

A New Regional Household Demand System for GTAP

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Abstract

The GTAP final demand system has some known defects: the computation of the equivalent variation is not exact but approximate; with non-standard demand parameters, the equivalent variation may be grossly in error; the decomposition of the equivalent variation contains a nuisance term. We find a further defect, that the top-level demand equations are invalid. We revise the system to remove these defects.

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1 Introduction

The GTAP model, versions 4.1 and lower, suffers from some defects in the implementation of the regional household demand system:

- The top level of the demand system assumes a fixed price for utility from private consumption. But with a private consumption demand system of the CDE form, the price of utility from private consumption depends on the level of private consumption expenditure. With households not price takers for utility from private consumption, the familiar Cobb-Douglas demand system does not apply. Accordingly, the top-level demand equations are invalid.
- The equivalent variation is wrongly computed in simulations with non-standard settings for the CDE expansion parameters. Even with the standard settings, in multi-step simulations the utility and equivalent variation computations are inexact.
- The welfare decomposition inherits the defects of the final demand and equivalent variation computations.

In removing these defects we revise in passing some minor misfeatures of the old treatment:

- We treat the entire final demand system as the demand system of a representative household, rather than a conglomeration of representative and region-wide demand systems (subsection 2.6).
- We provide a new facility for shifting the allocation of regional income exogenously by modifying rather than overriding the final demand system (subsection 2.12).
- We eliminate an uninterpretable “nuisance term” from the decomposition of equivalent variation (subsection 4.1).
- We reorganize the GTAP model source code to gather within a single module all equations derived from the top-level demand system (subsection 2.6).

The main disadvantage of the new treatment relative to the old is that its implementation and properties are somewhat more complex. It requires more equations and variables, mostly to support an exact calculation of the equivalent variation. Also, whereas the old treatment allocated regional income in fixed shares between private consumption expenditure, government household expenditure, and saving, the new treatment allows the shares to vary in response to changes in income and consumer prices.

This paper describes the new treatment. It should be accompanied by several program files—source code, `gtap.tab`, for the revised solution model, a Tablo stored input file `gtap.fts` showing a typical model condensation, and a GTAP test simulation command file `ghom.cmf` showing a typical model closure. It includes extensive listings from the source code. It does not describe but rather takes as given the standard GTAP model notation; it should accordingly be read in conjunction with the source code or the original GTAP model documentation (Hertel and Tsigas [5]).

Listings in this paper of old program code come from a GTAP model version 5 prerelease, incorporating Ken Itakura’s admirable reorganization of the code structure, but no relevant changes in the model theory over version 4.1, and used in the August 2000 GTAP short course.

We use, often without further notice, the convention that a lower-case symbol denotes the percentage change in the corresponding upper-case symbol; so, for a variable X , x denotes percentage change in X , $x = (1/100)dX$.

2 The top level of the regional household demand system

2.1 The old treatment

In the GTAP model as originally implemented (Hertel and Tsigas [5]), in each region a *regional household* allocates regional income so as to maximize *per capita* aggregate utility according to a Cobb-Douglas utility function. The maximand is described as “aggregate” utility because it comprises both government and private sector behavior. The arguments in the utility function are *per capita* utility from private consumption, *per capita* utility from government consumption, and *per capita* real saving. We refer to these as the *top-level commodities* of the final demand system.

Real saving is a single commodity, defined as saving deflated by a saving price. Utility from government consumption is a Cobb-Douglas aggregate of government consumption of individual commodities. *Per capita* utility from private consumption is aggregated from *per capita* private consumption of individual commodities following Hanoch’s ([3]) constant difference elasticity (CDE) demand system.

It may be noted that utility from private consumption, unlike utility from government consumption and real saving, is defined on a *per capita* basis. This is necessary because the private consumption demand system is non-homothetic. The allocation of private consumption expenditure across commodities depends on the total sum to be allocated, but the appropriate total variable is not economy-wide but *per capita* private consumption expenditure.

The CDE demand system is characterized by an implicit expenditure-cum-indirect-utility function,

$$1 = \sum_i B_i U^{\Upsilon_i E_i} \left(\frac{P_i}{X} \right)^{\Upsilon_i}, \quad (1)$$

where i is an index over commodities, U_i utility, P_i the price of commodity i , X , expenditure, and B_i , Υ_i , and E_i , various parameters. Following Hanoch [3], we call the B_i distribution parameters, the Υ_i substitution parameters, and the E_i expansion parameters. Constraints on the parameters are:

$$\begin{aligned} \forall i, B_i &> 0, \\ \forall i, E_i &> 0, \end{aligned}$$

and either

$$\forall i, \quad \Upsilon_i < 0$$

or

$$\forall i, \quad 0 < \Upsilon_i < 1.$$

Although we are not required to do so by theory, in standard GTAP data bases we normalize the expansion parameters so that their share-weighted sum is equal to one,

$$\sum_i S_i E_i = 1,$$

where S_i denotes the share of commodity i in private consumption expenditure.

This suffices to specify the final demand system; it remains to work out the implications of these specifications. This is done briefly in Hertel and Tsigas ([5]), but for the purposes of this paper we find it convenient to provide a more detailed derivation.

We adopt a two-stage budgeting approach, first allocating income between the top-level commodities, then allocating private and government consumption expenditure across individual commodities. For present purposes, it suffices to develop the top level of the demand system, that is, the allocation of income between the top-level commodities.

We write the top-level utility function as

$$U = CU_P^{B_P} U_G^{B_G} U_S^{B_S}, \quad (2)$$

where U denotes *per capita* aggregate utility, U_P , *per capita* utility from private consumption, U_G , *per capita* government consumption, and U_S , *per capita* real saving. B_P , B_G , and B_S are distribution parameters.

We define a saving price P_S , and postulate the existence of suitable price indices P_G and P_P for utility from government and private consumption. Then given income Y , the regional household maximizes U subject to the budget constraint

$$N(P_P U_P + P_G Q_G + P_S Q_S) = Y,$$

where N denotes population.

Since the utility function is Cobb-Douglas, we expect the regional household to allocate regional income in fixed shares between the top-level commodities:

$$Y_P = \frac{B_P}{B} Y, \quad (3)$$

$$Y_G = \frac{B_G}{B} Y, \quad (4)$$

$$Y_S = \frac{B_S}{B} Y, \quad (5)$$

where B denotes the sum of the distribution parameters, $B = B_P + B_G + B_S$, Y_P private consumption expenditure, $Y_P = P_P N U_P$, Y_G government consumption expenditure, $Y_G = P_G N U_G$, and Y_S saving, $Y_S = P_S N U_S$. Then

$$P_P N U_P = \frac{B_P}{B} Y,$$

$$P_G N U_G = \frac{B_G}{B} Y,$$

$$P_S N U_S = \frac{B_S}{B} Y.$$

To allow for exogenous shocks in the allocation of saving, we define “slack variables” K_S and K_G for saving and utility from government consumption, initially equal to one. We insert these into the corresponding demand equations:

$$P_G N U_G = K_G \frac{B_G}{B} Y,$$

$$P_S N U_S = K_S \frac{B_S}{B} Y.$$

Rewriting in terms of Q_G , government consumption, $Q_G = N U_G$, and Q_S , aggregate saving, $Q_S = N U_S$, we obtain

$$P_G Q_G = K_G \frac{B_G}{B} Y,$$

$$P_S Q_S = K_S \frac{B_S}{B} Y.$$

Differentiating and rearranging, we obtain

$$q_G = y - p_G + \kappa_G,$$

$$q_S = y - p_S + \kappa_S.$$

These appear in the old code as:

Equation GOVERTU
computation of utility from government consumption (HT 39) #
(all,r,REG)
ug(r) = y(r) - pgov(r) + govelack(r);

and

Equation SAVINGS
regional demand for savings (HT 38) #
(all,r,REG)
qsave(r) = y(r) - psave(r) + saveslack(r);

In the presence of shocks to the slack variables, the top-level demand system is no longer operative; the budget constraint however must still be observed. Accordingly, we include in the model not the demand equation for utility from private consumption but instead the budget constraint

$$Y_P = Y - Y_G - Y_S.$$

For no very compelling reason, we express government consumption expenditure Y_G as the sum of expenditures on individual commodities, $Y_G = \sum_i Y_{Gi}$, where Y_{Gi} denotes government consumption expenditure on commodity i , $Y_{Gi} = P_{Gi}Q_{Gi}$, where P_{Gi} denotes the price of commodity i when purchased for government consumption, and Q_{Gi} government consumption of commodity i . Then

$$\begin{aligned} Y_P &= Y - \sum_i Y_{Gi} - Y_S \\ &= Y - \sum_i P_{Gi}Q_{Gi} - P_S Q_S, \end{aligned}$$

or, in percentage change form,

$$Y_P y_P = Y y - \sum_i Y_{Gi} (p_{Gi} + q_{Gi}) - Y_S (p_S + q_S).$$

This appears in the old code as:

```
Equation PRIVATEXP
# private consumption expenditure (HT 8) # (all,r,REG)
PRIVEXP(r)*yp(r)
= INCOME(r)*y(r)
- SAVE(r)*[psave(r) + qsave(r)]
- sum(i,TRAD_COMM, VGA(i,r)*[pg(i,r) + qg(i,r)])
;
```

Finally, we compute utility. Recalling equation (2), we have

$$\begin{aligned} U &= CU_P^{B_P} U_G^{B_G} U_S^{B_S} \\ &= CU_P^{B_P} \left(\frac{Q_G}{N} \right)^{B_G} \left(\frac{Q_S}{N} \right)^{B_S}. \end{aligned}$$

Differentiating, we obtain

$$\begin{aligned} u &= B_P u_P + B_G (q_G - n) + B_S (u_S - n) \\ &= B \left[\frac{B_P}{B} u_P + \frac{B_G}{B} (q_G - n) + \frac{B_S}{B} (q_S - n) \right] \\ &= B \left[\frac{Y_P}{Y} u_P + \frac{Y_G}{Y} (q_G - n) + \frac{Y_S}{Y} (q_S - n) \right], \end{aligned}$$

using equations (3-5). Then, setting $B = 1$, we have

$$Y u = Y_P u_P + Y_G (q_G - n) + Y_S (q_S - n).$$

This appears in the old code as:

```
Equation UTILITY
# computation of per capita regional utility (HT 37) #
(all,r,REG)
= INCOME(r)*u(r)
+ GOVEXP(r)*[ug(r) - pop(r)]
+ SAVE(r)*[qsave(r) - pop(r)]
;
```

2.2 Defects in the old treatment

The top-level demand system just derived appears plausible enough. Indeed, its plausibility has been demonstrated over several years by its universal acceptance by GTAP developers, instructors, students, and users, the present author included. Curiously enough however it is also invalid. Indeed, except for the private consumption expenditure equation (an accounting identity), all the model equations derived above are invalid.

