as they are provided in identical units (*i.e.*, international dollars) and facilitate comparison of prices and quantities for disaggregate commodities across countries.

The ICP data record final consumption of 26 goods and services in 114 countries, with countries ranging in per capita expenditure from Malawi to the United States. In keeping with the additive nature of AIDADS, the 26 goods and services are aggregated into three broad aggregate goods: food (FOOD); other non-durables (ONONDUR); and services (SERVICES). Because of the dynamic nature of decisions with respect to durable goods, and given the cross-section nature of the data, durable goods have been omitted from this analysis. In other words, we focus only on the allocation of expenditures across non-durables and services.⁵ Budget shares are constructed by dividing nominal expenditure on each aggregate good by total nominal expenditure. The price of each good equals the ratio of nominal expenditure for that good to real expenditure for the same good. Total nominal expenditure per capita serves as the per capita expenditure term in AIDADS. Table 1 provides summary statistics of the ICP data used for estimation.

5. Moreover, initial estimation of AIDADS with data that included durables resulted in an empirical model that would not converge, nor would it converge after numerous attempts to resolve the issue (e.g. changing starting points of the optimization program, changing the bounds on the choice variable set, etc).

On the expenditure distribution side of the data base, quintiles and deciles, are obtained from an updated release of Deninger and Squire's (1996) World Income Inequality Database (WIID). Only expenditure or consumption based quintiles and deciles are used.⁶ Table 2 shows the quintile and decile values, year of coverage, and measurement details.⁷ The household survey data show the minimum, average and maximum value of expenditure for each percentile of the population in the three focus countries. As these are rather voluminous, they are not presented here. However, these data are drawn from household surveys for Indonesia (1993), Thailand (1996) and Philippines (1999).⁸

ECONOMETRIC RESULTS

Estimated parameters of AIDADS are shown in Table 3. Beginning with the estimates of subsistence quantities, γ_i , note that the estimate for services is at its lower bound of zero, while those for food and other non-durables are positive. The estimates of γ_i suggest, as one might expect, that food and other non-durables are a required part of the subsistence bundle of goods, while services are not strictly required for survival. Premultiplying the γ_i s by their respective, country-specific, prices and summing over the three goods permits us to establish the cost of the subsistence bundle. This survival level of expenditure on non-durables is equal to \$14, \$15 and \$26, respectively, for Indonesia, the Philippines and Thailand. The estimates of γ_i upon which these subsistence bundles are based reflect the level of real expenditure of the poorest household in the sample, i.e. those households on the extreme lower end of the expenditure distribution (on a global scale). Not surprisingly, these "survival" expenditures are drastically lower than poverty line(s) previously reported in the literature.⁹ The invariance of these subsistence purchases to expenditure will have important implications for the overall behavior of consumption at the lowest expenditure levels as will be shown below.

6. The updated WIID is a compilation of Gini coefficients and quintile and decile data for various countries over time.

7. The year of coverage often deviates from 1996, but usually by no more than five years, while quintiles are measured in different units across countries (*i.e.*, households versus individuals, gross versus net of taxes). Because expenditure distributions tend to change slowly over time, the mismatch between years is assumed unimportant. Due to the high correlation between expenditure, gross and net of taxes, and for households versus individuals, this mismatch in the data is also assumed away.

8. One may wonder why these household survey data were not directly incorporated into the analysis. Sala-i-Martin (2006) outlines three reasons why one should not use household survey means in such analysis. Albeit weak, his first point is that the literature uses "...population-weighted distribution...", the implication being that comparison to the literature requires use of data similar in nature. Second, and perhaps more persuasively, survey means have poorly understood properties; the notion being that differences in survey methodologies and strategies results in possible misleading summary statistics from household based surveys. Thirdly, surveys are not available for all countries and all time periods. This latter point highlights the difficulties one might encounter in attempting to estimate a global demand system using household based survey data. We would add to Sala-i-Martin's (2006) the fact that not all surveys collect the information needed to estimate demand systems; some surveys do not (or cannot) collect price data, while others only collect partial information on the household's consumption bundle, both of which make it nearly impossible to estimate a useful demand system.

9. For instance, Ravallion *et al.* (1991) conclude that \$23 per month (in 1985 PPP units), or \$276 per annum, is a reasonable lower bound to the poverty line.

Next, turn to the two sets of parameters describing the behavior of marginal budget shares. For FOOD, the estimated value of α_i indicates that, at subsistence expenditure levels, 73 percent of an additional dollar of expenditure is devoted to food, as opposed to 18 percent for other non-durables and just nine percent for services. This highlights the critical role of food in the budget decisions of very poorest households. The estimates of β_i provide the target value towards which the marginal budget share evolves as expenditure rises without bound. Not surprisingly, this is zero for food – at some point the household is satiated with respect to food – but over two-thirds for services; at extremely high expenditure levels slightly over two-thirds each additional dollar of expenditure is allocated to services.

The value of the marginal budget shares, fitted budget shares and Engel elasticities for all three goods, calculated at the means of the data, are presented in Table 4. As expected, when evaluated at the means of the data, the marginal budget share for food is low (0.068), while that for services is large (about 0.6). This highlights the danger of using the more restrictive LES specification when one is interested in the behavior of households in poverty. By restricting the marginal budget share to be constant, the LES is likely to understate the marginal budget share on food at the subsistence level by a full order of magnitude (0.73 versus 0.068 at mean prices and expenditure).

Ideally, we would like to be able to compare the recovered and observed budget shares across the expenditure spectrum. However, in our experience, attempts to do so are tenuous at best. In particular, there is an inherent discordance between the ICP data and the household survey data. Firstly, the definition of specific goods and services differs. Secondly, the data collection methods differ; ICP builds on the national accounts, while the household data are based on surveys implemented using a sampling approach. (Sala-i-Martin (2006) devotes considerable discussion to these issues.) As such, any comparison between actual budget shares from the household surveys and recovered budget shares tends to be dominated by differences in the measurement of disaggregated spending, and as such, is not terribly informative. Moreover, our primary interest lies in how one might use the recovered shares to undertake policy analysis. As foreshadowed above, the first step in this regard is a post-estimation calibration scheme which turns a global demand system into national demand systems. We turn attention next to this scheme.

POST-ESTIMATION CALIBRATION

While it is a useful analytical construct, the assumption of globally common preferences and the subsequent invariance of the AIDADS parameters across countries is somewhat troubling for policy analysis. And so, as is commonly done with micro-simulation analysis, it is useful to have a strategy for post-estimation calibration, in which the international demand system is “nationalized” by forcing the calibrated system to pass through the observed per capita expenditure levels. In this context post-estimation calibration of AIDADS is achieved by first rescaling α_i and β_i , and then re-computing a value of κ . However, in keeping with our focus on internationally comparable

measures of poverty, we do not alter the subsistence quantities, γ_i , which we assume to be a function of human needs and, as such, is invariant across regions.

Our calibration scheme works as follows. First, given that the subsistence parameters are invariant, the subsistence shares (i.e. price times γ_i divided by expenditure) are known and constant. Thus, it makes sense to parse the AIDADS equation in share form into two components – a subsistence share and a discretionary share (the latter could also be referred to as the super-numeracy share). We then calculate the fitted discretionary budget shares at per capita expenditure as:

$$\hat{\delta}_{it} = \frac{\hat{\alpha}_i + \hat{\beta}_i \exp(\bar{u}_t)}{1 + \exp(\bar{u}_t)} \left(1 - \frac{\mathbf{p}'_t \hat{\boldsymbol{\gamma}}}{\bar{y}_t}\right), \quad (2)$$

where $\hat{\alpha}_i$ and $\hat{\beta}_i$ are estimated values, $\hat{\boldsymbol{\gamma}}$ is the vector of estimated subsistence quantities, and $\bar{u}_t$ is the value of utility arising from choice of the optimal consumption bundle at the per capita level of expenditure, $\bar{y}_t$. The calculated value of $\hat{\delta}_{it}$ is simply the value of the discretionary budget share at the per capita level of expenditure, calculated using the estimated values of AIDADS parameters.

Next, add the estimation residuals for the i th equation in the t th observation back to $\hat{\delta}_{it}$ to obtain “observed” discretionary budget shares as follows:

$$\delta_{it} = \hat{\delta}_{it} + (\bar{w}_{it} - \hat{w}_{it}) \quad (3)$$

where $\hat{w}_{it}$ is the fitted value of the budget share evaluated at the per capita level of expenditure, calculated using the econometrically estimated value of the AIDADS parameters. Here, the regression errors (stated in terms of budget shares) are added back into the discretionary portion of AIDADS. Doing so assumes that all of the regression error is attributable to an imperfectly observable discretionary budget share.

The next step in the calibration scheme is to adjust the original estimates of α_i and β_i with the ratio of fitted to “observed” discretionary budget shares as follows: $\tilde{\alpha}_{it} = \hat{\alpha}_i \hat{\delta}_{it} / \delta_{it}$ and $\tilde{\beta}_{it} = \hat{\beta}_i \hat{\delta}_{it} / \delta_{it}$. Next, normalize the values of $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$ to ensure they sum to unity: $\alpha_{it}^C = \tilde{\alpha}_{it} / \sum_j \tilde{\alpha}_{jt}$ and $\beta_{it}^C = \tilde{\beta}_{it} / \sum_j \tilde{\beta}_{jt}$, where the superscript “C” denotes the parameters have been calibrated. These country specific, calibrated values of α_i and β_i are then fixed and used in calibrating utility and the κ parameter by solving a non-linear system of equations for each country. This system contains the defining equation of utility for AIDADS and the AIDADS system with quantities fixed at their observed per capita consumption levels, but evaluated at the per capita level of expenditure.¹⁰ Hence, each country for which post-estimation calibration is undertaken has their

10. This normalization is required, as integration of the calibrated demand system back to an underlying dual function requires a constant of integration. Adjusting κ accordingly provides such normalization.

own values of α_i , β_i and κ . These calibrated parameters are such that they yield the observed per capita budget share at the per capita level of expenditure.¹¹ Moreover, this calibration scheme results in a calibrated set of budget shares across all expenditure levels within each country's expenditure distribution.

Table 5 shows the estimated and calibrated values of the AIDADS model. Recall that the subsistence parameters, γ_i , are assumed to be invariant internationally and are not calibrated. In light of the subsequent trade policy experiment which we will explore, we focus our attention here on the behavior of food expenditures across the expenditure spectrum. In this regard, note that since the original estimate of β_i for food is zero, then so too are the calibrated values of β_i . However, relative to its estimated value the calibrated value of α_i has increased for Indonesia and the Philippines, but fallen for Thailand. The calibrated values of κ have also changed relative to original estimate, with Indonesia's and Thailand's calibrated κ values being smaller than that estimated for all countries, while the calibrated value of κ is larger for the Philippines.

These calibrated values are not terribly useful in conveying the impact of calibration. Instead it is more instructive to compare the recovered and calibrated budget shares for food across the expenditure distribution in the three focus countries. Figures 1, 2 and 3 plot the recovered and calibrated budget shares for food, the observed per capita budget share for food at the per capita level of expenditure, the calibrated value of α_i (for the respective focus country), and a break-down of the budget shares showing the subsistence $(p_i\gamma_i/y)$ and discretionary shares $((\alpha_i + \beta_i \exp(u))/(1 + \exp(u))(1 - \mathbf{p}'\boldsymbol{\gamma}/y))$ to illustrate how these individual components adjust to changes in expenditure, thereby influencing the overall budget share.

