# A Modified, Implicit, Directly Additive Demand System

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**Abstract**

A recently developed demand system, nicknamed AIDADS (An Implicit, Directly Additive Demand System), offers an approach to capturing consumer preferences across a wide range of expenditure levels. AIDADS generalizes the LES by assuming marginal budget shares vary with utility and hence with expenditure. Like the LES, AIDADS includes subsistence parameters that define minimum consumption levels. Here we present a modified AIDADS (MAIDADS) that replaces the constant subsistence parameters with functions that also vary with utility; these transformed subsistence levels are referred to as minimum consumption quantities. This model is applied to the 1996 International Consumption Project data. As these data span a wide range of expenditure levels, MAIDADS offers a viable alternative to the estimation of a "global demand system." Results suggest minimum consumption quantities for staple grains, livestock, other food products, alcohol and tobacco, clothing and footwear, and transport and transport services vary with expenditure, while those for rent and fuel, and household furnishings and operations are constant (and equal to zero) across expenditure levels. Only the minimum consumption quantity for staple grains declines with expenditure.

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## Abstract

A recently developed demand system, nicknamed AIDADS (An Implicit, Directly Additive Demand System), offers an approach to capturing consumer preferences across a wide range of expenditure levels. AIDADS generalizes the LES by assuming marginal budget shares vary with utility and hence with expenditure. Like the LES, AIDADS includes subsistence parameters that define minimum consumption levels. Here we present a modified AIDADS (MAIDADS) that replaces the constant subsistence parameters with functions that also vary with utility; these transformed subsistence levels are referred to as minimum consumption quantities. This model is applied to the 1996 International Consumption Project data. As these data span a wide range of expenditure levels, MAIDADS offers a viable alternative for the estimation of a "global demand system." Results suggest minimum consumption quantities for staple grains, livestock, other food products, alcohol and tobacco, clothing and footwear, and transport and transport services vary with expenditure, while those for rent and fuel, and household furnishings and operations are zero, and invariant across expenditure levels. Only the minimum consumption quantity for staple grains declines with expenditure.


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## Introduction

The choice of functional form has long vexed economists undertaking empirical work related to producer and consumer behavior. Over time, however, attention has focused on functional forms which embody more general properties with respect to parameters outside the economic agents' control (i.e., prices, income or perhaps output levels). While these flexible functional forms prove useful, they are often difficult to estimate. However, in some cases, subtle modifications to the more restrictive – yet widely used – functional forms can prove effective in relaxing an overly restrictive or untenable assumption of an existing functional form. A good example of this is the displacement of the origin in the Cobb-Douglas utility function by introducing subsistence quantities, resulting in the Linear Expenditure System. The purpose of this paper is to motivate, develop and illustrate one such generalization in the context of consumer demand systems. Specifically, the model developed here is a generalization of Rimmer and Powell's (1992, 1996) AIDADS model.

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Beginning as early as Houthakker (1957, 1965), demand analysts have strived to develop increasingly flexible representations of consumer preferences and resultant demand systems. For example, John Muellbauer and Angus Deaton worked to develop formulations that embody convenient aggregation properties. These efforts began with price independent (PI) preference structures, followed by PIGL and PIGLOG preference structures, the latter of which has its roots in Working's (1943) specification of demand as a function of expenditure. The development of PIGLOG structures was a watershed for the empiricist and it led to Deaton and Muellbauer's (1980) Almost Ideal Demand System (AIDS).

Generalizations of the AIDS model have followed, such as Cooper and McLaren's (1992) modified AIDS model (MAIDS), Banks *et al.*'s (1997) quadratic AIDS (QUAIDS) model and Lewbel's (2003) rational rank four AIDS model (RAIDS). Recent applications of the AIDS model in this journal include use in a dynamic context (e.g., Mantzouneas *et al.* 2004, Eakins and Gallagher 2003; Klonaris and Hallam, 2003; Fanelli and Mazzocchi, 2002; Chambers and Nowman, 1997), as well as to decompose changes in consumer demand (Karagiannis and Velentzas, 2004) and investigate the impacts of "sin" taxes (Jones and Giannoni, 1996).[1] Beyond its convenient aggregation properties, one can also directly test the value of these generalizations because AIDS is nested within these models. Such tests can also lead to useful economic information, such as the uncovering the rank of a demand system and obtaining a better understanding of consumer preferences.

Another useful example has its roots in the Cobb-Douglas (CD) preference structure. Stone (1954) took the CD as a starting point in his development of the linear expenditure system (LES). In turn, the LES was extended by Howe *et al.* (1979) to include a term that is quadratic in discretionary expenditure (i.e., the QES).[2] Other generalizations of the LES have focused on introducing marginal budget shares which vary with expenditure (e.g., Gamaletos, 1973; Lluch *et al.*, 1977), or with demographic variables (Raper *et al.*, 2002) More recently, Rimmer and Powell (1992, 1996) introduced a variant of the LES that allows the marginal budget shares to vary with

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[1] Related studies include Moro and Sckokai's (2002) use of an inverse QUAIDS model, Duffy's (2001) use of Barten's general model, which nests a linearized AIDS model as a special case, and Selvanathan and Selvanathan (2006) use of the Rotterdam model.

[2] Ryan and Wales (1999) developed further generalizations and extensions of the QES. These extensions merge the QES with other demand systems, such as the Translog, Normalized Quadratic and Generalized Leontief models. These resulting models offer Engel curves that are quadratic in expenditure, but also carry with them more flexible properties than the QES.

expenditure. This system, which is nicknamed AIDADS, is based on the assumption of direct, implicit additivity (see Hanoch, 1975). As with the LES, the AIDADS subsistence parameters (in the LES terminology) are constant. In this paper, AIDADS is modified by replacing the constant subsistence parameters with functions which vary monotonically with utility, and hence with expenditure. The result is a modified AIDADS (MAIDADS) model that allows the parameters used to represent minimum consumption quantities to vary across expenditure levels. Because these parameters no longer represent a subsistence level of consumption per se, we refer to them as minimum consumption quantities.

The next section of the paper provides a brief overview of the AIDADS model – its general characteristics, as well as conditions needed for it to satisfy regularity. Following this, the modified AIDADS model is developed and discussed. The econometric methods and data used to estimate MAIDADS are included in subsequent sections, followed by empirical results and conclusions.

**An Implicitly, Directly Additive Demand System (AIDADS)**

Rimmer and Powell (1992, 1996) introduced an implicitly, directly additive demand system that they nicknamed AIDADS. They view AIDADS as a generalization of the LES that overcomes one drawback of the LES – namely, constancy of the marginal budget shares. Using a modest number of parameters ($3n + 1$, where $n$ is the number of commodities), they obtain flexibility of marginal budget shares, which is the primary strength of AIDADS. AIDADS is particularly useful in poverty analysis (see Hertel *et al.*

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2004) as it devotes two-thirds of its parameters to characterizing consumption behavior at extremely low levels of per capita income.