The error in the derivation lies in the assumption that the regional household can obtain utility from private consumption at some fixed price P_P ; equivalently, that the budget constraint for the top level of the demand system is of the form

$$P_P U_P + P_G U_G + P_S U_S = X \quad (6)$$

(writing X for *per capita* income, $X = Y/N$). This assumption is in fact false, and its falsity infects the old model equations derived above.

With no fixed price for utility from private consumption, the top-level budget shares are not fixed. Accordingly, the equations for government consumption and saving are invalid. Furthermore, the fixed share assumption is used in deriving the utility equation, and that too is invalid.

Recalling that utility from private consumption and utility from government consumption are defined within the private and government consumption demand subsystems, the general form of the top-level budget constraint is

$$E_P(\mathbf{P}_P, U_P) + E_G(\mathbf{P}_G, U_G) + R_S U_S = X, \quad (7)$$

where E_P and E_G are expenditure functions and $\mathbf{P}_P$ and $\mathbf{P}_G$ are price vectors for private and government consumption. It might so happen that the expenditure functions were of the form

$$\begin{aligned} E_P(\mathbf{P}_P, U_P) &= \Pi_P(\mathbf{P}_P)U_P, \\ E_G(\mathbf{P}_G, U_G) &= \Pi_G(\mathbf{P}_G)U_G \end{aligned} \quad (8)$$

for suitable functions $\Pi_P(\mathbf{P}_P)$ and $\Pi_G(\mathbf{P}_G)$. If so, we could set $P_P = \Pi_P(\mathbf{P}_P)$ and $P_G = \Pi_G(\mathbf{P}_G)$, and replace the general budget constraint equation (7) with the simpler equation (6). In fact, the government consumption expenditure function is of the required form, but the private consumption expenditure function is not; so we cannot use the simpler equation for the budget constraint.

At this point a little theory comes in handy.

Proposition 1 *For any demand system, the expenditure function is of the form $E(\mathbf{P}, U) = \Pi(\mathbf{P})U$ if and only if the system is homothetic.*

Proof. Sufficiency is easy. For necessity, use Shephard's lemma. For each commodity i , the budget share is equal to the elasticity of Π with respect to the price of i ; so the budget shares are independent of utility; so the system is homothetic.

Now as Hanoch [3] shows, the CDE is in general non-homothetic. Indeed, this is a requirement for any empirically satisfactory demand system (see, for example, Deaton and Muellbauer [1] p. 144), and part the reason for adopting the CDE in GTAP (Hertel and Tsigas [5], p. 49). So in GTAP, the private consumption expenditure function is not of the form (8), and the budget constraint is not equation (6).

With the more general form of the budget constraint, we can still solve the top-level allocation problem, but the standard Cobb-Douglas fixed shares property no longer applies:

Proposition 2 *In a Cobb-Douglas demand system, with utility function*

$$U = C \prod_i Q_i^{B_i}, \quad (9)$$

with expenditures on individual commodities convex in quantities, the budget shares

$$S_i = \frac{X_i}{X} = \frac{\Phi_i^{-1} B_i}{\sum_j \Phi_j^{-1} B_j}, \quad (10)$$

where X_i denotes expenditure on commodity i , and Φ_i denotes the elasticity of expenditure on commodity i with respect to quantity of commodity i .

The proof is a straightforward exercise in constrained optimisation.

Note that with expenditures proportional to quantities, the Φ_i are unity, so the equation reduces to the standard Cobb-Douglas fixed-shares equation.

Now according to Hanoch [3], for a CDE demand system, the utility elasticity of expenditure,

$$\Phi = \sum_i S_i E_i, \quad (11)$$

In general, with non-uniform E_i , the system is non-homothetic, so the budget shares S_i are variable, so Φ too is variable. Then, from equation (10), the budget shares are variable. This suffices to show, as noted above, that the model equations derived in subsection 2.1, with the exception of the accounting identity for private consumption expenditure, are invalid.

2.3 A digression on the Gorman conditions

There is obviously some relation between our concerns about the top-level demand system and Gorman's [2] result on two-stage budgeting. The relation however is not a simple one.

The Gorman conditions apply in the context of *weak separability*. A system is said to be *weakly separable* if the utility function can be represented in the form

$$U(\mathbf{Q}) = U_\bullet(U_1(\mathbf{Q}_1), \dots, U_G(\mathbf{Q}_G)),$$

where $\mathbf{Q}_1, \dots, \mathbf{Q}_G$ is a partition of the quantity vector $\mathbf{Q}$ into subvectors representing groups of commodities. The function $U_\bullet$ is the top-level utility function, and the $U_1, \dots, U_G$ are *lower-level utilities* or *subutilities*. In a weakly separable system, the Gorman conditions are the necessary and sufficient conditions for the existence of a top-level demand system

$$\max U = U_\bullet(U_1^*, \dots, U_G^*) \text{ subject to } \sum_{i=1}^G P_i^* U_i^* = X, \quad (12)$$

where P_i^* and U_i^* are price and quantity indices for the i 'th lower-level subsystem, and U_i^* an aggregator function, such that the solution for the top-level system is consistent with the solution for the overall system. Note that the quantity indices U_i^* may or may not be the subutilities U_i . Also, the aggregator function U_i^* may or may not be of the same form as the top-level utility function $U_\bullet$.

This is related to but distinct from our concerns with the old GTAP treatment. The error in the old derivation is that the assumption that utility from private consumption could act as a quantity index for private consumption in the top-level demand system (12). As we have shown, if we try to use utility from private consumption as the quantity index, we find there is no corresponding price index. This does not mean that suitable price and quantity indices for private consumption cannot be found; it means only that if they exist, the quantity index must be something other than utility from private consumption.

On the other hand, even if suitable price and quantity indices did exist, that would not necessarily validate the old top-level system. It would show that we could specify a top-level demand system that treated private consumption as an ordinary good, but it would not guarantee that the aggregator function U_i^* in that system was of the Cobb-Douglas form, nor consequently that the demands had the Cobb-Douglas fixed budget shares property. In short, the requirements for the validity of the old treatment are more stringent than the Gorman conditions.

Gorman [2] shows that a top-level system of the desired form can be constructed under either of two alternative conditions. One alternative is that the lower-level systems are homothetic. Under this alternative, the quantity indices are just the lower-level utilities, and the aggregator function is just the top-level utility function. The other alternative is that the top-level utility function is additive, and the lower-level systems admit indirect utility functions of the *Gorman generalized polar form*,

$$\Psi_i(\mathbf{P}_i, X_i) = F_i \left(\frac{X_i}{M_i(\mathbf{P}_i)} \right) + A_i(\mathbf{P}_i). \quad (13)$$

Under this alternative, the quantity indices, $U_i^* = X_i/M_i(\mathbf{P}_i)$, the price indices, $P_i^* = M_i$, and the aggregator function, $U_\bullet^*(U_1^*, \dots, U_G^*) = \sum_{i=1}^G F_i(U_i^*)$.

As we have seen already, the GTAP final regional household demand system does not meet the first condition, that the lower-level demand systems be homothetic. It seems obvious, but it is not easily proven, that except in degenerate cases, the CDE and Gorman generalized polar forms are incompatible.

Conjecture 1 *If a demand system is both a CDE system and a Gorman generalized polar form, then it is a CES system.*

As noted above, satisfying the Gorman conditions is not enough to validate the old top-level demand system. In particular, solutions involving the Gorman generalized polar form typically do not validate the old top-level demand system. For example, except for degenerate cases of the Gorman generalized polar form ($A_i \equiv 0$, F_i the identity function), the Gorman quantity indices are not the subutilities.

Assuming that the conjecture is accurate, it is true but misleading to say that the old treatment is erroneous because the CDE system does not satisfy the Gorman conditions. It is true because, if the old treatment were valid, the Gorman conditions would necessarily be satisfied. It is misleading because the Gorman conditions are a red herring in this context. To show that the logic behind the old derivation is invalid, we do not need to refer to the Gorman conditions; it is sufficient, and simpler, to show that the private consumption demand system is non-homothetic. Nevertheless, as we show below (subsection 2.5), although the Gorman result is not useful in refuting the old treatment, it is potentially useful in remedying its defects.

2.4 Possible remedies

There are several different approaches we might take to remedy the defects of the old treatment.

1. We might retain the basic premises of the old treatment, in particular, the CDE form for the private consumption demand system, and correct the errors in the derived equations.
2. We might adopt a new functional form for the private consumption demand system, that would allow us to retain fixed budget shares in the top-level system.
3. We might abandon the concept of a top-level demand system. Rather than representing the allocation of regional income as optimizing behavior by a fictitious regional household, we might simply impose some arbitrary rule. There would not necessarily be a concept of regional welfare, but instead a purely descriptive treatment of macroeconomic behavior. This might be a simple rule such as the fixed shares rule, or some more complex empirically motivated treatment.