For Indonesia (Figure 1), the calibrated food expenditure shares now pass through the per capita based budget share, and vary only slightly from the recovered shares. Specifically, the calibrated shares are rotated in a clockwise manner around the per capita based budget share. Relative to the recovered shares, this means the calibrated shares are larger than the recovered shares at low expenditure levels, while the reverse is true at high expenditure levels. However, the difference between the calibrated and recovered shares is slight, which makes this rotation difficult to observe. The difference between the calibrated α_i for food (the horizontal line in Figure 1) and the calibrated budget share for food in the poorest household in recovered expenditure distribution for Indonesia is about eight percent, with the calibrated share being less than the calibrated value of α_i . Recall, α_i is a measure of the limiting behaviour of marginal expenditures as total expenditure approaches our estimate of subsistence expenditure. As the poorest household in the expenditure distribution for Indonesia has an expenditure level well above the subsistence expenditure at local (i.e. Indonesian) prices, this difference is not surprising. Since Engel's law suggests the marginal budget share for food falls as expenditure grows, it is hardly surprising that the calibrated per capita

11. The only potential problem with this procedure occurs if the actual consumption level for a given good is not larger than the subsistence level. This suggests that it may be of interest to constrain the subsistence levels to be strictly less than the lowest level of observed consumption.

budget share for food is well below the limiting behaviour given by α_i . Comparing the subsistence and discretionary shares for food in Indonesia, we see that the majority of the change in food's budget share is driven by discretionary expenditure. As the subsistence share has expenditure in the denominator, this share falls with the level of expenditure; for Indonesia, this means that as expenditure approaches the level of the wealthiest household, the subsistence share becomes nearly zero.

Somewhat different results are obtained for the Philippines, where calibrated shares differ markedly from the recovered shares (see Figure 2). Here, the calibrated shares have shifted upwards relative to the recovered shares so that they pass through the observed national per capita budget share. Moreover, the difference between the recovered and calibrated shares grows as one progresses upwards through the Philippines' expenditure distribution. Calibrated food budget shares are very close to the calibrated value of α_i for the Philippines (the horizontal line in Figure 2), suggesting that total expenditure on non-durables by the poorest household in the Philippines' recovered expenditure distribution is closer to subsistence expenditure (at local prices) than in Indonesia. The subsistence share of food at the subsistence level of expenditure is much higher in The Philippines than in Indonesia -- approximately 16 percent of total non-durable expenditure in the poorest household. As with Indonesia, it falls towards zero at the highest expenditure levels. It is also interesting to note that the behaviour of the discretionary shares across the expenditure spectrum is rather different from Indonesia. Discretionary shares for food in the Philippines initially rise with expenditure, thereupon reaching a maximum, before beginning to fall after about 5.2 on the log expenditure scale. As with Indonesia, food's budget share is almost entirely accounted for by discretionary expenditures on food at high levels of expenditure.

For Thailand the difference between calibrated and recovered shares is much more pronounced (Figure 3). As expected, Thailand's calibrated food budget shares pass through the per capita based budget share for food. Moreover, the calibrated shares have shifted down, relative to the recovered shares, in near parallel fashion. Also note that the gap between the calibrated value of α_i for food and the food budget share for the poorest household in Thailand's expenditure distribution is even more pronounced than in Indonesia. And, as with Indonesia, the subsistence shares and calibrated discretionary shares fall as expenditure increases, with the discretionary share accounting for a larger portion of food's budget share as one moves up the expenditure distribution.

What should be clear from the preceding discussion is that our calibration strategy does not affect the subsistence shares. Calibration only plays a role in changing the location and shape of the discretionary shares; it does so by altering the values of α_i , β_i and κ , and the subsequent value of utility when the utility function is evaluated, subject to the budget constraint, using the calibrated parameters. For the particular estimates presented above, only the term $(\alpha_i + \beta_i \exp(u))/(1 + \exp(u))$ is altered, and that drives the changes in the discretionary share. It is important to recognize, however, that any exogenous policy shock to either prices or expenditure (income) levels will alter both the subsistence and discretionary shares. For instance, a price increase arising from trade liberalization would increase the subsistence share for food, but may increase or decrease the discretionary share. The latter is qualified as the price increase will decrease the $(1 - p'\gamma/y)$

component of the discretionary share but increase the $(\alpha_i + \beta_i \exp(u))/(1 + \exp(u))$ component (recall that the value of β_i for food is zero); depending on the size of these changes, the discretionary share may increase or decrease. As such, it is difficult to say *a priori* if food's share of expenditure will rise or fall in the wake of a price hike. Of even greater importance is how such a price shock might affect the poorest households and hence the incidence of poverty. In the tradition of micro-simulation, we turn next to an exploration of these questions by simulating the impacts of a five percent global food price rise using the calibrated demand system. We will focus particular attention on the resulting changes in consumer demand and poverty.

POVERTY IMPACTS OF A GLOBAL FOOD PRICE RISE

There has been considerable interest recently in the potential impacts of multilateral trade liberalization on poverty in developing countries. The clear consensus is that agricultural reforms in the rich countries will cause world farm and food prices to rise, as farm subsidies are eliminated and rich country border protection is reduced (Beghin et al., Cline; Anderson and Martin). Such a food price rise may be expected to benefit agricultural producers in the developing world, while hurting consumers. The net outcome is therefore ambiguous (Hertel and Winters). Providing a comprehensive analysis of the trade/poverty debate is beyond the scope of this paper. However, the framework developed offers an important improvement in the way the consumption impacts of such a price hike are evaluated. Instead of simply assuming that the poor consume the *per capita* bundle of goods and services (Cline) or that they consume only food (Anderson, Martin and van der Mensbrugghe), we are now in a position to assess the differential impact of a food price increase *across the entire expenditure (income) spectrum*.

For illustrative purposes, we apply a five percent increase in the price of food and examine its impact on the demand for food and household welfare within each focus country. The change in welfare at the shocked prices is then used to assess the impact of the price shock on a modified Foster, Greer and Thorbecke (1984) measure of poverty. Traditionally, the FGT measure is defined as:

$$P_{\delta} = N^{-1} \sum_{n=1}^N \left(1 - \frac{y_n}{z}\right)^{\delta} \mathbf{I}(y_n \leq z)$$

where $\delta \in \{0,1,2\}$ ¹², N is the number of observations (i.e. households), y_n is the n th household's expenditure, z is a poverty line threshold level of expenditure and $\mathbf{I}(y_n \leq z)$ is an indicator function assuming a value of unity if the condition is true, and zero otherwise. P_0 measures the proportion of people in the population who are at or below the poverty line threshold, P_1 is the poverty gap (i.e., the per capita expenditure short fall of those in poverty, expressed as a share of the poverty line threshold level of expenditure), and P_2 is a poverty measure which "...is sensitive to distribution among the poor" (Deaton 2000, p.147). In some respects, P_2 could be viewed as akin to a partial Herfindahl index. It is a partial measure because it only reflects the concentration of expenditure amongst the poor.

For purposes of this paper, we redefine P_{δ} as:

$$P_{\delta} = N^{-1} \sum_{n=1}^N \left(1 - \frac{e(\tilde{\mathbf{p}}, u_n)}{e(\tilde{\mathbf{p}}, \bar{u})}\right)^{\delta} \mathbf{I}(u_n \leq \bar{u}) \quad (4)$$

12. To avoid confusion with a parameter of AIDADS, we use δ as the subscript to the FGT measure (i.e., P_{δ}) rather than the traditional α .

where $e(\tilde{\mathbf{p}}, u_n)$ is the calibrated AIDADS expenditure function evaluated at the price vector $\tilde{\mathbf{p}}$ and calibrated utility for the n th household (u_n), while $e(\tilde{\mathbf{p}}, \bar{u})$ is the calibrated AIDADS expenditure function evaluated using the price vector $\tilde{\mathbf{p}}$ and the poverty level of utility, $\bar{u}$. When $\tilde{\mathbf{p}}$ is set equal to the base (*shocked*) price vector, $e(\tilde{\mathbf{p}}, u_n)$ and $e(\tilde{\mathbf{p}}, \bar{u})$ measures household expenditure on non-durables and the poverty level of expenditure before (*after*) the price shock, respectively. The use of utility in the indicator function is advantageous as utility will vary with expenditure and prices, and consumer demands are free to change as well; hence, our version of FGT enables poverty analysis calculation of the change in P_δ arising from the shock, as well as the change in consumption of goods and services in the consumer's bundle. This is the key point. To be clear, since we deal with a price change, it is the poverty level of expenditure, $e(\tilde{\mathbf{p}}, \bar{u})$, that adjusts after the price shock, but the poverty level of utility does not change.

A natural question to next ask is how one establishes the poverty level of utility. We use two approaches to establishing the poverty level of utility. The first approach assumes one has a poverty level of expenditure. In this case, country specific poverty levels of utility can be calculated by maximizing the AIDADS utility function, using the calibrated AIDADS parameters, subject to the budget constraint evaluated at local prices and the poverty level of expenditure. The resulting solution will include the optimal consumption bundle at the poverty level of expenditure and local prices, but also the poverty level of utility (i.e. the utility of the consumption bundle purchased at local prices with the poverty level of expenditure). The advantage of such an approach is that the resulting poverty levels of utility (across countries) reflect inter-country price level differences that otherwise would not be accounted for if one used a poverty level of expenditure only. We use this approach to calculate poverty levels of utility associated with one and two dollar a day poverty thresholds; specifically, we use a one dollar a day poverty level of expenditure (i.e. \$365 per annum) in calculating the poverty level of utility for Indonesia and the Philippines, and the two dollar a day (i.e. \$730 per annum) threshold for Thailand.¹³ The one and two dollar a day poverty thresholds of expenditure are employed in order to have some measure of consistency with the second way in which a poverty level of utility can be established.

In particular, the World Bank's World Development Indicators (WDI) provides national poverty rates (NPRs), defined as the proportion of national populations which fall below nationally defined poverty levels. These percentages are 15.7, 36.8 and 13.0 percent in Indonesia, the Philippines and Thailand, respectively. Using the recovered approximation to the expenditure distribution and its support, it is easy to determine the poverty level of utility and expenditure. The share of a country's population is summed until the cumulative sum just exceeds the national level of poverty. The utility level of the household group just below the point at which the cumulative sum just exceeds the NPR is the poverty level of utility. We can then map back from that household

13. The one and two dollar a day poverty lines have been the subject of some discussion in the literature. Sala-i-Martin (2006) notes that the World Bank's definition of the poverty line was stated as \$1.02 per day (in 1985 prices) in 1990, but at \$1.08 (in 1993 prices) in 2000. The issue is what base year is used to define the poverty line, and the extent of price inflation since that base year was established. Nevertheless, as our approach could accommodate *any* poverty line, we do not address whether one ought to use one dollar a day, \$1.02 per day or \$1.08 per day, and focus instead on how one might use the approach with any particular poverty line definition.

group to the expenditure level consistent with these NPR-based thresholds; these values are \$331 for Indonesia, \$343 for the Philippines and \$629 for Thailand. Moreover, it should now be clear why we choose the one dollar a day poverty level of expenditure for Indonesia and the Philippines, and two dollars a day for Thailand in our first approach to defining the poverty level of utility; doing so makes the analysis based on the two poverty level of utility approaches more comparable in terms of poverty level of expenditure.

The average percent change in demand for food when the price of food increases by five percent ranges from -3.8 percent for Thailand to -4.1 percent of the Philippines. To better illustrate these reductions in demand for food across the focus countries, Figure 4 shows the level of demand for food in the focus countries before and after the price shock. Price shock induced reductions in quantity demanded vary not only across focus countries, but also across expenditure levels within each focus country. For instance, Indonesia, the Philippines and Thailand have per capita expenditure levels of \$655, \$763 and \$1,454, respectively. Figure 4 shows that Thailand (a wealthier country compared to Indonesia and the Philippines) generally has smaller changes in the quantity of food demanded, regardless of where one is located in the expenditure distribution. Changes in demand for food in Indonesia and the Philippines are larger compared to Thailand, but also reflect considerable within-country variation.