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AIDADS was investigated extensively by Cranfield *et al.* (2000). Here we draw on that work to summarize the properties of AIDADS. AIDADS is an implicit functional form. Thus, the relationship between utility and consumption levels is expressed in terms of an identity (see Hanoch, 1975 for further details). The relevant identity is:

$$\sum_{i=1}^{n} \frac{\alpha_i + \beta_i e^u}{1+e^u} \ln\left(\frac{x_i - \gamma_i}{Ae^u}\right) = 1 \qquad (1)$$

where $\alpha_i \geq 0$, $\beta_i \geq 0$, $\gamma_i \geq 0$ and $A > 0$ are constant parameters, $e^x$ is the exponential operator, *ln*(.) is the natural logarithm operator, *u* denotes utility arising from the consumption bundle, $x_i (> \gamma_i)$ is the level of consumption of the *i*-th good, and $\Sigma_i \alpha_i = \Sigma_i \beta_i = 1$. (This identity is valid only over the region where $x_i > \gamma_i$. Hence, the relationship between utility and consumption is considered to be defined only over this region.) Rimmer and Powell (1996) are more general in their specification of this function, but (1) is the form that has been used for most empirical work. Note that if $\alpha_i = \beta_i$ then equation (1) becomes equivalent to the LES. Like the LES, the $\gamma_i$s in AIDADS can be viewed as subsistence parameters. (The further simplification of setting $\gamma_i = 0$ along with $\alpha_i = \beta_i$ yields demands equivalent to Cobb-Douglas.) It is for this reason that AIDADS is viewed as a generalization of the LES.

The implicit model of consumer behavior is that of maximizing utility, *u*, subject to (1) and a budget constraint. The resulting first-order optimality conditions for the $x_i$ can then be re-arranged to express the AIDADS model in budget share form:

$$s_i = \frac{\gamma_i p_i}{c} + \frac{\alpha_i + \beta_i e^u}{1+e^u}\left(1 - \sum_{i=1}^{n} \frac{p_i}{c}\gamma_i\right), \tag{2}$$

where $p_i$ are prices of the goods in the consumption set and $c$ is total expenditure on all final goods and services.

Combining (1) and (2) and rearranging, we obtain the following restatement of (1) that is stated in terms of prices, expenditure, and utility (i.e., consumption levels now only appear implicitly):

$$\sum_{i=1}^{n} \frac{\alpha_i + \beta_i e^u}{1+e^u} \ln\left[\frac{1}{p_i}\,\frac{\alpha_i + \beta_i e^u}{1+e^u}\left(c - \sum_{i=1}^{n} p_i \gamma_i\right)\right] - \ln(A) - u = 1. \tag{3}$$

From the consumer's perspective, the only variable in this problem is *u*. Solutions to the consumer's problem with AIDADS preferences will be unique if and only if the solution to (3) is unique. Based on examination of this relationship, Powell *et al.* (2002) find restrictions involving relative prices and differences in the parameters are needed to define the region over which AIDADS is regular (i.e., utility is strictly increasing in the level of consumption of each good and the upper level sets for utility are convex). One extremely useful relationship they develop is the following:

$$\sum_{i=1}^{n} (\beta_i - \alpha_i)\ln(p_i) \geq -\frac{(1-e^u)^2}{e^u} + \sum_{i=1}^{n} \ln\left(\frac{\alpha_i + \beta_i e^u}{1+e^u}\right)(\beta_i - \alpha_i). \tag{4}$$

Some important insights can be obtained from this inequality. First, violations of the relationship cannot occur in the LES case (i.e. when $\alpha_i = \beta_i$). Second, violations are driven by differences in the relative prices or in the discretionary budget shares, both of which depend upon inputs to the consumer choice problem that are not *parameters* of the AIDADS relationship. Third, the subsistence quantities, $\gamma_i$, are not involved. (Note, however, that extreme relative prices suggest that the quantity level for the good with the high price may be near its subsistence value.) Regularity, during empirical exercises where relative price changes are typically modest, is likely to be maintained. In the empirical section of this paper we examine the magnitudes of relative price changes that are needed to violate (4) and find that they are quite large and far beyond the typical changes that would be encountered in the course of routine economic analysis.

**A Modification of AIDADS**

The goal of generalizing AIDADS is to increase the flexibility of the price and expenditure effects as we move across the expenditure spectrum. One approach to the generalization of AIDADS is to allow the subsistence quantities, $\gamma_i$, to change as a function of the utility level. To reflect the notion that a utility varying $\gamma_i$ does not represent physiologically based subsistence needs, we refer to the $\gamma_i$s, which vary with utility, as minimum consumption quantities. A simple approach is to choose $\gamma_i$ as follows:

$$\gamma_i(u) = \frac{\delta_i + \tau_i e^{\omega u}}{1 + e^{\omega u}} \tag{5}$$

where $\delta_i$, $\tau_i$, and $\omega$ are positive constants. We refer to (5) as the i-th minimum consumption quantity. The magnitudes of $\delta_i$ and $\tau_i$ prove useful when characterizing the minimum consumption quantity's pattern of adjustment, provided that (4) is satisfied. That is, $\delta_i$ is the asymptotic limit on the minimum consumption of good *i* as expenditure approaches the value of the minimum consumption bundle, and $\tau_i$ is the asymptotic limit as expenditure increases without bound. If $\delta_i < \tau_i$ ($\delta_i > \tau_i$), then the minimum consumption level increases (decreases) with expenditure. Note that if $\delta_i = \tau_i$, the minimum consumption level is constant.

The use of minimum consumption quantities that vary with the level of utility is a bit at variance with the usual interpretation of $\gamma_i$ as the consumption levels essential for human survival. The motivation for the alternative treatment is that as a country develops, the bundle of goods that is viewed as necessary also seems to increase. For example, in most developed countries, a telephone is considered a virtual necessity despite the fact that it is not essential to survival. When $\delta_i < \tau_i$, the necessity of the *i*-th good grows with total expenditure as we would expect for livestock products, while when $\delta_i > \tau_i$, the necessity of the *i*-th good falls with expenditure as we would expect for staple grains. The parameters $\delta_i$ may still be interpreted as the levels of consumption needed for human survival, while the levels of $\tau_i$ may be interpreted as the consumption levels deemed "necessary" by a very wealthy consumer. The purpose of $\omega$ is to allow the transition from the low to high subsistence bundle to occur at a different rate than the transition from the low to high discretionary budget share.

The defining equation for the MAIDADS model appears as:

$$\sum_{i=1}^{n} \frac{\alpha_i + \beta_i e^u}{1+e^u} \ln\left( x_i \quad \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right) \quad \ln(A) \quad u = 1. \tag{6}$$

Maximizing utility, subject to this defining equation of utility and a budget constraint results in the following demand system:

$$s_i = \frac{p_i}{c} \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} + \frac{\alpha_i + \beta_i e^u}{(1+e^u)} \left( 1 - \sum_{j=1}^{n} \frac{p_j}{c} \frac{\delta_j + \tau_j e^{\omega u}}{1+e^{\omega u}} \right). \tag{7}$$

Clearly, if $\delta_i = \tau_i$ for all $i$, then MAIDADS collapsese to the AIDADS model.

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Following Hanoch (1975) and Rimmer and Powell (1992), the partial elasticities of substitution remain as they do for AIDADS:

$$\sigma_{ij} = \frac{(x_i - \gamma_i(u))(x_j - \gamma_j(u))}{x_i x_j} \times \frac{c}{c - \Sigma_{i=1}^{n} p_i \gamma_i(u)}. \tag{8}$$

These are unchanged from the case where the $\gamma_i$s are not functions of $u$, which is appropriate since these partial elasticities are evaluated with constant utility. While the form of (8) is unchanged from AIDADS, note that the values nonetheless depend on the role of the $\gamma_i(u)$ and hence are functions of utility. Thus, these elasticities change not only with $x_i$, but also with $u$. Depending on how the parameters of the $\gamma_i$ functions are realized (particularly $\omega$), the convergence of these substitution elasticities to one (i.e., Cobb-Douglas preferences) may be delayed or accelerated over the range of the expenditure.