Option (1) has the advantage of maximizing theoretical consistency with the old treatment. Its disadvantage is that the top-level demand equations become more complex, so that the top-level budget shares are no longer fixed. Options (2) and (3) let us keep the fixed budget shares property, but require changes in the basic theory.

In this paper we develop and implement option (1). This provides the first accurate implementation of the current basic theory, and may assist in any subsequent decision whether to change it. First however (subsection 2.5) we explore option (2) just far enough to establish its viability.

2.5 A digression on alternative private consumption demand systems

As shown above (subsection 2.3), unless we accept a homothetic private consumption demand system, we must accept some changes to the derived equations for the top-level demand system. Looking forward, we are not concerned to retain all aspects of the current top-level system, but would like to retain the fixed shares property. In this section we investigate whether we can find a new form for the private consumption demand system so as to retain the fixed shares property while accepting other changes in the top-level system.

Recalling equation (10),

$$S_i = \frac{X_i}{X} = \frac{\Phi_i^{-1} B_i}{\sum_j \Phi_j^{-1} B_j},$$

we see that even when the elasticities Φ are not all equal to one, the budget shares are constant provided that the elasticities are constant. This looks promising: with constant elasticities, we change some aspects of the top-level system but retain the fixed budget shares. As it turns out however, this approach imposes unacceptable restrictions on the form of the lower-level demand systems.

Proposition 3 *In any demand system, if the utility elasticity of expenditure is constant, the system is homothetic.*

Proof. Let $\bar{U}$ denote some arbitrary elasticity level, and Φ the constant utility elasticity. Then for any utility level U , $E(\mathbf{P}, U) = (U/\bar{U})^\Phi E(\mathbf{P}, \bar{U})$. But then we can write, for all $\mathbf{P}$, U , $E(\mathbf{P}, U) = \Pi(\mathbf{P})(U/\bar{U})^\Phi$, where $\Pi(\mathbf{P}) = E(\mathbf{P}, \bar{U})$. Then, by Shephard's lemma, for each commodity i , the budget share is equal to the elasticity of Π with respect to the price of i ; so the budget shares are independent of utility; so the system is homothetic.

Since homotheticity is empirically unacceptable, this idea does not help us find an acceptable form for the private consumption demand system.

We may also attempt to use the Gorman [2] conditions for two stage budgeting to find a functional form for the private consumption demand system that lets us preserve the top-level demand system. This is a somewhat subtle strategy. We have seen above (subsection 2.3) that there is no non-homothetic private consumption demand system that, in conjunction with a Cobb-Douglas top-level utility function $U_\bullet$, leads to fixed top-level budget shares. There may however be a non-homothetic private consumption demand system that, in conjunction with a top-level utility function not of the Cobb-Douglas form, leads to a Cobb-Douglas top-level aggregator function $U_\bullet^*$. This is in fact feasible.

Of the two alternative conditions in [2], one entails a homothetic lower-level demand systems, which is unacceptable. Under the other condition, we can obtain a top level aggregator function $U_\bullet^*$ of the Cobb-Douglas form provided that $F_i \equiv \log$ in equation (13). That is, the lower level demand systems must admit indirect utility functions of the form

$$\Psi_i(\mathbf{P}_1, X_i) = \log \frac{X_i}{M_i(\mathbf{P}_1)} + A_i(\mathbf{P}_1).$$

This allows both the Cobb-Douglas demand system used in GTAP for government consumption, and (with non-zero A_P) a non-homothetic private consumption demand system.

Surprisingly then, we can keep the fixed-shares property of the Cobb-Douglas utility function, but to do so we need to replace the Cobb-Douglas utility function itself, adopting instead the additive form.

2.6 A new treatment

In the old treatment 2.1, aggregate utility is attributed to a single regional household encompassing both the private and government sectors. Since however aggregate utility is a *per capita* variable, we might better describe it as an attribute of a representative regional household. It is unclear how this representative household can choose a level of utility from government consumption, that being an economy-wide variable. Since however the government consumption demand system is homothetic, we can redefine utility from government consumption as a *per capita* variable without changing any model results for any other variable.

Since saving and utility from private consumption are already *per capita* variables, this lets us treat the entire final demand system as the demand

system of a representative regional household.

We maintain the old functional forms for the top and lower levels of the final demand system. We do not therefore develop a new approach to the final demand system, but merely derive more accurately the implications of the old approach.

From equation (10), we have, in percentage change form,

$$x_i - x = b_i - \phi_i - \sum_j S_j(b_j - \phi_j), \quad (14)$$

where x_i denotes *per capita* expenditure on i . The index i ranges over P , private consumption, G , government consumption, and S , saving. The parameter S_i represents the share $\Phi_i^{-1}B_i / \sum_j \Phi_j^{-1}B_j$; by equation (10), this is just the expenditure share.

For saving there is a single price, and for government consumption, a well-behaved price index, so Φ_G and Φ_S are constant, and ϕ_G and ϕ_S may be omitted from the percentage change equation. So equation (14) reduces to

$$x_P - x = -(1 - S_P)\phi_P + b_P - b_{AV} \quad (15)$$

$$x_G - x = S_P\phi_P + b_G - b_{AV} \quad (16)$$

$$x_S - x = S_P\phi_P + b_S - b_{AV} \quad (17)$$

where b_{AV} is the percentage change in a weighted average of the distribution parameters,

$$b_{AV} = \sum_i S_i b_i. \quad (18)$$

We modify the module structure within the GTAP model source code, to bring within the regional household module all equations derived from the top level of the final demand system, rather than leaving them scattered across the regional household, government household, and investment and saving modules.

2.7 Shared variables

To implement the revised system, we begin by defining some new cross-module variables. For the new theory, we need a variable representing the elasticity of private consumption expenditure with respect to utility from private consumption,

476 Variable (all,r,REG)
477 uepriv(r)
478 #elasticity of cost wrt utility from private consumption#;
to be shared between the private household and regional household modules.
It is also convenient to define a private consumption price index pp ,

473 Variable (all,r,REG)
474 ppriv(r)
475 #price index for private consumption expenditure in region r#;
and a variable representing government consumption expenditure,

470 Variable (all,r,REG)
471 yg(r)
472 #regional government consumption expenditure, in region r#;

At the same time, some variables previously shared between modules now become localised to individual modules. Utility from government consumption, ug , the prices of composite commodities in government consumption, pg , and quantities of composite commodities consumed by government, qg , become local to the government consumption module. Utility from private consumption, up , becomes local to the private consumption module.

2.8 The utility elasticity of private consumption expenditure

Within the private consumption module, we first calculate the level of the elasticity of private consumption expenditure. We apply CDE elasticity formula (11) to the private consumption system, and implement:

805 Coefficient (all,r,REG)
806 UELASPRIV(r)
807 #elasticity of cost wrt utility from private consumption#;
808 Formula (all,r,REG)
809 UELASPRIV(r) = sum(i,TRAD_COMM, CONSHR(i,r)*INCPAR(i,r));

We then calculate the percentage-change variable. To that end, we convert equation (11) to percentage-change form, obtaining:

$$\begin{aligned} \phi &= \sum_i S_{Li} s_i \\ &= \sum_i S_{Li} (p_i + q_i - x) \end{aligned} \quad (19)$$

where ϕ denotes the percentage change in the utility elasticity, S_{Ei} the expansion-parameter-weighted budget share of commodity i ,

$$S_{Ei} = \frac{S_i E_i}{\sum_j S_j E_j} = \frac{S_i E_i}{\Phi}, \quad (20)$$

and s_i the percentage change in the unweighted share.

To apply this to the private consumption utility elasticity, we identify the commodities i with the traded commodities, the budget shares S_i with the commodity shares in private consumption expenditure, p_i and q_i with prices and quantities in private consumption, and x with private consumption expenditure. We compute the expansion-parameter-weighted budget shares S_{Ei} according to equation (20),

```

918 Coefficient (all,i,TRAD_COMM)(all,r,REG)
919   XWCONSHR(i,r)
920   #expansion-parameter-weighted consumption share#;
921 Formula (all,i,TRAD_COMM)(all,r,REG)
922   XWCONSHR(i,r) = CONSHR(i,r)*INCPAR(i,r)/UELASPRIV(r);

```

and the private consumption utility elasticity according to equation (19):

```

924 Equation UTILELASPRIV
925 #elasticity of expenditure wrt utility from private consumption#
926 (all,r,REG)
927   uepriv(r)
928   = sumf(i,TRAD_COMM, XWCONSHR(i,r))*[pp(i,r) + qp(i,r) - yp(r)];

```

In these equations, as income increases, the budget share of commodities with higher expansion parameters increases. Then because of the expansion-parameter weighting of XWCONSHR, the utility elasticity uepriv also increases with income.