To better illustrate what is driving the changes in demand arising from the five percent increase in price, Figure 5 plots the uncompensated price elasticity for food in the Philippines and its components based on the calibrated demand system. These components include the expenditure (Engel) elasticity, budget share, the negative of the product of the share and Engel elasticity and the compensated price elasticity (i.e. the components of the Slutsky equation are plotted across the expenditure spectrum). What is clear is that the expenditure effect (i.e. the negative of the Engel elasticity times the budget share) dominates the compensated price effect, and is the most significant driver of changes in the uncompensated price elasticity. Moreover, the uncompensated own-price elasticity for food becomes more inelastic as expenditure grows. Consequently, the relative change in demand falls as expenditure grows. However, because the level of demand increases from low to high expenditure levels, these smaller relative changes in demand actually translate into larger absolute changes in demand at higher expenditure levels.

To relate the price shock impact back to the fundamental parameters of AIDADS, Figures 6, 7 and 8 show the breakdown of the *change in food's budget share* into the change in the subsistence share and the change in the discretionary share spent on food, across expenditure levels in the focus countries. Recall that the subsistence share will rise for any price shock, whereas the discretionary share may increase or decrease, depending on the size of change in $(1 - \mathbf{p}'\boldsymbol{\gamma}/y)$ versus $\alpha_i/(1 + \exp(u))$ (where we have reflected the fact that β_i is zero in the calibrated demand system). Further insight can be gained by noting that when β_i equals zero (as is the case for food), the impact of a price change on the discretionary share can be expressed as:

$$\frac{\partial \hat{\delta}_{jt}}{\partial p_{it}} = \frac{\hat{\alpha}_i}{1 + \exp(\bar{u}_t)} \left\{ -\gamma_i + \left(1 - \frac{\mathbf{p}'_t \hat{\boldsymbol{\gamma}}}{\bar{y}_t}\right) \frac{-\alpha_i}{1 + \exp(\bar{u}_t)} \frac{\partial u_t}{\partial p_{it}} \right\}. \quad (5).$$

Since $\alpha_i/(1+\exp(\bar{u}_t))$ is positive, the sign of (5) depends on the two terms within the brackets $\{\}$, but since $\partial u_t/\partial p_{it}$ is negative, the term in $\{\}$ can be either positive or negative.

In Indonesia (see Figure 6), the *change in food's budget share* initially increases as expenditure increases, reaches a maximum and then declines. As is evident in Figure 6, the *change in the subsistence share* becomes smaller as one progresses through the expenditure levels, while the *change in the discretionary share* increases from a very small level, to a maximum and then declines. The latter effect suggests that 6.8 on the natural log of expenditure scale is the critical point after which the reduction in $(1-\mathbf{p}'\boldsymbol{\gamma}/y)$ arising from the price increase overwhelms the increase in $\alpha_i/(1+\exp(u))$ and the change in the overall budget share for food begins to drop. As is clear from Figure 6, the change in the subsistence share dominates at low expenditure levels, as associated points in Figure 6 lie above those representing the change in the discretionary budget share for food. However, the change in overall food budget share is dominated by the change in discretionary share at higher expenditure levels.

The change in food's budget share for the Philippines (Figure 7) is somewhat different; specifically, the share initially falls, reaches a local minimum, rises to a maximum and then falls again. This wave pattern of adjustment reflects two competing sets of changes. On the one hand, the change in the subsistence share falls through the entire range of expenditure (as expected). On the other hand, the change in the discretionary budget share is initially negative, but increases, becomes positive, reaches a maximum and then begins to fall. Based on this, we may conclude that at low levels of expenditure, the role of the subsistence parameter for food overwhelms the positive effect of the second term in brackets in equation 5.

Figure 8 illustrates that results are also different for Thailand, where the change in food's budget share arising from the five percent food price rise is positive, but decreasing in expenditure. However, as before, the change in food's share in the total non-durables budget reflects a diminishing role of the subsistence expenditure, and variable role for discretionary expenditure. In particular, the latter increases, reaches a maximum and then decreases as expenditure grows, while the subsistence share falls as expenditure grows.

Three points emerge from this analysis. First, the change in quantity of food demanded, as a result of the price shock, is larger for households with higher expenditure levels. Second, food's share of total expenditure increases with the price increase (as one might naturally expect). Lastly, the change in food's budget share is dominated by changes in subsistence expenditure shares at low income levels, but changes in discretionary expenditure shares at higher income levels.

Recognize too that the increase in food price erodes a household's purchasing power. The impact of this purchasing power change is captured in Figure 9, which shows the compensating variation (CV) associated with the shock to food price calculated using the calibrated AIDADS expenditure function. As expected given a price increase, and regardless of the focus country, CV increases as expenditure increases. However, when CV is expressed as a percent of initial

expenditure, as shown in Figure 10, it is clear that the price shock has a much larger relative impact on poorer households than on wealthier households.

Table 6 summarizes the impact of the price shock on the modified FGT poverty measures developed here. Across all focus countries, and regardless of how the poverty level of utility is established, the FGT poverty measures increase with a five percent increase in the price of food price. In percentage terms, the increase in P_8 is greatest in Indonesia, followed by Thailand and then the Philippines. And, while the size of the percent change varies across the approaches to establishing the poverty level of utility, the magnitudes of these changes are generally the same (except for P_1 in Thailand). Nevertheless, results suggest that the five percent food price increase generates a greater incidence and intensity of poverty in Indonesia than in Thailand or the Philippines. The larger percent changes in P_0 and P_1 in Indonesia drive the greater incidence of poverty, while the larger percent in P_2 drive the greater intensity of poverty.

SUMMARY AND CONCLUSIONS

This paper has developed a means by which one can recover an approximation to the distribution of expenditure, estimate parameters of a demand system and recover unobservable levels of consumption in a manner that replicates the means by which aggregate economic data are collected, namely, as the sum of disaggregate expenditure and demand levels. The proposed approach takes advantage both of international cross-section data, as well as data from household expenditure surveys.

As has been recently pointed out by Sala-i-Martin, there are fundamental inconsistencies between household survey data and national accounts data. This complicates the measurement of poverty, as well as the assessment of changes in poverty due to (e.g.) trade reforms. By recovering the demand for goods and services across the expenditure distribution within a set of countries in a manner which is consistent with the national accounts, we are able to overcome this inconsistency. Specifically, we calibrate the underlying global demand system parameters to in order to replicate observed per capita levels of demand. Consequently, our approach allows for analysis, not only at the per capita level, but also analysis of the impact of policies on the distribution of welfare measures across individuals within the population. Moreover, since the calibrated values pass through the observed of per capita budget share, one can be more confident that welfare changes, especially when stated in relative terms, reflect economic factors and are not an artifact of statistical measurement errors.

In order to illustrate how this approach can be used for poverty analysis, we examine the impact of a five percent rise in the world price of food, as a consequence of rich country trade reforms. This is a topic that has received considerable attention recently. However, these studies have not been able to come to grips with the differential consumption impacts of this price increase across the income spectrum, within a theoretically consistent framework. We analyze these differential impacts in considerable detail, decomposing the households' responses into subsistence and discretionary components. At the lowest expenditure levels, the impact of higher food prices on subsistence expenditures dominates the change in total food expenditures. However, this changes as

one moves to higher expenditure levels, giving rise non-monotonic changes in food expenditure shares across the income spectrum. Not surprisingly, the food price increase has an adverse impact on consumers in the countries examined, with the largest welfare losses felt by the poorest households in the Philippines. At the same time, the five percent food price rise increases the incidence and intensity of poverty in the focus countries considered here (i.e. the Philippines, Indonesia and Thailand). In percentage terms, these poverty increases are largest for Indonesia, followed by Thailand and the Philippines.

While our partial equilibrium analysis does not account for the impact of higher world food prices on household incomes, the framework that we outline here could be readily incorporated into a general equilibrium model aimed at assessing the poverty impacts of trade reforms. Indeed, such a step would enhance the credibility of such analyses, which are often viewed as being overly simplistic in their treatment of household expenditures.

Table 1. ICP Data Summary Statistics

	FOOD	ONONDUR	SERVICE
Budget shares			
Mean	0.368	0.244	0.388
Standard deviation	0.029	0.005	0.021
Prices			
Mean	0.634	0.601	0.520
Standard deviation	0.074	0.226	0.198
Per capita expenditure ('000 of international dollars)			
Mean	48.97		
Standard deviation	4292.68		

Table 2. Decile and Quintile Measures Used for Estimation¹

Country	Expenditure Class										Year	Type
	1	2	3	4	5	6	7	8	9	10		
	Observations for which Quintile Data are Available and Quintile Values											
Ecuador	5.4	9.4	14.2	21.3	49.7						1995	E
Estonia	6.91	11.44	15.87	22.01	43.77						1995	E
Jordan	7.6	11.4	15.5	21.1	44.4						1997	E
Kazakhstan	6.7	11.5	16.4	23.1	42.3						1996	E
Kenya	5.0	9.7	14.2	20.9	50.2						1994	E
Korea	7.5	12.9	17.4	22.9	39.3						1993	E
Mauritius	6.70	11.60	15.70	22.60	43.40						1991	C
Mongolia	7.3	12.2	16.6	23.0	40.9						1995	E
Morocco	6.5	10.6	14.8	21.3	46.6						1999	E
Panama	3.6	8.1	13.6	21.9	52.8						1997	E
Poland	8.50	12.88	16.98	22.42	39.21						1993	E
Turkey	5.8	10.2	14.8	21.6	47.7						1994	E
Turkmenist	6.1	10.2	14.7	21.5	47.5						1998	E
Yemen	6.1	10.9	15.3	21.6	46.1						1992	E
	Observations for which Decile Data are Available and Decile Values											
Albania	3.68	5.02	6.01	7.02	8.06	9.11	10.64	12.54	15.12	22.80	1996	C
Armenia	1.68	2.88	3.71	4.51	5.48	6.79	8.53	11.12	15.62	39.69	1996	C
Bangladesh	2.93	4.21	5.09	5.96	6.91	7.93	9.34	11.48	14.96	31.18	1996	C
Belarus	3.01	4.69	5.82	6.89	8.01	9.16	10.55	12.36	15.36	24.15	1996	C
Bolivia	1.07	2.17	3.19	4.18	5.37	6.74	8.58	11.39	16.71	40.62	2000	C
Bulgaria	3.10	5.00	6.15	7.19	8.16	9.31	10.58	12.19	14.62	23.69	1997	C
Cameroon	1.51	2.44	3.24	4.20	5.41	6.89	8.96	11.85	16.70	38.80	1996	C
CotedIvoir	1.96	3.09	4.17	5.32	6.41	7.82	9.60	11.83	16.50	33.31	1995	C
Egypt	1.2	2.4	3.3	4.2	5.2	6.3	7.8	10.1	14.6	44.8	1997	C
Guinea	1.17	2.08	2.93	3.87	4.94	6.32	8.17	10.77	15.76	43.99	1994	C
Hungary	3.83	5.44	6.50	7.45	8.36	9.33	10.42	11.95	14.40	22.32	1997	C
Jamaica	1.16	2.22	3.19	4.13	5.34	7.01	9.12	12.42	18.35	37.04	1996	C