Now let us consider the MAIDADS Engel elasticities, which are expressed as:

$$\begin{aligned}\eta_i = \frac{c}{p_i x_i}\Bigg\{ & \frac{\alpha_i + \beta_i e^u}{1+e^u} + \left(c - \sum_{j=1}^{n} p_j \frac{\delta_j + \tau_j e^{\omega u}}{1+e^{\omega u}}\right)\frac{(\beta_i - \alpha_i)e^u}{(1+e^u)^2}\lambda \\ & + \frac{(\tau_i - \delta_i)\omega e^{\omega u}}{(1+e^{\omega u})^2} p_i \lambda - \frac{\alpha_i + \beta_i e^u}{1+e^u}\sum_{j=1}^{n}\frac{(\tau_j - \delta_j)\omega e^{\omega u}}{(1+e^{\omega u})^2} p_j \lambda \Bigg\},\end{aligned} \tag{9}$$

where $\lambda$ is defined as:

$$\lambda = -\left\{ \sum_{i=1}^{n} \left[ \frac{(\beta_i - \alpha_i)e^u}{(1+e^u)^2} \ln\left( x_i - \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right) - \right. \right.$$

$$\left. \left. \frac{\alpha_i + \beta_i e^u}{1+e^u} \left( x_i - \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right)^{-1} \frac{(\tau_i - \delta_i)\omega e^{\omega u}}{(1+e^{\omega u})^2} \right] - 1 \right\}^{-1} \tag{10}$$

$$\times \left( c - \sum_i p_i \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right)^{-1}.$$

(See Appendix A for derivation of the Engel elasticity.) In equation (9), the term multiplying the expression in brackets is one over the expenditure share on the ***i***-th good. Inside the brackets, the first term is the discretionary budget share of the ***i***-th good. The second term in the brackets is total discretionary expenditure times the derivative of the discretionary budget share of the ***i***-th good with respect to total expenditure. To this point, the elasticity is identical to AIDADS except that in the second term, the minimum consumption quantity is a function of utility. The next term in the brackets is the derivative of the ***i***-th minimum consumption share with respect to total expenditure, and the fourth term is the derivative of total discretionary expenditure with respect to total expenditure. The sign of the first term in brackets is positive, but the signs of the other three terms in the brackets are ambiguous. Thus, the effect of utility on the Engel elasticity is determined on empirical grounds.

The modification of AIDADS does not affect the first-order conditions with respect to the consumption variables. However, as with AIDADS, regularity is dependent on the defining equation of utility. To see this, use (7) to substitute for the consumption levels in (6):

$$\sum_{i=1}^{n} \frac{\alpha_i + \beta_i e^u}{1+e^u} \ln\left[ \frac{1}{p_i} \frac{\alpha_i + \beta_i e^u}{1+e^u} \left( c - \sum_{i=1}^{n} p_i \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right) \right] - \ln(A) - u = 1. \tag{11}$$

As with AIDADS, the only concern about regularity is the uniqueness of solutions to (11). Re-deriving the equivalent of (4), we find that the new region of regularity has become:

$$\left(c-\sum_{i=1}^{n} p_i\gamma_i\right)^{-1}\sum_{i=1}^{n}\frac{\alpha_i+\beta_i e^u}{1+e^u}\sum_{j=1}^{n}\frac{(\tau_j-\delta_j)\omega e^{\omega u}}{\left(1+e^{\omega u}\right)^2}$$
$$\geq\sum_{i=1}^{n}(\beta_i-\alpha_i)\left[\ln\left(\frac{\alpha_i+\beta_i e^u}{1+e^u}\right)-\ln(p_i)\right]-\frac{(1+e^u)^2}{e^u}. \tag{12}$$

Similar to (4), this condition depends not only upon the values of the parameters in MAIDADS, but also upon the levels of prices, income (or expenditure), and utility. (In the empirical results, we find that the magnitudes of the relative price changes that are required to violate either (4) in the case of AIDADS or (12) in the case of MAIDADS is over six orders of magnitude – far larger than the size of relative price changes typically encountered in empirical analyses.) We turn next to estimation of MAIDADS, which nests AIDADS as a special case (when $\delta_i = \tau_i$). Hence, we only discuss estimation of the former.

**Estimation Framework**

One challenge in estimating MAIDADS (or AIDADS, for that matter) is that the unobservable level of utility is an argument in the demand function. That is, unlike the case where utility is an explicit function of consumption levels, the utility variable, *u*, does not drop out of the system of demand equations. Thus, the utility variable remains explicit in the demand system, and the value of utility at each observation is estimated in addition to the system parameters. This is accomplished by making the MAIDADS defining equations explicit in the estimation problem. For this reason, we estimate

MAIDADS using maximum likelihood techniques in the context of a nonlinearly constrained mathematical programming model.

To estimate MAIDADS, it is useful to define notation representing the data: $p_{it}$ and $x_{it}$ denote the price and quantity for the $i$-th good for the $t$-th observation, respectively, $c_t$ denotes total expenditure for the $t$-th observation, and the observed expenditure shares are denoted by $s_{it}$ (= $p_{it}x_{it}/c_t$). It is useful to distinguish between the observed budget share, $s_{it}$, and the fitted budget share $\hat{s}_{it}$. Fitted utility for the $t$-th observation is denoted by $\hat{u}_t$, and should be viewed as an observation-specific parameter that is estimated as part of the demand system. Equations that define the fitted shares are a part of the system, and are comprised of two parts: a minimum quantity ("subsistence") share – the first term on the right hand side of (13), and a discretionary budget share – the second term on the right hand side of (13):

$$\hat{s}_{it} = \frac{p_{it}}{c_t}\frac{\delta_i + \tau_i e^{\omega u_t}}{1+e^{\omega u_t}} + \frac{\alpha_i + \beta_i e^{u_t}}{(1+e^{u_t})}\left(1-\sum_{j=1}^{n}\frac{p_{jt}}{c_t}\frac{\delta_j + \tau_j e^{\omega u_t}}{1+e^{\omega u_t}}\right), \tag{13}$$

and error terms for the shares, denoted by $v_{it}$, are set equal to the difference between the observed and fitted shares:

$$v_{it} = s_{it} - \hat{s}_{it}\,. \tag{14}$$

In addition to (13) and (14), equations are required to define the relationship between fitted utility, the parameter values, and the fitted shares. Following Cranfield *et al*. (2000), this relationship is defined in terms of the fitted shares as follows:

$$\sum_{i=1}^{n}\frac{\alpha_i + \beta_i e^{\hat{u}_t}}{1+e^{\hat{u}_t}}\ln\left(\frac{\hat{s}_{it}c_t}{p_{it}} - \frac{\delta_i + \tau_i e^{\omega\hat{u}_t}}{1+e^{\omega\hat{u}_t}}\right) - \hat{u}_t = \kappa \tag{15}$$

where $\kappa = \ln(A) + 1$.