2.9 Regional household preliminaries

Inside the regional household module, we write the new regional household demand system. We begin by calculating shares in regional expenditure, the S_i of equation (10) applied to the top level of the demand system.

```

1075 Coefficient (all,r,REG)
1076   XSHRPRIV(r) #private expenditure share in regional income#;
1077 Formula (all,r,REG)
1078   XSHRPRIV(r) = PRIVEXP(r)/INCOME(r);
1079

```

```

1080 Coefficient (all,r,REG)
1081   XSHRGOV(r) #government expenditure share in regional income#;
1082 Formula (all,r,REG)
1083   XSHRGOV(r) = GOVEXP(r)/INCOME(r);
1084
1085 Coefficient (all,r,REG)
1086   XSHRSARE(r) #saving share in regional income#;
1087 Formula (all,r,REG)
1088   XSHRSARE(r) = SAVE(r)/INCOME(r);

```

Next we declare the distribution parameters, the b_i of equation (14):

```

2082 Variable (all,r,REG)
2083   dppriv(r) #private consumption distribution parameter#;
2084 Variable (all,r,REG)
2085   dpgov(r) #government consumption distribution parameter#;
2086 Variable (all,r,REG)
2087   dpsave(r) #saving distribution parameter#;

```

and declare and calculate an average distribution parameter, the b_{AV} of equation (18):

```

2105 Variable (all,r,REG)
2106   dpav(r) #average of the distribution parameters#;
2107 Equation DPARAV #average distribution parameter# (all,r,REG)
2108   INCOME(r)*dpav(r)
2109   = PRIVEXP(r)*dppriv(r) + GOVEXP(r)*dpgov(r) + SAVE(r)*dpsave(r);

```

2.10 Regional household demand equations

We can now write down the regional household demand equations, corresponding to equations (15)–(17):

```

2111 Equation PRIVCONSEXP #private consumption expenditure# (all,r,REG)
2112   yp(r) - y(r)
2113   = -[1.0 - XSHRPRIV(r)]*uepriv(r) + dppriv(r) - dpav(r);
2114
2115 Equation GOVCONSEXP #government consumption expenditure# (all,r,REG)
2116   yg(r) - y(r) = XSHRPRIV(r)*uepriv(r) + dpgov(r) - dpav(r);
2117
2118 Equation SAVING #saving# (all,r,REG)
2119   psave(r) + qsave(r) - y(r)
2120   = XSHRPRIV(r)*uepriv(r) + dpsave(r) - dpav(r);

```

In this treatment, as income increases, the increase in uepriv leads to a shift away from private consumption toward government consumption and

saving. This indeed is the essential feature of the new treatment, not altogether welcome in its implications for model applications but necessary for coherence in the theoretical structure.

Comparing the new program with the old, it may be noted that we have moved the government utility equation in from the government household module, and the saving equation in from the investment and saving module. All these equations derive from the same optimization problem, so any change in theory that affects one is likely to affect all. Accordingly, they are best kept together. We also rename the government household module the “overnment consumption” module, since with the new definition of utility from government consumption as a *per capita* variable, the concept of a government household is no longer needed.

Along the way we replace the old `govslack` and `saveslack` variables with variables `dpgov` and `dpsave`, and also add a new variable `dppriv`, representing parameters in the utility function. The new treatment lets us shift the allocation of regional income exogenously without invalidating the demand system or the welfare measures.

2.11 Regional household utility

Now we calculate utility for the regional household. Differentiating the utility function, equation (9), we apply the envelope theorem (or Shepard’s lemma and Roy’s identity) to obtain

$$U = \Phi^{-1} \left[x - \sum_i S_i p_i \right] + \sum_i B_i (\log Q_i) b_i + c,$$

where the parameter Φ denotes the elasticity of the cost of utility with respect to utility, and the variables p_i are percentage changes in expenditure-weighted price indices associated with the expenditure functions X_i . We remark that the initial settings of $\log Q_i$ are arbitrary, in that they are not constrained by the observed state of the economy as recorded in the data base, and do not affect the positive properties of the demand system. They affect only the effect on utility of changes in preferences. Once the initial settings have been made however, theory dictates how the coefficients should be updated.

By adjusting the settings of $\log Q_i$, we can make utility increasing in the distribution parameters, decreasing, or locally invariant. We can also make it increasing with respect to some of the distribution parameters and decreasing with respect to others.

The requirements for implementing distribution terms in the equation are somewhat onerous, in that we would need to store and update both the distribution parameters B_i and the quantities Q_i —even though these are not required for any positive variables. It is not clear that the implementation would be any great service to users; more likely, to most users the utility effects of any distribution parameter shocks would be an undesired side effect. For these reasons, and also because as noted above the magnitudes and even the direction of the effects are arbitrary, we choose not to implement them.

We reconcile this non-implementation of the distribution parameter effects with the underlying theory, by postulating that the general scaling factor in the utility function (C in equation (9)) adjusts endogenously to compensate them.

This treatment does have one disadvantage: it makes utility, U , a path-dependent variable, in simulations in which the distribution parameters B_i change, since the required change in the general scaling factor C depends on the quantities Q_i . If this turns out to be a problem for users, we may need to revisit the treatment.

Dropping the distribution parameter variables b_i and the general scaling variable c and rearranging, we obtain

$$\begin{aligned} x &= \sum_i S_i p_i + \Phi u \\ &= p + \Phi u, \end{aligned} \tag{21}$$

where p denotes the percentage change in a price index for disposition of income,

$$p = \sum_i S_i p_i. \tag{22}$$

For the utility elasticity Φ we have, by the envelope theorem,

$$\Phi^{-1} = \sum_i \Phi_i^{-1} B_i. \tag{23}$$

Since we can rescale the distribution parameters B_i without observable effect, the initial value of Φ is arbitrary. Accordingly we read it from data. Within the simulation however it must be updated consistently with theory. Differentiating equation (23), we obtain

$$\begin{aligned} \phi &= \sum_i S_i \phi_i - \sum_i S_i b_i \\ &= \sum_i S_i \phi_i - b_{AV} \end{aligned}$$

where b_{AV} is the percentage change in a weighted average of the distribution parameters, as defined in equation (18).

Since the subutility elasticities ϕ_i are identically unity for government consumption and saving, but not for private expenditure, this simplifies to

$$\phi = S_P \phi_P - b_{AV}, \quad (24)$$

implemented as:

```

2131 Variable (all,r,REG)
2132   uelas(r) #elasticity of cost of utility wrt utility#;
2133 Equation UTILTELASTIC #elasticity of cost of utility wrt utility#
2134   (all,r,REG)
2135   uelas(r) = XSHRPRIV(r)*uepriv(r) - dpav(r);
```

We can now compute the utility elasticity coefficient:

```

2137 Coefficient (all,r,REG)
2138   UTILELAS(r) #elasticity of cost of utility wrt utility#;
2139 Read
2140   UTILELAS from file GTAPDATA header "UE";
2141 Update (all,r,REG)
2142   UTILELAS(r) = uelas(r);
```

To calculate the income disposition price index p we need its components p_i , specifically the private consumption price index p_P , the government consumption price index p_G , and the saving price p_S .

```

901 Equation PHLDINDEX
902 # computes change in price index for private consumption expenditure #
903   (all,r,REG)
904   ppriv(r) = sum(i,TRAD_COMM, CONSHR(i,r)*pp(i,r));
```

The government consumption price index p_G is already available in the model as p_{GOV} , so also is the saving price p_S , as $psave$. These given, we can compute the price index for disposition of income by the regional household, according to equation (22):

```

2144 Variable (all,r,REG)
2145   p(r) #price index for disposition of income by regional household#;
2146 Equation PRICEINDEXREG
2147   #price index for disposition of income by regional household#
2148   (all,r,REG)
2149   p(r)
2150     = XSHRPRIV(r)*ppriv(r)
2151       + XSHRGGOV(r)*pgov(r)
2152       + XSHRSVE(r)*psave(r)
2153       ;
```

We note in passing that p is a suitable region-wide outlays-oriented price deflator. For example, it would be a suitable deflator for use in calculating real factor income, $pfactreal$.

Now finally we compute regional household utility, according to equation (21):

```

2155 Equation UTILITY #regional household utility# (all,r,REG)
2156   y(r) - pop(r) = p(r) + UTILELAS(r)*u(r);
```

2.12 Shifting distribution without affecting the utility elasticity

In this treatment, changes in the distribution parameters do not affect the current level of utility (utility is compensated as described in subsection 2.11), but they do generally affect the marginal utility of income. It would be nice to be able to modify the top-level income distribution without changing the marginal utility of income. To that end we provide the distribution parameters with item-specific and generic shift variables.

Since changes to the distribution parameters do not affect income X nor (by the endogeneity of the general shift factor, C , as described in subsection 2.11) utility, U , fixing the marginal utility of income is equivalent to fixing the elasticity of utility with respect to income. From the equation (24), the distribution parameters affect the utility elasticity through the average distribution parameter b_{AV} . So to change the distribution parameters without affecting the marginal utility of income, the user keeps b_{AV} exogenous. The user may swap b_{AV} either with one of the b_i , or with a generic distribution parameter scaling factor. To support this, we write each distribution parameter as the product of a specific and the generic scaling factor.

```

2089 Variable (all,r,REG)
2090   dpfpriv(r) #private-consumption-specific distparam shift#;
2091 Variable (all,r,REG)
2092   dpfgov(r) #government-consumption-specific distparam shift#;
2093 Variable (all,r,REG)
2094   dpfsave(r) #saving-specific distparam shift#;
2095 Variable (all,r,REG)
2096   dpsht(r) #generic distparam shift#;
2097
2098 Equation DPAPRPRIV #private consumption distribution parameter# (all,r,REG)
2099   dpfpriv(r) = dpfpriv(r) + dpsht(r);
2100 Equation DPARGOV #government consumption distribution parameter# (all,r,REG)
```

```

2101 dpgov(r) = dpfgov(r) + dpshift(r);
2102 Equation DPARSAVE #saving distribution parameter# (all,r,REG)
2103 dpsave(r) = dpfsave(r) + dpshift(r);

```

If no exogenous change in distribution is desired, the user may make the distribution parameters endogenous, the average of the distribution parameters endogenous, and the distribution parameter shift variables exogenous. To shift say the saving share in income without changing the utility elasticity,

- make exogenous:
 - the saving distribution parameter `dpssave`,
 - the average distribution parameter `dpav`,
 - the private consumption distribution parameter shift `dpfpriv`, and
 - the government consumption distribution parameter shift `dpfgov`;
- make endogenous:
 - the private consumption distribution parameter `dppriv`,
 - the government consumption distribution parameter `dpgov`,
 - the saving distribution parameter shift `dpfsave`, and
 - the distribution parameter generic shift `dpshift`; and
- shock the saving distribution parameter `dpssave` as desired.