Kyrgyz	2.37	3.68	4.55	5.38	6.40	7.58	9.18	11.52	15.99	33.35	1997	C
Latvia	3.66	5.26	6.36	7.47	8.42	9.38	10.63	12.16	14.41	22.25	1996	C
Lithuania	2.96	4.52	5.57	6.61	7.64	8.75	10.23	12.07	14.99	26.65	1996	C
Macedonia	2.60	4.36	5.44	6.54	7.55	8.91	10.38	12.56	15.49	26.16	1997	C
Madagascar	1.96	3.23	4.23	5.22	6.32	7.63	9.43	12.19	16.30	33.48	1997	C
Mali	1.15	2.05	2.91	3.98	5.21	6.70	8.58	11.27	15.96	42.20	1994	C
Mexico	1.24	2.42	3.43	4.39	5.42	6.80	8.52	11.12	16.06	40.60	1998	C
Moldova	2.01	3.72	4.84	5.97	7.11	8.58	10.45	12.86	16.37	28.09	1997	C
Nepal	2.24	3.33	4.09	4.90	5.89	6.96	8.49	10.63	14.54	38.94	1996	C
Nigeria	1.71	2.91	3.78	4.69	5.72	7.05	8.80	11.40	15.96	37.98	1996	C
Pakistan	3.58	4.58	5.26	5.86	6.51	7.25	8.19	9.49	11.90	37.36	1997	C
Peru	2.21	3.40	4.46	5.48	6.65	7.96	9.50	11.74	16.00	32.59	1997	C
Russia	0.76	1.53	2.49	3.94	5.47	7.32	9.64	12.47	17.42	38.97	1996	E
Senegal	2.71	3.77	4.59	5.37	6.15	7.11	8.42	10.40	14.05	37.43	1994	C
Slovakia	4.93	6.40	7.26	8.05	8.87	9.78	10.85	12.04	13.90	17.92	1993	C
SriLanka	3.24	4.42	5.23	5.97	6.82	7.82	9.18	11.15	14.61	31.58	1995	C
Tajikistan	2.99	4.67	5.72	6.77	7.75	8.74	9.98	11.78	14.86	26.72	1999	C
Tanzania	1.80	2.91	3.82	4.76	5.84	7.16	8.87	11.38	15.98	37.47	1993	C
Ukraine	2.16	3.62	4.75	5.74	6.83	8.02	9.54	11.83	15.19	32.32	1995	C
Uzbekistan	1.29	2.90	4.17	5.41	6.67	8.16	9.95	12.36	16.54	32.56	2001	C
Vietnam	3.05	4.33	5.19	5.98	6.88	8.03	9.43	11.64	15.41	30.05	1998	C
Zambia	1.25	2.22	3.06	3.95	5.00	6.29	7.97	10.54	15.47	44.25	1996	C
Zimbabwe	0.53	1.05	1.54	2.10	2.76	3.58	4.67	6.41	9.92	67.44	1995	C

1. Year denotes the year in which the underlying survey was implemented; Type denotes whether the data are based on expenditure (E) or consumption (C)

Table 3. Estimated AIDADS Parameters

	FOOD	ONONDUR	SERVICE
α	0.730	0.181	0.090
β	0.000	0.311	0.689
γ	0.346	0.039	0.000
κ	2.783		

Table 4. Marginal Budget Shares, Fitted Budget Shares and Engel Elasticities, Evaluated at the Means of the Data.

	FOOD	ONONDUR	SERVICE
Marginal budget share	0.068	0.298	0.633
Fitted budget share	0.259	0.264	0.476
Engel Elasticity	0.263	1.129	1.329

Table 5. Estimated and Calibrated AIDADS Parameters

	FOOD	ONONDUR	SERVICE
α			
Estimated	0.730	0.181	0.090
Indonesia-calibrated	0.738	0.164	0.098
Philippines-calibrated	0.731	0.226	0.043
Thailand-calibrated	0.589	0.244	0.166
β			
Estimated	0.000	0.311	0.689
Indonesia-calibrated	0.000	0.271	0.729
Philippines-calibrated	0.000	0.540	0.460
Thailand-calibrated	0.000	0.247	0.753
γ			
Estimated	0.346	0.039	0.000
Indonesia	0.346	0.039	0.000
Philippines	0.346	0.039	0.000
Thailand	0.346	0.039	0.000
κ			
Estimated	2.783		
Indonesia	2.740		
Philippines	3.127		
Thailand	2.358		

Table 6. Foster-Greer-Thorbecke P_δ Measures of Poverty (percentage change in parentheses)

	WDI		One/two dollars a day	
	Base	Shock	Base	Shock
Indonesia ^a				
P ₀	15.93	17.04 (6.98)	20.74	22.59 (8.93)
P ₁	2.79	3.19 (14.14)	4.24	4.73 (11.68)
P ₂	0.69	0.82 (18.96)	1.19	1.37 (15.42)
Philippines ^a				
P ₀	37.03	38.15 (3.00)	39.26	40.37 (2.83)
P ₁	14.36	15.05 (4.84)	15.79	16.51 (4.51)
P ₂	7.21	7.65 (6.16)	8.13	8.60 (5.78)
Thailand ^b				
P ₀	13.33	14.07 (5.56)	20.00	21.11 (5.56)
P ₁	2.23	2.48 (11.10)	4.23	4.57 (7.92)
P ₂	0.52	0.59 (15.20)	1.22	1.35 (10.67)

a. Based on a one dollar a day poverty level of expenditure

b. Based on a two dollar a day poverty level of expenditure

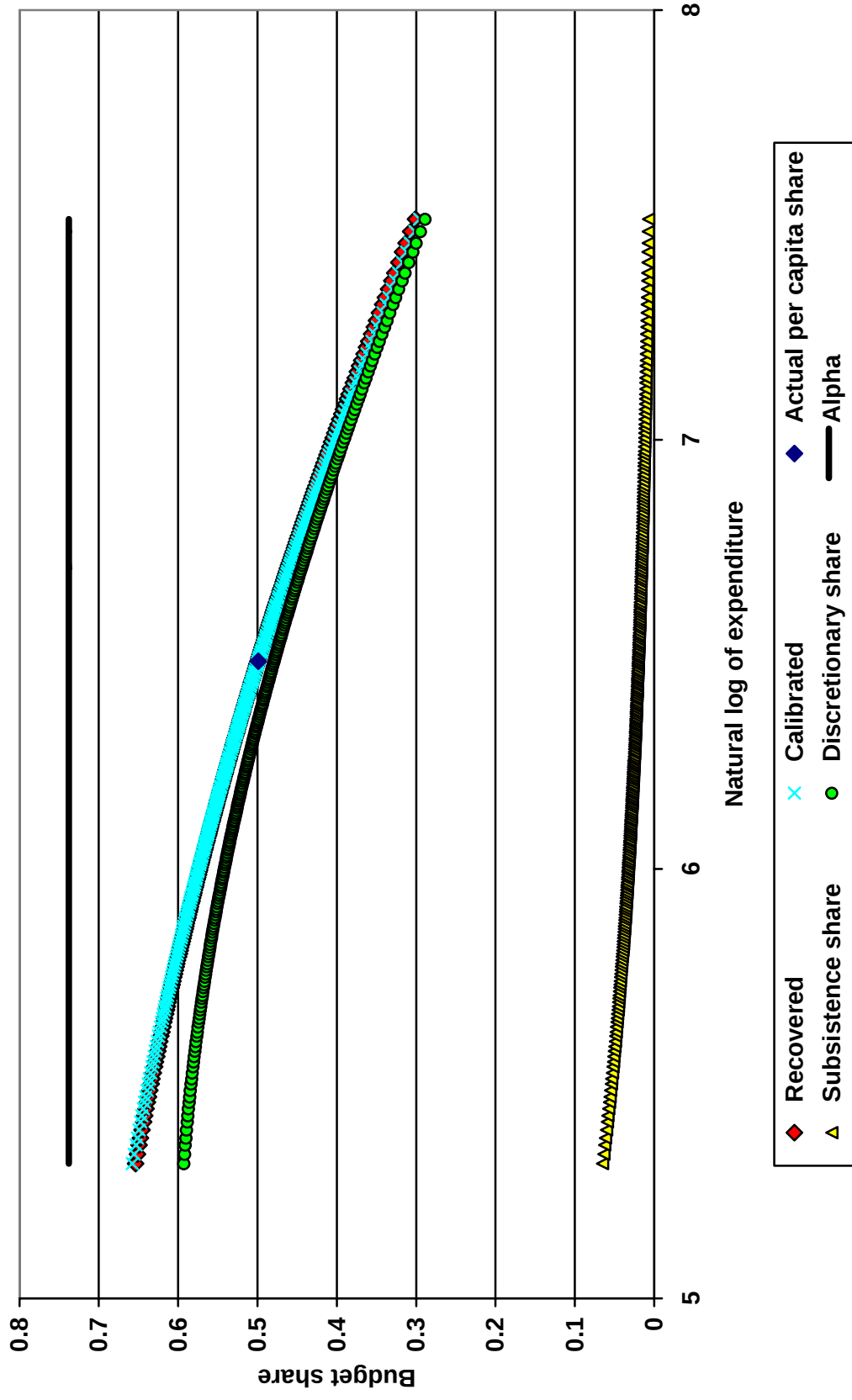


Figure 1: Recovered and Calibrated Food Budget Shares for Indonesia.

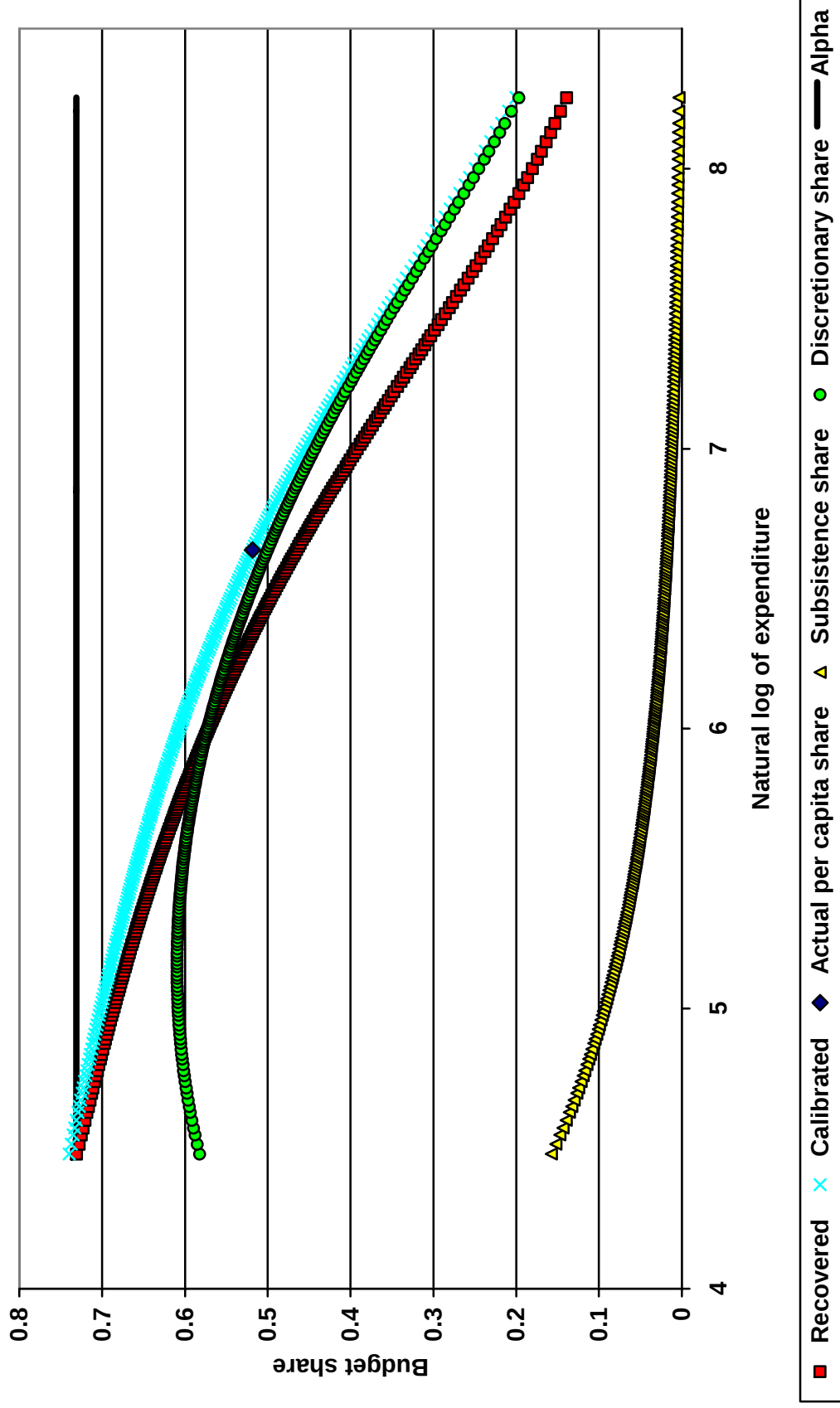


Figure 2: Recovered and Calibrated Food Budget Shares for the Philippines.