By the adding up property of expenditure shares, $\sum_{i=1}^{n} v_{it} = 0$ for all $t$, so the resulting covariance matrix is singular. Dropping the last equation from each observation allows one to define $\boldsymbol{\Sigma}$, an ($n$-1)x($n$-1) covariance matrix, in terms of the $n$-1 vector of share equation errors. Upon concentration, and ignoring terms that are independent of the unknown parameters, the log-likelihood function can be written as $-0.5\ln\left|\hat{\boldsymbol{\Sigma}}\right|$, where $\hat{\boldsymbol{\Sigma}}$ is the estimate of $\boldsymbol{\Sigma}$ with typical element $\hat{\Sigma}_{ij} = T^{-1}\sum_{t=1}^{T} v_{it}v_{jt}$ for all $i \neq n, j \neq n$. Evaluation of the objective function is simplified by noting that positive definiteness of $\hat{\boldsymbol{\Sigma}}$ allows us to use the decomposition $\hat{\boldsymbol{\Sigma}} = \boldsymbol{R}^t\boldsymbol{R}$, where $\boldsymbol{R}$ is an upper triangular matrix of conformable dimension. This relationship between the residuals and the $\boldsymbol{R}$ matrix is imposed via the equations:

$$T^{-1}\sum_{t=1}^{T} v_{it}v_{jt} = \sum_{k=1}^{n-1} r_{ki}r_{kj} \ \forall\ i \neq n, j \neq n, \tag{16}$$

with upper trianguarity of $\boldsymbol{R}$ imposed via the restriction $r_{kl} = 0$ for all $k > l$. Thus, the objective function (to be minimized because the equivalent of a concentrated log-likelihood function is used) of the optimization problem used here is:

$$-0.5\ln\prod_{i=1}^{n-1} r_{ii}^2 . \tag{17}$$

The choice variables in the optimization problem are: $\alpha_i$, $\beta_i$, $\kappa(=1+\ln(A))$, $\hat{u}_t$, $\hat{s}_{it}$, $v_{it}$, $r_{ij}$ for all $i < j$, $\delta_i$, $\tau_i$ and $\omega$. The constraints in this minimization problem include (13)-(16), $r_{kl} = 0$ for all $k > l$, $0 \leq \alpha_i, \beta_i \leq 1$ for all $i$, and $\sum_{i=1}^{n}\alpha_i = \sum_{i=1}^{n}\beta_i = 1$, and $\delta_i \geq 0$ and $\tau_i \geq 0$.

To prevent the estimation procedure from attempting mathematically impossible operations, and to ensure the properties of demand are satisfied, the logarithm term in the defining equation of utility must be positive. As $(\alpha_i + \beta_i \exp(u_t))/(1+\exp(u_t))$ is bounded between zero and one, and $p_i \in \Re_{++}$, then discretionary expenditure must be positive. Consequently, the following constraint is also included:

$$0.99\, c_t \geq \mathbf{p}_t' \boldsymbol{\gamma}(u) \quad \forall t. \tag{18}$$

While the scaling factor (i.e. the 0.99 on the left hand side of (18)) on expenditure is somewhat arbitrary, experiments with this value suggest results are robust to our choice.

Since MAIDADS is a non-linear model, and the constraint set has non-linear equality and linear inequality constraints, using starting values that are at least feasible, and preferably close to optimal, greatly reduces the computational burden of finding an optimal solution. In addition, appropriate choice of upper and lower bounds on the parameters, fitted budget shares, utility levels, and error terms helps to reduce the space over which the solution algorithm searches for an optimal solution. In this regard, we follow the solution strategy outlined in Cranfield *et al.* (2002). The exception to this relates to starting values and bounds on the subsistence function parameters. Lower bounds on $\delta_i$ and $\tau_i$ are set at zero, while upper bounds are set so as not to be active in the optimal solution. A zero lower bound, and an upper bound that is set large enough to be inactive at an optimum, is placed on $\omega$. Estimates of $\gamma_i$ (i.e., a constant subsistence parameter) are used as the starting values for $\delta_i$ and $\tau_i$, while the starting value for $\omega$ is arbitrarily set at 0.01. This mathematical programming problem is implemented in the General Algebraic Modeling System (GAMS) and solved using the MINOS solver.

AIDADS is estimated by imposing $\delta_i = \tau_i$ on the MAIDADS estimation framework outlined above.

**Data**

The 1996 International Comparisons Project (ICP) data are used for this analysis. These data are useful in analyzing international demand patterns since they are provided in identical units (*i.e.*, international dollars). The raw data are composed of real and nominal expenditure on 26 final goods and services in 114 countries which range in expenditure levels from Malawi to the USA. For estimation, the data are aggregated into nine goods: grains, livestock, other food, alcohol and tobacco, clothing and footwear, rent and fuel, household furnishings and operations, transport and transport services and other expenditures. (This aggregation is similar to Cranfield *et al*. 2000. The main exception being that food is more disaggregated here.) Expenditure on each aggregate good is computed as the sum of nominal expenditure on each good in the aggregate group. Total per capita expenditure equals total nominal expenditure divided by population. Unit prices for each good equals nominal expenditure divided by real expenditure. Nominal expenditure is defined in exchange rate converted US dollars, while real expenditure is defined in purchasing power parity converted international dollars. Finally, budget shares are computed as the ratio of nominal expenditure on the good to total nominal expenditure.

Table 1 provides summary statistics (i.e., means and standard deviations) for the prices and budget shares of each good. Table 2 provides mean value and standard deviations for per capita expenditure across the whole sample, and for sub-groupings based on the World Bank's 1996 income based classification of countries (i.e., low income, lower-middle income, upper-middle income and high income countries). The mean value of per capita expenditure increases as one moves from low to high income countries, as does the standard deviation of the within-group per capita level of expenditure. Almost one-third of the countries in the sample are lower-middle income countries (38 countries), followed by low income (29 countries), high income (28 countries) and lastly upper-middle income (19 countries). A complete listing of the countries in the sample, arranged according to their income grouping, is provided in Appendix B.

**Results**

In the results reported below, equation (18), which was included to ensure discretionary expenditure was positive, was not active – the strong inequality always prevailed; so discretionary expenditure was strictly positive in the solution to the mathematical programming problem used for estimation. For comparative purposes, Table 3 displays the estimated parameters for AIDADS and MAIDADS. Recall that the AIDADS estimates of the subsistence parameters, $\gamma_i$, do not vary with utility (or expenditure), moreover, the estimates for AIDADS are in-keeping with those previously reported (see, for instance, Rimmer and Powell 1992, 1996; Cranfield *et al.* 2000, 2002). Focusing on the AIDADS estimates at the top of Table 3, we see that, for all food goods, the values of $\alpha_i$ and $\beta_i$ indicate that marginal budget shares for the respective food goods fall as one progresses through higher levels of expenditure, as predicted by Engel's Law. Moreover, except for alcohol and tobacco, and clothing and footwear, the share of each additional

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dollar of expenditure devoted to non-food goods rises in expenditure. For four goods (grains, other food, clothing and footwear and rent and fuel), the subsistence parameters are strictly positive, indicating that these are indeed subsistence goods, while the $\gamma_i$s for all other goods are zero.

As with the AIDADS model, the values of the estimated $\alpha_i$ and $\beta_i$ in the MAIDADS model (bottom panel in Table 3) suggest the marginal budget shares for the food goods fall as expenditure rises, while the marginal budget shares for four of the non-food goods rise with expenditure (the exceptions again being alcohol and tobacco, and clothing and footwear). For non-food goods, the MAIDADS estimates of $\alpha_i$ and $\beta_i$ have similar magnitudes to those from the AIDADS model. However, the magnitudes of $\alpha_i$ for grains and other food and $\beta_i$ for livestock and other foods are substantively different, suggesting that the minimum consumption quantity parameters reflect some of the change in expenditure share as we move across the expenditure spectrum. In addition to these differences in magnitude, the value of $\beta_i$ for other food is zero with AIDADS but non-zero for MAIDADS, which has important implications for the asymptotic Engel elasticity as expenditure grows, as is illustrated below.

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Estimates of $\delta_i$ and $\tau_i$ (i.e., parameters of the minimum consumption quantities) are both zero for rent and fuel, household furnishings and operations, and other expenditure. For grains, $\hat{\delta}_i > \hat{\tau}_i$, indicating that the minimum consumption quantity for this good falls as per capita expenditure increases. In other words, consumer in wealthy countries consume more processed food products and will likely consider these to be necessities in lieu of grains consumption. In contrast, estimates of $\delta_i$ and $\tau_i$ indicate that minimum consumption quantities for livestock, other food, alcohol and tobacco, clothing and footwear, and transport and transport services increase with per capita expenditure.