Because the marginal utility of income depends on the budget shares S_i , this approach keeps it fixed only locally, not globally. Furthermore, it introduces a path dependency into the model, affecting `dpshift` and the endogenous distribution parameter variables (in the example shown above, `dppriv` and `dpgov`). If path dependency is considered objectionable, the user may prefer to leave `dpshift` exogenous, and apply to it a shock calculated *ex ante* outside the model.

2.13 Government consumption

Following the redefinition of utility from government consumption, `ug`, we rewrite the government consumption demand equation as a relation between *per capita* government consumption and utility from government consumption:

```

788 Equation GOVDMDS
789 # government consumption demands for composite commodities (HT 41) #
790 (all,i,TRAD_COMM)(all,r,REG)
791 ug(i,r) - pop(r) = ug(r) - [pg(i,r) - pgov(r)];

```

We define the relation between utility from government consumption and *per capita* government consumption expenditure:

```

793 Equation GOVU
794 # computation of utility from government consumption in r #
795 (all,r,REG)
796 yg(r) - pop(r) = pgov(r) + ug(r);

```

Besides making these substantive changes, we remove all references to the government household from the source code.

2.14 Changes to the data file

As described in subsection 2.11, we read a new coefficient `UTITLEAS` from the data file. To do this we need a new data file array `UE`, with dimension `REG`. The new array records, for each region, the elasticity of income with respect to utility.

The data in this array do not affect the positive results, but only the utility variable `u`. Accordingly their setting is rather arbitrary. We set each element equal to unity; this means that initially a one per cent increase in consumption of all top-level commodities (viz., utility from private consumption, utility from government consumption, and saving) leads to a one per cent increase in utility.

3 Equivalent variation

We derive and evaluate the old treatment (subsection 3.1), develop a new treatment (subsection 3.2), and implement it (subsections 3.3–3.6).

By definition, the equivalent variation (EV),

$$EV = Y_{EV} - \bar{Y},$$

where Y_{EV} denotes regional income required to achieve current utility at initial prices, and $\bar{Y}$ denotes initial regional income. Differentiating, we obtain:

$$dEV = \frac{1}{100} Y_{EV} y_{EV}. \quad (25)$$

This equation provides a starting point for both the old and new treatments.

3.1 The old treatment and its defects

In the old treatment, EV is calculated according to the equation

$$\begin{aligned} & \text{Equation EVREG} \\ & \# \text{ regional EV, the money metric welfare change (HT 67) \#} \\ & (\text{all.r,REG}) \\ & \text{EV}(x) \\ & = [\text{REGEXP}(x)/100] * [\text{URATIO}(x) * \text{POPRAPIO}(x)] * [\text{u}(x) + \text{pop}(x)]; \end{aligned}$$

In mathematical notation, we may write this as

$$dEV = \frac{1}{100} \bar{Y} U_R N_R (n + u), \quad (26)$$

where $U_R = \bar{U}/U$ is the ratio of initial to current utility, $N_R = \bar{N}/N$ the ratio of initial to current population, and n the percentage change in population. So far as we are aware, no derivation of this equation is available in the original GTAP documentation (Hertel and Tsigas [5]) or earlier GTAP technical papers or working papers. We now provide one, in order to explore the conditions under which the equation is valid.

Proposition 4 *Equation (26) is a valid first-order approximation for small changes in U , provided that initially the utility elasticity of income, Φ , is equal to one.*

Proof. Recall equation (25),

$$dEV = \frac{1}{100} Y_{EV} y_{EV}.$$

Now

$$y_{EV} = n + x_{EV},$$

where x_{EV} denotes percentage change in *per capita* expenditure required to achieve current utility at initial prices. Also, adapting equation (21), we have

$$x_{EV} = \Phi_{EV} u,$$

where Φ_{EV} denotes the utility elasticity of income, evaluated at current utility and initial prices. So

$$y_{EV} = n + \Phi_{EV} u,$$

25

and

$$dEV = \frac{1}{100} Y_{EV} (n + \Phi_{EV} u).$$

Although this equation is suitable for implementation, it does not lead directly to the GTAP 4.1 equation (26). To derive that we need to replace Y_{EV} with an expression involving $\bar{Y}$. Now

$$\begin{aligned} Y_{EV} &= N X_{EV} \\ &= N_R \bar{N} X_{EV}, \end{aligned}$$

where X_{EV} denotes *per capita* expenditure required to achieve current utility at initial prices; and

$$X_{EV} = U_R^{\Phi_{ARC}} \bar{X},$$

where Φ_{ARC} denotes the arc elasticity of income with respect to utility along the arc between $(\mathbf{P}, \bar{U})$ and $(\mathbf{P}, U)$. So

$$\begin{aligned} Y_{EV} &= N_R U_R^{\Phi_{ARC}} \bar{N} \bar{X} \\ &= N_R U_R^{\Phi_{ARC}} \bar{Y}, \end{aligned}$$

and

$$dEV = \frac{1}{100} N_R U_R^{\Phi_{ARC}} \bar{Y} (n + \Phi_{EV} u).$$

Suppose that initially Φ is equal to one. Now Φ_{EV} is initially equal to Φ , so Φ_{EV} also is initially equal to one. Also, by the mean value theorem and continuity of Φ , Φ_{ARC} is arbitrarily close to one for sufficiently small changes in U . So, to a first-order approximation,

$$dEV = \frac{1}{100} N_R U_R \bar{Y} (n + u),$$

as was to be shown.

Next we examine the conditions under which the utility elasticity of income Φ is initially equal to one. Although this elasticity does not appear explicitly in the old treatment, it is implied by the equation for utility.

Equation UTILITY

computation of per capita regional utility (HT 37)

$$\begin{aligned} & (\text{all.r,REG}) \\ & \text{INCOME}(x) * u(x) \\ & = \text{PRIVEXP}(x) * u_p(x) \\ & + \text{GOVEXP}(x) * [\log(x) - \text{pop}(x)] \\ & + \text{SAVE}(x) * [\text{qsave}(x) - \text{pop}(x)] \\ & ; \end{aligned}$$

26

Ignoring problems associated with unitary *versus* representative households, we may write this in mathematical notation as

$$u = S_P q_P + S_G q_G + S_S q_S, \quad (27)$$

where the S_P etc. are budget shares, and the q_P etc. are *per capita* quantities demanded.

Proposition 5 *Under the assumptions of the old treatment, the utility elasticity of income is equal to one if and only if the share-weighted sum of the CDE expansion parameters is equal to one.*

Proof. We have, from general theory, and from the treatment of saving and government consumption,

$$\begin{aligned} q_S &= x_S - p_S, \\ q_G &= x_G - p_G, \\ q_P &= \Phi_P^{-1}(x_P - p_P). \end{aligned}$$

With the fixed-expenditure-shares top-level demand equations in the old system, this simplifies to

$$\begin{aligned} q_S &= x - p_S, \\ q_G &= x - p_G, \\ q_P &= \Phi_P^{-1}(x - p_P). \end{aligned}$$

Substituting into equation (27), we obtain

$$u = S_P \Phi_P^{-1}(x - p_P) + S_G(x - p_G) + S_S(x - p_S).$$

Recalling equation (21),

$$u = \Phi^{-1}(x - p),$$

where

$$p = S_P p_P + S_G p_G + S_S p_S,$$

we find (ignoring the pathological case $S_P = 0$) that the two equations for u are consistent if and only if $\Phi_P = 1$. If however we ignore the effects of price changes, but consider income changes only, we find that the equations are consistent provided

$$\Phi^{-1} = S_P \Phi_P^{-1} + S_G + S_S.$$

We interpret the value of Φ from this equation as the value implicit in the model. Then $\Phi = 1$ if and only if $\Phi_P = 1$ (again ignoring the case $S_P = 0$). But by equation 11, Φ_P is the expenditure-share-weighted sum of the CDE expansion parameters. So $\Phi = 1$ if and only if the expenditure-share-weighted sum of the CDE expansion parameters is equal to one; as was to be shown.

In constructing the standard GTAP data bases, we have normalized the expansion parameters so that their expenditure-share-weighted sum is indeed equal to one. This however is not a theoretical requirement of the CDE, so it is a defect of the old system that it miscalculates the equivalent variation with data bases in which the expansion parameters are not normalized. Furthermore, in multi-step simulations the expansion parameters do not generally remain normalized, so even with initially normalized parameters, the calculation of equivalent variation is not exact.

The old treatment works reasonably well with standard GTAP data bases in simulations where utility changes are small. A treatment that works well with non-standard data bases and large utility changes is of course even better.

3.2 A new treatment

We seek a new computation for the equivalent variation that does not assume a unit elasticity of income with respect to utility, and is consistent with the new implementation of the regional household demand system.

We cannot implement equation (25) for EV directly, since we do not have an explicit functional form for the regional household expenditure function. Indeed, we do not have an explicit functional form even for the private consumption expenditure function. We can however compute the expenditure function indirectly, by implementing the demand system and solving for expenditure Y given utility U . It is then easy to solve for EV .

The regional demand system already present in the model gives the relation between expenditure Y , current utility U , and current prices $\mathbf{P}$. To find the expenditure Y_{EV} required to achieve current utility U at initial prices $\bar{\mathbf{P}}$, we implement a *shadow* demand system with the same utility level as the ordinary system, but with prices held at initial levels. The expenditure level in this shadow system is just the Y_{EV} required to calculate EV .