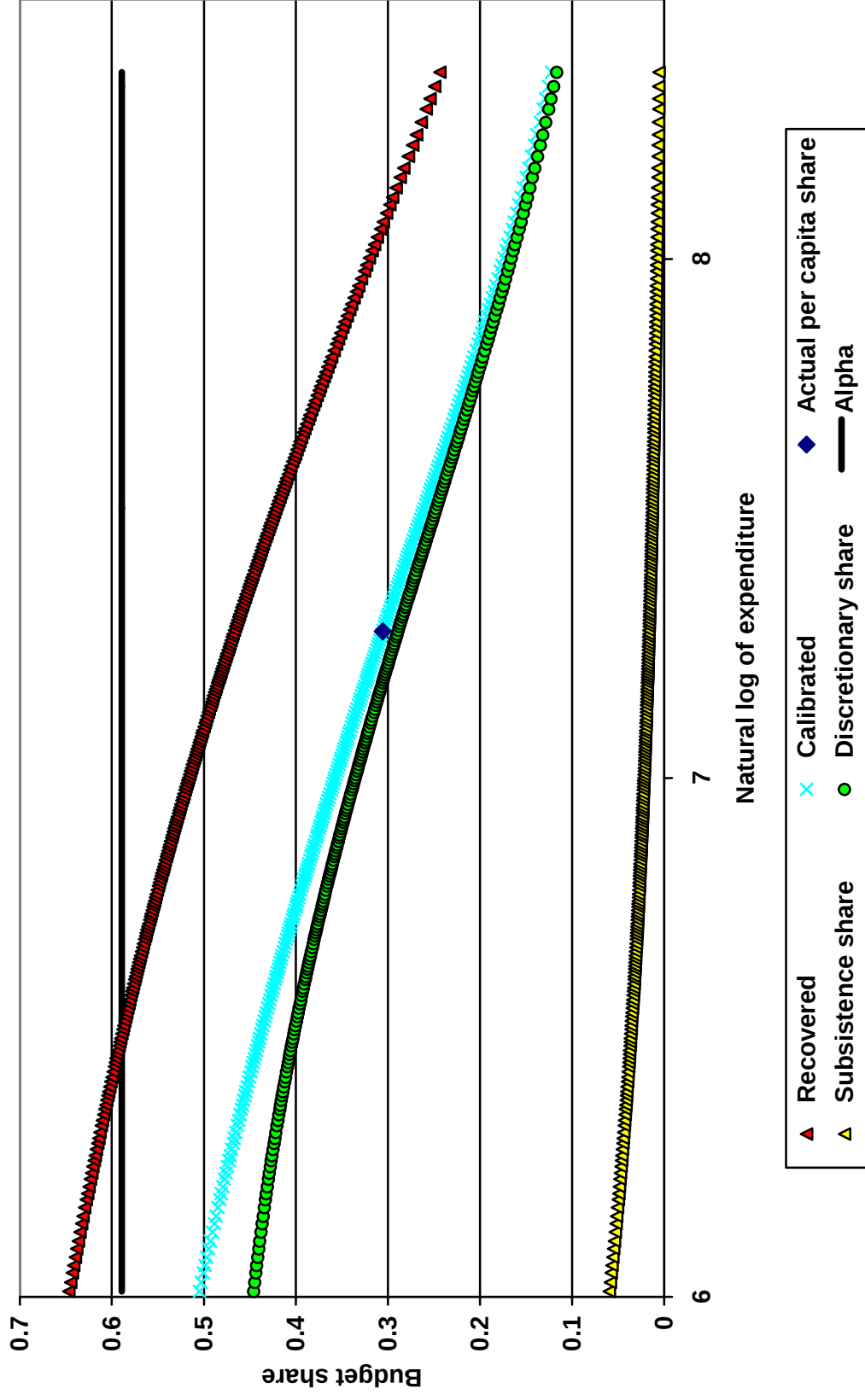


Figure 3: Recovered and Calibrated Food Budget Shares for Thailand.

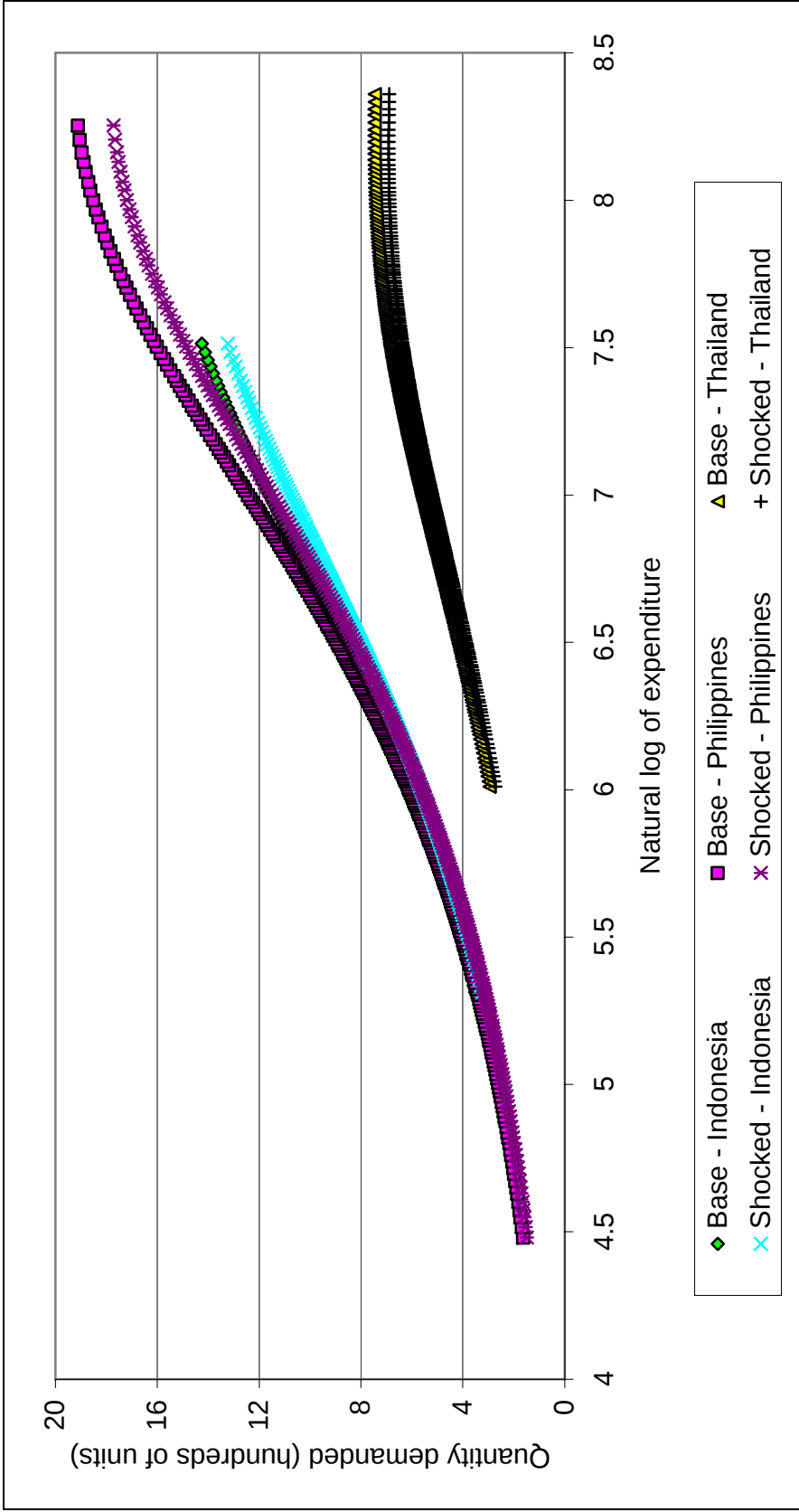


Figure 4: Demand for Food across Expenditure Distributions within Each Focus Country, Under the Base Case Scenario and with a Five Percent Shock to Price, Calculated Using the Calibrated AIDADS Model.

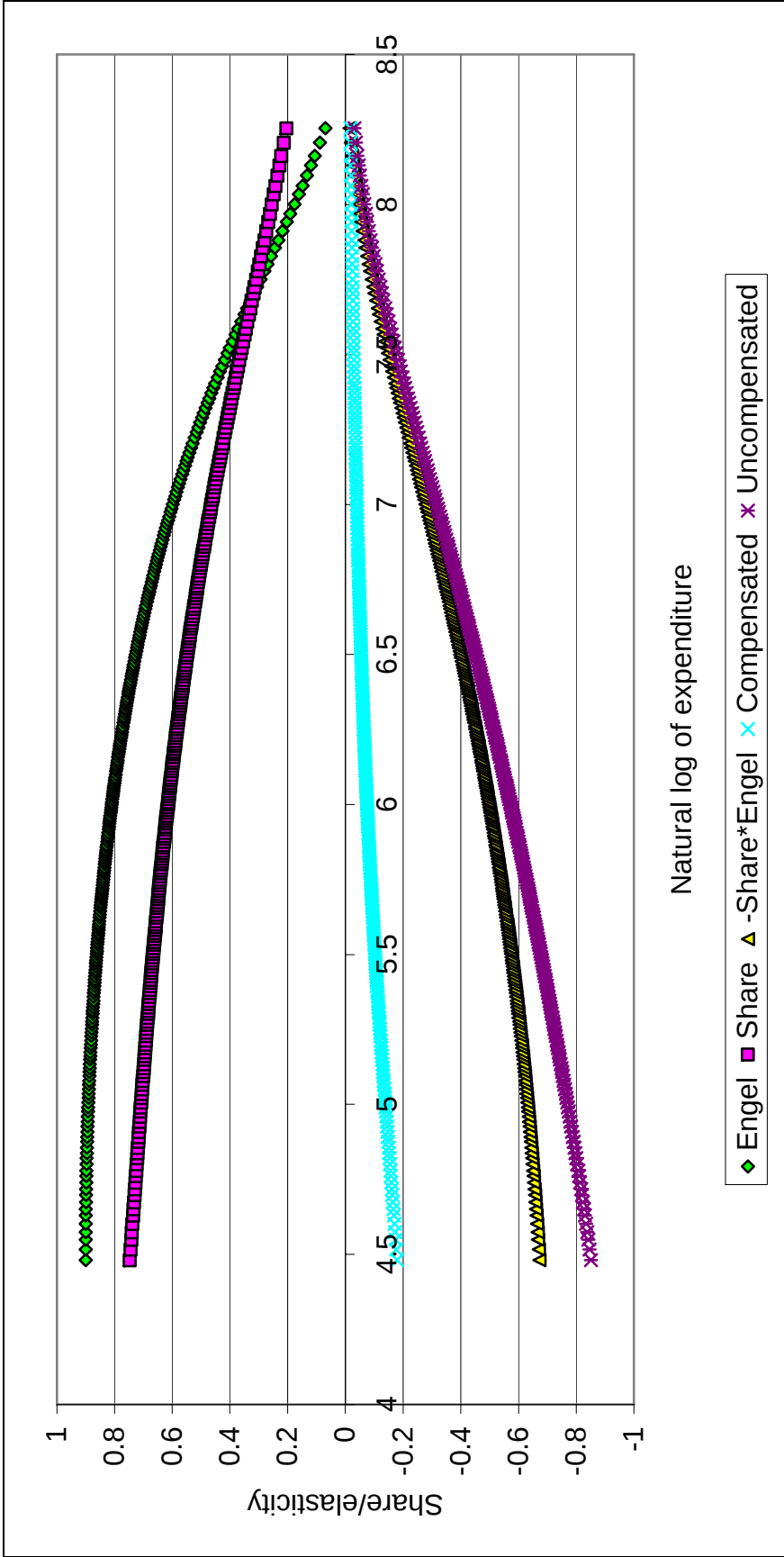


Figure 5: Uncompensated Own-Price Demand Elasticity for Food in the Philippines and its Components, Based on the Calibrated Demand System.