To illustrate the pattern of subsistence level adjustment, Figure 1 plots the calculated values of $\gamma_i(\hat{u})$ for the food goods as utility varies from its minimum to maximum fitted value. We focus on the food goods as they offer a contrast with respect to the pattern of adjustment in the minimum consumption quantity. The plots of the minimum consumption functions for livestock and other food show that the minimum consumption quantities rise with utility, while the minimum consumption quantity for grains falls as utility rises. This seems plausible as we expect consumers in wealthier countries will consider larger quantities of livestock products and other foods essential to their diets, while the quantity of grains that is considered essential falls. Further, the plots of the minimum consumption functions for these goods illustrate the asymptotic nature of the minimum consumption quantities at the extreme levels of utility.

But which model is preferred in a statistical sense? Is the AIDADS model preferred over the MAIDADS model, or vice versa? In the context of nested models, such questions can typically be answered by calculating a likelihood ratio test statistic based on the value of the log of the likelihood functions for the restricted and unrestricted models. In the current context, such an approach is at odds with the fact that the parameters have been estimated with bounds. Moreover, the key to comparing AIDADS to the MAIDADS model relates to equality of a set of parameters that, in the case of the MAIDADS model, involve weak inequality constraints which are active in some cases. Nevertheless, we proceed with the computation of this test statistic, in the hopes of gaining some insight into the relative performance of these two models.

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The log of the likelihood function value for the AIDADS, and MAIDADS models are 972.99 and 981.88, respectively, which gives rise to a likelihood ratio test (LRT) statistic of 17.78. To assess the significance of the restrictions which give rise to the AIDADS model using the LRT statistic, we need to know the degrees of freedom. In the MAIDADS model, the minimum consumption quantity is constant with both $\delta_i$ and $\tau_i$ at their lower bounds for three goods, while the minimum consumption quantity is not constant for six goods. In this sense, it seems appropriate to use just six degrees of freedom. Given this interpretation, one concludes that the restrictions which give rise to the AIDADS model are rejected at the five percent level, thus favoring MAIDADS over the AIDADS model. Again, however, it should be emphasized that these comparisons using the likelihood ratio test are clouded by the fact that the relevant restriction involves weak inequalities which are active in some instances.

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Having concluded that MAIDADS is likely preferred over AIDADS on statistical grounds, it remains to be determined whether there is an economic difference between these models. Table 4 reports the marginal budget shares, fitted budget shares and Engel elasticities, calculated at the means of the data for the AIDADS and MAIDADS models. Focusing on the Engel elasticities, it would appear that the economic behaviour characterized via Engel elasticities differs primarily for grains and livestock commodities. The Engel elasticity for grains is only on-sixth of the AIDADS estimate, and the Engel elasticity for livestock is more than one third again as high for MAIDADS as for AIDADS. The Engel elasticities for all other goods share at least one significant digit between the two models.

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Having compared the Engel elasticities at the mean of the data, we now compare them across the full range of price and expenditure levels in the sample. Here, we are particularly interested in consumption behavior at the lowest expenditure levels, as needed for poverty analysis. Figures 2 and 3 show the Engel elasticities for the three food demands from the AIDADS and MAIDADS models, respectively. To aid in our comparison, these figures show not only the Engel elasticity at each observation's respective price and expenditure levels, but also polynomial fitted values of the Engel elasticities across expenditure levels. The Engel elasticities for livestock and other food estimated using the AIDADS model exhibit a monotonic, downward trend throughout the sample. This pattern is especially clear for livestock which is a luxury (Engel elasticity in excess of one) at low expenditure levels, but which becomes a necessity as expenditure increases. Other food is a necessity throughout the range of expenditure, but while its Engel elasticity is over 0.8 at low expenditure levels, at high expenditure levels, it is exceedingly small. The Engel elasticity for grains shows an initial rising trend, reaches a maximum at about 0.5, and then falls to less than 0.1 at the highest level of per capita expenditure. Powell *et al.* (2002) note that AIDADS has asymptotic Cobb-Douglas behaviour as expenditure grows. Hence, we expect that the Engel elasticities should converge to one as expenditure increases. However, the exception to this rule is when $\beta_i = 0$. In this case, the ***i***-th good drops out of the limiting Cobb-Douglas, and we expect the asymptotic Engel elasticity to be zero. This is the case for grains and other food. However, for livestock, it appears that the curvature of the polynomial trend line hints that this Engel elasticity will turn up eventually

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Figure 3 shows the plots of the Engel elasticities for the food goods based on the MAIDADS model. Relative to the corresponding plot for AIDADS, we see a bit more variation as we move across the expenditure spectrum. Focusing on the lower end of the expenditure spectrum, the Engel elasticity for other food now initially increases from about 0.7 to 0.9 and then declines to about 0.2 at high expenditure levels. While the Engel elasticities for grains decline over the entire expenditure spectrum, those for livestock actually rise at the higher expenditure levels. Livestock is again a luxury at low expenditure levels and a necessity at higher expenditure levels. As with AIDADS, the limiting functional form of MAIDADS as expenditure increases without bound is Cobb-Douglas. However as we observed with AIDADS, the Engel elasticities only converge to one for goods with strictly positive $\beta_i$. This is the case for livestock and other food, and we observe that at the highest expenditure levels, the polynomial trend lines have clearly turned upwards. However for grains $\beta_i = 0$, and the trend line is converging to its asymptotic value of zero.

Compared to the AIDADS model, the Engel elasticities for the food goods in MAIDADS appear rather different. For grains, the introduction of minimum consumption quantities which vary with utility has a profound influence on the pattern of Engel elasticity response at very low expenditure levels. With AIDADS, grains' Engel elasticities initially increased with expenditure; with MAIDADS, grains' Engel elasticities fall monotonically in expenditure. In the case of other food, comparing Figure 2 to Figures 1 and 3 illustrates that as the minimum consumption quantity begins to increase dramatically, the MAIDADS Engel elasticity for other food begins to change its direction of movement (from a downward trend to an upward trend). A similar effect is noted for livestock. Taken together, these comparisons point to the added flexibility MAIDADS brings to estimating Engel elasticities when expenditure varies widely, as is the case with international cross-section data such as used in this study.

## Conclusion

In this paper we develop a modified version of Rimmer and Powell's (1992, 1996) AIDADS model. Following their lead, AIDADS is modified by replacing the constant subsistence parameters with functions which vary with utility, and hence expenditure. The result is a modified AIDADS (MAIDADS) model that allows minimum consumption quantities to vary across expenditure levels. This model is estimated using the 1996 International Consumption Project database. These data span a wide range of expenditure levels, and countries at various stages of development, thereby presenting a significant challenge to those estimating global demand systems. Likelihood ratio tests suggest the MAIDADS model is preferred to the AIDADS model.

The MAIDADS model also has important economic implications – particularly for those seeking to understand consumption behavior at very low levels of per capita income (or expenditure). Here, MAIDADS predicts considerably more responsiveness of staple grains demands to expenditure growth, as compared to the AIDADS–based estimates. On the other hand, the expenditure-responsiveness of livestock and other food demands at low expenditure levels – while still relatively high – is less responsive to expenditure growth in MAIDADS than that predicted by the AIDADS demand system. As a consequence, the composition of food demand for the poorest households is rather different under the two demand systems. This has important implications for poverty analysis (Cranfield *et al.* 2006), as well as for predictions of global food demand following income growth in the poorest countries of the world. It is in the context of these types of issues that a more flexible demand system, such as that proposed in this paper, can offer significant advantages.