Recalling the equation (21) for percentage change in income, $x = p + \Phi u$, we have for the shadow demand system, $x_{EV} = \Phi u$, or more fully, since all variables are evaluated at initial prices, $x_{EV} = \Phi(\bar{\mathbf{P}}, U)u = \Phi_{EV}$, where Φ_{EV} denotes the elasticity of income with respect to utility, evaluated at initial

prices. To calculate the elasticity Φ_{EV} through a multi-step simulation we need to track the corresponding percentage change variable ϕ_{EV} . To do that we need to include in the shadow system most of the top-level regional household demand system.

Furthermore, as shown by equation (24), $\phi = Sp\phi_P - b_{AV}$, the regional household elasticity ϕ depends on the private consumption elasticity ϕ_P . To compute that elasticity, we need to include the private consumption demand system. Altogether then the shadow demand system includes three parts: a private consumption demand system, a top-level regional household demand system, and equations relating income to the equivalent variation.

3.3 Shared variables

We begin by declaring the variables we wish to compute, the regional and world-wide equivalent variations.

```

2498 Variable (Change)(all,r,REG)
2499 EV(r) #equivalent variation, $ US million#;
2500 Variable (Change)
2501 WEV #equivalent variation for the world#;
```

There are also three variables shared between different parts of the EV module. The private consumption shadow demand system computes the shadow elasticity of private consumption expenditure with respect to utility from private consumption, $uepriv_{ev}$, for use in the top-level shadow demand system. The top-level shadow demand system computes shadow private consumption expenditure, yp_{ev} , for use in the private consumption shadow demand system. It also computes shadow income y_{ev} for use in computing equivalent variation EV.

```

2509 Variable (all,r,REG)
2510 uepriv_{ev}(r)
2511 #elasticity of cost wrt utility from private consen, for EV calc.#;
2512 Variable (all,r,REG)
2513 y_{ev}(r) #regional household income, in region r, for EV calc.#;
2514 Variable (all,r,REG)
2515 yp_{ev}(r)
2516 #regional private consumption expenditure, in region r, for EV calc.#;
```

3.4 The shadow private consumption demand system

The task for the shadow private consumption demand system is to compute the shadow elasticity $\phi_P(\mathbf{P}, U)$ of private consumption expenditure with

respect to utility from private consumption. Recalling equation (19), we have, with fixed prices,

$$\phi_P = \sum_i S_{Ei}(q_{Pi} - x_P), \quad (28)$$

where S_{Ei} denotes the expansion-parameter-weighted budget share, $S_i E_i / \Phi_P$, of commodity i in the shadow private consumption demand system. So to compute the shadow elasticity, we need shadow system values for the consumption shares S_{Ei} and the private consumption demands q_{Pi} . To compute the private consumption demands we need the expenditure elasticities, and to compute them and the expansion-parameter-weighted budget shares, we need the ordinary budget shares. To compute the ordinary budget shares, we need to record shadow private consumption expenditures for individual commodities.

We begin by declaring the shadow private consumption demand variables:

```

2524 Variable (all,i,TRAD_COMM)(all,r,REG)
2525 q_{pev}(i,r)
2526 #private hhd demand for commodity i in region r, for EV calc.#;
```

We then define the shadow private consumption expenditure levels:

```

2528 Coefficient (all,i,TRAD_COMM)(all,r,REG)
2529 VPAEV(i,r)
2530 #private hhd expend. on i in r valued at agent's prices, for EV calc.#;
2531 Formula (initial) (all,i,TRAD_COMM)(all,r,REG)
2532 VPAEV(i,r) = VPA(i,r);
2533 Update (all,i,TRAD_COMM)(all,r,REG)
2534 VPAEV(i,r) = q_{pev}(i,r);
```

and the shadow private consumption budget shares:

```

2536 Coefficient (all,r,REG)
2537 VPAREGEV(r) #private consumption expenditure in region r, for EV calc.#;
2538 Formula (all,r,REG)
2539 VPAREGEV(r) = sum{i,TRAD_COMM, VPAEV(i,r)};
2540
2541 Coefficient (all,i,TRAD_COMM)(all,r,REG)
2542 CONSHREV(i,r)
2543 #share of private hhd consn devoted to good i in r, for EV calc.#;
2544 Formula (all,i,TRAD_COMM)(all,r,REG)
2545 CONSHREV(i,r) = VPAEV(i,r)/VPAREGEV(r);
```

We calculate the expenditure elasticities as in the ordinary demand system, but using the shadow budget shares `CONSHREV` instead of the ordinary shares `CONSHR`:

```

2547 Coefficient (all,i,TRAD_COMM)(all,r,REG)
2548 EYEV(i,x)
2549 #pend. elast. of private hhld demand for i in r, for EV calc.#;
2550 Formula (all,i,TRAD_COMM)(all,r,REG)
2551 EYEV(i,x)
2552 = [1.0/[sumfn, TRAD_COMM, CONSHREV(n,r)*INCPAR(n,r)]]
2553 * [
2554     INCPAR(i,r)*[1.0 - ALPHA(i,r)]
2555     + sumfn, TRAD_COMM, CONSHREV(n,r)*INCPAR(n,r)*ALPHA(n,r)
2556 ]
2557 + ALPHA(i,r)
2558 - sumfn, TRAD_COMM, CONSHREV(n,r) * ALPHA(n,r)
2559 ;

```

We can then compute the shadow private consumption demands:

```

2561 Equation PRIVDNDSEV
2562 #private hhld demands for composite commodities, for EV calc.#
2563 (all,i,TRAD_COMM)(all,r,REG)
2564 qpev(i,r) - pop(r) = EYEV(i,r)*[ypev(r) - pop(r)];

```

Also we compute the shadow expenditure-share-weighted budget shares S_{Ei} :

```

2566 Coefficient (all,r,REG)
2567 UELASPRIVEV(r)
2568 #elast. of cost wrt utility from private consen, for EV calc.#;
2569 Formula (all,r,REG)
2570 UELASPRIVEV(r) = sum{i,TRAD_COMM, CONSHREV(i,r)*INCPAR(i,r)};
2571 Coefficient (all,i,TRAD_COMM)(all,r,REG)
2572 XWCONSHREV(i,r)
2573 #expansion-parameter-weighted consumption share, for EV calc.#;
2574 Formula (all,i,TRAD_COMM)(all,r,REG)
2575 XWCONSHREV(i,r) = CONSHREV(i,r)*INCPAR(i,r)/UELASPRIVEV(r);
2576

```

and finally, following equation (28), the shadow elasticity of private consumption expenditure with respect to utility from private consumption:

```

2578 Equation UTILELASPRIVEV
2579 #elasticity of cost wrt utility from private consen, for EV calc.#
2580 (all,r,REG)
2581 ueprivev(r)
2582 = sum{i,TRAD_COMM, XWCONSHREV(i,r)*[qpev(i,r) - ypev(r)]};

```

3.5 The shadow top-level regional household demand system

The tasks of the shadow top-level regional household demand system are to compute shadow income and shadow private consumption expenditure, given utility. To compute shadow income, it tracks the elasticity of income with respect to utility. Recalling equation (24), $\phi = S_P \phi_P - b_{AV}$, and equation (15), $x_P - x = -(1 - S_P)\phi_P + b_P - b_{AV}$, we see that we must also compute the share of private consumption in income, S_P .

First therefore we track shadow values for private consumption expenditure and income, and calculate the shadow private consumption expenditure share:

```

2591 Coefficient (all,r,REG)
2592 PRIVEXPEV(r)
2593 #private consumption expenditure in region r, for EV calc.#;
2594 Formula (initial) (all,r,REG)
2595 PRIVEXPEV(r) = PRIVEXP(r);
2596 Update (all,r,REG)
2597 PRIVEXPEV(r) = ypev(r);
2598 !< PRIVEXPEV should agree with VPAREGEV. >!
2599
2600 Coefficient (all,r,REG)
2601 INCOMEEV(r) #regional income, for EV calc.#;
2602 Formula (initial) (all,r,REG)
2603 INCOMEEV(r) = INCOME(r);
2604 Update (all,r,REG)
2605 INCOMEEV(r) = yev(r);
2606
2607 Coefficient (all,r,REG)
2608 XSHRPRIVEV(r)
2609 #private expenditure share in regional income, for EV calc.#;
2610 Formula (all,r,REG)
2611 XSHRPRIVEV(r) = PRIVEXPEV(r)/INCOMEEV(r);

```

This lets us calculate percentage changes in shadow private consumption expenditure, along the lines of equation (15):

```

2613 Equation PCONSEXPEV #private consumption expenditure, for EV calc.#
2614 (all,r,REG)
2615 ypev(r) - yev(r)
2616 = -[1.0 - XSHRPRIVEV(r)]*ueprivev(r) + dppriv(r) - dpav(r);

```

and the shadow elasticity of income with respect to utility, along the lines of equation (24):

```

2618 Variable (all,r,REG)
2619      uelasev(r)
2620      #elasticity of cost of utility wrt utility, for EV calc.#;
2621      Equation UTILITELASTICEV
2622      #elasticity of cost of utility wrt utility, for EV calc.#
2623      (all,r,REG)
2624      uelasev(r) = XSHAPRIVEV(r)*ueprivev(r) - dpav(r);

2626 We can now track the level of the shadow elasticity:
2627      UTILELASEV(r)
2628      #elasticity of cost of utility wrt utility, for EV calc.#;
2629      Formula (initial) (all,r,REG)
2630      UTILELASEV(r) = UTILELAS(r);
2631      Update (all,r,REG)
2632      UTILELASEV(r) = uelasev(r);

```

This finally lets us write down the linearized equation for the expenditure function:

```

2634 Equation INCOME_EQUIV #equivalent income# (all,r,REG)
2635      yev(r) - pop(r) = UTILELASEV(r)*u(r);

```

enabling the system to compute shadow income `yev` given utility `u` along the lines of equation (21).