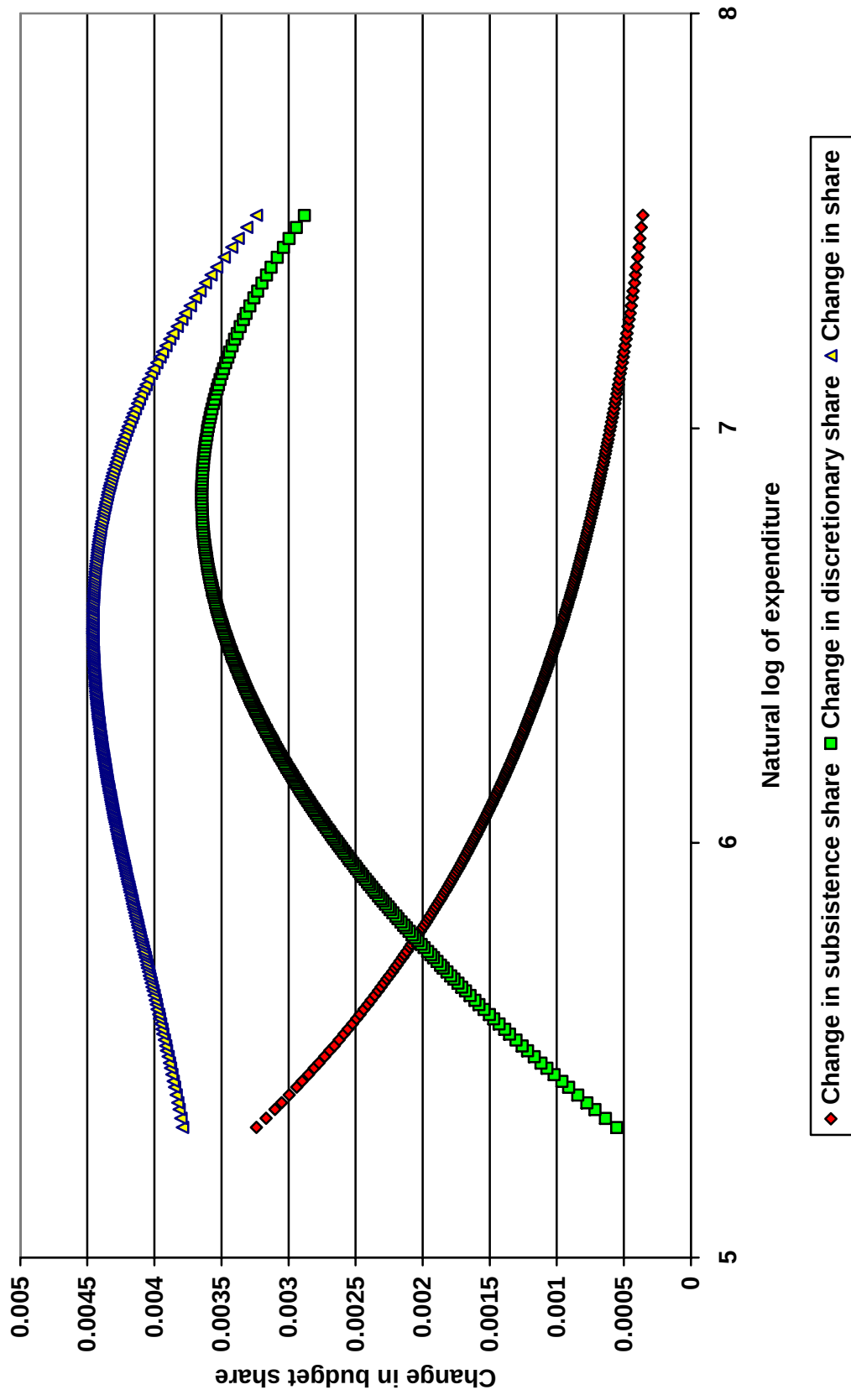


Figure 6: Change in Subsistence, Discretionary and Total Budget Share for Food in Indonesia Arising from a Five Percent Price Increase.

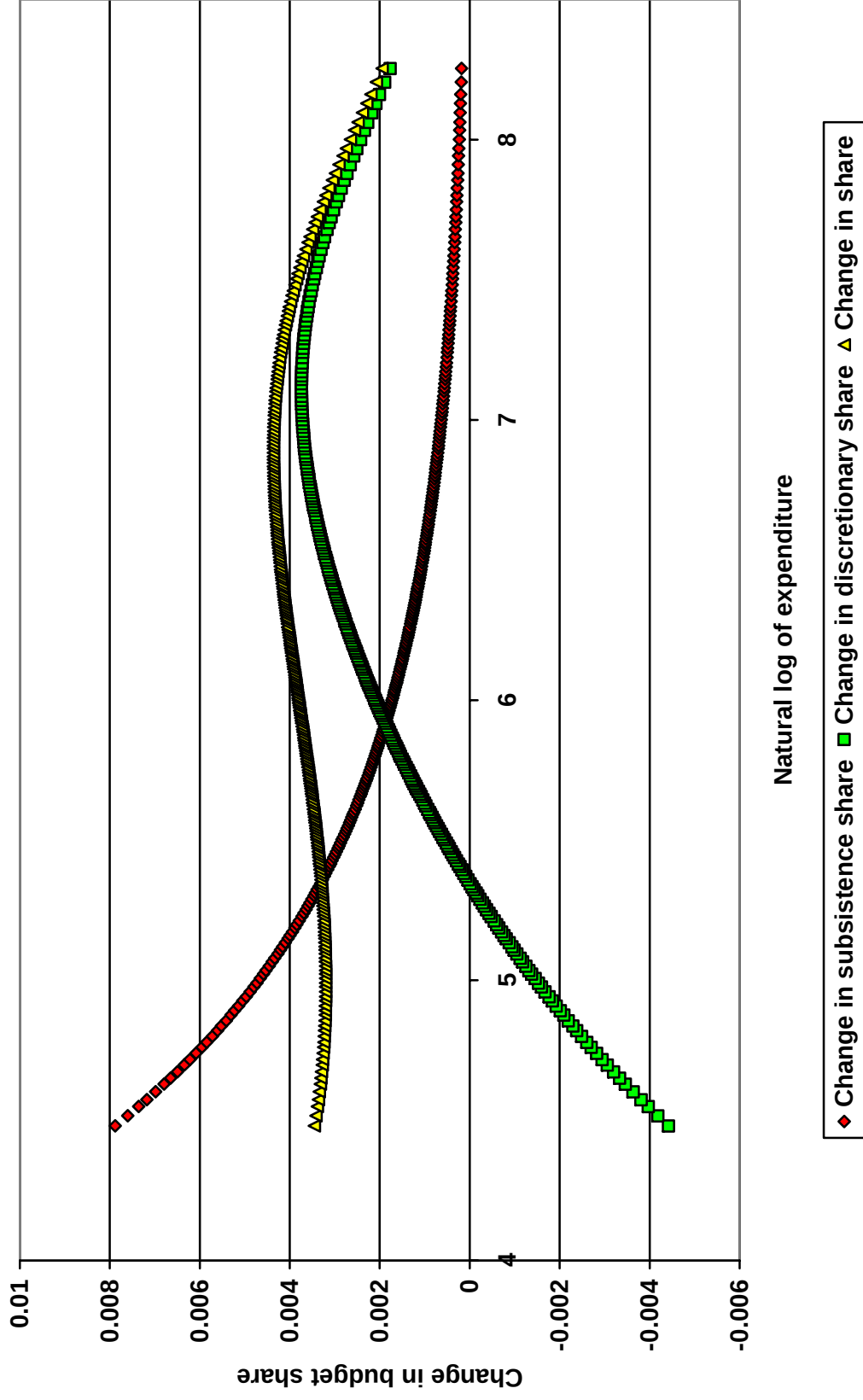


Figure 7: Change in Subsistence, Discretionary and Total Budget Share for Food in the Philippines Arising from a Five Percent Price Increase.

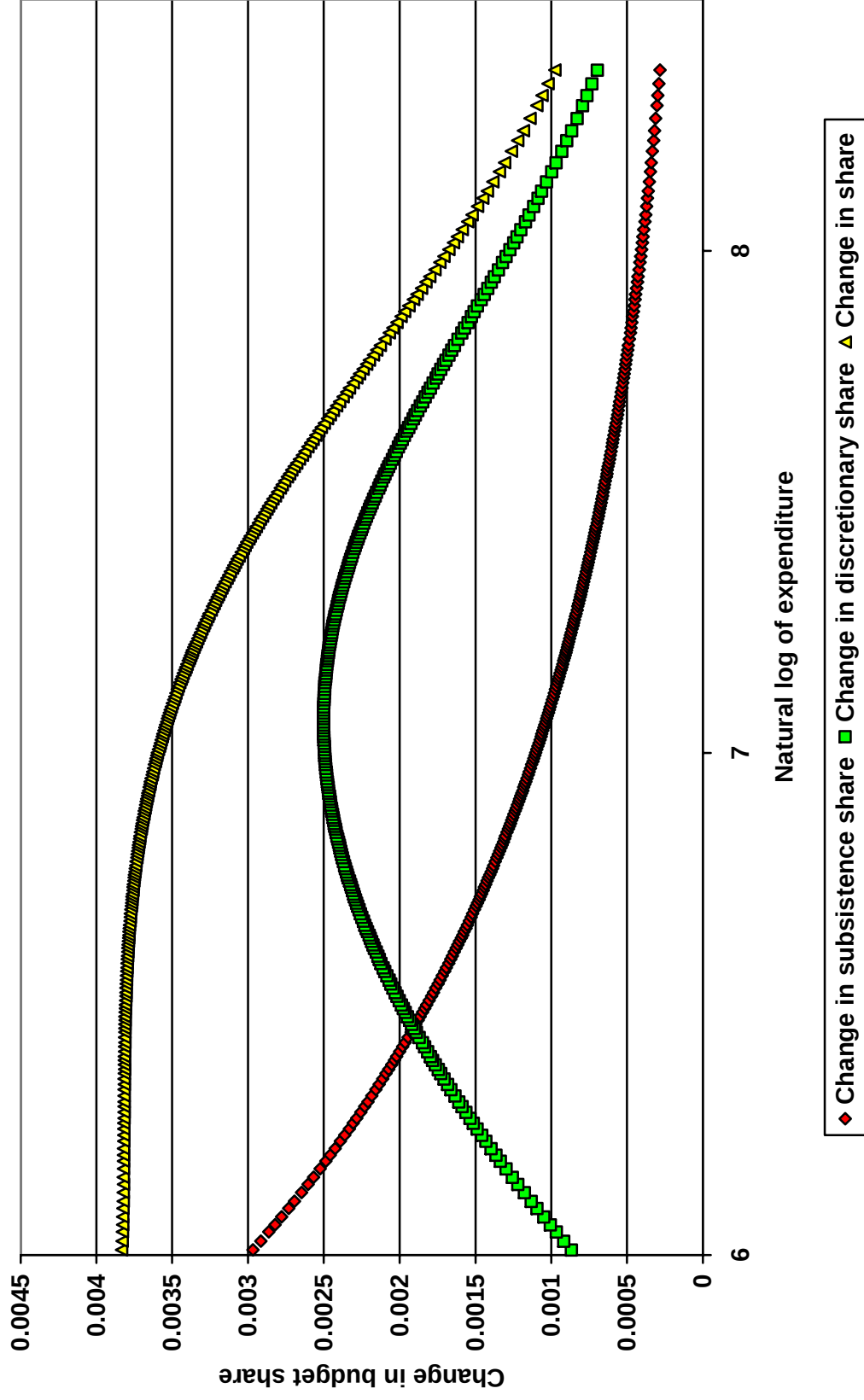


Figure 8: Change in Subsistence, Discretionary and Total Budget Share for Food in Thailand Arising from a Five Percent Price Increase.