**Table 1. Summary statistics of the 1996 ICP budget shares and prices.**

| | Shares | Prices ($) |
|---|---|---|
| Grains | 0.080 | 0.696 |
| | (0.075) | (0.373) |
| Livestock | 0.107 | 0.615 |
| | (0.052) | (0.287) |
| Other Food | 0.132 | 0.671 |
| | (0.073) | (0.279) |
| Alcohol & Tobacco | 0.037 | 0.660 |
| | (0.021) | (0.407) |
| Clothing & Footwear | 0.069 | 0.705 |
| | (0.028) | (0.383) |
| Rent & Fuel | 0.137 | 0.612 |
| | (0.059) | (0.559) |
| Household Furnishings & Operations | 0.061 | 0.729 |
| | (0.027) | (0.436) |
| Transport & Transport Services | 0.093 | 0.687 |
| | (0.037) | (0.516) |
| Other Expenditures | 0.283 | 0.550 |
| | (0.117) | (0.413) |

**Table 2. Summary statistics of the 1996 ICP per capita expenditures**

| | Mean ($) | Standard deviation ($) | Number of countries |
|---|---|---|---|
| Low income countries | 373.21 | 184.96 | 29 |
| Lower middle income countries | 1419.90 | 666.73 | 38 |
| Upper middle income countries | 4154.45 | 1853.79 | 19 |
| High income countries | 17002.70 | 5613.31 | 28 |
| Entire sample | 5436.75 | 7325.27 | 114 |

**Table 3. Estimated Parameters of AIDADS and MAIDADS**

| | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | $\delta_i$ | $\tau_i$ |
|---|---|---|---|---|---|
| Estimated AIDADS model ( $\kappa = 2.138$ ) | | | | | |
| Grains | 0.090 | 0.000 | 0.539 | n.a. | n.a. |
| Livestock | 0.191 | 0.008 | 0.000 | n.a. | n.a. |
| Other Food | 0.218 | 0.000 | 0.233 | n.a. | n.a. |
| Alcohol & Tobacco | 0.048 | 0.027 | 0.000 | n.a. | n.a. |
| Clothing & Footwear | 0.081 | 0.046 | 0.102 | n.a. | n.a. |
| Rent & Fuel | 0.094 | 0.208 | 0.025 | n.a. | n.a. |
| Household Furnishings & Operations | 0.061 | 0.069 | 0.000 | n.a. | n.a. |
| Transport & Transport Services | 0.079 | 0.126 | 0.000 | n.a. | n.a. |
| Other Expenditures | 0.138 | 0.517 | 0.000 | n.a. | n.a. |
| Estimated MAIDADS model ( $\kappa = 1.377$ , $\omega = 2.314$ ) | | | | | |
| Grains | 0.160 | 0.000 | n.a. | 0.372 | 0.000 |
| Livestock | 0.194 | 0.027 | n.a. | 0.000 | 1.560 |
| Other Food | 0.185 | 0.008 | n.a. | 0.297 | 3.265 |
| Alcohol & Tobacco | 0.042 | 0.026 | n.a. | 0.000 | 0.675 |
| Clothing & Footwear | 0.096 | 0.054 | n.a. | 0.056 | 0.144 |
| Rent & Fuel | 0.090 | 0.205 | n.a. | 0.000 | 0.000 |
| Household Furnishings & Operations | 0.062 | 0.075 | n.a. | 0.000 | 0.000 |
| Transport & Transport Services | 0.068 | 0.121 | n.a. | 0.000 | 0.553 |
| Other Expenditures | 0.104 | 0.485 | n.a. | 0.000 | 0.000 |

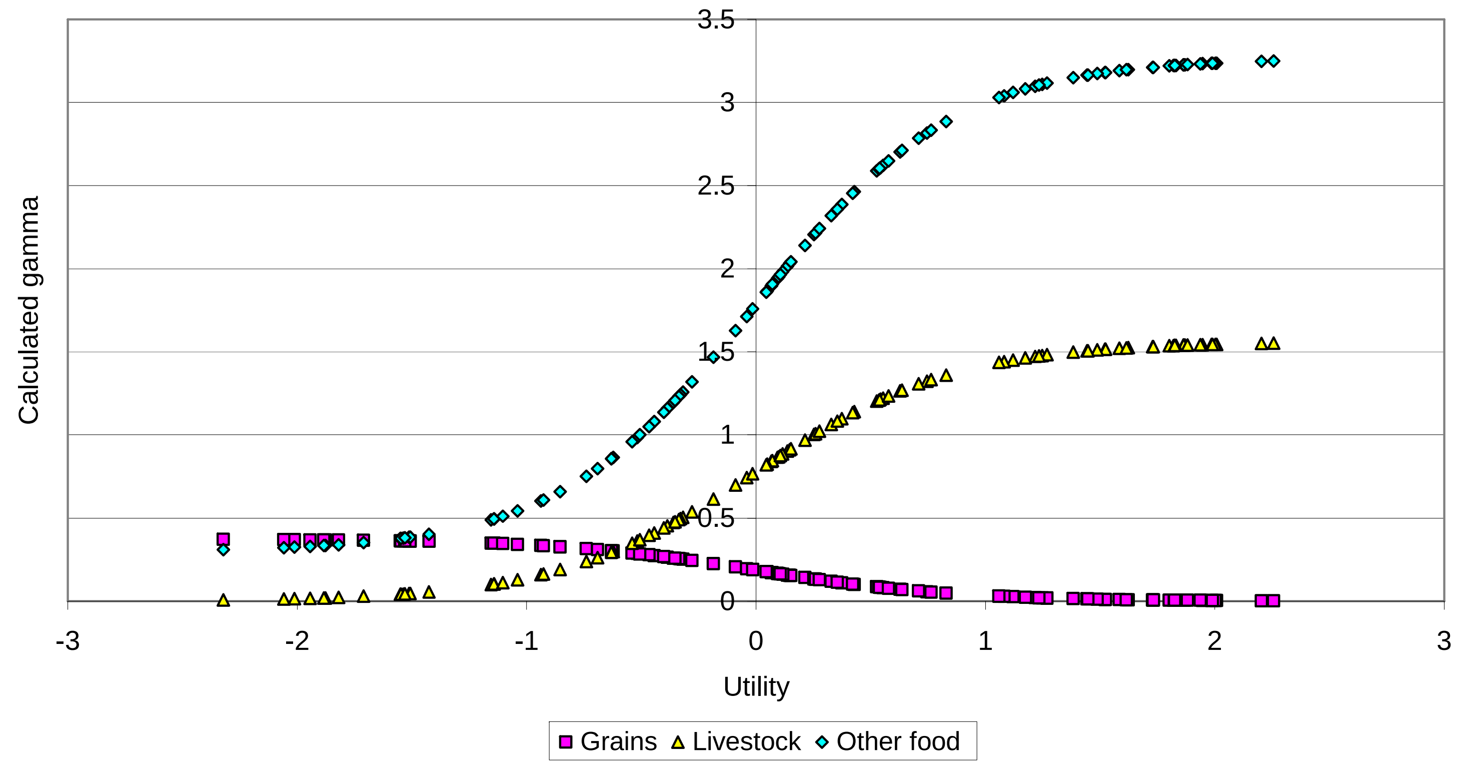


**Figure 1. Estimated minimum consumption quantities from MAIDADS.**

**Table 4. Estimated Marginal Budget Shares, Fitted Budget shares and Engel Elasticities for AIDADS and MAIDADS, Evaluated at Sample Means of the Data**

| | MBS | $\hat{s}_i$ | Engel |
|---|---|---|---|
| AIDADS | | | |
| Grains | 0.009 | 0.042 | 0.207 |
| Livestock | 0.026 | 0.079 | 0.325 |
| Other Food | 0.021 | 0.088 | 0.240 |
| Alcohol & Tobacco | 0.029 | 0.035 | 0.832 |
| Clothing & Footwear | 0.049 | 0.060 | 0.815 |
| Rent & Fuel | 0.197 | 0.162 | 1.219 |
| Household Furnishings & Operations | 0.068 | 0.065 | 1.045 |
| Transport & Transport Services | 0.121 | 0.106 | 1.144 |
| Other Expenditures | 0.480 | 0.363 | 1.322 |
| MAIDADS | | | |
| Grains | 0.001 | 0.035 | 0.032 |
| Livestock | 0.031 | 0.077 | 0.404 |
| Other Food | 0.016 | 0.084 | 0.192 |
| Alcohol & Tobacco | 0.027 | 0.035 | 0.768 |
| Clothing & Footwear | 0.054 | 0.061 | 0.888 |
| Rent & Fuel | 0.201 | 0.165 | 1.216 |
| Household Furnishings & Operations | 0.074 | 0.067 | 1.108 |
| Transport & Transport Services | 0.120 | 0.108 | 1.118 |
| Other Expenditures | 0.475 | 0.369 | 1.289 |