3.6 Computing EV

Implementing equation (25), we have:

```

2643 Equation EVREG #regional EV (HT 67)# (all,r,REG)
2644      EV(r) = [INCOMEEV(r)/100.0]*yev(r);

```

World equivalent variation, `WEV`, is simply the summation of the regional equivalent variations:

```

2646 Equation EWORLD #EV for the world (HT 68)#
2647      WEV = sum{r, REG, EV(r)};

```

4 Decomposing the equivalent variation

The old decomposition of the equivalent variation has a few minor defects. It is consistent with the old computation of the equivalent variation, which

as we have seen can be grossly in error for some scalings of the CDE expansion parameters. Since the computed equivalent variation can be grossly in error, its decomposition can be likewise. Also, the decomposition includes a “nuisance term” that we would like to eliminate, a term typically small in value that lacks an intuitive definition but appears as an undesired artefact of the derivation.

Despite these defects, the general approach is sound, and in many practical applications the errors are minor. We need make only a few changes to obtain a valid decomposition. Since the derivation is lengthy and the changes are small, we rely heavily on the old documentation ([6]).

The core of the old derivation is an equation of the form

$$Y(y - p) = D, \quad (29)$$

where D stands for a rather lengthy expression decomposing real regional income into components related to factor endowments, technological change, allocative efficiency, and terms of trade, and p is the income deflator derived in equation (22). The expression D is valid, and obtaining it accounts for most of the work in the derivation. After obtaining D , the old approach proceeds to convert the real income decomposition to a utility decomposition using an invalid equation for utility taken from the old solution program. In so doing, it introduces the nuisance term, a term in utility u .

4.1 A revised treatment

Hanslow [4] presents a general welfare decomposition applicable to many CGE models. For convenience, we base our derivation on the GTAP-specific Huff and Hertel ([6]) approach. As revised, the results are consistent with the Hanslow decomposition.

In revising the decomposition, we take for our starting point equation (29). Substituting into equation (21), $x = p + \Phi(\mathbf{P}, U)u$, we obtain

$$\begin{aligned} u &= (\Phi(\mathbf{P}, U))^{-1}(x - p) \\ &= (\Phi(\mathbf{P}, U))^{-1}(y - p - n) \\ &= (\Phi(\mathbf{P}, U))^{-1}(Y^{-1}D - n) \end{aligned}$$

Also from equation (21), we can obtain

$$y_{EV} = \Phi(\bar{\mathbf{P}}, U)u + n.$$

Substituting for u from the previous equation, we obtain

$$\begin{aligned} y_{EV} &= \frac{\Phi(\bar{\mathbf{P}}, \bar{\mathbf{U}})}{\Phi(\mathbf{P}, \mathbf{U})} (Y^{-1}D - n) + n \\ &= \frac{\Phi_{EV}}{\Phi} (Y^{-1}D - n) + n \end{aligned}$$

Then substituting into equation (25), we obtain the decomposition schema

$$d_{EV} = \frac{1}{100} \frac{\Phi_{EV}}{\Phi} \left[\frac{Y_{EV}}{Y} D - Y_{EV}n \right] + \frac{1}{100} Y_{EV}n.$$

This schema does have one objectionable feature, the presence of a nuisance term involving population growth n . The coefficient on n ,

$$\frac{1}{100} \left[\frac{\Phi_{EV}}{\Phi} - 1 \right] Y_{EV},$$

would typically be small but non-zero. We can remove this nuisance by modifying the schema, so that it decomposes not regional income y but income per household, x . Accordingly we write

$$Y(x - p) = D^*,$$

where D^* represents a decomposition not of real regional income but of real per-household income; that is, of utility. Then proceeding as before, we obtain

$$\begin{aligned} u &= \Phi^{-1} Y^{-1} D^*, \\ y_{EV} &= \frac{\Phi_{EV}}{\Phi} Y^{-1} D^* + n, \end{aligned}$$

and

$$d_{EV} = \frac{1}{100} \frac{\Phi_{EV}}{\Phi} \frac{Y_{EV}}{Y} D^* + \frac{1}{100} Y_{EV}n. \quad (30)$$

Now instead of a nuisance term, we have an interpretable term involving population growth n .

4.2 Decomposing utility

Based on Huff and Hertel [6], we have a decomposition of real regional income:

```
(all,r,REG)
  INCOME(r)*[u(r) + pop(r)]
=  sum(i, ENDW_COMM, VOA(i,r)*qo(i,r)) - VDEP(r)*kb(r)
+  sum(j, PROD_COMM, VOA(j,r)*ao(j,r))
+  sum(j, PROD_COMM, VVA(j,r)*ava(j,r))
+  sum(j, PROD_COMM, sum(i, ENDW_COMM, VFA(i,j,r)*afe(i,j,r)))
+  sum(j, PROD_COMM, sum(i, TRAD_COMM, VFA(i,j,r)*af(i,j,r)))
+  sum(s, REG, sum(i, TRAD_COMM, sum(m, MARG_COMM,
  VTMFSD(m,i,s,r)*atmfed(m,i,r,s)
  )))
+  sum(i, NSAV_COMM, [VOM(i,r) - VOA(i,r)]*qo(i,r))
+  sum(i, ENDW_COMM, sum(j, PROD_COMM,
  [VFA(i,j,r) - VFM(i,j,r)]*qfe(i,j,r)
  )))
+  sum(j, PROD_COMM, sum(i, TRAD_COMM,
  [VIFA(i,j,r) - VIFM(i,j,r)]*qfm(i,j,r)
  )))
+  sum(j, PROD_COMM, sum(i, TRAD_COMM,
  [VDFA(i,j,r) - VDFM(i,j,r)]*qfd(i,j,r)
  )))
+  sum(i, TRAD_COMM, [VIPA(i,r) - VIPM(i,r)]*qpm(i,r))
+  sum(i, TRAD_COMM, [VDPA(i,r) - VDPM(i,r)]*qpd(i,r))
+  sum(i, TRAD_COMM, [VIGA(i,r) - VIGM(i,r)]*qgm(i,r))
+  sum(i, TRAD_COMM, [VDGA(i,r) - VDMG(i,r)]*qgd(i,r))
+  sum(i, TRAD_COMM, sum(s, REG,
  [VXWD(i,r,s) - VXMD(i,r,s)]*qxs(i,r,s)
  )))
+  sum(i, TRAD_COMM, sum(s, REG,
  [VIMS(i,s,r) - VIWS(i,s,r)]*qxs(i,s,r)
  )))
+  sum(i, TRAD_COMM, sum(s, REG, VXWD(i,r,s)*pfob(i,r,s)))
+  sum(m, MARG_COMM, VST(m,r)*pm(m,r))
-  sum(i, TRAD_COMM, sum(s, REG, VXWD(i,s,r)*pfob(i,s,r)))
-  sum(m, MARG_COMM, VTMD(m,r)*pt(m))
+  NETINV(r)*pcgds(r) - SAVE(r)*psave(r)
;
```

This is a equation from Huff and Hertel [6], modified to conform to the new (in GTAP 5) treatment of international margins. The right hand side is the expression represented above as D . Rearranging, substituting for the

coefficient `INCOME` on `pop`, and introducing from GTAP 5 new notation for tax revenue coefficients, we obtain a decomposition

```
(all,r,REG)
  INCOME(r)*u(r)
=  sum{i,ENDW_COMM, VOA(i,r)*[qo(i,r) - pop(r)]}
  - VDER(r)*[kb(r) - pop(r)]
  + sum{j,PROD_COMM, VOA(j,r)*ao(j,r)}
  + sum{j,PROD_COMM, VVA(j,r)*ava(j,r)}
  + sum{j,PROD_COMM, sum{i,ENDW_COMM, VFA(i,j,r)*afe(i,j,r)}}
  + sum{j,PROD_COMM, sum{i,TRAD_COMM, VFA(i,j,r)*af(i,j,r)}}
  + sum{s,REG, sum{i,TRAD_COMM, sum{m,MARG_COMM,
    VTMFSD(m,i,s,r)*atmfed(m,i,r,s)
  }}}
  + sum{i,NSAV_COMM, PTAX(i,r)*[qo(i,r) - pop(r)]}
  + sum{i,ENDW_COMM, sum{j,PROD_COMM,
    ETAX(i,j,r)*[qfe(i,j,r) - pop(r)]
  }}
  + sum{j,PROD_COMM, sum{i,TRAD_COMM,
    IFTAX(i,j,r)*[qfm(i,j,r) - pop(r)]
  }}
  + sum{j,PROD_COMM, sum{i,TRAD_COMM,
    DFTAX(i,j,r)*[qfd(i,j,r) - pop(r)]
  }}
  + sum{i,TRAD_COMM, IPTAX(i,r)*[qpm(i,r) - pop(r)]}
  + sum{i,TRAD_COMM, DPTAX(i,r)*[qpd(i,r) - pop(r)]}
  + sum{i,TRAD_COMM, IGTAX(i,r)*[qgm(i,r) - pop(r)]}
  + sum{i,TRAD_COMM, DGTAX(i,r)*[qgd(i,r) - pop(r)]}
  + sum{i,TRAD_COMM, sum{s,REG,
    XTAXD(i,r,s)*[qxs(i,r,s) - pop(r)]
  }}
  + sum{i,TRAD_COMM, sum{s,REG,
    MTAX(i,s,r)*[qxs(i,s,r) - pop(r)]
  }}
  + sum{i,TRAD_COMM, sum{s,REG, VXWD(i,r,s)*pfob(i,r,s)}}
  + sum{m,MARG_COMM, VST(m,r)*pm(m,r)}
  - sum{i,TRAD_COMM, sum{s,REG, VXWD(i,s,r)*pfob(i,s,r)}}
  - sum{m,MARG_COMM, VTMD(m,r)*pt(m)}
  + NETINV(r)*pcgds(r)
  - SAVE(r)*psave(r)
```

;

Here the right hand side is the expression referred to above as D^* .