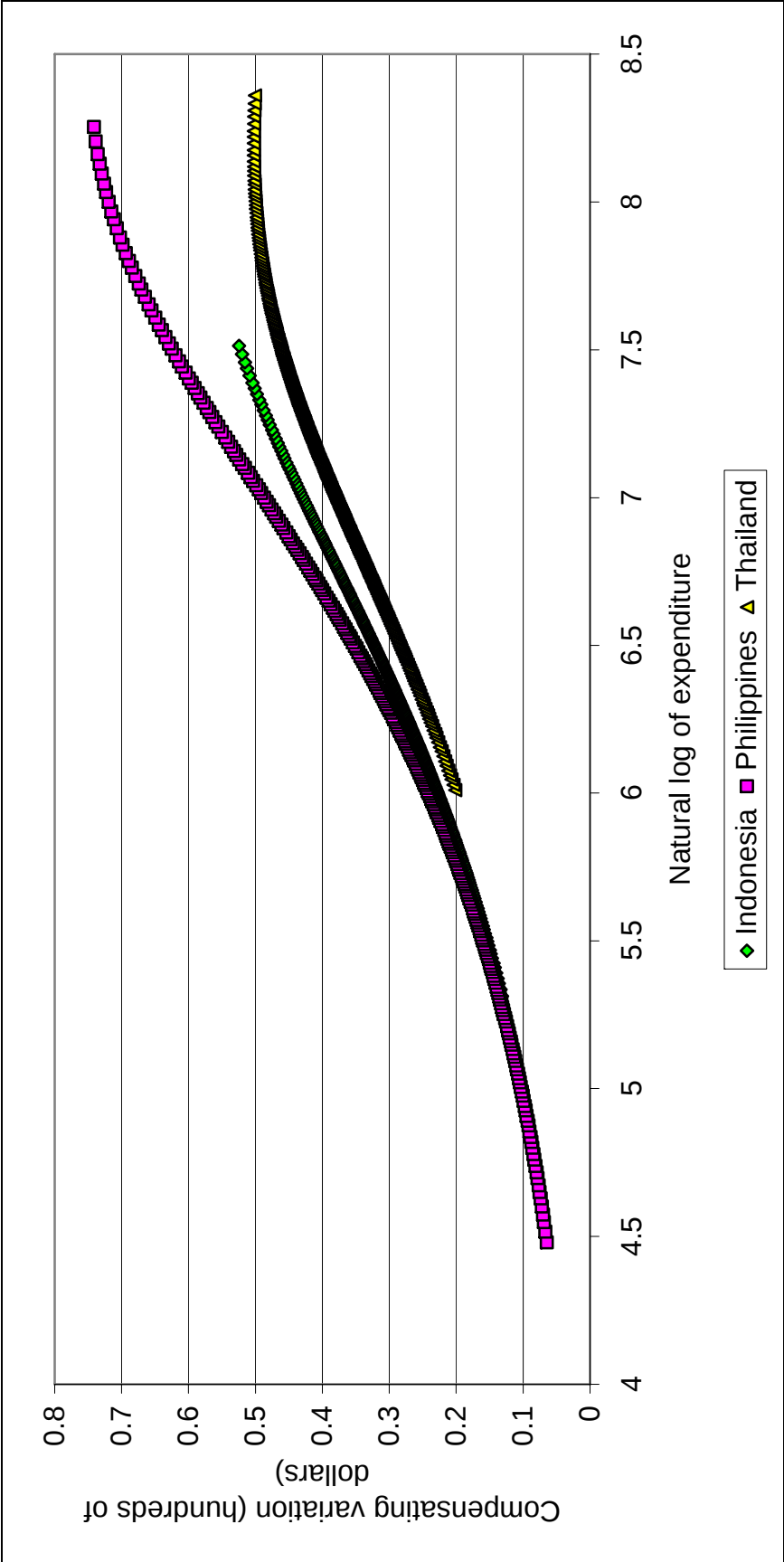


Figure 9: Compensating Variation Arising from the Five Percent Increase in Food Price.

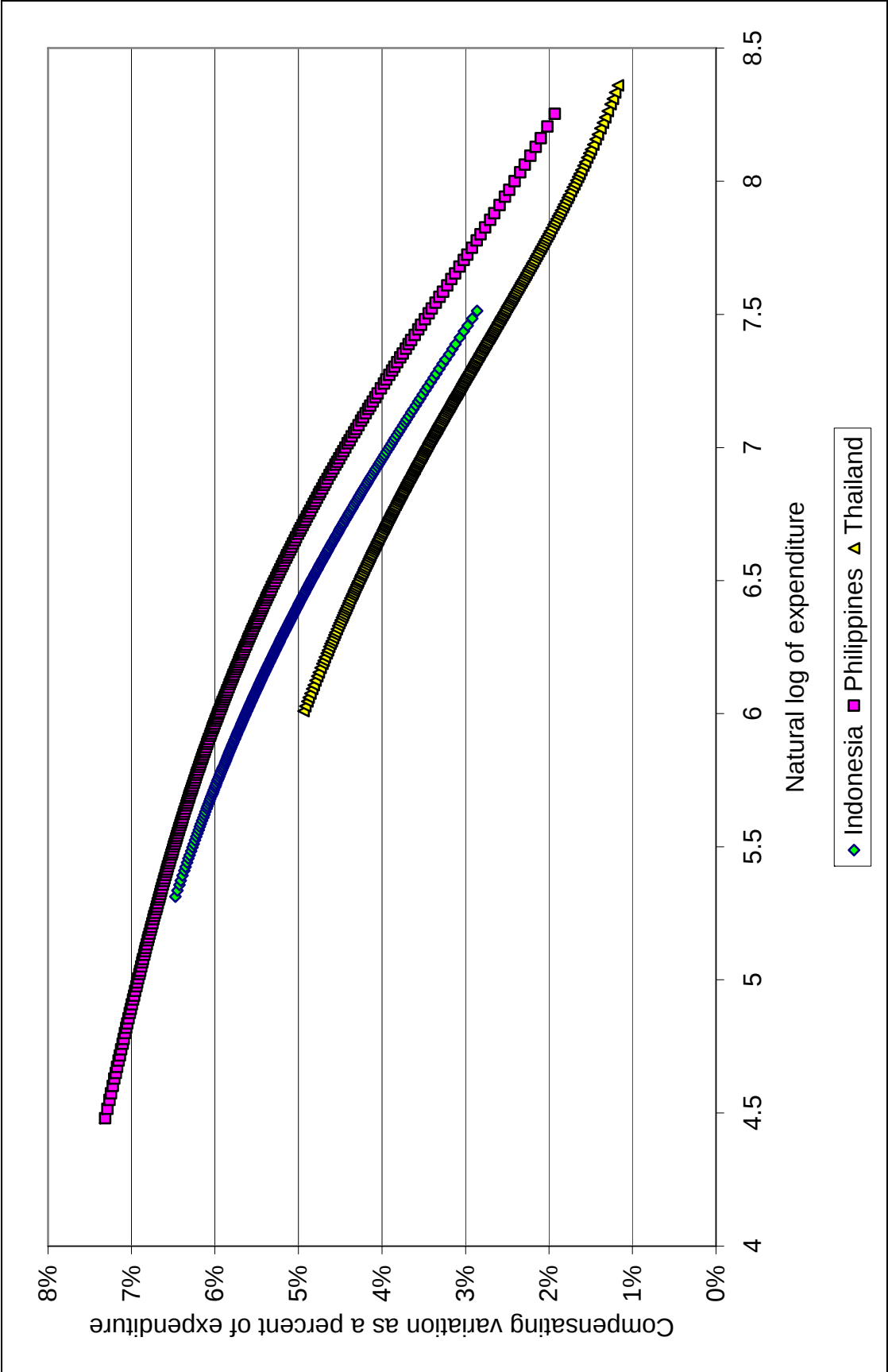


Figure 10: Compensating Variation Arising from the Five Percent Increase in Food Price as a Percent of Expenditure.

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TECHNICAL APPENDIX FOR: POVERTY ANALYSIS USING AN INTERNATIONAL CROSS-COUNTRY DEMAND SYSTEM

In what follows we outline the empirical model used in Cranfield, Hertel and Preckel. This method allows for recovery of expenditure distributions and household level demands using inequality measures, information from household level surveys and per capita data. While somewhat specific to the available data, the methodology may be readily modified to suit different contexts. Key to our approach is availability of information on the distribution of expenditure. If summary statistics on the distribution of expenditure are not available for a country, no attempt is made to recover an expenditure distribution, nor household demands for that country. The demand system for those countries simply relates per capita demand to prices and per capita expenditure. However, for some observations, we have expenditures at the quintile, decile or percentile level. In these instances, the model recovers an approximation to the expenditure distribution and household demands at different points in each country's expenditure distribution. For purposes of this paper, we focus our attention on three countries where percentile data are available to us (Thailand, Indonesia and the Philippines). The percentiles are used to identify the support of the recovered distribution of expenditure, and to recover the expenditure distributions in these focus countries.

The set of observations (i.e., countries) is denoted by T , with t indexing individual national observations. This set can be parsed into mutually exclusive and exhaustive subsets based on whether expenditure distribution data are available and the nature of these inequality data. Specifically, let T_H denote the subset of observations for which percentiles are available, T_D the subset of observations for which deciles are available, T_Q the subset of observations for which quintiles are available and T_N the subset of observations for which no distributional data are available. Thus $T = T_H \cup T_D \cup T_Q \cup T_N$.

When inequality information is available, the support of each country's expenditure distribution is parsed into expenditure classes, indexed by c . The total number of classes depends on the corresponding observation and the nature of the inequality data. If percentile data are available, the support of the expenditure distribution is separated into percentiles, denoted by the set C_H . If deciles are available, the support of the expenditure distribution is separated into ten expenditure classes, denoted by the set C_D . If quintiles are available, the support of the expenditure distribution is demarked into quintiles, denoted by the set C_Q . Further, each expenditure class is sub-divided into three expenditure levels, denoted by $l \in L = \{1, 2, 3\}$ (calculation of these expenditure levels is discussed later). Thus, for $t \in T_H$, there are initially three hundred points in the support of the recovered expenditure distribution (one hundred expenditure classes, each containing three expenditure levels).¹⁴ For $t \in T_D$, there are thirty points in the support of the recovered expenditure distribution, while for $t \in T_Q$, there are fifteen points in the support of the recovered expenditure

¹⁴ These three hundred points are trimmed to reflect the fact that survey based observations at extremely low and high levels of household expenditure are unreliable sources of information, due mainly to the scant number of observations at these extremes.

distribution. As such, the resolution of the recovered expenditure varies by country and the nature of the distribution information that is used. As previously noted, if no inequality information is available, then no attempt is made to recover the distribution of expenditure.