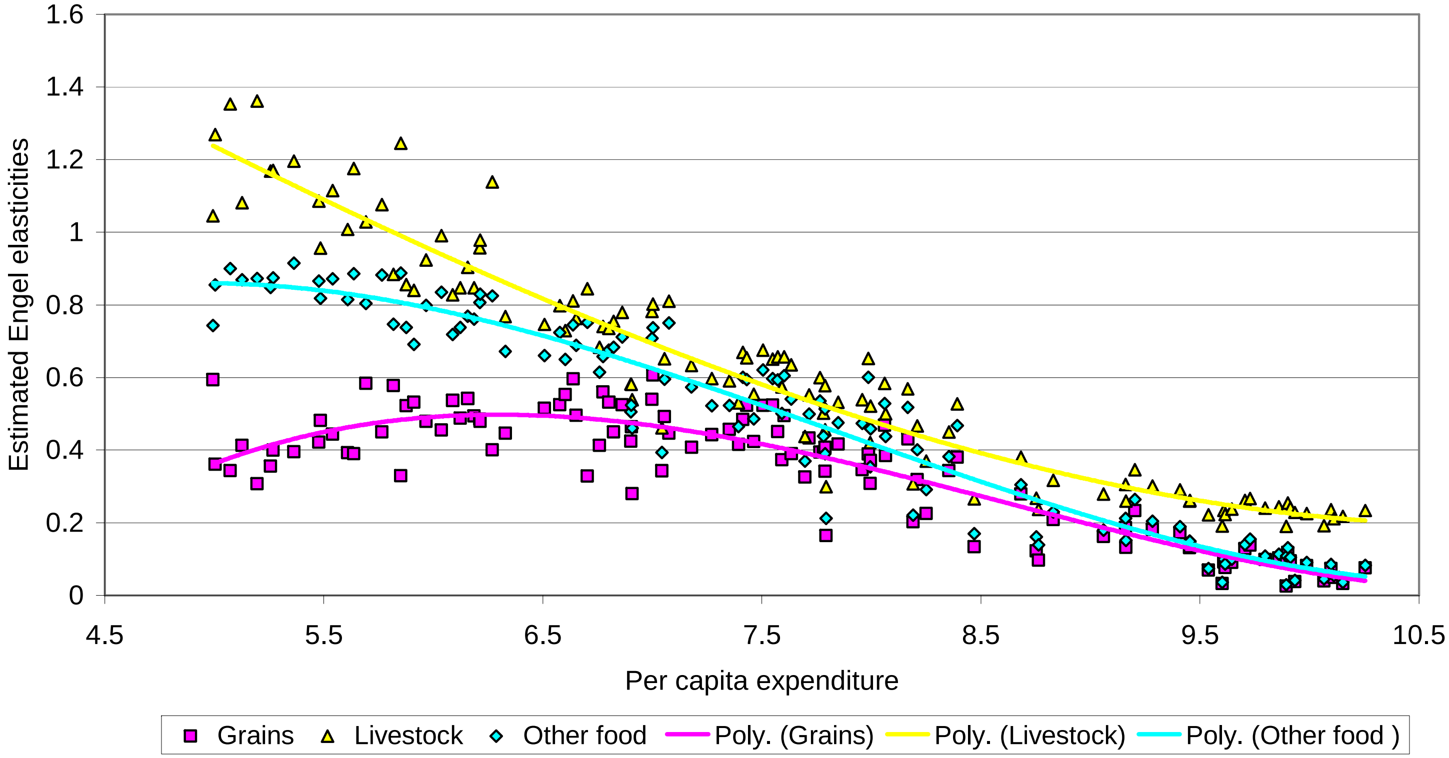


**Figure 2: Estimated Engel elasticities from the AIDADS at observed price and expenditure levels (points) and smoothed (lines)**

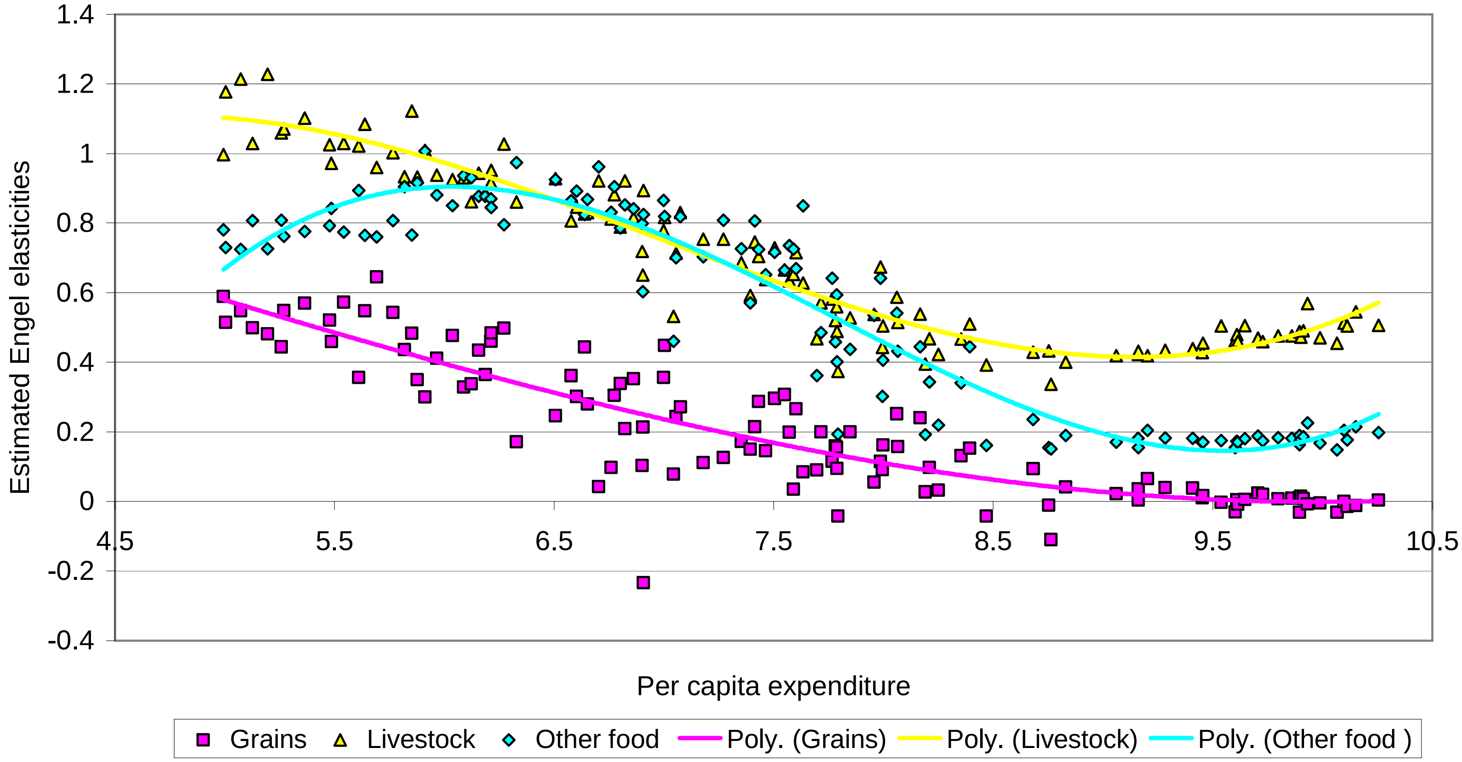


**Figure 3: Estimated Engel elasticities from MAIDADS model evaluated at observed price and expenditure levels (points) and smoothed (lines)**