Observe that unlike Hanslow [4] we do not introduce into the decomposition a new term involving population. Instead we incorporate the population variable into the terms involving quantity variables. We prefer this approach for several reasons.

- Looking forward to the equivalent variation decomposition, it does not create there a nuisance term involving population growth. There is indeed still a population growth term. It is however no longer a nuisance term but an interpretable term, expressing the intuition that in the absence of imbalances in growth, income grows equiproportionally with population.
- It does lead to a change in the interpretation of the endowment terms. We recognize now an increase in utility not arising from growth in total endowments, but from growth in endowments *per capita*. While change admittedly is bad, this change is not very bad, since the new endowment terms are as readily interpretable as the old ones.
- It leads also to a reinterpretation of the allocative efficiency effects, but here the change is for the better. With balanced growth in a distorted economy, the old decomposition reported an allocative efficiency improvement associated with every taxed flow, and an allocative efficiency deterioration associated with every subsidised flow. Intuitively however, balanced growth involves no change in allocative efficiency. The new decomposition here conforms to intuition better than the old.

4.3 Implementation

To implement the new treatment, we need to define the new population growth term in the decomposition, and revise the old terms. The old terms included a factor representing

$$\frac{\bar{U}^N \bar{Y}}{\bar{U}^N}.$$

The new terms include instead a factor representing

$$\frac{\Phi_{EV} Y_{EV}}{\Phi Y}.$$

Since the numerator

$$\frac{U}{U} \frac{N}{N} \bar{Y}$$

in the old factor is an approximation to Y_{EV} , provided that the elasticity of income with respect to utility is initially equal to one, and since

$$\frac{\Phi_{EV}}{\Phi}$$

in the new factor is approximately equal to one, for small changes, the old factor may be considered an approximation to the new one.

To implement the new treatment, we first compute the equivalent variation scaling factor described above:

```

3010 Coefficient (all,r,REG)
3011 EVSCALEFACT(r) #equivalent variation scaling factor#;
3012 Formula (all,r,REG)
3013 EVSCALEFACT(r)
3014 = [UTILELASEV(r)/UTILELAS(r)]*[INCOMEVEV(r)/INCOME(r)];

```

We then revise the decomposition-based computation of equivalent variation, using equation (30) and the utility decomposition obtained in subsection 4.2.

```

3026 Variable (Linear,Change)(all,r,REG)
3027 EV_ALT(r) # regional EV computed in alternative way #;
3028 Equation EV_DECOMPOSITION
3029 # decomposition of Equivalent Variation #
3030 (all,r,REG)
3031 EV_ALT(r)
3032 = [0.01*EVSCALEFACT(r)]
3033 * [

```

```

3034     sum{i,NSAV_COMM, PTAX(i,r)*[qo(i,r) - pop(r)]}
3035     + sum{i,ENDW_COMM, sum{j,PROD_COMM,
3036       ETAX(i,j,r)*[qfe(i,j,r) - pop(r)]}
3037     }
3038     + sum{j,PROD_COMM, sum{i,TRAD_COMM,
3039       IFTAX(i,j,r)*[qfm(i,j,r) - pop(r)]}
3040     }
3041     + sum{j,PROD_COMM, sum{i,TRAD_COMM,
3042       DFTAX(i,j,r)*[qfd(i,j,r) - pop(r)]}
3043     }
3044     + sum{i,TRAD_COMM, IFTAX(i,r)*[qpm(i,r) - pop(r)]}
3045     + sum{i,TRAD_COMM, DFTAX(i,r)*[qpd(i,r) - pop(r)]}
3046     + sum{i,TRAD_COMM, ICTAX(i,r)*[qgm(i,r) - pop(r)]}

```

```

3047     + sum{i,TRAD_COMM, DGTAX(i,r)*[qgd(i,r) - pop(r)]}
3048     + sum{i,TRAD_COMM, sum{s,REG,
3049       XTAXD(i,r,s)*[qxs(i,r,s) - pop(r)]}
3050     }
3051     + sum{i,TRAD_COMM, sum{s,REG,
3052       MTAX(i,s,r)*[qxs(i,s,r) - pop(r)]}
3053     }
3054     + sum{i,ENDW_COMM, VOA(i,r)*[qo(i,r) - pop(r)]}
3055     - VDEP(r)*[kb(r) - pop(r)]
3056     + sum{i,PROD_COMM, VOA(i,r)*ao(i,r)}
3057     + sum{j,PROD_COMM, VVA(j,r)*ava(j,r)}
3058     + sum{i,ENDW_COMM, sum{j,PROD_COMM, VFA(i,j,r)*afe(i,j,r)}}
3059     + sum{j,PROD_COMM, sum{i,TRAD_COMM, VFA(i,j,r)*af(i,j,r)}}
3060     + sum{m,MARG_COMM, sum{i,TRAD_COMM,
3061       sum{s,REG, VTMFSD(m,i,s,r)*atmfed(m,i,s,r)}}}
3062     + sum{i,TRAD_COMM, sum{s,REG, VAWD(i,r,s)*pfob(i,r,s)}}
3063     + sum{m,MARG_COMM, VST(m,r)*pm(m,r)}
3064     + NETINV(r)*pcgds(r)
3065     - sum{i,TRAD_COMM, sum{s,REG, VAWD(i,s,r)*pfob(i,s,r)}}
3066     - sum{m,MARG_COMM, VTMD(m,r)*pt(m)}
3067     - SAVE(r)*psave(r)
3068   ]
3069   + 0.01*INCOMEVEV(r)*pop(r);

```

Consistency between this and the standard equivalent variation computation is a check on the validity of the decomposition.

Finally we compute various components of the change in equivalent variation. We compute first the population term:

```

3077 Variable (Linear,Change) (all,r,REG) CNTpop(r)
3078 #contribution to EV in region r of change in population#;
3079 Equation CNT_WEV_pop (all,r,REG)
3080 CNTpop(r) = 0.01*INCOMEVEV(r)*pop(r);

```

The other terms derive from the utility decomposition. For example, for the allocative efficiency effect associated with production subsidies and income taxes, we have corresponding to the first term in the decomposition:

```

3082 Variable (Linear,Change) (all,r,REG) CNTqor(r)
3083 #contribution to EV in region r of output changes#;
3084 Equation CNT_WEV_qor (all,r,REG)
3085 CNTqor(r)
3086 = sum{i,NSAV_COMM, 0.01*EVSCALEFACT(r)*PTAX(i,r)*[qo(i,r) - pop(r)]};

```

The code for the remaining components is not reproduced here but may be found in the associated program source file.

5 Properties of the final demand system

In the new treatment, the model should display several easily checked properties:

- All variables except utility from private consumption (ϕ_P or `uelaspriv`) are invariant with respect to rescalings of the CDE expansion parameters.
- All variables except aggregate utility (ϕ or `uelas`) are invariant with respect to changes in the initial level of the elasticity of income with respect to utility.
- In quantity homogeneity tests—that is, simulations in which uniform shocks are applied to population, factor endowments, and any other exogenous quantity variables—all components of the EV decomposition except the population component are zero.

The first property is not obvious, since rescalings of the expansion parameters do affect the utility elasticity of private consumption expenditure, and, in the linearised equation system, changes in the utility elasticity of private consumption expenditure affect the top-level allocation of income. Nevertheless, as the following proposition shows, it does apply:

Proposition 6 *With a top-level Cobb-Douglas demand system and a bottom-level CDE system, with distribution parameters calibrated to a given initial equilibrium, rescaling the CDE expansion parameters has no effect on quantities demanded.*

Proof. Suppose that the CDE expansion parameters E_i are rescaled by a common positive factor K . Now changing the maximand from U to U^K does not affect quantities demanded, but as we see by substituting into equation (1), maximizing U^K with the old expansion parameters is equivalent to maximizing U with the new expansion parameters. So rescaling the CDE expansion parameters does not affect the private consumption demand system. It does affect the top level of the final demand system, since the elasticity of private consumption expenditure with respect to utility from private consumption is linearly homogeneous in the CDE expansion parameters. To calibrate to the observed income allocation, however, when we rescale the expansion parameters by a factor K , we need also to multiply by K the top-level distribution factor for private consumption. With that adjustment, the new demand system is equivalent to the old.

6 Future work

Later versions of this paper may include more detailed derivations and some illustrative applications. We may revise the program `decomp.tab`, used to prepare a ViewHAR-friendly variation decomposition report, to work with the revised `gtap.tab`. We may assess the empirical merits of the new, more complex top-level demand system.

References

- [1] A. Deaton and J. Muellbauer. *Economics and Consumer Behavior*. Cambridge, 1980.
- [2] W.M. Gorman. Separable utility and aggregation. *Econometrica*, 27:469–81, 1959.
- [3] G. Hanoch. Production and demand models in direct or indirect implicit additivity. *Econometrica*, 43:395–419, 1975.
- [4] K. Hanslow. A general welfare decomposition for CGE models. Paper presented to the Third Annual Conference on Global Economic Analysis, Melbourne, 2000.
- [5] T.W. Hertel and M.E. Tsigas. *Global Trade Analysis: Modeling and Applications*, chapter 3. Structure of GTAP. Cambridge University Press, 1997.
- [6] K. Huff and T.W. Hertel. Decomposing welfare changes in GTAP. Technical Paper 5, Center for Global Trade Analysis, Purdue University, West Lafayette, IN., 1996.