The optimization problem underpinning the estimation framework takes the form of a non-linear programming problem. The objective function consists of a term representing the information recovery process and a term representing the concentrated log-likelihood function. The measure of information recovery is the maximum entropy metric defined across shares of each observation's population at each expenditure level in the respective expenditure distribution. The objective function and choice variables of the problem are expressed as:

$$\begin{aligned} \text{Max} \quad & - \left(\Lambda + 0.5 \times \ln \prod_{i=1}^{n-1} r_{ii}^2 \right) \\ & \alpha_i, \beta_i, \gamma_i, \kappa, \\ & u_{tcl}, v_{ti}, \rho_{tcl} \end{aligned} \tag{A.1}$$

where Λ denotes the entropy component of the objective function and is expressed as $\sum_{t \in T_H} \sum_{c \in C_H} \sum_{l \in L} \rho_{tcl} \ln \rho_{tcl} + \sum_{t \in T_D} \sum_{c \in C_D} \sum_{l \in L} \rho_{tcl} \ln \rho_{tcl} + \sum_{t \in T_Q} \sum_{c \in C_Q} \sum_{l \in L} \rho_{tcl} \ln \rho_{tcl}$, and the ρ_{tcl} terms are the proportion of country t 's total population with expenditure at the l th level of the c th expenditure class. Note that by assumption, $0 < \rho_{tcl} < 1$. This entropy component is broken into three parts to facilitate clarity of exposition. The first term is the entropy of the recovered expenditure distribution for observations where percentile data are available. The second and third terms measure the entropy of the recovered distributions for observations where deciles and quintiles are available, respectively.

The second part of the overall objective function represents a concentrated log-likelihood function. The r_{ii} terms are the diagonal elements of $\mathbf{R}$, which is an upper triangular matrix resulting from the following Cholesky decomposition of the covariance matrix: $\hat{\Sigma} = \mathbf{R}' \mathbf{R}$, where $\hat{\Sigma}$ is the covariance matrix for the demand system being estimated. The relationship between the residuals and elements of $\mathbf{R}$ can be defined by noting that each element in $\hat{\Sigma}$ must be the same regardless of whether it is computed as $T^{-1} \sum_{t=1}^T v_{it} v_{kt}$ or based on the matrix decomposition of $\hat{\Sigma}$, where v_{it} are the residuals of the i th good's demand equation in the t th observation. As such, the following constraint is included during estimation:

$$T^{-1} \sum_{t=1}^T v_{it} v_{jt} = \sum_{k=1}^{n-1} r_{ki} r_{kj} \quad \forall i \neq n, j \neq n. \tag{A.2}$$

(Since AIDADS satisfies the adding up property of demand, the residuals must sum to zero, in which case the covariance matrix is singular. Equation A.2 reflects this fact by dropping the last equation's residual in the summation terms.)

One of the advantages of the entropy framework developed by Cranfield *et al.* (2004) is that it enables recovery of a distribution of expenditure that exactly matches the known inequality and expenditure moment information used during estimation. Furthermore, the problem is structured such that the recovered population shares are also used to recover unobserved budget shares for each good at each expenditure level in the distribution, such that the weighted sum of the recovered budget shares (with population shares serving as weights) equals the observed (per capita based) budget shares, up to a random error term. To see the logic underlying this process, note that per capita demands can be defined as follows: $\bar{x}_{it} = Z^{-1} \sum_{z=1}^Z x_{itz}$, where z indexes individuals in country t , x_{itz} is consumption of the i th good by individual z in country t , and $\bar{x}_{it}$ is per capita demand. Disaggregate demands are assumed to be recoverable such that they add up to the observed level of per capita demand up to an error term with known properties: $\bar{x}_{it} = \sum_c \sum_l \rho_{tcl} x_{itcl} + v_{it}$, where x_{itcl} is demand for the i th good by a household at the l th expenditure level in the c th expenditure class of the t th observation's expenditure distribution and v_{it} is an independently distributed normal error term with mean vector zero and finite covariance matrix. The inclusion of the random error term implicitly assumes errors in aggregation. Moreover, the error terms included in the consumption adding-up constraints are used to define the terms in the Cholesky factorization in the objective function. As such, estimation via maximum likelihood minimizes these errors.

In the present analysis, these residuals are defined using an AIDADS based approach to the consumption adding-up constraints:

$$\bar{w}_{it} - v_{it} = \begin{cases} \frac{p_{it}\gamma_i}{\bar{y}_t} + \frac{\alpha_i + \beta_i \exp(u_t)}{1 + \exp(u_t)} \left(1 - \frac{\mathbf{p}'_t \boldsymbol{\gamma}}{\bar{y}_t} \right) & \forall i, t \in T_N \\ \frac{p_{it}}{\bar{y}_t} \sum_{c \in C_K} \sum_{l \in L} \rho_{tcl} \left(\gamma_i + \frac{1}{p_{it}} \frac{\alpha_i + \beta_i \exp(u_{tcl})}{1 + \exp(u_{tcl})} (y_{tcl} - \mathbf{p}'_t \boldsymbol{\gamma}) \right) & \forall i, t \in T_K, K \end{cases} \quad (\text{A.3})$$

where $K = \{H, D, Q\}$ indexes the index used to delineate observations with different types of inequality information and $\bar{w}_{it}$ is the per capita budget share for the i th good in the t th observation.. As mentioned, equation (A.3) serves to ensure that the recovered disaggregate demands add-up to the observed per capita level, in expectation. Specifically, for each observation, disaggregate demands at each expenditure level, y_{tcl} , are assumed to add up to the known level of economy wide demand with population fractions ρ_{tcl} used as weights. Note too that the AIDADS model (stated in consumption level form) has been substituted in directly for the unobservable disaggregate demands (x_{itcl}). Hence, this constraint allows for estimation of the AIDADS parameters.

As with traditional, maximum likelihood estimation of AIDADS (see, for example, Cranfield *et al.* 2002), the defining equation of utility is also included in the scheme used here to permit estimation of the levels of utility. As with equation (A.3), the constraints representing the defining

equation of utility differ according to the data that is available for the respective observation. Equation (A.4) shows the set of utility function constraints:

$$\kappa = \begin{cases} \sum_{i=1}^n \frac{\alpha_i + \beta_i \exp(u_t)}{1 + \exp(u_t)} \ln \left(\frac{1}{p_{it}} \frac{\alpha_i + \beta_i \exp(u_t)}{1 + \exp(u_t)} (\bar{y}_t - \mathbf{p}'_t \boldsymbol{\gamma}) \right) - u_t & \forall t \in T_N \\ \sum_{i=1}^n \frac{\alpha_i + \beta_i \exp(u_{tcl})}{1 + \exp(u_{tcl})} \ln \left(\frac{1}{p_{it}} \frac{\alpha_i + \beta_i \exp(u_{tcl})}{1 + \exp(u_{tcl})} (y_{tcl} - \mathbf{p}'_t \boldsymbol{\gamma}) \right) - u_{tcl} & \forall t \in T_K, \\ & c \in C_K, l, K \end{cases} . \quad (\text{A.4})$$

Note first that the AIDADS model in levels form has been substituted into the defining equation of utility (this is the term in the $\ln(\cdot)$). As Cranfield *et al.* (2002) report, doing so greatly facilitates estimation of AIDADS, and allows one to include the defining equation of utility for AIDADS in implicit form. The first line of this constraint represents observations for which no inequality data are available; as such, it is included using per capita expenditure and utility levels for the “average” consumer in the respective observations. The last line in this constraint represents utility functions for those observations for which inequality data are available. In the latter instance, expenditure is indexed on the expenditure class and level, as is utility.

Equation (A.5) reflects the fact that the sum of the population fractions, ρ_{tcl} , across all expenditure levels in each expenditure class in each observation must equal unity:

$$\sum_{l=1}^L \rho_{tcl} = \begin{cases} 1/90 & \forall t \in T_H, c \in C_H \\ 1/10 & \forall t \in T_D, c \in C_D \\ 1/5 & \forall t \in T_Q, c \in C_Q \end{cases} . \quad (\text{A.5})$$

By definition, when percentile data are available, the sum of ρ_{tcl} across levels in an expenditure class must sum to 1/100. However, the focus countries’ percentile data are drawn from household survey data. Survey data such as these can be fraught with problems related to observations at the extreme levels of expenditure. In particular, there tend to be fewer observations at extremely low and high expenditure levels. Consequently, the tails of the expenditure distribution may be difficult to accurately identify. To remedy potential problems arising from the tails of the distributions, the lower and upper five percentiles are dropped during estimation. As such, the population shares within each expenditure class must sum to 1/90. However, in those cases where we work with deciles and quintiles, there is no such extreme point problem and so the sum of ρ_{tcl} across expenditure levels within each class must sum to 1/10 and 1/5, respectively.

Since the recovered expenditure distributions are driven by known information (in this case per capita expenditure and inequality information in the form of percentiles, deciles and quintiles), it

is important that the recovered expenditure distribution “give back” exactly what is known. That is, the location and scale parameters of the recovered expenditure distribution should exactly match the known location and scale parameters of the data. In this regard, equation (A.6) defines an expenditure adding up condition for the recovered expenditure distribution:

$$\sum_{l=1}^L \rho_{tcl} y_{tcl} = \begin{cases} \bar{y}_{tc}/90 & \forall t \in T_H, c \in C_H \\ DECILE_{tc} \bar{y}_t & \forall t \in T_D, c \in C_D \\ QUINTILE_{tc} \bar{y}_t & \forall t \in T_Q, c \in C_Q \end{cases} \quad (A.6)$$

where $\bar{y}_{tc}$ is the observed average level of household expenditure in the c th expenditure class of the t th observation, while $DECILE_{tc}$ and $QUINTILE_{tc}$ are the decile and quintile values for the respective observation-expenditure class combinations. In observations where deciles or quintiles are available, equation (A.6) requires the share-weighted sum of the expenditure levels within an expenditure class to sum to the product of per capita expenditure and the class’s quintile or decile value (see Cranfield *et al.* 2004 for details). Stated another way, when percentile data are available, the recovered expenditure distributions must add back to the observed expenditure class means.

In addition to the constraints discussed above, the following bounds and constraints are placed on the choice variables (either to prevent unbounded problems in the optimization program or arising from the need for theoretical consistency of AIDADS): $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \beta_i = 1$, $\alpha_i \in [0, 0.9]$ for all i , $\beta_i \in [0, 0.8]$ for all i , $\gamma_i \in [0, 0.5]$ for all i , $\kappa \in (-\infty, \infty)$, $u_{t_N} \in [-4, 2]$, $u_{tcl} \in [-7, 3]$ for all $t \notin T_N, c, l$, $v_{ti} \in [-1, 1]$ for all i, t , $\rho_{tcl} \in [10^{-9}, \omega^{-1}]$ for all $t \in T_K, c, l, K \in \{H, D, Q\}$, where ω is the product of the cardinality of the sets T_K and L , and $y_{cIt} - \mathbf{p}'_t \boldsymbol{\gamma} \geq \theta$ for all $t \notin T_N, c, l$, where $\theta > 0$.¹⁵

In countries for which deciles and quintiles are available, the lower and upper bounds of each expenditure class are calculated using the methods reported in Cranfield *et al.* (2004). In particular, within each expenditure class, an expenditure level is placed at the conditional mean of that expenditure class (i.e. expenditure at $l=2$ equals the conditional mean for that expenditure class), and at one-third of the class’s interval above and below the mid point to provide expenditure levels at $l=3$ and $l=1$, respectively. For observations where percentile data are available, the minimum and maximum bounds of the expenditure class is known. However, applying the one-third rule as above resulted in an infeasible solution. As such, the lower bound of each expenditure class for observations in the set T_H are defined as $\bar{y}_{ct} - (\bar{y}_{ct} - y_{ct}^{MIN})/\sqrt{2}$, where $\bar{y}_{ct}$ is the average level of

¹⁵ This value is set slightly above zero to ensure this constraint is not active in the optimal solution, which would imply no discretionary expenditure.

expenditure in the ct th expenditure class of the t th observation, and y_{ct}^{MIN} is the minimum level of expenditure in the ct th expenditure class of the t th observation. The upper bound of each expenditure class is defined as $\bar{y}_{ct} + (y_{ct}^{MAX} - \bar{y}_{ct})/\sqrt{2}$, where y_{ct}^{MAX} is the maximum level of expenditure in the ct th expenditure class of the t th observation.

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