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**Appendix A- Derivation of the Engel Elasticity for MAIDADS**

Derivation of the Engel elasticities for MAIDADS is approached from the consumer's utility maximization problem. Since the defining equation for MAIDADS is implicit in utility, it is treated as a constraint, along with the budget constraint, in the consumer's problem:

$$\begin{aligned}
&\underset{u,x}{\text{maximize}} \quad u \\
&\text{subject to}: \sum_{i=1}^{n} \frac{\alpha_i + \beta_i e^u}{1+e^u} \ln\left( x_i - \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right) - \ln(A) - u = 1 \\
&\qquad \sum_{i=1}^{n} p_i x_i = c \\
&\qquad x_i > \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \qquad i = 1,2,\ldots,n.
\end{aligned} \tag{A1}$$

Let $\mu$ and $\lambda$ denote the Lagrange multipliers associated with the first and second constraints in (A1). (While the final set of constraints are required to ensure the defining constraint is defined, they will not be active provided that the prices of all consumption goods are strictly positive, and provided expenditure is sufficient to purchase the utility-varying minimum consumption bundle.) The first-order condition with respect to $x_i$ is:

$$-\mu \frac{\alpha_i + \beta_i e^u}{1+e^u} \frac{1}{x_i - \dfrac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}}} - \lambda p_i = 0\,. \tag{A2}$$

A2 can be manipulated to show that

$$\lambda = -\mu \left( c - \sum_i p_i \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}} \right)^{-1}. \tag{A3}$$

The first-order condition with respect to $u$ is:

$$1-\mu\left\{\sum_{i=1}^{n}\left[\frac{(\beta_i-\alpha_i)e^u}{(1+e^u)^2}\ln\left(x_i-\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)\right.\right.$$
$$\left.\left.-\frac{\alpha_i+\beta_i e^u}{1+e^u}\left(x_i-\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)^{-1}\frac{(\tau_i-\delta_i)\omega e^{\omega u}}{(1+e^{\omega u})^2}\right]-1\right\}=0. \tag{A4}$$

This means that the Lagrange multiplier on the utility defining constraint is:

$$\mu=\left\{\sum_{i=1}^{n}\left[\frac{(\beta_i-\alpha_i)e^u}{(1+e^u)^2}\ln\left(x_i-\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)-\right.\right.$$
$$\left.\left.\frac{\alpha_i+\beta_i e^u}{1+e^u}\left(x_i-\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)^{-1}\frac{(\tau_i-\delta_i)\omega e^{\omega u}}{(1+e^{\omega u})^2}\right]-1\right\}^{-1}. \tag{A5}$$

Combining (A3) and (A5), the Lagrange multiplier on the budget constraint can be expressed as:

$$\lambda=-\left\{\sum_{i=1}^{n}\left[\frac{(\beta_i-\alpha_i)e^u}{(1+e^u)^2}\ln\left(x_i-\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)-\right.\right.$$
$$\left.\left.\frac{\alpha_i+\beta_i e^u}{1+e^u}\left(x_i-\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)^{-1}\frac{(\tau_i-\delta_i)\omega e^{\omega u}}{(1+e^{\omega u})^2}\right]-1\right\}^{-1} \tag{A6}$$
$$\times\left(c-\sum_i p_i\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}\right)^{-1}$$

From (A2) and (A3), we can denote the demand for the *i*-th good as follows:

$$x_i=\frac{\delta_i+\tau_i e^{\omega u}}{1+e^{\omega u}}+\frac{\alpha_i+\beta_i e^u}{(1+e^u)p_i}\left(c-\sum_{j=1}^{n}p_j\frac{\delta_j+\tau_j e^{\omega u}}{1+e^{\omega u}}\right). \tag{A7}$$

Following Hanoch (1975) and Rimmer and Powell's (1992) derivation strategies, the Engel elasticity is then expressed as:

$$\eta_i = \frac{c}{p_i x_i}\left\{\frac{\alpha_i + \beta_i e^u}{1+e^u} + \left(c - \sum_{j=1}^{n} p_j \frac{\delta_j + \tau_j e^{\omega u}}{1+e^{\omega u}}\right)\frac{(\beta_i - \alpha_i)e^u}{(1+e^u)^2}\lambda \right.$$
$$\left. + \frac{(\tau_i - \delta_i)\omega e^{\omega u}}{\left(1+e^{\omega u}\right)^2} p_i \lambda - \frac{\alpha_i + \beta_i e^u}{1+e^u}\sum_{j=1}^{n}\frac{(\tau_j - \delta_j)\omega e^{\omega u}}{\left(1+e^{\omega u}\right)^2} p_j \lambda \right\}. \qquad \text{(A8)}$$

**Appendix B – Country listing according to income category**

| Low income | Lower middle income | Upper middle income | High income |
|---|---|---|---|
| Albania | Belarus | Antigua and Barbuda | Australia |
| Armenia | Belize | Argentina | Austria |
| Azerbaijan | Bolivia | Bahrain | Bahamas |
| Bangladesh | Botswana | Barbados | Belgium |
| Benin | Bulgaria | Brazil | Bermuda |
| Cameroon | Dominica | Chile | Canada |
| Congo | Ecuador | Czech Republic | Denmark |
| Cote d'Ivoire | Estonia | Gabon | Finland |
| Egypt | Fiji | Greece | France |
| Georgia | Grenada | Hungary | Germany |
| Guinea | Indonesia | Korea | Hong Kong |
| Kenya | Iran | Mauritius | Iceland |
| Kyrgyz | Jamaica | Mexico | Ireland |
| Madagascar | Jordan | Oman | Israel |
| Malawi | Kazakhstan | Slovenia | Italy |
| Mali | Latvia | St. Kitts and Nevis | Japan |
| Mongolia | Lebanon | St. Lucia | Luxembourg |
| Nepal | Lithuania | Trinidad and Tobago | The Netherlands |
| Nigeria | Macedonia | Uruguay | New Zealand |
| Pakistan | Moldova | | Norway |
| Senegal | Morocco | | Portugal |
| Sierra Leone | Panama | | Qatar |
| Sri Lanka | Peru | | Singapore |
| Tajikistan | Philippine | | Spain |
| Tanzania | Poland | | Sweden |
| Vietnam | Romania | | Switzerland |
| Yemen | Russia | | United Kingdom |
| Zambia | Slovakia | | USA |
| Zimbabwe | St.Vincent and the Grenadines | | |
| | Swaziland | | |
| | Syria | | |
| | Thailand | | |
| | Tunisia | | |
| | Turkey | | |
| | Turkmenistan | | |
| | Ukraine | | |
| | Uzbekistan | | |
| | Venezuela | | |